# The 2-Loop Radiative Gluon Jet Function

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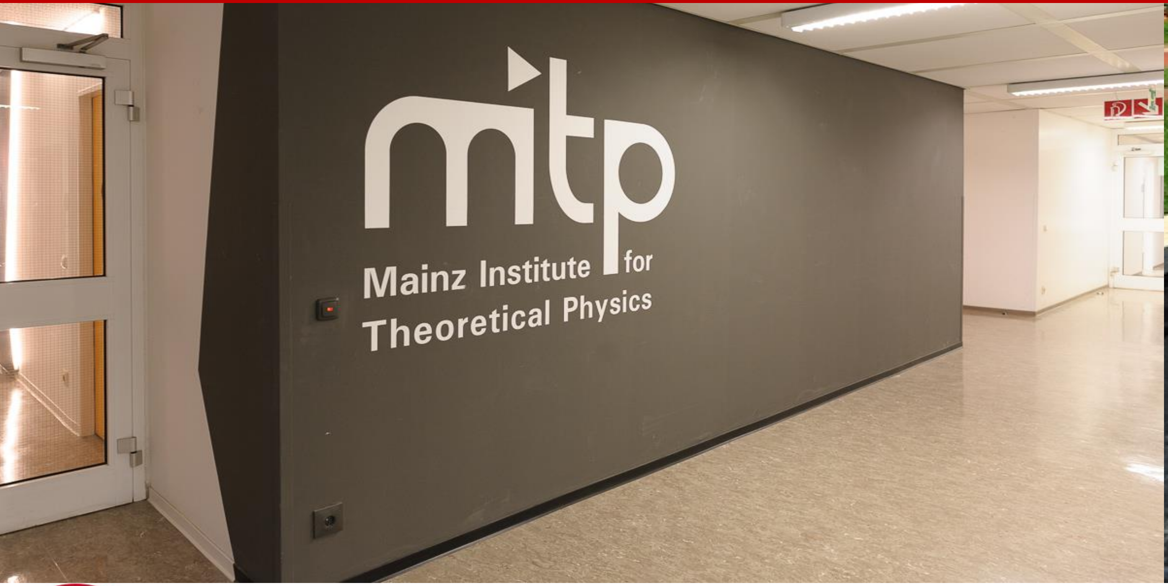
Z. Liu, B. Meçaj, M. Neubert, MS, X. Wang, arXiv: 1912.08818, 2003.03393, 2009.06779 and work in preparation





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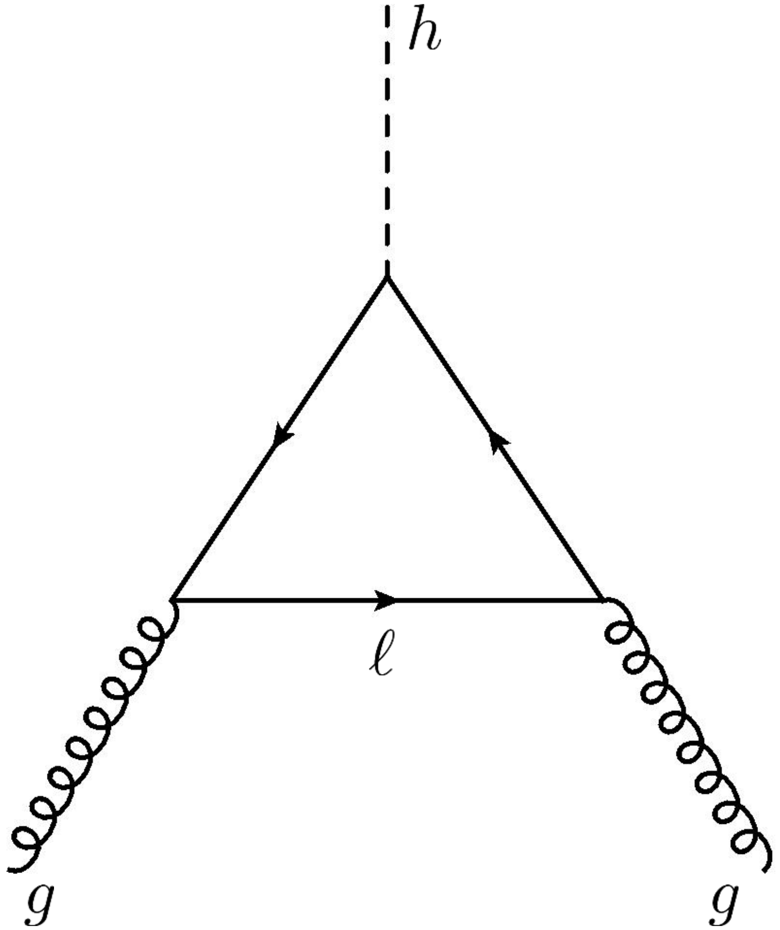
# Motivation and Introduction



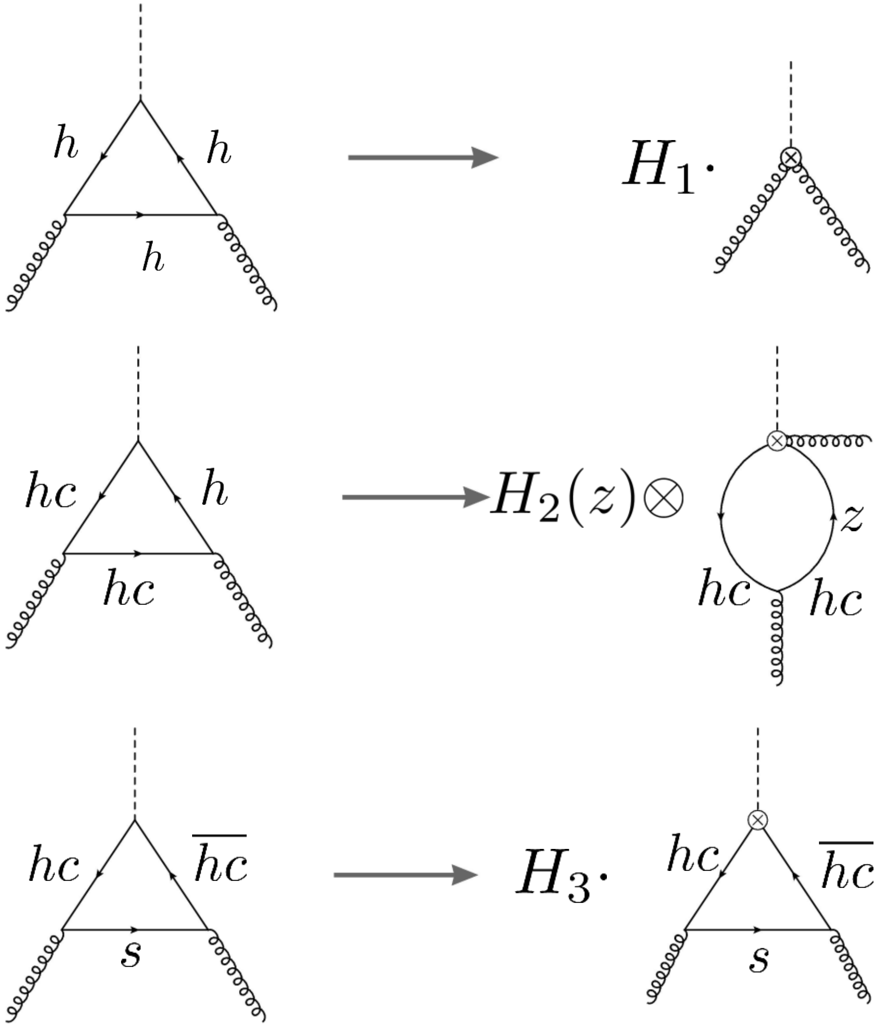

- Regard Higgs production/decay into two gluons via b-quark loop
- Not major contribution, but rich and interesting mathematical structure
- Work in SCET framework:

➢ decompose momenta into light-cone components:

$$\ell^\mu = (n_1 \cdot \ell)\frac{n_2^\mu}{2} + (n_2 \cdot \ell)\frac{n_1^\mu}{2} + l_\perp^\mu$$

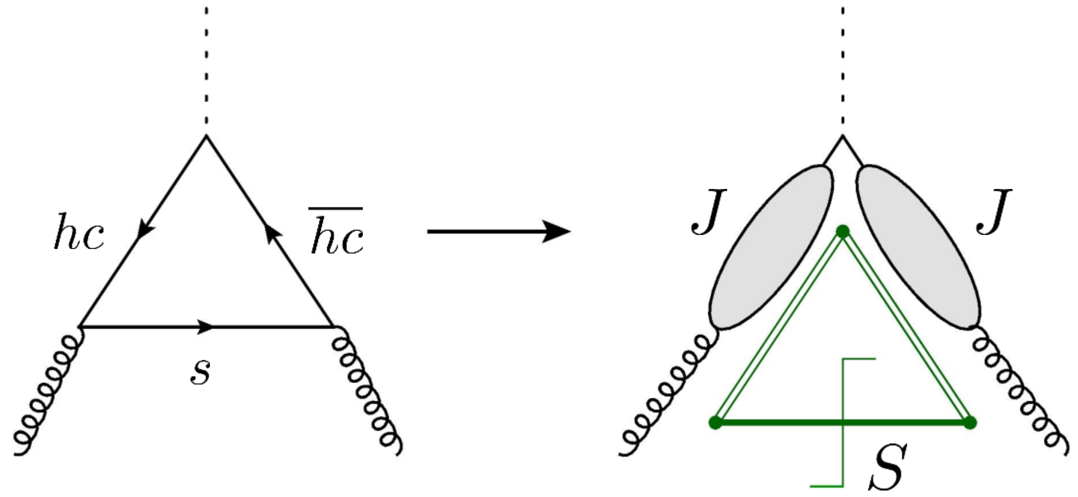
➢ $n_1^2 = n_2^2 = 0,\ n_1 \cdot n_2 = 2$

# Motivation and Introduction




- By applying method of regions, we identify the relevant modes and construct the relevant operators
- In third term insert two subleading SCET Lagrangians to couple hc-quarks to soft quarks [see also Neubert, Liu (2019)]

# Motivation and Introduction




- By applying method of regions, we identify the relevant modes and construct the relevant operators
- In third term insert two subleading SCET Lagrangians to couple hc-quarks to soft quarks [see also Neubert, Liu (2019)]
- In $O_3$ integrate out hc-momenta in matching of SCET-I to SCET-II

➢ $$\langle gg|O_3|h\rangle = \frac{ig^{\mu\nu}}{\pi} T_F \delta_{ab} \int_{-\infty}^{\infty} \frac{d\ell_+}{\ell_+} \int_{-\infty}^{\infty} \frac{d\ell_-}{\ell_-} J(\bar{n}_1 \cdot k_1 \ell_+) J(-\bar{n}_2 \cdot k_2 \ell_-) \mathcal{S}(\ell_+ \ell_-)$$

# The radiative Jet Function

- Compare with abelian case: The photon Jet Function from $h \to \gamma\gamma$
  - Exchange radiated gluon by radiated photon
  - Photon Jet Function also appears in B-meson decay $B^- \to \gamma \ell^- \bar{\nu}$
  - B-meson decay rate is proportional to

$$J \otimes \phi = \int_0^\infty \frac{d\omega}{\omega} J(-2E_\gamma \omega, \mu)\, \phi_+^B(\omega, \mu)$$

[Bosch, Lange, Neubert (2004)]

Jet Function — B-meson LCDA

  - Can deduce renormalisation of $J^\gamma$ based on RG consistency

# The radiative Jet Function

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- With $h \to gg$ factoriation as a starting point, we define the Jet Function by

$$\int d^D y e^{ip_s^- \cdot y} \langle g(k_2, a) | \mathrm{T} \left\{ \chi_{n_2}^{\beta k}(0) \left[ \bar{\chi}_{n_2}(y) \mathcal{A}\!\!\!/_{n_2}^{\perp}(y) \right]^{\gamma l} \right\} |0\rangle$$

$$= g_s T_{kl}^a \left[ \frac{\not{n}_2}{2} \not{\epsilon}_\perp^*(k_2) \right]^{\beta\gamma} \frac{i\bar{n}_2 \cdot k_2}{\bar{n}_2 \cdot k_2 \, n_2 \cdot p_s^- + i0} J\left(p^2\right)$$

- and $p = p_s + k_2, \;\; n_1 \cdot p = p_s^-, \;\; n_2 \cdot p = k_2^+$
- $p_s$ is carried away by the (multipole expanded) soft quark soft function
- k is collinear momentum of radiated gluon

# The radiative Jet Function

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- Comparison with radiative Photon Jet Function

$$\int d^D y e^{ip_s^- \cdot y} \langle \gamma(k_2)| \mathrm{T} \left\{ \chi_{n_2}^{\beta k}(0) \left[ \bar{\chi}_{n_2}(y) \left( \not{A}_{n_2}^{\perp}(y) + \not{\mathcal{A}}_{n_2}^{\perp}(y) \right) \right]^{\gamma l} \right\} |0\rangle$$

$$= e_b \delta^{kl} \left[ \frac{\not{n}_2}{2} \not{\epsilon}_\perp^*(k_2) \right]^{\beta\gamma} \frac{i\bar{n}_2 \cdot k_2}{\bar{n}_2 \cdot k_2 \, n_2 \cdot p_s^- + i0} J\left(p^2\right)$$

[Liu, Neubert (2020)]

# Calculation of Jet Function

- Using light-cone gauge $\bar{n}_2 \cdot G_{n_2} = 0$ results in gluon propagator (Covariant gauge delivers same result)

$$P^{\mu\nu}(l) = \frac{-i}{l^2+i0}\left[g^{\mu\nu} - \frac{\bar{n}_2^\mu l^\nu + \bar{n}_2^\nu l^\mu}{\bar{n}_2 \cdot l}\right]$$

but simplifies Wilson lines and gauge-invariant building blocks

$$W_{n_2}(x) = P \exp\left[ig_s \int_{-\infty}^{0} ds\, \bar{n}_2 \cdot G_{n_2}(x + s\bar{n}_2)\right] = 1$$

$$\mathcal{G}^\mu_{n_2}(x) = g_s \int_{-\infty}^{0} ds\ \bar{n}_{2,\nu} G^{\nu\mu}_{n_2}(x + s\bar{n}_2) = g_s G^\mu(x)$$

- To project out Dirac structure use

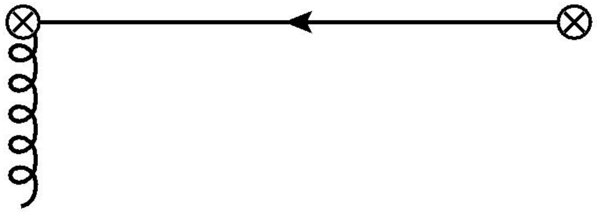
$$\mathrm{Tr}\left[\frac{\not{n}_2}{2}\gamma_\perp^\mu \gamma_\perp^\nu \frac{\not{\bar{n}}_2}{2}\right] = 2g_\perp^{\mu\nu}$$

# Calculation of Jet Function

- Practical workflow:
  - Draw all relevant Feynman diagrams
  - Perform IBP to reduce expression to Master Integrals
  - After implementing relations between MIs we find same MIs as in abelian case
  - MIs are calculated by direct Feynman parametrisation or dimensional recurrence relations

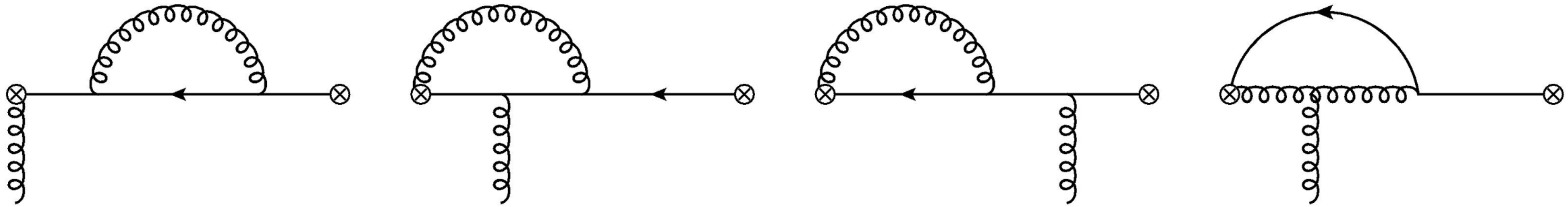
# Calculation of Jet Function

- At Leading order:



$$J^{(0)}(p^2) = 1$$

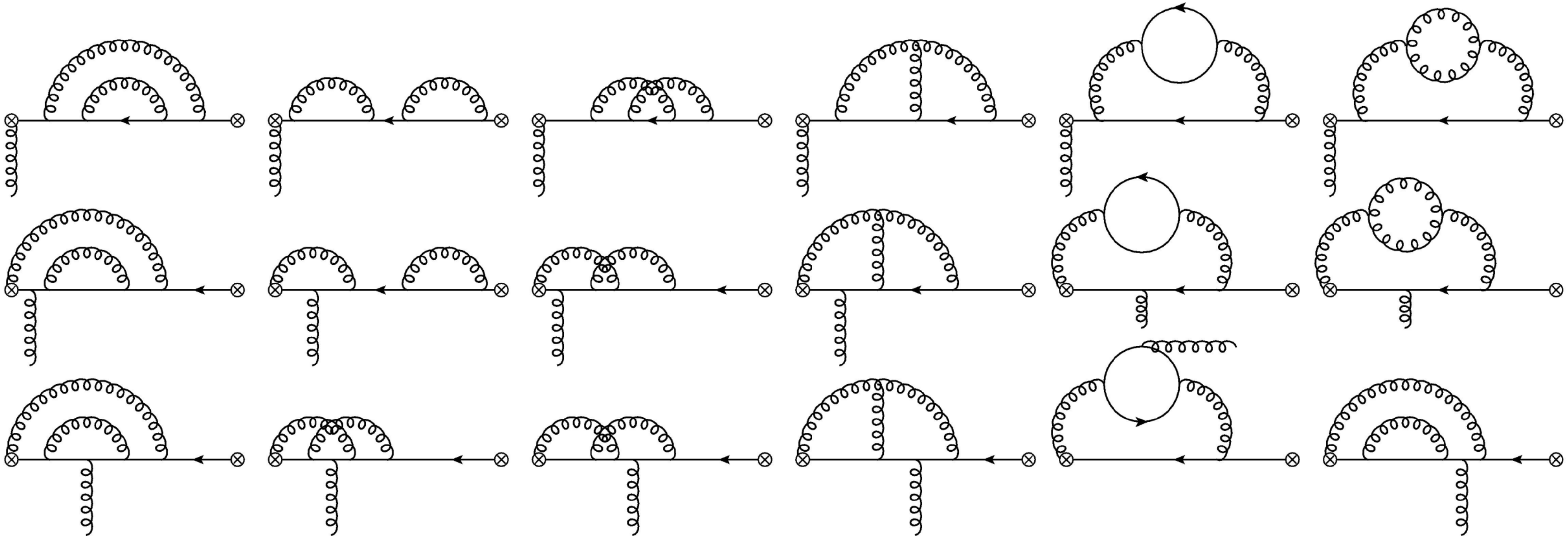
- At NLO:



$$J^{(1)}(p^2) = \frac{\alpha_s}{4\pi}(C_F - C_A)\left(\frac{-p^2}{\mu^2}\right)^{-\epsilon} e^{\epsilon\gamma_E}\,\frac{\Gamma(1+\epsilon)\Gamma^2(-\epsilon)}{\Gamma(2+2\epsilon)}\,(2-4\epsilon-\epsilon^2)$$

# Calculation of Jet Function

- At NNLO:



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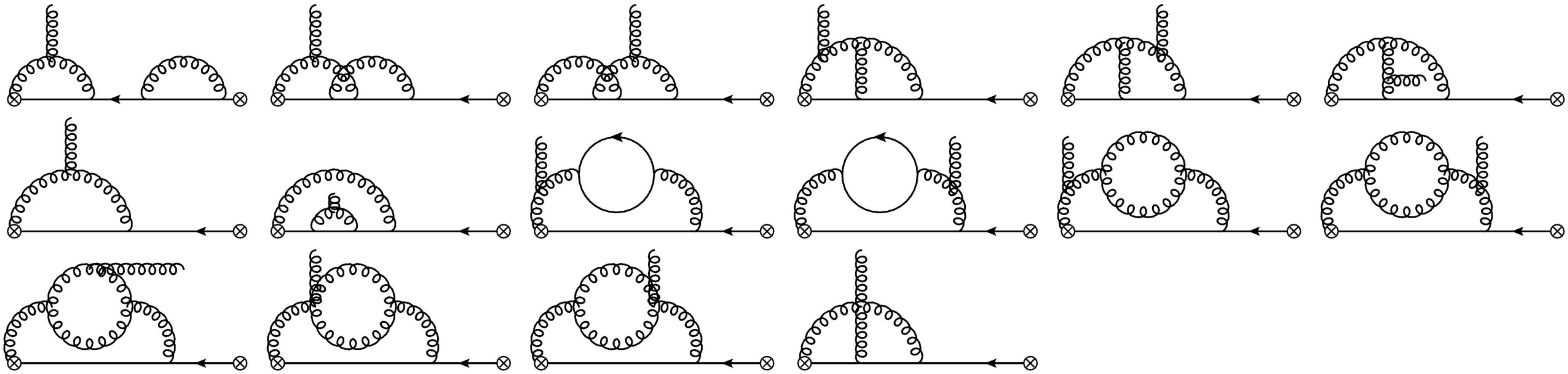

# Calculation of Jet Function

- At NNLO:

$$J^{(2)}(p^2) = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{-p^2}{\mu^2}\right)^{-2\epsilon} \left[C_F^2 K_{FF} + C_F C_A K_{FA} + C_A^2 K_{AA} + C_F T_F n_f K_{Fn} + C_A T_F n_f K_{An}\right]$$

- with

$$K_{FF} = \frac{2}{\epsilon^4} - \frac{1}{\epsilon^2}\left(2 + \frac{\pi^2}{3}\right) - \frac{1}{\epsilon}\left(4 + \frac{\pi^2}{2} + \frac{46\zeta_3}{3}\right) - \frac{13}{2} - \frac{\pi^2}{6} - 39\zeta_3 + \frac{\pi^4}{5}$$

$$K_{FA} = -\frac{4}{\epsilon^4} + \frac{11}{6\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{139}{18} + \frac{\pi^2}{2}\right) + \frac{1}{\epsilon}\left(\frac{319}{27} - \frac{\pi^2}{18} + \frac{80\zeta_3}{3}\right) + \frac{1087}{162} - \frac{83\pi^2}{54} + \frac{485\zeta_3}{18} - \frac{49\pi^4}{360}$$

$$K_{AA} = \frac{2}{\epsilon^4} - \frac{11}{6\epsilon^3} - \frac{1}{\epsilon^2}\left(\frac{103}{18} + \frac{\pi^2}{6}\right) - \frac{1}{\epsilon}\left(\frac{413}{54} - \frac{11\pi^2}{18} + \frac{34\zeta_3}{3}\right) + \frac{100}{81} + \frac{47\pi^2}{27} + \frac{259\zeta_3}{18} - \frac{23\pi^4}{360}$$

$$K_{Fn} = -\frac{2}{3\epsilon^3} - \frac{10}{9\epsilon^2} - \frac{1}{\epsilon}\left(\frac{20}{27} - \frac{\pi^2}{9}\right) + \frac{230}{81} + \frac{5\pi^2}{27} + \frac{64\zeta_3}{9}$$

$$K_{An} = \frac{2}{3\epsilon^3} + \frac{10}{9\epsilon^2} + \frac{1}{\epsilon}\left(\frac{11}{27} - \frac{2\pi^2}{9}\right) - \frac{491}{81} - \frac{10\pi^2}{27} - \frac{106\zeta_3}{9}$$

# Comparison at bare Level

- Compare with bare photon Jet Function that appears in $h \to \gamma\gamma$ and $B^- \to \ell^- \bar{\nu}\gamma$

$$J_g^{(0)} = J_\gamma^{(0)}$$

$$J_g^{(1)} = J_\gamma^{(1)} \text{ under } (C_F \leftrightarrow C_F - C_A)$$

$$J_g^{(2)} \text{and } J_\gamma^{(2)} \text{ not related by simple colour factor relation}$$

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- Does a simple replacement also hold for the NLO renormalisation, i.e. $Z_J^g = Z_J^\gamma \big|_{C_F \to C_F - C_A}$?

# Comparison at bare Level

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- Does a simple replacement also hold for the NLO renormalisation, i.e. $Z_J^g = Z_J^\gamma \big|_{C_F \to C_F - C_A}$?

➢ No, the analytical structure at NLO is too trivial to deduce correct non-local term

- For 1-Loop renormalisation use two different methods
  1) Conjecture based on renormalisation of third term of factorisation theorem
  2) Direct Calculation

# NLO Renormalisation, Conjecture Method

- Reminder: $\mathcal{M}_{h\to gg} = \mathcal{M}_0(T_1 + T_2 + T_3)\,,\, T_3 = H_3 O_3$
- $Z_{gg}^{-1} T_3$ is RG-invariant (up to cutoff corrections)
- $H_3$, its renormalisation and RGE are known from $h \to \gamma\gamma$
- Now assume $Z_J^g = Z_J^\gamma$ up to colour factors of local and non-local term:

$$Z_J^g(p^2, xp^2;\mu) = \left[1 + f_1(C_F, C_A)\,\frac{\alpha_s}{4\pi}\left(\frac{-2}{\epsilon^2} + \frac{2}{\epsilon}\ln\frac{-p^2}{\mu^2}\right)\right]\delta(1-x) + f_2(C_F, C_A)\,\frac{\alpha_s}{2\pi\epsilon}\Gamma(1,x) + \mathcal{O}(\alpha_s^2)$$

- Find with $f_1(C_F, C_A) = C_F - C_A$ and $f_2(C_F, C_A) = C_F - \frac{C_A}{2}$ the deduced $Z_S$ renormalises the soft function in non-trivial way

# NLO Renormalisation, Direct Calculation

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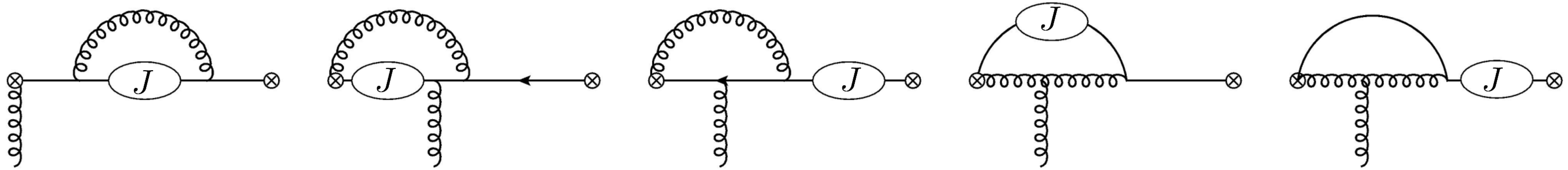
- Method first used by Bodwin et al. for Soft Function [Bodwin et al. (2021)]




- Blob = Jet Function at one-loop-level

- Only use structure of Jet Function, not exact result, i.e. use $J^{(n)}(p^2) \propto \left(-p^2\right)^{-n\epsilon}$

➢ $$Z_J(p^2, xp^2; \mu) = \left[1 + \frac{(C_F - C_A)\alpha_s}{4\pi}\left(\frac{-2}{\epsilon^2} + \frac{2}{\epsilon}\ln\frac{-p^2}{\mu^2}\right)\right]\delta(1-x) + \frac{\left(C_F - \frac{C_A}{2}\right)\alpha_s}{2\pi\epsilon}\Gamma(1,x) + \mathcal{O}(\alpha_s^2)$$

➢ $$\Gamma(y,x) = \left[\frac{\theta(y-x)}{y(y-x)} + \frac{\theta(x-y)}{x(x-y)}\right]_+$$ is the Lange-Neubert kernel

[Lange, Neubert (2003), Grozin, Neubert (1997)]

# Renormalised Jet Function

- Renormalised Jet Function at NLO:

$$J(p^2,\mu) = \int_0^\infty \mathrm{d}x\, Z_J(p^2, xp^2;\mu) J(xp^2) = 1 + \frac{(C_F - C_A)\alpha_s}{4\pi}\left(\ln^2\frac{-p^2}{\mu^2} - 1 - \frac{\pi^2}{6}\right) + \mathcal{O}(\alpha_s^2)$$

- From here we can derive RGE and anomalous dimension:

$$\frac{\mathrm{d}}{\mathrm{d}\ln(\mu)} J(p^2,\mu) = -\int_0^\infty \mathrm{d}x \gamma_J(p^2, xp^2;\mu) J(xp^2,\mu)$$

$$\gamma_J(p^2, xp^2;\mu) = \frac{\alpha_s}{\pi}\left[(C_F - C_A)\ln\frac{-p^2}{\mu^2}\delta(1-x) + \left(C_F - \frac{C_A}{2}\right)\Gamma(1,x)\right] + \mathcal{O}(\alpha_s^2)$$

# Renormalisation at NNLO

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- At NNLO the renormalisation factor is given by

$$\int_0^\infty \mathrm{d}x\, Z_J^{(2)}(p^2, xp^2) = -\left[ J^{(2)}(p^2) + \Delta J^{(2)}(p^2) + \int_0^\infty \mathrm{d}x\, Z_J^{(1)}(p^2, xp^2)\, J^{(1)}(xp^2) \right]_{\mathrm{div}}$$

Renormalisation of $\alpha_s$

Renormalisation of $J^{(1)}$ also removes divergences

- In general:

$$Z_J^{(2)}(p^2, xp^2) = \left[ \frac{\Delta\Gamma_0^2}{8\epsilon^4} - \frac{\Delta\Gamma_0}{4\epsilon^3}\left(\Delta\Gamma_0 L_p - \gamma_0' - \frac{3}{2}\beta_0\right) + \frac{1}{8\epsilon^2}\left(\Delta\Gamma_0 L_p - \gamma_0'\right)\left(\Delta\Gamma_0 L_p - \gamma_0' - 2\beta_0\right) - \frac{\Delta\Gamma_1}{\epsilon^2} + \frac{\Delta\Gamma_1 L_p - \gamma_1'}{4\epsilon} \right] \delta(1-x) + \frac{\Gamma_1^F - \frac{\Gamma_1^A}{2}}{2\epsilon}\Gamma(1,x) + \frac{C(x)}{\epsilon}$$

Non-local term, to be determined

- $\Delta\Gamma$=difference of cusp an. dim. in fundamental and adjoint, and $\gamma_1'$ to be determined

# Refactorisation Theorems

- Can check that the same refactorisation theorems hold in gluon case compared to $h \to \gamma\gamma$ for bare quantities as a further cross-check

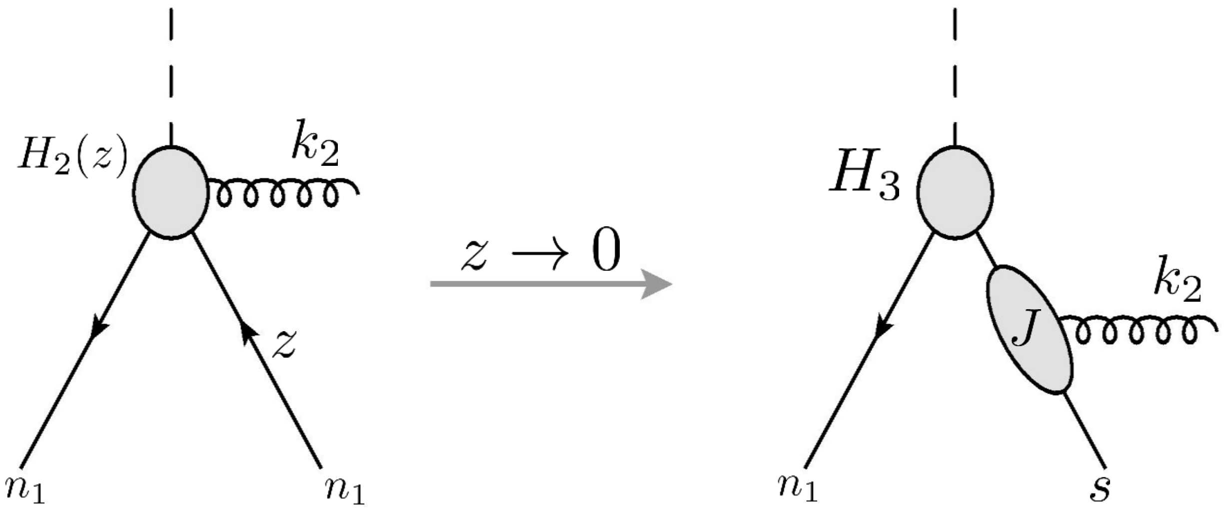
[Liu, Meçaj, Neubert, Wang (2020)]

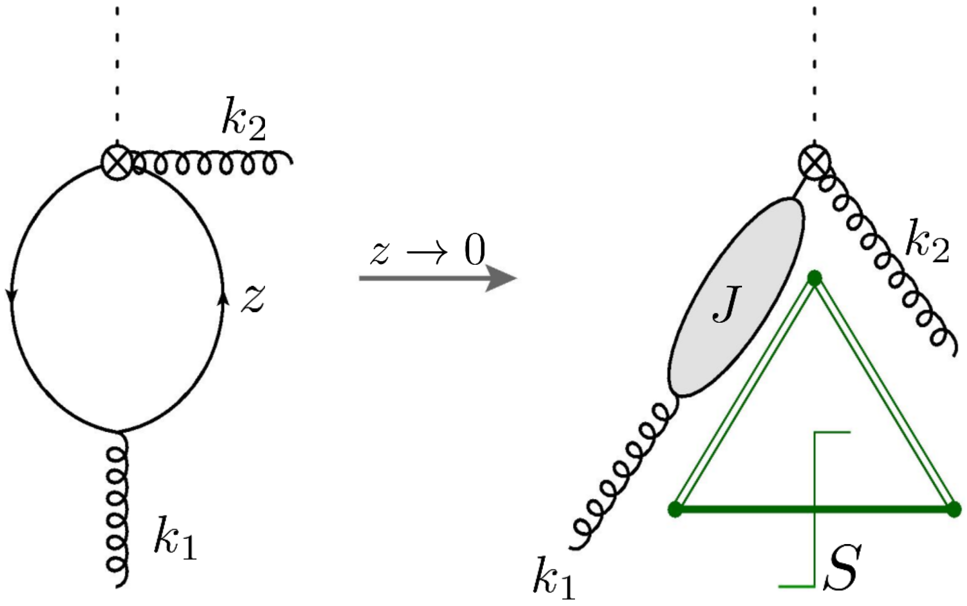
  - $$[\![\overline{H}_2(z)]\!] = -H_3 J(z\, m_h^2)$$
  - $$[\![O_2(z)]\!] = -\frac{1}{2}\int_0^\infty \frac{d\ell_+}{\ell_+} J(-m_h \ell_+) S(z\, m_h \ell_+)$$

- with $[\![f(z)]\!] = \lim_{z\to 0} f(z)$ and $\overline{H}_2(z) = z H_2(z)$

# Refactorisation Theorems

- Can check that the same refactorisation theorems hold in gluon case compared to $h \to \gamma\gamma$ for both bare and renormalised quantities
  - $[\![\overline{H}_2(z)]\!] = -H_3 J(z\, m_h^2)$, $\qquad [\![O_2(z)]\!] = -\frac{1}{2}\int_0^\infty \frac{d\ell_+}{\ell_+} J(-m_h\ell_+) S(z\, m_h \ell_+)$



# Summary and Outlook

- Radiative gluon jet function is an interesting object to study, appears in $h \to gg$ factorisation theorems
- Calculated jet function up to NNLO (2-Loop)
- Renormalised jet function up to NLO using two different methods
- Confirmed refactorisation theorems also for $h \to gg$
- Renormalisation at 2-Loop order is ongoing work

# Summary and Outlook

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  fac…
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**Thank you for your attention!**

# Backup Slides

# Photon Jet Function at 2-Loop

- $$J_\gamma^{(2)}(p^2) = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{-p^2}{\mu^2}\right)^{-2\epsilon} C_F \left(C_F K_F^\gamma + C_A K_A^\gamma + T_F n_f K_n^\gamma\right)$$

- and

$$K_F^\gamma = \frac{2}{\epsilon^4} + \frac{1}{\epsilon^2}\left(-2 - \frac{\pi^2}{3}\right) + \frac{1}{\epsilon}\left(-4 - \frac{\pi^2}{2} - \frac{46\zeta_3}{3}\right) - \frac{13}{2} - \frac{\pi^2}{6} - 39\zeta_3 - \frac{\pi^4}{5}$$

$$K_A^\gamma = \frac{11}{6\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{67}{18} - \frac{\pi^2}{6}\right) + \frac{1}{\epsilon}\left(\frac{103}{27} - \frac{11\pi^2}{36} - 7\right) - \frac{695}{162} - \frac{103\pi^2}{108} - \frac{14\zeta_3}{9} - \frac{43\pi^4}{180}$$

$$K_n^\gamma = -\frac{2}{3\epsilon^3} - \frac{10}{9\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{20}{27} + \frac{\pi^2}{9}\right) + \frac{230}{81} + \frac{5\pi^2}{27} + \frac{64\zeta_3}{9}$$