A NEW WILSON LINE-BASED CLASSICAL ACTION FOR GLUODYNAMICS

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in collaboration with:
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based on 2102.1137

supported by:
NATIONAL SCIENCE CENTRE POLAND,
GRANT NO. DEC-2018/31/D/ST2/02731

AGH

RADCOR/LoopFest 2021

PLAN

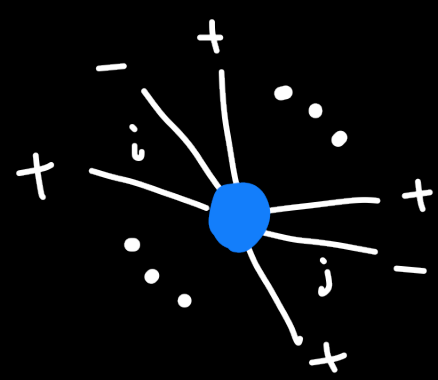
INTRODUCTION

Cachazo-Svrcek-Witten (CSW) method

MHV vertices

The Maximally Helicity Violating (MHV) amplitudes can be treated as interaction vertices.

[F. Cachazo, P. Svrcek, E. Witten, 2004]



$$= g^{n-2}\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

↑ off-shell spinor products

spinor products (on-shell)

$$\langle ij\rangle = \epsilon_{\alpha\beta}\lambda_i^\alpha\lambda_j^\beta$$ ← + helicity spinor

$$[ij] = \epsilon_{\dot\alpha\dot\beta}\tilde\lambda_i^{\dot\alpha}\tilde\lambda_j^{\dot\beta}$$ ← − helicity spinor

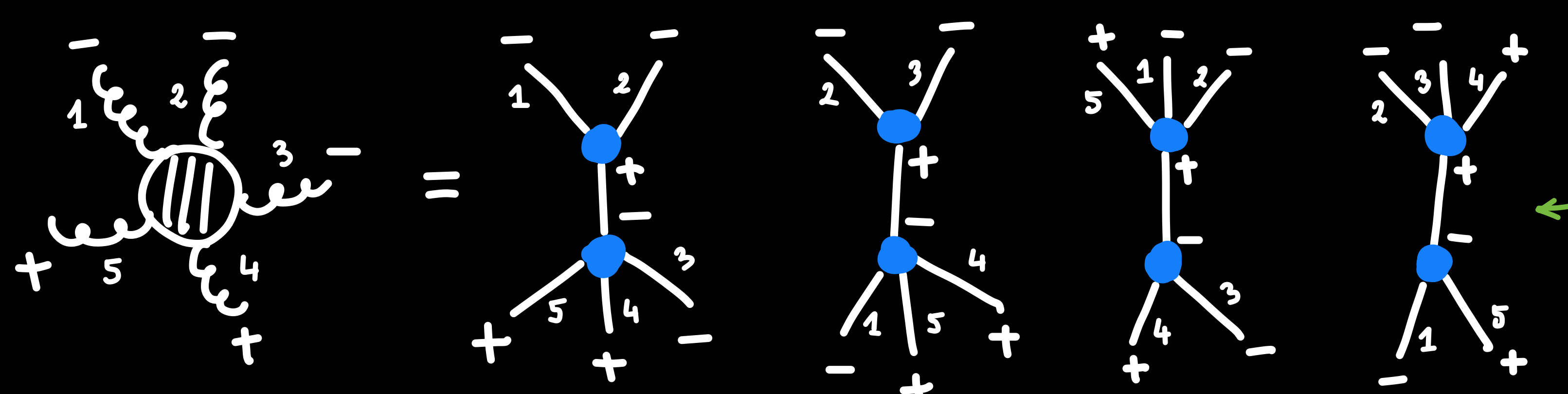
off-shell continuations

$$\lambda^*_{i\,\alpha} = (k_i)_{\alpha\dot\alpha}\tilde\eta_j^{\dot\alpha} = (k_i^\mu\sigma_\mu)_{\alpha\dot\alpha}\tilde\eta_j^{\dot\alpha}$$

↑ $(\frac{1}{2},0)\times(0,\frac{1}{2})$ representation of k^μ, $k^2\neq 0$

↑ auxiliary spinor

example: tree-level NMHV



Lagrangian for the CSW method (1)

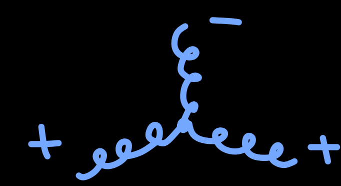
Yang-Mills theory on the light cone

[J. Scherk, J.H. Schwarz, 1975]

Set the light cone gauge $A^+ = 0$ and integrate out A^- :

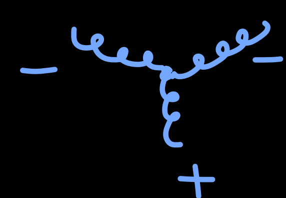
$$\hat{A} \equiv t^a A_a(x^+, \mathbf{x})$$

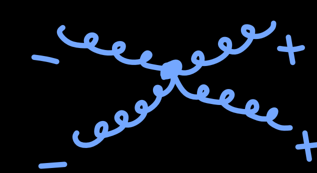
$$\mathbf{x} \equiv (x^-, x^\bullet, x^\star)$$



$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[A^\bullet, A^\star\right] = \int dx^+ \int d^3\mathbf{x} \left\{ -\mathrm{Tr}\,\hat{A}^\bullet \Box \hat{A}^\star - 2ig\,\mathrm{Tr}\,\partial_-^{-1}\partial_\bullet \hat{A}^\bullet \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right] - 2ig\,\mathrm{Tr}\,\partial_-^{-1}\partial_\star \hat{A}^\star \left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right] - 2g^2\,\mathrm{Tr}\left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right] \partial_-^{-2} \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right] \right\}$$

\+ −
helicity field





double-null coordinates

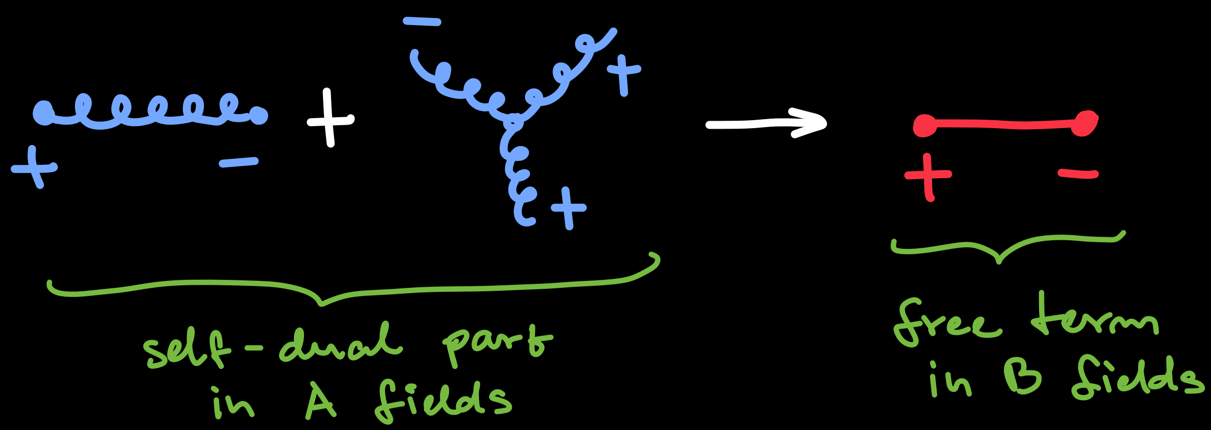
$$v^+ = v\cdot\eta = \frac{1}{\sqrt{2}}(v^0 + v^3) \qquad v^\bullet = v\cdot\varepsilon_\perp^+ = \frac{1}{\sqrt{2}}(v^1 + iv^2) \qquad \eta = \frac{1}{\sqrt{2}}(1,0,0,-1) \qquad \varepsilon_\perp^+ = \frac{-1}{\sqrt{2}}(0,1,+i,0)$$

$$v^- = v\cdot\tilde{\eta} = \frac{1}{\sqrt{2}}(v^0 - v^3) \qquad v^\star = v\cdot\varepsilon_\perp^- = \frac{1}{\sqrt{2}}(v^1 - iv^2) \qquad \tilde{\eta} = \frac{1}{\sqrt{2}}(1,0,0,1) \qquad \varepsilon_\perp^- = \frac{-1}{\sqrt{2}}(0,1,-i,0)$$

MHV action

Apply the canonical field transformation (at equal LC time) $\{A^\bullet, A^\star\} \rightarrow \{B^\bullet, B^\star\}$

such that:



[P. Mansfield, 2006]

and

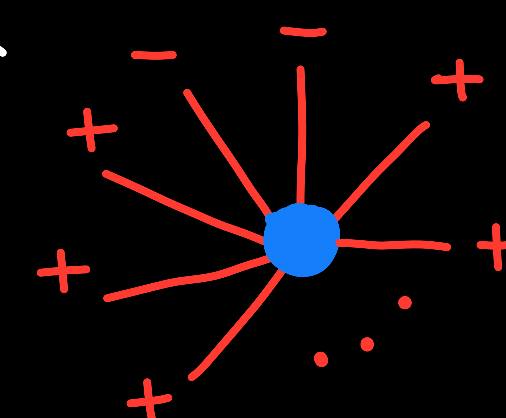
$$\partial\_A^\star_a\left(x^+;\mathbf{x}\right) = \int d^3\mathbf{y}\,\frac{\delta B^\bullet_c\left(x^+;\mathbf{y}\right)}{\delta A^\bullet_a\left(x^+;\mathbf{x}\right)}\,\partial\_B^\star_c\left(x^+;\mathbf{y}\right)$$

the solutions:

$$\widetilde{A}^\bullet_a[B^\bullet](x^+;\mathbf{P}) = \sum_{n=1}^{\infty}\int d^3\mathbf{p}_1\ldots d^3\mathbf{p}_n\,\widetilde{\Psi}_n^{ab_1\ldots b_n}(\mathbf{P};\mathbf{p}_1,\ldots,\mathbf{p}_n)\prod_{i=1}^{n}\widetilde{B}^\bullet_{b_i}(x^+;\mathbf{p}_i)$$

$$\widetilde{A}^\star_a[B^\bullet,B^\star](x^+;\mathbf{P}) = \sum_{n=1}^{\infty}\int d^3\mathbf{p}_1\ldots d^3\mathbf{p}_n\,\widetilde{\Omega}_n^{ab_1b_2\cdots b_n}(\mathbf{P};\mathbf{p_1},\mathbf{p_2},\ldots,\mathbf{p_n})\widetilde{B}^\star_{b_1}(x^+;\mathbf{p}_1)\prod_{i=2}^{n}\widetilde{B}^\bullet_{b_i}(x^+;\mathbf{p}_i)\,,$$

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[B^\bullet,B^\star\right] = \int dx^+\left(-\int d^3\mathbf{x}\,\mathrm{Tr}\,\hat{B}^\bullet\,\square\,\hat{B}^\star + \mathcal{L}^{(\mathrm{LC})}_{--+} + \ldots + \mathcal{L}^{(\mathrm{LC})}_{--+\ldots+} + \ldots\right)$$



MHV vertices

List of papers on the subject (incomplete list)

CSW method

- F. Cachazo, P. Svrcek, E. Witten, JHEP 09 (2004) 006
- F. Cachazo, P. Svrcek, E. Witten, JHEP 10 (2004) 074
- G. Georgiou, V. Khoze, JHEP 05 (2004)
- G. Georgiou, E.W.N. Glover, V.V. Khoze, JHEP 07 (2004)
- J.-B. Wu, C.-J. Zhu, JHEP 07 (2004) 032
- L.J. Dixon, E.W.N. Glover, V.V. Khoze, JHEP 12 (2004) 015
- K. Risager, JHEP 12 (2005) 003
- A. Brandhuber, B. Spence, G. Travaglini, JHEP 01 (2006) 142
- M. Kiermaier, S.G. Naculich, JHEP 05 (2009) 072
- T. Adamo, L. Mason, Phys.Rev.D 86 (2012) 065019

Lagrangian formulation of CSW at loop level

- A. Brandhuber, B. Spence, G. Travaglini, JHEP 02 (2007) 088
- A. Brandhuber, B. Spence, G. Travaglini, K. Zoubos, JHEP 07 (2007) 002
- J.H. Ettle, C.-H. Fu, J.P. Fudger, P. Mansfield, T.R. Morris, JHEP 05 (2007) 011
- R. Boels, C. Schwinn, JHEP 07 (2008) 007
- C.-H. Fu, J.P. Fudger, P. Mansfield, T.R. Morris, Z. Xiao, JHEP 06 (2009) 035
- H. Elvang, D.Z. Freedman, M. Kiermaier, JHEP 06 (2012) 015

Lagrangian formulation of CSW

- P. Mansfield, JHEP 03 (2006) 037
- J.H. Ettle, T.R. Morris, JHEP 08 (2006) 003
- A. Gorsky, A. Rosly, JHEP 01 (2006) 101
- J.H. Ettle, T.R. Morris, Z. Xiao, JHEP 08 (2008) 103
- T.R. Morris, Z. Xiao, JHEP 12 (2008) 028
- H. Feng, Y.-T. Huang, JHEP 04 (2009) 047
- C.-H. Fu, JHEP 04 (2010) 044
- S. Buchta, S. Weinzierl, JHEP 09 (2010) 071

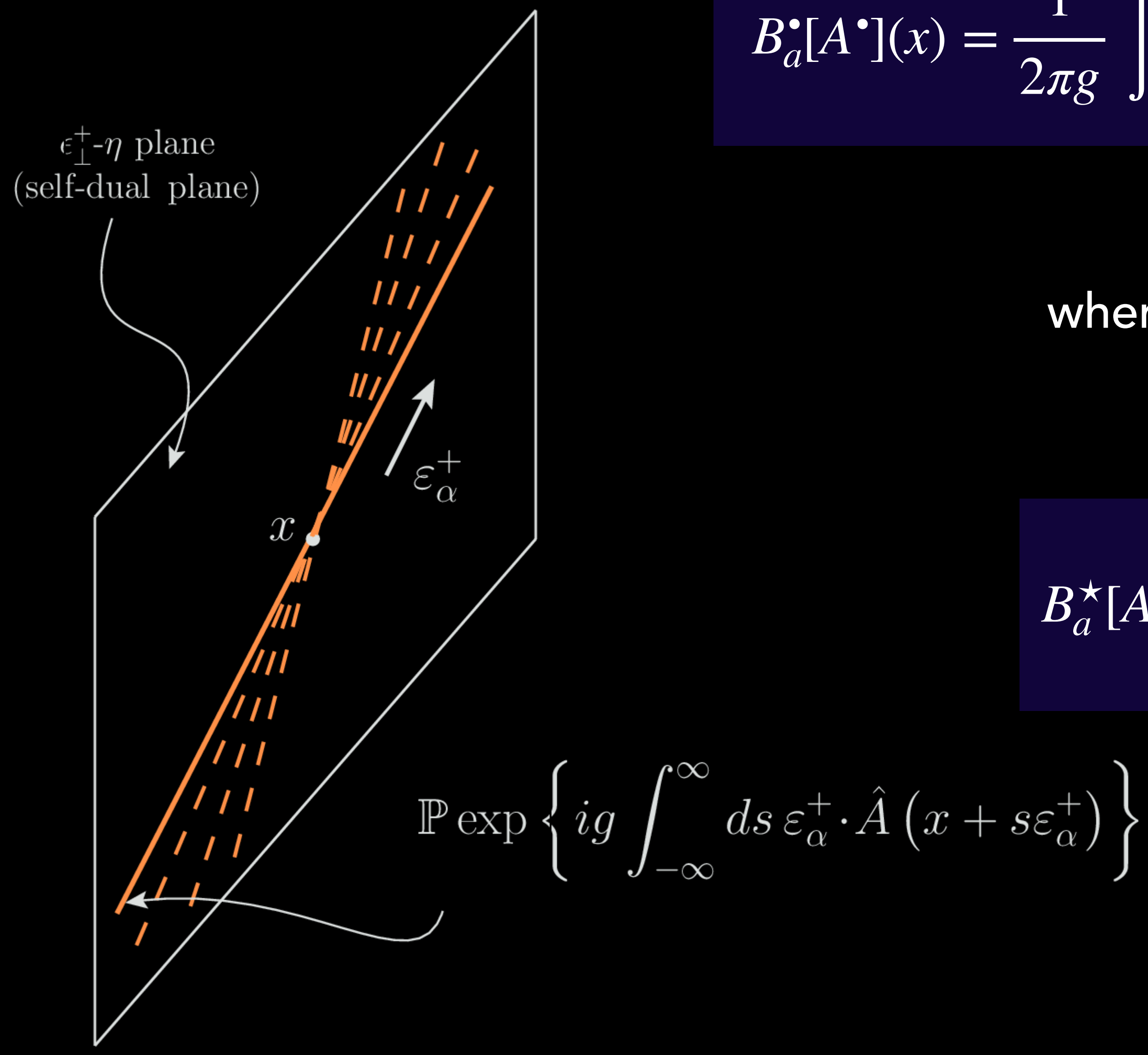
On Wilson lines in MHV Lagrangian

- P.K, A. Stasto, JHEP 09 (2017)
- H. Kakkad, P.K, A. Stasto, Phys.Rev.D 102 (2020) 9

Inverse solutions to field transformations

[PK, A. Stasto, 2017]
[H. Kakkad, PK, A. Stasto, 2020]

In order to understand the field transformations better, solve for B fields:



$$B_a^\bullet[A^\bullet](x) = \frac{1}{2\pi g}\int_{-\infty}^{\infty} d\alpha\, \mathrm{Tr}\left\{t^a \partial_- \mathbb{P}\exp\left[ig\int_{-\infty}^{\infty} ds\; \underbrace{\varepsilon_\alpha^+ \cdot \hat{A}}_{A^\bullet}\left(x+s\varepsilon_\alpha^+\right)\right]\right\}$$

where $\varepsilon_\alpha^{\pm\,\mu} = \varepsilon_\perp^{\pm\,\mu} - \alpha\eta^\mu$ $\qquad x \equiv (x^+, x^-, x^\bullet, x^\star)$

$$B_a^\star[A^\bullet, A^\star](x) = \int d^3\mathbf{y}\left[\frac{\partial_-^2(y)}{\partial_-^2(x)}\frac{\delta B_a^\bullet[A^\bullet](x^+;\mathbf{x})}{\delta A_c^\bullet(x^+;\mathbf{y})}\right]A_c^\star(x^+;\mathbf{y})$$

functional derivative of the Wilson line

Can one generalize to transformations supported on both self-dual and anti-self-dual plane?

New fields $Z^\bullet, Z^\star$

[H. Kakkad, PK, A. Stasto, 2021]

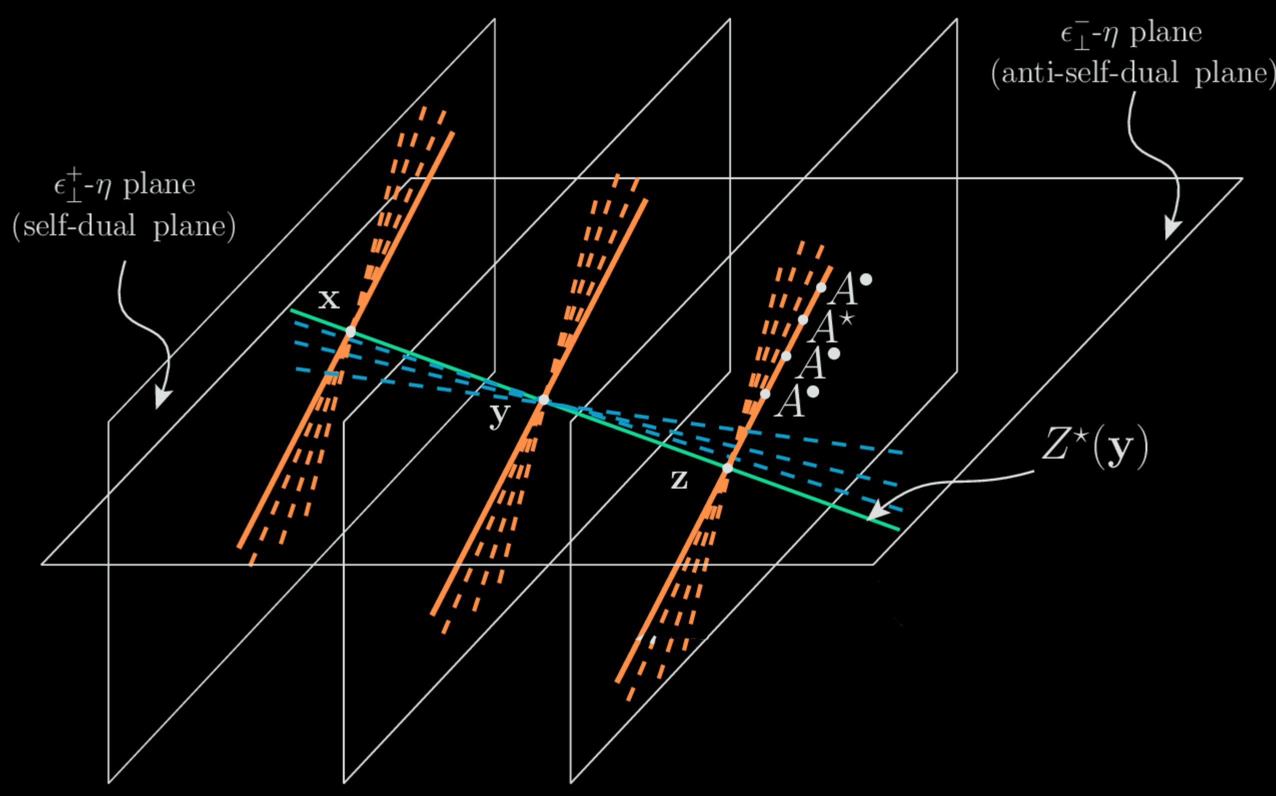
Introduce a canonical transformation $\{A^\bullet, A^\star\} \to \{Z^\bullet, Z^\star\}$ given by the generating functional:

$$\mathcal{G}[A^\bullet, Z^\star](x^+) = -\int d^3\mathbf{x}\ \mathrm{Tr}\, \hat{\mathcal{W}}^{-1}_{(-)}[Z](x)\ \partial_- \hat{\mathcal{W}}_{(+)}[A](x)$$

where

$$\mathcal{W}^a_{(\pm)}[K](x) = \frac{1}{2\pi g}\int_{-\infty}^{\infty} d\alpha\, \mathrm{Tr}\left\{t^a \partial_- \mathbb{P}\exp\left[ig\int_{-\infty}^{\infty} ds\ \varepsilon^{\pm}_{\alpha}\cdot \hat{K}\left(x + s\varepsilon^{\pm}_{\alpha}\right)\right]\right\}$$

Wilson line on self-dual or anti-self-dual plane



Relations between A and Z fields:

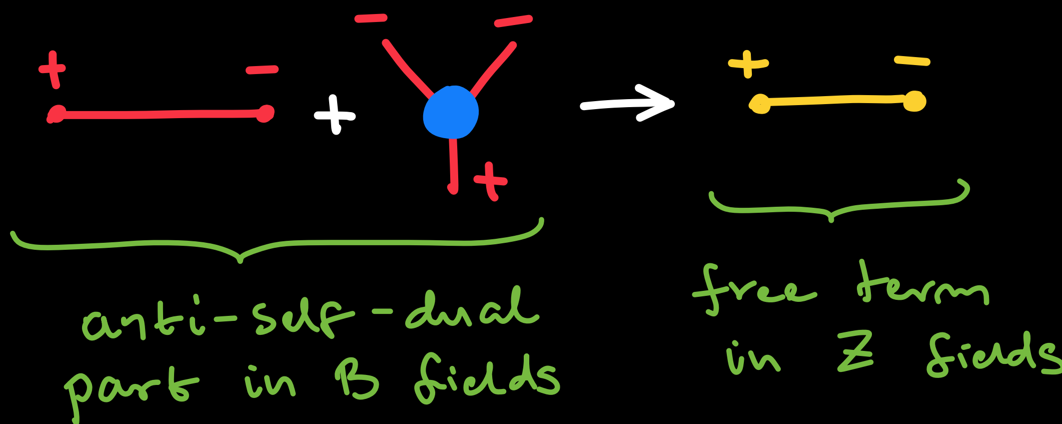
$$\partial_- A^\star_a(x^+, \mathbf{y}) = \frac{\delta\, \mathcal{G}[A^\bullet, Z^\star](x^+)}{\delta A^\bullet_a\left(x^+, \mathbf{y}\right)}$$

$$\partial_- Z^\bullet_a(x^+, \mathbf{y}) = -\frac{\delta\, \mathcal{G}[A^\bullet, Z^\star](x^+)}{\delta Z^\star_a\left(x^+, \mathbf{y}\right)}$$

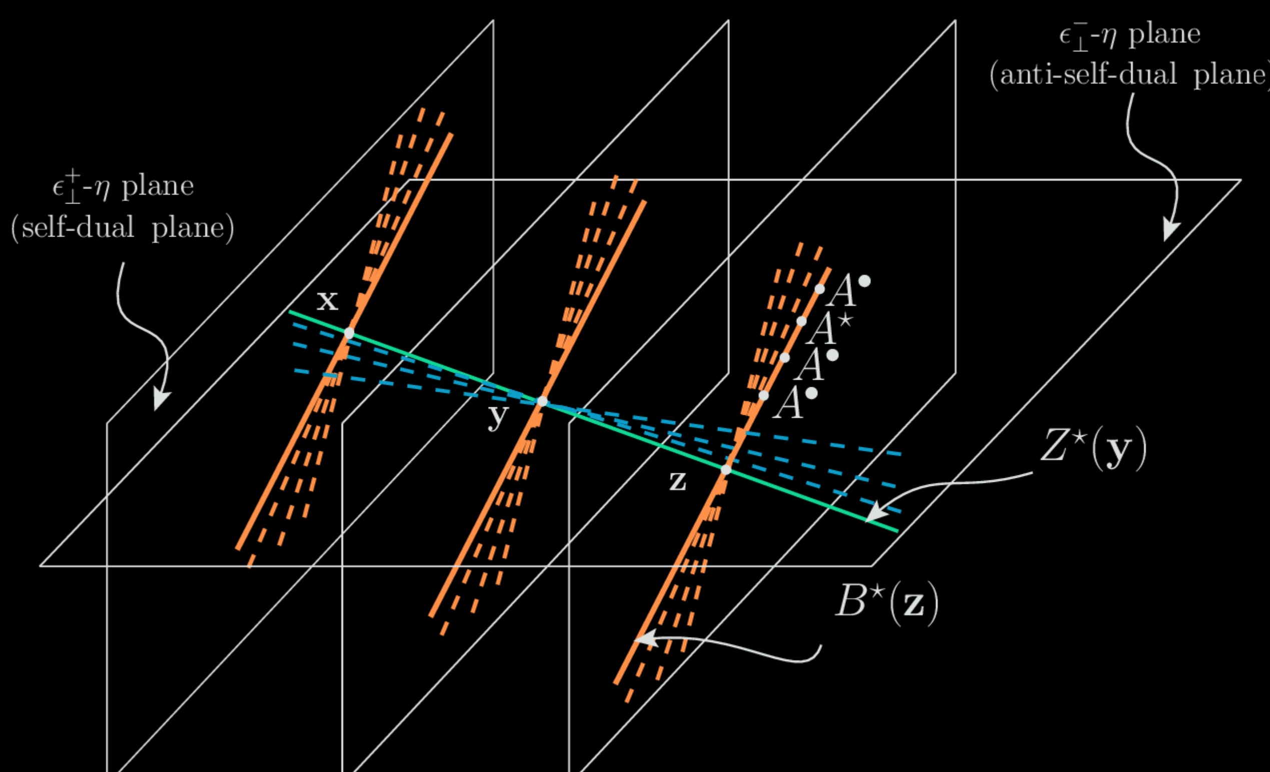
Canonical transformation of the B fields in the MHV action

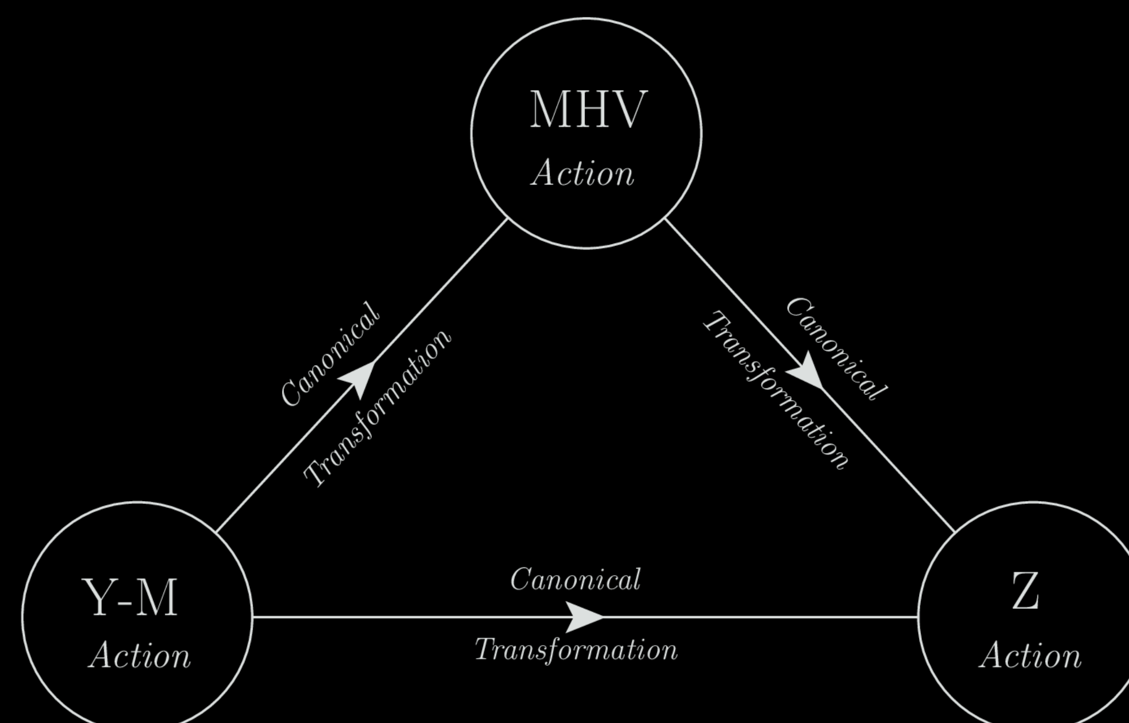
[H. Kakkad, PK, A. Stasto, 2021]

It turns out, that the new fields can be introduced also from the MHV action :



$$
\begin{cases}
Z^{\star}_{a}[B^{\star}](x) = \mathscr{W}^{a}_{(-)}[B](x) \\
Z^{\bullet}_{a}[B^{\bullet}, B^{\star}](x) = \int d^3\mathbf{y} \left[\dfrac{\partial_-^2(y)}{\partial_-^2(x)} \dfrac{\delta Z^{\star}_{a}[B^{\star}](x^+;\mathbf{x})}{\delta B^{\star}_{c}(x^+;\mathbf{y})} \right] B^{\bullet}_{c}(x^+;\mathbf{y})
\end{cases}
$$





Z-field action

[H. Kakkad, PK, A. Stasto, 2021]

Solving the field transformation relations we get the following action:

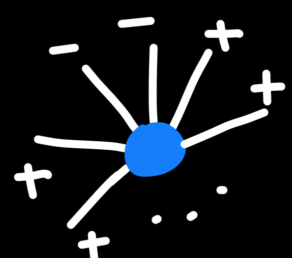
$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[Z^{\bullet},Z^{\star}\right] = \int dx^{+}\Bigg\{-\int d^{3}\mathbf{x}\,\mathrm{Tr}\,\hat{Z}^{\bullet}\,\Box\,\hat{Z}^{\star}$$

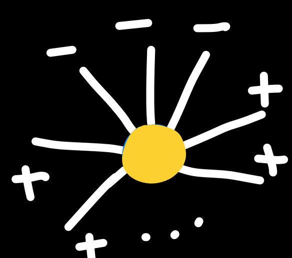
$$+\mathcal{L}^{(\mathrm{LC})}_{--++}+\mathcal{L}^{(\mathrm{LC})}_{--+++}+\mathcal{L}^{(\mathrm{LC})}_{--++++}+\dots$$

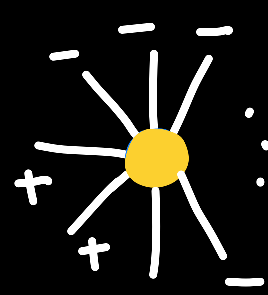
$$+\mathcal{L}^{(\mathrm{LC})}_{---++}+\mathcal{L}^{(\mathrm{LC})}_{---+++}+\mathcal{L}^{(\mathrm{LC})}_{---++++}+\dots$$

$$\vdots$$

$$+\mathcal{L}^{(\mathrm{LC})}_{---\dots-++}+\mathcal{L}^{(\mathrm{LC})}_{---\dots-+++}+\mathcal{L}^{(\mathrm{LC})}_{---\dots-++++}+\dots\Bigg\}$$





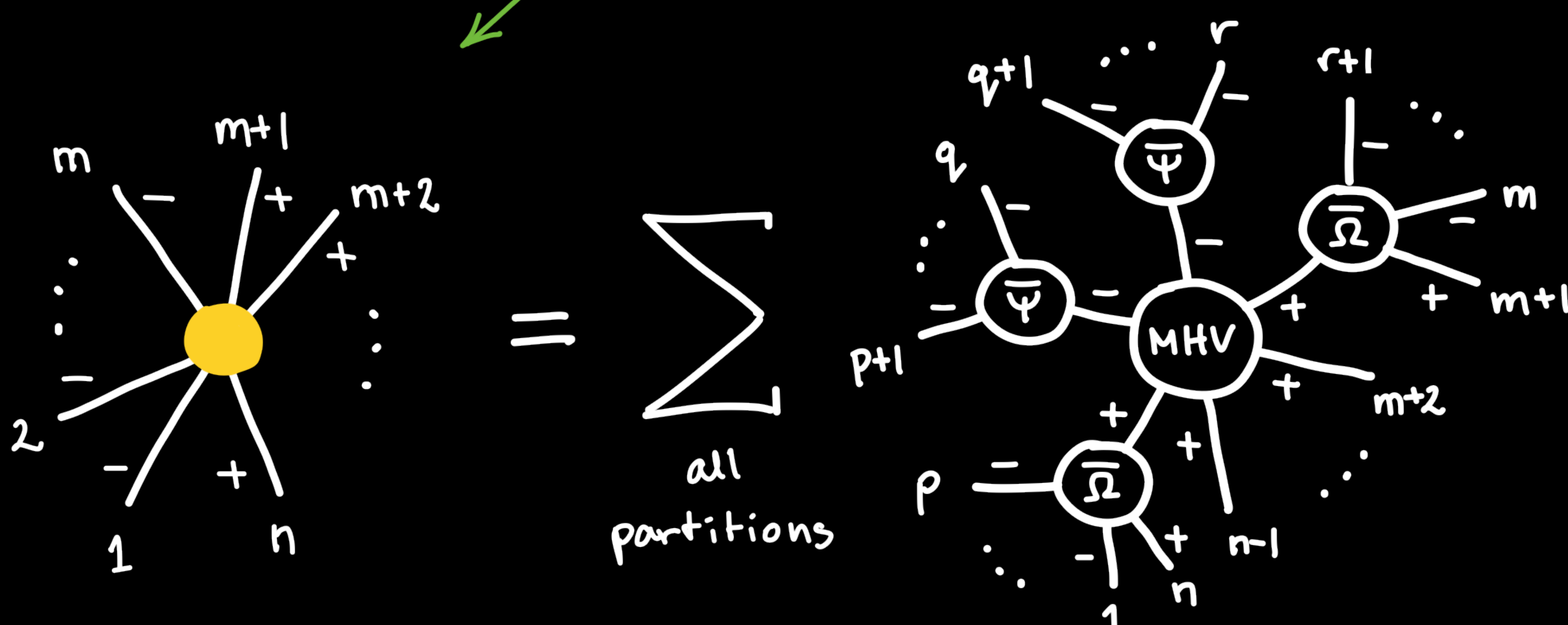


No triple gluon vertices!

Master formula for a vertex

[H. Kakkad, PK, A. Stasto, 2021]

$$\mathcal{L}^{(\mathrm{LC})}_{\underbrace{- \cdots -}_{m}\underbrace{+ \cdots +}_{n-m}} = \int d^3\mathbf{y}_1 \dots d^3\mathbf{y}_n \, \mathcal{U}^{b_1\dots b_n}_{-\dots-+\dots+}(\mathbf{y}_1,\cdots \mathbf{y}_n) \prod_{i=1}^{m} Z^{\star}_{b_i}(x^+;\mathbf{y}_i) \prod_{j=1}^{n-m} Z_{b_j}(x^+;\mathbf{y}_j)$$



Inverse Wilson line kernels

$$\overline{\widetilde{\Psi}}{}^{a\{b_1\cdots b_n\}}_{n}(\mathbf{P};\{\mathbf{p}_1,\dots,\mathbf{p}_n\}) = -(-g)^{n-1}\frac{\widetilde{v}_{(1\cdots n)1}}{\widetilde{v}_{1(1\cdots n)}}\frac{\delta^3(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{P})\ \mathrm{Tr}(t^a t^{b_1}\cdots t^{b_n})}{\widetilde{v}_{21}\widetilde{v}_{32}\cdots\widetilde{v}_{n(n-1)}}$$

$$\overline{\widetilde{\Omega}}{}^{ab_1\{b_2\cdots b_n\}}_{n}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\dots,\mathbf{p}_n\}) = n\left(\frac{p_1^+}{p_{1\cdots n}^+}\right)^2 \overline{\widetilde{\Psi}}{}^{ab_1\cdots b_n}_{n}(\mathbf{P};\mathbf{p}_1,\dots,\mathbf{p}_n)\,.$$

$$\widetilde{v}_{ij} = p_i^+\left(\frac{p_j^\star}{p_j^+}-\frac{p_i^\star}{p_i^+}\right) = -(\varepsilon_i^-\cdot p_j) \sim [ij]$$

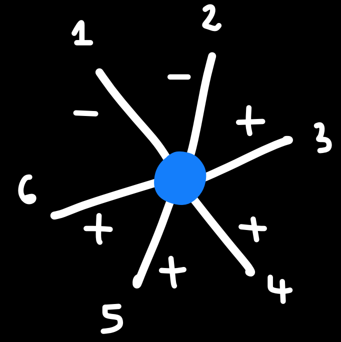
Example of amplitude calculation (1)

6-point amplitudes

[H. Kakkad, PK, A. Stasto, 2021]

$$\mathscr{A}(1^{\pm},2^{+},3^{+},4^{+},5^{+},6^{+}) = 0$$

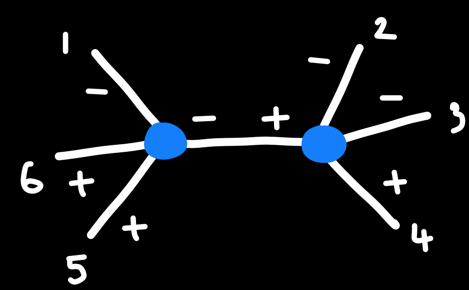
$$\mathscr{A}(1^{-},2^{-},3^{+},4^{+},5^{+},6^{+}) =$$

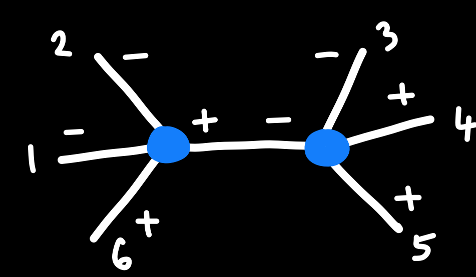


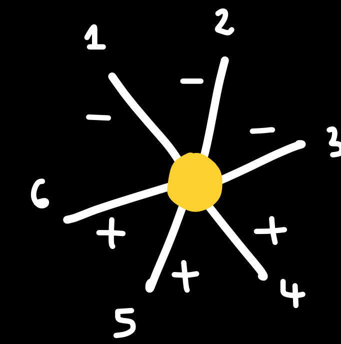
$$= g^4 \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{u}_{21}^4}{\widetilde{u}_{16}\widetilde{u}_{65}\widetilde{u}_{54}\widetilde{u}_{43}\widetilde{u}_{32}\widetilde{u}_{21}}$$

MHV

$$\mathscr{A}(1^{-},2^{-},3^{-},4^{+},5^{+},6^{+}) =$$

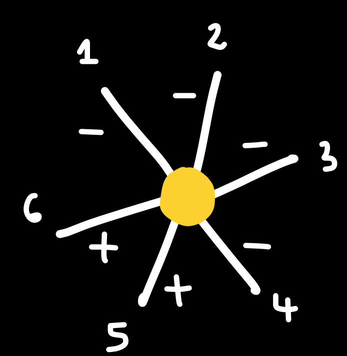






NMHV

$$\mathscr{A}(1^{-},2^{-},3^{-},4^{-},5^{+},6^{+}) =$$



$$= g^4 \left(\frac{p_5^+}{p_6^+}\right)^2 \frac{\widetilde{v}_{65}^4}{\widetilde{v}_{16}\widetilde{v}_{65}\widetilde{v}_{54}\widetilde{v}_{43}\widetilde{v}_{32}\widetilde{v}_{21}}$$

$\overline{\text{MHV}}$

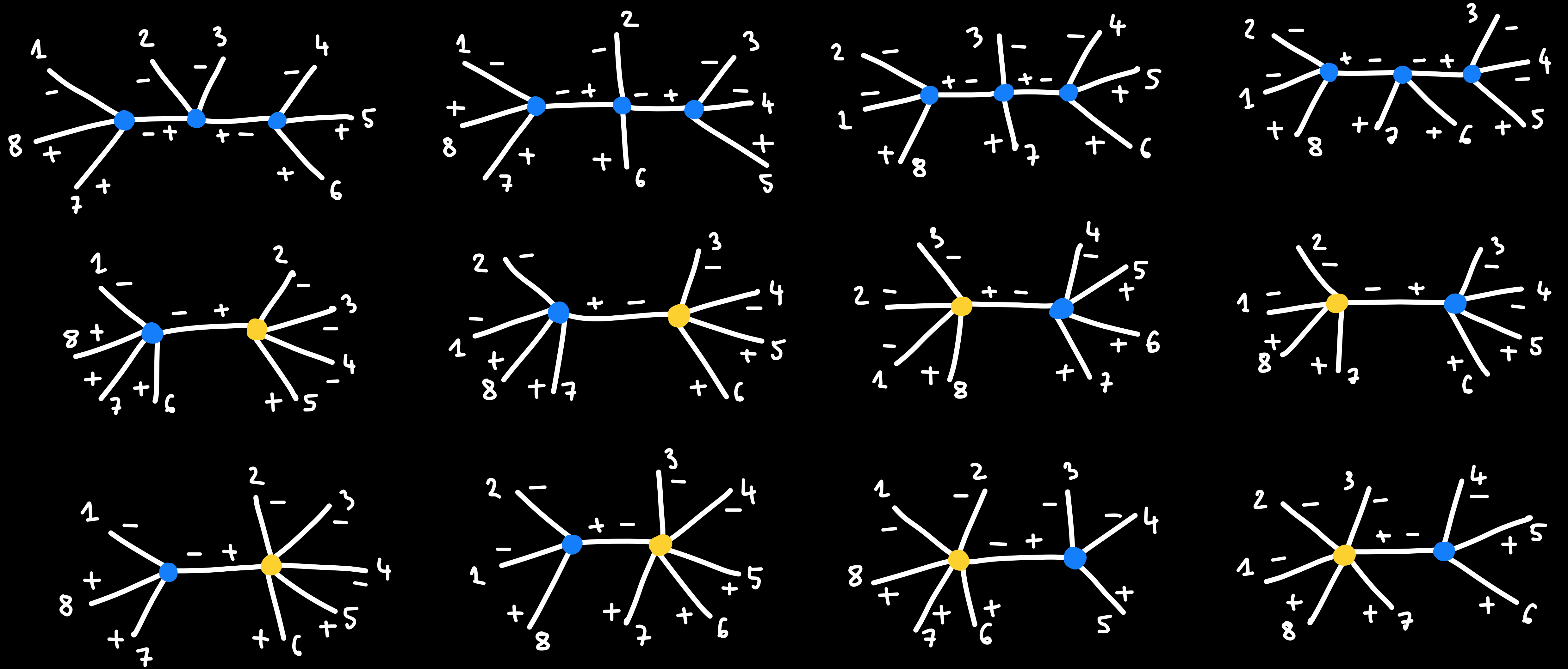
$$\widetilde{v}_{ij} = -(\varepsilon_i^- \cdot p_j) = p_i^+ \left(\frac{p_j^\star}{p_j^+} - \frac{p_i^\star}{p_i^+}\right) \sim [ij]$$

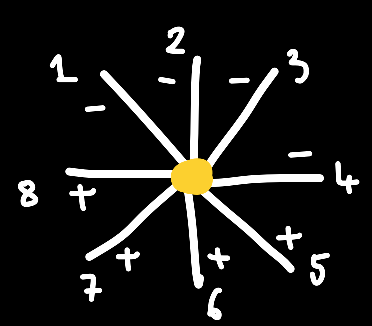
$$\widetilde{u}_{ij} = -(\varepsilon_i^+ \cdot p_j) = p_i^+ \left(\frac{p_j^\bullet}{p_j^+} - \frac{p_i^\bullet}{p_i^+}\right) \sim \langle ij \rangle$$

Example of amplitude calculation (2)

8-point NNMHV amplitude $\mathscr{A}(1^-,2^-,3^-,4^-,5^+,6^+,7^+,8^+)$

[H. Kakkad, PK, A. Stasto, 2021]





Summary: 13 diagrams

vertices used: 4p-MHV, 5p-MHV, 5p-$\overline{\text{MHV}}$, 6p-NMHV, 8p-NNMHV

CONCLUSIONS

Summary:

- A new classical action for gluodynamics was derived basing on canonical field transformation on the light cone Yang-Mills action.
- The new fields have very interesting geometric structure given by the Wilson line functionals.
- It allows for pretty convenient tree amplitude calculation, with fewer diagrams.

Work in progress:

- Quantum level action beyond all-plus one loop amplitudes.
- Better understanding of the geometry of the theory.
- Relation to twistor formulation?