how the Cosmic Microwave Background got its spots

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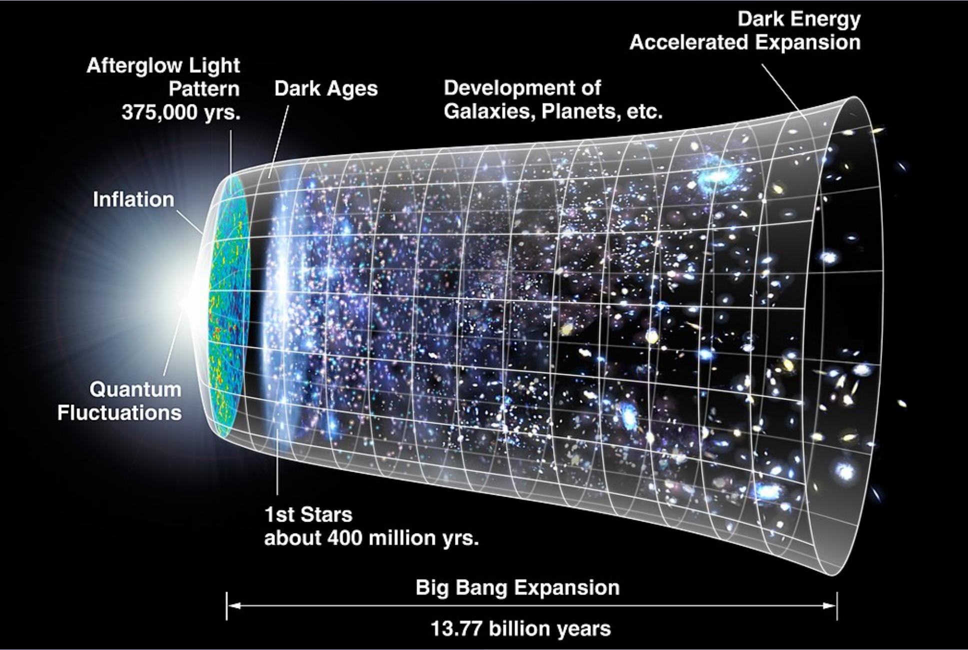
Outline

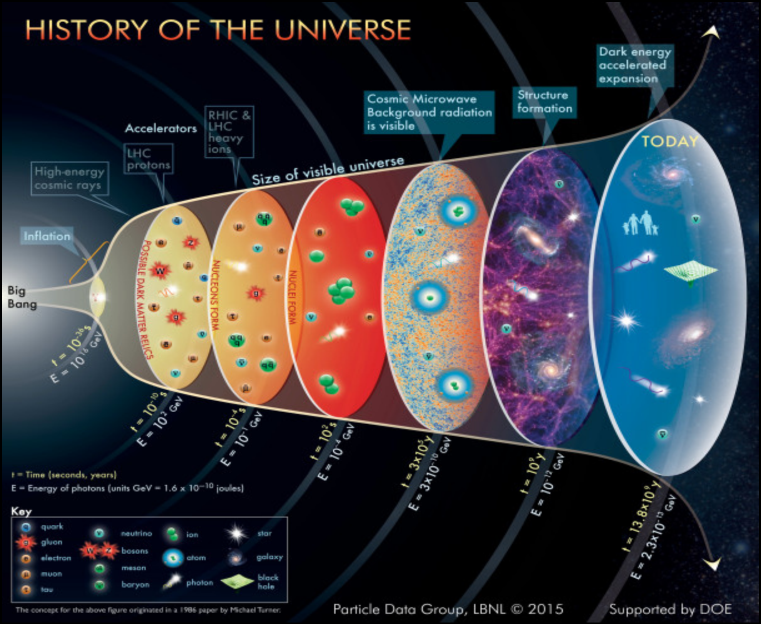
1. Introduction: history and cosmological parameters
2. What is the Cosmic Microwave Background (CMB)?
   - observables: spectrum, temperature, polarization, lensing
   - power spectra
3. How does the CMB get its spots?
   - sound waves in plasma
   - free streaming and anisotropies today
   - power spectrum prediction
4. What does the CMB tell us about the contents and evolution of the universe?
   - dependence of power spectrum on parameters
   - constraints from Planck

references: • Dodelson, *Modern Cosmology*; • W. Hu, `background.uchicago.edu`;
• Planck Mission, `www.cosmos.esa.int/web/planck`

History of the Universe: Astronomer's view



History of the Universe: Particle physicist's view



Cosmological parameters

Minimal ("vanilla") cosmological parameters

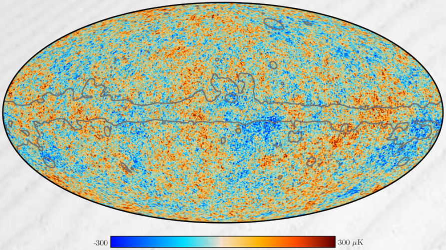
- "Initial" quantum fluctuations $A_s(k/k_0)^{n_s-1}$
  - amplitude A_s at characteristic length scale
  - scale-dependence n_s of amplitude (scale-invariant if 1)
- Contents of the universe today
  - baryonic matter: density fraction Ω_b
  - cold dark matter: density fraction Ω_c
- Expansion rate of universe today: $H_0 = 100h$ km/sec/Mpc
- Optical depth τ to CMB

Beyond the minimal model

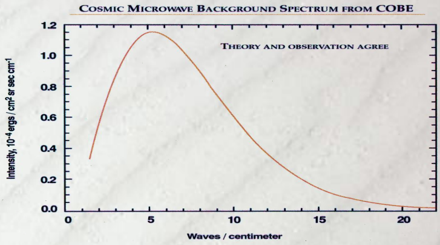
- Spatial curvature density fraction $\Omega_K = -\frac{\kappa}{H_0^2}$
- Massive neutrinos: density fraction Ω_ν
- Dark energy equation of state $P/\rho = w_0 + (1-a)w_a$
- Inflation: tensor:scalar ratio r, non-Gaussianity, isocurvature

Part II: What is the CMB?

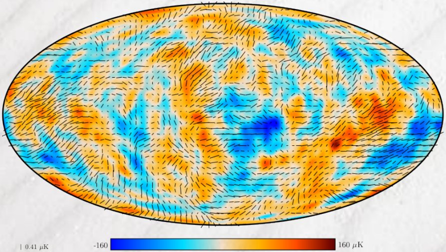
CMB observables



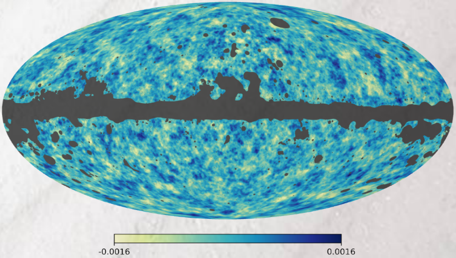
temperature map



energy spectrum

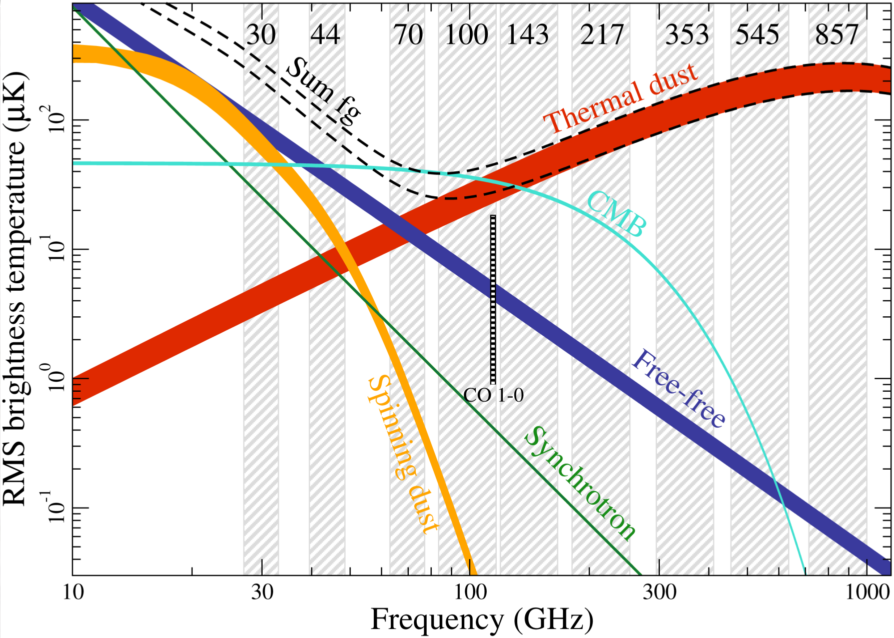


polarization map

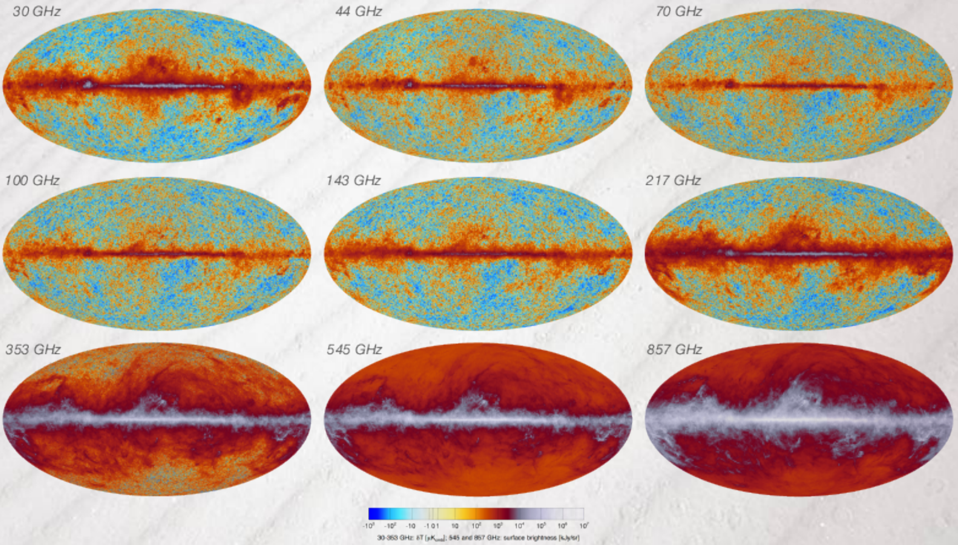


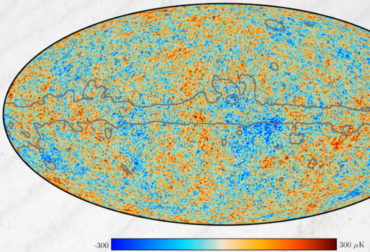
lensing map

CMB foregrounds I



CMB foregrounds II





$$\Delta T = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{k})$$

ℓ: angular scale of variation

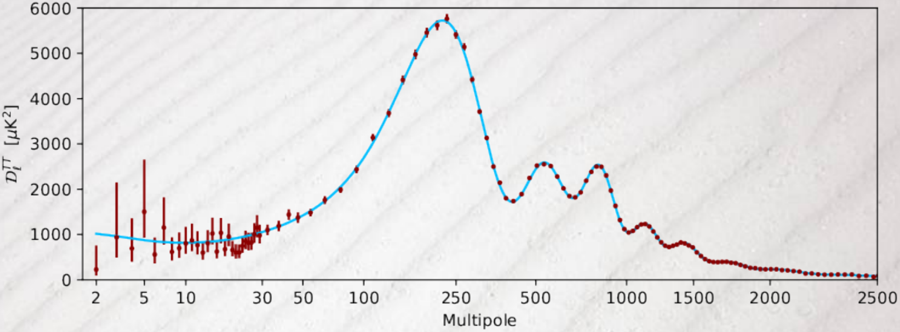
m: locations of peaks and troughs

ΔT from quantum fluctuations: we can only predict probabilities

$$\langle a_{\ell m} a_{\ell' m'} \rangle = \mathcal{C}_\ell \delta^{(K)}_{\ell\ell'} \delta^{(K)}_{mm'}$$

That is, the $a_{\ell m}$ in our universe are randomly drawn from a Gaussian probability distribution with mean 0 and variance $\mathcal{C}_\ell$.
$\mathcal{C}_\ell$ is called the power spectrum, $\mathcal{D}_\ell = \ell(\ell+1)\mathcal{C}_\ell/(2\pi)$.

CMB power spectrum II



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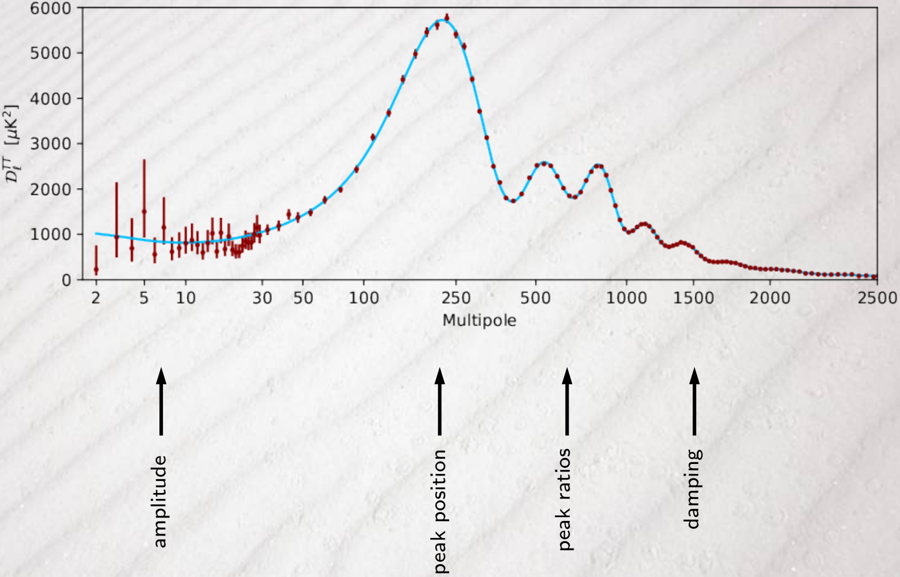
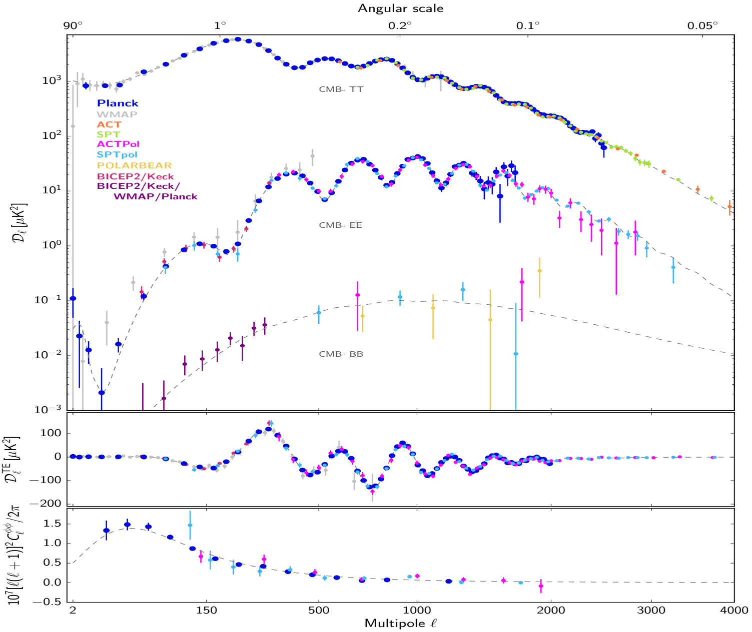




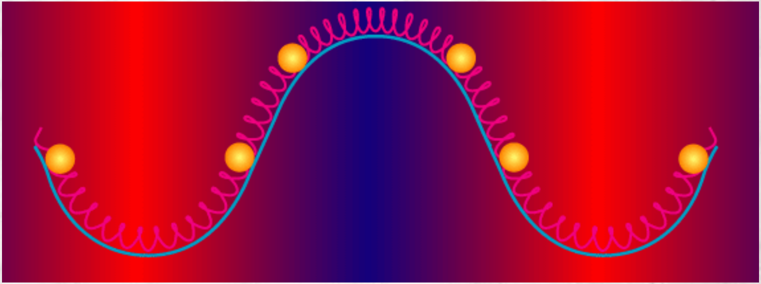
Angular scale
90°
1°
0.2°
0.1°
0.05°
Planck
WMAP
ACT
SPT
ACTPol
SPTpol
POLARBEAR
BICEP2/Keck
BICEP2/Keck/
WMAP/Planck
CMB- TT
CMB- EE
CMB- BB
$\mathcal{D}_\ell\,[\mu K^2]$
$\mathcal{D}_\ell^{TE}\,[\mu K^2]$
$10^7[\ell(\ell+1)]^2 C_\ell^{\phi\phi}/2\pi$
Multipole ℓ
2
150
500
1000
2000
3000
4000

Part III: How does the CMB get its spots?

Overview: How the CMB got its spots

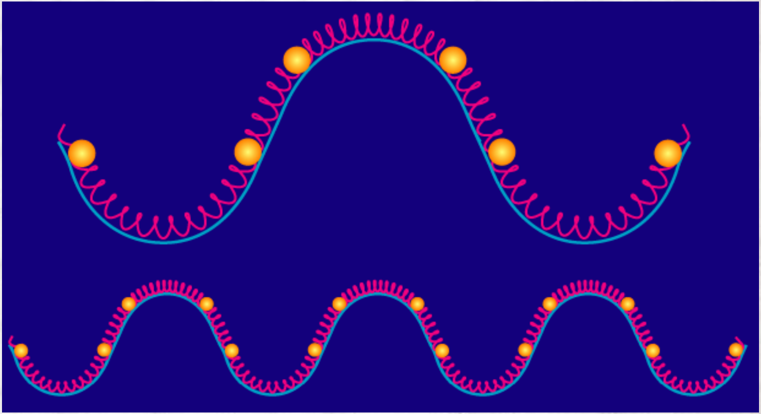
1. Before atoms form, the photons and baryons are tightly coupled. Their density perturbations oscillate under the competing influences of pressure and gravity.
2. A mode of wavelength λ oscillates with period λ/c_s where c_s is the sound speed. Large modes oscillate slowly.
3. Atoms form at time t_*, suddenly decoupling photons and baryons. Modes which happen to be at their peaks or troughs at t_* correspond to peaks in the power spectrum.
4. Afterwards, photons stream freely while baryons cluster.
5. The mapping of 3-D temperature fluctuations at t_* to 2-D anisotropies depends on the expansion of the universe after t_*.

Oscillation of the baryon-photon plasma I



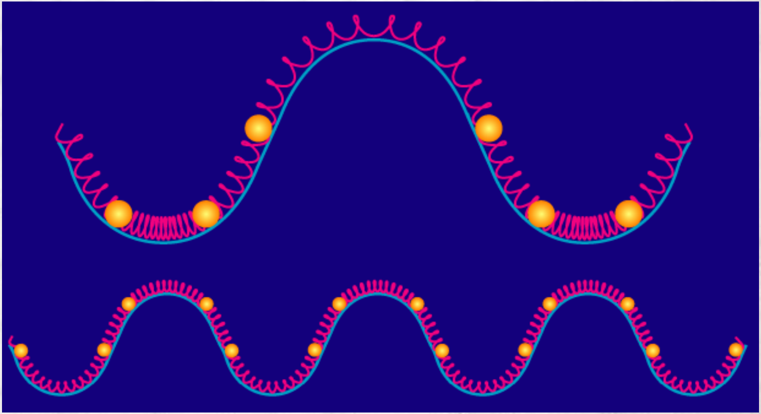
- Balls represent baryons.
    - Springs represent the pressure.
    - The blue curve is the gravitational potential.
- Colors represent photon temperature, with blue being hot.

Oscillation of the baryon-photon plasma II



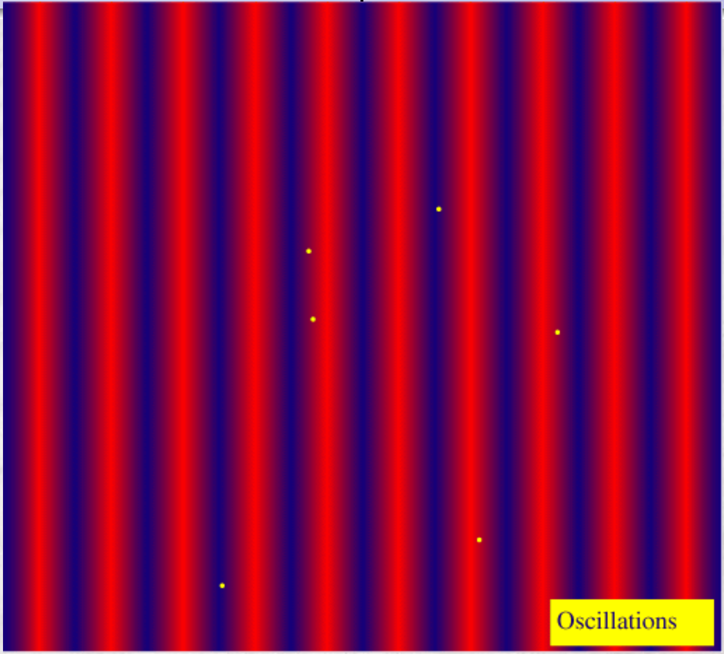
Longer-wavelength modes oscillate more slowly.

Oscillation of the baryon-photon plasma III



Atoms form at t_*, decoupling the photons and stopping oscillation.

From oscillations to anisotropies



Once more, but with math!

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Tight-coupling era: temperature moments related to baryonic fluid

- $\Theta_0(k, \mathfrak{t}) \propto \delta\rho_b$
- $\Theta_1(k, \mathfrak{t}) \propto \hat{k} \cdot \vec{v}_b$
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Evolution of Θ_0 in tight-coupling era, with $R = \frac{3\rho_b}{4\rho_\gamma}$, $c_s^2 = \frac{1}{3(1+R)}$:

$$\left(\frac{d^2}{d\mathfrak{t}^2} + \frac{\dot{R}}{1+R} \frac{d}{d\mathfrak{t}} + k^2 c_s^2 \right) (\Theta_0 + \Phi) = \frac{k^2}{3} \left(\frac{1}{1+R} \Phi - \Psi \right)$$

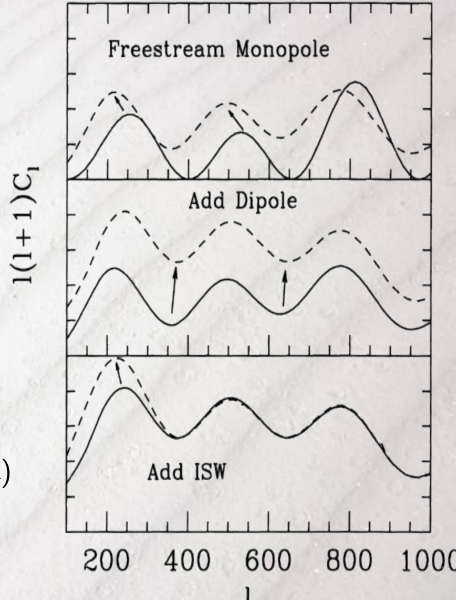
Free streaming and anisotropy

Θ_ℓ today from free-streaming of $\Theta_0(t_*)$, $\Theta_1(t_*)$ with $\Delta t = t_0 - t_*$:

$$\Theta_\ell(k, t_0) = [\Theta_0(k, t_*) + \Psi(k, t_*)] j_\ell(k\Delta t)$$

$$+ 3\Theta_1(k, t_*) j_{\ell-1}(k\Delta t)$$
$$- 3\Theta_1(k, t_*) \frac{(\ell+1)}{k\Delta t} j_\ell(k\Delta t)$$

$$+ \int_0^{t_0} dt e^{-\tau(t)} \frac{d[\Psi - \Phi]}{dt} j_\ell(k\Delta t)$$



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So, we can transfer an initial perturbation $\delta_{\rm in}(k)$ to late-time temperature perturbations $\Theta_\ell(k, \mathfrak{t})$. Since our equations of motion are linear, this lets us transform any combination of $\delta_{\rm in}(\vec{k})$ to final $\Theta_\ell(k, \mathfrak{t})$ using the transfer function

$$T_\ell(k) = |\Theta_\ell(k, \mathfrak{t}_0)/\delta_{\rm in}(k)|$$

Power spectrum I

But wait! If $\delta_{\rm in}(\vec{k})$ comes from a quantum fluctuation, then we can't predict its value, only it's probability distribution. Yet another power spectrum:

$$\left\langle \delta_{\rm in}(\vec{k})\delta_{\rm in}(\vec{k}')^* \right\rangle = (2\pi)^3 \delta^{(D)}(\vec{k} - \vec{k}') P_{\delta_{\rm in}}(k)$$

. Here $\frac{k^3}{2\pi^2} P_{\delta_{\rm in}}(k) \propto A_s (k/k_0)^{n_s - 1}$. Once again it is the variance of the probability distribution of the amplitude squared as a function of scale, but in 3 dimensions this time.

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Recall the CMB power spectrum: $\mathcal{C}_\ell\delta^{(K)}_{\ell\ell'}\delta^{(K)}_{mm'} = \langle a_{\ell m}a_{\ell' m'}\rangle$

Power spectrum II

If we knew $\Theta(\vec{x}, \hat{p}, t)$, then $a_{\ell m} = \int d^2\hat{p} Y^*_{\ell m}(\hat{p})\Theta(\vec{x}, \hat{p}, t_0)$.

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But this means that $\langle a_{\ell m} a_{\ell' m'} \rangle$ is related to
$\left\langle \Theta(\vec{k}, \hat{p}, t_0)\Theta(\vec{k}', \hat{p}', t_0) \right\rangle = (2\pi)^3 \delta^{(D)}(\vec{k} - \vec{k}') P_{\delta_{\rm in}}(k) T_\ell(k)^2$.

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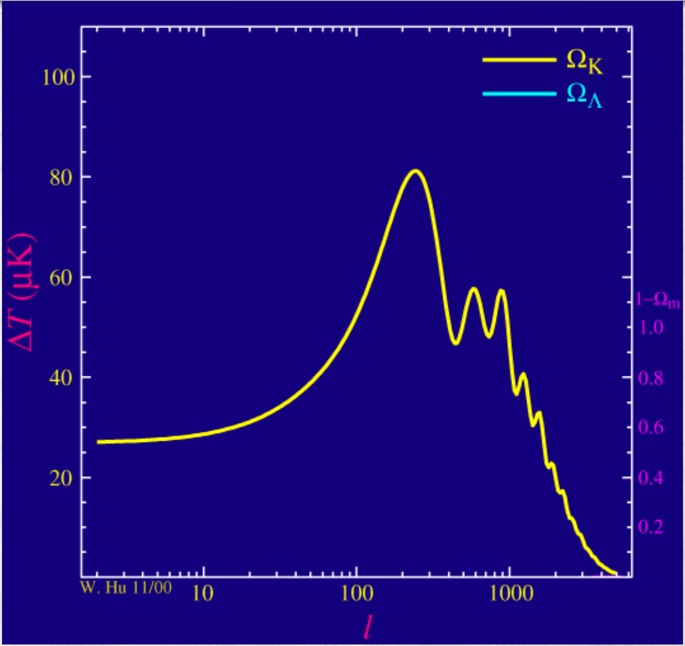
$$\Rightarrow \mathcal{C}_\ell = \sum_{\ell', m'} \langle a_{\ell m} a_{\ell' m'} \rangle = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\delta_{\rm in}}(k) T_\ell(k)^2$$

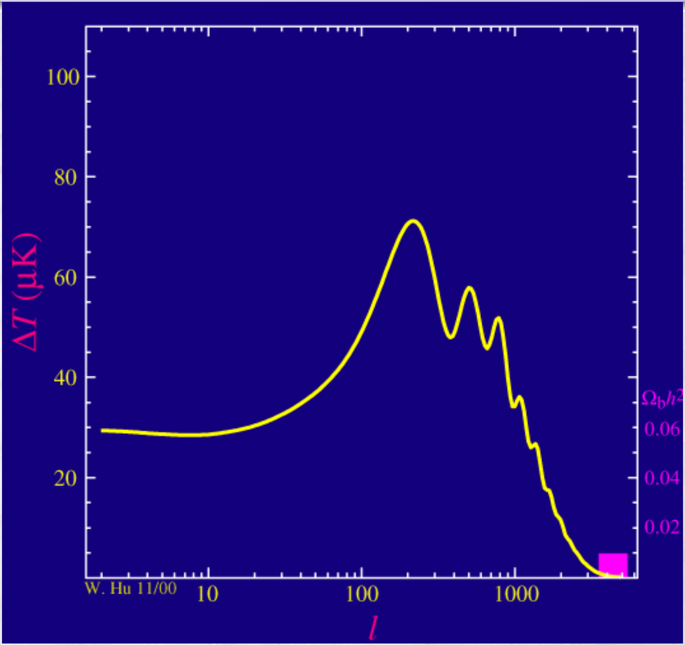
This is our final answer! Given the initial power spectrum (A_s, n_s), the composition of the late universe (Ω_m, Ω_b), the late-time the expansion rate (h, $w(a)$, Ω_K), and optical depth (τ), we can predict $\mathcal{C}_\ell$ and compare it to measurements.

Part IV: CMB as a probe of fundamental physics

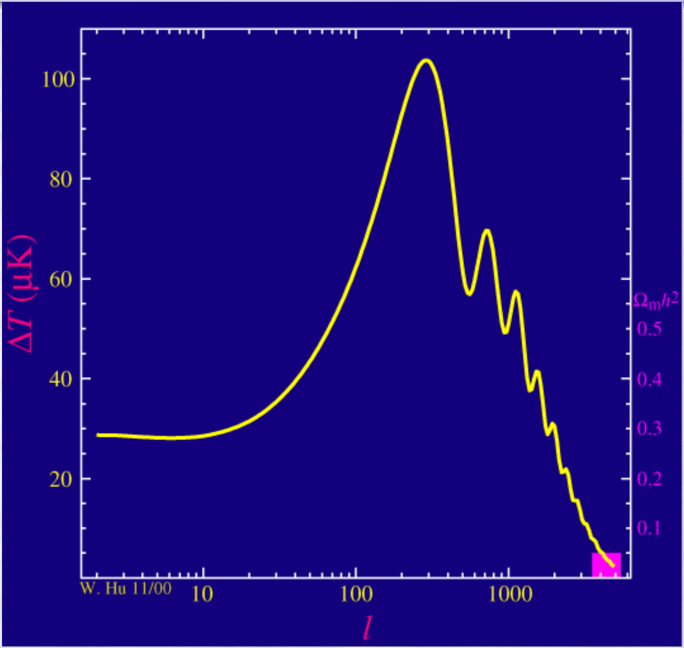
Position of the first acoustic peak



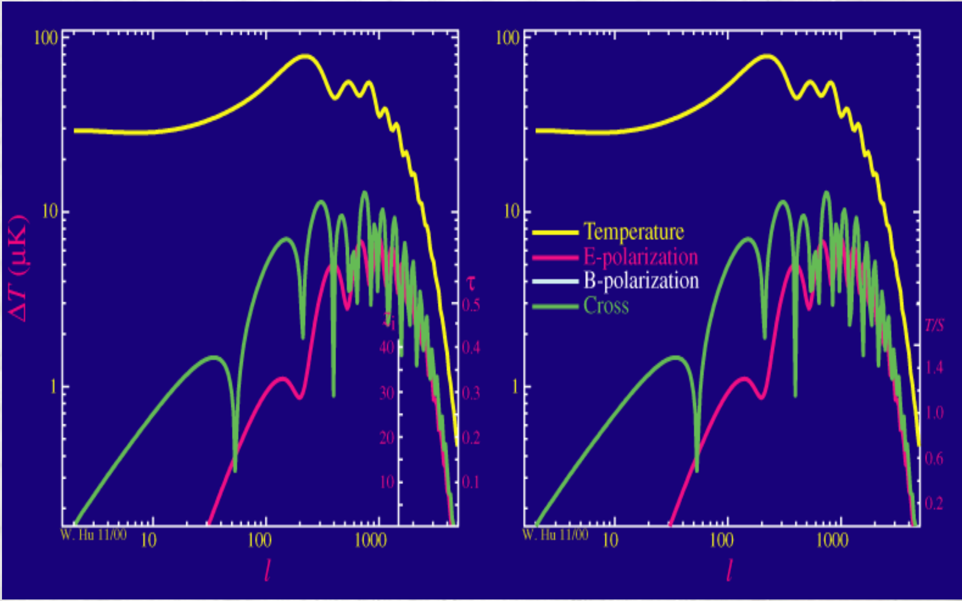
Relative peak heights I



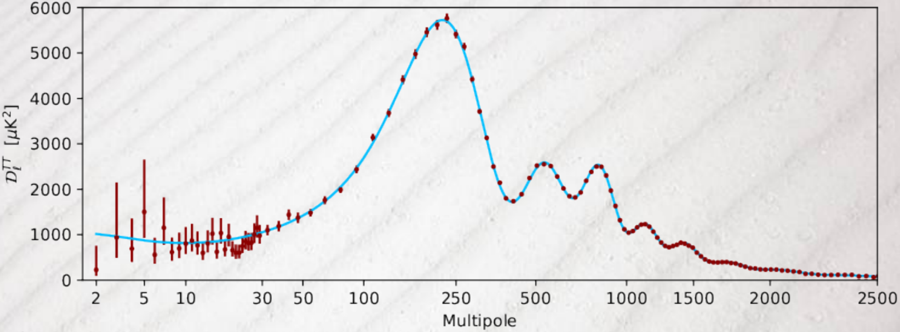
Relative peak heights II



Polarization



Data constraints



- $\Omega_c h^2 = 0.1200 \pm 0.0012$
- $\Omega_b h^2 = 0.02237 \pm 0.00015$
- $h = 0.6736 \pm 0.0054$
- $\log(10^{10} A_s) = 3.044 \pm 0.014$
- $n_s = 0.9649 \pm 0.0042$
- $\tau = 0.0544 \pm 0.0073$

Conclusions

1. The CMB is a rich source of information on high-energy physics as well as the late-time evolution of the universe. It forms the foundation of the standard cosmological model.

2. Temperature and polarization anisotropies can be characterized by power spectra, quantifying the spots in the CMB. Features in the power spectra arise through acoustic oscillations of the baryon-photon plasma.

3. Quantifying the effect of each element of the cosmological model on the CMB power spectra provides powerful constraints on the cosmological parameters, including possible new physics.