

# Automation of Beam and Jet functions at NNLO

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# Outline

- Motivation
- Jet functions
  - General set-up
  - Results
- Beam functions
  - General set-up
  - Results
- Outlook

# Higher order QCD corrections

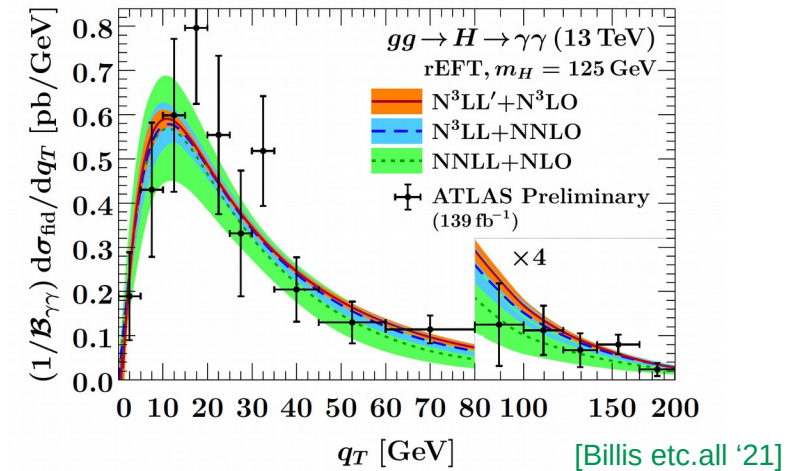
- Explosion of higher order QCD corrections in the last two decades.
- NNLO predictions are available for many processes in the SM and even beyond.  
(See talks from Monday)
  - Public codes are available like **MCFM**, **MATRIX** etc.
- Complete N<sup>3</sup>LO corrections are also available for a few processes (inclusive Higgs, DY) and for a few observables (eg. Rapidity, P<sub>T</sub>).  
[Mistlberger, Dulat, Duhr '20  
Cieri, Chen, Gehrmann, Glover, Huss '18,'21 ]
- Efforts towards N<sup>4</sup>LO level are also ongoing.  
[Das, Moch, Vogt '20,  
Agarwal, Manteuffel, Panzer, Schabinger '21]

# Resummation

- Fixed order calculations are often plagued by **large logarithms**

Resummation is needed for these large logarithms to all orders in perturbation theory!

- Resummation has been proved useful in correctly describing observables in lepton and hadron colliders.



- **SCET** has emerged as an important tool in studying the **IR sector** of QCD and resum these large logarithms in a systematic framework.
- The backbone relies on the underlying **factorization theorems**.

# Factorization

- Factorization in **SCET** for different observables

$$d\sigma \simeq H(\mu_f) \prod_i B_i(\mu_f) \otimes \prod_j J_j(\mu_f) \otimes S(\mu_f)$$

Hard interaction

ISR

FSR

Soft radiation

- Each functions can be calculated perturbatively
- Resummation is performed by calculating them at their characteristic scales and **evolving them to a common scale.**

# RG Evolution

- An example- RGE of the Hard function:

$$\frac{d \ln H(Q, \mu)}{d \ln \mu} = \gamma_H(Q, \mu)$$

[Becher & Neubert '10]

- Hard anomalous dimension has the structure

$$\gamma_H(Q, \mu) = \Gamma_{cusp}(\alpha_S) \ln \frac{Q}{\mu} + \gamma_{(nc),H}(\alpha_S)$$

$$H(Q, \mu) = H(Q, \mu_H) U(\mu_H, \mu)$$

Boundary term

Evolution kernel

$$U(\mu_H, \mu) = \exp \left[ \int_{\mu_H}^{\mu} d \ln \mu' \gamma_H(Q, \mu') \right]$$

- Boundary term is free from logarithms  $\mu_H \simeq Q$
- Evolution kernel resums logarithms  $\ln^n(Q/\mu)$  to all orders

# Ingredients for Resummation

- We need to have all the anomalous dimensions & matching coefficients

$$\underbrace{\Gamma_{cusp}, \gamma_{(nc),H}, c_H}_{\text{Observable-independent}}, \underbrace{\gamma_{(nc),B}, c_B, \gamma_{(nc),J}, c_J, \gamma_{(nc),S}, c_S}_{\text{Observable-dependent}}$$

- Hard anomalous dimensions are known to three-loops [Becher & Neubert '09, Almelid, Duhr, Gardi '15]
- Beam, Jet and Soft quantities are computed on a case-by-case basis
- Need two-loop matching coefficients to achieve NNLL' accuracy

	$\Gamma_{cusp}, \beta$	$\gamma_{(nc)H,B,J,S}$	Boundary term ( $c_{H,B,J,S}$ )
NLL	2-loop	1-loop	1
NNLL'	2-loop	1-loop	$\alpha_S$
NNLL	3-loop	2-loop	$\alpha_S$
NNLL'	3-loop	2-loop	$\alpha_S^2$

# Soft Function

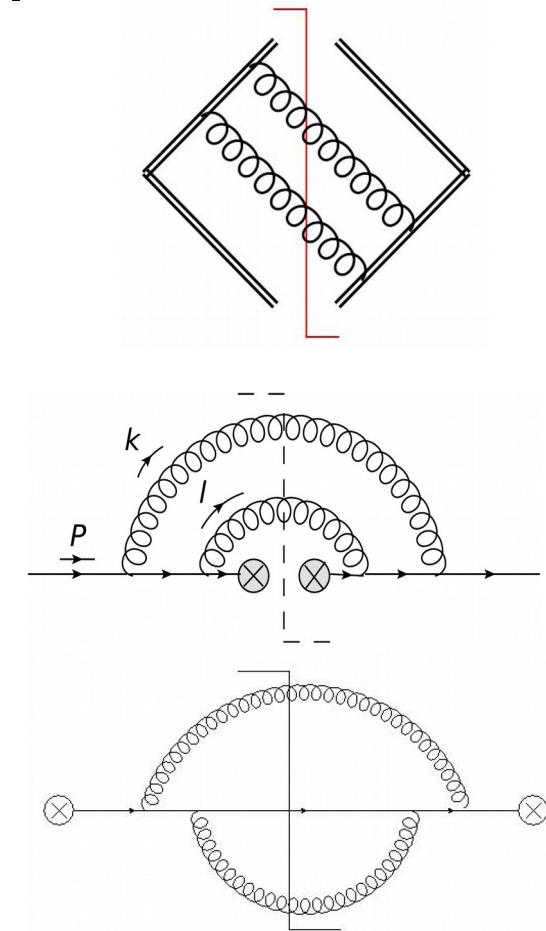
- An automated framework exists to calculate Soft functions up to two loop
- **SoftSERVE** Version-1.0.1 [Bell, Rahn, Talbert '18, '20]
  - NNLO di-jet soft functions
  - A large class of SCET-I and SCET-II observables has been calculated
    - Thrust, C-parameter, Angularities, Hemisphere masses ...
    - Threshold resummation,  $P_T$  resummation, Jet-veto resummation ...
  - Available at <https://softserve.hepforge.org>
- In progress [Bell, Dehnadi, Mohrmann, Rahn]
  - Arbitrary light-like directions (for SCET-I observables)
    - N-jettiness, N-angularities, invariant mass of boosted tops ...





# Automation: Beam and Jet functions

- Set up a general framework to automatically calculate Beam and Jet functions for a **general class of observables**.
- **Soft functions**  $\longrightarrow$  2-particle final state
- **Beam functions**  $\longrightarrow$  2-particle final state
  - Non-trivial matching onto pdfs
- **Jet functions**  $\longrightarrow$  3-particle final state
  - Complicated divergence structures



# Jet Function

- Definition

$$J_q(\tau) \sim \sum_{i \in X} \delta\left(Q - \sum_i \bar{n} \cdot k_i\right) \delta^{(d-2)}\left(\sum_i k_{\perp}^i\right) \langle 0 | \chi | X \rangle \langle X | \bar{\chi} | 0 \rangle \mathcal{M}(\tau, \{k_i\})$$

- Collinear field operators

$$\chi = W^\dagger \frac{\not{n} \not{\eta}}{4} \psi$$

- Automation exists at NLO

[K. Brune's master thesis '18,  
Basdev-Sharma, Herzog, Velzen, Waalewijn '20]

# Jet Function : NLO

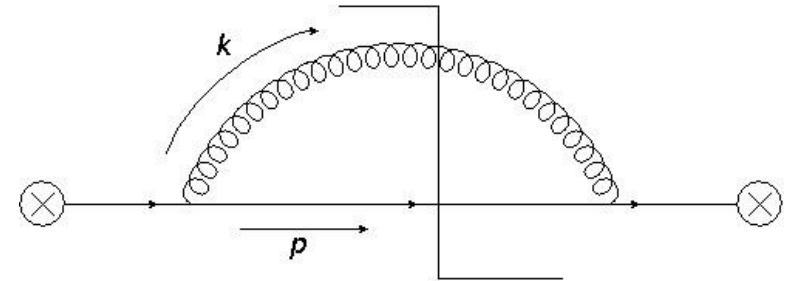
- Matrix element: LO splitting kernel

$$P_{qg}^{(0)}(z) = C_F \frac{1}{z} \left[ (1 - \epsilon)z^2 + 2\bar{z} \right]$$

[Altarelli & Parisi '77]

- Phase space:

$$z = \frac{k_-}{Q}, \quad k_T = \sqrt{k_+ k_-}, \quad t_k = \frac{1 - \cos(\theta_k)}{2}$$



$$\left\{ k_-, k_T, \theta_k \right\} \rightarrow \left\{ z, k_T, t_k \right\}$$

# Jet Function : NLO

- Generic Measurement function for one emission (in Laplace space)

$$\mathcal{M}_1(\tau, k) = \exp \left[ -\tau k_T \left( \frac{k_T}{zQ} \right)^n f(z, t_k) \right]$$

- Examples:

- Thrust:  $n = 1, f(z, t_k) = \frac{1}{1-z}$

- Angularity:  $n = 1 - A, f(z, t_k) = 1 + \left( \frac{z}{1-z} \right)^{1-A}$

Non-zero  
in the singular limit  
of ME

- Master Formula

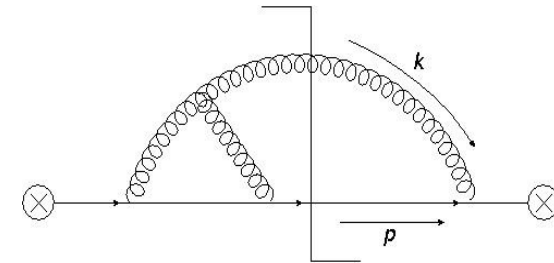
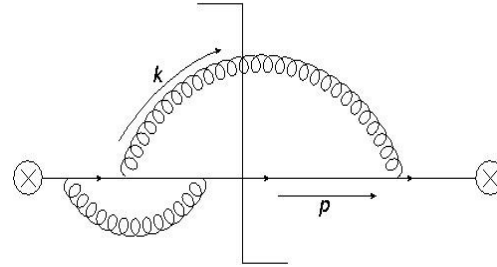
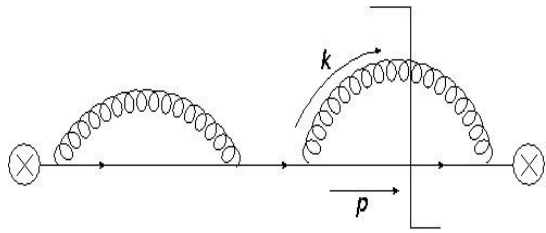
$$J_q^{(1)}(\tau) \sim \Gamma\left(\frac{-2\epsilon}{1+n}\right) \int_0^1 dz z^{-1-\frac{2n\epsilon}{1+n}} \left[ z P_{qg}^{(0)} \right] \int_0^1 dt_k (4t_k \bar{t}_k)^{-\frac{1}{2}-\epsilon} f(z, t_k)^{\frac{2\epsilon}{1+n}}$$

**All singularities are factorised !**

# Jet Function : NNLO(Real-Virtual)

- Matrix Element:

related to NLO collinear splitting kernel -  $P_{qg}^{(1)}(z)$  [Furmanski, Petronzio '80]



- Phase Space

2-particle phase space

$$\{k_-, k_T, \theta_k\} \rightarrow \{z, k_T, t_k\}$$

- Measurement

$$\mathcal{M}_1(\tau, k) = \exp \left[ -\tau k_T \left( \frac{k_T}{zQ} \right)^n f(z, t_k) \right]$$

**Similar to NLO !**

# Jet Function : NNLO(Real-Virtual)

- Master Formula

$$J_q^{(2),RV}(\tau) \sim V(\epsilon) \Gamma\left(\frac{-4\epsilon}{1+n}\right) \int_0^1 dz \int_0^1 dt_k z^{-1-\frac{4n\epsilon}{1+n}} \mathcal{W}(z, t_k) f(z, t_k)^{\frac{4\epsilon}{1+n}}$$

- Divergences:

- Virtual corrections -  $V(\epsilon) \sim \frac{1}{\epsilon^2} + \mathcal{O}(\epsilon^{-1})$
- Phase space singularities -  $\Gamma\left(\frac{-4\epsilon}{1+n}\right) z^{-1-\frac{4n\epsilon}{1+n}}$

ME w/o divergent term  
PSP, Jacobian

**All phase space singularities are factorised !**

# Jet Function : NNLO (Real-Real)

- Matrix element: LO triple collinear splitting kernel

$$P_{q' \bar{q}' q}^{(0), C_F T_F}, P_{q \bar{q} q}^{(0), id}, P_{ggq}^{(0), C_F^2}, P_{ggq}^{(0), C_A C_F}$$

[Catani, Grazzini '98]

- Complicated divergence structures

$$\frac{1}{s_{12}^2 s_{123}^2 (z_1 + z_2)^2}, \frac{1}{s_{13} s_{23} z_1 z_2}, \frac{1}{s_{13} s_{123} z_1 z_2}, \frac{1}{s_{13} s_{123} z_2 \bar{z}_3}, \dots$$

# Jet Function: NNLO (R-R): $C_F T_F n_f$

- Divergence structure

$$\frac{1}{s_{12}^2 s_{123}^2 (z_1 + z_2)^2}$$

- Phase Space:

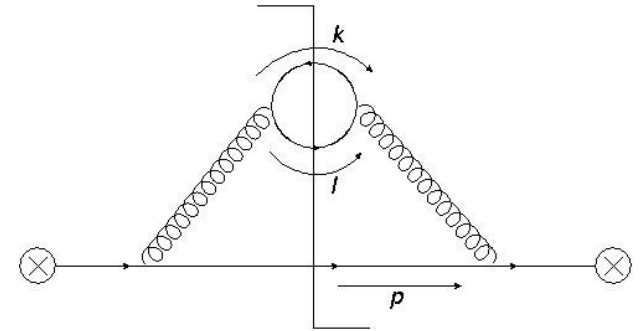
3-particle phase space

$$a = \frac{k_- l_T}{l_- k_T}, \quad b = \frac{k_T}{l_T},$$

$$z_{12} = \frac{k_- + l_-}{Q}, \quad q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$

$$\left\{ k_-, k_T, l_-, l_T, \theta_k, \theta_l, \theta_{kl} \right\} \rightarrow \left\{ a, b, z_{12}, q_T, t_k, t_l, t_{kl} \right\}$$

Remap  $\{a, b\}$  to unit hypercube  $\longrightarrow$  4 sectors





# NNLO : Measurement

- Generic Measurement function for two emissions (in Laplace space):

$$\mathcal{M}_2(\tau, \{k, l\}) = \exp \left[ - \tau q_T \left( \frac{q_T}{z_{12} Q} \right)^n \mathcal{F}(z_{12}, a, b, t_{kl}, t_k, t_l) \right]$$

- Due to complicated structures of the matrix element, the exact form could be different  $\longrightarrow$  depends on parametrisation & ME divergences

$$\mathcal{F}(z_{12}, a, b, t_{kl}, t_k, t_l) \longrightarrow \text{ensure it does NOT vanish in the singular limits of ME !}$$

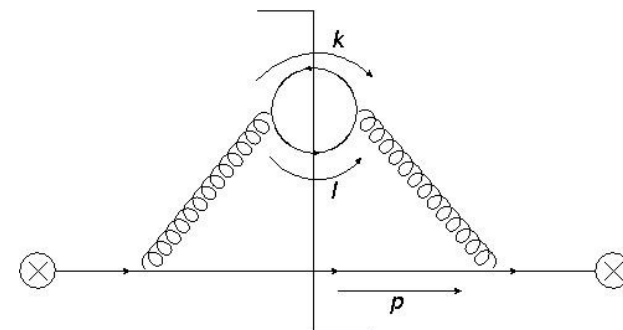
# Jet Function: NNLO(R-R): $C_F T_F n_f$

- Additional non-linear transformation

$$\{a, t_{kl}\} \rightarrow \{u, v\}$$

Removes overlapping in collinear divergence

between k and l  $(\bar{a}^2 + 4at_{kl})^{-1} \rightarrow u^{-1}$



- Master formula

$$J_q^{(2), C_F T_F} \sim \Gamma\left(\frac{-4\epsilon}{1+n}\right) \int_0^1 dz_{12} \int_0^1 du z_{12}^{-1-\frac{4n\epsilon}{1+n}} u^{-1-2\epsilon} \int_0^1 dv \int_0^1 db \int_0^1 dt_l \int_0^1 dt_5 \mathcal{W}(z_{12}, u, v, b, t_l, t_5) \times \mathcal{F}(z_{12}, u, v, b, t_l, t_5)^{\frac{4\epsilon}{1+n}}$$

Triple collinear       $\bar{n}$  collinear      Double collinear      Finite function

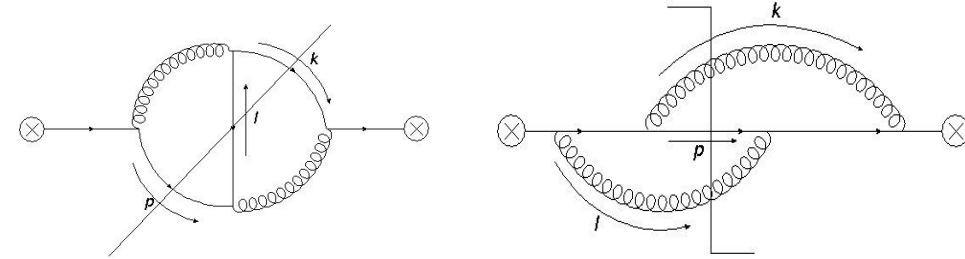
**All singularities factorise**

Exploit k-l symmetry  
4 sectors  $\rightarrow$  2 sectors

# Jet Function : NNLO (R-R): $C_F^2$

- Divergence structures

$$\frac{1}{s_{12}s_{13}\bar{z}_2\bar{z}_3}, \frac{1}{s_{12}s_{123}\bar{z}_2\bar{z}_3}, \dots, \frac{z_3}{s_{13}s_{23}z_1z_2}, \frac{1}{s_{13}s_{123}z_1z_2}, \dots$$



- Complications due to many overlapping divergences in ME

- Additional complications from measurement

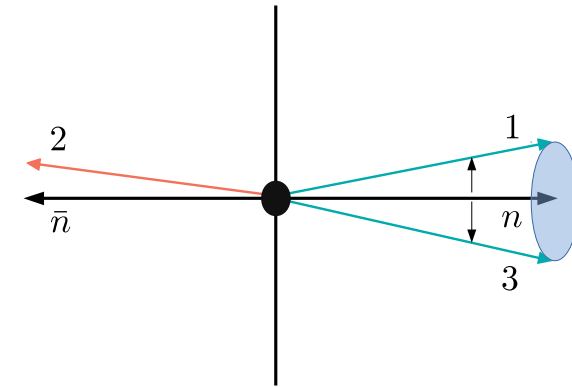
$$\mathcal{F} \sim (a + b)(1 + ab)\bar{z} + az(\bar{b}^2 + 4b\bar{t}_{kp})$$

Must stay non-zero in the physical limits

$$\left. \begin{array}{l} a \rightarrow 0, b \rightarrow 0 \\ a \rightarrow 0, \bar{z} \rightarrow 0 \\ \bar{b} \rightarrow 0, \bar{z} \rightarrow 0, \bar{t}_{kp} \rightarrow 0 \end{array} \right\}$$

- Strategy:

- Smart **selector functions**
- Use of complicated **non-linear transformations**
- **Sector decomposition**

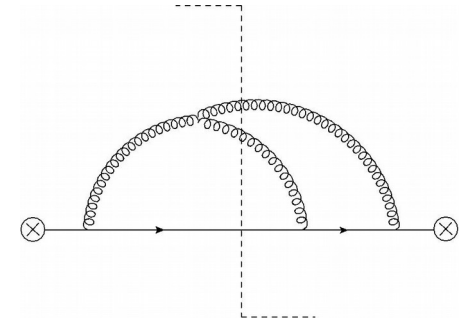
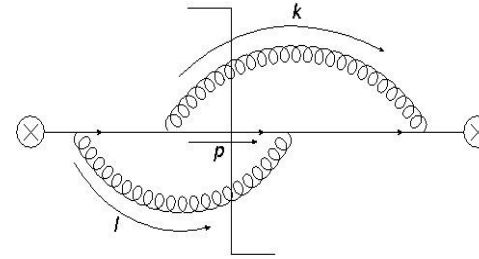
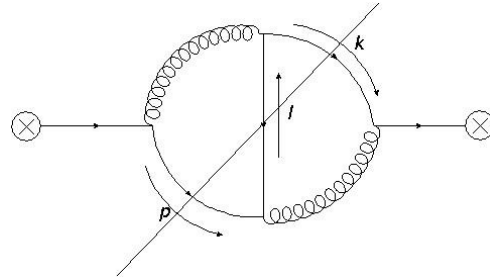
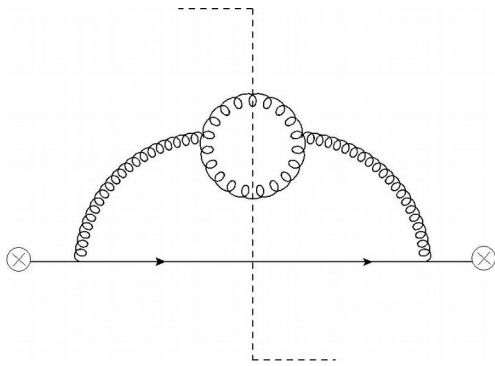


~40 sectors

# Jet Function : NNLO (R-R): $C_A C_F$

- Divergence structures:

$$\frac{1}{s_{12}s_{13}z_2\bar{z}_3}, \quad \frac{1}{s_{12}s_{123}z_2\bar{z}_3}, \quad \frac{1}{s_{13}s_{123}z_2\bar{z}_3}$$



- Calculation follows similar to  $C_F^2$  structures.

~ new 35 sectors

# Jet Function : Renormalization

- RGE for **SCET-I** type observables:

$$\frac{d \ln J(\tau, \mu)}{d \ln \mu} = \frac{2(1+n)}{n} \Gamma_{cusp} L + 2\gamma_{(nc), J}$$

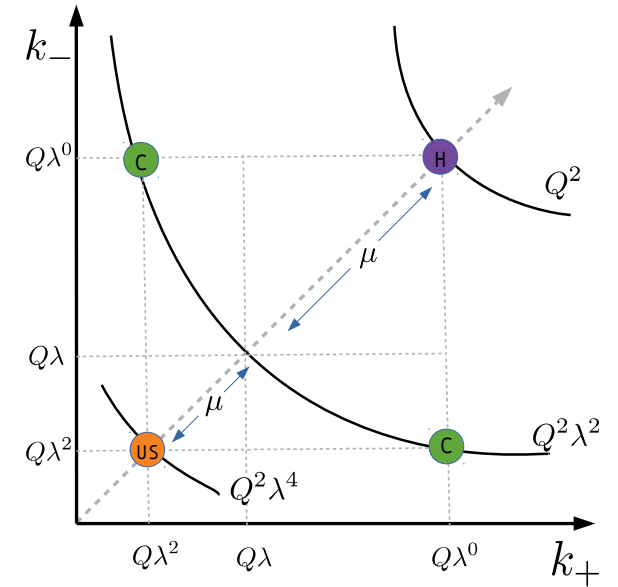
- Solution:

$$J(\tau, \mu) = 1 + \left( \frac{\alpha_S}{4\pi} \right) \left[ \sum_{i=1}^2 a_i(n) L^i + C_1 \right] + \left( \frac{\alpha_S}{4\pi} \right)^2 \left[ \sum_{i=1}^4 b_i(n) L^i + C_2 \right]$$


$$L = \ln(\bar{\tau} \mu)$$

- Extract anomalous dimensions and matching coefficients

$$\{\Gamma_0, \Gamma_1, \gamma_0, \gamma_1, C_1, C_2\}$$



# Results: Thrust

Implementation  **pySecDec** [G. Heinrich et.al.]

- Thrust

Intermediate checks with **MB.m**  
[M. Czakon '05]

$$\omega_0 = \sum_i k_{i+}$$

[1. Becher & Neubert '06]

$\gamma_1$	Analytical[1]	This work
$C_F T_F n_f$	-13.35	-13.35(1)
$C_F^2$	10.61	10.48(18)
$C_A C_F$	-3.26	-3.22(11)

$C_2$	Analytical[1]	This work
$C_F T_F n_f$	-10.79	-10.79(3)
$C_F^2$	4.66	4.53(33)
$C_A C_F$	-2.16	-2.03(32)

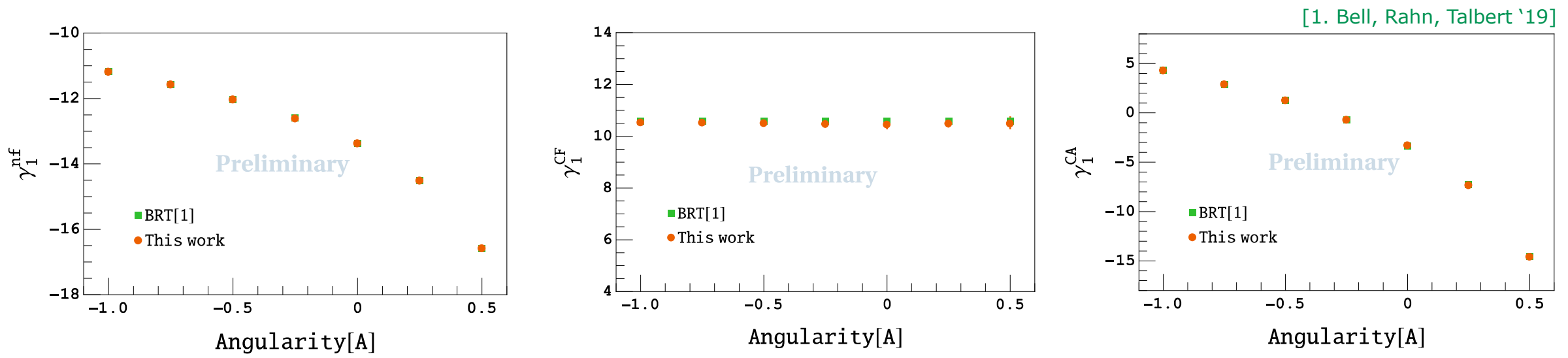
# Results: Angularity

- Angularity

$$\omega_A = \sum_i k_{i+}^{1-A/2} k_{i-}^{A/2}$$

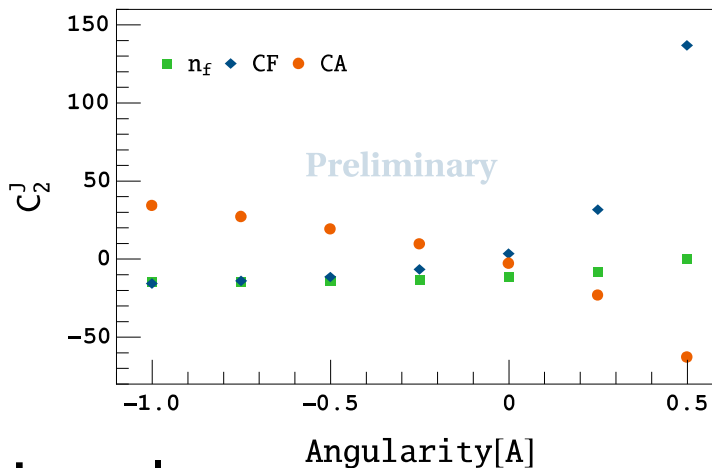
- Additional complications depending on Angularity values

- Check: Anomalous dimensions @ 2 loops (against **SoftSERVE**)



# Results: Angularity

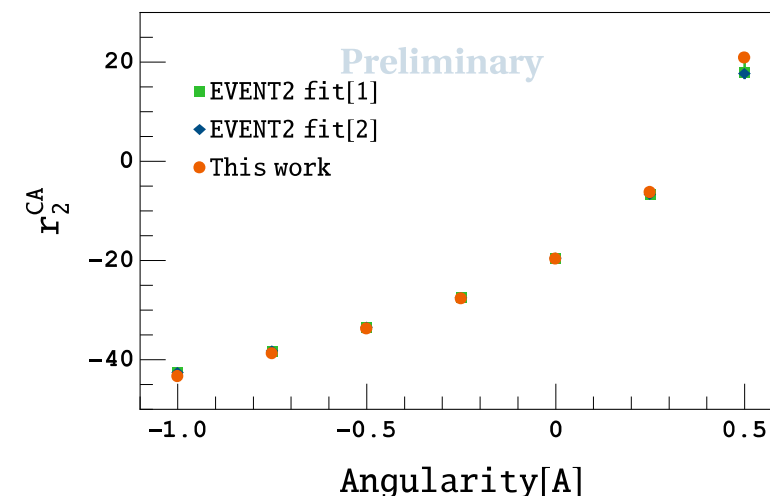
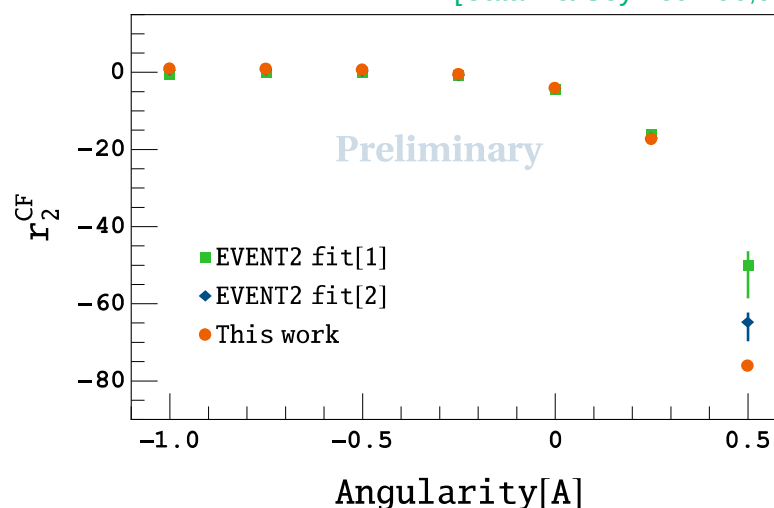
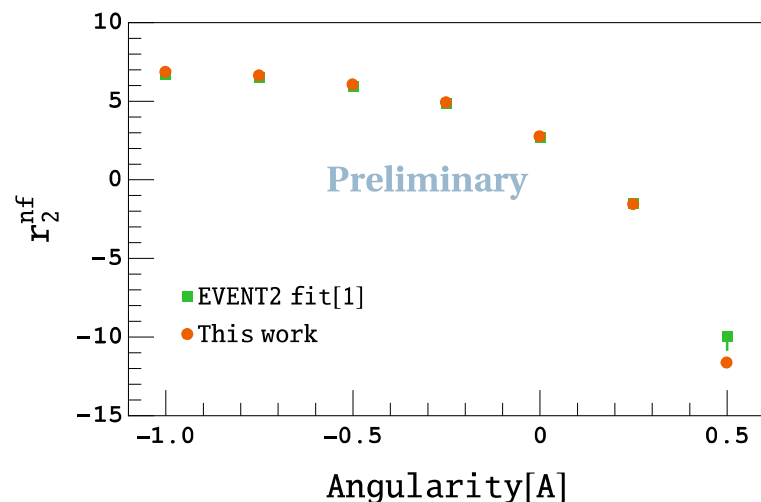
- Matching coefficients at two loops



- Comparison with non-singular remainder coefficients  $r_2$  from **EVENT2** fit

[Catani & Seymour '96,97]

[1. Bell, Hornig, Lee, Talbert '18  
2. Bell, Hornig, Lee, Talbert (unpublished) ]





# Beam Function

Proton matrix element of renormalized operator composed of partonic fields (with additional dependence on measurement)

$$\mathcal{B}_{qq}(\tau, x, \mu) = \sum_{i \in X_c} \delta\left((\bar{n} \cdot P)(1-x) - \bar{n} \cdot k_{X_c}\right) \langle P | \bar{\chi} | X_c \rangle \frac{\bar{n}}{2} \langle X_c | \chi | P \rangle \mathcal{M}(\tau, \{k_i\})$$

Contains non-perturbative contributions

- In the limit  $\tau \gg \Lambda_{QCD}$

$$\mathcal{B}_{qq}(\tau, x, \mu) = \sum_i \mathcal{I}_{qi}(\tau, x, \mu) \otimes f_{iq}(x, \mu)$$

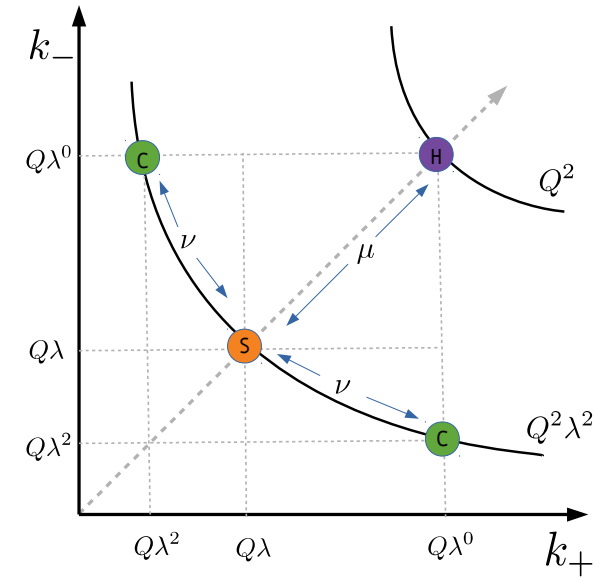
IR-divergent

IR-finite  
Matching coefficients

IR-divergent.  
Partonic PDF

# Rapidity Divergences

- **SCET-II observables** :
  - Suffers from additional rapidity divergences
  - Rapidity logs can be resummed to all orders
  - Follow the **collinear-anomaly** approach. [Becher & Neubert, '10]
  - Introduce analytic regulators at the phase space level



[Becher & Bell, '11]

Single emission PSP

$$\left(\frac{\nu}{k_-}\right)^\alpha$$

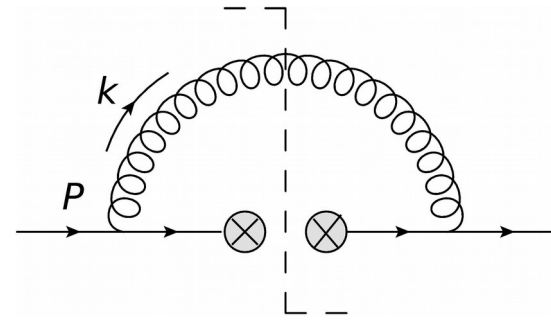
Double emission PSP

$$\left(\frac{\nu}{k_-}\right)^\alpha \left(\frac{\nu}{l_-}\right)^\alpha$$

# Beam Function : NLO

- Automated formalism is available at NLO [K. Brune's master thesis '18]

- Matrix Element: related to collinear splitting kernel  $P_{qg}^{(0)}(x)$



- Phase Space: 1-particle phase space

$$\bar{x} = \frac{k_-}{P_-}, \quad k_T = \sqrt{k_+ k_-}, \quad t_k = \frac{1 - \cos(\theta_k)}{2} \quad \longrightarrow \quad \{k_-, k_T, \theta_k\} \rightarrow \{x, k_T, t_k\}$$

- Generic Measurement function for single emission

$$\mathcal{M}_1(\tau, k_i) = \exp \left[ -\tau k_T \left( \frac{k_T}{(1-x)P_-} \right)^n f(t_k) \right]$$

Non-zero  
in the singular limit  
of ME

- $P_T$  - resummation:

$$n = 0, \quad f(t_k) = -2i (1 - 2t_k)$$

- $P_T$  - Veto:

$$n = 0, \quad f(t_k) = 1$$

- Master Formula

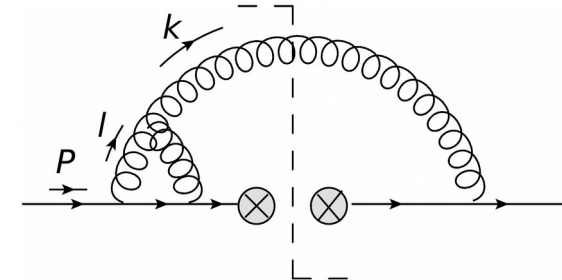
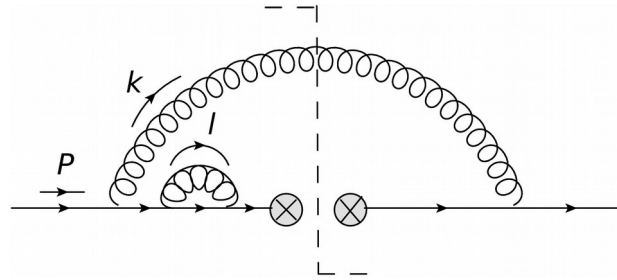
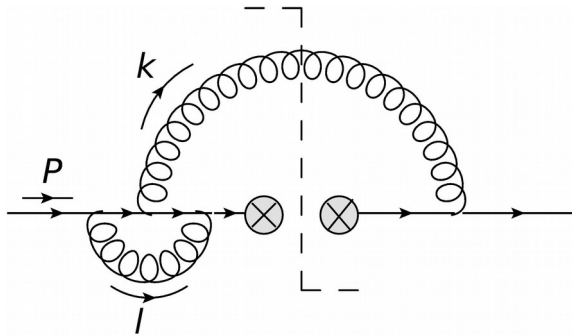
$$\mathcal{B}_{qq}^{(1)}(\tau, x) \sim \Gamma \left( \frac{-2\epsilon}{1+n} \right) \bar{x}^{-1 - \frac{2n\epsilon}{1+n} - \alpha} \left[ \bar{x} P_{qg}^{(0)}(x) \right] \int_0^1 dt_k (4t_k \bar{t}_k)^{-\frac{1}{2} - \epsilon} f(t_k)^{\frac{2\epsilon}{1+n}}$$

**All singularities are factorised !**

# Beam Function : Real-Virtual

- Matrix Element:

related to NLO collinear splitting kernel -  $P_{qg}^{(1)}(x)$



- Phase Space & measurement function follow NLO type

- Master formula

$$\mathcal{B}_{qq}^{(2),RV}(\tau, x, \mu) \sim V(\epsilon) \Gamma\left(\frac{-4\epsilon}{1+n}\right) \bar{x}^{-1-\frac{4n\epsilon}{1+n}-\alpha} \mathcal{W}(x) \int_0^1 dt_k (4t_k \bar{t}_k)^{-\frac{1}{2}-\epsilon} f(t_k)^{\frac{4\epsilon}{1+n}}$$

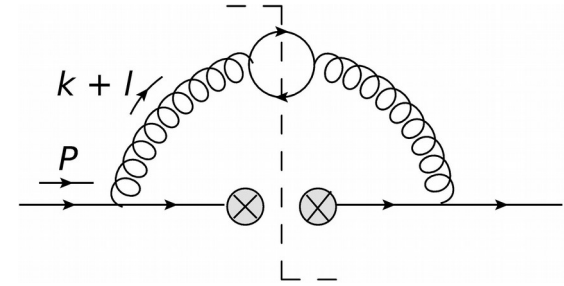
$\sim \epsilon^{-2} + \mathcal{O}(\epsilon^{-1})$

**All Phase space singularities are factorised !**

# Beam Function: NNLO (R-R) $C_F T_F n_f$

- Matrix Element:

related to the collinear splitting kernel -  $P_{\bar{q}' q' q}(x)$



- Phase Space:

2-particle phase space

$$a = \frac{k_- l_T}{l_- k_T}, \quad b = \frac{k_T}{l_T}, \quad x_{12} = \frac{k_- + l_-}{P_-}, \quad q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$

$$\{k_-, k_T, l_-, l_T, \theta_k, \theta_l, \theta_{kl}\} \rightarrow \{a, b, x_{12}, q_T, t_k, t_l, t_{kl}\}$$

Remap **{a,b}** to unit hypercube

- Avoid distribution valued functions  $\longrightarrow$  work in **Laplace-Mellin space**.

- Master formula

$$\mathcal{B}_{qq}^{(2),nf}(\tau, N_1) \sim C(\epsilon) \left(\frac{\nu}{q_-}\right)^{2\alpha} \int_0^1 dx_{12} db du dv dt_l dt_5 x_{12}^{-1-2\alpha} u^{-1-2\epsilon} \bar{x}_{12}^{N_1-1+2\alpha} \times \mathcal{G}(x_{12}, b, u, v, t_l, t_5) \mathcal{F}(x_{12}, b, u, v, t_l, t_5)^{4\epsilon}$$

**All singularities factorise !**

Finite function

Non-zero in the singular limit of ME

# Beam Function: NNLO (R-R) $C_F^2$

- Matrix Element:

related to the collinear splitting kernel -  $P_{ggq}^{CF^2}(x)$

- Phase Space:

- Two different parametrizations

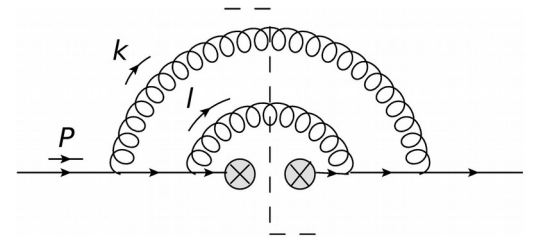
$$(s_{12}s_{13}x_1x_2)^{-1} \quad : \quad x_1 = \frac{k_-}{P_-}, \quad x_2 = \frac{l_-}{P_-},$$

$$b = \frac{k_T}{l_T}, \quad q_T = k_T + l_T$$

- Master formula:

$$\mathcal{B}_{qq}^{(2),CF} \sim \mathcal{C}(\epsilon) \left(\frac{\nu}{q_-}\right)^{2\alpha} \int dx_1 dx_2 db dt_{kl} dt_l dt_5 x_1^{-1-\alpha} x_2^{-1-\alpha} b^{-1-2\epsilon} \bar{x}_1^{N_1-1+2\alpha} \bar{x}_2^{N_1-1+2\alpha} \times \mathcal{G}(x_1, x_2, b, t_l, t_{kl}, t_5) \mathcal{F}(x_1, x_2, b, t_l, t_{kl}, t_5)^{4\epsilon}$$

- $(s_{123}s_{13}x_1x_2)^{-1}$  : so far we did not resolve overlapping divergences



# Renormalization

- **Collinear anomaly** approach

$$\left[ \mathcal{S}(\bar{\tau}, \mu, \nu) \mathcal{I}_{qq}(N_1, \bar{\tau}, \mu, \nu) \bar{\mathcal{I}}_{\bar{q}\bar{q}}(N_2, \bar{\tau}, \mu, \nu) \right]_{q^2} \stackrel{\alpha \equiv 0}{=} \left( \bar{\tau}^2 q^2 \right)^{2F_{q\bar{q}}(\bar{\tau}, \mu)} I_{qq}(N_1, \bar{\tau}, \mu) \bar{I}_{\bar{q}\bar{q}}(N_2, \bar{\tau}, \mu)$$

- RGE: Anomaly coefficients

$$\frac{d}{d \ln \mu} F_{q\bar{q}}(\bar{\tau}, \mu) = -\Gamma_{cusp}(\alpha_S)$$

[Becher & Neubert, '10]

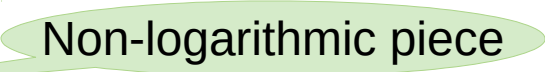
$$F_{q\bar{q}}(\bar{\tau}, \mu) = -\left(\frac{\alpha_S}{4\pi}\right) \left[ \Gamma_0 L - d_1 \right] - \left(\frac{\alpha_S}{4\pi}\right)^2 \left[ \beta_0 \Gamma_0 L^2 + \Gamma_1 L + \beta_0 d_1 L - d_2 \right] + \mathcal{O}(\alpha_S^3)$$

- RGE: Matching coefficients

$$L = \ln(\bar{\tau}\mu)$$

$$\frac{d}{d \ln \mu} I_{qq}(N_1, \bar{\tau}, \mu) = \left[ 2\Gamma_{cusp}(\alpha_S)L + 2\gamma_I(\alpha_S) \right] I_{qq}(N_1, \bar{\tau}, \mu) - 2 \sum_i I_{qi}(N_1, \bar{\tau}, \mu) P_{iq}(N_1, \mu)$$

- solution  renormalized matching kernel

Finite remainder coefficient  $\tilde{I}_{qq}(N_1)$   Non-logarithmic piece

# Result: $P_T$ -Resummation

- $P_T$ -resummation:

$$\omega_{p_T} = -2i \sum_i |\vec{k}_{i,T}| \cos(\theta_i)$$

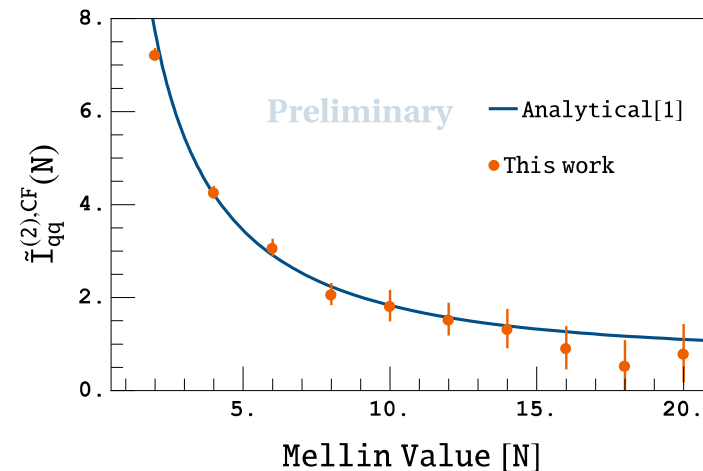
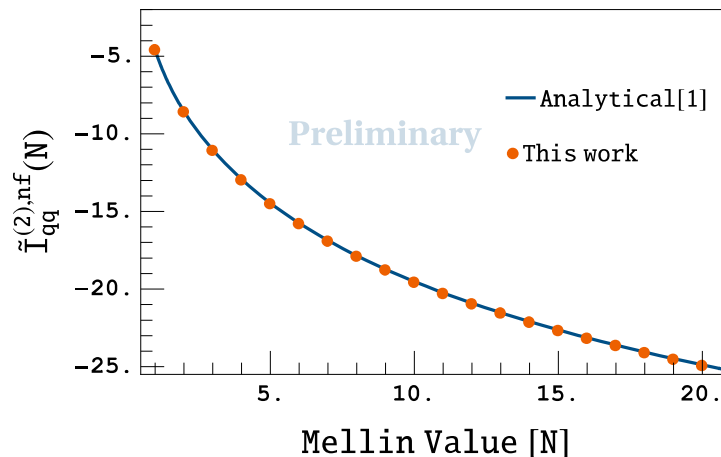
- Calculated the anomaly coefficients for two color structures.

$$d_1 = 0$$

$d_2$	Analytical[1]	This Work
$d_2^{n_f}$	4.148	4.147(4)
$d_2^{C_F}$	0	0.02(6)

[1. Gehrmann, Lübbert, Yang '14]

- Renormalized matching kernels:





# Result: $P_T$ -Veto

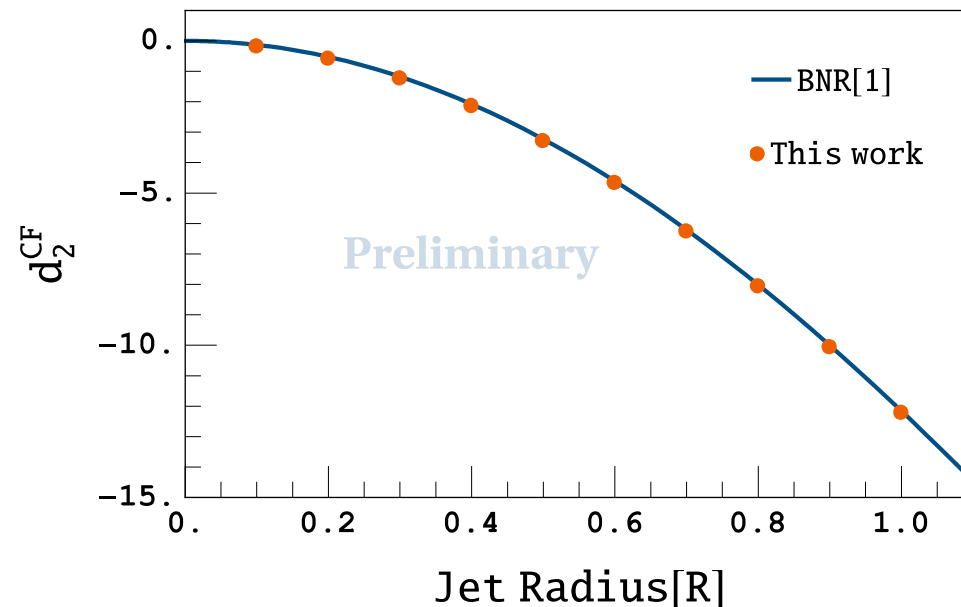
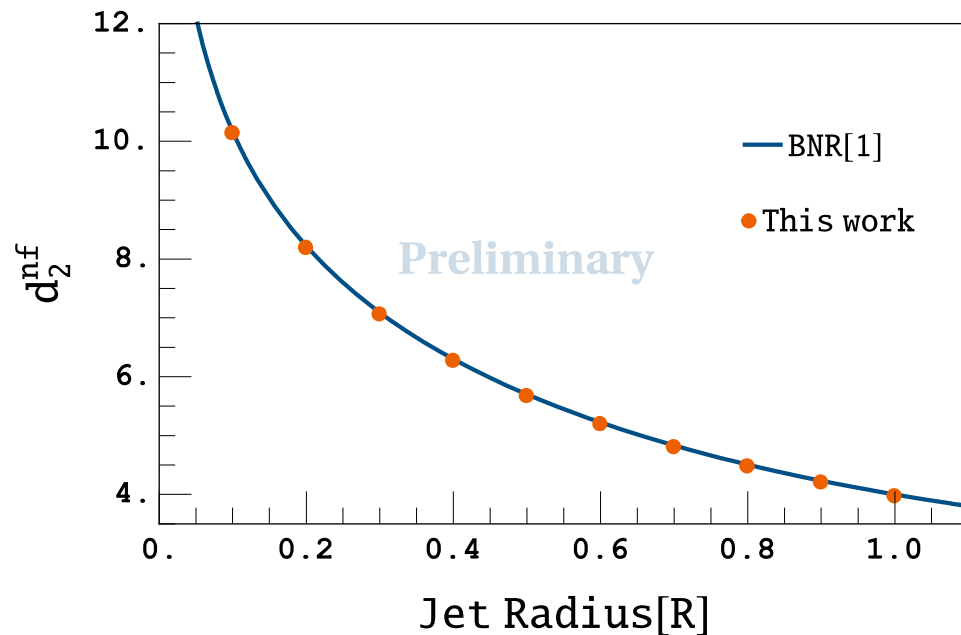
- $P_T$ -veto:

Distance measure 
$$\Delta^2 = \frac{1}{4} \ln^2 \left( \frac{l_+ k_-}{l_- k_+} \right) + \Theta_{lk}^2$$

$$\omega_{p_T-veto} = \Theta(\Delta - R) \max(\{k_{i,T}\}) + \Theta(R - \Delta) \left| \sum_i \vec{k}_{i,T} \right|$$

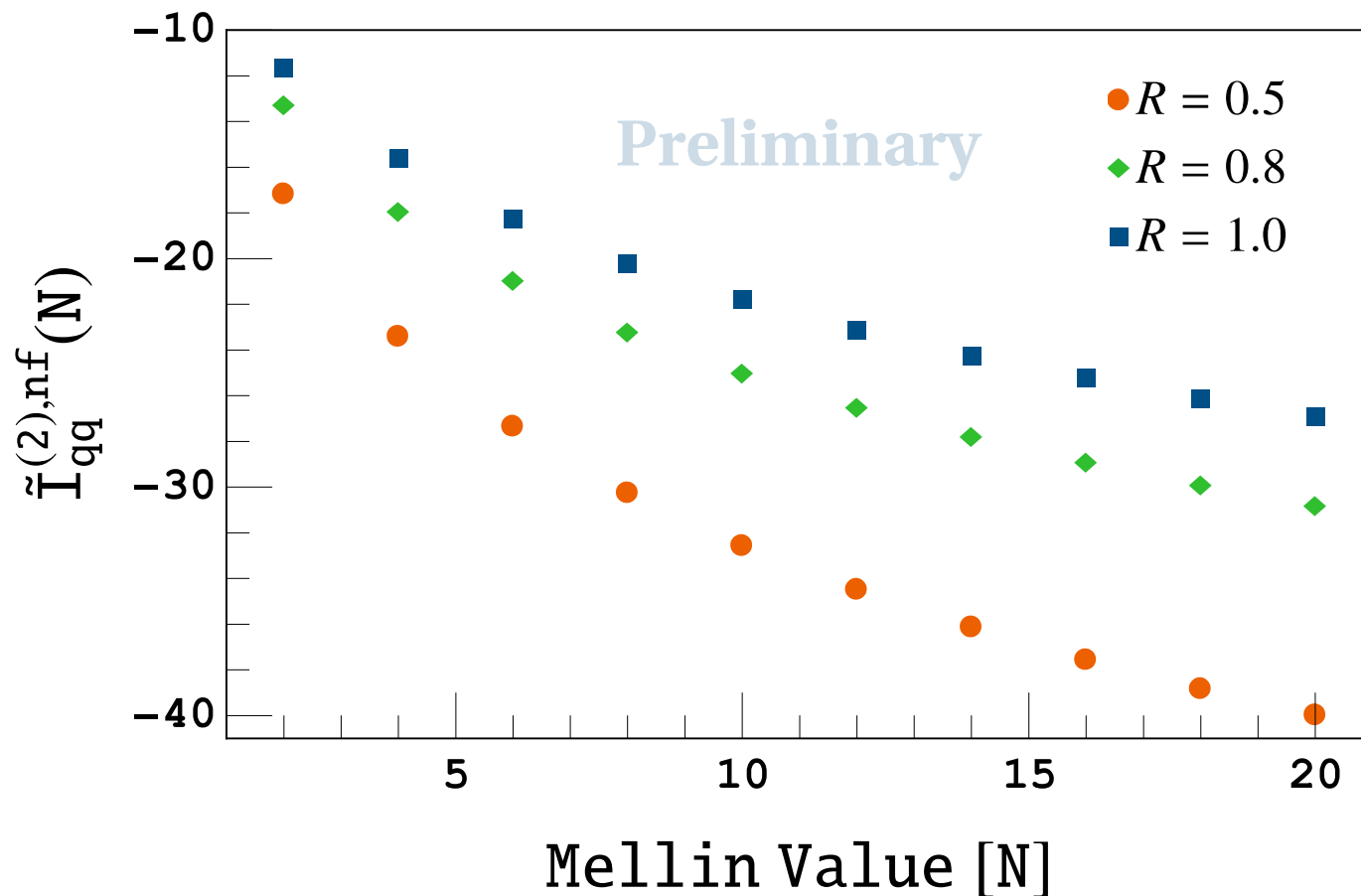
- Anomaly coefficients: depends on the jet radius at 2-loop

[1. Becher, Neubert, Rothen '13]



# Result: $P_T$ -Veto

- Matching coefficient at two loops



# Outlook

- Developed an automated way to calculate **Jet** and **Beam** functions for a **wide class of observables** at NNLO.
- Using a suitable phase-space parameterization(s) we are able to completely disentangle **IR divergences** into **monomial form**.
- We have presented preliminary results for **event shape observables** and  **$P_T$ -resummation** and  **$P_T$ -veto**.

# Outlook

- **Jet Functions:**
  - More observables are on the way (SCET-II observables, Jet algorithms, EEC ...)
  - Extend the method to calculate gluon Jet functions.
- **Beam Functions:**
  - Plans to extend to SCET-I observables (beam thrust, rapidity-dependent jet vetos ...).
  - Extend the method to remaining matching kernels.

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  - More observables are on the way (SCET-II observables, Jet algorithms, EEC ...)
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**Thank you for your attention!**

# Backup: $P_T$ -Veto Bare results

- The bare results for  $P_T$ -veto Beam function have been calculated for  $C_F^2$  color structure.

