Complete Analytic Four-Loop Cusp and Collinear Anomalous Dimensions

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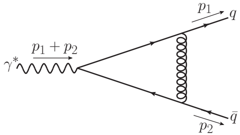
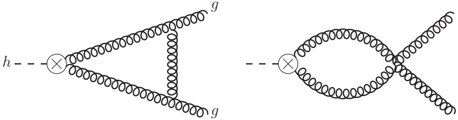
The Basic Quark and Gluon Form Factors

- In this talk, I focus on the basic $q\bar{q}\gamma^*$ and ggH vertices
- consider perturbative expansion, *e.g.* at 1-loop:





- calculate form factors through 4 loop QCD in dim. reg. ($d = 4 - 2\epsilon$)
- they provide virtual corrections to Drell-Yan and Higgs production at N^4LO
- after UV renormalization, the amplitudes contain soft and collinear poles in ϵ
- leading poles through to $1/\epsilon^2$ predicted by cusp anomalous dimension Γ_{cusp}
- $1/\epsilon$ pole involves collinear anomalous dimensions

Evolution Equation for the Form Factor

- The evolution of the form factor $\mathcal{F}$ can be written as [Magnea, Sterman 1990]

$$Q^2 \frac{\partial}{\partial Q^2} \ln \mathcal{F}(\alpha_s, Q^2/\mu^2, \epsilon) = \frac{1}{2} K(\alpha_s, \epsilon) + \frac{1}{2} G(Q^2/\mu^2, \alpha_s, \epsilon)$$

  where
  - μ is the renormalization scale
  - K contains all infrared poles in ϵ
  - G contains all dependence on Q^2 and is finite for $\epsilon \to 0$
- renormalization scale dependence must cancel between K and G:

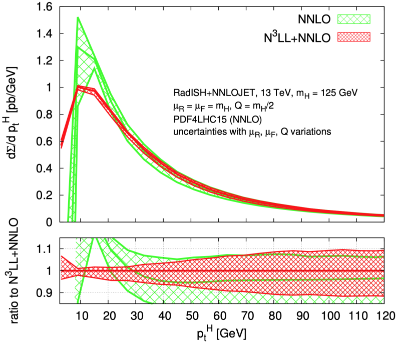
$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s, \epsilon) \frac{\partial}{\partial \alpha_s}\right) G(Q^2/\mu^2, \alpha_s, \epsilon) = \Gamma_{\text{cusp}}(\alpha_s)$$

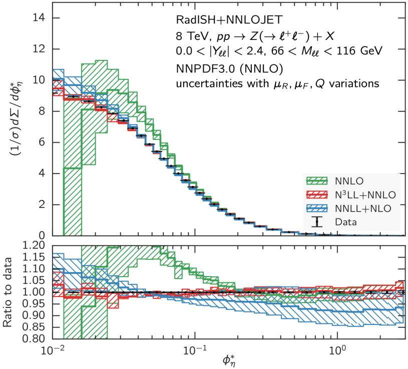
$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s, \epsilon) \frac{\partial}{\partial \alpha_s}\right) K(\alpha_s, \epsilon) = -\Gamma_{\text{cusp}}(\alpha_s)$$

- $\Gamma_{\text{cusp}} = \sum_{l=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^l \Gamma_l$ is the cusp anomalous dimension
- At l loop order, Γ_l encodes the soft poles which are not just the exponentiated poles from lower loops
- Γ_{cusp} universal quantity, appears also in splitting functions and Wilson loops

Distributions @ LHC

[Bizon, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 2018]





- low p_T region requires to supplement fixed-order with resummations
- N^3LL resummation requires cusp anomalous dimension at 4 loops

Color Representation Dependence

- Cusp anomalous dimension Γ^r_{cusp} is representation r dependent (*i.e.* different for quarks and gluons)
- IR poles of general scattering amplitudes follow simple $T_i T_j$ pattern observed through to 2 loops [Catani '98], considered also for higher loops [Becher, Neubert '09; Gardi, Magnea '09] ("dipole conjecture")
- In particular, the cusp anomalous dimension was hoped to fulfill quadratic Casimir scaling $\Gamma^g_{\text{cusp}} \underbrace{=}_{?} C_A/C_F \Gamma^q_{\text{cusp}}$ (observed @ 3 loops by the time the dipole conjecture was proposed)
- As became clear in the following, picture needs to be generalized to account for more complicated Casimir operators, *e.g.* [Caron-Huot '15; Almelid *et. al.* '16; Henn *et. al.* '16; Boels *et. al.* '16; Moch *et. al.* '17, '18; Lee *et. al.* '19; Henn *et. al.* '19; Catani *et. al.* '19; Becher, Neubert '19]
- Upshot: only generalized Casimir scaling for the quark and gluon cusp predicted; coefficients of new Casimir operators for quarks and gluons must coincide (see also [Korchemsky '89])

Status @ 4 loops

- Cusp anomalous dimensions ($1/\epsilon^2$, weight 6):
  - $\mathcal{N} = 4$ SYM non-planar:
    [Boels, Kniehl, Tarasov, Yang '12, '15], [Boels, Huber, Yang '17], [Henn, Korchemsky, Mistlberger '19], [Huber, von Manteuffel, Panzer, RMS, Yang '19]
  - QCD accurate numerical fits for quarks and gluons:
    [Moch, Ruijl, Ueda, Vermaseren, Vogt '17,'18]
  - QCD analytical:
    [Beneke, Braun '95, Grozin, Henn, Korchemsky, Marquard '15, Henn, Smirnov, Smirnov, Steinhauser '16, von Manteuffel, RMS '16, '19, Grozin '18, Lee, Smirnov, Smirnov, Steinhauser '19, Brüser, Grozin, Henn, Stahlhofen '19, Henn, Peraro, Stahlhofen, Wasser '19], [Henn, Korchemsky, Mistlberger '19]
    First complete and conjecture-free analytic results: [von Manteuffel, Panzer, RMS '20]
  - note: various independent approaches (form factors, splitting functions, Wilson lines, . . .)
- Collinear anomalous dimension ($1/\epsilon$, weight 7):
  - $\mathcal{N} = 4$ SYM planar: [Dixon '17]
  - QCD full matter dependence for quarks and gluons: [von Manteuffel, Panzer, RMS '20]
  - QCD accurate numerical fits for quarks and gluons: [Das, Moch, Vogt '19, '20]
  - Complete (conjectural) analytic results in both $\mathcal{N} = 4$ SYM and QCD now available!
    [Agarwal, von Manteuffel, Panzer, RMS '21]
- Quark form factors (ϵ^0):
  - n_f^3, planar, n_f^2, quartic, $n_f n_{q\gamma}$: [Henn, Smirnov, Smirnov, Steinhauser, Lee '16, '16], [von Manteuffel, RMS '16, '19, '20], [Lee, Smirnov, Smirnov, Steinhauser '17,'19]
- Gluon form factors (ϵ^0):
  - n_f^3, n_f^2: [von Manteuffel, RMS '16, '19]

Form Factor Approach
[Andreas von Manteuffel, Erik Panzer, RMS]

Analytical calculation of $q\bar{q}\gamma$ and ggH amplitudes to four-loop QCD:

1. Feynman diagrams (`QGraf`) + numerator algebra (`Form`)
2. reduction to finite master integrals (`Finred`)
3. solve master integrals by direct integration (`HyperInt`)
4. extract anomalous dimensions from poles

Complexity:

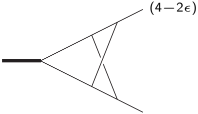
- $O(50000)$ Feynman diagrams
- $O(10^9)$ integrals in diagrams, one scale, up to 6 irreducible scalar products
- 100 different 12-line topologies, 10 integral families (18 indices)
- $O(25000)$ sectors, $O(2000)$ non-shiftable, max $O(10^8 - 10^9)$ equations/sector
- 294 master integrals

Checks:

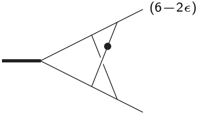
- use R_ξ gauge and show ξ independence
- calculate redundant sets of master integrals, check consistency
- numerical checks of master integrals with `Fiesta`

Part 1: The Method of Finite Integrals

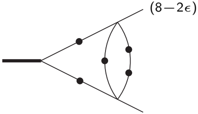
theorem: always possible to decompose wrt basis of finite integrals



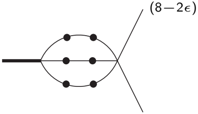
$$= -\frac{4(1-4\epsilon)}{\epsilon(1-\epsilon)q^2}$$



$$-\frac{2(2-3\epsilon)(5-21\epsilon+14\epsilon^2)}{\epsilon^4(1-\epsilon)^2(2-\epsilon)^2q^2}$$



$$+\frac{4(2-3\epsilon)(7-31\epsilon+26\epsilon^2)}{\epsilon^4(1-2\epsilon)(1-\epsilon)^2(2-\epsilon)^2q^2}$$



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basis consists of standard Feynman integrals, but

- in shifted dimensions
- with additional dots (propagators taken to higher powers)

Practical algorithm for basis construction

Algorithm: construction of finite basis

- systematic scan for finite integrals with dim-shifts and dots (with `Reduze 2`)
- IBP + dimensional recurrence for actual basis change

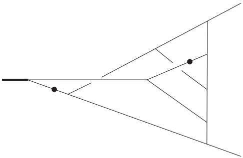
remarks:

- computationally expensive part shifted to IBP solver
- efficient, easy to automate
- any dim-shift good, *e.g.* shifts by [Tarasov '96], [Lee '10]
- see also [Bern, Dixon, Kosower '93] for dim-shifted one-loop integrals

Analytical integration @ 4-loops

[von Manteuffel, Panzer, RMS '15]

a non-planar 12-line topology @ 4-loops:



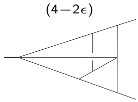
$$= \frac{18}{5}\zeta_2^2\zeta_3 - 5\zeta_2\zeta_5 + \Big(24\zeta_2\zeta_3 + 20\zeta_5 - \frac{188}{105}\zeta_2^3 - 17\zeta_3^2 + 9\zeta_2^2\zeta_3$$

$$- 47\zeta_2\zeta_5 - 21\zeta_7 + \frac{6883}{2100}\zeta_2^4 + \frac{49}{2}\zeta_2\zeta_3^2 + \frac{1}{2}\zeta_3\zeta_5 - 9\zeta_{5,3}\Big)\epsilon + \mathcal{O}\left(\epsilon^2\right)$$

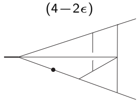
- numerical result with `Fiesta` [A. Smirnov]: straightforward confirmation

Weight Drops in a Conventional Basis

- consider sector with a conventional choice for the 2 master integrals:



$$= \frac{1}{\epsilon^8}\left(\frac{1}{144}\right) + \frac{1}{\epsilon^6}\left(-\frac{1}{24}\zeta_2\right) + \frac{1}{\epsilon^5}\left(-\frac{29}{24}\zeta_3\right) + \frac{1}{\epsilon^4}\left(-\frac{71}{16}\zeta_2^2\right) + \frac{1}{\epsilon^3}\left(-\frac{1819}{24}\zeta_5 + \frac{23}{6}\zeta_2\zeta_3\right)$$
$$+ \frac{1}{\epsilon^2}\left(\frac{1285}{24}\zeta_3^2 - \frac{80579}{1008}\zeta_2^3\right) + \frac{1}{\epsilon}\left(-\frac{434203}{192}\zeta_7 + \frac{7139}{24}\zeta_2\zeta_5 + \frac{54139}{120}\zeta_2^2\zeta_3\right)$$
$$+ \frac{2023}{12}\zeta_{5,3} + \frac{30581}{4}\zeta_3\zeta_5 + \frac{6829}{24}\zeta_2\zeta_3^2 - \frac{45893321}{100800}\zeta_2^4 + \mathcal{O}(\epsilon) \quad \text{and}$$

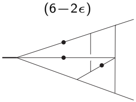


$$= \frac{1}{\epsilon^8}\left(-\frac{1}{144}\right) + \frac{1}{\epsilon^7}\left(-\frac{1}{12}\right) + \frac{1}{\epsilon^6}\left(\frac{1}{24}\zeta_2 - \frac{7}{36}\right) + \frac{1}{\epsilon^5}\left(\frac{29}{24}\zeta_3 + \frac{1}{2}\zeta_2 - \frac{1}{72}\right)$$
$$+ \frac{1}{\epsilon^4}\left(\frac{71}{16}\zeta_2^2 + \frac{29}{2}\zeta_3 + \frac{39}{16}\zeta_2 + \frac{335}{144}\right) + \frac{1}{\epsilon^3}\left(\frac{1819}{24}\zeta_5 - \frac{23}{6}\zeta_2\zeta_3 + \frac{213}{4}\zeta_2^2 + \frac{1211}{24}\zeta_3 + \frac{431}{48}\zeta_2\right.$$
$$\left.+\frac{47}{18}\right) + \frac{1}{\epsilon^2}\left(-\frac{1285}{24}\zeta_3^2 + \frac{80579}{1008}\zeta_2^3 + \frac{1819}{2}\zeta_5 - 46\zeta_2\zeta_3 + \frac{25787}{160}\zeta_2^2 + \frac{417}{8}\zeta_3 - \frac{1175}{48}\zeta_2 - \frac{7277}{72}\right)$$
$$+ \frac{1}{\epsilon}\left(\frac{434203}{192}\zeta_7 - \frac{7139}{24}\zeta_2\zeta_5 - \frac{54139}{120}\zeta_2^2\zeta_3 - \frac{1285}{2}\zeta_3^2 + \frac{80579}{84}\zeta_2^3 + \frac{5571}{2}\zeta_5 - \frac{9005}{24}\zeta_2\zeta_3 + \frac{967}{480}\zeta_2^2\right.$$
$$\left.- \frac{4045}{8}\zeta_3 - \frac{733}{24}\zeta_2 + \frac{57635}{72}\right) - \frac{2023}{12}\zeta_{5,3} - \frac{30581}{4}\zeta_3\zeta_5 - \frac{6829}{24}\zeta_2\zeta_3^2 + \frac{45893321}{100800}\zeta_2^4 + \frac{434203}{16}\zeta_7$$
$$- \frac{7139}{2}\zeta_2\zeta_5 - \frac{54139}{10}\zeta_2^2\zeta_3 - \frac{10706}{3}\zeta_3^2 + \frac{7987951}{3360}\zeta_2^3 + \frac{1309}{12}\zeta_5 - \frac{30317}{24}\zeta_2\zeta_3 - \frac{43847}{96}\zeta_2^2 + \frac{32335}{24}\zeta_3$$
$$+ \frac{2553}{4}\zeta_2 - \frac{334727}{72} + \mathcal{O}(\epsilon)\,.$$

need to compute additional terms of epsilon expansion (weight 9) to obtain cusp (weight 6) :(

Choosing a Good Finite Basis

- better basis choice avoids substantial weight mismatches between basis and physics result
- here, “better” means finite and close to a uniform weight basis [RMS ’18]
- in our example, using a suitable finite integral basis including



$$= -\frac{3}{2}\zeta_3^2 - \frac{4}{3}\zeta_2^3 + 10\zeta_5 + 2\zeta_2\zeta_3 - \frac{1}{5}\zeta_2^2 - 6\zeta_3 + \mathcal{O}(\epsilon)$$

  we need only 1 term (weight 6) for the cusp (weight 6)

- in many cases, our choice of basis allows us to avoid the calculation of complicated topologies altogether (more on this later)

Part 2: Physics Results

RESULTS: 4 LOOP CUSP ANOMALOUS IN $\mathcal{N} = 4$ SYM

- [Boels, Huber, Yang '17] found a compact representation for the $\mathcal{N} = 4$ Sudakov form factor:

$$F^{(4)} = 2\Big[8I^{(1)}_{\mathrm{p},1} + 2I^{(2)}_{\mathrm{p},2} - 2I^{(3)}_{\mathrm{p},3} + 2I^{(4)}_{\mathrm{p},4} + \frac{1}{2}I^{(5)}_{\mathrm{p},5} + 2I^{(6)}_{\mathrm{p},6} + 4I^{(7)}_{\mathrm{p},7} + 2I^{(9)}_{\mathrm{p},8} - 2I^{(10)}_{\mathrm{p},9} + I^{(12)}_{\mathrm{p},10}$$
$$+ I^{(12)}_{\mathrm{p},11} + 2I^{(13)}_{\mathrm{p},12} + 2I^{(14)}_{\mathrm{p},13} - 2I^{(17)}_{\mathrm{p},14} + 2I^{(17)}_{\mathrm{p},15} - 2I^{(19)}_{\mathrm{p},16} + I^{(19)}_{\mathrm{p},17} + I^{(21)}_{\mathrm{p},18} + \frac{1}{2}I^{(25)}_{\mathrm{p},19} + 2I^{(30)}_{\mathrm{p},20} + 2I^{(13)}_{\mathrm{p},21}$$
$$+ 4I^{(14)}_{\mathrm{p},22} - 2I^{(14)}_{\mathrm{p},23} - I^{(14)}_{\mathrm{p},24} + 4I^{(17)}_{\mathrm{p},25} - I^{(17)}_{\mathrm{p},26} - 2I^{(17)}_{\mathrm{p},27} - 2I^{(17)}_{\mathrm{p},28} - I^{(19)}_{\mathrm{p},29} - I^{(19)}_{\mathrm{p},30} + I^{(19)}_{\mathrm{p},31} - \frac{1}{2}I^{(30)}_{\mathrm{p},32}\Big]$$
$$+ \frac{48}{N_c^2}\Big[\frac{1}{2}I^{(21)}_{1} + \frac{1}{2}I^{(22)}_{2} + \frac{1}{2}I^{(23)}_{3} - I^{(24)}_{4} + \frac{1}{4}I^{(25)}_{5} - \frac{1}{4}I^{(26)}_{6} - \frac{1}{4}I^{(26)}_{7} + 2I^{(27)}_{8} + I^{(28)}_{9}$$
$$+ 4I^{(29)}_{10} + I^{(30)}_{11} + I^{(27)}_{12} - \frac{1}{2}I^{(28)}_{13} + I^{(29)}_{14} + I^{(29)}_{15} + I^{(30)}_{16} + I^{(30)}_{17} + I^{(30)}_{18} + I^{(22)}_{19} + I^{(22)}_{20}$$
$$- I^{(24)}_{21} + \frac{1}{4}I^{(24)}_{22} + \frac{1}{2}I^{(28)}_{23}\Big], \quad \text{(master integrals } I \text{ conjectured to be uniform weight)}$$

- calculation of ϵ expansion suggests I and $F^{(4)}$ are UT [Huber, von Manteuffel, Panzer, RMS, Yang '19]:

$$F^{(4)} = \frac{1}{\epsilon^8}\left(\frac{2}{3}\right) + \frac{1}{\epsilon^6}\left(\frac{2}{3}\zeta_2\right) + \frac{1}{\epsilon^5}\left(-\frac{38}{9}\zeta_3\right) + \frac{1}{\epsilon^4}\left(\frac{5}{18}\zeta_2^2\right) + \frac{1}{\epsilon^3}\left(\frac{1082}{15}\zeta_5 + \frac{23}{3}\zeta_3\zeta_2\right)$$
$$+ \frac{1}{\epsilon^2}\left(\frac{10853}{54}\zeta_3^2 + \frac{95477}{945}\zeta_2^3\right) + \frac{1}{N_c^2}\left[\frac{1}{\epsilon^2}\left(18\zeta_3^2 + \frac{372}{35}\zeta_2^3\right)\right] + \mathcal{O}\left(\epsilon^{-1}\right).$$

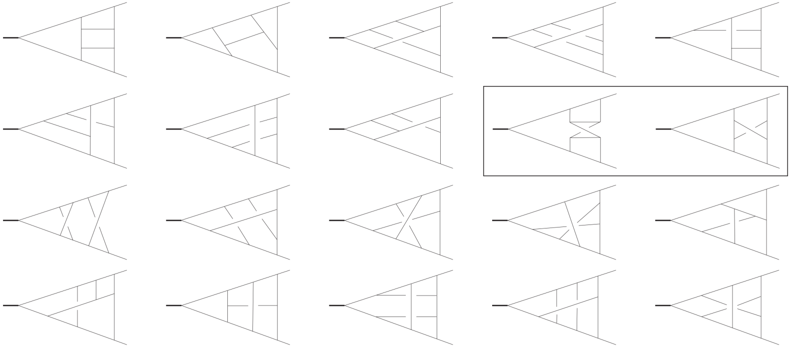
- this results in the following cusp anomalous dimension:

$$\Gamma_4^{\mathcal{N}=4} = \frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-384\zeta_3^2 - \frac{7936}{35}\zeta_2^3\right) + C_A^4\left(-16\zeta_3^2 - \frac{20032}{105}\zeta_2^3\right),$$

[Henn, Korchemsky, Mistlberger '19], [Huber, von Manteuffel, Panzer, RMS, Yang '19]

Results: Topologies for QCD
[von Manteuffel, Panzer, RMS '20]

irreducible twelve-propagators topologies:



⇐ top-level topologies contributing to Γ_4^q and Γ_4^g

- before reduction, total of 100 twelve-propagator topologies
- first two rows are linearly reducible
- last two rows not linearly reducible out of the box
- use variable changes to render non-linearly reducible topologies linearly reducible
- hardest topologies do not contribute to cusp due to our basis choice

Results: Reduced QCD Form Factors

[von Manteuffel, Panzer, RMS '20]

- quark form factor:

$$\bar{\mathcal{F}}_4^q(\epsilon) = N_f^3 C_F\, c_1^q(\epsilon) + N_f^2 C_A C_F\, c_2^q(\epsilon) + N_f^2 C_F^2\, c_3^q(\epsilon) + N_{q\gamma} N_f \frac{d_F^{abc} d_F^{abc}}{N_F}\, c_4^q(\epsilon) + N_f \frac{d_F^{abcd} d_F^{abcd}}{N_F}\, c_5^q(\epsilon)$$
$$+ N_f C_A^2 C_F\, c_6^q(\epsilon) + N_f C_A C_F^2\, c_7^q(\epsilon) + N_f C_F^3\, c_8^q(\epsilon) + N_{q\gamma} C_A \frac{d_F^{abc} d_F^{abc}}{N_F}\, c_9^q(\epsilon) + N_{q\gamma} C_F \frac{d_F^{abc} d_F^{abc}}{N_F}\, c_{10}^q(\epsilon)$$
$$+ C_A^3 C_F\, c_{11}^q(\epsilon) + C_A^2 C_F^2\, c_{12}^q(\epsilon) + C_A C_F^3\, c_{13}^q(\epsilon) + C_F^4\, c_{14}^q(\epsilon) + \frac{d_A^{abcd} d_F^{abcd}}{N_F}\, c_{15}^q(\epsilon),$$

- gluon form factor:

$$\bar{\mathcal{F}}_4^g(\epsilon) = N_f^3 C_A\, c_1^g(\epsilon) + N_f^3 C_F\, c_2^g(\epsilon) + N_f^2 C_A^2\, c_3^g(\epsilon) + N_f^2 C_A C_F\, c_4^g(\epsilon) + N_f^2 C_F^2\, c_5^g(\epsilon) + N_f^2 \frac{d_F^{abcd} d_F^{abcd}}{N_A}\, c_6^g(\epsilon)$$
$$+ N_f C_A^3\, c_7^g(\epsilon) + N_f C_A^2 C_F\, c_8^g(\epsilon) + N_f C_A C_F^2\, c_9^g(\epsilon) + N_f C_F^3\, c_{10}^g(\epsilon) + N_f \frac{d_A^{abcd} d_F^{abcd}}{N_A}\, c_{11}^g(\epsilon)$$
$$+ C_A^4\, c_{12}^g(\epsilon) + \frac{d_A^{abcd} d_A^{abcd}}{N_A}\, c_{13}^g(\epsilon).$$

Results: 4 loop cusp anomalous dimensions in QCD

[Henn, Korchemsky, Mistlberger '19], [von Manteuffel, Panzer, RMS '20]

$$\begin{aligned}
\Gamma_4^r = {} & N_f^3 C_R \left(\frac{64}{27}\zeta_3 - \frac{32}{81}\right) \\
& + N_f^2 C_A C_R \left(-\frac{224}{15}\zeta_2^2 + \frac{2240}{27}\zeta_3 - \frac{608}{81}\zeta_2 + \frac{923}{81}\right) \\
& + N_f^2 C_F C_R \left(\frac{64}{5}\zeta_2^2 - \frac{640}{9}\zeta_3 + \frac{2392}{81}\right) \\
& + N_f C_A^2 C_R \left(\frac{2096}{9}\zeta_5 + \frac{448}{3}\zeta_3\zeta_2 - \frac{352}{15}\zeta_2^2 - \frac{23104}{27}\zeta_3 + \frac{20320}{81}\zeta_2 - \frac{24137}{81}\right) \\
& + N_f C_A C_F C_R \left(160\zeta_5 - 128\zeta_3\zeta_2 - \frac{352}{5}\zeta_2^2 + \frac{3712}{9}\zeta_3 + \frac{440}{3}\zeta_2 - \frac{34066}{81}\right) \\
& + N_f C_F^2 C_R \left(-320\zeta_5 + \frac{592}{3}\zeta_3 + \frac{572}{9}\right) \\
& + N_f \frac{d_F^{abcd} d_R^{abcd}}{N_R} \left(-\frac{1280}{3}\zeta_5 - \frac{256}{3}\zeta_3 + 256\zeta_2\right) \\
& + \frac{d_A^{abcd} d_R^{abcd}}{N_R} \left(-384\zeta_3^2 - \frac{7936}{35}\zeta_2^3 + \frac{3520}{3}\zeta_5 + \frac{128}{3}\zeta_3 - 128\zeta_2\right) \\
& + C_A^3 C_R \left(-16\zeta_3^2 - \frac{20032}{105}\zeta_2^3 - \frac{3608}{9}\zeta_5 - \frac{352}{3}\zeta_3\zeta_2 + \frac{3608}{5}\zeta_2^2 + \frac{20944}{27}\zeta_3 - \frac{88400}{81}\zeta_2 + \frac{84278}{81}\right)
\end{aligned}$$

where $R = F, A$ for $r = q, g$. Note the quartic Casimir (dd) contributions.

Results: 4 loop collinear anomalous dimensions in QCD

[von Manteuffel, Panzer, RMS '20], [Agarwal, von Manteuffel, Panzer, RMS '21]

$$\gamma_4^q = \frac{d_A^{abcd} d_F^{abcd}}{N_F} \Big[3484\zeta_7 + 1024\zeta_5\zeta_2 - \frac{736}{5}\zeta_3\zeta_2^2 - \frac{3344}{3}\zeta_3^2 + \frac{27808}{315}\zeta_2^3 - \frac{1840}{9}\zeta_5 - 1792\zeta_3\zeta_2 + \frac{224}{15}\zeta_2^2 - \frac{7808}{9}\zeta_3$$
$$- \frac{2176}{3}\zeta_2 + 192 \Big] + C_A^3 C_F \Big[\frac{1648}{3}\zeta_5\zeta_2 - \frac{45511}{6}\zeta_7 - \frac{4132}{15}\zeta_3\zeta_2^2 + \frac{5126}{9}\zeta_3^2 - \frac{77152}{315}\zeta_2^3 + \frac{175166}{27}\zeta_5 + \frac{15400}{9}\zeta_3\zeta_2$$
$$+ \frac{186742}{135}\zeta_2^2 - \frac{1751224}{243}\zeta_3 + \frac{1062149}{729}\zeta_2 + \frac{7179083}{26244} \Big] + C_A^2 C_F^2 \Big[17220\zeta_7 - 4208\zeta_5\zeta_2 - \frac{128}{5}\zeta_3\zeta_2^2 - \frac{14204}{3}\zeta_3^2 - \frac{43976}{35}\zeta_2^3$$
$$+ \frac{10708}{9}\zeta_5 + \frac{4192}{9}\zeta_3\zeta_2 - \frac{48680}{27}\zeta_2^2 + \frac{259324}{27}\zeta_3 - \frac{93542}{27}\zeta_2 + \frac{29639}{18} \Big] + C_A C_F^3 \Big[-21840\zeta_7 + 4128\zeta_5\zeta_2 + \frac{512}{5}\zeta_3\zeta_2^2$$
$$+ 6440\zeta_3^2 + \frac{634376}{315}\zeta_2^3 - 1952\zeta_5 - \frac{3976}{3}\zeta_3\zeta_2 + \frac{8668}{5}\zeta_2^2 - 6520\zeta_3 + 2334\zeta_2 - \frac{2085}{2} \Big] + C_F^4 \Big[11760\zeta_7 - 768\zeta_5\zeta_2$$
$$+ \frac{256}{5}\zeta_3\zeta_2^2 - 2304\zeta_3^2 - \frac{33776}{35}\zeta_2^3 - 5040\zeta_5 - 240\zeta_3\zeta_2 - \frac{1368}{5}\zeta_2^2 + 4008\zeta_3 - 900\zeta_2 + \frac{4873}{12} \Big] + N_f\text{-terms}$$

$$\gamma_4^g = \frac{d_A^{abcd} d_A^{abcd}}{N_A} \Big[3484\zeta_7 + 1024\zeta_5\zeta_2 - \frac{736}{5}\zeta_3\zeta_2^2 - \frac{3344}{3}\zeta_3^2 + \frac{39776}{315}\zeta_2^3 + \frac{2720}{9}\zeta_5 - 2336\zeta_3\zeta_2 - \frac{1808}{15}\zeta_2^2 - \frac{12512}{9}\zeta_3$$
$$+ 64\zeta_2 + \frac{128}{9} \Big] + C_A^4 \Big[-\frac{2671}{6}\zeta_7 - \frac{2212}{15}\zeta_3\zeta_2^2 - \frac{896}{3}\zeta_5\zeta_2 - \frac{286}{9}\zeta_3^2 - \frac{674696}{945}\zeta_2^3 + \frac{19232}{27}\zeta_5 + \frac{1588}{3}\zeta_3\zeta_2$$
$$+ \frac{249448}{135}\zeta_2^2 + \frac{36380}{243}\zeta_3 - \frac{1051411}{729}\zeta_2 + \frac{10672040}{6561} \Big] + N_f\text{-terms}$$

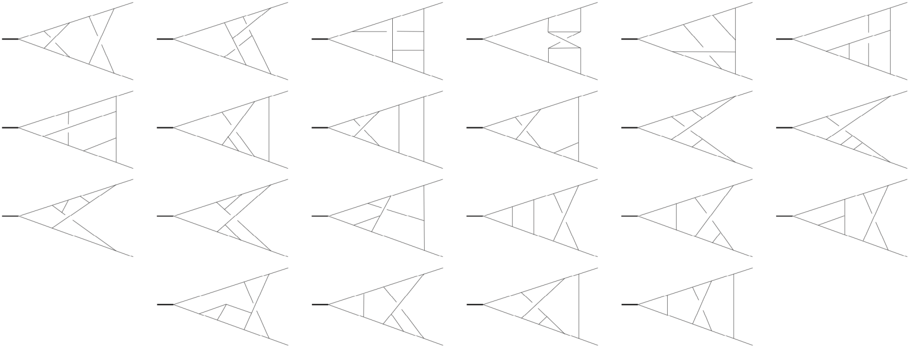
pySecDec [Borowka, Heinrich, Jahn, Jones, Kerner '17] played a crucial role in our analysis, as its quasi-Monte Carlo integrator allowed us to evaluate a single, missing finite integral coefficient to such high precision (ten digits) that we could PSLQ its maximal weight (seven) part.

Results: 4 loop form factors in QCD

- all planar master integrals available: [Henn, Smirnov, Smirnov, Steinhauser '16], [von Manteuffel, RMS '19]
- form factor with most non-planar topologies so far: n_f^2 terms in ggH (163 master integrals): result in terms of multiple zeta values [von Manteuffel, RMS '19]

TABLE I. Complexity of various form factor contributions.

	$\bar{\mathcal{F}}_4^q\vert_{N_f^2}$	$\bar{\mathcal{F}}_4^q\vert_{N_{q\gamma}N_f}$	$\bar{\mathcal{F}}_4^g\vert_{N_f^2}$
# diagrams	71	226	2554
# planar twelve-line top.	0	4	21
# nonplanar twelve-line top.	0	3	19
# nonequivalent top. in red.	158	923	1781
# integrals in amp. ($\xi \neq 1$)	$\mathcal{O}(10^5)$	$\mathcal{O}(10^6)$	$\mathcal{O}(10^7)$
max. # inverse propagators	5	5	6



Conclusions

method of finite integrals:

- simple and efficient method for singularity resolution in multi-loop integrals
- analytical integrations: finite basis integrals from dim-shifted, dotted Feynman integrals
- numerical integrations: much faster and more stable evaluations than before, ideal for both `Fiesta` and `pySecDec` runs

4 loop results:

- complete *ab initio* analytic calculation of cusp anomalous dimensions in QCD
- complete conjectural analytic calculation of collinear anomalous dimensions in QCD
- conjectural part of collinear calculation independently confirmed, paper coming soon
- progress towards finite parts of virtual amplitudes: full matter dependence coming soon, significant additional computer time for complete analytic results [Henn, Smirnov, Smirnov '13]