MIXED QCD-EW CORRECTIONS TO $p + p \to l^+ \nu_l + X$ AT THE LHC

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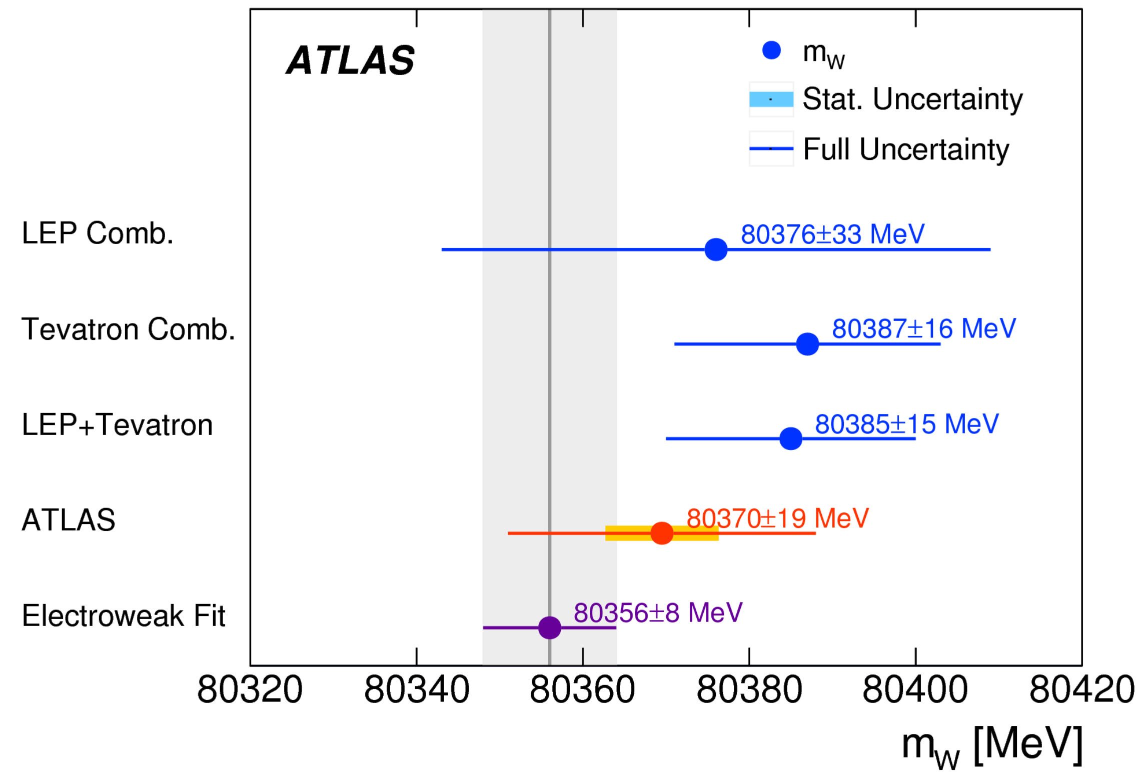
Introduction

The Drell-Yan process is of the primary importance for the LHC precision physics program given the large production rates and clean experimental signatures

At hadron colliders, studies of resonant production of Z and W bosons lead to the precise measurement of EW parameters as the **W boson mass**

Very accurate SM predictions are required to achieve a control over the theory systematic in the extraction of the W mass at the level of $\mathcal{O}(10\,\mathrm{MeV})$



QCD corrections dominant effects. They are known up to

- NNLO for differential cross sections
  [Anastasiou, Dixon, Melnikov, Petriello (2003)], [Melnikov, Petriello (2006)] [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)] [Catani, Ferrera, Grazzini (2010)]
- N3LO for inclusive cross section (for γ^* and W production)
  [Duhr, Dulat, Mistlberger (2020)]

NLO EW corrections are known since long *[S. Dittmaier and M. Kramer (2002)], [Baur,Wackeroth (2004)], [Baur, Brein, Hollik, Schappacher, Wackeroth (2002)], [Zykunov (2006,2007)]*
and nowadays automatised in different available generators *[Les Houches 2017, 1803.07977]*

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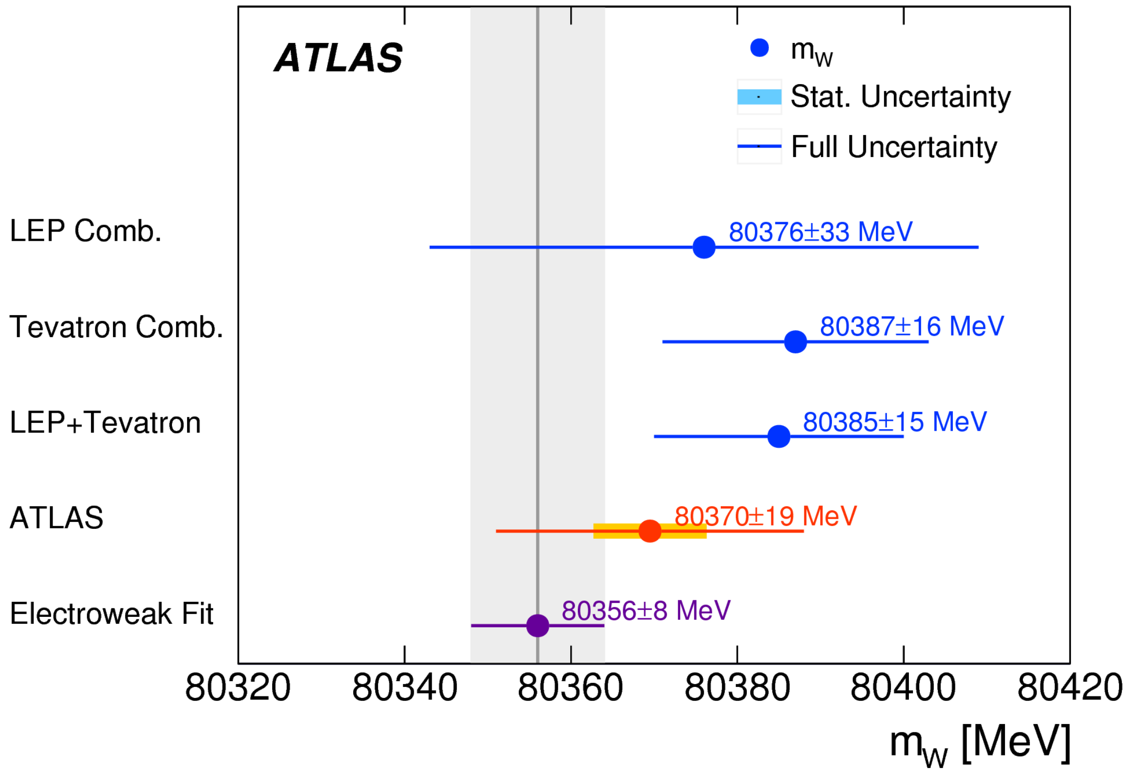

Given the (sub)per-mille accuracy target, mixed QCD-EW corrections become relevant

Recently, quite a hot topic: at least 4 talks this conference!

Mixed QCD-EW corrections: state of the art

The computation of fully differential mixed QCD-EW corrections to the production of an electro-weak boson is a complicated task

Theoretical developments

- progress on two-loop master integrals
  [Bonciani, Di Vita, Mastrolia, Schubert (2016)], [Heller, von Manteuffel, Schabinger (2019)] [Hasan, Schubert (2020)]
- renormalization
  [Dittmaier, Schmidt, Schwarz (2020)]
- 2-loop amplitudes for $2 \to 2$ neutral current DY
  [Heller, von Manteuffel, Schabinger, Spiesberger (2020)]

see von Manteuffel's talk

On-shell Z/W production ($2 \to 1$ process)

- analytical mixed QCD–QED corrections to the inclusive production of an on- shell Z
  [De Florian, Der, Fabre (2018)]
- fully differential mixed QCD–QED corrections to the production of an on-shell Z
  [Delto, Jaquier, Melnikov, Röntsch (2019)]
- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections
  [Bonciani, Buccioni, Rana, Vicini (2020)]

see Rana's talk

- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections
  [F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch (2020)],[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)]

see Behring's talk

Beyond on-shell computations

- dominant Mixed QCD-EW corrections in Pole Approximation for neutral- and charged- DY processes
  [Dittmaier, Huss, and Schwinn (2014,2015)]
- neutrino-pair production including NNLO QCD-QED corrections
  [Cieri, Der, De Florian, Mazzitelli (2020)]

Mixed QCD-EW corrections to $p + p \to l^+ \nu_l + X$

The computation of fully differential mixed QCD-EW corrections to the production of an electro-weak boson is a complicated task

This talk: we present the **first (*almost*) exact fully differential** computation of the mixed corrections to the $2 \to 2$ charged current DY process

The complexity of the computation is similar to that of NNLO QCD corrections for a $2 \to 2$ with (many) scales. The complete computation requires

- double real emission tree-level diagrams
- single real emission one-loop diagrams
- two-loop virtual and one-loop squared diagrams

All contributions are separately infrared divergent (IR):

1. we need a **suitable subtraction formalism** to handle IR singularities
2. the two-loop virtual amplitude represents the **bottle neck** (still not available)

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- double real emission tree-level diagrams
- single real emission one-loop diagrams
- two-loop virtual and one-loop squared diagrams

→ We consistently included all the contributions

1. achieving the cancellation of IR singularities within the q_T subtraction formalism
2. approximating only the finite part of the two-loop virtual

In the following, we will focus on one key observable, namely **the transverse momentum of the charged lepton**

Outline

- Handling of IR singularities
- Hard-Virtual coefficient
- Numerical results
- Conclusions

Handling of IR singularities

q_T subtraction for NNLO QCD-EW correction (and NLO-EW)

General ideas

- Do not reinvent the wheel: start from the well established experience at NNLO QCD
- IR structure is associated to only QCD-QED subpart: **recycle** NNLO QCD results via a careful **abelianisation procedure**
  [de Florian, Rodrigo, Sborlini (2016)], [de Florian, Der, Fabre (2018)]
- We rely on the q_T subtraction formalism and on the recent developments to heavy quarks
  [Catani, Grazzini (2007)], [Catani, Torre, Grazzini (2014)] **see Devoto's talk**
- Final state is colour neutral: purely soft contributions have a **much simpler structure**

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$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)}$$

$\mathcal{O}(\alpha_s^m \alpha^n)$ term

$$d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{\mathrm{cut}}}$$

q_T := transverse momentum of the dilepton finale state
Q := invariant mass of the dilepton finale state

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Since $q_T/Q > r_{\rm cut}$, the integration of double real and real-virt terms present **only NLO (single unresolved) IR singularities**

We use **massive CS dipoles** to subtract them
[Catani. Seymour (1998)], [Dittmaier(1999)], [Catani, Dittmaier, Seymour, Trocsanyi (2002)]

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Subtraction counterterm at small $r_{\rm cut}$ (double unresolved limits) derived from NNLO computation of heavy quarks

- NNLO QCD color singlet ingredients sufficient to deal with **initial state radiation**
- Production of on-shell Z or neutrino pair production *[Cieri, de Florian, Der, Mazzitelli (2020)]*

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Subtraction counterterm at small $r_{\rm cut}$ (double unresolved limits) derived from NNLO computation of heavy quarks

- The mass of the lepton is the physical regulator of the final state collinear radiation
  Pure soft ingredients: we need only $\mathcal{O}(\alpha)$ coefficients!

$$\Gamma_t = -\frac{1}{4}\left\{ e_\ell^2(1 - i\pi) + \sum_{i=1,2} e_i e_3 \ln \frac{(2p_i \cdot p_3)^2}{Q^2 m_\ell^2} \right\}$$

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Hard-Collinear coefficient:
contains the genuine two-loop virtual contribution plus finite contributions that restore unitarity

only missing ingredient!

Hard-Virtual coefficient

Hard–Virtual coefficient

$$d\sigma^F_{(N)NLO} = \mathscr{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jet(s)}_{(N)LO} - d\sigma^{CT} \right]$$

$$\mathscr{H}^F = [\, H^F \;\; C_1 C_2]$$

Process dependent hard-virtual functions: universal relation with the all-order virtual amplitude
[Catani, Cieri, de Florian, Ferrera Grazzini (2013)]

$$|\tilde{\mathscr{M}} > = (1 - \tilde{I})\,|\mathscr{M} >$$

$$H^F \sim < \tilde{\mathscr{M}} \,|\, \tilde{\mathscr{M}} >$$

Process independent (universal) **collinear functions** known up N3LO
[Catani,Grazzini (2011)],
[Catani, Cieri, de Florian, Ferrera, Grazzini (2012)
[Luo, Yang, Zhu, Zhu (2019)]
[Ebert, Mistlberger, Vita (2020)]

computed with abelianisation

In order to expose the *irreducible* virtual contribution, we introduce the decomposition

$$\mathscr{H}^{(m,n)} = H^{(m,n)}\delta(1 - z_1)\delta(1 - z_2) + \delta\mathscr{H}^{(m,n)}$$

$$H^{(0,1)} \equiv \frac{2\mathrm{Re}\left(\mathscr{M}^{(0,1)}_{\mathrm{fin}}\mathscr{M}^{(0,0)*}\right)}{|\mathscr{M}^{(0,0)}|^2} \qquad H^{(1,1)} \equiv \frac{2\mathrm{Re}\left(\mathscr{M}^{(1,1)}_{\mathrm{fin}}\mathscr{M}^{(0,0)*}\right)}{|\mathscr{M}^{(0,0)}|^2}$$

Hard-Virtual coefficient: IR structure and finite amplitudes

$$\mathcal{M}^{(1,0)}_{\rm fin} = \mathcal{M}^{(1,0)} + \frac{1}{2}\left(\frac{\alpha_s}{\pi}\right) C_F \left[\frac{1}{\epsilon^2} + \left(\frac{3}{2} + i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right]\mathcal{M}^{(0)}$$

$$\mathcal{M}^{(0,1)}_{\rm fin} = \mathcal{M}^{(0,1)} + \frac{1}{2}\left(\frac{\alpha}{\pi}\right)\left\{\left[\frac{1}{\epsilon^2} + \left(\frac{3}{2} + i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right]\frac{e_u^2 + e_d^2}{2} - \frac{2\Gamma_t}{\epsilon}\right\}\mathcal{M}^{(0)}$$

$$\begin{aligned}
\mathcal{M}^{(1,1)}_{\rm fin} = \mathcal{M}^{(1,1)} &- \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\left\{\frac{1}{8\epsilon^4}(e_u^2 + e_d^2)C_F + \frac{1}{2\epsilon^3}C_F\left[\left(\frac{3}{2} + i\pi\right)\frac{e_u^2 + e_d^2}{2} - \Gamma_t\right]\right\}\mathcal{M}^{(0)} \\
&+ \frac{1}{2\epsilon^2}\left\{\left(\frac{\alpha}{\pi}\right)\frac{e_u^2 + e_d^2}{2}\mathcal{M}^{(1,0)}_{\rm fin} + C_F\left(\frac{\alpha_s}{\pi}\right)\mathcal{M}^{(0,1)}_{\rm fin}\right. \\
&\quad \left. + C_F\left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\left[\left(\frac{7}{12}\pi^2 - \frac{9}{8} - \frac{3}{2}i\pi\right)\frac{e_u^2 + e_d^2}{2} + \left(\frac{3}{2} + i\pi\right)\Gamma_t\right]\mathcal{M}^{(0)}\right\} \\
&+ \frac{1}{2\epsilon}\left\{\left(\frac{\alpha}{\pi}\right)\left[\left(\frac{3}{2} + i\pi\right)\frac{e_u^2 + e_d^2}{2} - 2\Gamma_t\right]\mathcal{M}^{(1,0)}_{\rm fin} + \left(\frac{\alpha_s}{\pi}\right)C_F\left[\frac{3}{2} + i\pi\right]\mathcal{M}^{(0,1)}_{\rm fin}\right. \\
&\quad \left. + \frac{1}{8}C_F\left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right)\left[\left(\frac{3}{2} - \pi^2 + 24\zeta(3) + \frac{2}{3}i\pi^3\right)\frac{e_u^2 + e_d^2}{2} - \frac{2}{3}\pi^2\Gamma_t\right]\mathcal{M}^{(0)}\right\}
\end{aligned}$$

Hard-Virtual coefficient: Pole Approximation

The Pole Approximation (PA) is a systematic expansion around the resonance pole with respect to the parameter Γ_W/M_W.

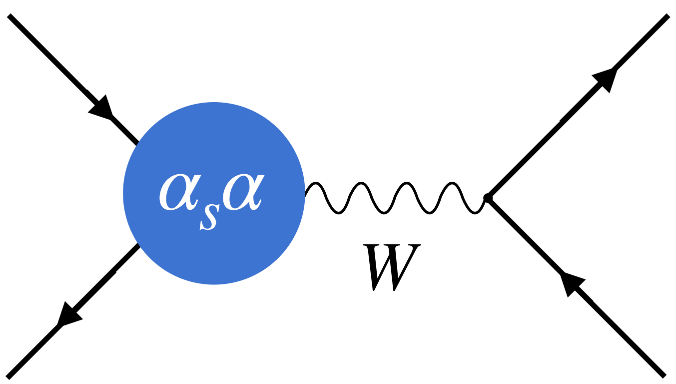
Beyond the narrow width approximation, the PA:

- keeps dominant (logarithmic) terms in Γ_W/M_W
- the structure of the IR singularities resembles that of the full computation

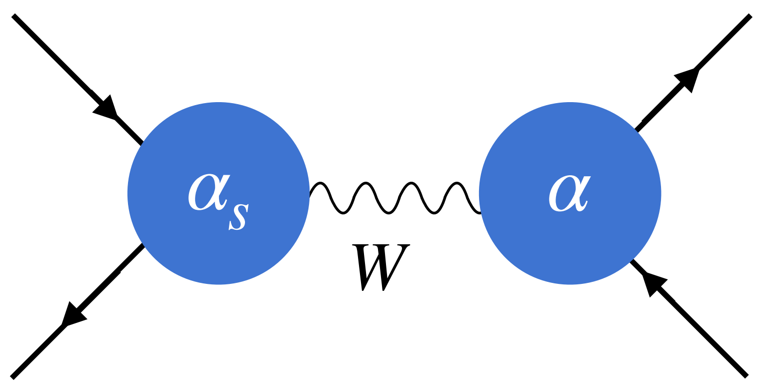
Factorisable corrections

The contributions are evaluated on-shell, only the resonant propagator is kept exact

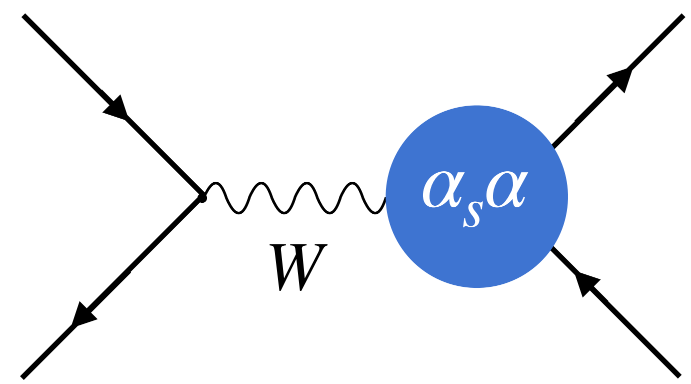
They corresponds to corrections to the production and/or decay vertex



Initial-Initial: extracted from mixed QCD-EW form factor for the W boson
[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)]



Initial-Final: computed using the one-loop provider RECOLA



Final-Final: finite renormalisation constant
[Dittmaier, Huss, and Schwinn (2015)]

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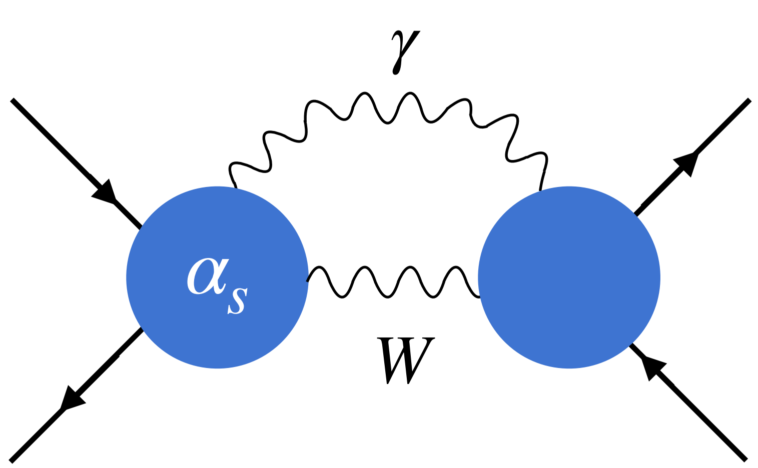
Non-Factorisable corrections

The contributions are evaluated on-shell, only the resonant propagator is kept exact

Correspond to box topologies containing a soft photon linking production and decay. The factorisable corrections are subtracted in order to avoid double-counting

Notice that no logs of the lepton mass can be generated by these contributions



$$= \mathscr{F}_{\text{nf}}^{(1,1)} \mathscr{M}_{\text{PA}}^{(0)} = \delta_{\text{nf}}^{(0,1)} \delta^{(1,0)} \mathscr{M}_{\text{PA}}^{(0)}$$

[Dittmaier, Huss, and Schwinn (2014)]

Hard-Virtual coefficient: Pole Approximation

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Remarks

At variance with the computation carried out in [Dittmaier, Huss, and Schwinn (2015)]

- we use the PA only for the (double) virtual-tree interference
- we include all factorisable and non-factorisable contributions which ensure the **correct structure** of the IR singularities
- power-corrections of the mass of the lepton are neglected in some of the two-loop contributions

Hard-Virtual coefficient: re-weighting

$$H_{\rm PA}^{(m,n)} = \frac{2{\rm Re}\left(\mathcal{M}_{\rm fin}^{(m,n)}\mathcal{M}^{(0,0)*}\right)_{\rm PA}}{|\mathcal{M}^{(0,0)}|^2}, \quad \text{for } m = 0{,}1,\ n = 1$$

Remark: since the Hard-Virtual term is eventually multiplied by $d\sigma_{\rm LO}$, the above definition corresponds to **compute the virtual-tree interference in PA**

We consider **alternative definitions** which differ for terms beyond the accuracy of the PA

- at NLO-EW ($m = 0, n = 1$)

$$H_{\rm PA,rwg}^{(0,1)} = \frac{2{\rm Re}\left(\mathcal{M}_{\rm fin}^{(0,1)}\mathcal{M}^{(0,0)*}\right)_{\rm PA}}{|\mathcal{M}_{\rm PA}^{(0,0)}|^2}$$

Cancellation of IR poles is exact

Effectively re-weights the virtual in PA with the exact Born amplitude

- at NNLO QCD-EW ($m = 1, n = 1$)

$$H_{\rm PA,rwg_B}^{(1,1)} = H_{\rm PA}^{(1,1)} \times \frac{|\mathcal{M}^{(0,0)}|^2}{|\mathcal{M}_{\rm PA}^{(0,0)}|^2} = \frac{2{\rm Re}\left(\mathcal{M}_{\rm fin}^{(1,1)}\mathcal{M}^{(0,0)*}\right)_{\rm PA}}{|\mathcal{M}_{\rm PA}^{(0,0)}|^2}$$

$$H_{\rm PA,rwg_V}^{(1,1)} = H_{\rm PA}^{(1,1)} \times \frac{H^{(0,1)}}{H_{\rm PA}^{(0,1)}} = \frac{2{\rm Re}\left(\mathcal{M}_{\rm fin}^{(1,1)}\mathcal{M}^{(0,0)*}\right)_{\rm PA}}{|\mathcal{M}^{(0,0)}|^2} \times \frac{2{\rm Re}\left(\mathcal{M}_{\rm fin}^{(0,1)}\mathcal{M}^{(0,0)*}\right)}{2{\rm Re}\left(\mathcal{M}_{\rm fin}^{(0,1)}\mathcal{M}^{(0,0)*}\right)_{\rm PA}}$$

Effectively re-weights with the exact one-loop EW virtual amplitude

Numerical Results

IMPLEMENTATION & SETUP

Our calculation has been implemented in the **MATRIX** framework

[Grazzini, Kallweit, Wiesemann (2017)]

- Efficient multi-channel integrator MUNICH by S.Kellweit
- Automatic implementation of dipole subtraction
- Interfaced to OpenLoops and Recola for the evaluation of required tree-level and one-loop matrix elements
- q_T subtraction is implemented as a slicing

Setup similar to *[Dittmaier, Huss, and Schwinn (2015)]*

Physical Parameters (G_μ complex mass scheme)

- $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}
- $M_{W,\mathrm{OS}} = 80.385$ GeV $\quad M_{Z,\mathrm{OS}} = 91.1876$ GeV
- $\Gamma_{W,\mathrm{OS}} = 2.085$ GeV $\quad \Gamma_{Z,\mathrm{OS}} = 2.4952$ GeV
- $m_\mu = 105.658369$ MeV $\quad M_t = 173.3$ GeV $\quad M_H = 125.9$ GeV
- $\mu_F = \mu_R = M_W$ $\quad$ pdf set: `NNPDF31_nnlo_as_0118_luxqed`

Fiducial cuts

- $p_{T,\mu} > 25$ GeV $\quad |y_\mu| < 2.5$ $\quad p_{T,\nu_\mu} > 25$ GeV
- *no lepton-photon recombination (**bare muon**)*

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Smallness of the mass is an extra challenge!

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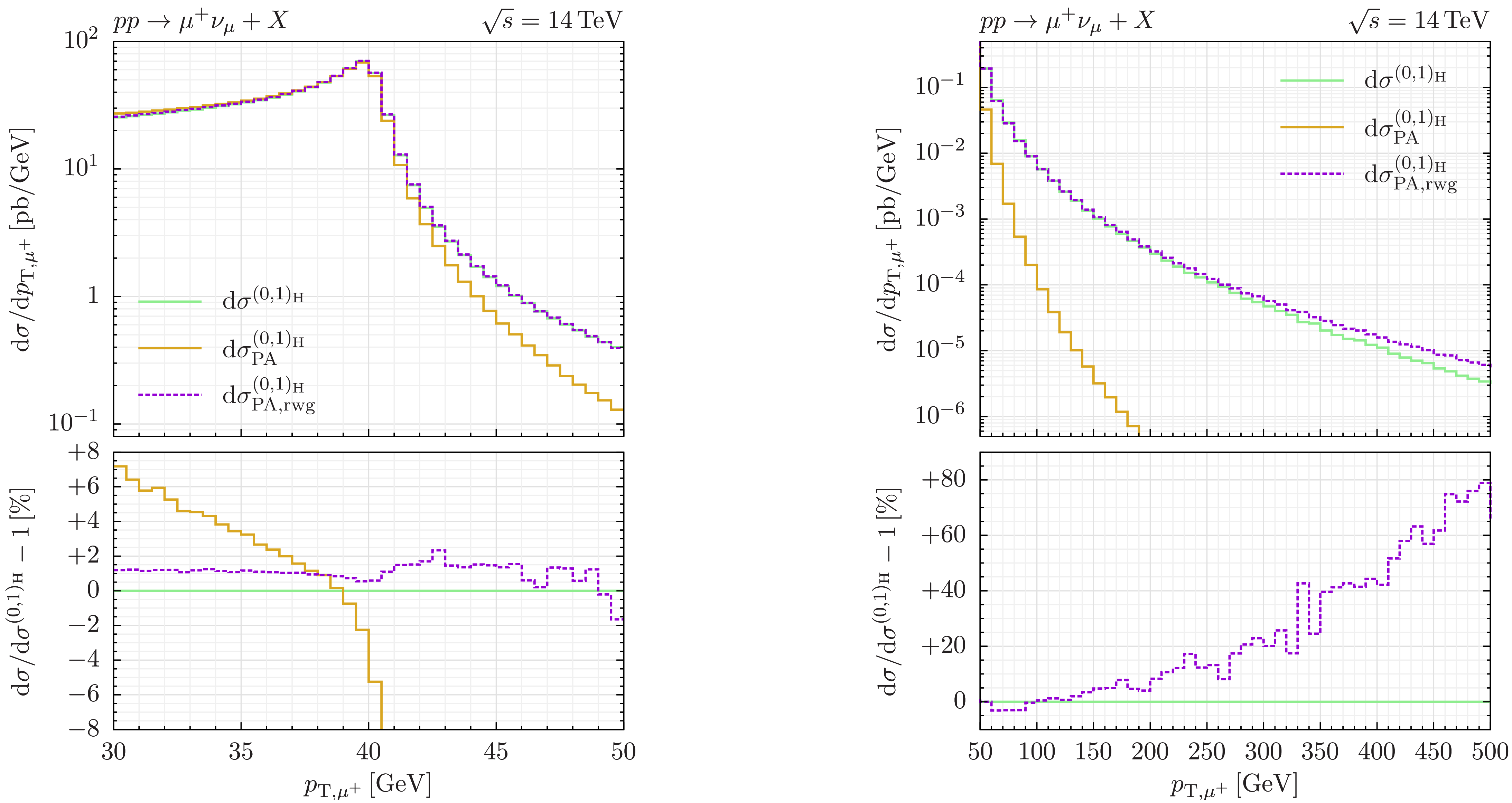
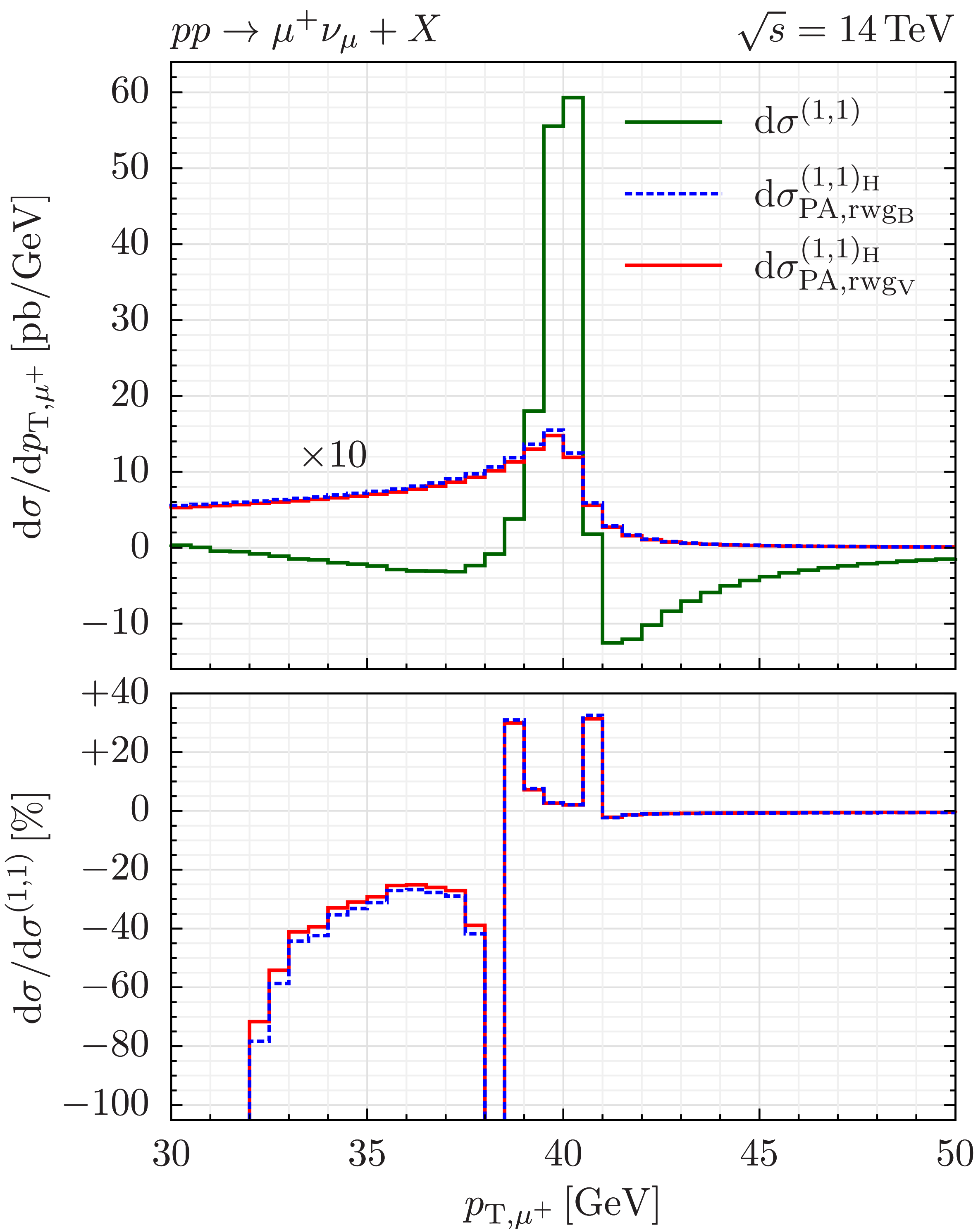
VALIDATION of the POLE APPROXIMATION @NLO-EW



- The Pole Approximation **supplemented** with the **re-weighting**
  - agrees with the exact result at the **percent level** both below and above the **peak**
  - good modelling (**correct order of magnitude)** of the hard-virtual at high pT
  - difference with exact coefficient: $\mathcal{O}(20\%)$ at 300 GeV, $\mathcal{O}(80\%)$ at 500 GeV with PA systematically overshooting the exact result (**Sudakov Logs**)

VALIDATION of the POLE APPROXIMATION @NNLO QCD-EW

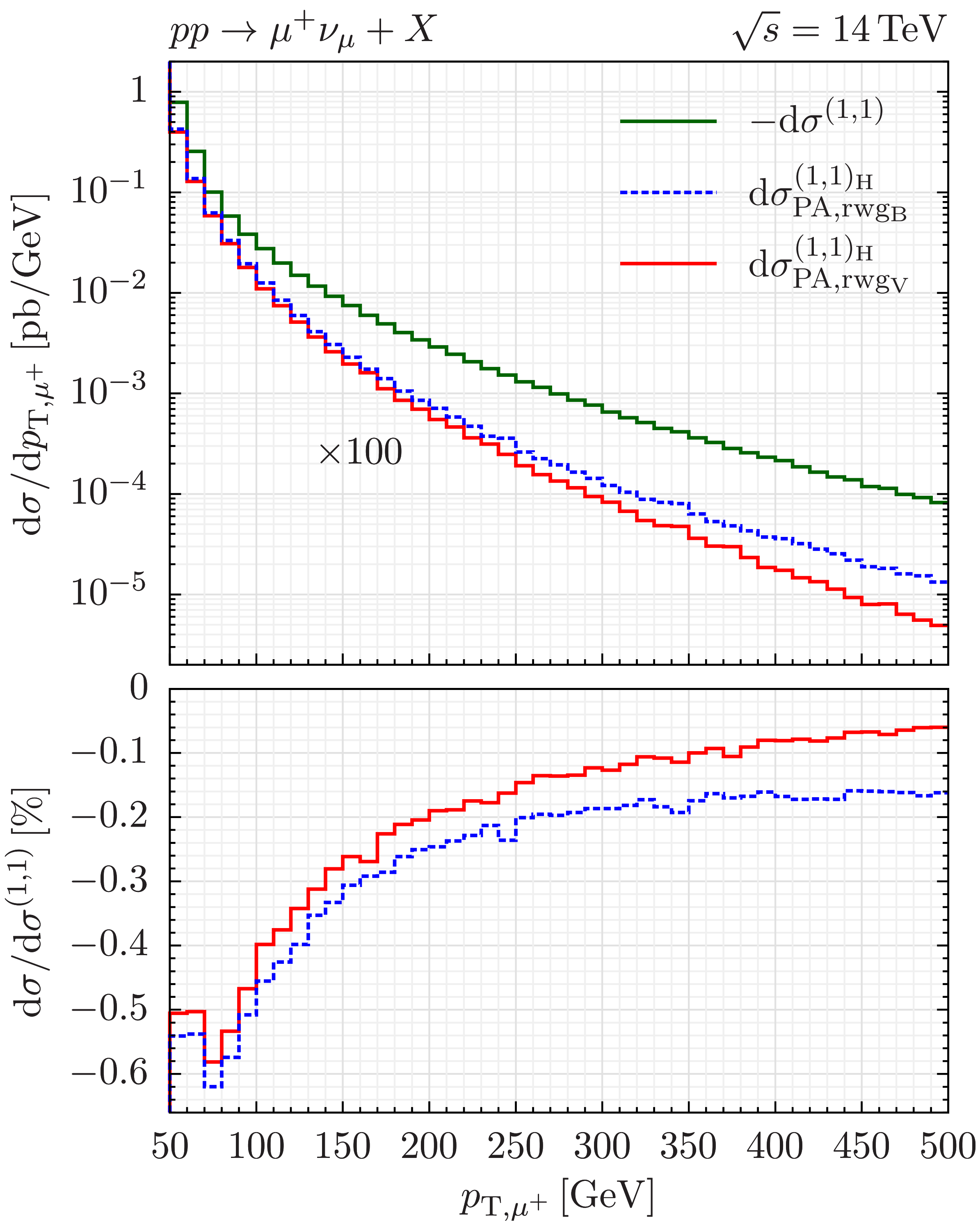


The comparison between the two approximations $\mathrm{rwg_B}$ and $\mathrm{rwg_V}$ allows us to gauge the uncertainties associated to the mis-modelling of the hard-virtual coefficient $H^{(1,1)}$

Around the peak region (low-p_T)

- the two approximations are very to close to each other , consistently with the expectation that PA should work well
- the relative impact of $H^{(1,1)}$ is rather modest/small but for the regions in which the mixed corrections change sign and/or are vanishing

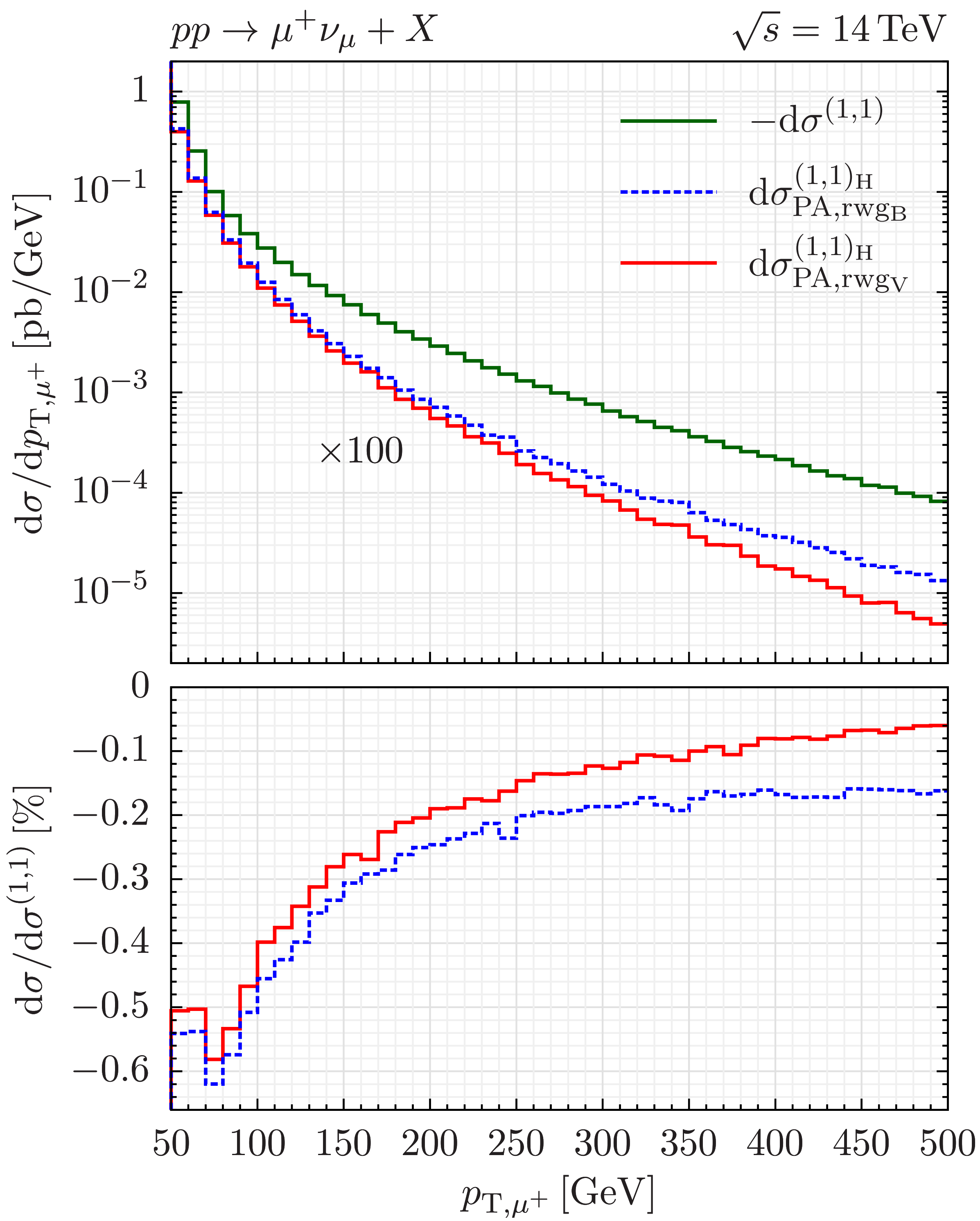
VALIDATION of the POLE APPROXIMATION @NNLO QCD-EW



At high-p_T

- the two approximations starts to differs (a factor ~ 2 at $p_T = 500\,\mathrm{GeV}$)
- the PA re-weighted with the NLO-EW coefficient $H^{(0,1)}$ ($\mathrm{rwg_V}$) displays a softer spectrum going at higher p_T as expected since it includes exact Sudakov Logs
- the relative impact of $H^{(1,1)}$ **is always smaller than** $1\,\%$ (and becomes smaller as p_T increases)

VALIDATION of the POLE APPROXIMATION @NNLO QCD-EW



Physical explanation: at high-p_T, resonant Born-like topologies **are suppressed** and the cross section is dominated by real contributions where an on-shell W boson recoils against an hard QCD or QED emission

Furthermore, we find that the dominant contribution is given here by the *qg* **channel**, which is computed exactly

The result: p_T of the muon

We present our prediction for the $\mathcal{O}(\alpha_s\alpha)$ correction as

- absolute correction
- normalised correction with respect to the LO cross section
- normalised correction with respect to the NLO QCD cross section

We compare our results with the naive factorised ansantz given by the formula

$$\frac{d\sigma^{(1,1)}_{\text{fact}}}{dp_T} = \left(\frac{d\sigma^{(1,0)}}{dp_T}\right) \times \left(\frac{d\sigma^{(0,1)}_{q\bar{q}}}{dp_T}\right) \times \left(\frac{d\sigma_{\text{LO}}}{dp_T}\right)^{-1}$$

Remark

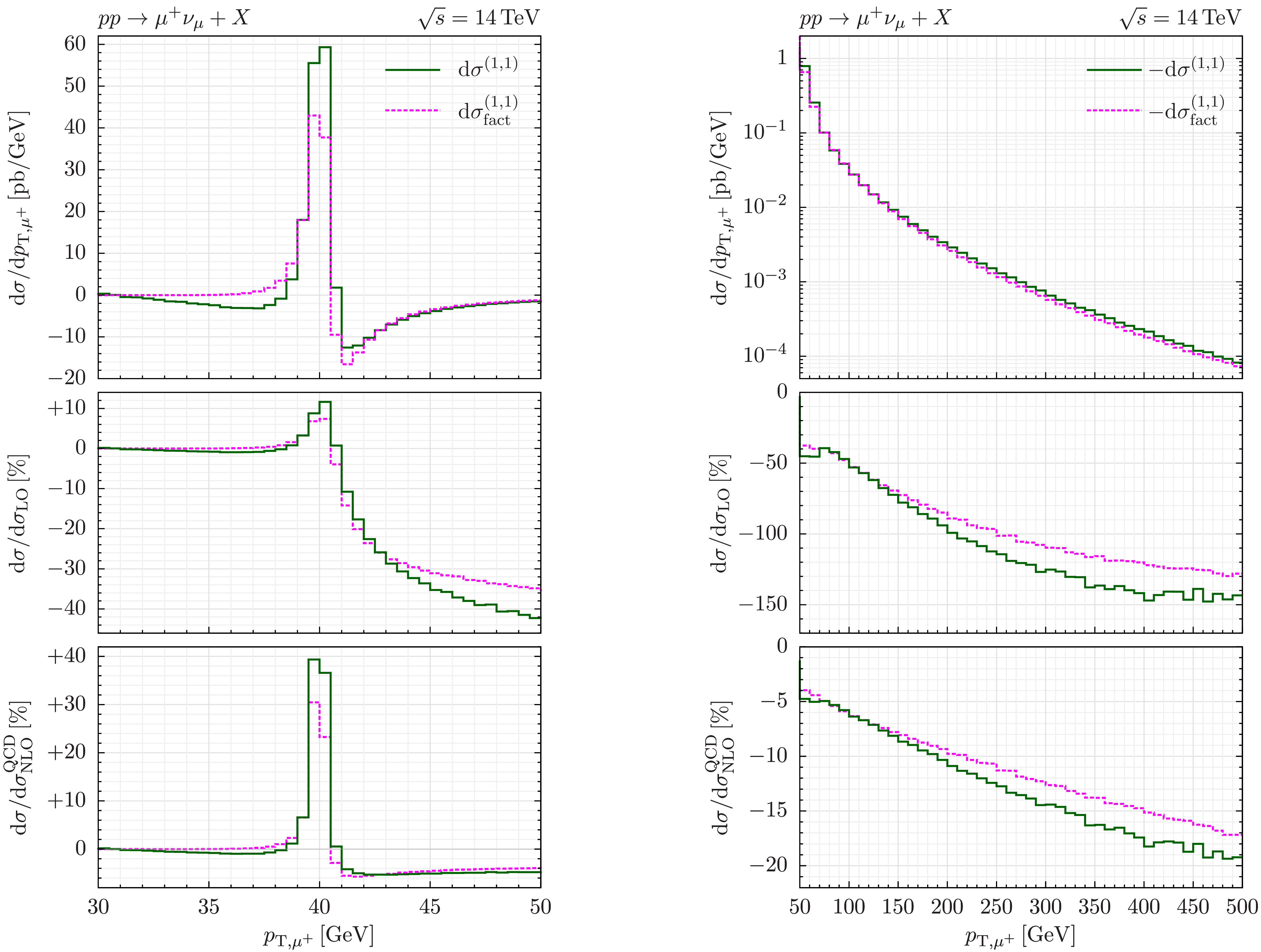
A factorised approach is justified if the dominant sources of QCD and EW corrections factorise with respect to the hard W production subprocesses.

At NLO, gluon/photon initiated channels open up populating the tail of the p_T spectrum, thus leading to large corrections (***giant K-factors***)

We do not include the **photon-induced** channels in the NLO-EW differential K-factor to avoid the multiplication of two giant K-factors of QCD and EW origin, which is not expected to work

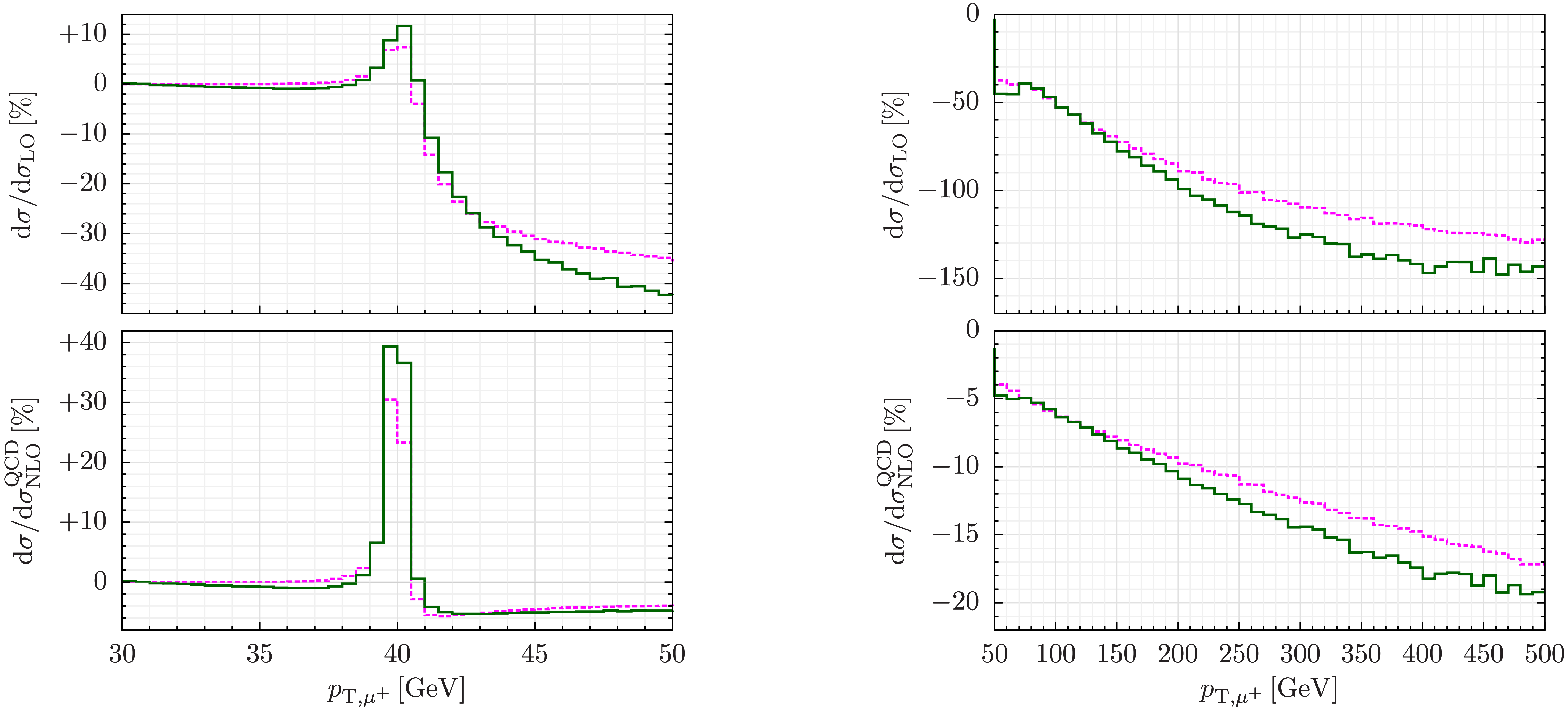
[Lindert, Grazzini, Kallweit, Pozzorini, Wiesemann (2019)]

The result: p_T of the muon



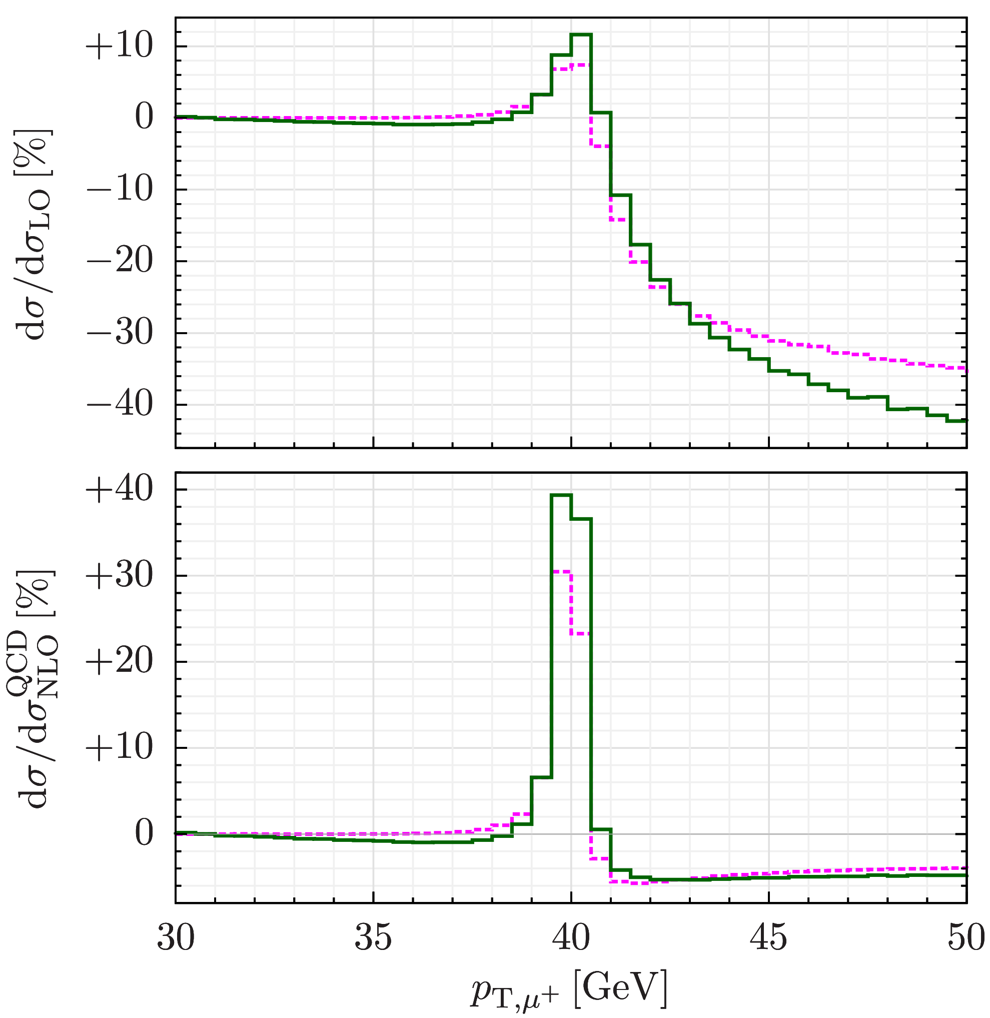
The result: p_T spectrum

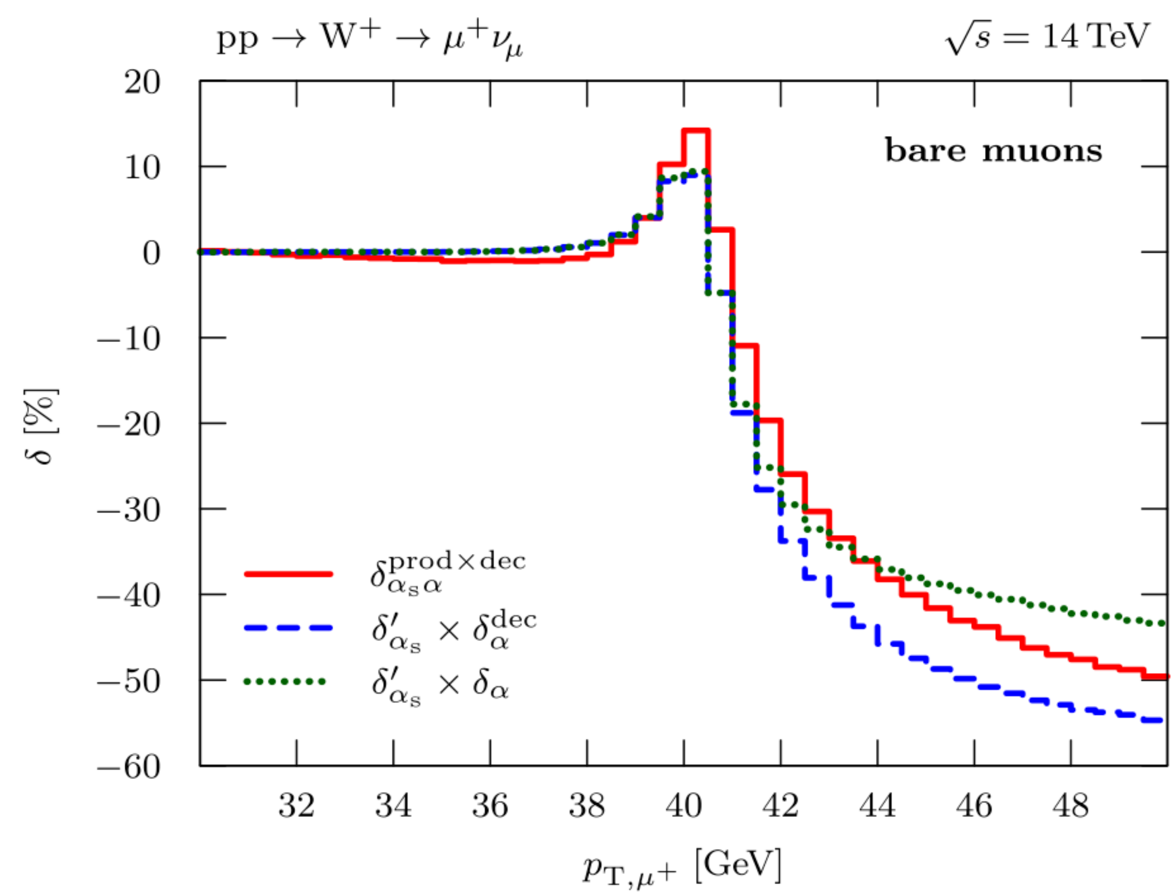
- Negative correction in the tail as large as $\mathcal{O}(20\%)$ of the NLO QCD at $p_T = 500\,\mathrm{GeV}$
- The factorised anzatz shows a harder spectrum, but overall decently reproduce the complete result



The result: p_T spectrum

- Negative correction in the tail as large as $\mathcal{O}(20\%)$ of the NLO QCD at $p_T = 500\,\text{GeV}$
- The factorised anzatz shows a harder spectrum, but overall decently reproduce the complete result
- Around the peak, qualitative agreement with the result of **[Dittmaier, Huss, and Schwinn (2015)]**



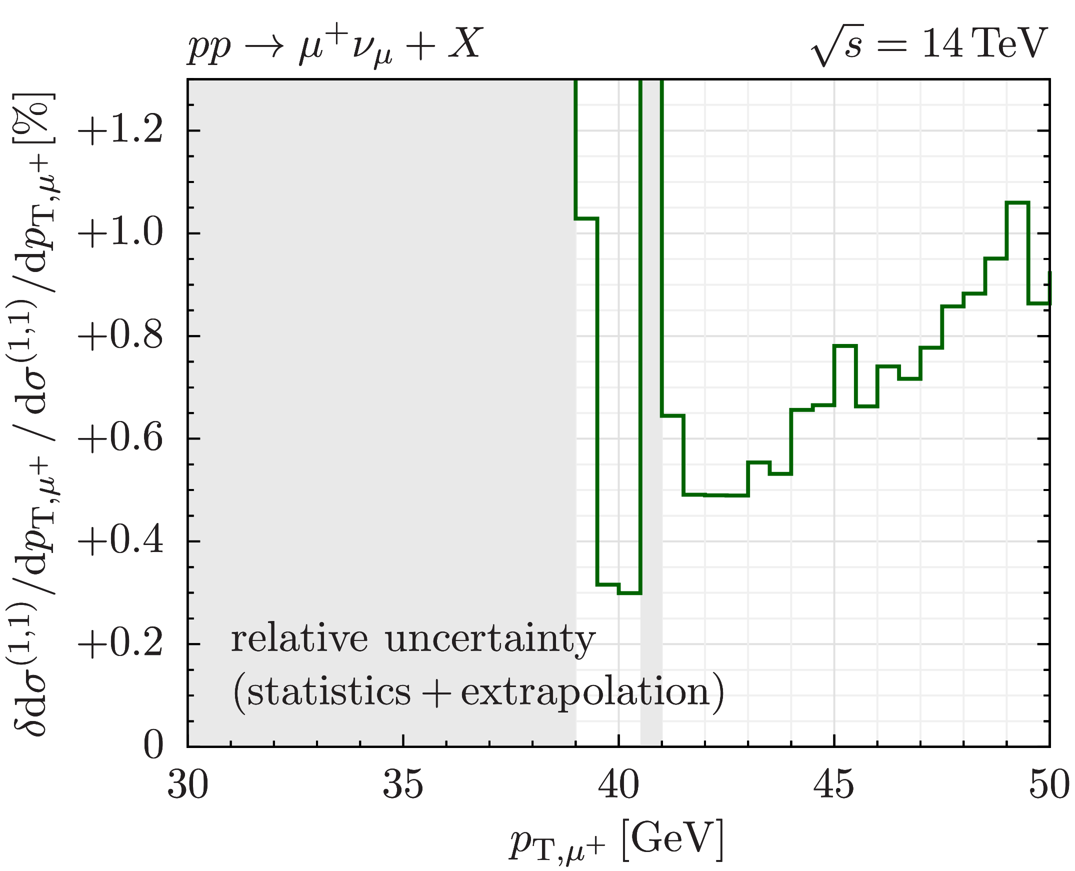


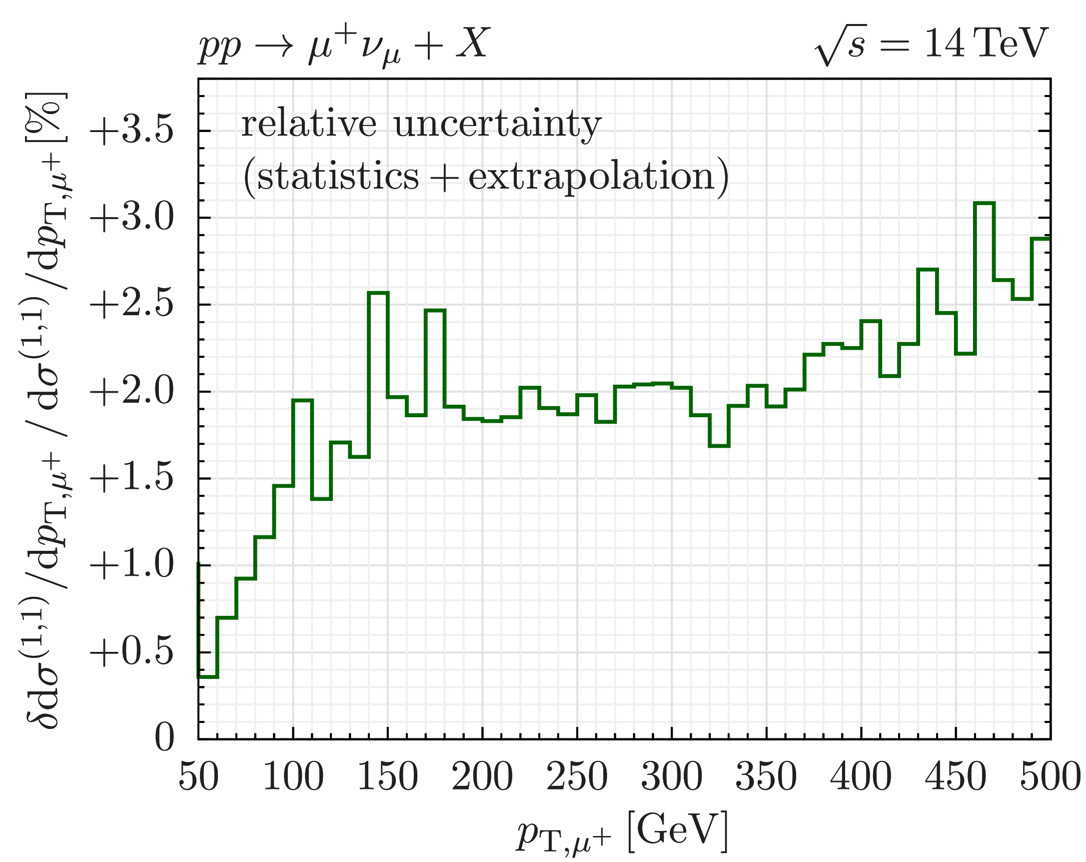
Systematic uncertainties

q_T subtraction is implemented as a **slicing:** the computation is carried out keeping a finite value of $r_{\text{cut}} = q_{T,\text{cut}}/Q$

Our best prediction is obtained by an extrapolation procedure for $r_{\text{cut}} \to 0$, varying r_{cut} in the range $[10^{-4}, 10^{-2}]$. It is applied both at the level of the total cross section and <u>at the level of individual bins of differential distributions</u>.

We have a rather good control over the total systematic (statics+extrapolation), both in the peak (sub-percent) and in the tail region (few percent), which is sufficient for the phenomenology





CONCLUSIONs

- We have presented a new computation of the mixed QCD-EW corrections to the $2 \to 2$ charged Drell-Yan process with massive lepton

- For the first time, all real and virtual contributions are consistently included but for the finite part of the two-loop amplitude, which is computed in the pole approximation and improved through a re-weighting procedure

- The cancellation of the IR singularities is achieved with q_T subtraction. The extension for the mixed QCD-EW case can be worked out applying a careful abelianisation procedure to the NNLO QCD calculation for heavy quarks

- We have focused on the p_T of the muon, showing that our calculation is reliable in the entire region of the muon transverse momentum

- We believe that our calculation fills the gap in controlling the residual uncertainty coming from the mixed QCD-EW corrections for the considered process

BACKUP

q_T subtraction formalism: review (color singlet)

Consider the production of a vector boson (color singlet system) of mass M

$$h_1(P_1) + h_2(P_2) \to F(Q)$$

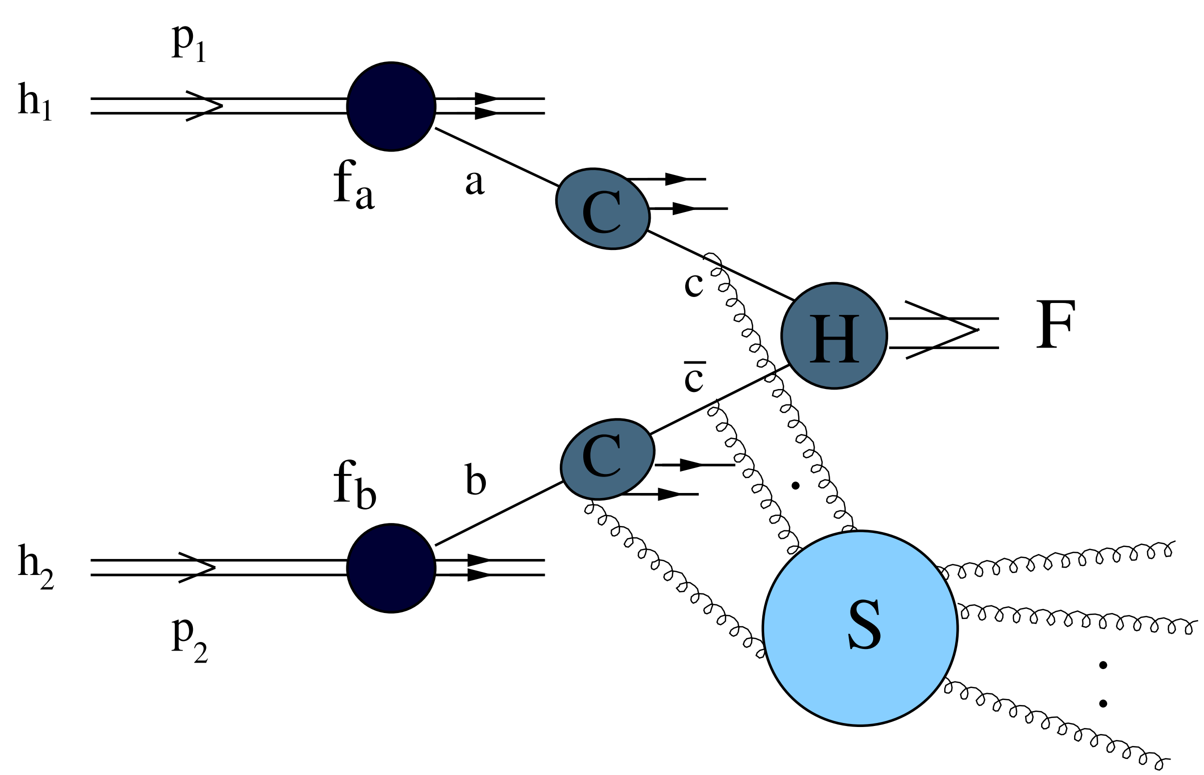
The transverse momentum of the color singlet q_T controls the the structure of the **infrared singularities** (**good resolution variable**)

1. At NLO: all the IR singularities are contained in the small q_T limit

2. At NNLO, disentanglement of the singularities:

   • $q_T > 0$, the structure of the divergence is as NLO for the process $F +$ jet

   • $q_T = 0$, the genuine NNLO singularities occur (**double unresolved emissions**)



q_T subtraction formalism: review (color singlet)

The singular part of the cross section in the small-q_T limit is controlled by the **transverse resummation formula**

[S. Catani, D. de Florian and M. Grazzini, Nucl. Phys. B 596 (2001) 299]

Sudakov Form Factor: large logs

$$d\sigma^{(sing)} = \frac{M^2}{s} \sum_c \sigma^{(0)}_{c\bar{c},V} \int_0^\infty db \frac{b}{2} J_0(bq_T) S_q(M,b)$$
$$\times \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} [H^F C_1 C_2]_{c\bar{c};a_1 a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2) dy dq_T^2$$

Hard-collinear function: $\delta(q_T^2)$ terms

Fixed-order expansion of this formula allows to build a subtraction scheme

[S. Catani, M. Grazzini, Phys.Rev.Lett. 98 (2007) 222002]

$$d\sigma^F_{(N)NLO} = \mathscr{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jet(s)}_{(N)LO} - d\sigma^{CT} \right]$$

Hard-collinear **auxiliary cross section**

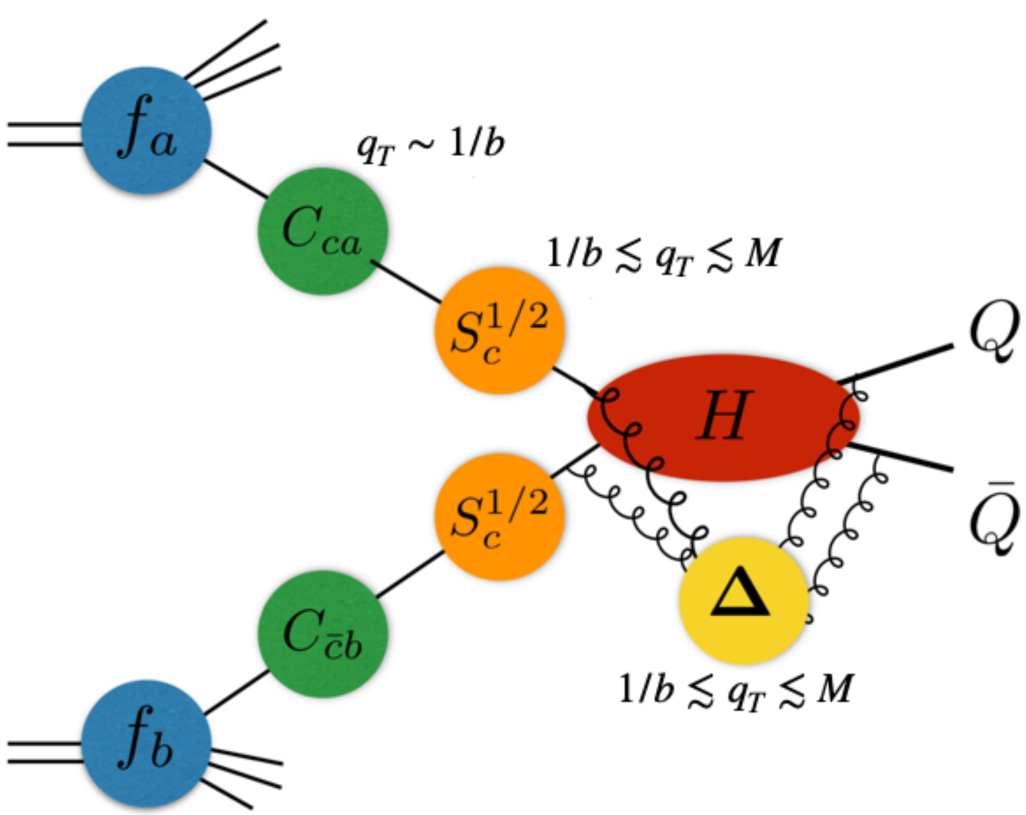
q_T subtraction formalism for heavy quarks production

q_T subtraction formalism extended to the case of heavy quarks production *[Catani, Grazzini, Torre (2014)]*

Successful employed for computation of NNLO QCD corrections to the production of

- a top pair *[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan (2019)]*
- a bottom pair production *[Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2021)]*
- a top pair and a Higgs (off-diagonal channels) *[Catani, Fabre, Grazzini, Kallweit, (2021)]*

The resummation formula shows a **richer structure** because of additional soft singularities (four coloured patrons at LO)



- Soft logarithms controlled by the **transverse momentum anomalous dimension** $\mathbf{\Gamma}_t$ known up to NNLO *[Mitov, Sterman, Sung(2009)], [Neubert et al (2009)]*
- Hard coefficient gets a **non-trivial** colour structure (matrix in colour-space)
- Non trivial azimuthal correlations
- Notice that is crucial that the final state is **massive:** the mass is the physical regulator of the final state collinear singularities

q_T subtraction formalism: review (color singlet)

q_T counterterm is by construction **non-local**

$$d\sigma^{CT} = d\sigma_{L0} \otimes \Sigma^F \left(\frac{q_T}{M}\right) d^2 q_T$$

q_T-subtraction is actually implemented as a **slicing** introducing a cutoff on the minimum allowed transverse momentum

$$\frac{q_T}{M} > r_{\text{cut}}$$

The real emission cross section and the counterterm are **integrated separately, giving rise to logs** in r_{cut}. Trade off between

- **Global cancellation** between large logs: choose r_{cut} relatively large
- The slicing is **exact** in the $r_{\text{cut}} \to 0$ limit; for finite r_{cut}, it introduces **power corrections:** choose r_{cut} relatively small

For color singlet production power corrections are known to be **quadratic** for inclusive cross sections: *[Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann (2016)], [Ebert, Moult, Stewart, Tackmann. Vita. Zhu (2019)], [Cieri, Oleari, Rocco (2019)]*

They might become more severe in presence of cuts (as **for symmetric cuts** on final states) *[Grazzini,Kallweit, Wiesemann (2017)], [Ebert, Michel, Stewart, Tackmann (2020)], [Alekhin, Kardos, Moch, Trócsányi (2021)]*

q_T subtraction @NLO−EW: validation

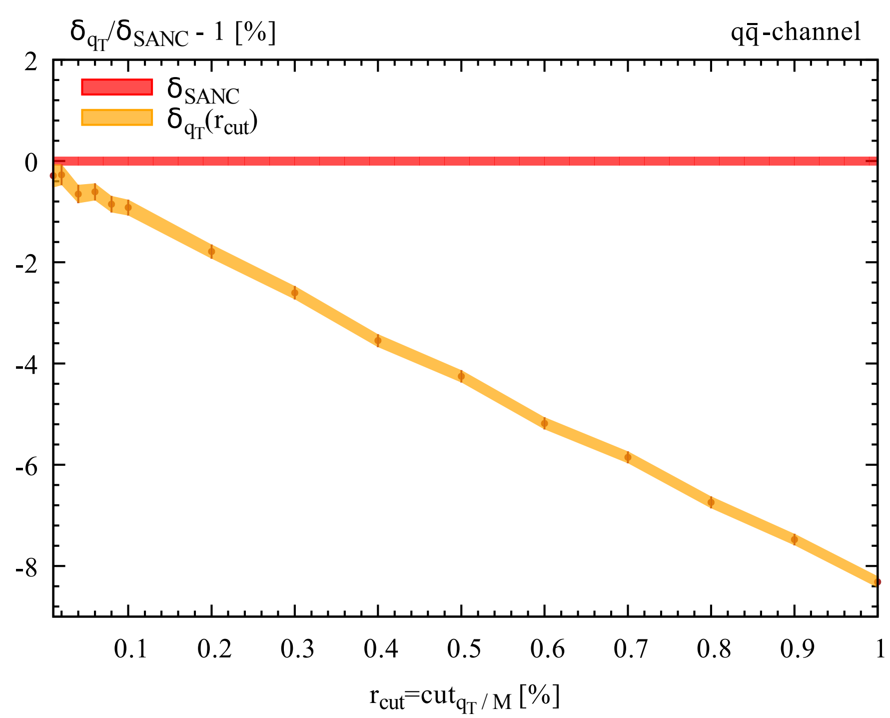
LB, M. Grazzini, F. Tramontano (2019)

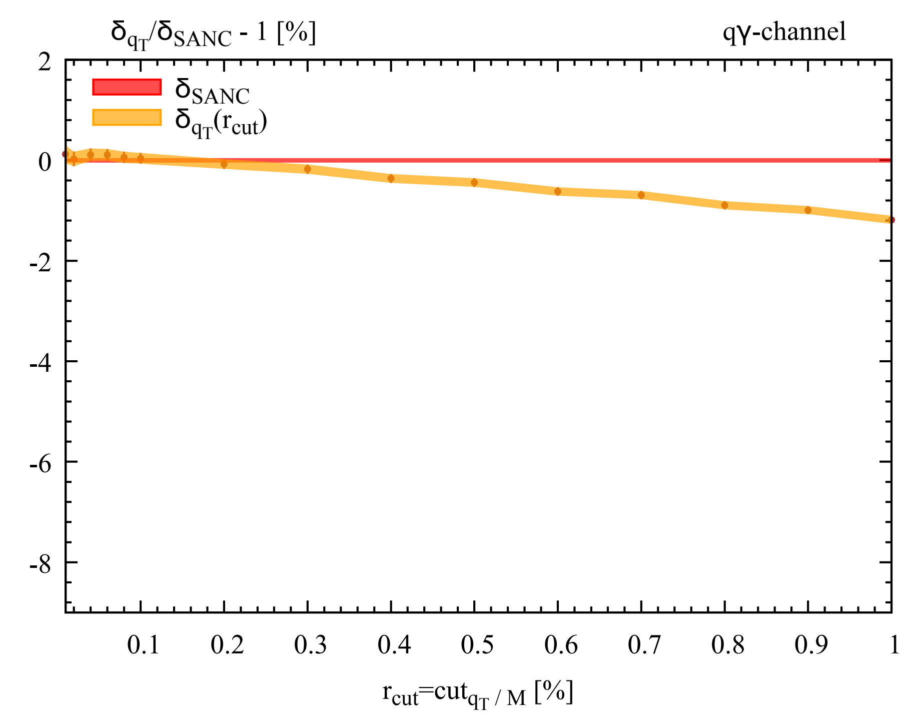
Proof-of-concept: NLO-EW corrections to neutral current Drell-Yan production with fiducial cuts (symmetric cuts) and for **physical muon** mass $m_\mu = 105.658369\,\text{MeV}$

$$M_{\ell^+\ell^-} > 50\,\text{GeV} \qquad p_{\ell^\pm} > 25\,\text{GeV} \qquad |y_{\ell^\pm}| < 2.5$$

	q_T + GoSam	SANC
$\sigma^{q\bar{q}}_{LO}$ (pb)	739.45 ± 0.02	739.17 ± 0.01
$\sigma^{\gamma\gamma}_{LO}$ (pb)	1.289 ± 0.005	1.29 ± 0.01
$\Delta\sigma_{q\bar{q}}$ (pb)	-29.18 ± 0.03	-29.23 ± 0.02
$\Delta\sigma_{q\gamma}$ (pb)	-0.777 ± 0.002	-0.78 ± 0.01

We study analytically the power corrections arising from soft (QED) radiation off massive final state: they are **linear** (even for inclusive setup)





Results: Fiducial Cross Sections

σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	5029.2	970.5(3)	−143.61(15)	251(4)	− 7.0(1.2)
qg	—	−1079.86(12)	—	−377(3)	39.0(4)
$q(g)\gamma$	—	—	2.823(1)	—	0.055(5)
$q(\bar{q})q'$	—	—	—	44.2(7)	1.2382(3)
gg	—	—	—	100.8(8)	—
tot	5029.2	−109.4(4)	−140.8(2)	19(5)	33.3(1.3)
$\sigma^{(m,n)}/\sigma_{\mathrm{LO}}$		−2.2 %	−2.8 %	+0.4 %	+0.6 %

- NLO and NNLO QCD corrections show large cancellations among the partonic channels (especially between $q\bar{q}$ and qg)
- NLO QCD and NLO EW corrections are of the same order
- Mixed QCD-EW corrections are dominated by the qg channel (exact) and are larger than NNLO QCD (for the particular chosen setup)

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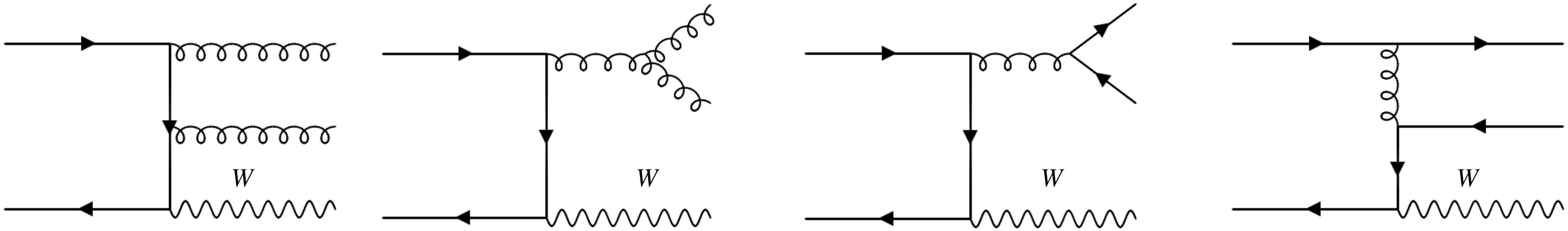
Remark: the pattern of QCD correction is sensitive to the choice of the scales. For example, for $\mu_F = \mu_R = m_W/2$ we find

$\sigma^{(m,n)}/\sigma_{\mathrm{LO}}$		+10 %	−2.9 %	+4.2 %	+0.8 %

Example of abelianisation at NNLO (ISR)

qq_x channel



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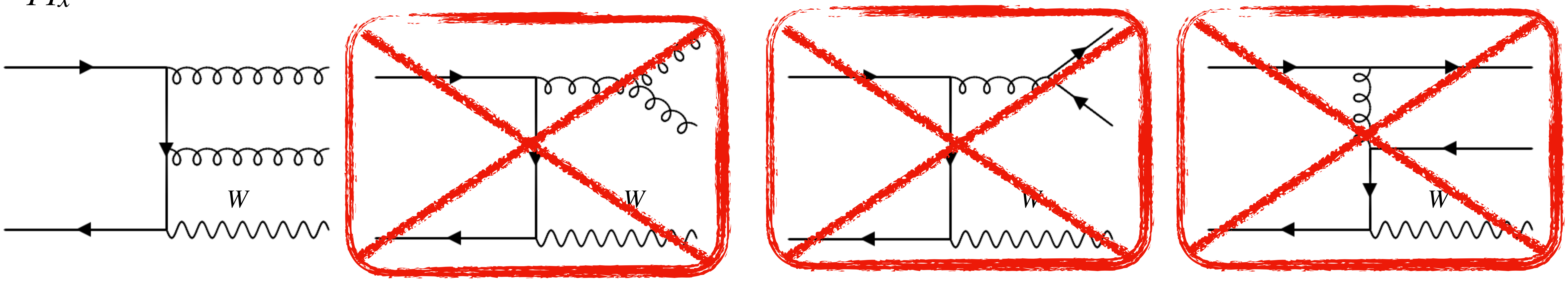
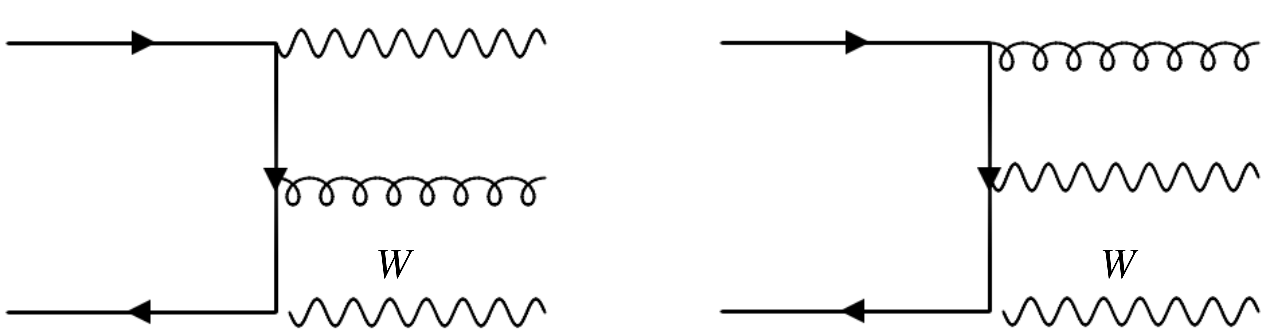


Color structure + symmetry factor (**identical gluons**)

$$\frac{1}{2N_C^2}\mathrm{Tr}[T^aT^aT^bT^b] = \frac{C_F^2}{2N_C} \qquad \frac{1}{2N_C^2}\mathrm{Tr}[T^aT^bT^aT^b] = \frac{1}{2N_C}C_F\left(C_F - \frac{C_A}{2}\right)$$

Photon-gluon replacement. Two **distinguishable** processes



$$\frac{1}{N_C^2}\mathrm{Tr}[T^aT^a]e_f^2 = \frac{C_F e_f^2}{N_C}$$

Replacement list:

$$C_A \to 0, \quad C_F^2 \to 2C_F e_f^2$$