Importing pertubative QCD methods into gravitational wave physics

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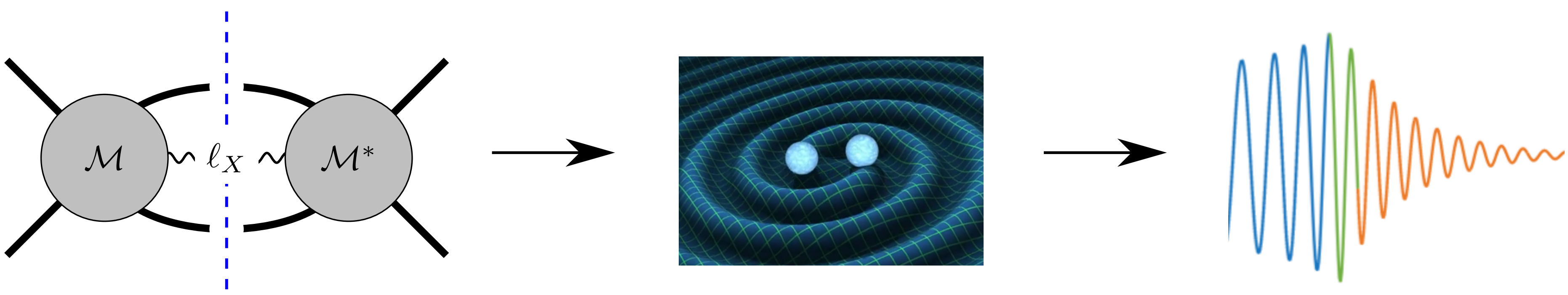
Talk at Radcor / LoopFest 2021

arXiv:2005.04236 (JHEP), Parra-Martinez, Ruf, MZ
arXiv:2101.07254 (PRL), Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, MZ

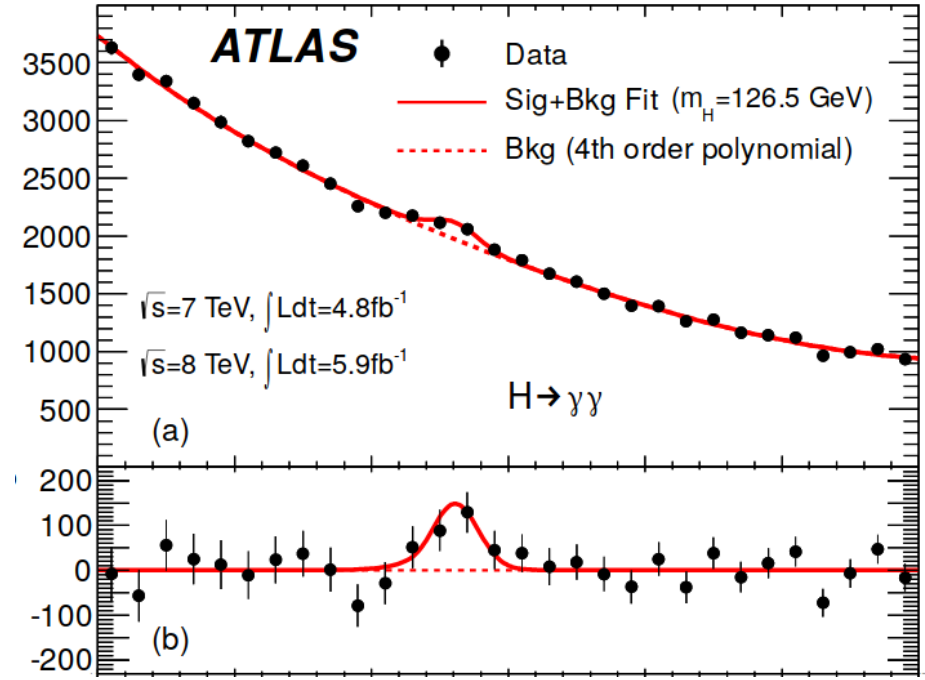


OUTLINE

1. Background - Precision GW Physics
2. The Amplitudes Approach
3. Collider methods - method of regions, differential equations, reverse unitarity...
4. Results & comparisons with numerical relativity

Background

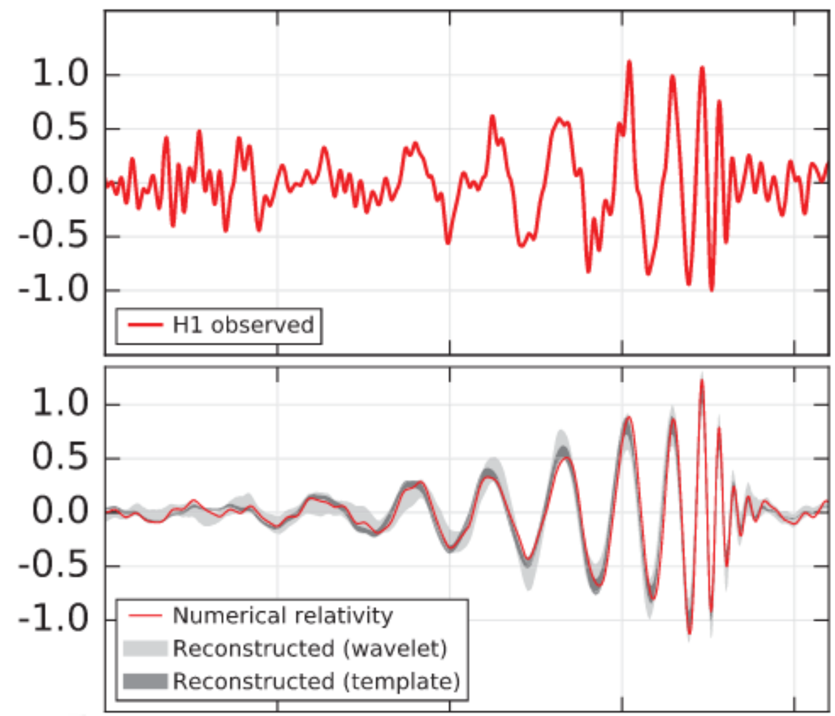
DISCOVERIES OF OUR TIMES



ATLAS Collaboration, arXiv:1207.7214

Two of the fundamental discoveries of our times: Higgs boson (2012), **gravitational waves (2015).** Spectacular confirmation of SM / GR.

DISCOVERIES OF OUR TIMES



LIGO & VIRGO collaborations, arXiv:1602.03837

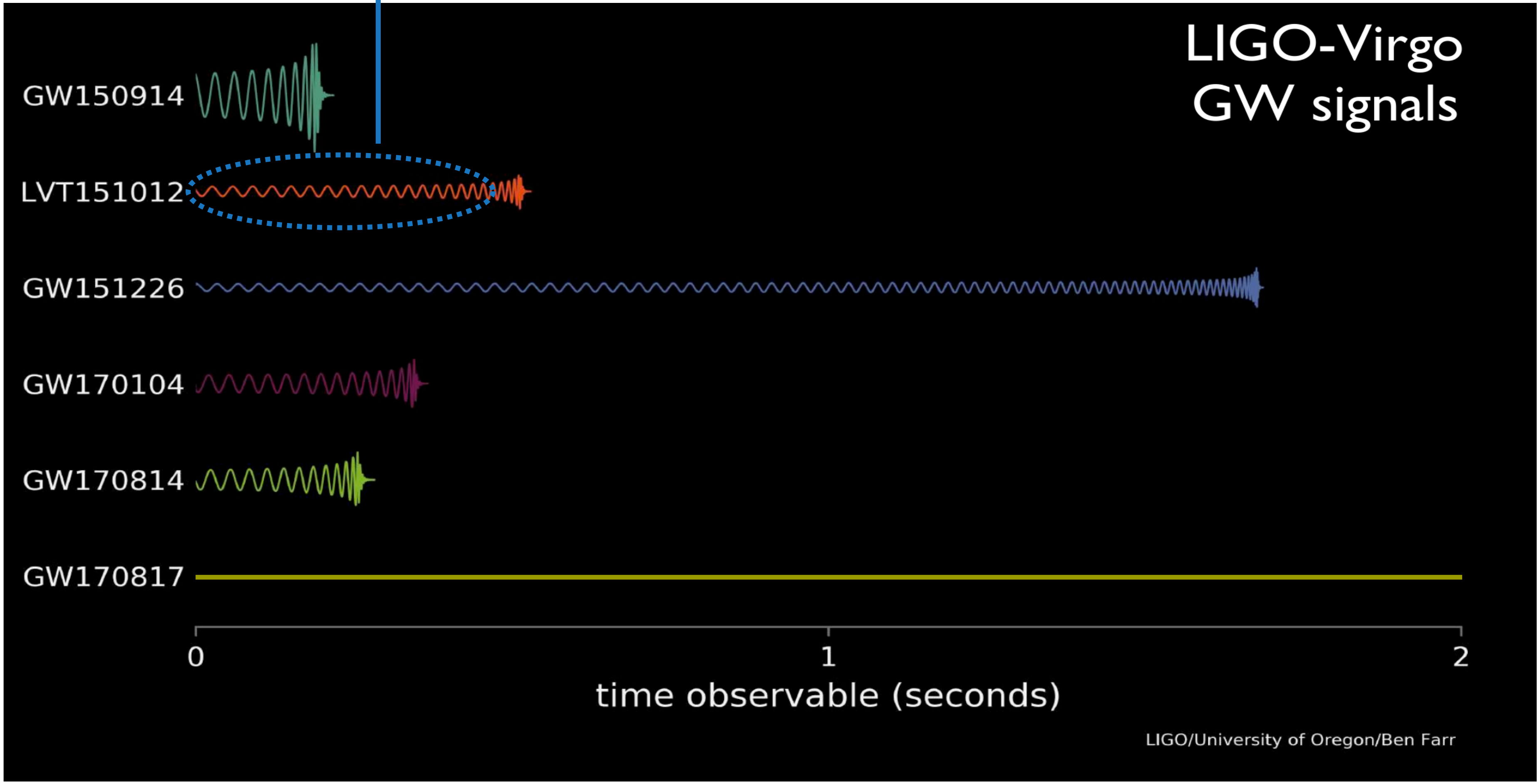
Two of the fundamental discoveries of our times: **Higgs boson (2012)**, gravitational waves (2015). Spectacular confirmation of SM / GR.

- **Intense precision theory efforts** on two types experiments - chance for cross-fertilization?

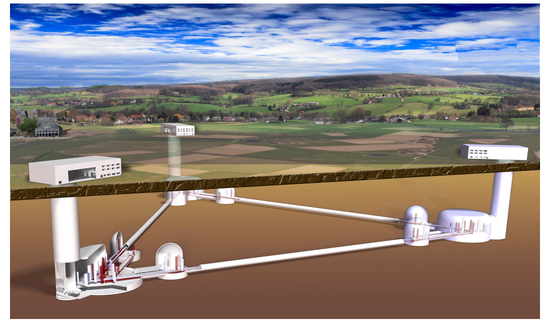
INSPIRAL WAVEFORMS

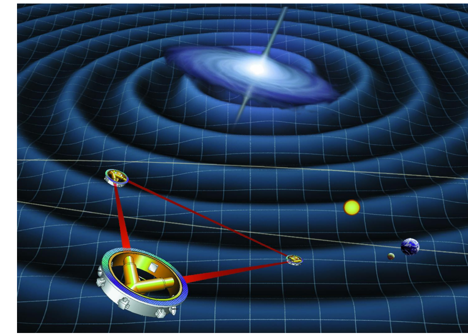
Inspiral → Merger → Ringdown

Weak-field perturbative expansions: Post-Newtonian, **Post-Minkoskian,** Self force, Effective one body...

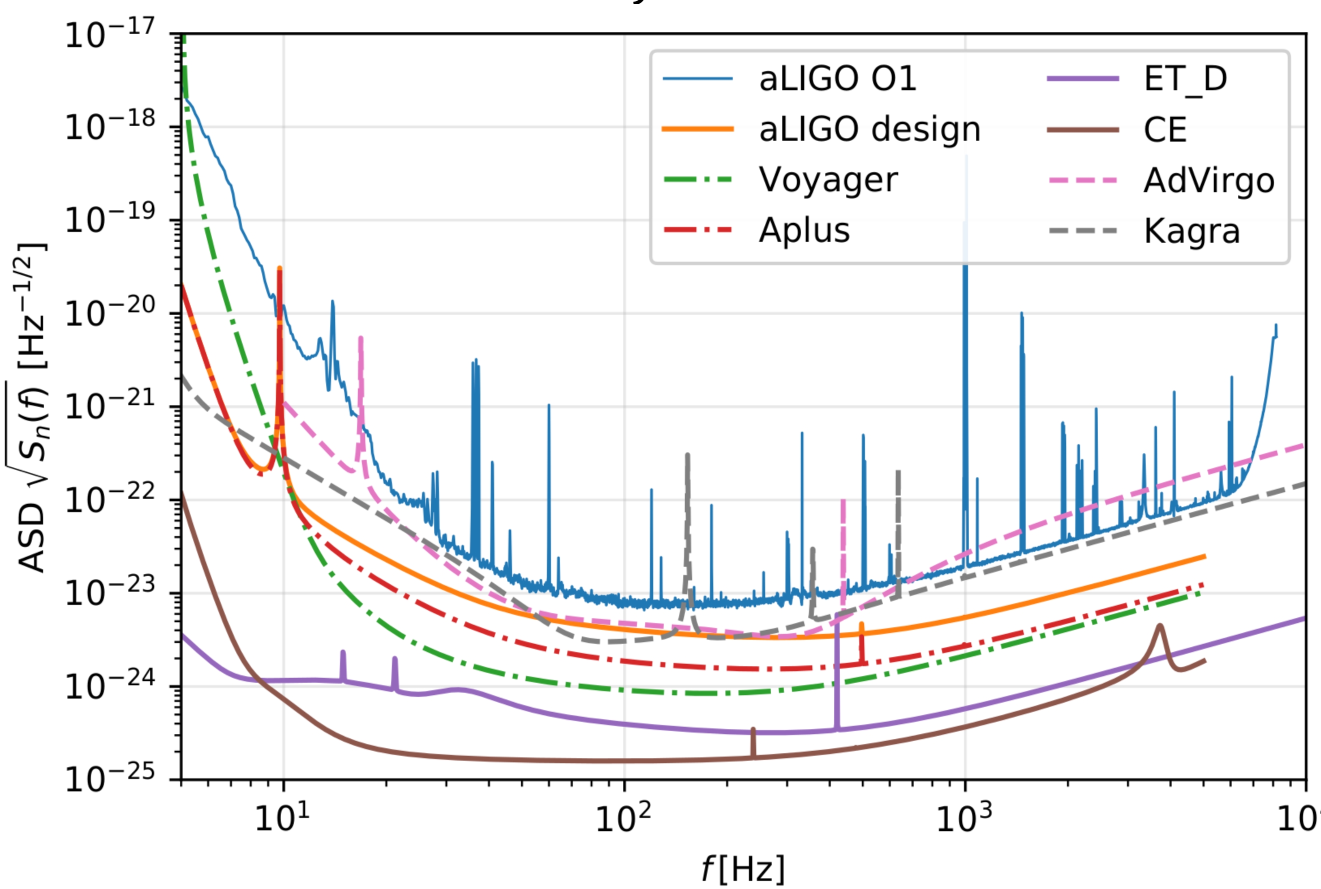


FUTURE GW DETECTORS

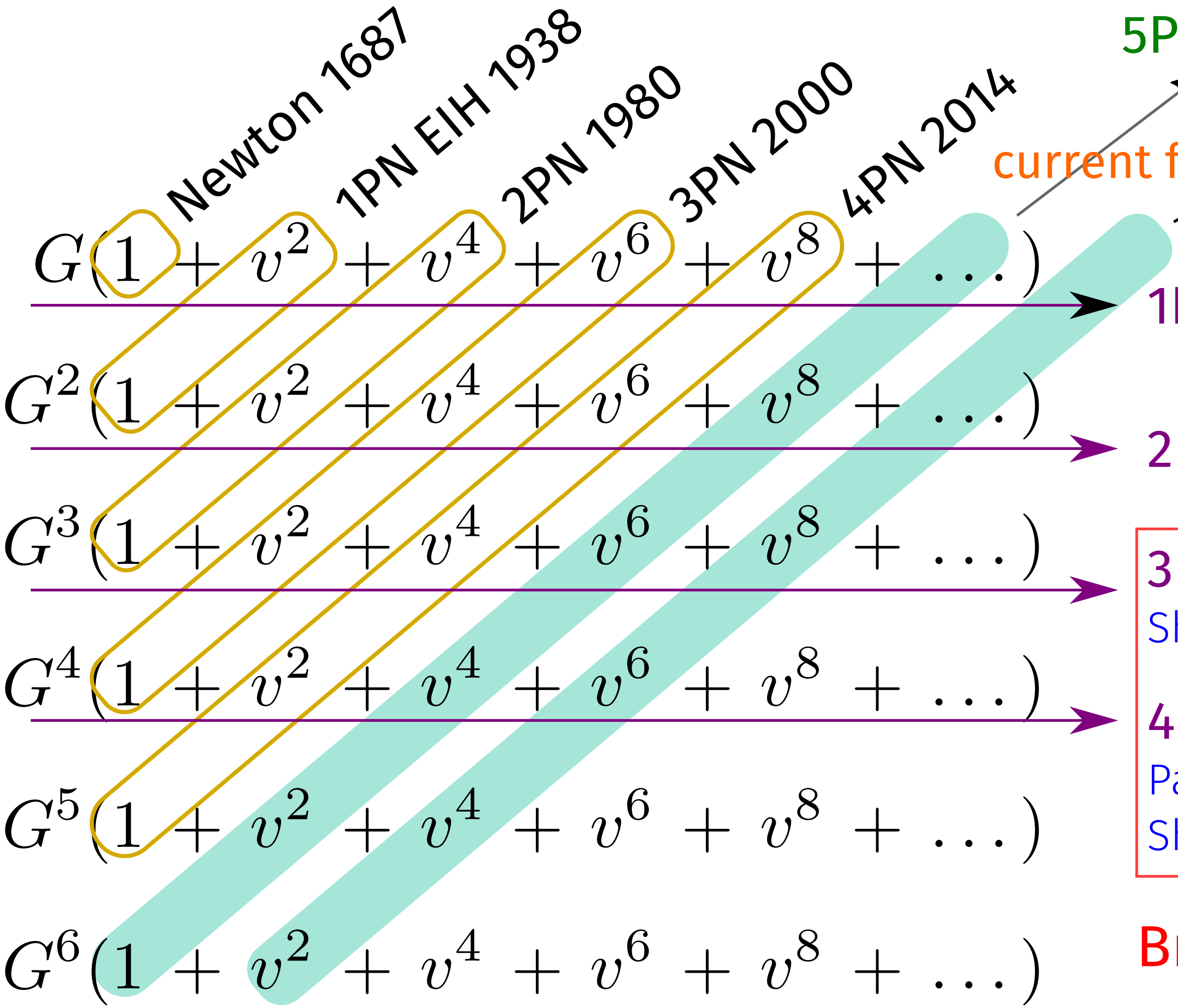




Plot: sensitivity of future GW detectors.



NEW RESULTS FOR CONSERVATIVE DYNAMICS



5PN: distinguish BH / NS

6PN: LISA, ET...

talk by Andreas Maier

1PM

2PM [Westpfahl 1985]

3PM [Bern, Cheung, Roiban, Shen, Solon, **MZ**, 2019]

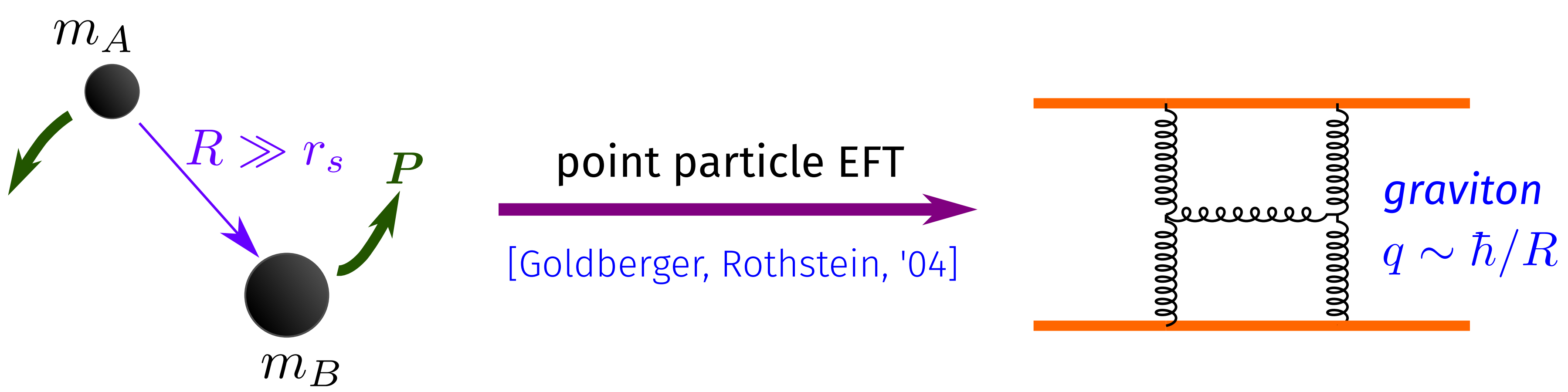
4PM (potential) [Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, **MZ**, 2021]

Breathrough after 30+ year hiatus.

[adapted from Mikhail Solon's slide]

The Amplitudes Approach

POINT PARTICLE EFFECTIVE FIELD THEORY



Massive particles (scalar field) coupled to gravity.

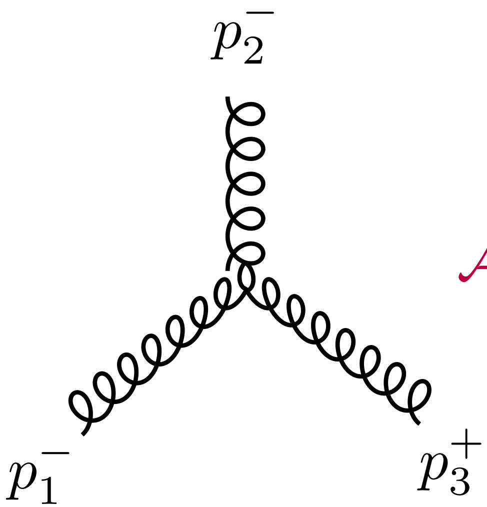
Lagrangian: $S = S_{\text{Einstein-Hilbert}} + S_{\text{point-particle}} + S_{\text{finite-size}}$

Point-particle: scalar field in dynamic metric.

Finite-size (tidal) effect: ***highly suppressed*** for compact objects $\sim \mathcal{O}(G^5)$, even more so for black holes $\sim \mathcal{O}(G^6)$.

GRAVITY AMPLITUDES FROM YANG-MILLS

- Gravity = (Yang-Mills)^2. 3-point amplitude example:



Yang-Mills *Gravity*

$$\mathcal{A}_3(1^-2^-3^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}, \quad \mathcal{M}_3(1^-2^-3^+) = \frac{\langle 12\rangle^6}{\langle 23\rangle^2\langle 31\rangle^2}$$

square!

- 4-points and above: generally a ***sum*** of YM × YM expressions.
  - Early example: *Kawai-Lewellen-Tye relations* (string theory)
  - Local Feynman diagram-like version: *Double copy from color-kinematic duality* [Bern, Carrasco, Johanson, 08]

CLASSICAL LIMITS FROM AMPLITUDES

- **Eikonal exponentiation:** scattering angle from saddle-point / stationary phase approximation.
  [Glauber, '59; Levy, Sucher, '69; Soldate, '87; 't Hooft, '87; Amati, Ciafoloni, Veneziano, '87, '88, '90, '07; Muzinich, Soldate, '88; Kobat, Ortiz, '92; Bern, Ita, Parra-Martinez, Ruf, '20; Parra-Martinez, Ruf, MZ, '20; Di Vecchia, Heissenberg, Russo, Veneziano, '21...]
- **Two-body potential**, defined in an EFT inspired by NRQED / NRQCD, has smooth classical limit.
  [Cheung, Rothstein, Solon, '18]
- Suitable **physical observables** from S-matrix have well-defined classical limits. [Kosower, Maybee, O'Connell, '18]
- Hyperbolic scattering can be **analytically continued** to elliptic orbits. [Kalin, Porto, '20]

MATCHING TO NON-RELATIVISTIC EFT

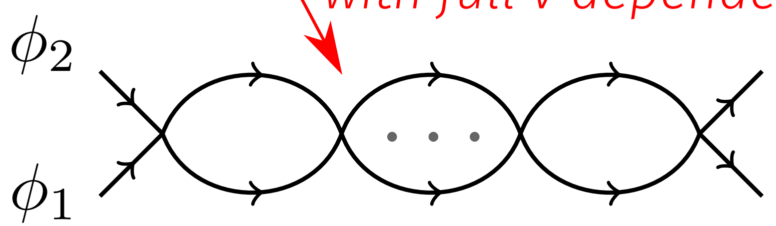
[Cheung, Rothstein, Solon, 1808.02489]

Lagrangian: two scalars, no antiparticles, no particle creation

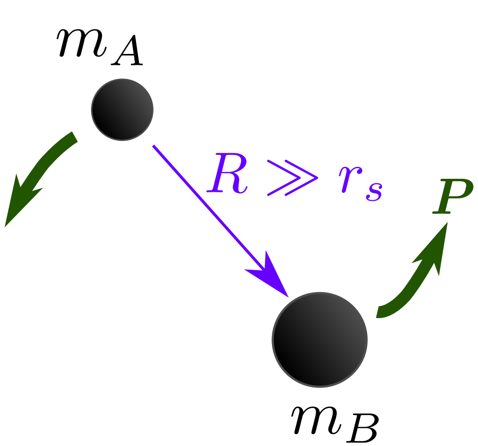
$$\mathcal{L} = \sum_{i=1,2} \int_{\boldsymbol{k}} \phi_i^\dagger(-\boldsymbol{k}) \left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right) \phi_i(\boldsymbol{k}) \quad \longleftarrow \text{kinetic term}$$

$$+ \text{ instantaneous potential } \sim V(\boldsymbol{k}, \boldsymbol{k}')\phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2$$

with full v dependence

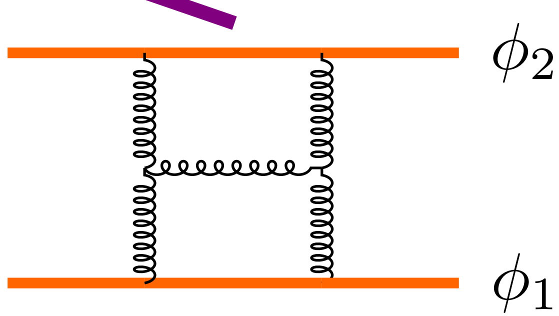


2. non-relativistic EFT



1. point particle EFT

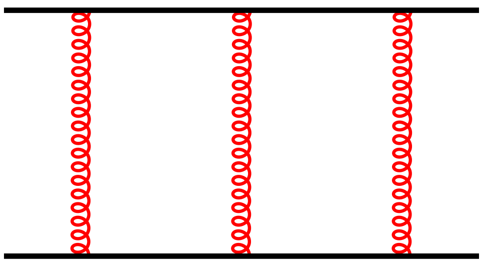
[Goldberger, Rothstein, '04]



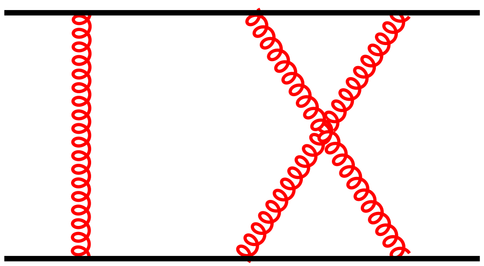
Importing collider methods

CHALLENGES IN LOOP INTEGRATION

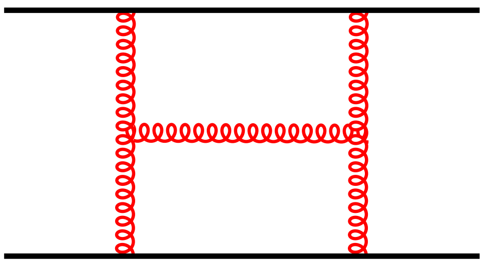
GW physics needs 3 loops and beyond, but exact evaluation is very difficult already at 2 loops: most results are planar, with $m_1 = m_2$.



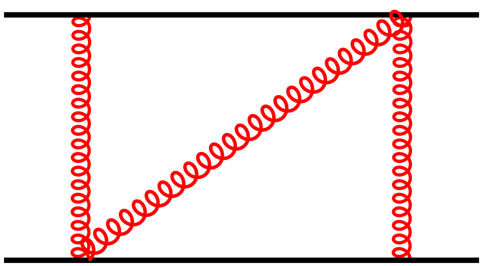
Smirnov, '01;
Henn, Smirnov, '13
Two-mass: Heller '21



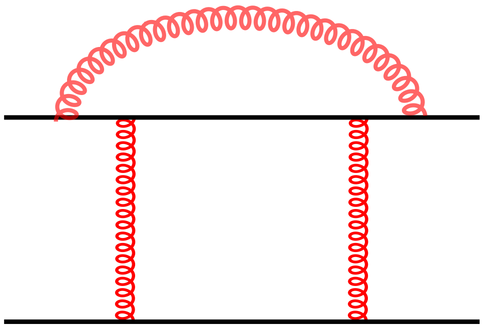
Heinrich, Smirnov, '04
(only the $1/\epsilon^2$ *pole)*



Leoni, Bianchi, '16;
Kreer, Weinzierl, '21
Talk by S. Weinzierl

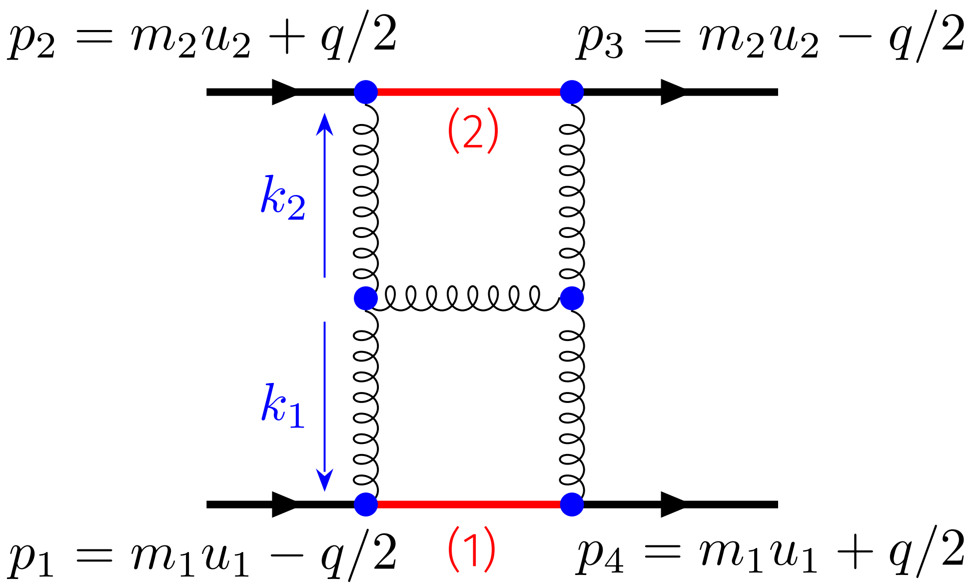


Heller, von Manteuffel, Schabinger, 19;
Broedel, Duhr, Dulat, Penante, Tancredi, '19



Talk by V. Smirnov, w/ Duhr, Tancredi

METHOD OF REGIONS



Soft region:

$$|k_1| \sim |k_2| \sim |q| \ll m_1, m_2, \sqrt{s}$$

Kinematics:

$$u_1^2 = u_2^2 = 1,\ u_1 \cdot q = u_2 \cdot q = 0,$$

$$q^2 = t, \quad u_1 \cdot u_2 = \sigma$$

σ: dimensionless velocity parameter - ***differential equations in σ***

IBP analytic - ***two decoupled systems:*** *integer and half-integer powers of t*

Expanding matter propagators (linearized):

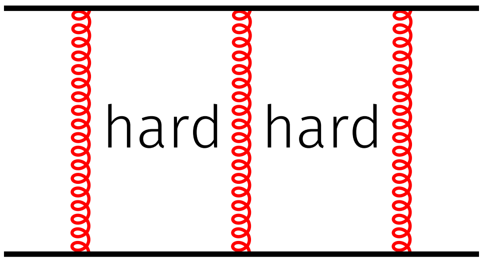
$$(1):\ (k_1 + p_1)^2 - m_1^2 \approx 2 m_1 u_1 \cdot k_1 + i0$$

$$(2):\ (k_2 + p_2)^2 - m_2^2 \approx 2 m_2 u_2 \cdot k_2 + i0$$

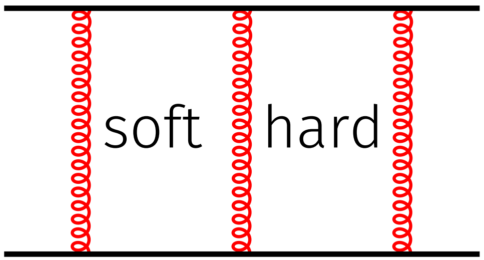
*(**Hard region** does not contribute to classical physics)*

NO NEED FOR (MIXED) HARD REGIONS

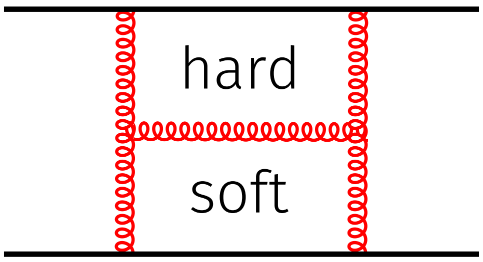
Hard gravitons are irrelevant for classical physics, sometimes after nontrivial cancellations.



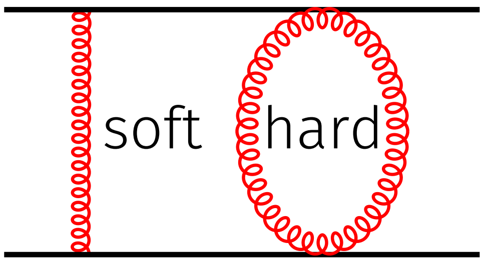
Contact potential, no long-range effects



Scaleless integral

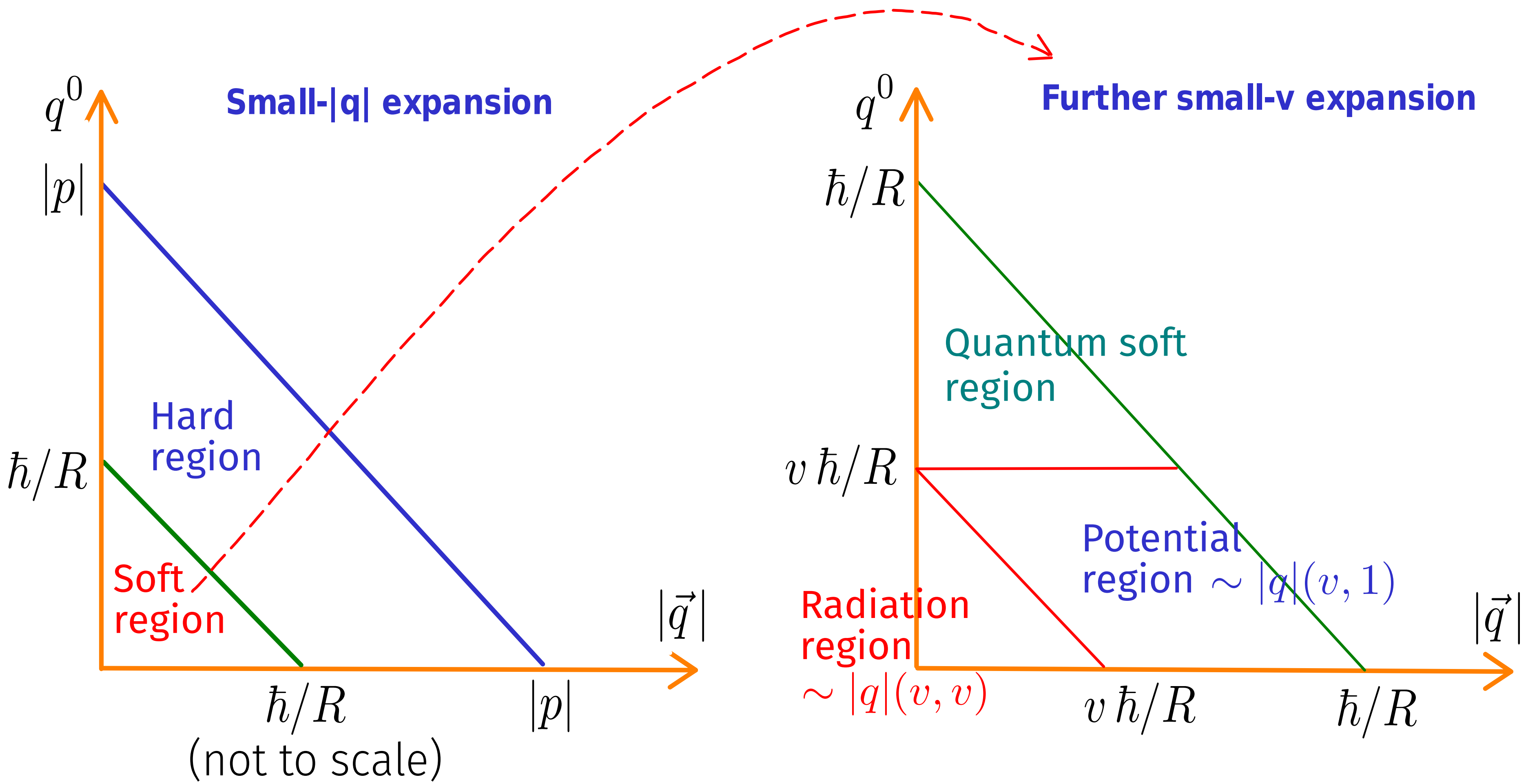


Suppressed in $\hbar \sim |q|/m,\ |q|/\sqrt{s}$



Suppressed after IR subtraction

FURTHER EXPANSION IN SMALL VELOCITY

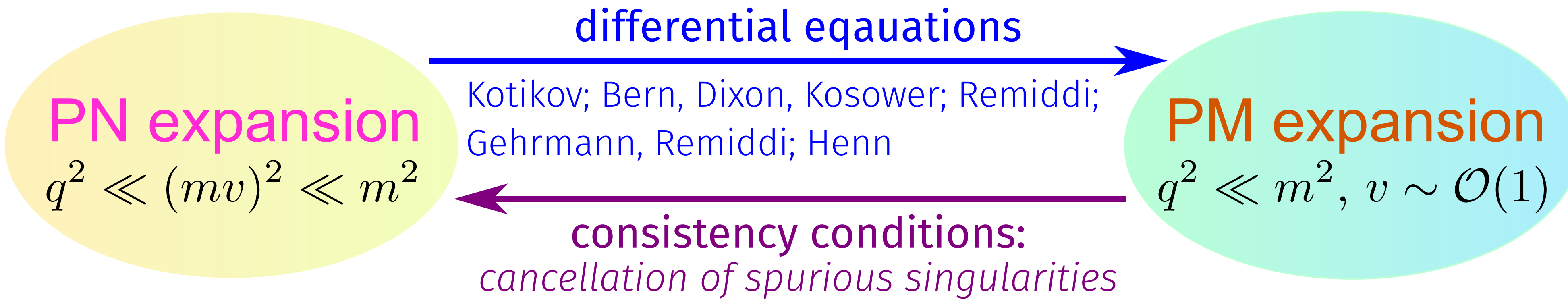


Potential region coincides with ***conservative dynamics*** up to 2 loops.

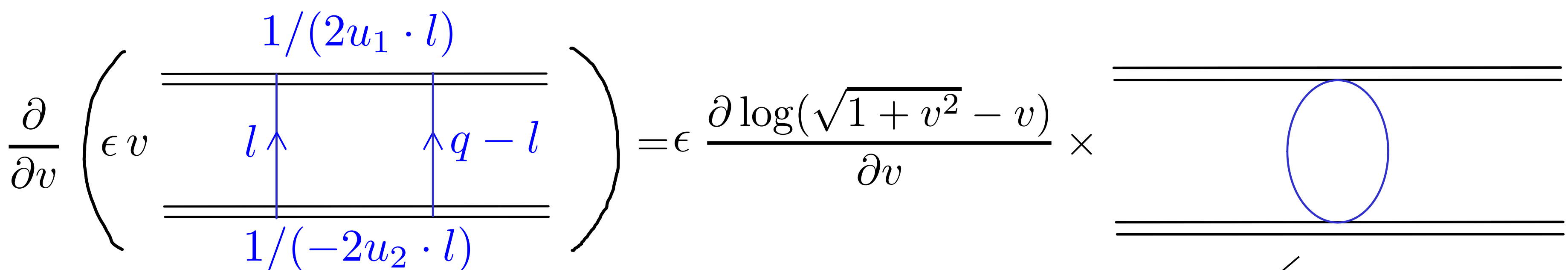
- suitable regularization needed: symmetrization prescription

[Cheung, Rothstein, Solon, '18; Cheung, Bern, Roiban, Shen, Solon, MZ, '19; Parra-Martinez, Ruf, MZ, '20]

DIFFERENTIAL EQUATIONS



Imported into post-Minkowskian gravity: [Parra-Martinez, Ruf, **MZ**, '20]
subsequently: [Kalin, Porto, '20; Di Vecchia, Heissenberg, Russo, Veneziano, '21; Herrmann, Parra-Martinez, Ruf, **MZ**, '21; Bjerrum-Bohr, Damgaard, Plante, Vanhove, '21]

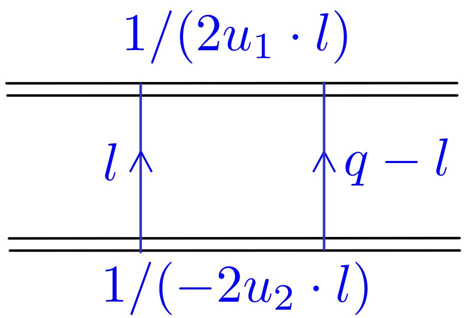


$$u_1^2 = u_2^2 = 1,\; u_1 \cdot u_2 = y = \sqrt{1+v^2}$$

Solution for box: constant in potential region. Logarithm in soft region.

v independent; zero in potential region

DIFFERENTIAL EQUATIONS



$u_1^2 = u_2^2 = 1,\ u_1 \cdot u_2 = y = \sqrt{1+v^2}$

Rationalization: $y = \dfrac{1+x^2}{2x}$

Physical region: $0 < x < 1$

Euclidean region: $-1 < x < 0$

Similar to letters in massive cusp anomalous dimension [Bruser, Dlapa, Henn, Yan, '20]
Talk by C. Dlapa

symbol letters: $x, 1 \pm x, 1 + x^2$

Canonical form: [Henn, '13]

rational matrix

$$\frac{\partial}{\partial x}\vec{I} = \epsilon \sum_r \frac{\partial \log W_r}{\partial x} \mathbb{M}^{(r)} \cdot \vec{I}$$

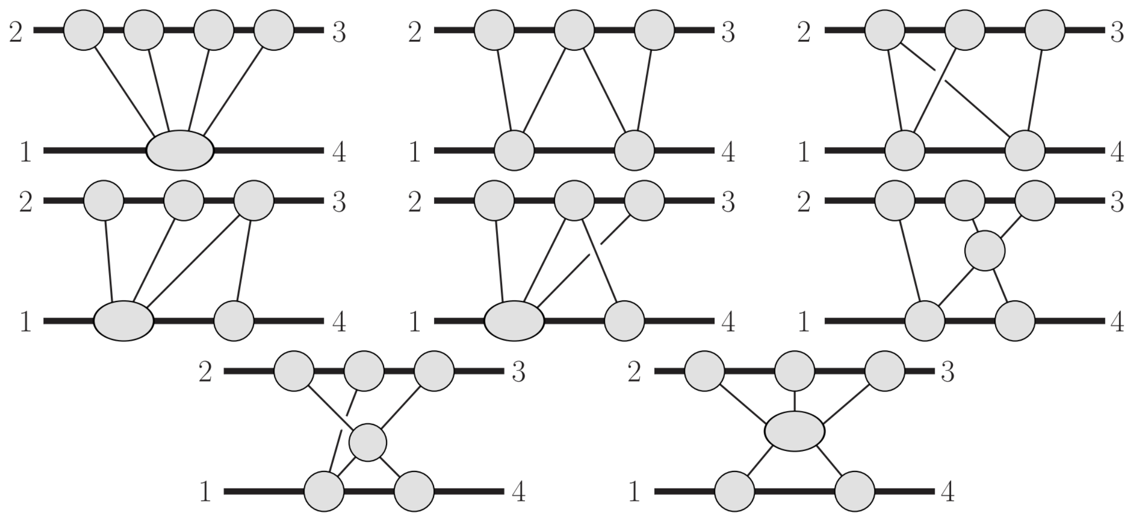
The last letter only appears at 3 loops. For the potential region, smooth static limit implies that $(1-x)$ is never a first entry.

Results and Comparisons

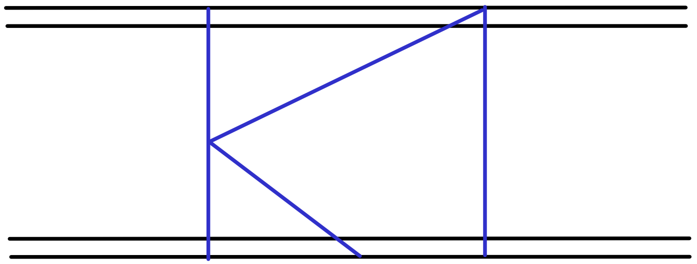
THE 4PM / 3-LOOP CALCULATION

[Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, **MZ**, '21 (PRL)]

- Amplitude integrand from 8 generalized unitarity cuts.



- In *potential region*, used `epsilon` [Prauso, '17] to find canonical form, except for one bottom-level sector (ellitpic, 3×3 system).



THE 4PM / 3-LOOP CALCULATION

[Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, **MZ**, '21 (PRL)]

$$\mathcal{M}_4(\boldsymbol{q}) = G^4 M^7 \nu^2 |\boldsymbol{q}| \left(\frac{\boldsymbol{q}^2}{4^{1/3}\tilde{\mu}^2}\right)^{-3\epsilon} \pi^2 \left[\mathcal{M}_4^p + \nu\left(\frac{\mathcal{M}_4^t}{\epsilon} + \mathcal{M}_4^f\right)\right] + \text{IR divs.}$$

$$\begin{aligned}\mathcal{M}_4^{\text{f}} &= h_4 + h_5 \log\left(\tfrac{\sigma+1}{2}\right) + h_6 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + h_7 \log(\sigma) - h_2 \frac{2\pi^2}{3} + h_8 \frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1} + h_9 \left[\operatorname{Li}_2\left(\tfrac{1-\sigma}{2}\right) + \frac{1}{2}\log^2\left(\tfrac{\sigma+1}{2}\right)\right] \\ &+ h_{10}\left[\operatorname{Li}_2\left(\tfrac{1-\sigma}{2}\right) - \frac{\pi^2}{6}\right] + h_{11}\left[\operatorname{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) - \operatorname{Li}_2\left(\tfrac{\sigma-1}{\sigma+1}\right) + \frac{\pi^2}{3}\right] + h_2 \frac{2\sigma(2\sigma^2-3)}{(\sigma^2-1)^{3/2}}\left[\operatorname{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \operatorname{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right)\right] \\ &+ \frac{2h_3}{\sqrt{\sigma^2-1}}\left[\operatorname{Li}_2\left(1-\sigma-\sqrt{\sigma^2-1}\right) - \operatorname{Li}_2\left(1-\sigma+\sqrt{\sigma^2-1}\right) + 5\operatorname{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - 5\operatorname{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) + 2\log\left(\tfrac{\sigma+1}{2}\right)\operatorname{arccosh}(\sigma)\right] \\ &+ h_{12}\mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{13}\mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right)\mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{14}\mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right),\end{aligned}$$

polylogarithms up to transcendental weight 2

complete elliptic integrals of the 1st & 2nd kind

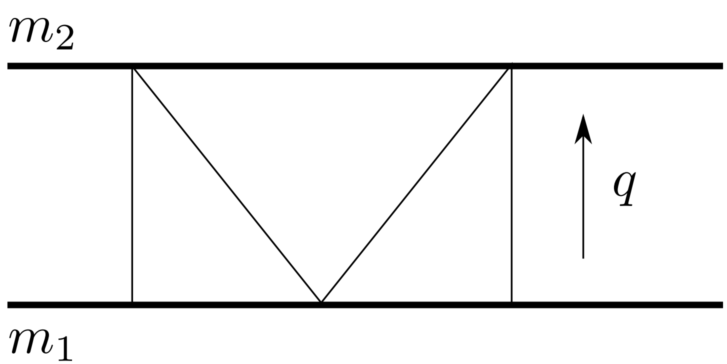
Rational prefactors:

$$h_1 = \frac{1151 - 3336\sigma + 3148\sigma^2 - 912\sigma^3 + 339\sigma^4 - 552\sigma^5 + 210\sigma^6}{12(\sigma^2-1)},$$

$$h_2 = \frac{1}{2}\left(5 - 76\sigma + 150\sigma^2 - 60\sigma^3 - 35\sigma^4\right),$$

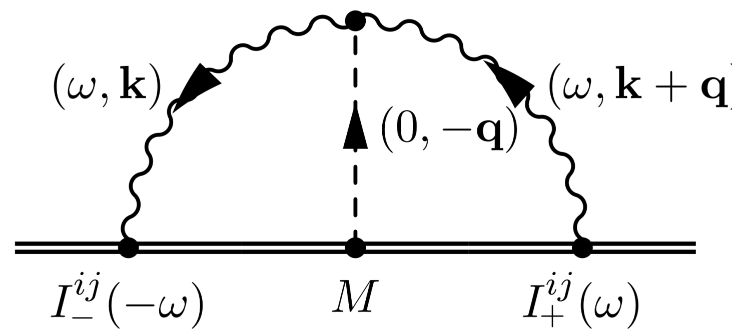
$$\vdots$$

IR DIVERGENCE IN 2-BODY POTENTIAL



potential region / near-zone

$+$



radiation region / far-zone

[Gally, Leibovich, Porto, Ross, '15]

Potential region: graviton dominated by spatial momenta

$$\mathcal{M}_4^{\rm pot}(\boldsymbol{q}) \propto \left[\frac{\mathcal{M}_4^t}{\epsilon} + \text{finite}\right] + \text{Iterations}$$

Next slides: preliminary NR comparison

Radiation region: small energy and spatial momenta $\sim 1/\lambda$

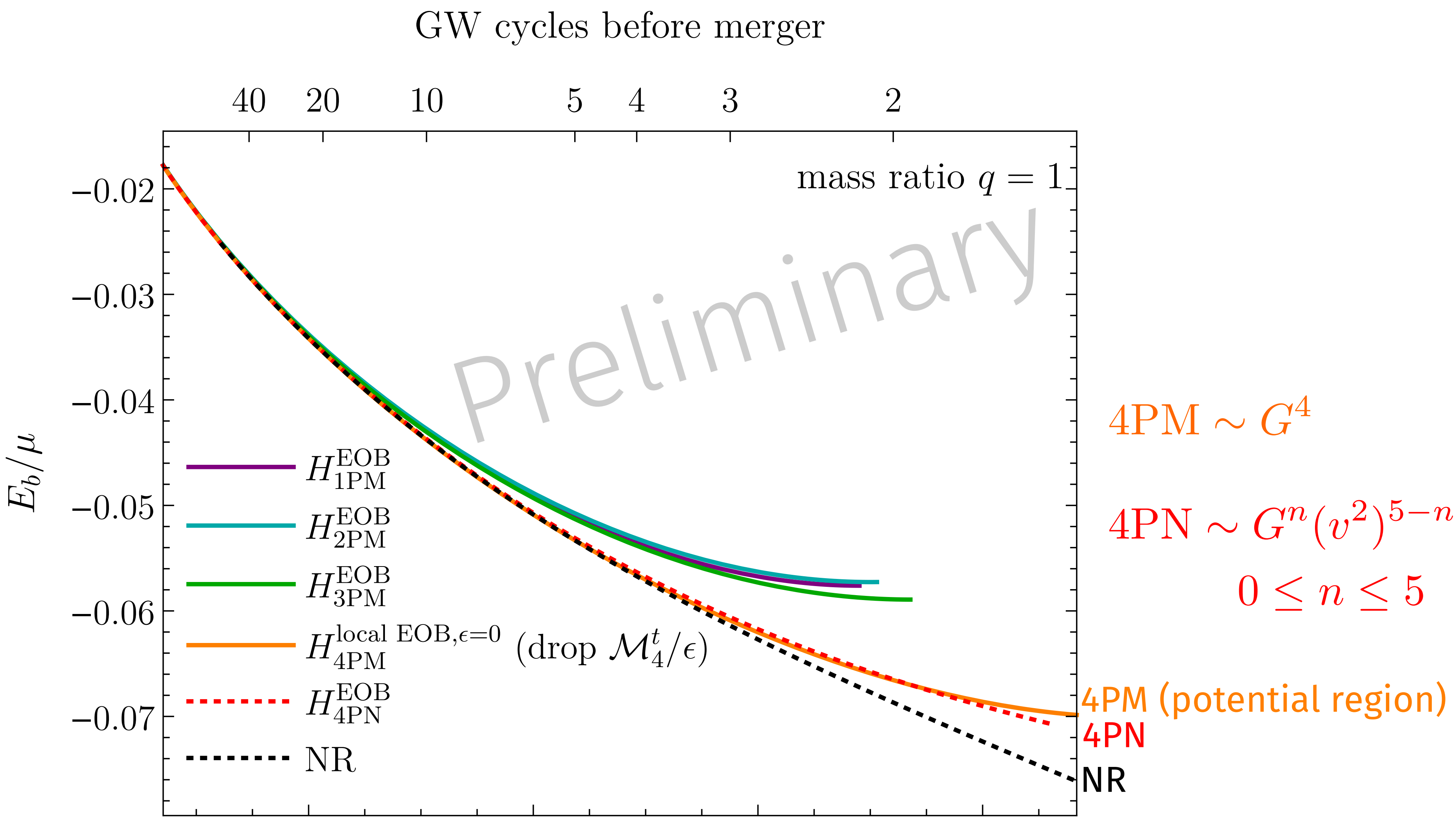
$$\mathcal{M}_4^{\rm rad}(\boldsymbol{q}) \propto \left[-\frac{\mathcal{M}_4^t}{\epsilon} + 2\log(v^2) + \text{finite}\right]$$

To be calculated in PM expansion!

Divergence cancels in the sum, leaving $\log(v)$ term analogous to Lamb shift in QED with $\log(\alpha)$ term.

4PM BINDING ENERGY VS. NUMERICAL RELATIVITY

[Khalil, Buonanno, Steinhoff, Vines, preliminary]



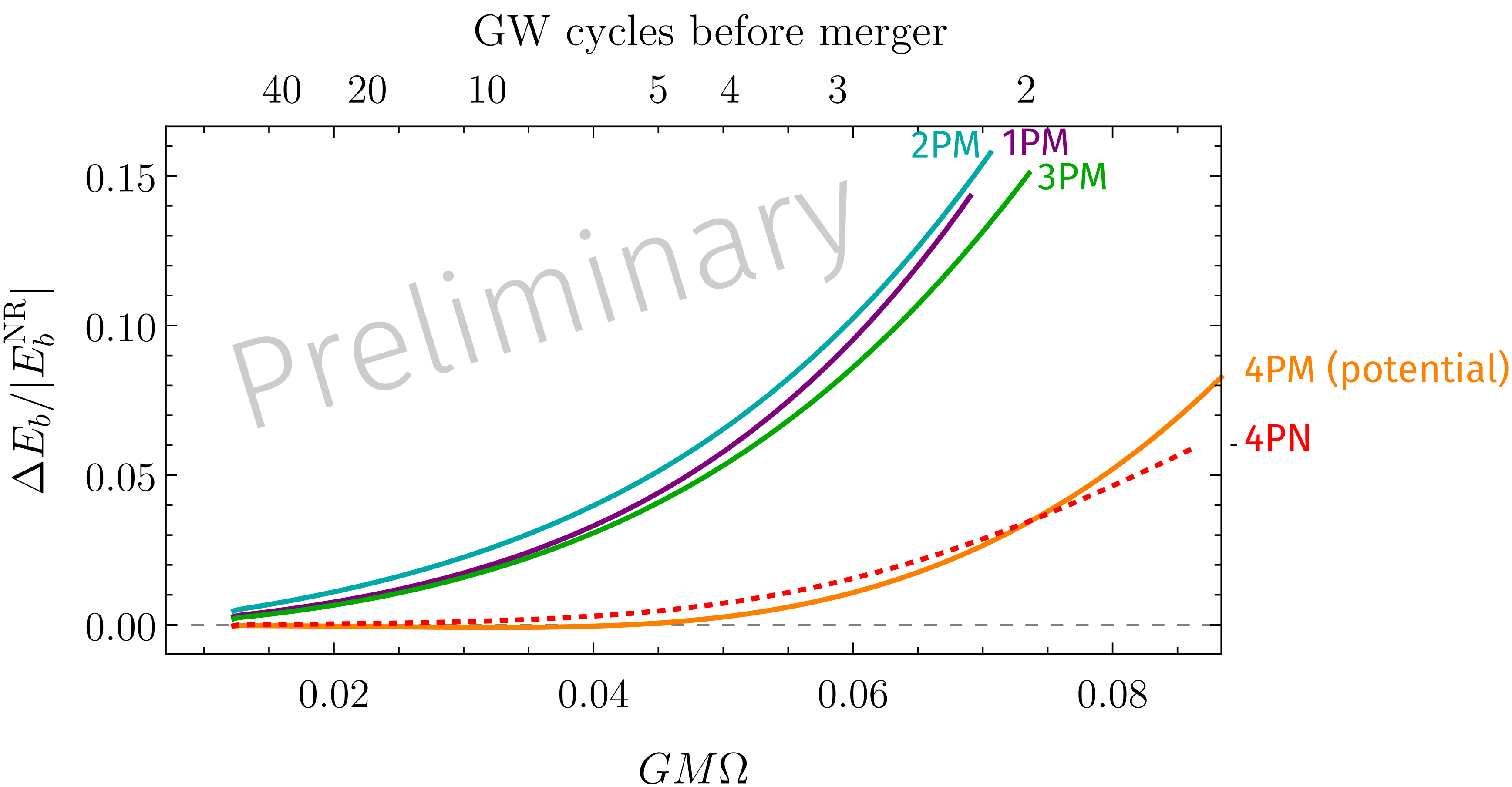
$4\text{PM} \sim G^4$

$4\text{PN} \sim G^n (v^2)^{5-n},$
$0 \le n \le 5$

4PM BINDING ENERGY VS. NUMERICAL RELATIVITY

[Khalil, Buonanno, Steinhoff, Vines, preliminary]

Same plot shown as relative deviation from NR.



Post-Minksowskian prediction starts to become competitive with post-Newtonian predictions!

DISCUSSIONS & OUTLOOK

- Obtained new results for *post-Minkowskian binary dynamics,* beyond the best classical calculations.

- Start to compete with post-Newtonian theory, and offers ***new analytic insights.***

- Relies on modern methods for ***scattering amplitudes*** and *advanced integration techniques* developed in QCD:
  - Method of regions
  - IBP & Differential equations
  - Polylogarithms & (iterated) elliptic integrals
  - Reverse unitarity   Talk by Julio Parra-Martinez

- Rich physics to be explored - spin, tidal effects, radiation reaction... Preparing for *coming decades of GW physics!*