BROOKHAVEN NATIONAL LABORATORY

# Nested soft-collinear subtractions for deep inelastic scattering

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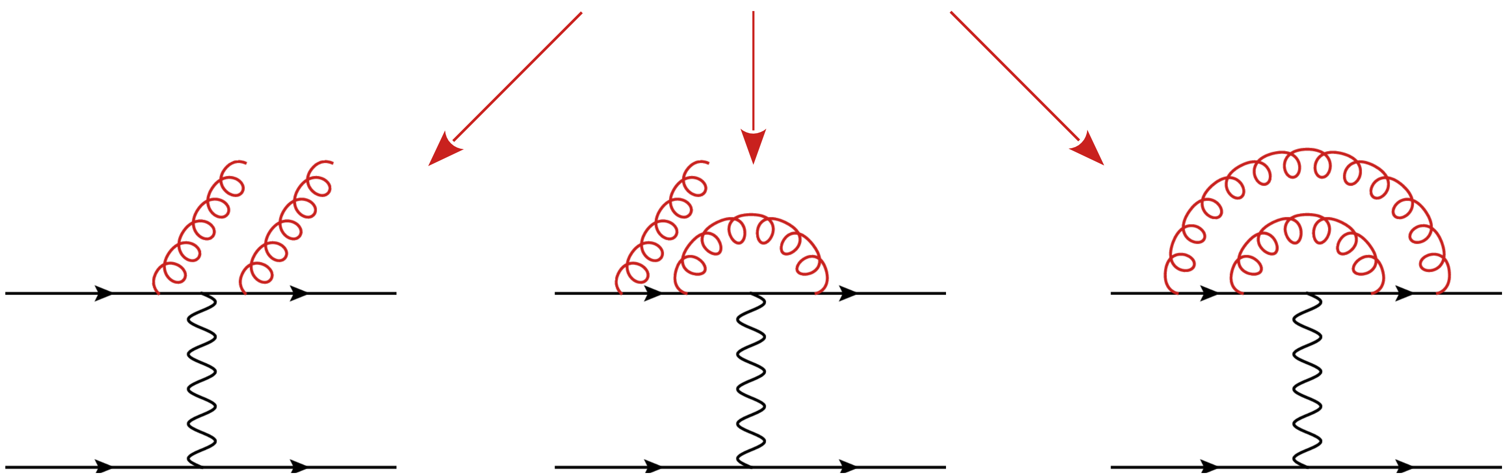
# IR finite differential cross section @NNLO QCD

- To test the SM, predictions for fully-differential cross sections through higher orders in QCD are needed.
- Contributions to next-to-next-to-leading order partonic cross section

$$\mathrm{d}\hat{\sigma}_{\mathrm{nnlo}} = \mathrm{d}\hat{\sigma}_{\mathrm{rr}} + \mathrm{d}\hat{\sigma}_{\mathrm{rv}} + \mathrm{d}\hat{\sigma}_{\mathrm{vv}} + \mathrm{d}\hat{\sigma}_{\mathrm{pdf}}$$



double real emission contributions | real-virtual contributions | double virtual contribution

# IR finite differential cross section @NNLO QCD

- To test the SM, predictions for fully-differential cross sections through higher orders in QCD are needed.
- Contributions to next-to-next-to-leading order partonic cross section

contain **explicit** infrared poles in $1/\epsilon$

$$\mathrm{d}\hat{\sigma}_{\mathrm{nnlo}} = \mathrm{d}\hat{\sigma}_{\mathrm{rr}} + \mathrm{d}\hat{\sigma}_{\mathrm{rv}} + \mathrm{d}\hat{\sigma}_{\mathrm{vv}} + \mathrm{d}\hat{\sigma}_{\mathrm{pdf}}$$

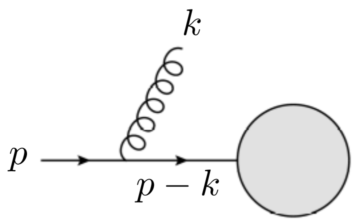
contain infrared **singularities** that become poles in $1/\epsilon$ **only** upon phase space integration

- In dimensional regularization ( $d = 4 - 2\epsilon$ ) the explicit poles of 1-loop and 2-loop amplitudes are known independent of the hard matrix element [Catani '98; Becher, Neubert '09]

$$\mathcal{M}_{\text{1-loop}}(\{p\}) = \left[\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_i \left(\frac{1}{\epsilon^2} + \frac{g_i}{\mathbf{T}_i^2}\frac{1}{\epsilon}\right) \sum_{j\neq i} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \left(\frac{\mu^2}{-s_{ij}}\right)^{\epsilon}\right] \mathcal{M}_{\text{tree}}(\{p\}) + \mathcal{M}^{\text{fin}}_{\text{1-loop}}(\{p\})$$

# Singularities of real emission contributions

- Singularities of QCD amplitudes come in two varieties: soft $(E \to 0)$ and collinear $(\vec{p}_i \parallel \vec{p}_j)$

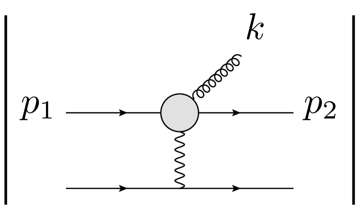



$$\sim \frac{1}{(p-k)^2} \sim \frac{1}{E_p \times \boxed{E_k} \times \boxed{(1-\vec{n}_p\cdot\vec{n}_k)}} \longrightarrow \infty \quad \begin{cases} \text{for} & E_k \to 0 \\ \text{for} & \vec{n}_p \parallel \vec{n}_k \end{cases}$$

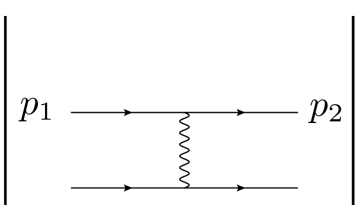
soft singularity collinear singularity

- The corresponding limits of amplitudes are generic and independent of a hard process

- For example, the soft $(E_k \to 0)$ limit of a single real emission DIS amplitude is [Altarelli, Parisi, '77]




$$\left| \cdots \right|^2 \underset{E_k \to 0}{\approx} 2C_F\, g_{s,b}^2 \times \boxed{\frac{p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}} \times \left| \cdots \right|^2$$




eikonal function

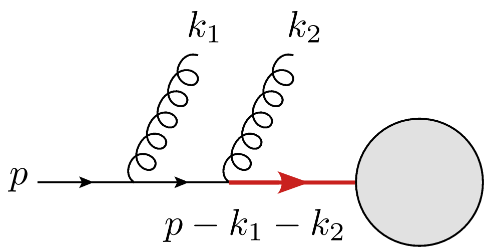
- Two new genuine NNLO singularities: double soft and triple collinear

- Factorization formulas for double soft and triple collinear singularities are known [Catani, Grazzini '99; ...]

- They are structurally similar to the NLO case.

# Entangled soft and collinear limits

- Entangled soft/collinear limits




$$\sim \frac{1}{(p-k_1-k_2)^2} \sim \frac{1}{2p\cdot k_1 + 2p\cdot k_2 - 2k_1\cdot k_2} \xrightarrow[\substack{k_1 \parallel p \\ k_2 \to 0}]{} \infty$$

- For a **given amplitude** it can be checked explicitly that entangled **soft/collinear** singularities do not occur
- This observation is general thanks to a phenomenon known as **colour coherence** (a soft gluon does not resolve details of a collinear splitting) [Caola, Melnikov, Röntsch, '17]

## As a result ...

... known soft and collinear limits of amplitudes are sufficient to construct all relevant subtraction terms;

... soft and collinear limits can be treated independently;

... can be used to extend FKS subtraction @NLO to NNLO [Frixione, Kunszt, Signer '96].

**→ Nested soft-collinear subtraction scheme**

[Caola, Melnikov, Röntsch, '17]

# How to regulate and extract singularities without integration?

- Soft and collinear singularities turn into $1/\epsilon$ poles upon phase space integration.

$$\int \frac{\mathrm{d}^{d-1}k}{2E}|M(\{p\},k)|^2 \sim \int \frac{\mathrm{d}E}{E^{1+\epsilon}}\,\frac{\mathrm{d}\theta}{\theta^{1+2\epsilon}} \times |M(\{p\})|^2 \sim \frac{1}{\epsilon^2}$$

- We would like to extract singularities without integration over resolved phase space. Currently two approaches used: **slicing** and **subtraction**.
- To illustrate the basic idea of **subtraction**, consider an integral

$$I = \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}\mathrm{F}(x)$$

where $\mathrm{F}(0)$ is finite. We then write

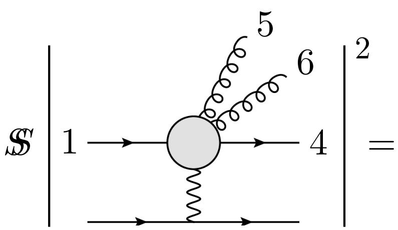
$$I = \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}\left[\mathrm{F}(x) - \mathrm{F}(0)\right] + \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}\mathrm{F}(0) = \underbrace{\int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}\left[\mathrm{F}(x) - \mathrm{F}(0)\right]}_{\text{regulated, finite in the } \epsilon \to 0 \text{ limit}} - \underbrace{\frac{1}{\epsilon}\mathrm{F}(0)}_{\text{extracted } 1/\epsilon \text{ pole}}$$

**regulated, finite in the $\boldsymbol{\varepsilon} \to 0$ limit**

**extracted $1/\boldsymbol{\varepsilon}$ pole**

# Regulating the double soft singularity

- As an example we consider the double soft singularity ( $E_5 \sim E_6 \to 0$ ). The action of $\mathbb{S}$ on the differential cross section is defined as

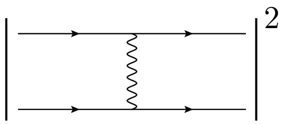



$$= \mathbb{S}\left[\mathcal{N}\int \mathrm{dLips}\ (2\pi)^d\delta^{(d)}\left(p_1+p_2-\sum_{i=3}^{6}p_i\right)|M^{\mathrm{tree}}(\{p\},p_5,p_6)|^2\mathcal{O}(p_3,p_4,p_5,p_6)\right]$$

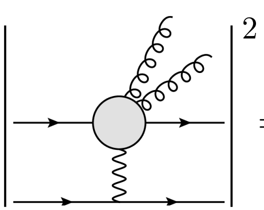
$$\equiv g_{s,b}^4 \times \underbrace{\mathrm{Eikonal}(1,4,5,6)}\times \underbrace{\mathcal{N}\int \mathrm{dLips}\ (2\pi)^d\delta^{(d)}\left(p_1+p_2-p_3-p_4\right)|M^{\mathrm{tree}}(\{p\})|^2\mathcal{O}(p_3,p_4)}$$

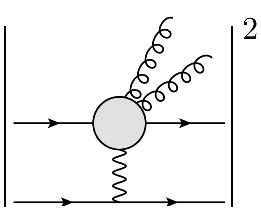
independent of the hard matrix element and the observable

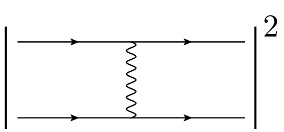
LO differential cross-section, independent of gluons 5 & 6

$\equiv$ 


- We insert unity decomposed as $I = (I - \mathbb{S}) + \mathbb{S}$ into the phase space




$= (I - \mathbb{S})\times$ 


$+\ g_{s,b}^2 \times \mathrm{Eikonal(1{,}4{,}5{,}6)}\times$ 

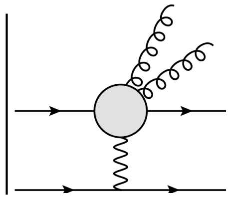
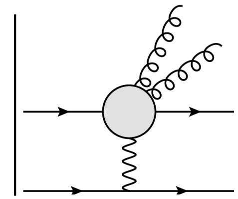

Double-soft singularity regularized but still contains single soft and collinear singularities.

Subtraction term; soft gluons decouple; integrate analytically over phase space of gluons 5 and 6

[Caola, Delto, Frellesvig, Melnikov '18]

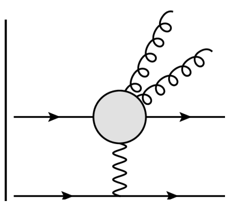
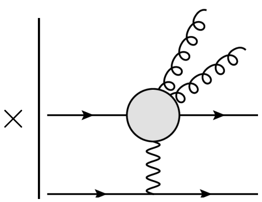
# … continue iteratively for other singularities

- Gluons ordered in energy → only one single soft singularity (e.g. for $E_6 \to 0$); insert $I = (I - S_6) + S_6$

$(I - \mathbb{S})\times$  $^2 = (I - \mathbb{S})(I - S_6)\times$  $^2$ + (Subtraction terms)

- Due to the absence of entangled soft and collinear singularities we can continue with collinear singularities in the same way.

- In case of collinear singularities introduction of partition functions [Frixione, Kunszt, Signer '96] and splitting of the angular phase space into sectors [Czakon '10 '11; Czakon, Heymes '14] to separate overlapping singularities.

 $^2 = \sum_{ij} (I - \mathbb{S})(I - S_6)(I - \mathbb{C})(I - C_{61}) \times w_i \times \theta_j \times$  $^2$ + (Subtraction terms)

*triple collinear singularity* → $(I - \mathbb{C})$

*double collinear singularity; e.g. (6//1)* → $(I - C_{61})$

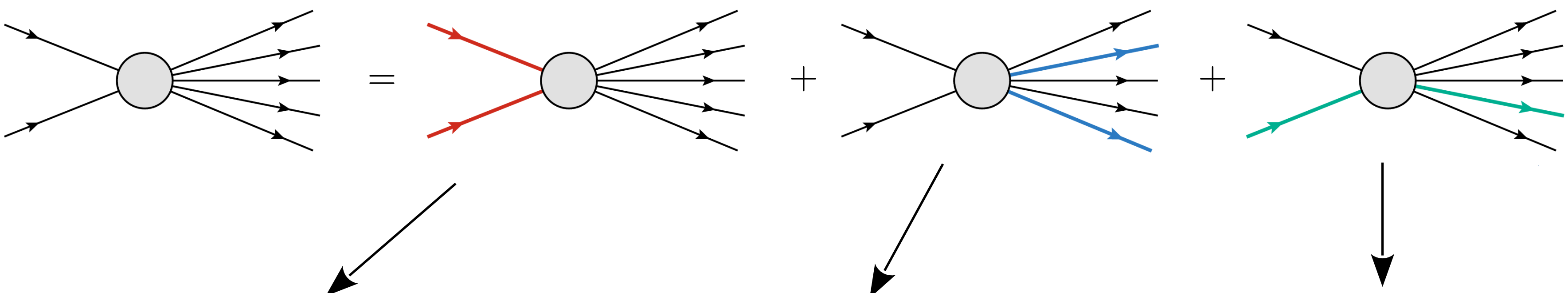
*partition function* → $w_i$

*sector; angular ordering* → $\theta_j$

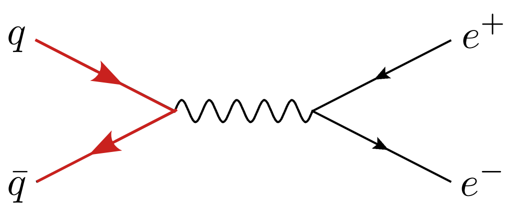
- Subtraction terms can be integrated analytically over the phase space of one or both gluons and $1/\epsilon$ poles can be extracted **explicitly**.

# The role of deep inelastic scattering

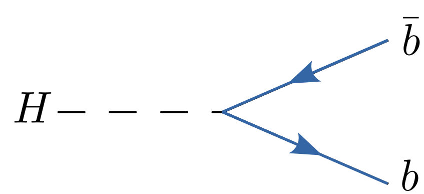
- Most complex singular contributions (both soft and collinear) only depend on the properties of two external partons
- Separation of complex $pp \to N$ processes into simpler building blocks



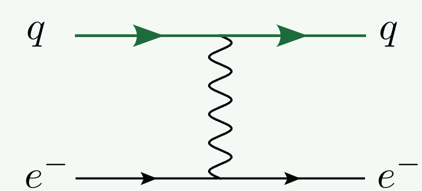

**Drell-Yan process**
both **initial state momenta**
[Caola, Melnikov, Röntsch '19]




$H \to b\bar{b}$ **decay**
both **final state momenta**
[Caola, Melnikov, Röntsch '19]




**Deep inelastic scattering**
one **initial** and one **final state momenta**
[**KA**, Caola, Melnikov, Röntsch '19]




this talk

- Focus on simpler processes → check against analytical results possible

# Fully regulated double-real contribution

$$
\begin{aligned}
F_{\rm LM}(1,4,5,6) = {} & \langle \mathbb{S} F_{\rm LM}(1,4,5,6)\rangle + \langle [I-\mathbb{S}] S_6 F_{\rm LM}(1,4,5,6)\rangle \\
& + \sum_{\substack{i,j\in\{1,4\}\\ i\neq j}} \Big\langle [I-\mathbb{S}][I-S_6]\Big[C_{5i} w^{5i,6j} + C_{6i} w^{5j,6i} + \Big(\theta_i^{(a)} C_{5i} + \theta_i^{(c)} C_{6i}\Big) w^{5i,6i}\Big] \\
& \qquad\qquad \times [\mathrm{d}p_5][\mathrm{d}p_6] F_{\rm LM}(1,4,5,6)\Big\rangle \\
& + \sum_{i\in\{1,4\}} \Big\langle [I-\mathbb{S}][I-S_6]\Big[\theta_i^{(b)} C_{56} + \theta_i^{(d)} C_{56}\Big][\mathrm{d}p_5][\mathrm{d}p_6] w^{5i,6i} F_{\rm LM}(1,4,5,6)\Big\rangle \\
& - \sum_{\substack{i,j\in\{1,4\}\\ i\neq j}} \Big\langle [I-\mathbb{S}][I-S_6] C_{5i} C_{6j} [\mathrm{d}p_5][\mathrm{d}p_6] w^{5i,6j} F_{\rm LM}(1,4,5,6)\Big\rangle \\
& + \sum_{i\in\{1,4\}} \Big\langle [I-\mathbb{S}][I-S_6]\Big[\theta_i^{(a)} \mathbb{C}_i [I-C_{5i}] + \theta_i^{(b)} \mathbb{C}_i [I-C_{56}] + \theta_i^{(c)} \mathbb{C}_i [I-C_{6i}] \\
& \qquad\qquad + \theta_i^{(d)} \mathbb{C}_i [I-C_{56}]\Big][\mathrm{d}p_5][\mathrm{d}p_6] w^{5i,6i} F_{\rm LM}(1,4,5,6)\Big\rangle
\end{aligned}
$$

**subtraction terms**

$$
\begin{aligned}
& + \sum_{\substack{i,j\in\{1,4\}\\ i\neq j}} \Big\langle [1-\mathbb{S}][1-S_6][1-C_{6j}][1-C_{5i}]\ [\mathrm{d}p_5][\mathrm{d}p_6] w^{5i,6j} F_{\rm LM}(1,4,5,6)\Big\rangle \\
& + \sum_{i\in\{1,4\}} \Big\langle [1-\mathbb{S}][1-S_6][1-\mathbb{C}_i]\Big(\theta^{(a)}[1-C_{6i}] + \theta^{(b)}[1-C_{56}] \\
& \qquad\qquad + \theta^{(c)}[1-C_{5i}] + \theta^{(d)}[1-C_{56}]\Big)\ [\mathrm{d}p_5][\mathrm{d}p_6] w^{5i,6i} F_{\rm LM}(1,4,5,6)\Big\rangle
\end{aligned}
$$

**regulated matrix elements**

# Pole structure @NNLO

- Analytic integration of subtraction terms is possible (analytic simplifications after recombining subtractions terms)

$$\Big\langle [1-\mathbb{S}][1-S_6]\Big[C_{54}w^{54,61}+C_{64}w^{51,64}+\Big(\theta^{(a)}C_{64}+\theta^{(c)}C_{54}\Big)w^{54,64}\Big][\mathrm{d}g_5][\mathrm{d}g_6]F_{LM}(1,4,5,6)\Big\rangle$$

$$=\frac{[\alpha_s]C_F}{\epsilon}\Big\langle \sum_{i=1,4}(I-S_5)(I-C_{5i})w^{5i}\Big[\Big(\frac{1}{\epsilon}+Z^{2,2}\Big)(2E_4)^{-2\epsilon}-\frac{1}{\epsilon}(2E_5)^{-2\epsilon}\Big]\Big[w^{51}_{\mathrm{dc}}+w^{54}_{\mathrm{tc}}\Big(\frac{\rho_{54}}{4}\Big)^{-\epsilon}\Big]F_{LM}(1,4,5)\Big\rangle$$

$$+\frac{[\alpha_s]^2C_F^2}{\epsilon^3}\Big\langle\Big[\Big(\frac{1}{\epsilon}+Z^{2,2}\Big)(2E_4)^{-2\epsilon}(2E_{max})^{-2\epsilon}-\frac{1}{2\epsilon}(2E_{max})^{-4\epsilon}\Big]$$
$$\times\Big[\langle\Delta_{51}\rangle_{S_5}-\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}-\frac{2^\epsilon}{2}\frac{\Gamma(1-\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1-3\epsilon)}\Big]F_{LM}(1,4)\Big\rangle$$
$$+\frac{[\alpha_s]^2C_F^2}{\epsilon^2}\Big[\frac{2^\epsilon}{2}\frac{\Gamma(1-\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1-3\epsilon)}\Big]\Big[\frac{1}{\epsilon}+Z^{2,2}\Big]\Big[\frac{1}{\epsilon}+Z^{4,2}\Big]\Big\langle(2E_4)^{-4\epsilon}F_{LM}(1,4)\Big\rangle$$
$$-\frac{[\alpha_s]^2C_F^2}{\epsilon^3}\Big[\frac{1}{2\epsilon}+Z^{2,4}\Big]\Big\langle\Big[\langle\Delta_{51}\rangle_{S_5}+\Big(\frac{2^\epsilon}{2}\frac{\Gamma(1-\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1-3\epsilon)}\Big)\Big](2E_4)^{-4\epsilon}F_{LM}(1,4)\Big\rangle$$

$$-\frac{[\alpha_s]^2C_F^2}{\epsilon^2}\Big[\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\Big]\int\mathrm{d}z\,\Big\langle\Big[\Big(\frac{1}{\epsilon}+Z^{2,2}\Big)(2E_4)^{-2\epsilon}-\frac{1}{\epsilon}(2E_1)^{-2\epsilon}(1-z)^{-2\epsilon}\Big]$$
$$\times(2E_1)^{-2\epsilon}(1-z)^{-2\epsilon}\bar{P}_{qq}(z)\frac{F_{LM}(z\cdot 1,4)}{z}\Big\rangle.$$

- The subtraction terms contains the regulated NLO differential cross section → poles cancel against similar terms from real virtual contributions
- Regular and "boosted" LO differential cross section → cancel against double virtual (and collinear renormalization contributions)
- Provide analytic formula for the finite remainder

# Finite contributions of integrated subtraction terms

- E.g. abelian boosted quark non-singlet contribution (multiplying the LO matrix element)
- This is one of the more complex contributions, still relatively compact

$$
\begin{aligned}
\mathcal{T}^{1}_{\rm ns} = {} & \frac{\pi^2(5-2z)}{3(z-1)} + 5z + 8\mathcal{D}_3(z) + 12\mathcal{D}_2(z)\ln\left(\frac{4E_1^2}{\mu^2}\right) \\
& + \ln\left(\frac{E_1}{E_4}\right)\Bigg(\ln\eta_{14}\left(-2z-2(z+1)\ln\left(\frac{4E_1^2}{\mu^2}\right)-4(z+1)\ln(1-z)+2\right) \\
& + (2-2z)\ln z - \frac{2(z^2+1)\ln(4E_1^2/\mu^2)\ln z}{z-1} - \frac{4(z^2+1)\ln(1-z)\ln z}{z-1}\Bigg) \\
& + \ln\eta_{14}\Bigg(-3z+2(z-1)\ln z+\ln\left(\frac{4E_1^2}{\mu^2}\right)\left(\frac{2(z^2+1)\ln z}{z-1}-3(z+1)\right) \\
& \quad + \ln(1-z)\left(\frac{4(z^2+1)\ln z}{z-1}-6(z+1)\right)+3\Bigg) + \ln z\Bigg(\frac{-22z^2-5z+17}{2(z-1)} \\
& \quad + \frac{\pi^2(2z^2+2)}{3(1-z)} + 4(z+1)\ln(1+z)\Bigg) + \ln^2\left(\frac{E_1}{E_4}\right)\Bigg(-z+(-z-1)\ln\left(\frac{4E_1^2}{\mu^2}\right) \\
& \quad - 2(z+1)\ln(1-z)+1\Bigg) - 2(z+1)\ln(1-z)\ln^2\eta_{14} \\
& + \ln\left(\frac{E_1}{E_{\rm max}}\right)\Bigg(-\frac{1}{2}\ln^2(2)(z+1)-2\ln^2\eta_{14}(z+1)+\frac{2}{3}\pi^2(z+1) \\
& \quad + \ln\eta_{14}\left(4(z-1)+4(z+1)\ln\left(\frac{4E_1^2}{\mu^2}\right)+8(z+1)\ln(1-z)\right)\Bigg) \\
& + \ln^2\left(\frac{4E_1^2}{\mu^2}\right)\left(\frac{1}{2}(-z-5)-2(z+1)\ln(1-z)+\frac{(3z^2+1)\ln z}{2(z-1)}\right) \\
& + \left(2(z-1)+\frac{(1-7z^2)\ln z}{2(z-1)}\right)\ln^2(1-z) - \frac{3(2z^2+2z-7)\ln^2 z}{4(z-1)} \\
& + \ln(1-z)\Bigg(\frac{\pi^2(27-29z^2)}{6(1-z)}+\frac{(-7z^2+2z-7)\ln z}{z-1}+\frac{-7z-(z+1)\ln^2(2)-46}{2} \\
& \quad + \frac{5(z^2+1)\ln^2 z}{2(z-1)}\Bigg) + \ln\left(\frac{4E_1^2}{\mu^2}\right)\Bigg(-3z+2\pi^2(z+1)+\frac{(-4z^2-2z+3)\ln z}{z-1} \\
& \quad + \ln(1-z)\left(-z+\frac{2(z^2+1)\ln z}{z-1}-5\right) - 6(z+1)\ln^2(1-z) \\
& \quad + \frac{(3z^2+1)\ln^2 z}{2(z-1)} - 10\Bigg) - 4(z+1)\ln^3(1-z) + \frac{1}{12}(z+1)\ln^3(z) \\
& + {\rm Li}_2(1-\eta_{14})\Bigg(-2z-4(z+1)\ln\left(\frac{E_1}{E_{\rm max}}\right)-2(z+1)\ln\left(\frac{4E_1^2}{\mu^2}\right) \\
& \quad - 8(z+1)\ln(1-z)+2\Bigg) + \mathcal{D}_1(z)\Bigg(8\ln\left(\frac{E_1}{E_4}\right)\ln\eta_{14}-16\ln\left(\frac{E_1}{E_{\rm max}}\right)\ln\eta_{14} \\
& \quad + 12\ln\eta_{14}+6\ln\left(\frac{4E_1^2}{\mu^2}\right)+\ln^2(2)+4\ln^2\left(\frac{E_1}{E_4}\right)+4\ln^2\eta_{14}+4\ln^2\left(\frac{4E_1^2}{\mu^2}\right) \\
& \quad + 16{\rm Li}_2(1-\eta_{14})-8\pi^2+26\Bigg) + \left(4(z+1)+\frac{4(z^2+1)\ln z}{z-1}\right){\rm Li}_2(-z) \\
& + {\rm Li}_2(z)\left(\frac{2(2z^2-5)}{z-1}-2(z+1)\ln\left(\frac{4E_1^2}{\mu^2}\right)+\frac{(3-5z^2)\ln(1-z)}{z-1}+\frac{(z^2+1)\ln z}{z-1}\right) \\
& + \frac{(9z^2+1)\,{\rm Li}_3(1-z)}{1-z} - \frac{8(z^2+1){\rm Li}_3(-z)}{z-1} + \frac{(1-3z^2)\,{\rm Li}_3(z)}{z-1} \\
& + \mathcal{D}_0(z)\Bigg\{4\ln\left(\frac{E_1}{E_4}\right)\ln\eta_{14}\ln\left(\frac{4E_1^2}{\mu^2}\right)+6\ln\eta_{14}\ln\left(\frac{4E_1^2}{\mu^2}\right)+2\ln^2\left(\frac{E_1}{E_4}\right)\ln\left(\frac{4E_1^2}{\mu^2}\right) \\
& \quad + \left(13-\frac{10\pi^2}{3}\right)\ln\left(\frac{4E_1^2}{\mu^2}\right)+\ln\left(\frac{E_1}{E_{\rm max}}\right)\Bigg(-8\ln\eta_{14}\ln\left(\frac{4E_1^2}{\mu^2}\right)+\ln^2(2) \\
& \quad + 4\ln^2\eta_{14}-\frac{4\pi^2}{3}\Bigg)+3\ln^2\left(\frac{4E_1^2}{\mu^2}\right)+\left(8\ln\left(\frac{E_1}{E_{\rm max}}\right)+4\ln\left(\frac{4E_1^2}{\mu^2}\right)\right){\rm Li}_2(1-\eta_{14}) \\
& \quad + 16\zeta_3\Bigg\} + \frac{(1-11z^2)\,\zeta_3}{z-1} - 2,
\end{aligned}
$$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# Numerical results

- Due to the focus on simpler processes as building blocks, results can be extensively tested against known analytic results. [Kazakov et al. '90; Zijlstra, van Neerven '92; Moch, Vermaseren '00]

- In the case of photon-induced deep-inelastic scattering with only up-quarks and gluons in the initial state and $\sqrt{s} = 100\,\mathrm{GeV},\ 10\,\mathrm{GeV} < Q < 100\,\mathrm{GeV},\ \mu_R = \mu_F = 100\,\mathrm{GeV}$ we obtain permille agreement for the NNLO contribution

$$\sigma = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{NLO}} + \boxed{\Delta\sigma_{\mathrm{NNLO}}}$$

| channel | numeric result (pb) | analytic result (pb) |
|---|---|---|
| $\sigma^{\mathrm{NNLO}}_{\mathrm{q,ns}}$ | $33.1(2) - 2.18(1)\cdot n_f$ | $33.1 - 2.17\cdot n_f$ |
| $\sigma^{\mathrm{NNLO}}_{\mathrm{q,s}}$ | $9.19(2)$ | $9.18$ |
| $\sigma^{\mathrm{NNLO}}_{\mathrm{g}}$ | $-142.4(4)$ | $-142.7$ |

[**KA**, Caola, Melnikov, Röntsch '19]

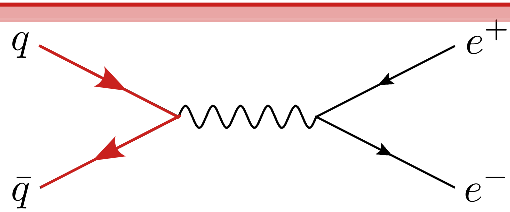
- In general, we find that we can get per mill precision on the NNLO total cross section, corresponding to a few percent precision on the NNLO coefficient, running for a few hours on an 8-core machine.

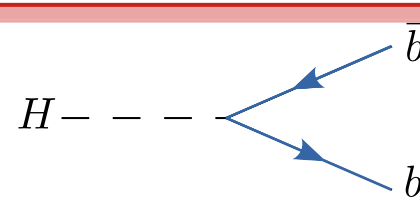
# The role of deep inelastic scattering

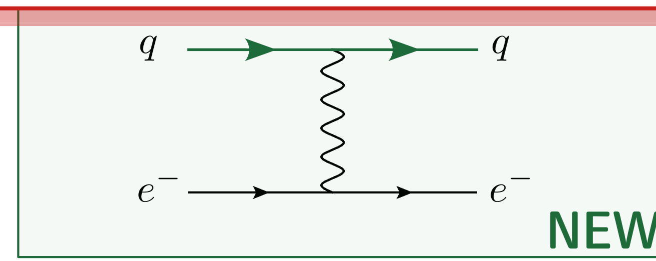
- Most complex singular contributions (both soft and collinear) only depend on the properties of two external partons
- Separation of complex $pp \to N$ processes into simpler building blocks

- Permille agreement for all basic processes against analytic results.
- Fast computation time for the NNLO cross section (Drell-Yan $\mathcal{O}(1h)$ , $H \to b\bar{b}$ decay $\mathcal{O}(< 1h)$ , DIS $\mathcal{O}(60h)$)
- **Building blocks under control! What is next?**

  $\to$ Natural next application: factorizable VBF @ NNLO QCD

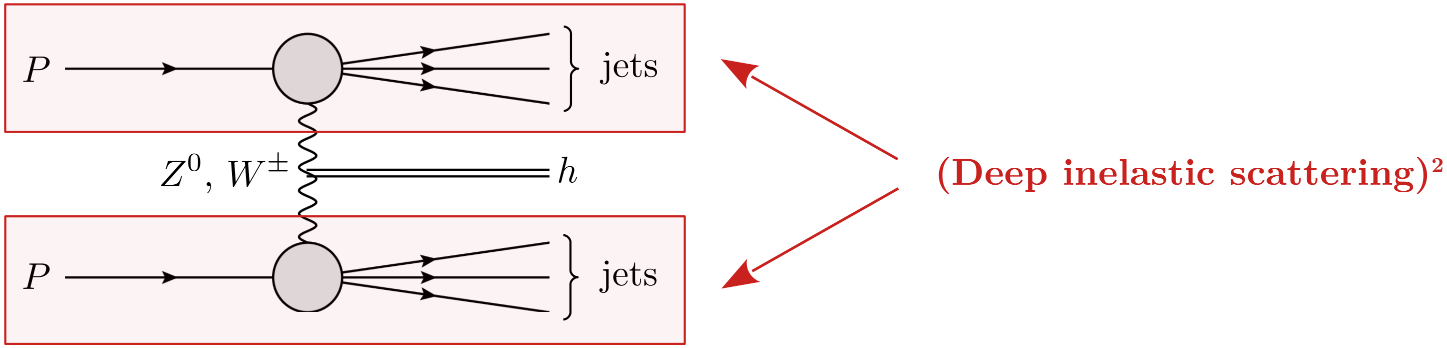



- Due to focus on simpler processes check against analytical results possible

# Vector boson fusion (developing a subtraction scheme)




- Challenging VBF phase space → requires efficient subtraction scheme

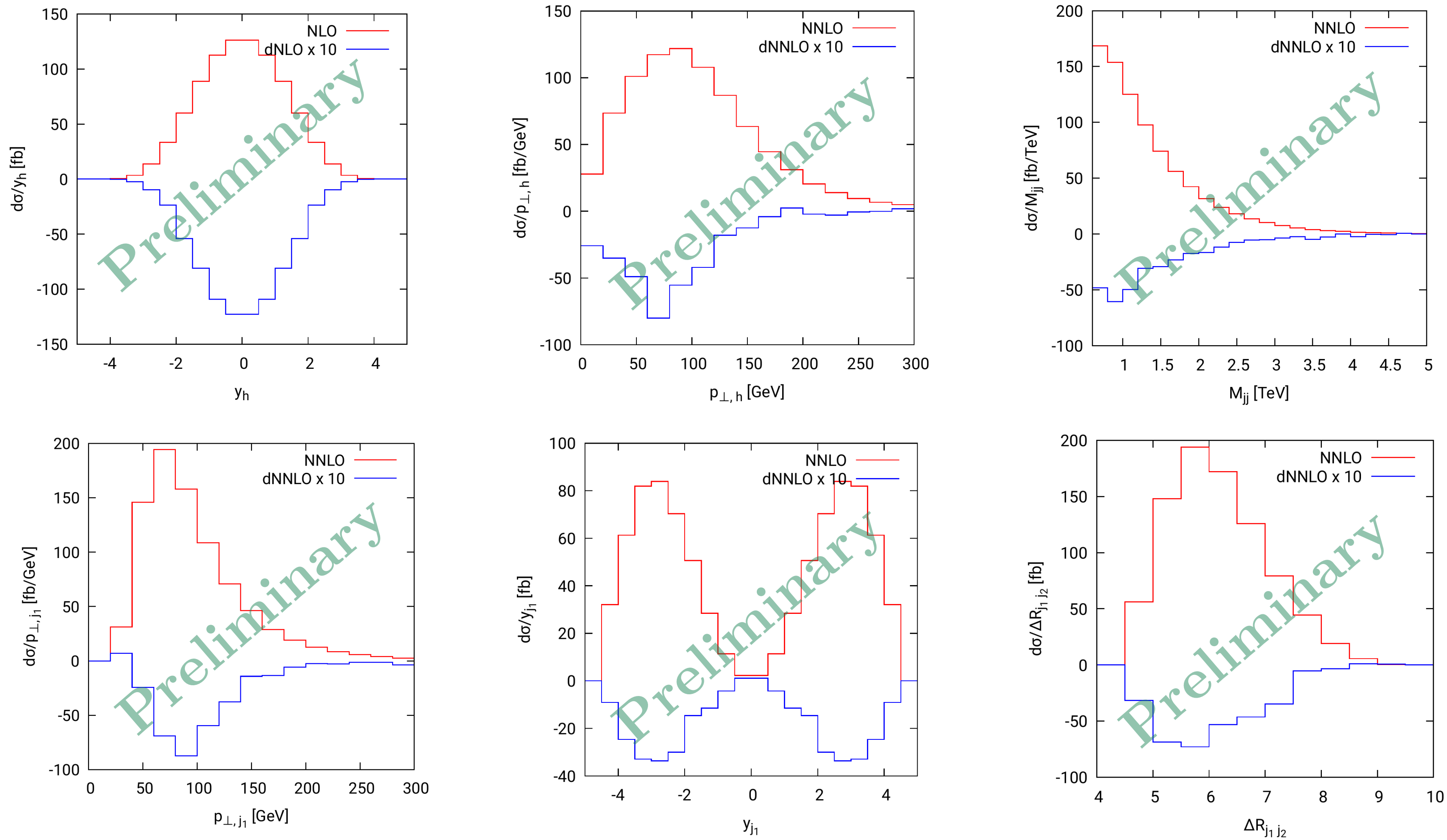
DIS: **partonic** COM frame ⟶ VBF: **hadronic** COM frame

- Integrated subtraction terms not Lorentz invariant → complex cancellation between analytically integrated subtractions and regulated real emission contributions.
- Can be checked against known fully differential descriptions @NNLO QCD [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15; Cruz-Martinez, Gehrmann, Glover, Huss '18]

# Vector boson fusion (physics)

- Important production channel of Higgs boson @LHC (second highest cross section @14TeV)
- Fast numerical implementation → study Higgs decay, anomalous couplings, non-factorizable contributions, ...

# Preliminary results




- Setup: 13 TeV, typical VBF cuts
- Computation time for ΔNNLO histograms around ~ 50.000 CPU hours

# Conclusion

- HL-LHC requires high precision theoretical predictions for collider processes.
- Despite progress with developing IR subtraction schemes, the “perfect” subtraction scheme is yet to come.
- The presented nested soft-collinear scheme for NNLO descriptions includes many of the desired properties from FKS @NLO.
- Development status: Complete set of analytic building blocks (obtained from studies of colour singlet production, decay and a DIS process) that can be used as building blocks to design subtractions for arbitrary LHC processes.
- Next steps: Application to more complex processes; in the pipeline: Higgs production in vector boson fusion.

## Construction of the nested soft-collinear subtraction scheme is based on …

… iterative extraction of soft and collinear singularities;

… partitioning of angular phase space into sectors to obtain well-defined sets of collinear limits;

… (not shown) the possibility to parametrize phase space in a way that makes analytic integration of subtraction terms possible.

# Backup

# Subtraction @NLO is …

| | |
|---|---|
| **… physically transparent** | "physical" singularities and clear mechanism of cancellation |
| **… local** | subtracted matrix elements are finite at any point in the phase-space |
| **… analytic** | analytic formulas for integrated subtraction terms |
| **… modular** | subtractions for complex processes are built from subtraction terms established in analyses of simpler processes (soft singularities are sensitive to pairs of emittors; collinear singularities factorize on external lines) |
| **… efficient** | efficient numerical evaluation (as result of local and analytic) |

## Situation @NNLO

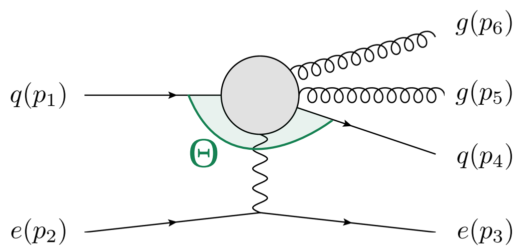
- Many subtraction schemes at NNLO [Gehrmann-de Ridder, Gehrmann, Glover '05; Czakon '10, '11; Cacciari et al '15; Somogyi, Trócsányi, Del Duca '05; Caola, Melnikov, Röntsch '17; Herzog '18; Magnea et al '18; …]
- None of the existing subtraction schemes satisfies all of the above criteria …

  **… but up to now this was not a problem for phenomenology.**
- For more complex processes, better subtraction schemes may become a necessity.

# The role of deep inelastic scattering

- Most complex singular contributions (both soft and collinear) only depend on the properties of two external partons
- Separation of complex $pp \to N$ processes into simpler building blocks

## Deep inelastic scattering




- Colour charged particles not back-to-back
- As a result: integrated subtraction terms are not numbers but functions in the opening angle $\Theta$
- First frame independent computation (using this scheme)
- Dependence on alpha-like parameter $E_{\max}$ that controls the “amount” of soft subtraction


this talk

- Focus on simpler processes → check against analytical results possible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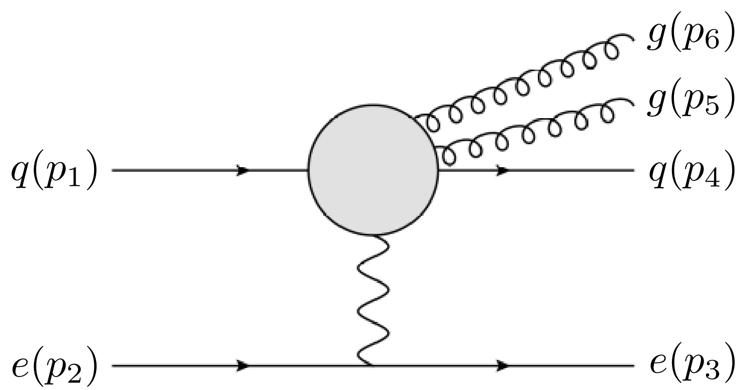
# Deep inelastic scattering @NNLO QCD




- We write the differential cross section as

Energy ordering

$$2s \cdot \mathrm{d}\sigma_{\mathrm{rr}} = \int [\mathrm{d}g_5][\mathrm{d}g_6] \boxed{\theta(E_5 - E_6)} F_{\mathrm{LM}}(1,4,5,6) \equiv \langle F_{\mathrm{LM}}(1,4,5,6) \rangle$$

with

$$F_{\mathrm{LM}}(1,4,5,6) = \mathcal{N} \int \mathrm{dLips}\ (2\pi)^d \delta^{(d)}(p_1 + p_2 - p_3 - p_4 - p_5 - p_6)$$
$$\times |M^{\mathrm{tree}}(\{p\}), p_5, p_6|^2 \times \mathcal{O}(p_3, p_4, p_5, p_6)$$

$$[\mathrm{d}g_i] = \frac{\mathrm{d}^{d-1} p_i}{(2\pi)^{d-1} 2E_i}\ \theta(E_{\mathrm{max}} - E_i)$$

# Different subtraction schemes and slicing methods

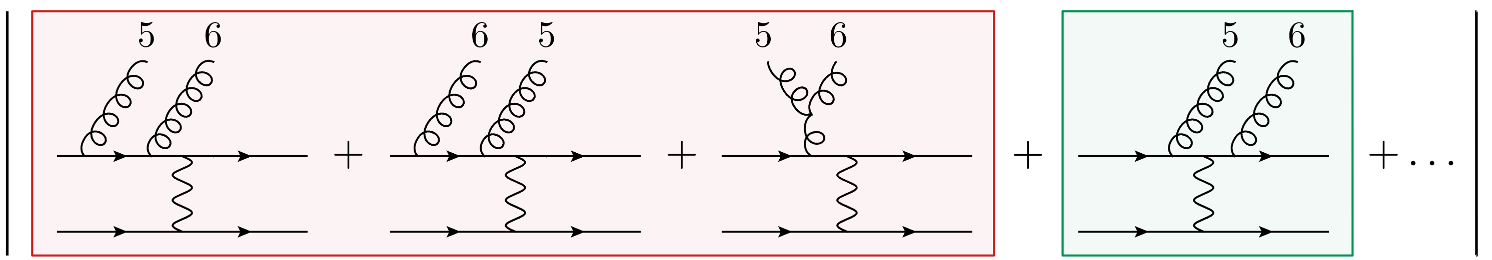
| | | |
|---|---|---|
| qt | slicing | [Catani, Grazzini] |
| Jettiness | slicing | [Boughezal et al., Gaunt et al.] |
| Antenna | subtraction | [Gehrmann-de Ridder, Gehrmann, Glover et al.] |
| Projection-to-Born | subtraction | [Cacciari et al.] |
| Colorful NNLO | subtraction | [Del Duca, Troscanyi et al.] |
| Stripper | subtraction | [Czakon] |
| Nested soft-collinear | subtraction | [Caola, Melnikov, Röntsch] |
| Local Analytic Sector | subtraction | [Magnea, Maina et al.] |
| Geometric | subtraction | [Herzog] |

| | Analytic | FS Colour | IS Colour | Local |
|---|---|---|---|---|
| Antenna | ✓ | ✓ | ✓ | ✗ |
| qT | ✓ | ✗ | ✓ | ✗ **(slicing)** |
| Colourful | ✓ | ✓ | ✗ | ✓ |
| Stripper | ✗ | ✓ | ✓ | ✓ |
| N-jettiness | ✓ | ✓ | ✓ | ✗ **(slicing)** |

Updated and adapted from [Nigel Glover, Amplitudes '15]

# Collinear singularities

$$\left|M^{\text{nnlo}}(\{p\}, p_5, p_6)\right|^2 =$$




- In the collinear limits, many different singular configurations exist, but collinear singularities factorize on external legs, therefore either **three partons** become collinear or **two pairs of partons** become collinear at once.

- To control which partons these are, the different configurations are separated by **introducing partition functions** (similarly to NLO)

$$1 = \boxed{w^{51,61}} + w^{54,64} + \boxed{w^{51,64}} + w^{54,61}$$

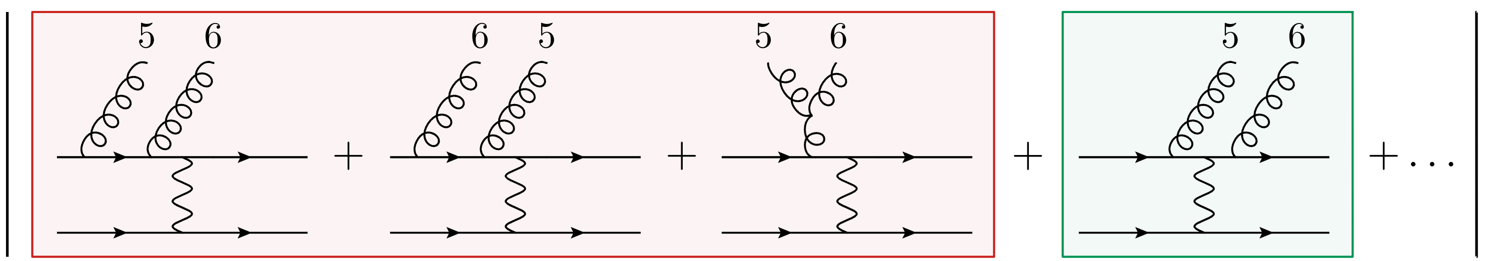
- Singularities in **double collinear sectors** are separated.

- Different collinear singularities in **triple collinear partitions** are isolated in the angular phase space.

- We separate them by **splitting the phase space** into different sectors.

# Partition functions

$$\left|M^{\rm nnlo}(\{p\},p_5,p_6)\right|^2 =$$




- The different configurations are separated by **introducing partition functions** in the phase space

$$1 = \boxed{w^{51,61}} + w^{54,64} + \boxed{w^{51,64}} + w^{54,61}$$

with

$$\lim_{5\|l} w^{5i,6j} \sim \delta_{li}\,, \quad \lim_{6\|l} w^{5i,6j} \sim \delta_{lj} \quad \text{and} \quad \lim_{5\|i}\lim_{6\|j} w^{5i,6j} = 1\,.$$

- One possible choice

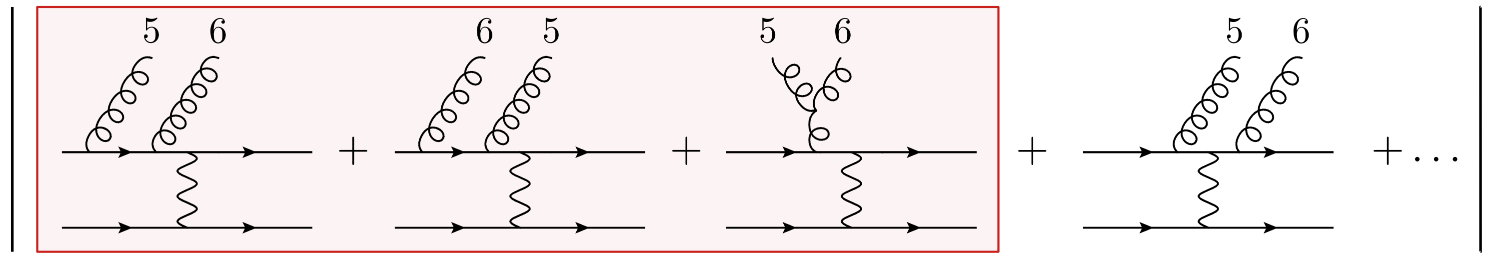
$$w^{51,61} = \frac{\rho_{54}\rho_{64}}{d_5 d_6}\left(1 + \frac{\rho_{51}}{d_{5641}} + \frac{\rho_{61}}{d_{5614}}\right), \quad w^{51,64} = \frac{\rho_{54}\rho_{61}\rho_{56}}{d_5 d_6 d_{5614}}\,,$$
$$w^{54,64} = \frac{\rho_{51}\rho_{61}}{d_5 d_6}\left(1 + \frac{\rho_{64}}{d_{5641}} + \frac{\rho_{54}}{d_{5614}}\right), \quad w^{54,61} = \frac{\rho_{51}\rho_{64}\rho_{56}}{d_5 d_6 d_{5641}}\,,$$

where

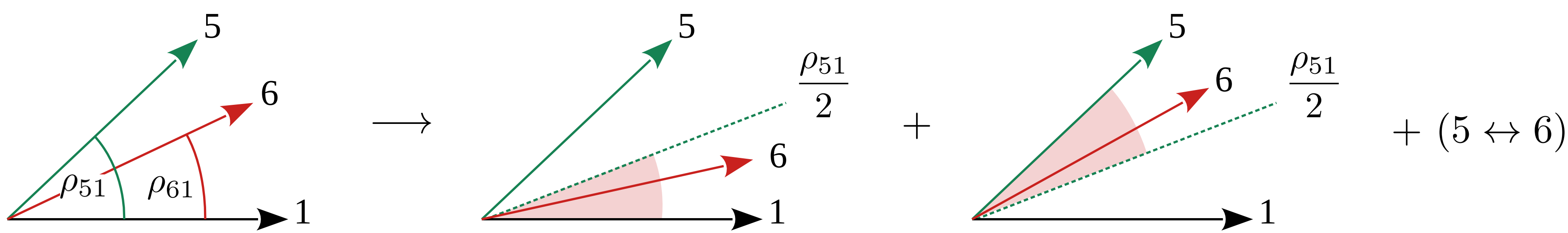
$$d_{i=5,6} \equiv \rho_{1i} + \rho_{4i}\,, \quad d_{5614} \equiv \rho_{56} + \rho_{51} + \rho_{64}\,, \quad d_{5641} \equiv \rho_{56} + \rho_{54} + \rho_{61}\,.$$

# Splitting of the angular phase space

$$\left|M^{\rm nnlo}(\{p\},p_5,p_6)\right|^2 =$$




- As example consider partition $\boxed{w^{51,61}}$: it is singular when $(5||1)$, $(6||1)$ and $(5||6)$




double-collinear $(6||1)$
triple-collinear $(6||5||1)$

double-collinear $(6||5)$
triple-collinear $(6||5||1)$

- In practice this is done by introducing the unity

$$1 = \theta\left(\rho_{61} < \frac{\rho_{51}}{2}\right) + \theta\left(\frac{\rho_{51}}{2} < \rho_{61} < \rho_{51}\right) + \theta\left(\rho_{51} < \frac{\rho_{61}}{2}\right) + \theta\left(\frac{\rho_{61}}{2} < \rho_{51} < \rho_{61}\right)$$

- To integrate singularities analytically it is crucial the the phase space is parameterized in such a way that all singularities are made explicit [Czakon]

# Subtraction terms before NLO regulation

- Single collinear finial state emission

$$\left\langle [I - \mathbb{S}][I - S_6]\left[C_{54}w^{54,61} + C_{64}w^{51,64} + \left(\theta^{(a)}C_{64} + \theta^{(c)}C_{54}\right)w^{54,64}\right][\mathrm{d}g_5][\mathrm{d}g_6]F_{LM}(1,4,5,6)\right\rangle$$
$$= \frac{[\alpha_s]C_F}{\epsilon}\left\langle \left[\left(\frac{1}{\epsilon} + Z^{2,2}\right)(2E_4)^{-2\epsilon} - (2E_5)^{-2\epsilon}\right]\left(w_{\mathrm{DC}}^{51} + w_{\mathrm{TC}}^{54}\left(\frac{\rho_{54}}{4}\right)^{-\epsilon}\right) \boxed{F_{LM}(1,4,5)}\right\rangle$$
$$- \frac{[\alpha_s]^2C_F^2}{\epsilon^3}\left(\frac{1}{2\epsilon} + Z^{2,4}\right)\left\langle \langle \Delta_{51}\rangle_{S_5}(2E_4)^{-4\epsilon}F_{LM}(1,4)\right\rangle .$$

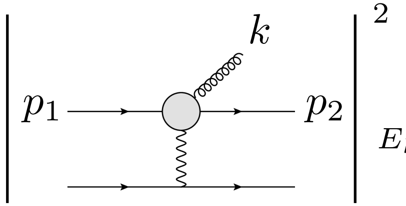
with

$$Z^{n,m} = -\frac{2}{m\epsilon} - \int_0^1 \mathrm{d}z\; z^{-n\epsilon}(1-z)^{-m\epsilon}P_{qq}(z) = \frac{3}{2} + \frac{1}{12}\left[6 + 21m + 15n - 4n\pi^2\right]\epsilon + \mathcal{O}(\epsilon^2)\,,$$
$$\langle \Delta_{51}\rangle_{S_5} = \left(-\frac{1}{\epsilon}\left[\frac{1}{8\pi^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\right]2^{-2\epsilon}\right)^{-1}\int \mathrm{d}\Omega_5^{(d-1)}\;\frac{\rho_{14}}{\rho_{15}\rho_{45}}\left[w_{\mathrm{DC}}^{51} + w_{\mathrm{TC}}^{54}\left(\frac{\rho_{54}}{4}\right)^{-\epsilon}\right] = \frac{3}{2} + \epsilon\left(\frac{\ln 2}{2} - 2\ln\eta_{14}\right) + \mathcal{O}(\epsilon^2)\,,$$
$$w_{\mathrm{DC}}^{51} = C_{64}w^{51,64}\,,$$
$$w_{\mathrm{TC}}^{54} = C_{64}w^{54,64}\,.$$

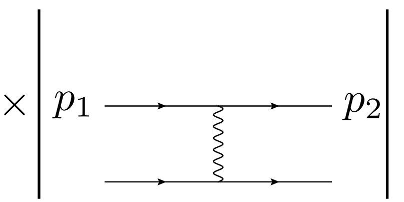
- The subtraction terms contains the **NLO differential cross-section** with **NLO singularities**

# Single and double soft limit

- Single soft at NLO




$$\approx_{E_k \to 0} 2C_F\ g_{s,b}^2 \times \underbrace{\frac{p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}}_{\text{Eikonal function}} \times$$




- Single soft at NNLO

$$S_6 F_{\mathrm{LM}}(1,4,5,6) = g_{s,b}^2 \times \frac{1}{E_6^2}\left[(2C_F - C_A)\frac{\rho_{14}}{\rho_{16}\rho_{46}} + C_A\left(\frac{\rho_{15}}{\rho_{16}\rho_{56}} + \frac{\rho_{45}}{\rho_{46}\rho_{56}}\right)\right] \times F_{\mathrm{LM}}(1,4,5)$$

- Double soft eikonal

$$\begin{aligned}
\text{Eikonal}(1,4,6,7) &= 4C_F^2 S_{14}(6) S_{14}(7) + C_A C_F\left[2S_{12}(6,7) - S_{11}(6,7) - S_{22}(6,7)\right], \\
S_{ij}(k) &= \frac{p_i \cdot p_j}{[p_i \cdot p_k][p_j \cdot p_k]}, \\
S_{ij}(k,l) &= S_{ij}^{\mathrm{so}}(k,l) - \frac{2[p_i \cdot p_j]}{[p_k \cdot p_l][p_i \cdot (p_k + p_l)][p_j \cdot (p_k + p_l)]} \\
&\quad + \frac{[p_i \cdot p_k][p_j \cdot p_l] + [p_i \cdot p_l][p_j \cdot p_k]}{[p_i \cdot (p_k + p_l)][p_j \cdot (p_k + p_l)]}\left(\frac{1-\epsilon}{[p_k \cdot p_l]^2} - \frac{1}{2} S_{ij}^{\mathrm{so}}(k,l)\right), \\
S_{ij}^{\mathrm{so}}(k,l) &= \frac{p_i \cdot p_j}{p_k \cdot p_l}\left(\frac{1}{[p_i \cdot p_k][p_j \cdot p_l]} + \frac{1}{[p_i \cdot p_l][p_j \cdot p_k]}\right) - \frac{[p_i \cdot p_j]^2}{[p_i \cdot p_k][p_j \cdot p_k][p_i \cdot p_l][p_j \cdot p_l]}.
\end{aligned}$$