Effective Field Theory

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Outline

1. Introduction
2. EFT Formalism
3. ‘Non-renormalisable theories’ are renormalisable
4. SMEFT
5. EFT in practice

Why EFT?

- **Heavy new physics** very probably exists (neutrino masses, baryogenesis, strong CP, GUT, SUSY, gravity. . .)
- How do we study new physics? Write down the Lagrangian and look at its interactions with the SM
- When does this not work so well?
  1. If we want to be agnostic about the new physics
  2. If the new physics is too heavy to produce in an experiment
- EFT approach: study how heavy physics **modifies interactions between light fields**, focus on what we know!

Integrating out heavy fields

- Define a set of light fields, $\{\phi\}$, and a set of heavy fields, $\{\omega\}$
- All fields in $\{\phi\}$ should have masses $\leq m$ and all fields in $\{\omega\}$ should have masses $\geq M$, with a **separation of scales**, $M \gg m$
- 'Integrating out' the set of heavy fields, $\{\omega\}$, at their mass scale generates new interactions among the ϕ fields:

$$\int \mathcal{D}\phi\mathcal{D}\omega \exp\left(i\int d^4x[\mathcal{L}(\phi)+\mathcal{L}(\phi,\omega)]\right)$$
$$\to \int \mathcal{D}\phi \exp\left(i\int d^4x[\mathcal{L}(\phi)+\delta\mathcal{L}(\phi)]\right) \quad (1)$$

- Trade $\mathcal{L}(\phi,\omega)$ for $\delta\mathcal{L}(\phi)$: **it is now as though the heavy fields, $\{\omega\}$, don't exist**
- This process can be repeated many times

Best-known example: Fermi theory

- Fermi observed β decay, $n \to p e^- \bar{\nu}_e$, and parameterised it by the four-fermion operator,

$$\mathcal{L} \supset -G_F(\bar{p}\Gamma n)(\bar{e}\Gamma \nu)\,, \tag{2}$$

where Γ is some combination of $\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$ (in fact $\Gamma = \gamma_\mu(1-\gamma_5)/2$)

- What do we learn?
  1. After measuring $G_F = 1/(\sqrt{2}v^2)$, we see that new physics must be at a scale $\Lambda \lesssim 4\pi/\sqrt{G_F} \approx 4$ TeV
  2. If this interaction is mediated at tree-level, it must be by a boson of electric charge 0 or ± 1

Fermi theory

- We now know the underlying physics, the electroweak theory
- Still, it is **much easier to calculate** with the Fermi theory and still very accurate
- Importantly, we can quantify the error in using the EFT compared to the full theory

Outline

Matching

- We trade heavy-light and heavy-heavy interactions for additional light-light interactions
- Basic requirement: amplitudes involving light fields in the initial and final state should be the same in the EFT as they are in the full theory
- Enforcing this is called **matching** the full theory on the EFT

Matching

- Always the caveat in physics: "... up to some order in perturbation theory"
- What are our small parameters?
  - Loops, as always, which give $g^2/(16\pi^2)$ for some coupling g
  - Powers of p/M or m/M, where M is the scale of heavy new physics
- Work in EFT to some given power of p/M and some loop order

Integrating out in practice

- Consider the propagator of a massive fermion with mass M and momentum p
- When $p^2 \ll M^2$ (need to make a Lorentz invariant statement!), we can **expand the propagator perturbatively**,

$$\frac{i}{\not{p} - M} = -\frac{i}{M} \sum_{n=0}^{\infty} \left(\frac{\not{p}}{M} \right)^n \qquad (3)$$

- This generates the series in powers of p/M which allows us to match a full theory onto an EFT in a well-defined way

Integrating out a LQ: the formal method

- Introduce scalar doublet leptoquark, $R \sim (3,2)_{1/6}$
- Baryon number conserving Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + (D_\mu R)^\dagger (D^\mu R) - M_R^2 R^\dagger R - \overline{d_R} Y_R (R^T \epsilon l_L) + h.c., \quad (4)$$

then the EOM is

$$\frac{\partial \mathcal{L}}{\partial R^\dagger} = D_\mu \frac{\partial \mathcal{L}}{\partial (D_\mu R)^\dagger} \Rightarrow -M_R^2 R - (\epsilon \overline{l_L^T}) Y_R^\dagger d_R = D^2 R$$
$$\Rightarrow R = -\frac{1}{M_R^2} (\overline{l_L^T} \epsilon) Y_R^\dagger d_R + \mathcal{O}(p^2/M_R^4) \quad (5)$$

- Insert this expression everywhere in the Lagrangian gives

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{2M_R^2} Y_{R,aj}^\dagger Y_{R,ib} (\overline{l_{La}} \gamma_\mu l_{Lb})(\overline{d_{Ri}} \gamma^\mu d_{Rj}) + \mathcal{O}(p^2/M_R^4) \quad (6)$$

Integrating out a LQ: a different method

- There's another, simpler way to see this: diagrammatically!

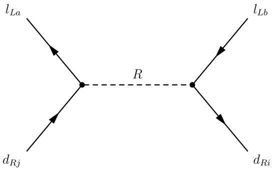

Figure 1: Integrating out the R

- We compute this tree-level diagram,

$$\delta\mathcal{L} \simeq \frac{(-i)^2}{i}\frac{\mathbf{i}}{-\mathbf{M_R^2}} Y^\dagger_{R,aj} Y_{R,ib} (\overline{l_{La}} d_{Rj})(\overline{d_{Ri}} l_{Lb}) + \mathcal{O}(p^2/M_R^4) \quad (7)$$

- We can use the Fierz identity to rewrite this as

$$\delta\mathcal{L} = -\frac{1}{2M_R^2} Y^\dagger_{R,aj} Y_{R,ib} (\overline{l_{La}}\gamma_\mu l_{Lb})(\overline{d_{Ri}}\gamma^\mu d_{Rj})\,, \quad (8)$$

as in the previous slide

Power counting

- The new light-light interactions, $\delta\mathcal{L}(\phi)$, has a particular form:

$$\mathcal{L}_{\text{EFT}} = \sum_{d>4;i} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad (9)$$

- New terms in the Lagrangian are **operators**, $Q_i^{(d)}$, of dimension $d > 4$ suppressed by the mass scale of the heavy fields
- The operators must be Lorentz singlets and must respect the gauge symmetry of the full theory
- The $C_i^{(d)}$ are called **Wilson coefficients** (WCs), characterise the strength of interactions between light (known) and heavy (unknown) physics
- E.g. in the Fermi theory, $Q = (\bar{p}\Gamma n)(\bar{e}\Gamma\nu)$, $C/\Lambda^2 = -G_F$

Power counting

- If an operator of dimension d_i is inserted into an amplitude, it scales as

$$\mathcal{M} \propto \left(\frac{|p|, m}{\Lambda} \right)^{d_i - 4} , \tag{10}$$

where p is the momentum of the process and $m \ll M$ the mass of a state in the EFT

- Key point: the larger the dimension, the more suppressed the effect of an operator is

- Often only work to dimension-6, i.e. $1/\Lambda^2$

Matching recap

- This procedure clearly ensures that the amplitudes are the same at tree-level
- Working order-by-order in p/M makes **perturbation theory manifest**, convenient for calculation
- It is more complicated (but still possible!) to do this at loop level, the classical EOMs only hold at tree-level
- The logic of matching can go either way
  - If you start off with a full theory, you can derive the WCs through this matching procedure
  - On the other hand, if you have measured a set of WCs, then you can calculate the parameters of the full theory

Outline

Renormalisation

- EFTs are sometimes also called 'non-renormalisable theories'
- But this is not true, **you can renormalise them**
- First, let's quickly review renormalisation

Lightning review of renormalisation

- Values of quantities evolve with the energy scale
- For instance, $\alpha_{em}(\mu = m_e) \simeq 1/137$ while $\alpha_{em}(\mu = m_Z) \simeq 1/128$
- Evolution is described by **R**enormalisation **G**roup **E**quations (RGEs), e.g.

$$\frac{d \log \alpha_{em}}{d \log \mu} = \frac{8}{3} \frac{\alpha_{em}}{4\pi} + \mathcal{O}\left(\left[\frac{\alpha_{em}}{4\pi} \right]^2 \right) , \tag{11}$$

for $m_e < \mu < m_\mu$

- This is obtained by computing the divergent parts of loops and introducing counterterms, see e.g. Schwartz Chapter 23

Renormalisation in EFTs

- In an EFT, there are three aspects to renormalisation of WCs
    1. Renormalisation of individual WCs
    2. Mixing of WCs of operators of the **same dimension**
    3. Mixing of WCs of operators of **different dimensions**

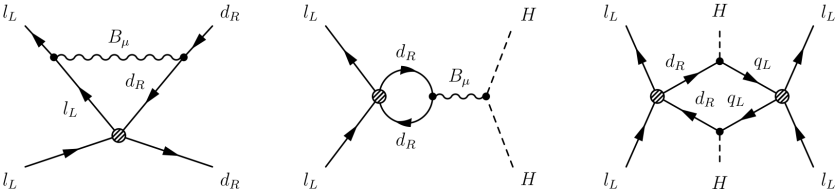

Figure 2: Diagrams corresponding to renormalisation of the three different types

Renormalisation in EFTs

- The renormalisation of a single WC behaves just like a coupling in the SM, have a β function,

$$\frac{dC}{d\log\mu} = \gamma\frac{C}{16\pi^2} + \mathcal{O}\left(\left[\frac{C}{16\pi^2}\right]^2\right) \quad (12)$$

- For mixing between WCs of the same dimension, this can be **generalised to a matrix equation**,

$$\frac{dC_i}{d\log\mu} = \gamma_{ij}\frac{C_j}{16\pi^2} + \mathcal{O}\left(\left[\frac{C}{16\pi^2}\right]^2\right) \quad (13)$$

- In this case we have counterterms for each WC of that dimension

Renormalisation in EFTs

- For mixing of WCs of operators of different dimensions, we can do something similar, e.g.

$$\frac{dC_i}{d\log\mu} = \gamma_{ijk}\frac{C_jC_k}{16\pi^2} + \dots, \qquad (14)$$

where $dim(Q_i) - 4 = (dim(Q_j) - 4) + (dim(Q_k) - 4)$

- However, this continues on for arbitrarily high dimensional operators: dim-5 mixing into dim-6, then into dim-7, dim-8 etc.
- Problem: we **need an infinite number of counterterms** and thus an infinite number of measurements to fix these!
- Obvious solution: if we only **work e.g. to dim-6**, don't need to worry about renormalising WCs of higher-dimensional operators

Operator mixing

- Not only is renormalisation possible, it can be very important
- Suppose matching a UV theory onto an EFT generates

$$C_1(\mu = \Lambda) \neq 0\,,\;\; C_2(\mu = \Lambda) = 0 \tag{15}$$

- The statement $C_2 = 0$ is scale-dependent: mixing from the RGE

$$\frac{dC_i}{d\log\mu} = \gamma_{ij}\frac{C_j}{16\pi^2} \tag{16}$$

for $i = 1, 2$ will generate

$$C_2(\mu = \mu_0) = -\gamma_{21}\frac{C_1}{16\pi^2}\log\frac{\Lambda}{\mu_0}\,. \tag{17}$$

- Although C_2 is loop-suppressed compared to C_1, it may still be more strongly experimentally constrained

Outline

Lots of EFTs

- There are many different EFTs with all kinds of acronyms: SMEFT, LEFT, HEFT, SCET, HQET, WET, χPT, . . .
- Want to focus on a particular favourite, the **S**tandard **M**odel **EFT** (SMEFT)
- This is the general EFT for new physics much heavier than the electroweak scale

The SMEFT, dim-5

- How do we define/construct an EFT? This is an interesting exercise
- The EFT is comprised of operators of dimension > 4, so the first place to start is dimension 5
- Question: how many unique, Lorentz-scalar, SM gauge-invariant dimension-5 operators made up of SM fields are there?
- Answer: **one** (ignoring flavour), the Weinberg operator:

$$\mathcal{L}_{d=5} = \frac{C^W_{\alpha\beta}}{\Lambda}(\overline{l^c_{L\alpha}}\tilde{H}^*)(\tilde{H}^\dagger l_{L\beta}) + h.c. = \frac{C^W_{\alpha\beta}}{\Lambda} Q_{W,\alpha\beta} + h.c.. \quad (18)$$

- The Weinberg operator generates **neutrino masses**, $m_\nu = -\frac{v^2}{2}\frac{C^W}{\Lambda}$, so $\frac{v}{\Lambda}C^W \lesssim 10^{-12}$

The SMEFT, dim-6

- Repeat the question: how many **unique** Lorentz-scalar, SM gauge-invariant dimension-6 operators made up of SM fields are there?
- We can construct a lot quite easily:
  - $(H^\dagger H)$ is a Lorentz-scalar gauge singlet, so have $Q_H = (H^\dagger H)^3$, and also $(H^\dagger H) \times \mathcal{L}_{SM}$, e.g.

$$Q_{eH} = (\overline{l_L} H e_R)(H^\dagger H)\,,\;\; Q_{uH} = (\overline{q_L} \tilde{H} u_R)(H^\dagger H)\,, Q_{dH} = (\overline{q_L} H d_R)(H^\dagger H)\,,$$
$$Q_{HX} = X_{\mu\nu} X^{\mu\nu}(H^\dagger H)\,,\;\; Q_{H\tilde{X}} = X_{\mu\nu}\tilde{X}^{\mu\nu}(H^\dagger H)\,,\;\; \text{for } X = B, W^a, G^A$$

- All bilinears of the form $(\overline{\psi}\gamma_\mu \psi)$ are gauge singlets, so have operators of the form

$$Q_{\psi\chi} = (\overline{\psi}\gamma_\mu\psi)(\overline{\chi}\gamma^\mu\chi) \tag{19}$$

for $\psi, \chi = l_L, e_R, q_L, u_R, d_R$

The Warsaw basis

- Our quick procedure gives 32 operators after thinking carefully about $SU(2)$ and $SU(3)$
- In total, the Warsaw basis [Grzadkowski+ JHEP **10** (2010) 085] has 63 operators (ignoring flavour), or 2499 including flavour
- Basis means **no operators related by EOMs, Fierz etc.**

The Warsaw basis

- With so many dim-6 operators, **almost all pheno is covered by the Warsaw basis**
- Since higher dimensional operators are more suppressed by powers of p/Λ, don't really need to go to dim > 6
- Ongoing exercise to **bound WCs of different SMEFT operators**
    - The dipole operators, $Q_{eB} = \overline{l_L}\sigma_{\mu\nu}e_R H B^{\mu\nu}$ and $Q_{eW} = \overline{l_L}\sigma_{\mu\nu}e_R \sigma^A H W^{A\mu\nu}$ induce flavour violating radiative decays and dipole moments, so are well-constrained
    - An operator like $Q_H = (H^\dagger H)^3$ isn't well-constrained because the Higgs sector isn't known precisely
- If we have a specific model in mind, that tells us how best to study it; if we are being agnostic, that tells us a bit about what new physics looks like

Outline

When is the EFT valid?

- Take the operator we have looked at, $Q_{ld,\alpha\beta\gamma\delta} = (\overline{l_{L\alpha}}\gamma_\mu l_{L\beta})(\overline{d_{R\gamma}}\gamma^\mu d_{R\delta})$
- For $\alpha = \beta = e$ and $\gamma = \delta = b$, LEP gives us [ALEPH Collab., EPJC **49** (2007) 411]

$$-\frac{1}{(0.8\ \text{TeV})^2} \leq \frac{C^{ld}_{eebb}}{\Lambda^2} \leq \frac{1}{(1.1\ \text{TeV})^2}\,, \tag{20}$$

while the LHC gives [Greljo+Marzocca, EPJC **77** (2017) 8]

$$-\frac{1}{(1.9\ \text{TeV})^2} \leq \frac{C^{ld}_{eebb}}{\Lambda^2} \leq \frac{1}{(2.0\ \text{TeV})^2}\,. \tag{21}$$

When is the EFT valid?

- There's a crucial difference between experiments: at LEP, $\sqrt{s} \leq 209$ GeV, while at the LHC, the dilepton pair invariant masses are as large as $m_{e^+e^-} \lesssim 3$ TeV
- We know that the EFT expansion only works if $M^2 \gg p^2$
- This seems to be the case for the LEP bound, but for LHC should by wary of EFT interpretations

Spurion analysis

- Taking just the kinetic terms of the SM, there is a $SU(3)^5$ **flavour symmetry**:

$$\mathcal{L}_{\text{kin}} = \sum_{\psi} \overline{\psi} i \not{D} \psi \,, \tag{22}$$

  is invariant under $\psi \to U_\psi \psi$, where U_ψ is a 3×3 special unitary matrix, for $\psi = l_L, e_R, q_L, u_R, d_R$

- We can see this explicitly:

$$l_L \to U_l l_L,\ \overline{l_L} \to \overline{l_L} U_l^\dagger,\ \ \Rightarrow \overline{l_L} i \not{D} l_L \to \overline{l_L} U_l^\dagger i \not{D} U_l l_L = \overline{l_L} i \not{D} l_L \,, \tag{23}$$

  since $\not{D}$ doesn't act on U_l and $U_l^\dagger U_l = \mathbb{1}$

- There is a $U(3)$ for each of the five fermions

Spurion analysis

- Introducing Yukawa couplings breaks this symmetry:

$$\overline{l_L} Y_e H e_R \to \overline{l_L} U_l^\dagger Y_e H U_e e_R \,. \tag{24}$$

- However, the symmetry is restored if the **Yukawa coupling is treated as a field, a spurion**, which also transforms under the symmetry as

$$Y_e \to U_l Y_e U_e^\dagger \,. \tag{25}$$

Spurion analysis

- Let's go back to the LQ model, with

$$\mathcal{L}_{\mathrm{LQ}} \supset -M_R^2 R^\dagger R - \overline{d_R} Y_R (R^T \epsilon l_L) + h.c.\,. \tag{26}$$

- This new interaction again breaks the symmetry, but it's restored if Y_R is a spurion with transformation

$$Y_R \to U_d Y_R U_l^\dagger,\ M_R^2 \to M_R^2\,. \tag{27}$$

- Let's see this explicitly:

$$\overline{d_R} Y_R (R^T \epsilon l_L) \to \overline{d_R} U_d^\dagger U_d Y_R U_l^\dagger (R^T \epsilon U_l l_L) = \overline{d_R} Y_R (R^T \epsilon l_L)\,. \tag{28}$$

Spurion analysis

- Now consider the SMEFT, with

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{\Lambda^2} C_{ab}^{Hl(1)} (\overline{l_{La}} \gamma_\mu l_{Lb})(H^\dagger \overleftrightarrow{D}^\mu H) \equiv \frac{C_{ab}^{Hl(1)}}{\Lambda^2} Q_{Hl(1),ab} \,. \tag{29}$$

- Under the $SU(3)^5$ flavour symmetry, we have

$$Q_{Hl(1),ab} \rightarrow U_l^\dagger Q_{Hl(1),ab} U_l \tag{30}$$

- If the full Lagrangian was invariant under this, so must the EFT be, so we need $C_{ab}^{Hl(1)} Q_{Hl(1),ab}$ to be invariant, i.e.

$$C_{ab}^{Hl(1)} \rightarrow U_l C_{ab}^{Hl(1)} Q_{Hl(1),ab} U_l^\dagger \,. \tag{31}$$

Spurion analysis

- So we want

$$C_{ab}^{Hl(1)} \to U_l C_{ab}^{Hl(1)} Q_{Hl(1),ab} U_l^\dagger \,. \tag{32}$$

- Since $Y_R \to U_d Y_R U_l^\dagger$, the simplest combination is

$$C_{ab}^{Hl(1)} \propto Y_R^\dagger Y_R \,. \tag{33}$$

- This is correct, indeed we have

$$\frac{1}{\Lambda^2} C_{ab}^{Hl(1)}(\mu) = \frac{g_1^2}{48\pi^2} (Y_R^\dagger Y_R)_{ab} \log \frac{M_R}{\mu} + \ldots \,, \tag{34}$$

  from the one-loop diagram when we looked at renormalisation
- This is a **powerful tool for understanding the dependence of WCs on the couplings** (and even masses) in a theory

Conclusions

Let's summarise the EFT procedure:

1. Start with full theory in the UV
2. At the scale of heavy new physics, Λ, match full theory onto EFT (integrate out heavy fields)
3. Renormalise theory, RG evolution down to next scale
4. At scale of heaviest field in the UV, integrate out that field, match onto your new EFT
5. Continue all the way down to QED

Advantages of EFT

- Can study new physics while being agnostic about the UV completion of the SM: focus on what we know!
- Easy mapping of new physics models onto WCs, allows for comparison between different new physics scenarios
- Simple way to organise computations in powers of p/M
- Is renormalisable, and once you compute running/mixing for an EFT, you have computed it for every theory which matches onto that EFT
- Spurion analysis gives useful information about the structure of WCs, and hence observables

Incomplete Bibliography

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Back-up slides

Back-up slides

Integrating out: the Fermi theory

- For the Fermi theory we have a W boson:

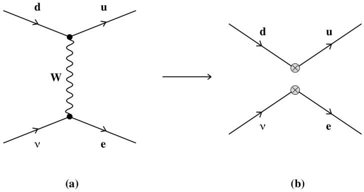

Figure 3: Integrating out the W boson, diagram from Buras lectures

- Fermi theory comes from the approximation

$$-\frac{i(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2})}{p^2 - m_W^2} = -\frac{i g_{\mu\nu}}{m_W^2} + \mathcal{O}(p^2/m_W^4) \qquad (35)$$

- Corresponds to tree-level matching at $\mathcal{O}(1/m_W^2)$
- For β decay, $p^2 \approx (m_p - m_n)^2$, so error is $\mathrm{MeV}^2/m_W^2 \approx 10^{-10}$

The full Warsaw basis, part 1

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

Figure 4: From Grzadkowski+

The full Warsaw basis, part 2

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Table 3: Four-fermion operators.

Figure 5: From Grzadkowski+

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- Consider how different bounds on SMEFT WCs tells us about a given UV completion
- The operator $Q_{eH,\alpha\beta} = (\overline{l_{L\alpha}} H e_{R\beta})(H^\dagger H)$ induces flavour-violating Higgs decays,

$$\Gamma(h \to \ell_\alpha^+ \ell_\beta^-) + \Gamma(h \to \ell_\alpha^- \ell_\beta^+) \simeq \frac{v^4 m_h}{16\pi\Lambda^4} \left(|C_{\alpha\beta}^{eH}|^2 + |C_{\beta\alpha}^{eH}|^2\right) \tag{36}$$

- Bounds from the LHC give

$$v^2 \sqrt{\left|\frac{C_{\alpha\beta}^{eH}}{\Lambda^2}\right|^2 + \left|\frac{C_{\beta\alpha}^{eH}}{\Lambda^2}\right|^2} \lesssim 2.3 \times 10^{-4} \tag{37}$$

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- Other operators induce flavour-violating Z decays,

$$\Gamma(Z \to \ell_\alpha^+ \ell_\beta^-) + \Gamma(Z \to \ell_\alpha^- \ell_\beta^+) \simeq \frac{\sqrt{2} v^4 m_Z^3 G_F}{12\pi\Lambda^4} \left(|C_{\alpha\beta}^{Hl(1)} + C_{\alpha\beta}^{Hl(3)}|^2 + |C_{\alpha\beta}^{He}|^2 \right) \quad (38)$$

- Bounds from the LHC give

$$v^2 \sqrt{\left| \frac{C_{\alpha\beta}^{Hl(1)} + C_{\alpha\beta}^{Hl(3)}}{\Lambda^2} \right|^2 + \left| \frac{C_{\alpha\beta}^{He}}{\Lambda^2} \right|^2} \lesssim 1.7 \times 10^{-3} \quad (39)$$

- So the bound here is milder than the bound on C^{eH}/Λ^2

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- But consider e.g. the type-III seesaw, wherein at tree-level,

$$\frac{C^{eH}}{\Lambda^2} = \frac{Y_\Sigma^\dagger Y_\Sigma Y_e^\dagger}{M_\Sigma^2}$$

$$\frac{C^{Hl(1)}}{\Lambda^2} = \frac{3 Y_\Sigma^\dagger Y_\Sigma}{4 M_\Sigma^2}\,, \quad \frac{C^{Hl(3)}}{\Lambda^2} = \frac{Y_\Sigma^\dagger Y_\Sigma}{4 M_\Sigma^2}\,, \quad \frac{C^{He}}{\Lambda^2} = 0\,.$$

- Then the bound on C^{eH}/Λ^2 from LFV Higgs decays turns into

$$\frac{(Y_\Sigma^\dagger Y_\Sigma)_{e\mu}}{M_\Sigma^2} \lesssim \frac{1}{(400\ \text{GeV})^2} \tag{40}$$

while the bound on $C^{Hl(1)}$, $C^{Hl(3)}$, C^{He} from LFV Z decays turns into

$$\frac{(Y_\Sigma^\dagger Y_\Sigma)_{e\mu}}{M_\Sigma^2} \lesssim \frac{1}{(6\ \text{TeV})^2} \tag{41}$$