Three-loop soft anomalous dimensions in QCD

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- **Higher-order soft-gluon corrections**
- **Three-loop soft anomalous dimensions**
- Γ_S **for processes with** W, Z, γ, H
- Γ_S **for single-top, top-pair, and** $2 \to 3$ **top processes**

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Soft-gluon corrections

very important because they are typically large and they dominate the perturbative corrections

consider partonic processes $p_a + p_b \to p_1 + p_2 + \cdots$

define $s = (p_a + p_b)^2$, $t = (p_a - p_1)^2$, $u = (p_b - p_1)^2$ **and** $s_4 = s + t + u - \sum m_i^2$

At partonic threshold $s_4 \to 0$

Soft corrections $\left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+$ **with** $k \le 2n - 1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

At NLL accuracy we need one-loop soft anomalous dimensions

At NNLL accuracy need two loops; at N^{3}LL accuracy need three loops

Finite-order expansions-no prescription needed

Approximate NNLO (aNNLO) and approximate N^{3}LO (aN^{3}LO) predictions for cross sections and differential distributions (single and double)

Soft-gluon Resummation

moments of the partonic cross section with moment variable N:

$\hat{\sigma}(N) = \int (ds_4/s)\, e^{-N s_4/s} \hat{\sigma}(s_4)$

factorized expression for the cross section in $4-\epsilon$ dimensions

$$\sigma^{ab\to 12}(N,\epsilon) = \mathrm{tr}\left\{ H^{ab\to 12}\left(\alpha_s(\mu_R)\right)\, S^{ab\to 12}\left(\frac{M}{N\mu_F}, \alpha_s(\mu_R)\right)\right\}$$

$$\times\, \psi_a\left(N_a, \mu_F, \epsilon\right)\, \psi_b\left(N_b, \mu_F, \epsilon\right) \prod J_i\left(N, \mu_F, \epsilon\right)$$

$H^{ab\to 12}$ **is hard function and** $S^{ab\to 12}$ **is soft function**

$S^{ab\to 12}$ **satisfies the renormalization group equation**

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S^{ab\to 12} = -\Gamma_S^{\dagger\, ab\to 12}\, S^{ab\to 12} - S^{ab\to 12}\, \Gamma_S^{ab\to 12}$$

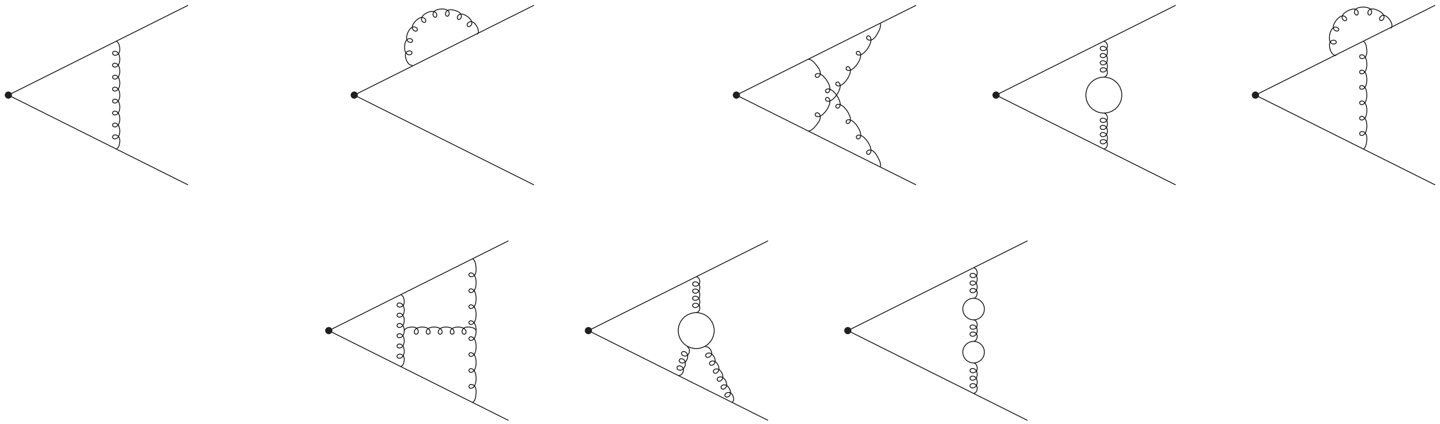
Soft anomalous dimension $\Gamma_S^{ab\to 12}$ controls the evolution of the soft function which gives the exponentiation of logarithms of N

Resummed cross section

After renormalization-group evolution of the functions in the factorized cross section, we have

$$
\begin{aligned}
d\hat{\sigma}^{ab\to 12}_{\rm resum}(N) = & \exp\left[\sum_{i=a,b} E_i(N_i)\right] \; \exp\left[\sum_{j={\rm f.s.}\; q,g} E'_j(N)\right] \\
& \times {\rm tr}\left\{ H^{ab\to 12}\left(\alpha_s(\sqrt{s})\right) \bar{P}\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \Gamma_S^{\dagger\, ab\to 12}\left(\alpha_s(\mu)\right)\right] \right. \\
& \left. \times \tilde{S}^{ab\to 12}\left(\alpha_s(\frac{\sqrt{s}}{\tilde{N}})\right) P\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \Gamma_S^{ab\to 12}\left(\alpha_s(\mu)\right)\right] \right\}
\end{aligned}
$$

Cusp anomalous dimension



A basic ingredient of soft anomalous dimensions

cusp angle $\theta = \cosh^{-1}(p_i \cdot p_j/\sqrt{p_i^2 p_j^2})$ **and** $\Gamma_{\rm cusp} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \Gamma_{\rm cusp}^{(n)}$

One loop

$$\Gamma_{\rm cusp}^{(1)} = C_F(\theta \coth\theta - 1)$$

In terms of $\beta = \tanh(\theta/2) = \sqrt{1 - \frac{4m^2}{s}}$ **we have** $\theta = \ln\left(\frac{1+\beta}{1-\beta}\right)$ **and**

$$\Gamma_{\rm cusp}^{(1)\,\beta} = C_F\left[-\frac{(1+\beta^2)}{2\beta}\ln\frac{(1-\beta)}{(1+\beta)} - 1\right] = -C_F\left(L_\beta + 1\right)$$

Cusp anomalous dimension

Two loops

$$\Gamma_{\rm cusp}^{(2)} = K_2\,\Gamma_{\rm cusp}^{(1)}+\frac{1}{2}C_F C_A \left\{1+\zeta_2+\theta^2-\coth\theta\left[\zeta_2\theta+\theta^2+\frac{\theta^3}{3}+{\rm Li}_2\left(1-e^{-2\theta}\right)\right]\right.$$
$$\left.+\coth^2\theta\left[-\zeta_3+\zeta_2\theta+\frac{\theta^3}{3}+\theta\,{\rm Li}_2\left(e^{-2\theta}\right)+{\rm Li}_3\left(e^{-2\theta}\right)\right]\right\}$$

where $K_2=C_A\left(\frac{67}{36}-\frac{\zeta_2}{2}\right)-\frac{5}{18}n_f$

In terms of β

$$\Gamma_{\rm cusp}^{(2)\,\beta} = K_2\,\Gamma_{\rm cusp}^{(1)\,\beta}+\frac{1}{2}C_F C_A\left\{1+\zeta_2+\ln^2\left(\frac{1-\beta}{1+\beta}\right)\right.$$
$$+\frac{(1+\beta^2)}{2\beta}\left[\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)-\ln^2\left(\frac{1-\beta}{1+\beta}\right)+\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)-{\rm Li}_2\left(\frac{4\beta}{(1+\beta)^2}\right)\right]$$
$$+\frac{(1+\beta^2)^2}{4\beta^2}\left[-\zeta_3-\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)-\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)-\ln\left(\frac{1-\beta}{1+\beta}\right){\rm Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right.$$
$$\left.\left.+{\rm Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]\right\}$$

Cusp anomalous dimension

Three loops

$$\Gamma_{\rm cusp}^{(3)} \quad = \quad K_3 \Gamma_{\rm cusp}^{(1)} + 2\, K_2 \left(\Gamma_{\rm cusp}^{(2)} - K_2\, \Gamma_{\rm cusp}^{(1)}\right) + C^{(3)}$$

where

$$K_3 \quad = \quad C_A^2 \left(\frac{245}{96} - \frac{67}{36}\zeta_2 + \frac{11}{24}\zeta_3 + \frac{11}{8}\zeta_4\right) + C_F n_f \left(-\frac{55}{96} + \frac{\zeta_3}{2}\right) + C_A n_f \left(-\frac{209}{432} + \frac{5\zeta_2}{18} - \frac{7\zeta_3}{12}\right) - \frac{n_f^2}{108}$$

and $C^{(3)}$ has a very long expression

Again, the result can be expressed in terms of β

For $n_f = 5$, we have the numerical result

$$\Gamma_{\rm cusp}^{(3)\,\beta} \approx 0.092\, \beta^2 + 2.803\, \Gamma_{\rm cusp}^{(1)\,\beta}$$

Cusp anomalous dimension - massless cases

If eikonal line i represents a massive quark and eikonal line j a massless quark, then we have simpler expressions

One loop

$$\Gamma_{\rm cusp}^{(1)\, m_i} = C_F \left[\ln\left(\frac{2p_i \cdot p_j}{m_i \sqrt{s}}\right) - \frac{1}{2}\right]$$

Two loops

$$\Gamma_{\rm cusp}^{(2)\, m_i} = K_2\, \Gamma_{\rm cusp}^{(1)\, m_i} + \frac{1}{4} C_F C_A (1-\zeta_3)$$

Three loops

$$\Gamma_{\rm cusp}^{(3)\, m_i} = K_3\, \Gamma_{\rm cusp}^{(1)\, m_i} + \frac{1}{2} K_2 C_F C_A (1-\zeta_3) + C_F C_A^2 \left(-\frac{1}{4} + \frac{3}{8}\zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8}\zeta_2\zeta_3 + \frac{9}{16}\zeta_5\right)$$

If both eikonal lines are massless, then

$$\Gamma_{\rm cusp}^{\rm massless} = C_F \ln\left(\frac{2p_i \cdot p_j}{s}\right) \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n K_n$$

Γ_S for some simple processes

processes with trivial color structure

Γ_S **vanishes for:**

Drell-Yan processes $q\bar{q} \to \gamma^*$, $q\bar{q} \to Z$

W**-boson production via** $q\bar{q}' \to W^{\pm}$

Higgs production via $b\bar{b} \to H$ **and** $gg \to H$

electroweak boson pair production $q\bar{q} \to \gamma\gamma$, $q\bar{q} \to ZZ$, $q\bar{q} \to W^+W^-$, $q\bar{q} \to \gamma Z$; $q\bar{q}' \to W^{\pm}\gamma$; $q\bar{q}' \to W^{\pm}Z$

charged Higgs production via $b\bar{b} \to H^-W^+$, $b\bar{b} \to H^+H^-$, $gg \to H^+H^-$

for Deep Inelastic Scattering (DIS)

At one loop: $\Gamma_S^{(1)\, q\gamma^* \to q} = C_F \ln(-t/s)$

At two loops: $\Gamma_S^{(2)\, q\gamma^* \to q} = K_2\, C_F \ln(-t/s)$

At three loops: $\Gamma_S^{(3)\, q\gamma^* \to q} = K_3\, C_F \ln(-t/s)$

Γ_S for large-p_T W, Z, γ, H production

Let $V = W$ **or** Z **or** γ **or** H

For the processes $qg \to W^\pm q'$ **and** $qg \to Zq$ **and** $qg \to \gamma q$ **and** $bg \to Hb$

At one loop: $\Gamma_S^{(1)\, qg \to Vq'} = C_F \ln\left(\frac{-u}{s}\right) + \frac{C_A}{2} \ln\left(\frac{t}{u}\right)$

At two loops: $\Gamma_S^{(2)\, qg \to Vq'} = K_2\, \Gamma_S^{(1)\, qg \to Vq'}$

At three loops: $\Gamma_S^{(3)\, qg \to Vq'} = K_3\, \Gamma_S^{(1)\, qg \to Vq'}$

For the processes $q\bar{q}' \to W^\pm g$ **or** $q\bar{q} \to Zg$ **or** $q\bar{q} \to \gamma g$ **or** $b\bar{b} \to Hg$

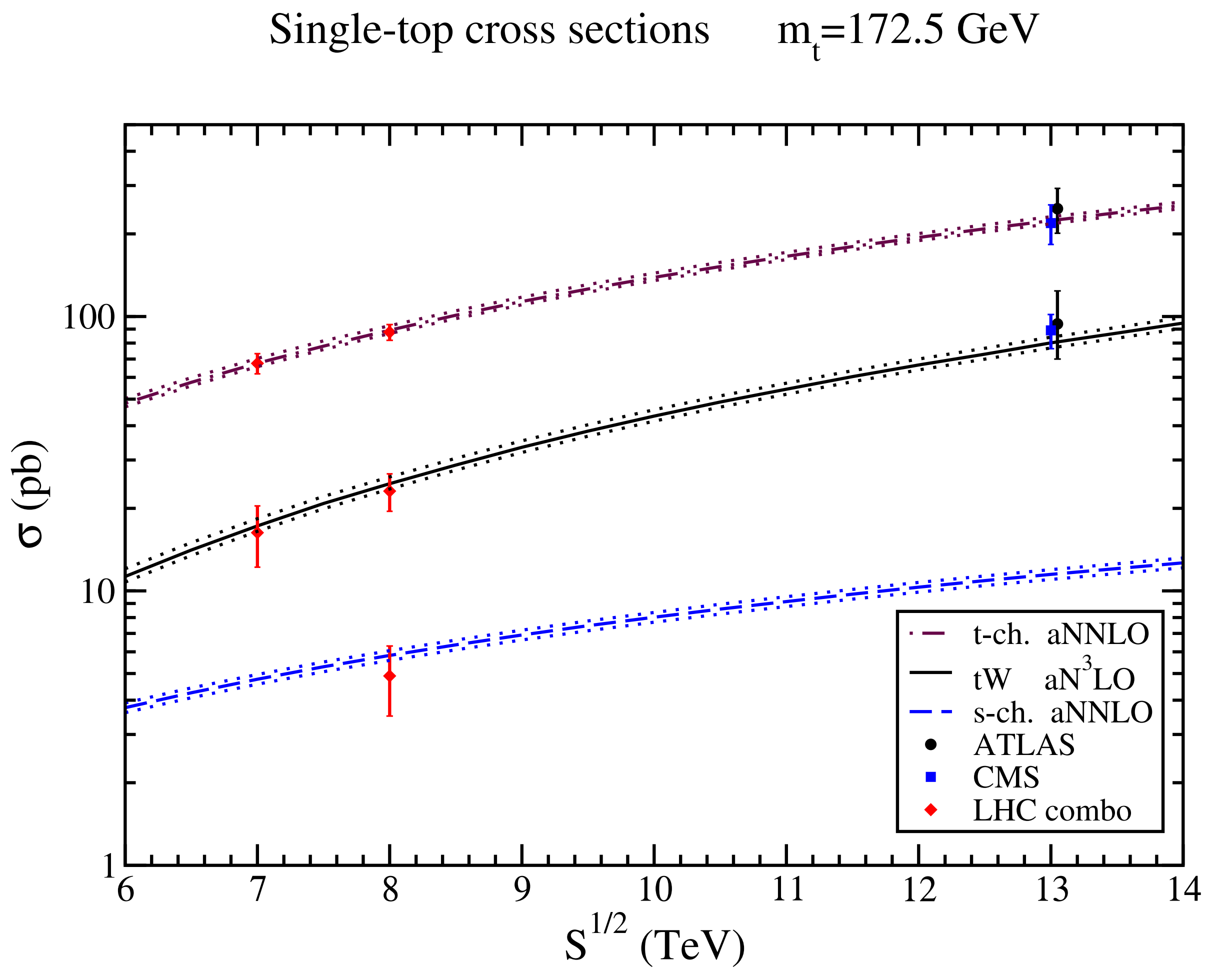
At one loop: $\Gamma_S^{(1)\, q\bar{q}' \to Vg} = \frac{C_A}{2} \ln\left(\frac{tu}{s^2}\right)$

At two loops: $\Gamma_S^{(2)\, q\bar{q}' \to Vg} = K_2\, \Gamma_S^{(1)\, q\bar{q}' \to Vg}$

At three loops: $\Gamma_S^{(3)\, q\bar{q}' \to Vg} = K_3\, \Gamma_S^{(1)\, q\bar{q}' \to Vg}$

same Γ_S **for reverse processes such as** $\gamma q \to qg$ **and** $\gamma g \to q\bar{q}$

Single-top production



Single-top t-channel production

$\Gamma_S^{bq\to tq'}$ **is a 2×2 matrix: use t-channel singlet-octet color basis**

At one loop

$$\Gamma_{S\,11}^{(1)bq\to tq'}=C_F\left[\ln\left(\frac{t(t-m_t^2)}{m_t s^{3/2}}\right)-\frac{1}{2}\right], \quad \Gamma_{S\,12}^{(1)bq\to tq'}=\frac{C_F}{2N_c}\ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right), \quad \Gamma_{S\,21}^{(1)bq\to tq'}=\ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right)$$

$$\Gamma_{S\,22}^{(1)bq\to tq'}=\left(C_F-\frac{C_A}{2}\right)\left[\ln\left(\frac{t(t-m_t^2)}{m_t\, s^{3/2}}\right)-\frac{1}{2}+2\ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right)\right]+\frac{C_A}{2}\left[\ln\left(\frac{u(u-m_t^2)}{m_t\, s^{3/2}}\right)-\frac{1}{2}\right]$$

At two loops

$$\Gamma_{S\,11}^{(2)bq\to tq'}=K_2\,\Gamma_{S\,11}^{(1)bq\to tq'}+\frac{1}{4}C_F C_A(1-\zeta_3), \qquad \Gamma_{S\,12}^{(2)bq\to tq'}=K_2\,\Gamma_{S\,12}^{(1)bq\to tq'}$$

$$\Gamma_{S\,21}^{(2)bq\to tq'}=K_2\,\Gamma_{S\,21}^{(1)bq\to tq'}, \qquad \Gamma_{S\,22}^{(2)bq\to tq'}=K_2\,\Gamma_{S\,22}^{(1)bq\to tq'}+\frac{1}{4}C_F C_A(1-\zeta_3)$$

At three loops

$$\Gamma_{S\,11}^{(3)bq\to tq'}=K_3\,\Gamma_{S\,11}^{(1)bq\to tq'}+\frac{1}{2}K_2 C_F C_A(1-\zeta_3)+C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right)$$

$$\Gamma_{S\,12}^{(3)bq\to tq'}=K_3\,\Gamma_{S\,12}^{(1)bq\to tq'}+X_{S\,12}^{(3)bq\to tq'}, \qquad \Gamma_{S\,21}^{(3)bq\to tq'}=K_3\,\Gamma_{S\,21}^{(1)bq\to tq'}+X_{S\,21}^{(3)bq\to tq'}$$

$$\Gamma_{S\,22}^{(3)bq\to tq'}=K_3\,\Gamma_{S\,22}^{(1)bq\to tq'}+\frac{1}{2}K_2 C_F C_A(1-\zeta_3)+C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right)+X_{S\,22}^{(3)bq\to tq'}$$

Single-top s-channel production

$\Gamma_S^{q\bar{q}'\to t\bar{b}}$ **is a 2×2 matrix: use s-channel singlet-octet color basis**

At one loop

$$\Gamma_{S\,11}^{(1)q\bar{q}'\to t\bar{b}}=C_F\left[\ln\left(\frac{s-m_t^2}{m_t\sqrt{s}}\right)-\frac{1}{2}\right]\,,\quad \Gamma_{S\,12}^{(1)q\bar{q}'\to t\bar{b}}=\frac{C_F}{2N_c}\ln\left(\frac{t(t-m_t^2)}{u(u-m_t^2)}\right)\,,\quad \Gamma_{S\,21}^{(1)q\bar{q}'\to t\bar{b}}=\ln\left(\frac{t(t-m_t^2)}{u(u-m_t^2)}\right)$$

$$\Gamma_{S\,22}^{(1)q\bar{q}'\to t\bar{b}}=\left(C_F-\frac{C_A}{2}\right)\left[\ln\left(\frac{s-m_t^2}{m_t\sqrt{s}}\right)-\frac{1}{2}+2\ln\left(\frac{t(t-m_t^2)}{u(u-m_t^2)}\right)\right]+\frac{C_A}{2}\left[\ln\left(\frac{t(t-m_t^2)}{m_t\,s^{3/2}}\right)-\frac{1}{2}\right]$$

At two loops

$$\Gamma_{S\,11}^{(2)q\bar{q}'\to t\bar{b}}=K_2\,\Gamma_{S\,11}^{(1)q\bar{q}'\to t\bar{b}}+\frac{1}{4}C_F C_A(1-\zeta_3)\,,\qquad \Gamma_{S\,12}^{(2)q\bar{q}'\to t\bar{b}}=K_2\,\Gamma_{S\,12}^{(1)q\bar{q}'\to t\bar{b}}$$

$$\Gamma_{S\,21}^{(2)q\bar{q}'\to t\bar{b}}=K_2\,\Gamma_{S\,21}^{(1)q\bar{q}'\to t\bar{b}}\,,\qquad \Gamma_{S\,22}^{(2)q\bar{q}'\to t\bar{b}}=K_2\,\Gamma_{S\,22}^{(1)q\bar{q}'\to t\bar{b}}+\frac{1}{4}C_F C_A(1-\zeta_3)$$

At three loops

$$\Gamma_{S\,11}^{(3)q\bar{q}'\to t\bar{b}}=K_3\,\Gamma_{S\,11}^{(1)q\bar{q}'\to t\bar{b}}+\frac{1}{2}K_2 C_F C_A(1-\zeta_3)+C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right)$$

$$\Gamma_{S\,12}^{(3)q\bar{q}'\to t\bar{b}}=K_3\,\Gamma_{S\,12}^{(1)q\bar{q}'\to t\bar{b}}+X_{S\,12}^{(3)q\bar{q}'\to t\bar{b}}\,,\qquad \Gamma_{S\,21}^{(3)q\bar{q}'\to t\bar{b}}=K_3\,\Gamma_{S\,21}^{(1)q\bar{q}'\to t\bar{b}}+X_{S\,21}^{(3)q\bar{q}'\to t\bar{b}}$$

$$\Gamma_{S\,22}^{(3)q\bar{q}'\to t\bar{b}}=K_3\,\Gamma_{S\,22}^{(1)q\bar{q}'\to t\bar{b}}+\frac{1}{2}K_2 C_F C_A(1-\zeta_3)+C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right)+X_{S\,22}^{(3)q\bar{q}'\to t\bar{b}}$$

Associated tW production

At one loop

$$\Gamma_S^{(1)bg\to tW} = C_F\left[\ln\left(\frac{m_t^2-t}{m_t\sqrt{s}}\right)-\frac{1}{2}\right]+\frac{C_A}{2}\ln\left(\frac{u-m_t^2}{t-m_t^2}\right)$$

At two loops

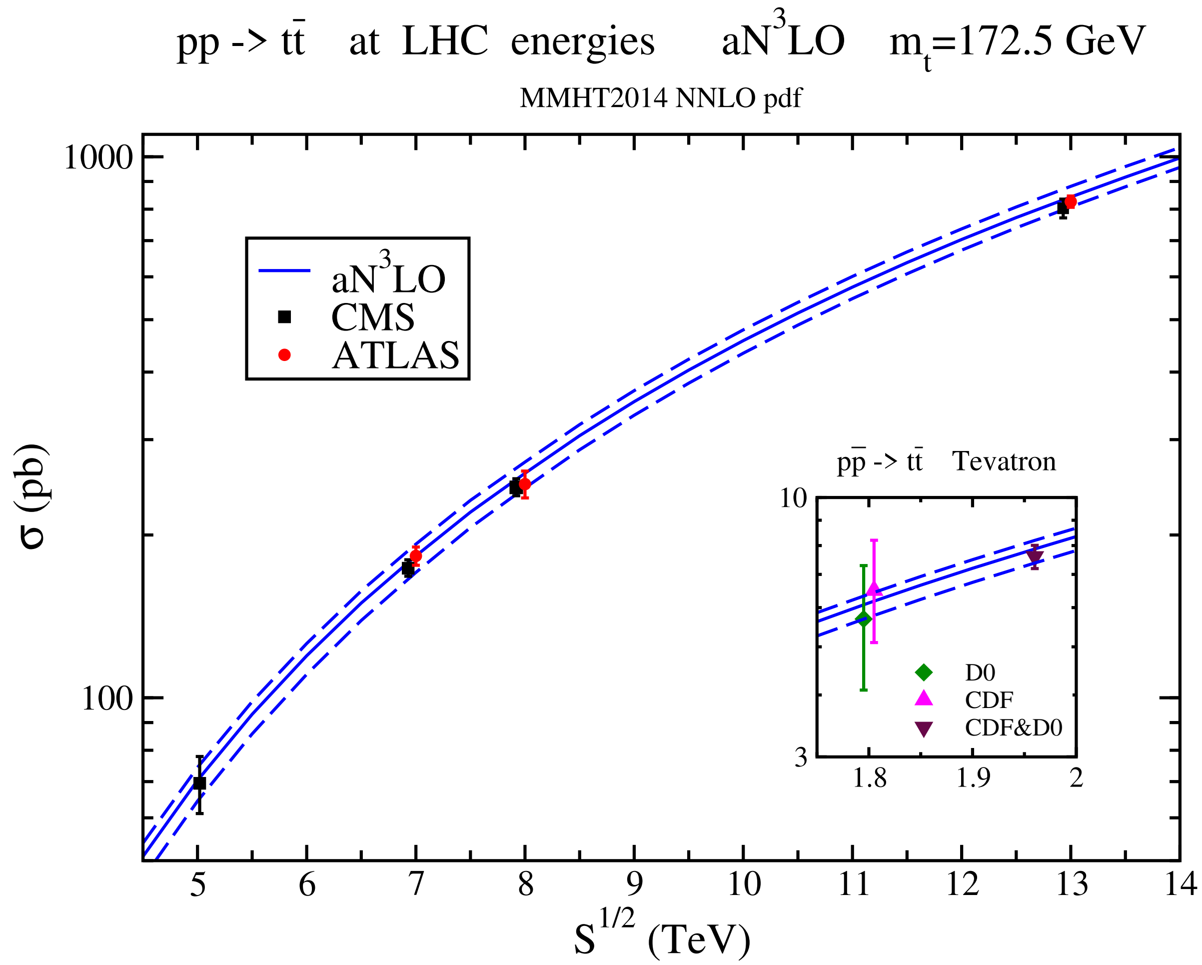
$$\Gamma_S^{(2)bg\to tW} = K_2\,\Gamma_S^{(1)bg\to tW}+\frac{1}{4}C_F C_A(1-\zeta_3)$$

At three loops

$$\Gamma_S^{(3)bg\to tW} = K_3\,\Gamma_S^{(1)bg\to tW}+\frac{1}{2}K_2 C_F C_A(1-\zeta_3)+C_F C_A^2\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right)$$

same Γ_S for $bg\to tH^-$ and for FCNC $qg\to tZ$, $qg\to tZ'$, $qg\to t\gamma$

Top-antitop pair production



Top-antitop pair production: $q\bar{q} \to t\bar{t}$ channel

$\Gamma_S^{q\bar{q}\to t\bar{t}}$ **is a 2×2 matrix: use s-channel singlet-octet color basis**

At one loop for $q\bar{q} \to t\bar{t}$

$$\Gamma_{S\,11}^{(1)\,q\bar{q}\to t\bar{t}} = \Gamma_{\rm cusp}^{(1)\,\beta}\,, \quad \Gamma_{12}^{(1)\,q\bar{q}\to t\bar{t}} = \frac{C_F}{C_A}\ln\left(\frac{t-m_t^2}{u-m_t^2}\right)\,, \quad \Gamma_{21}^{(1)\,q\bar{q}\to t\bar{t}} = 2\ln\left(\frac{t-m_t^2}{u-m_t^2}\right)$$

$$\Gamma_{22}^{(1)\,q\bar{q}\to t\bar{t}} = \left(1-\frac{C_A}{2C_F}\right)\Gamma_{\rm cusp}^{(1)} + 4C_F\ln\left(\frac{t-m_t^2}{u-m_t^2}\right) - \frac{C_A}{2}\left[1+\ln\left(\frac{s m_t^2 (t-m_t^2)^2}{(u-m_t^2)^4}\right)\right]$$

At two loops for $q\bar{q} \to t\bar{t}$

$$\Gamma_{S\,11}^{(2)\,q\bar{q}\to t\bar{t}} = \Gamma_{\rm cusp}^{(2)\,\beta}\,, \quad \Gamma_{12}^{(2)\,q\bar{q}\to t\bar{t}} = \left(K_2 - C_A\,N_2^\beta\right)\Gamma_{12}^{(1)\,q\bar{q}\to t\bar{t}}\,, \quad \Gamma_{21}^{(2)\,q\bar{q}\to t\bar{t}} = \left(K_2 + C_A\,N_2^\beta\right)\Gamma_{21}^{(1)\,q\bar{q}\to t\bar{t}}$$

$$\Gamma_{22}^{(2)\,q\bar{q}\to t\bar{t}} = K_2\Gamma_{22}^{(1)\,q\bar{q}\to t\bar{t}} + \left(1-\frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(2)\,\beta} - K_2\Gamma_{\rm cusp}^{(1)\,\beta}\right) + \frac{C_A^2}{4}(1-\zeta_3)$$

where

$$N_2^\beta = \frac{1}{4}\ln^2\left(\frac{1-\beta}{1+\beta}\right) + \frac{(1+\beta^2)}{8\beta}\left[\zeta_2 - \ln^2\left(\frac{1-\beta}{1+\beta}\right) - {\rm Li}_2\left(\frac{4\beta}{(1+\beta)^2}\right)\right]$$

At three loops for $q\bar{q} \to t\bar{t}$

$$\Gamma_{S\,22}^{(3)\,q\bar{q}\to t\bar{t}} = K_3\,\Gamma_{S\,22}^{(1)\,q\bar{q}\to t\bar{t}} + \left(1-\frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(3)\,\beta} - K_3\Gamma_{\rm cusp}^{(1)\,\beta}\right) + \frac{K_2}{2}C_A^2(1-\zeta_3)$$
$$+ C_A^3\left(-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right) + X_{S\,22}^{(3)\,q\bar{q}\to t\bar{t}}$$

where $X_{S\,22}^{(3)\,q\bar{q}\to t\bar{t}}$ denotes unknown three-loop contributions from four-parton correlations

other 3-loop matrix elements not fully known either but have analogous structure to that at two loops

Top-antitop pair production: $gg \to t\bar{t}$ channel

3×3 matrix for $gg \to t\bar{t}$ with color basis $c_1 = \delta^{ab}\delta_{12}$, $c_2 = d^{abc}T^c_{12}$, $c_3 = if^{abc}T^c_{12}$

$$\Gamma_S^{gg\to t\bar{t}} = \left[\begin{array}{ccc} \Gamma_{S\,11}^{gg\to t\bar{t}} & 0 & \Gamma_{S\,13}^{gg\to t\bar{t}} \\ 0 & \Gamma_{S\,22}^{gg\to t\bar{t}} & \Gamma_{S\,23}^{gg\to t\bar{t}} \\ \Gamma_{S\,31}^{gg\to t\bar{t}} & \Gamma_{S\,32}^{gg\to t\bar{t}} & \Gamma_{S\,22}^{gg\to t\bar{t}} \end{array}\right]$$

At one loop for $gg \to t\bar{t}$

$$\Gamma_{S\,11}^{(1)\,gg\to t\bar{t}} = \Gamma_{\rm cusp}^{(1)\,\beta}, \quad \Gamma_{S\,13}^{(1)\,gg\to t\bar{t}} = \ln\left(\frac{t-m_t^2}{u-m_t^2}\right), \quad \Gamma_{S\,31}^{(1)\,gg\to t\bar{t}} = 2\ln\left(\frac{t-m_t^2}{u-m_t^2}\right),$$

$$\Gamma_{S\,22}^{(1)\,gg\to t\bar{t}} = \left(1-\frac{C_A}{2C_F}\right)\Gamma_{\rm cusp}^{(1)\,\beta} + \frac{C_A}{2}\left[\ln\left(\frac{(t-m_t^2)(u-m_t^2)}{s\,m_t^2}\right)-1\right],$$

$$\Gamma_{S\,23}^{(1)\,gg\to t\bar{t}} = \frac{C_A}{2}\ln\left(\frac{t-m_t^2}{u-m_t^2}\right), \quad \Gamma_{S\,32}^{(1)\,gg\to t\bar{t}} = \frac{(N_c^2-4)}{2N_c}\ln\left(\frac{t-m_t^2}{u-m_t^2}\right)$$

Top-antitop pair production: $gg \to t\bar{t}$ channel

At two loops for $gg \to t\bar{t}$

$$\Gamma_{S\,11}^{(2)gg\to t\bar{t}} = \Gamma_{\rm cusp}^{(2)\,\beta}\,, \quad \Gamma_{S\,13}^{(2)gg\to t\bar{t}} = \left(K_2 - C_A N_2^{\beta}\right) \Gamma_{S\,13}^{(1)gg\to t\bar{t}}\,, \quad \Gamma_{S\,31}^{(2)gg\to t\bar{t}} = \left(K_2 + C_A N_2^{\beta}\right) \Gamma_{S\,31}^{(1)gg\to t\bar{t}}\,,$$

$$\Gamma_{S\,22}^{(2)gg\to t\bar{t}} = K_2\, \Gamma_{S\,22}^{(1)gg\to t\bar{t}} + \left(1 - \frac{C_A}{2C_F}\right) \left(\Gamma_{\rm cusp}^{(2)\,\beta} - K_2 \Gamma_{\rm cusp}^{(1)\,\beta}\right) + \frac{C_A^2}{4}(1-\zeta_3)\,,$$

$$\Gamma_{S\,23}^{(2)gg\to t\bar{t}} = K_2\, \Gamma_{S\,23}^{(1)gg\to t\bar{t}}\,, \quad \Gamma_{S\,32}^{(2)gg\to t\bar{t}} = K_2\, \Gamma_{S\,32}^{(1)gg\to t\bar{t}}$$

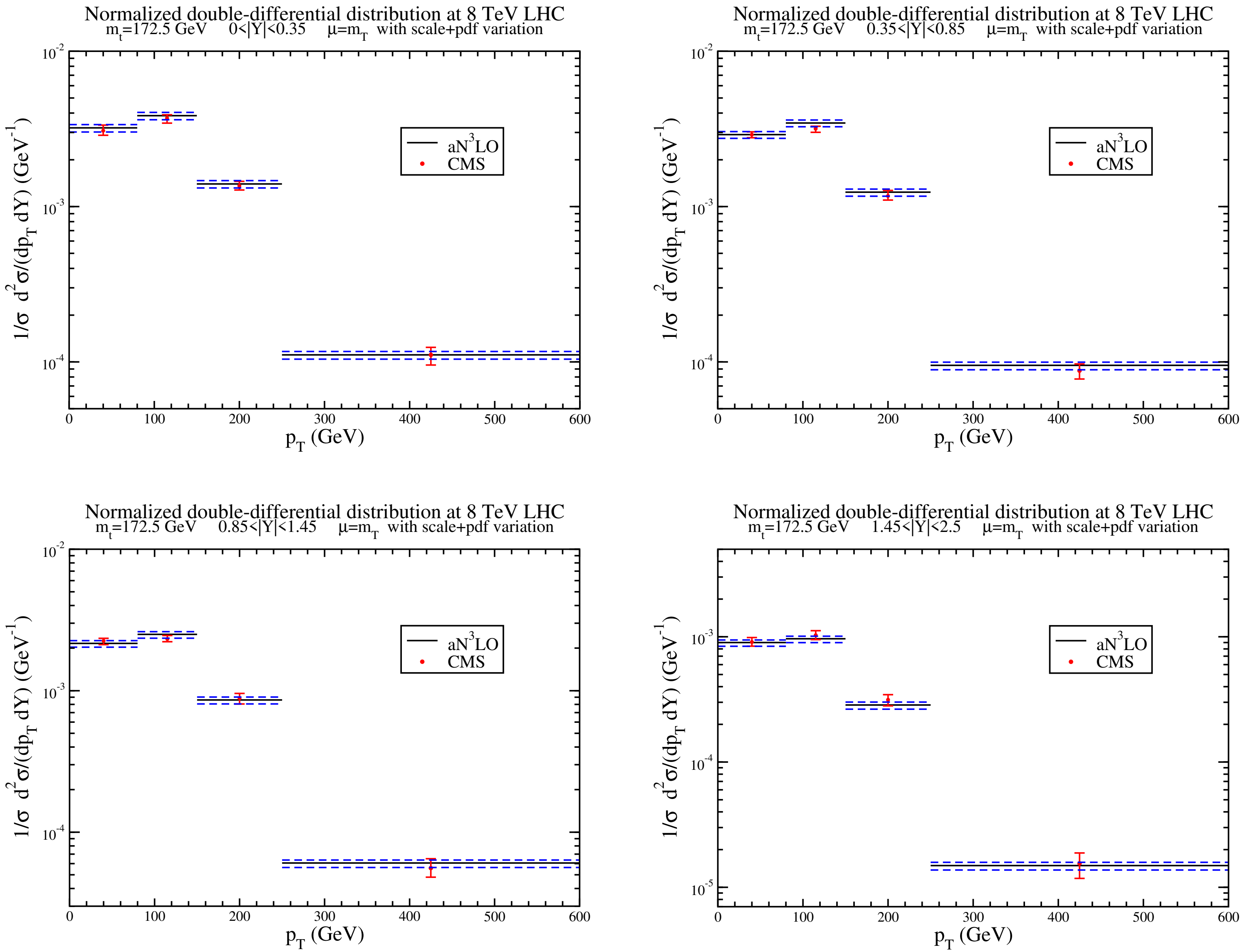
At three loops for $gg \to t\bar{t}$

$$\Gamma_{S\,22}^{(3)gg\to t\bar{t}} = K_3\, \Gamma_{S\,22}^{(1)gg\to t\bar{t}} + \left(1 - \frac{C_A}{2C_F}\right) \left(\Gamma_{\rm cusp}^{(3)\,\beta} - K_3 \Gamma_{\rm cusp}^{(1)\,\beta}\right) + \frac{K_2}{2} C_A^2 (1-\zeta_3)$$
$$+ C_A^3 \left(-\frac{1}{4} + \frac{3}{8}\zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8}\zeta_2\zeta_3 + \frac{9}{16}\zeta_5\right) + X_{S\,22}^{(3)gg\to t\bar{t}}$$

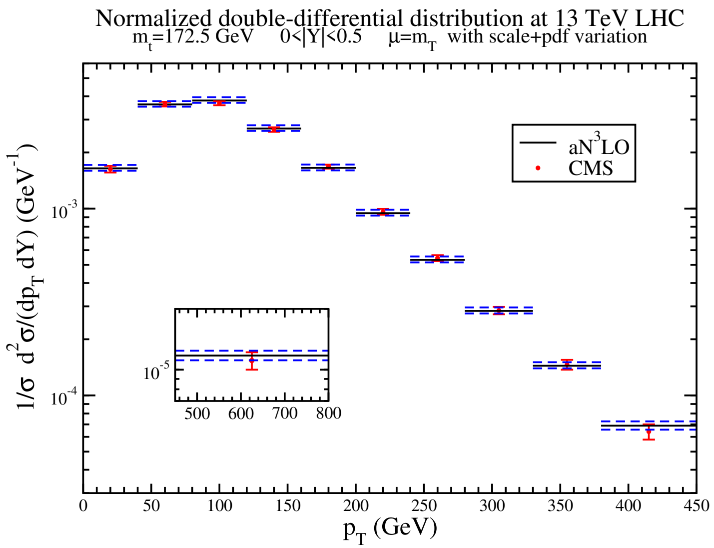
where $X_{S\,22}^{(3)gg\to t\bar{t}}$ denotes unknown three-loop contributions from four-parton correlations.

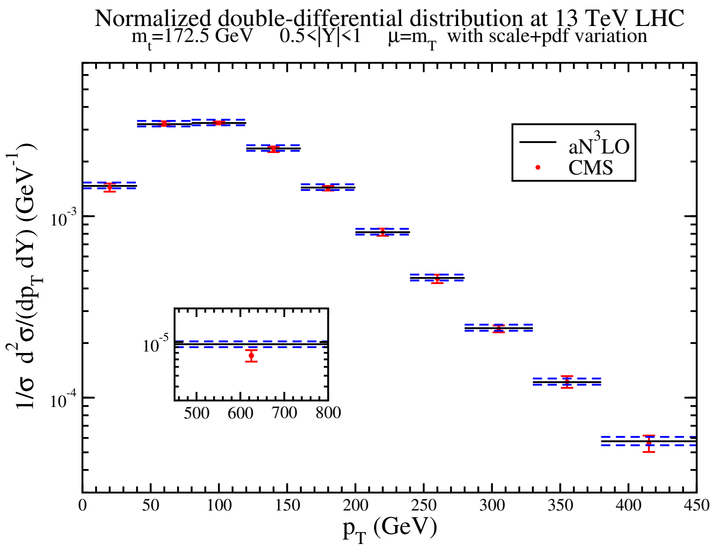
other 3-loop matrix elements not fully known either but have analogous structure to that at two loops

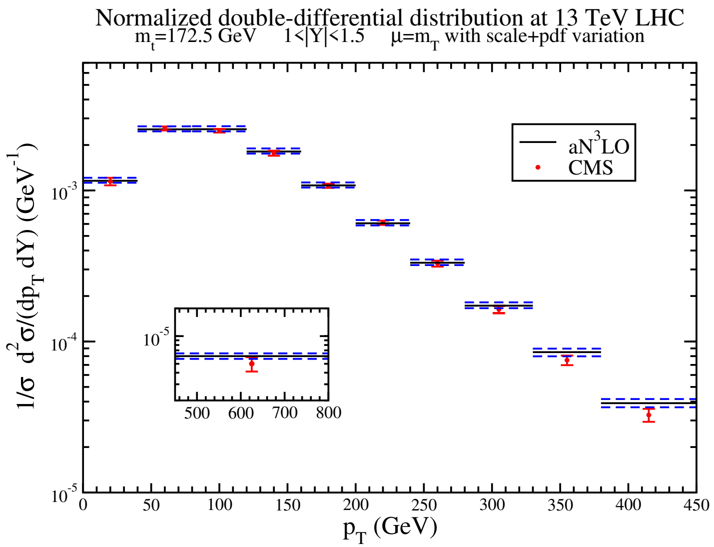
Top double-differential distributions in $t\bar{t}$ production

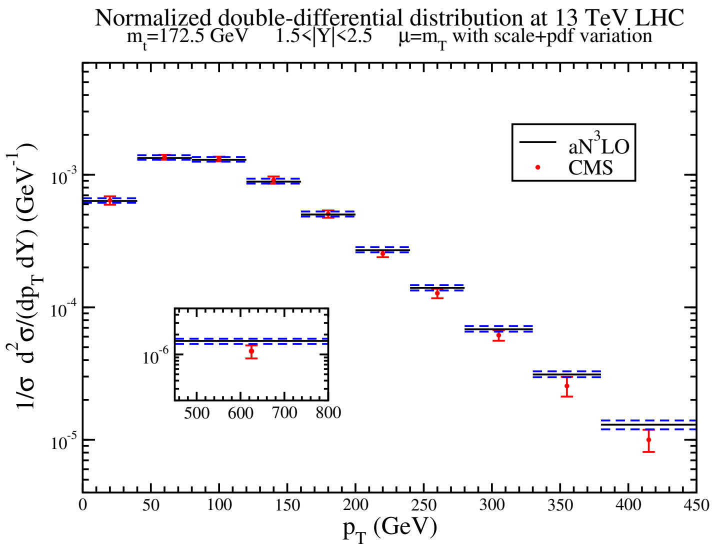


Top double-differential distributions in $t\bar{t}$ production









tqH, tqZ, $tq\gamma$, tqW production

consider $q\bar{q}' \to t\bar{b}H$ **as well as** $q\bar{q}' \to t\bar{b}Z$, $q\bar{q}' \to t\bar{b}\gamma$, $q\bar{q} \to t\bar{b}W^-$, $q\bar{q}' \to t\bar{q}''W^+$

use s**-channel singlet-octet color basis and further define**
$s' = (p_1 + p_2)^2$, $t' = (p_b - p_2)^2$, $u' = (p_a - p_2)^2$
At one loop

$$\Gamma_{S\,11}^{(1)\,q\bar{q}'\to t\bar{b}H} = C_F\left[\ln\left(\frac{s'-m_t^2}{m_t\sqrt{s}}\right)-\frac{1}{2}\right], \Gamma_{S\,12}^{(1)\,q\bar{q}'\to t\bar{b}H} = \frac{C_F}{2N_c}\ln\left(\frac{t'(t-m_t^2)}{u'(u-m_t^2)}\right), \Gamma_{S\,21}^{(1)\,q\bar{q}'\to t\bar{b}H} = \ln\left(\frac{t'(t-m_t^2)}{u'(u-m_t^2)}\right)$$

$$\Gamma_{S\,22}^{(1)\,q\bar{q}'\to t\bar{b}H} = C_F\left[\ln\left(\frac{s'-m_t^2}{m_t\sqrt{s}}\right)-\frac{1}{2}\right]-\frac{1}{N_c}\ln\left(\frac{t'(t-m_t^2)}{u'(u-m_t^2)}\right)+\frac{N_c}{2}\ln\left(\frac{t'(t-m_t^2)}{s(s'-m_t^2)}\right)$$

Two-loop and three-loop result structure as in s**-channel single top**

also consider processes $bq \to tq'H$ **as well as** $bq \to tq'Z$, $bq \to tq'\gamma$, $bq \to tqW^-$,
$qq \to tq'W^+$, **and use** t**-channel singlet-octet color basis**
At one loop

$$\Gamma_{S\,11}^{(1)\,bq\to tq'H} = C_F\left[\ln\left(\frac{t'(t-m_t^2)}{m_t s^{3/2}}\right)-\frac{1}{2}\right], \Gamma_{S\,12}^{(1)\,bq\to tq'H} = \frac{C_F}{2N_c}\ln\left(\frac{u'(u-m_t^2)}{s(s'-m_t^2)}\right), \Gamma_{S\,21}^{(1)\,bq\to tq'H} = \ln\left(\frac{u'(u-m_t^2)}{s(s'-m_t^2)}\right)$$

$$\Gamma_{S\,22}^{(1)\,bq\to tq'H} = C_F\left[\ln\left(\frac{t'(t-m_t^2)}{m_t s^{3/2}}\right)-\frac{1}{2}\right]-\frac{1}{N_c}\ln\left(\frac{u'(u-m_t^2)}{s(s'-m_t^2)}\right)+\frac{N_c}{2}\ln\left(\frac{u'(u-m_t^2)}{t'(t-m_t^2)}\right)$$

Two-loop and three-loop result structure as in t**-channel single top**

Summary

- **soft anomalous dimensions through three loops**

- Γ_S **for processes with** W**,** Z**,** γ**,** H

- **single-top production**

- **top-antitop pair production**

- $2 \to 3$ **processes involving tops**

- **soft-gluon corrections through aN^{3}LO are very significant**