

RADCOR-LoopFest 2021

Florida State University, USA

20 May, 2021

Virtual Conference

Radiative corrections in neutrino physics



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Outline

1) coherent elastic **neutrino-nucleus** scattering (CEvNS)

neutrino energy < 100 MeV

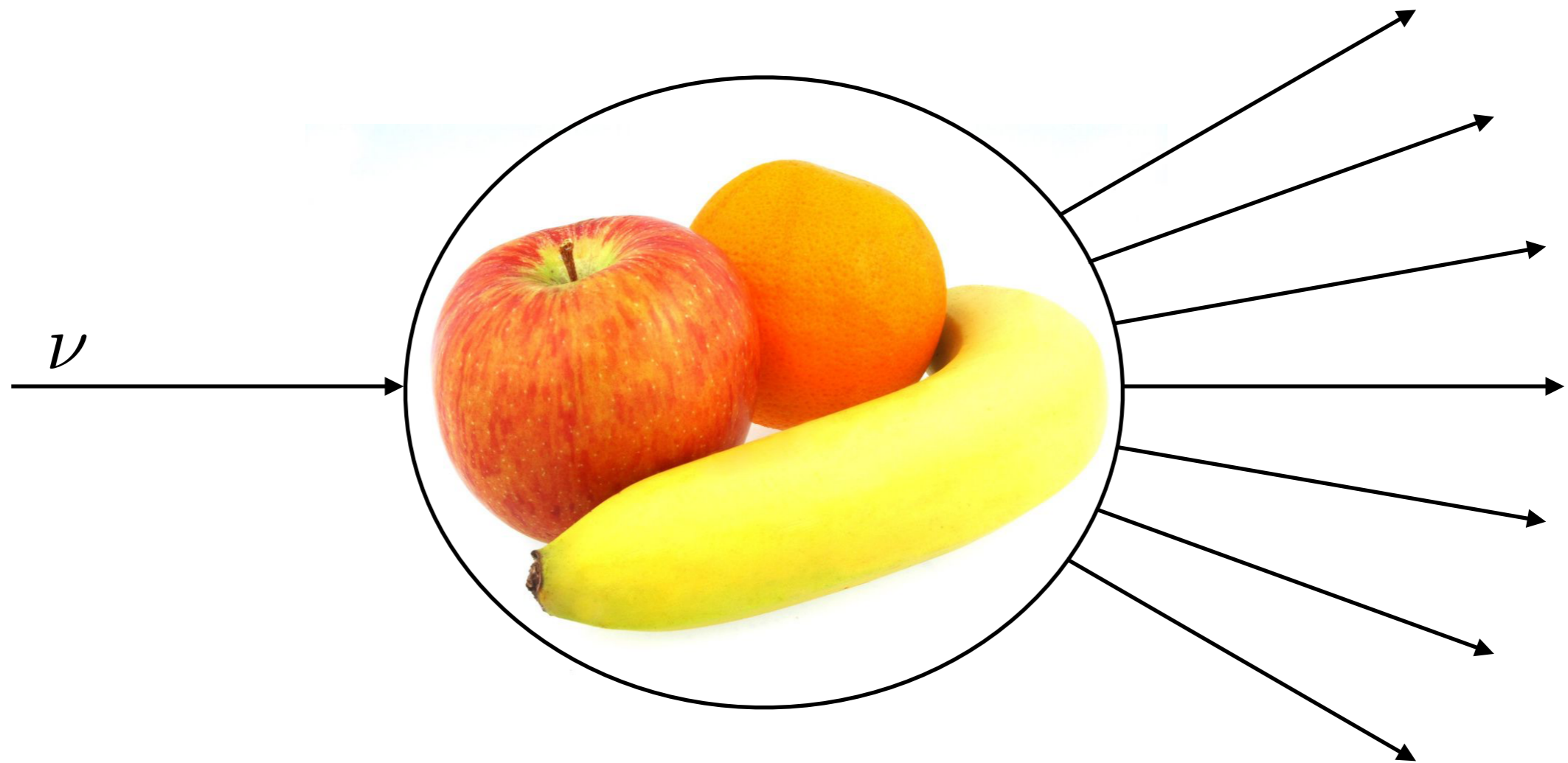
O.T., Pedro Machado, Vishvas Pandey and Ryan Plestid, JHEP 2102, 097 (2021)

2) charged-current elastic scattering on **nucleons**

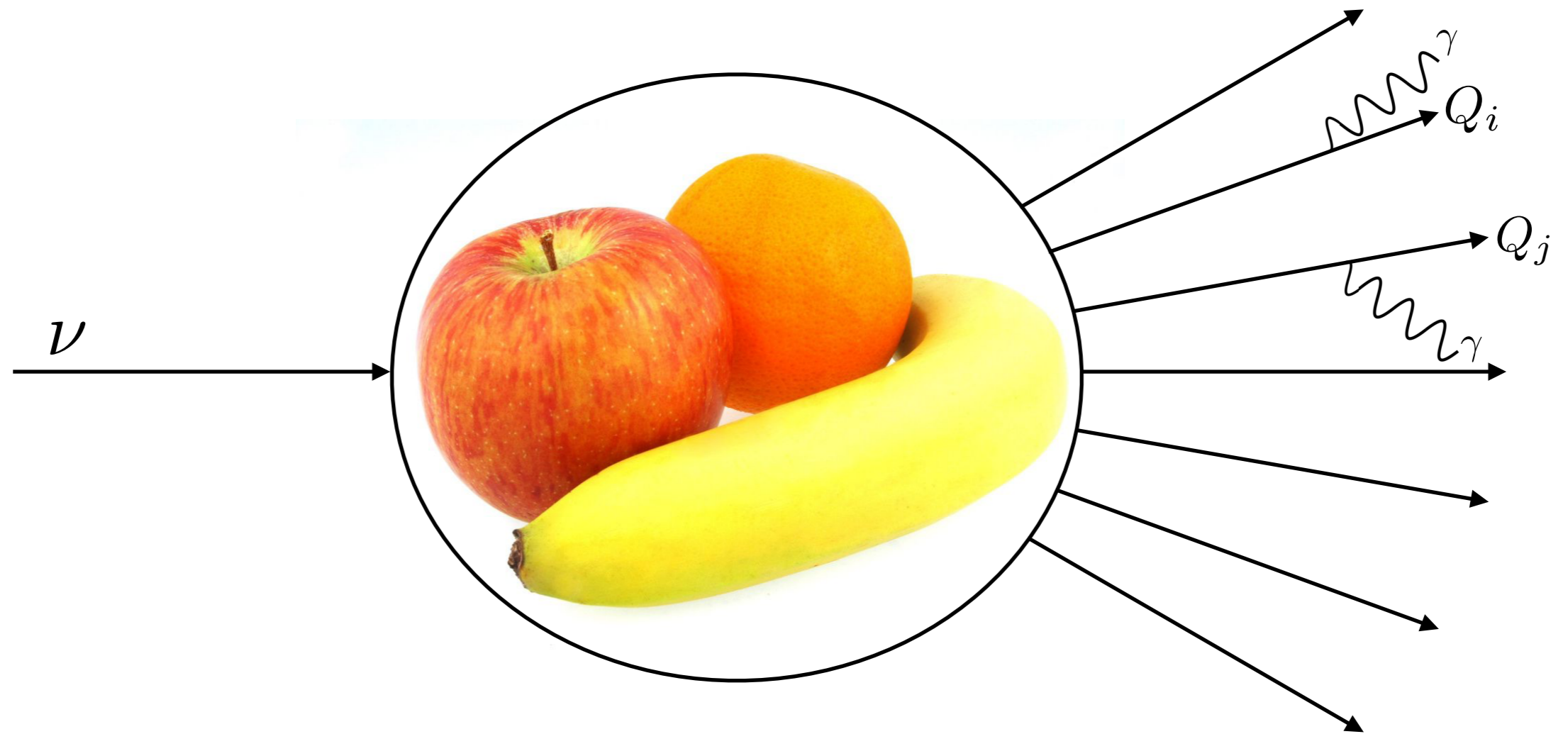
neutrino energy ~ 1 GeV

O.T., Qing Chen, Richard J. Hill and Kevin S. McFarland, arXiv: 2105.07939

Neutrino interactions

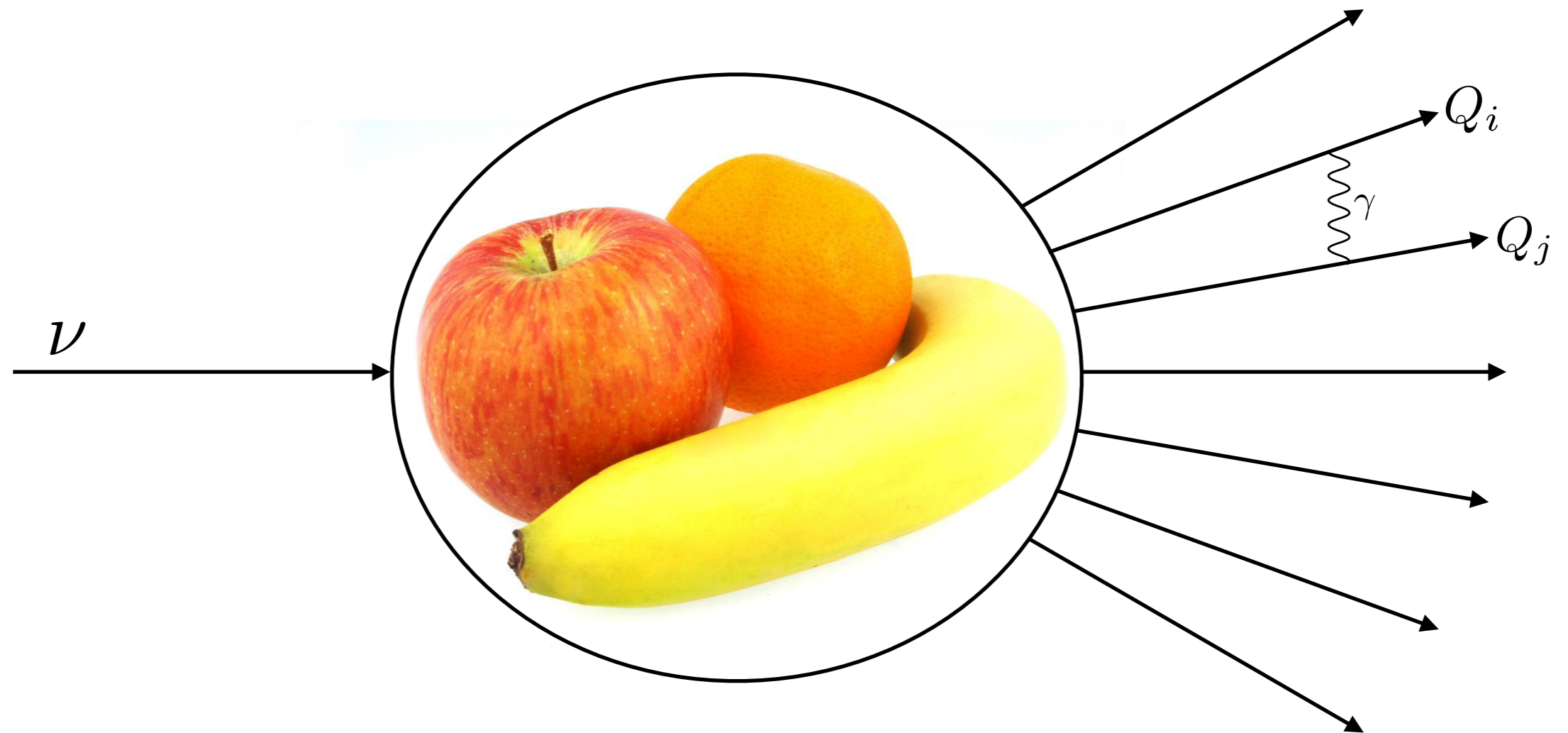


QED corrections



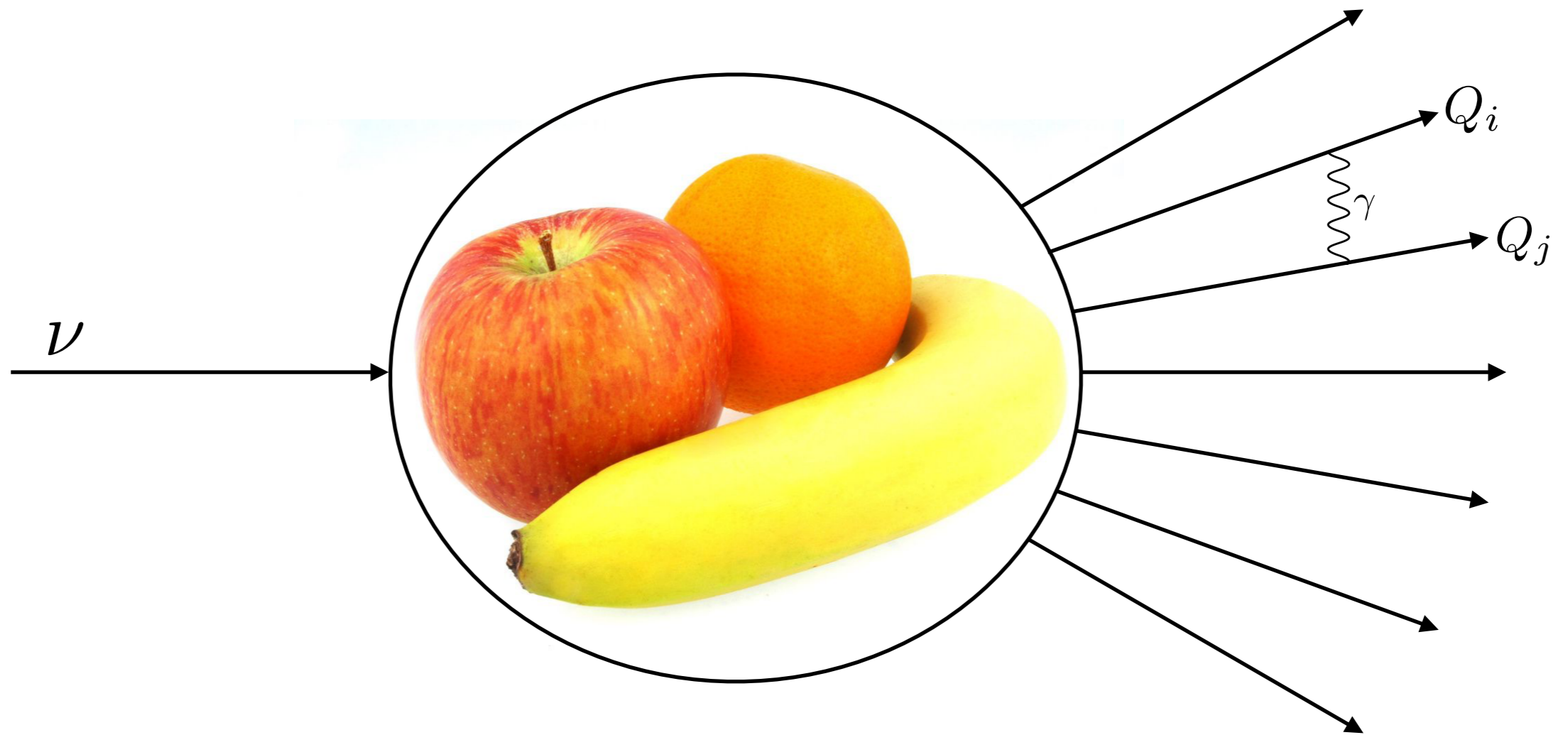
- all charged particles couple to real and virtual photons

QED corrections



- all charged particles couple to real and virtual photons

QED corrections



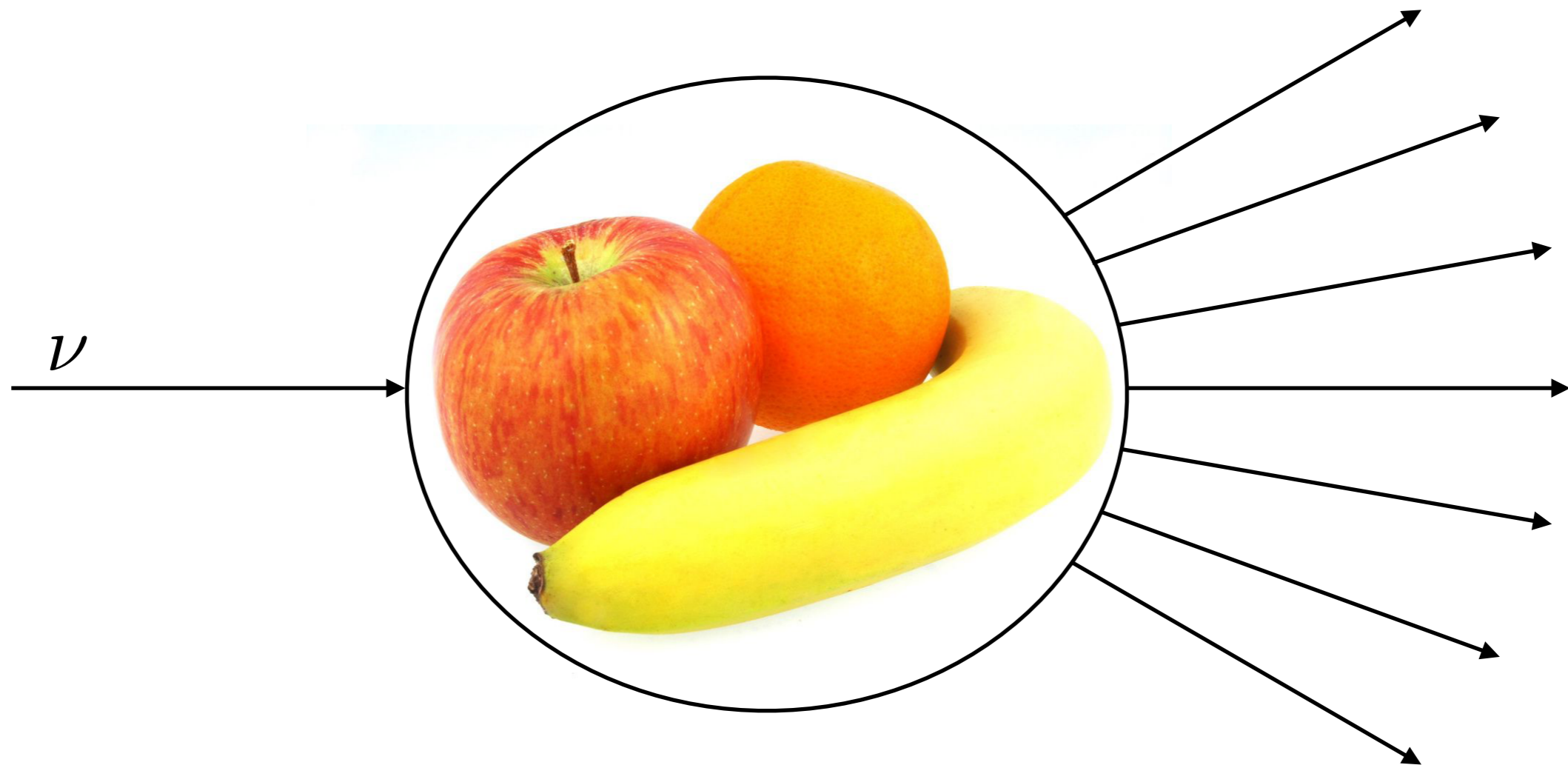
- $\frac{\alpha}{\pi} \sim 0.2\%$ suppression by electromagnetic coupling constant

QED corrections

CEvNS

neutral-current interactions

JHEP 2102, 097 (2021)



$X \frac{\alpha}{\pi} \sim 0.2 \%$ multiplied by kinematic-dependent factors

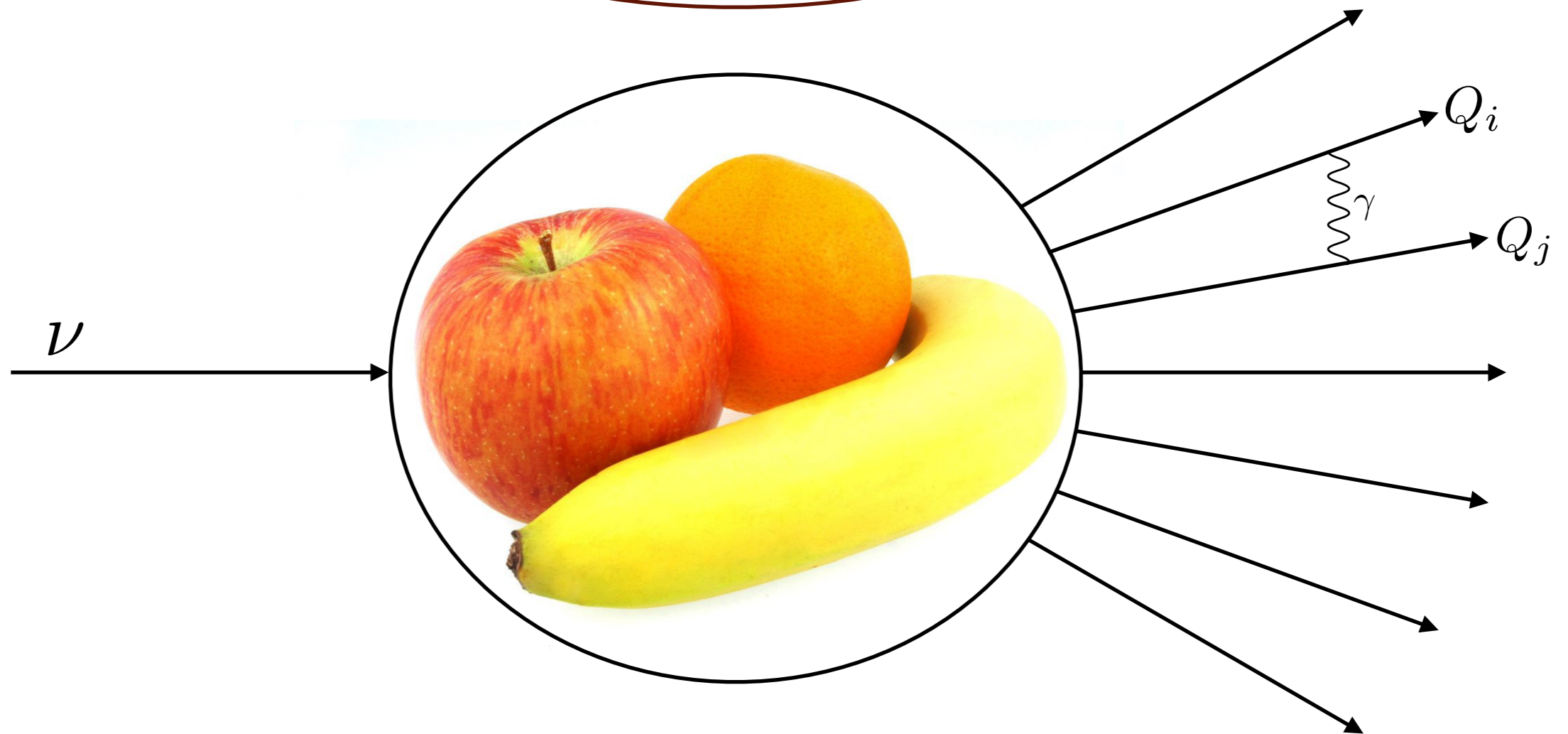
- kinematic dependence and factor X can enhance QED corrections

QED corrections

CCQE

$$m_e \ll m_\mu \ll E_\nu$$

arXiv: 2105.07939

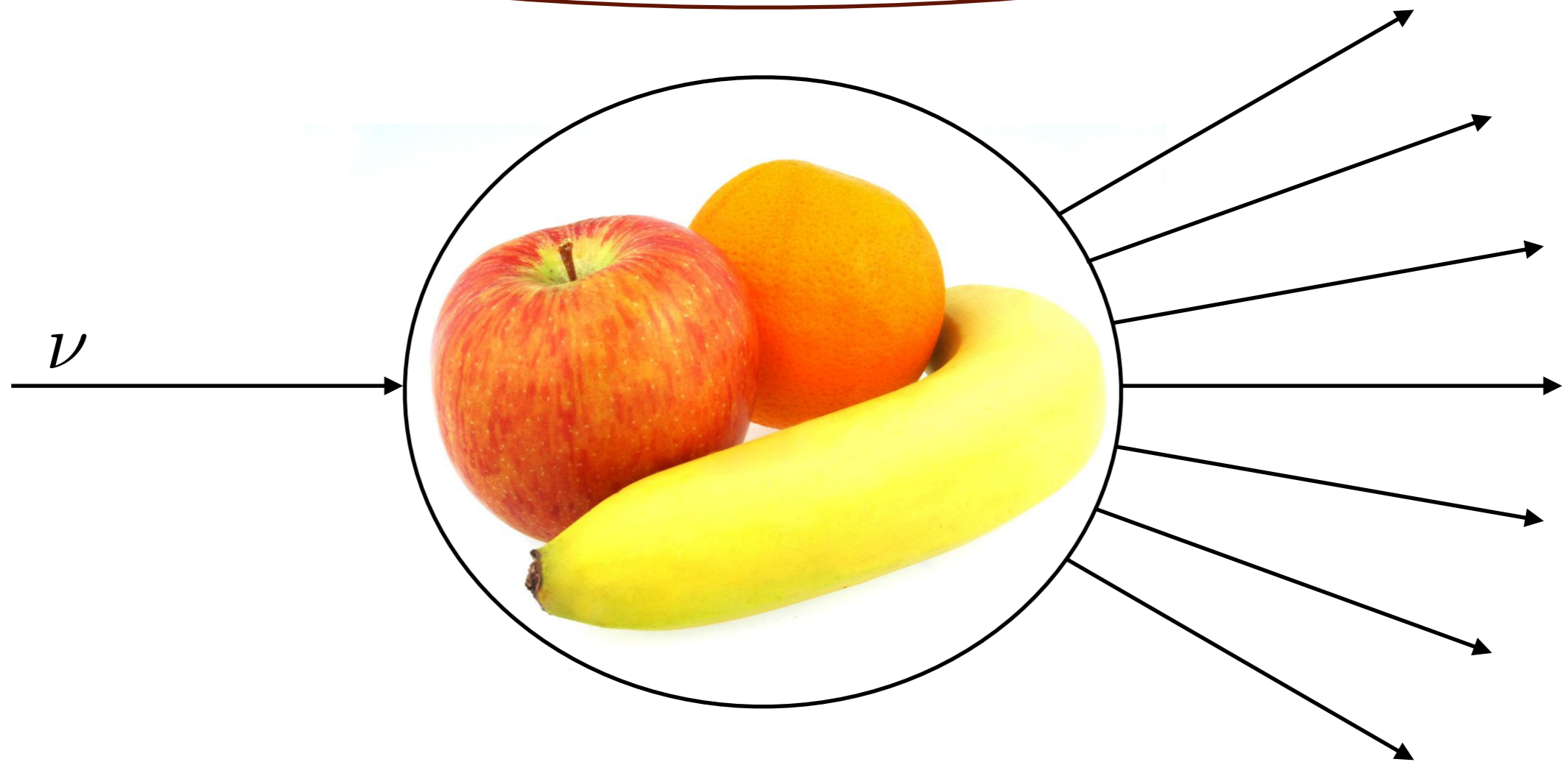


$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

- scale separation: large flavor-dependent logarithms for GeV energy

Electroweak corrections

$$m_e, m_\mu, M, E_\nu \ll M_W, M_Z, m_t, m_H$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \frac{1}{\sin^2 \theta_W}, \ln \frac{M_Z}{M}, \ln \frac{M_t}{M}, \dots$$

- electroweak corrections can be included in low-energy interactions

couplings of **effective Lagrangian** are precisely determined

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = -\bar{\nu}_l \gamma_\mu P_L \nu_l \cdot \bar{f} \gamma^\mu (c_L^{\nu_l f} P_L + c_R^{\nu_l f} P_R) f$$

$$\mathcal{L}_{\text{eff}}^{\text{CC}} = -2\sqrt{2}G_F \sum_{l \neq l'} \bar{\nu}_{l'} \gamma^\mu P_L \nu_l \bar{l} \gamma_\mu P_L l' - c^{qq'} \sum_{q \neq q'} \bar{l} \gamma^\mu P_L \nu_l \bar{q} \gamma_\mu P_L q'$$

Neutrino-lepton, neutrino-quark scattering

O.T. and Richard J Hill, Phys. Lett. B 805 (2020) 3, 135466

poster at Neutrino 2020:

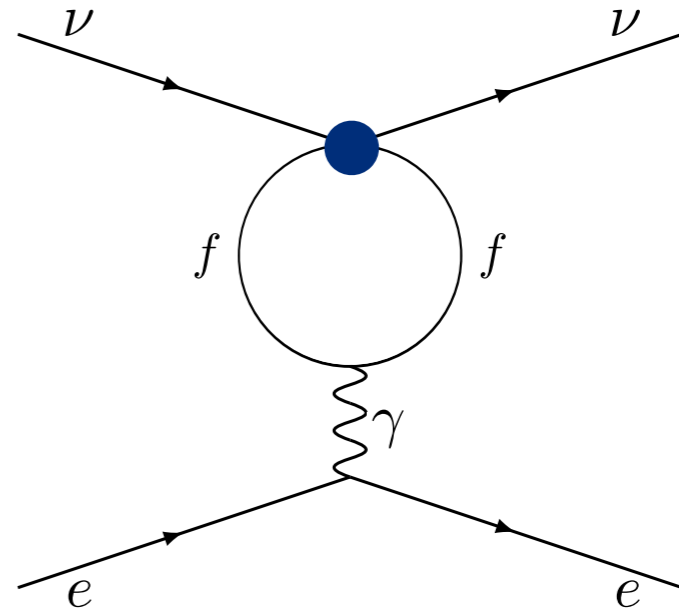
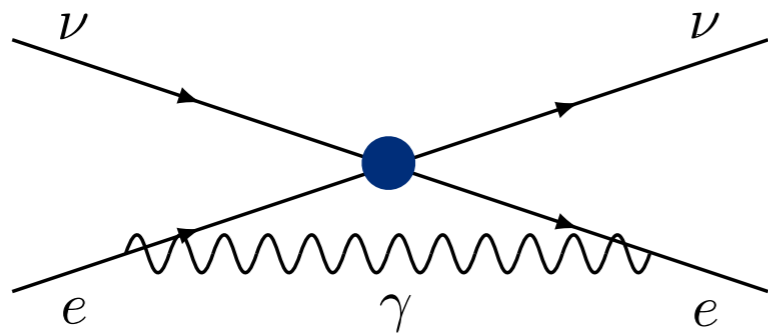
<https://youtu.be/mrW4aYjP57w>

known at permille level



leading in G_F terms with loop expansion in α , α_s within Standard Model

gauge-invariant matching and running, $\overline{\text{MS}}$ renormalization scheme



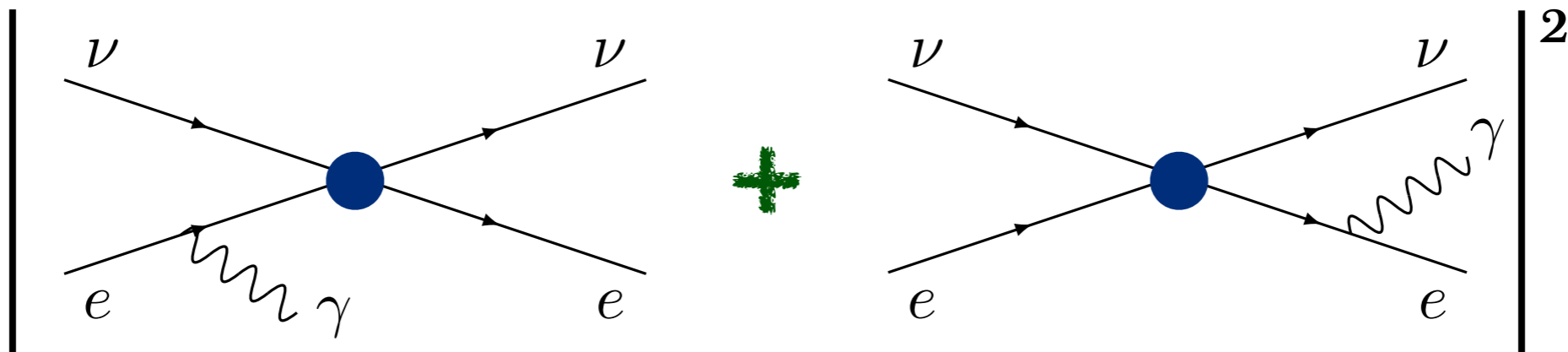
Neutrino-electron scattering

O.T. and Richard J Hill, Phys. Rev. D 101 (2020) 3, 033006

poster at Neutrino 2020:

<https://youtu.be/mrW4aYjP57w>

known at permille level



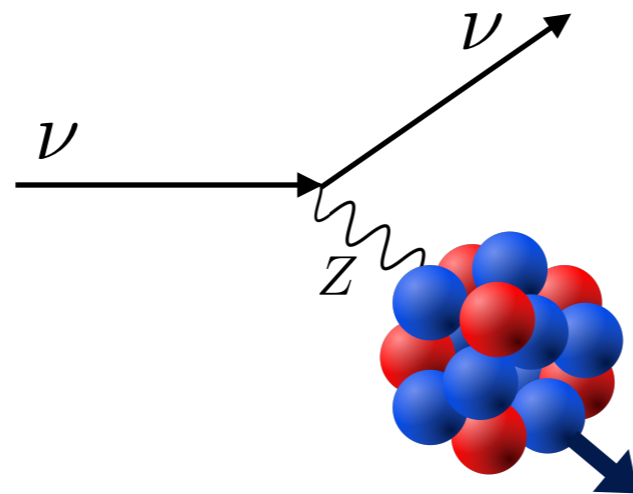
Coherent elastic neutrino-nucleus scattering

O.T., Pedro Machado, Vishvas Pandey and Ryan Plestid, JHEP 2102, 097 (2021)

Coherent elastic neutrino-nucleus scattering

- at low neutrino energies (<50 MeV) nuclear state is unchanged
nucleus recoils as a whole

Stodolsky (1966), Freedman (1974), Kopeliovich and Frankfurt (1974)



recoil nucleus energy T

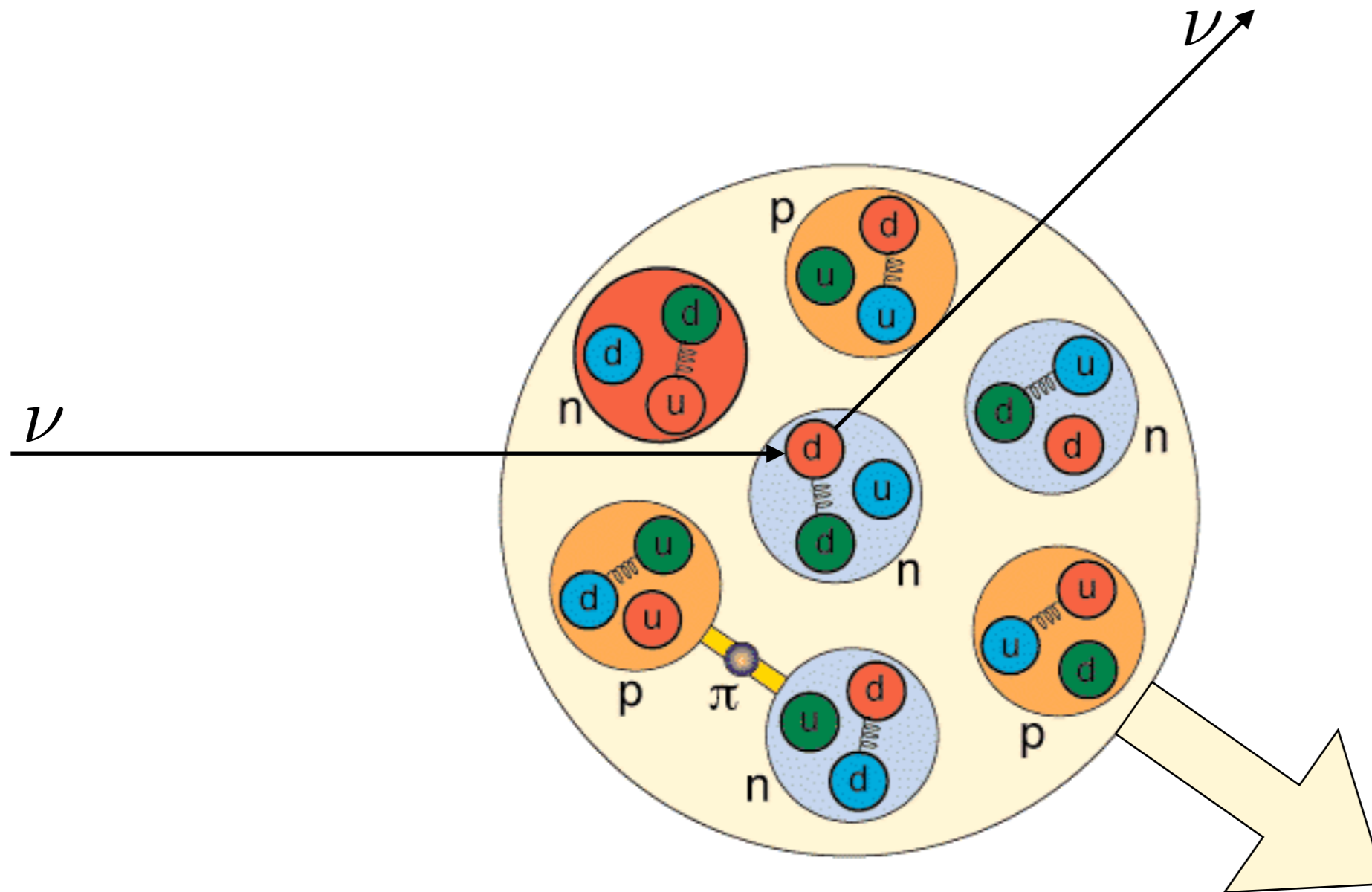
- large cross section scales as squared number of neutrons N^2

$$\frac{d\sigma}{dT} \approx \frac{G_F^2 M_A}{4\pi} \left(1 - \frac{M_A T}{2E_\nu^2} \right) \left(N - (1 - 4 \sin^2 \theta_W) Z \right)^2$$

- first detection in 2017 at SNS (Oak Ridge), measured on CsI and Ar
COHERENT, Science 357 (2017) 6356, 1123-1126
- rapidly developing field nowadays

- CEvNS enters precision era with π DAR sources

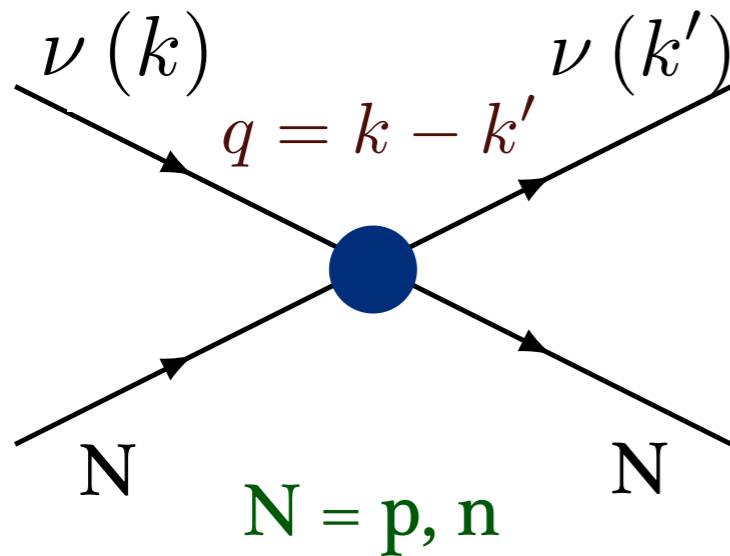
From quarks to nuclei



fafnir.phyast.pitt.edu

- scattering on quarks in nucleons in nucleus

From quarks to nucleons



momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- neutral-current nucleon matrix elements

$$P_{L,R} = \frac{1 \mp \gamma_5}{2}$$

$$\mathcal{M} \sim \bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \langle N | \sum_q \bar{q} \gamma^\mu (c_L^{\nu \ell q} P_L + c_R^{\nu \ell q} P_R) q | N \rangle$$

$$\mathcal{M} \sim G_E(Q^2), G_M(Q^2), \quad F_A(Q^2), F_P(Q^2)$$

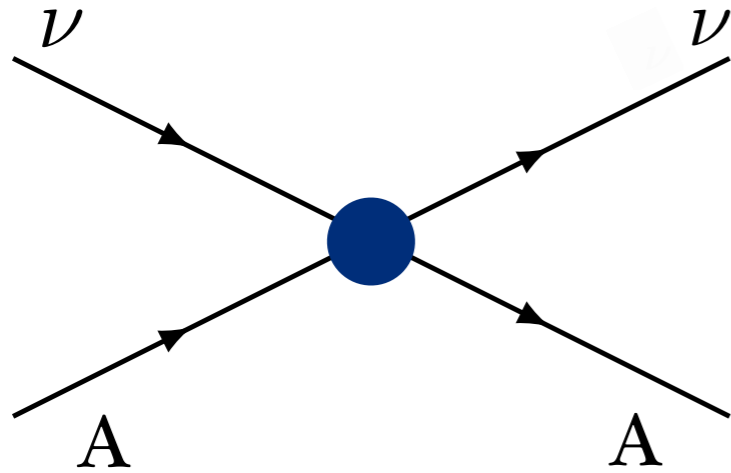
form factors:

electric and magnetic

axial and pseudoscalar

- form factors describe matrix elements of quark currents
- π DAR sources: only normalizations and charge radii

From nucleons to nuclei



- tree-level cross section

$$\frac{d\sigma}{dT} = \frac{G_F^2 M_A}{4\pi} \left(1 - \frac{T}{E_\nu} - \frac{M_A T}{2E_\nu^2} \right) F_W^2(Q^2)$$

spin-0 nuclei

- sum over nucleons with point-nucleon form factors f_p, f_n

$$F_W = \left(\frac{c_L^{\nu\ell u} + c_R^{\nu\ell u}}{\sqrt{2}G_F} G_E^{n,u} + \frac{c_L^{\nu\ell d} + c_R^{\nu\ell d}}{\sqrt{2}G_F} G_E^{n,d} \right) f_n + (n \leftrightarrow p)$$

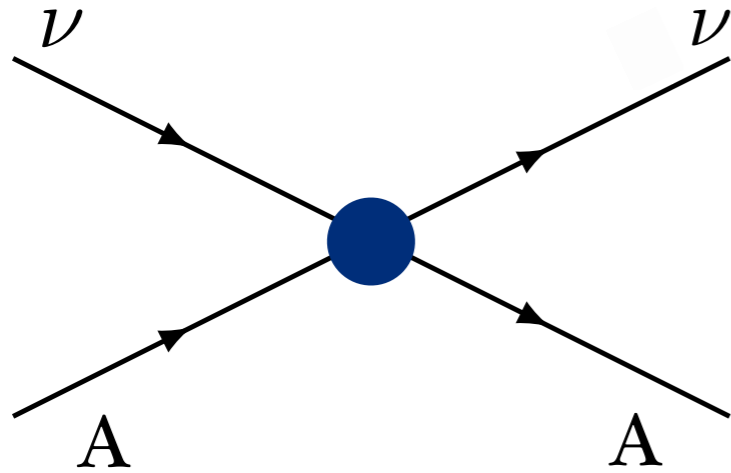
- flavor-independent form factor above GeV scale

- Q^2/M^2 corrections and spin-dependent terms are known

Hoferichter et al. (2020)

- point-nucleon form factors: distribution of nucleons in nuclei
- π DAR sources: factorization starting from quark level

CEvNS cross section on spin-0 nuclei

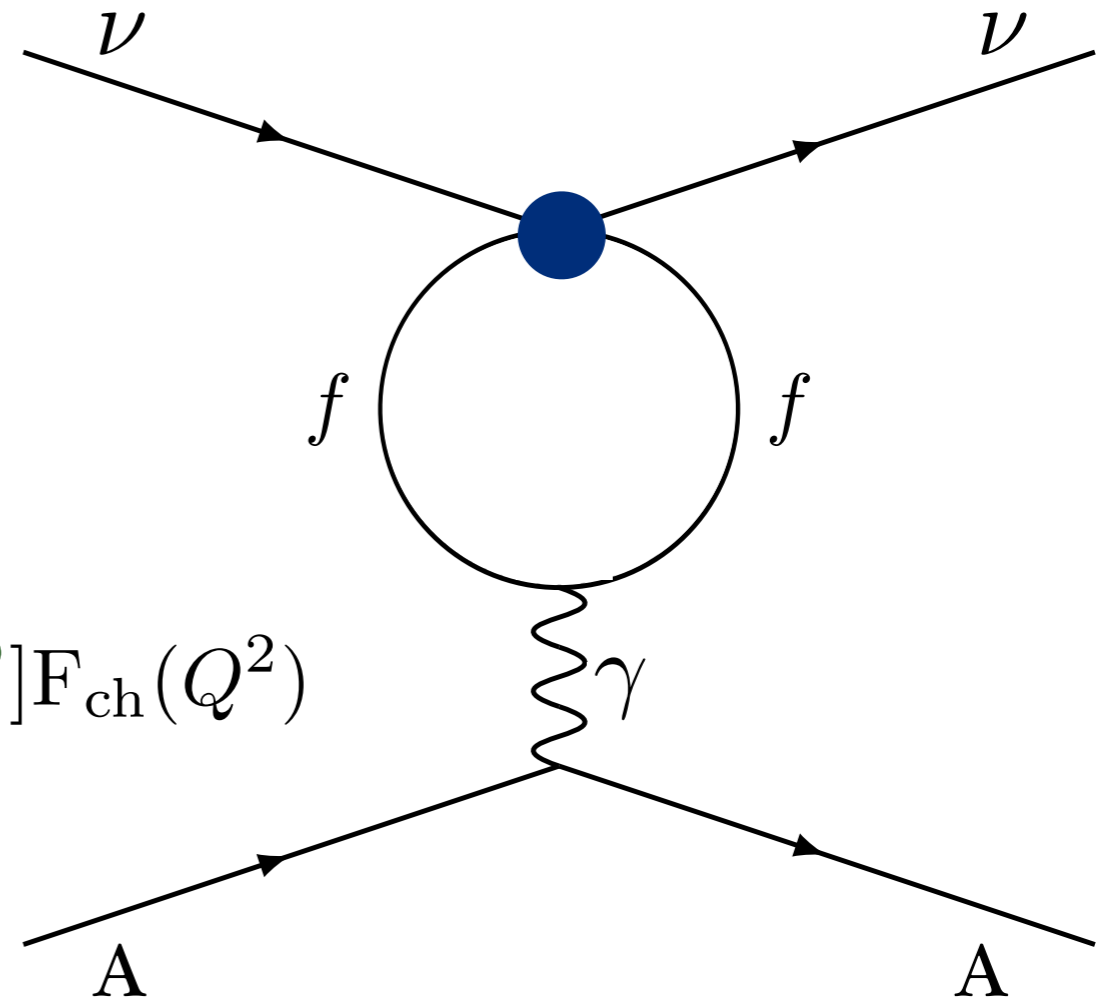


- tree-level cross section

$$\frac{d\sigma}{dT} = \frac{G_F^2 M_A}{4\pi} \left(1 - \frac{T}{E_\nu} - \frac{M_A T}{2E_\nu^2} \right) F_W^2(Q^2)$$

- effect of radiative corrections

$$F_W(Q^2) \rightarrow F_W(Q^2) + \frac{\alpha}{\pi} [\delta^{\nu e} + \delta^{\text{QCD}}] F_{\text{ch}}(Q^2)$$



- radiative corrections enter with the nucleus charge form factor

Neutrino scattering in EFT. Matching

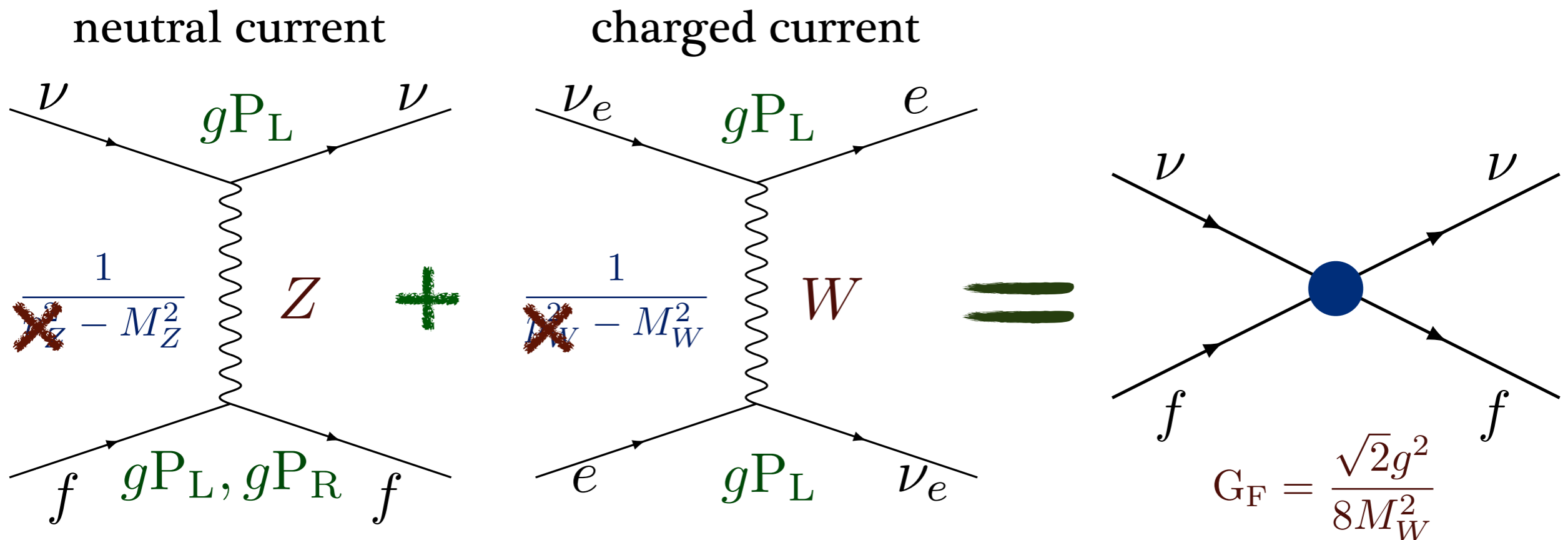
- tree-level matching to low-energy EFT

$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left(c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$

couplings to electron

$$c_R = 2\sqrt{2}G_F \sin^2 \theta_W \quad c_L = 2\sqrt{2}G_F (\sin^2 \theta_W - 0.5 + \delta_{\nu, \nu_e})$$

Weinberg (1967), 't Hooft (1971)



- charged current contributes to couplings with the same flavor

Virtual QED corrections. Fermion loop

- all charged fermions contribute to elastic scattering at one loop

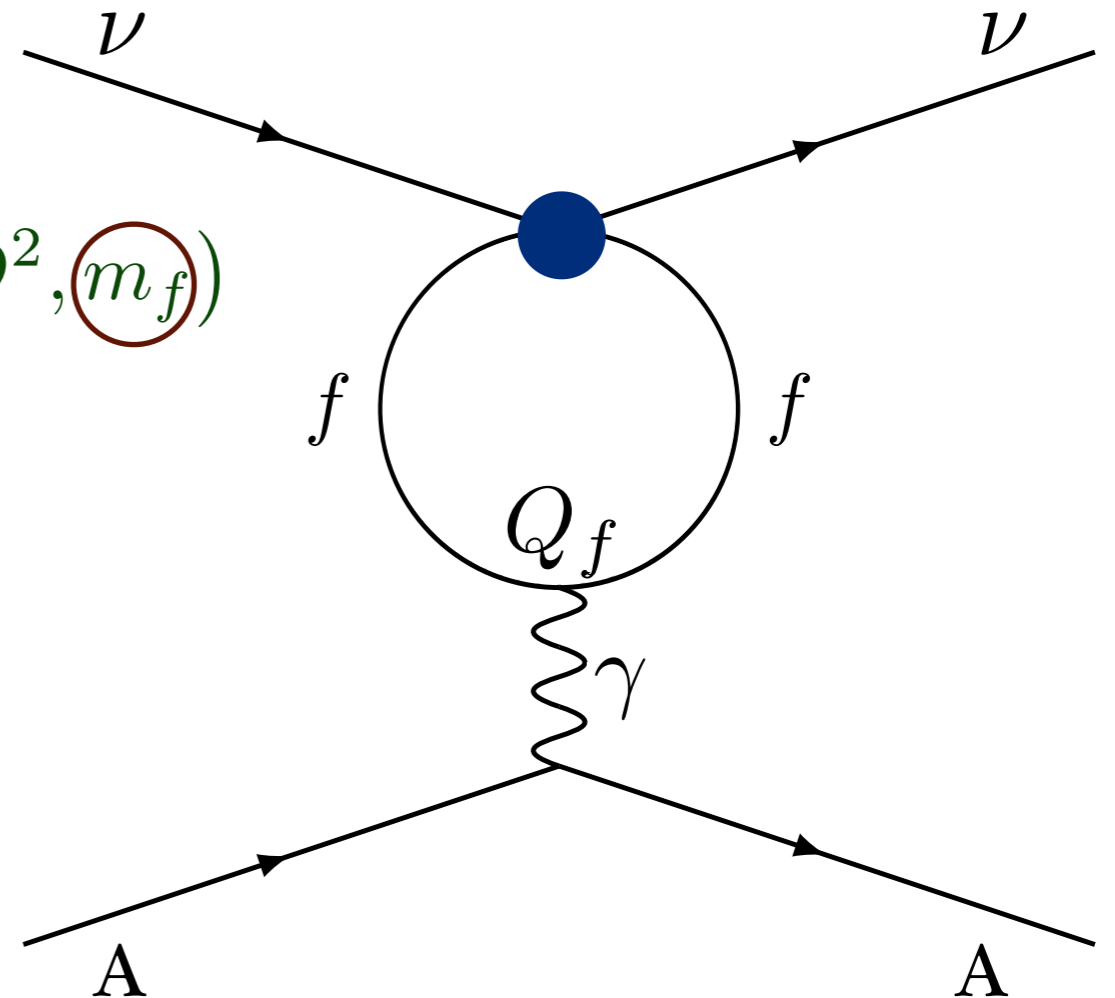
$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left(c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$

- lepton loops

$$\delta^{\nu_\ell} = - \sum_f \frac{c_L^{\nu_\ell f} + c_R^{\nu_\ell f}}{\sqrt{2} G_F} Q_f \Pi(Q^2, m_f)$$

- origin of flavor dependence

$$c_L^{\nu_e \mu} = c_L^{\nu_\mu e} \neq c_L^{\nu_\mu \mu} = c_L^{\nu_e e}$$



- lepton mass breaks "flavor universality"

Light-quark contribution

- description in terms of quarks is invalid at CEvNS kinematics

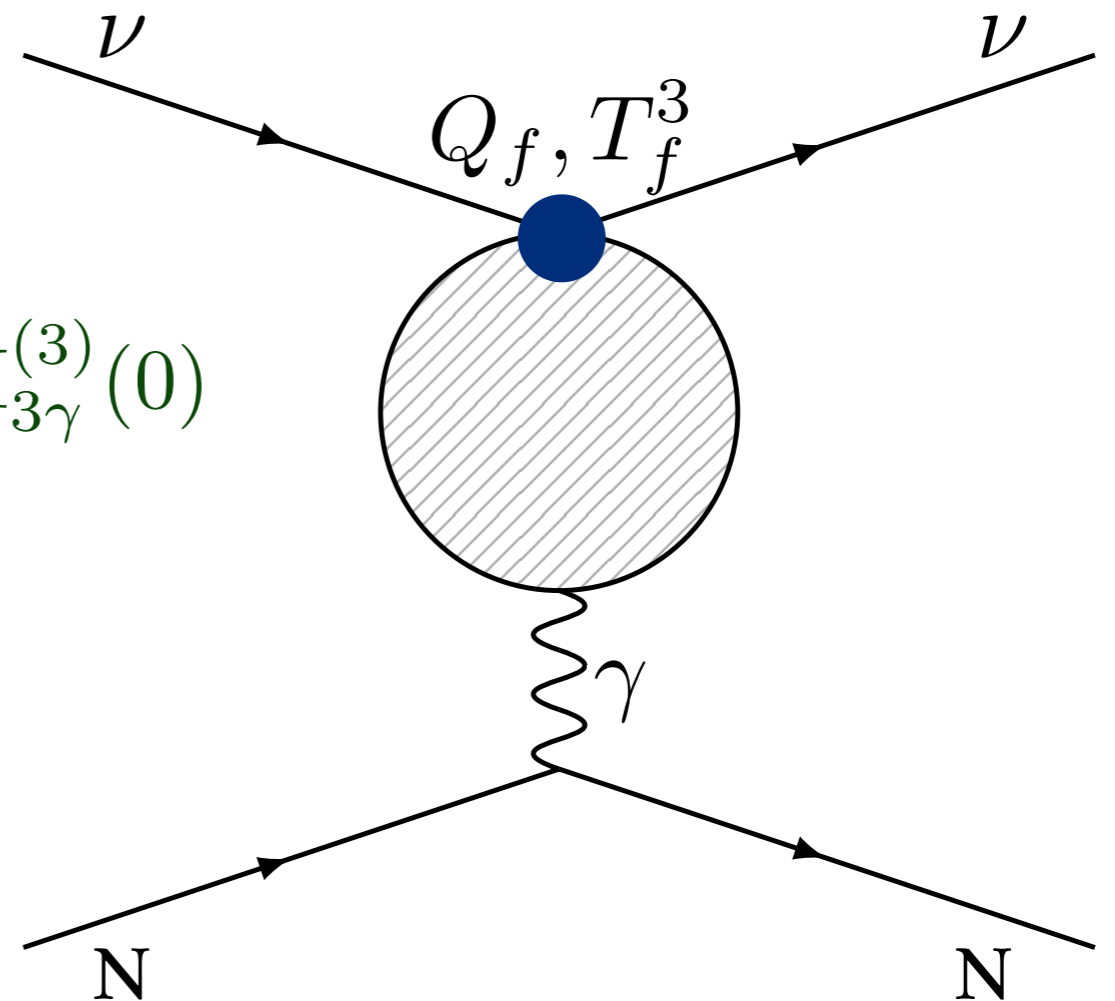
$$Q^2 \ll \Lambda_{\text{QCD}}^2$$

- light quarks

$$\delta^{\text{QCD}} = 4\Pi_{\gamma\gamma}^{(3)}(0) \sin^2 \theta_W - 2\Pi_{3\gamma}^{(3)}(0)$$

- chiral symmetry approximation

- flavor independent

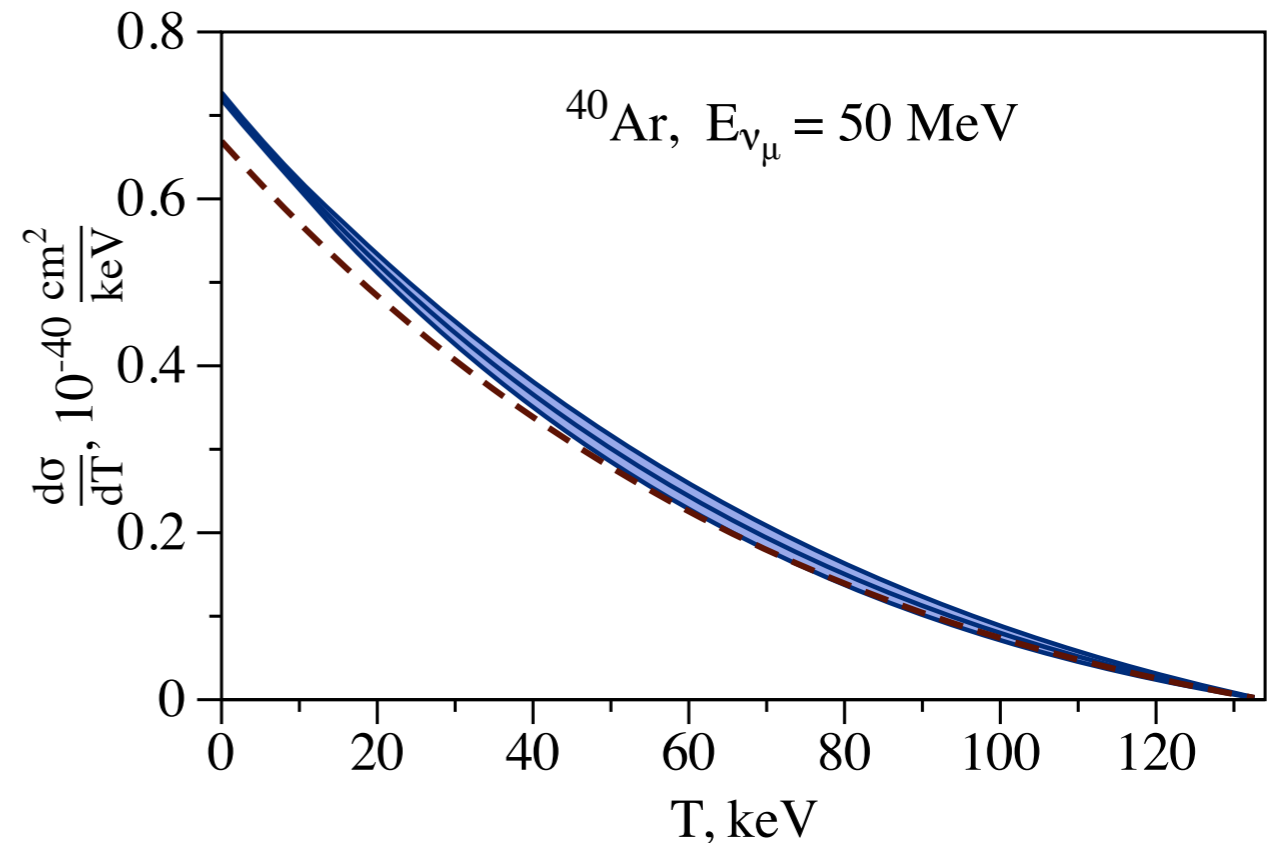
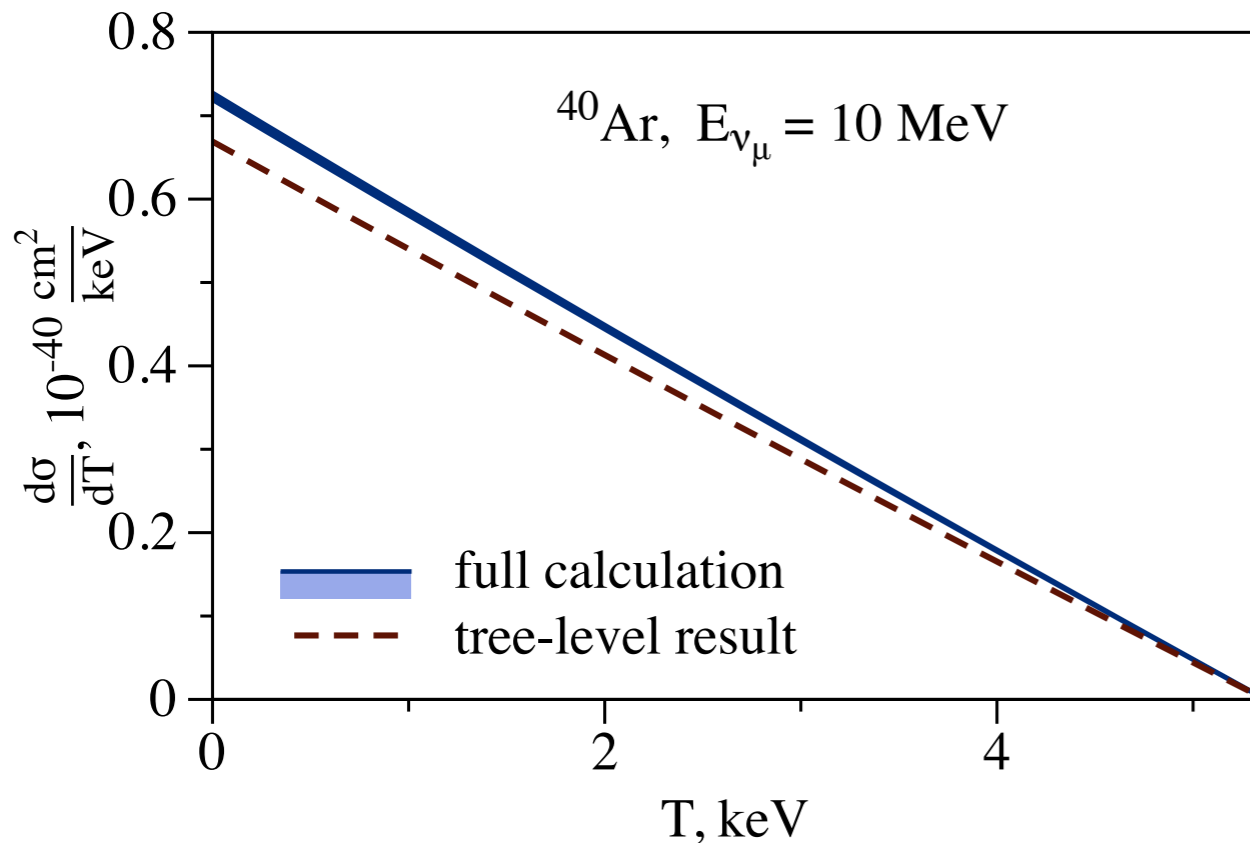


- non-perturbative light-quark contribution: error at low energy

Total and differential cross section

- recoil nucleus energy spectrum: one-loop vs tree level

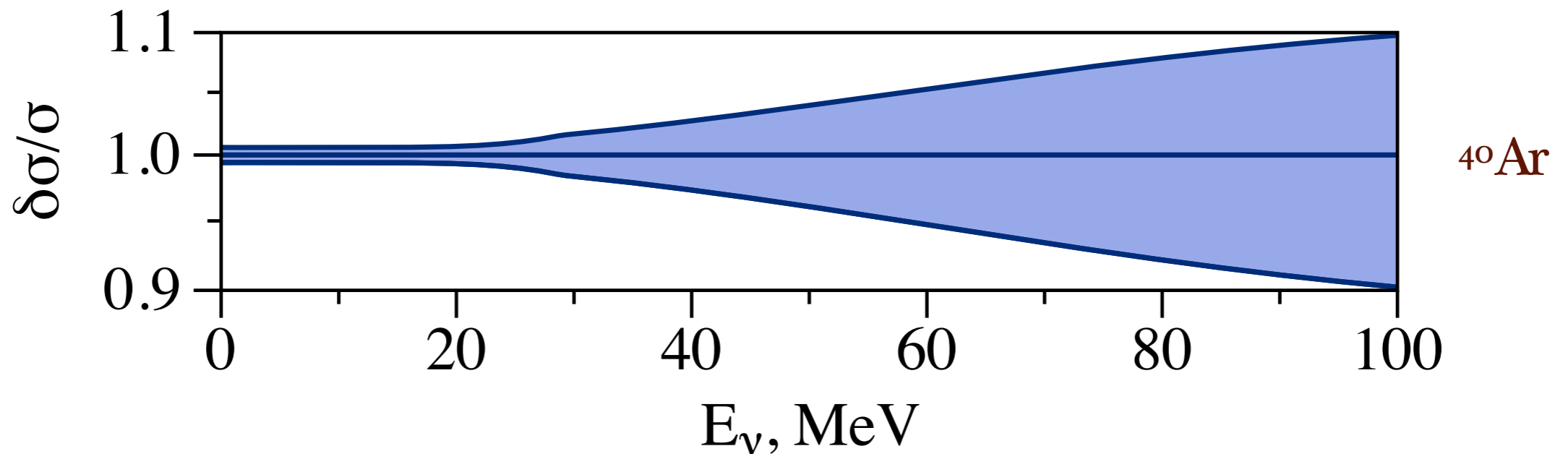
nuclear models for point-nucleon form factors:
Yang et al. (2019), Payne et al. (2019), Hoferichter et al. (2020), Van Dessel et al. (2020)



- % effect of radiative corrections on cross sections

Total cross section errors

- relative cross section error



- sources of uncertainty (%)

E_ν , MeV	Nuclear	Nucleon	Hadronic	Quark	Perturbative	Total
50	4	0.06	0.56	0.13	0.08	4.05
30	1.5	0.014	0.56	0.13	0.03	1.65
10	0.04	0.001	0.56	0.13	0.004	0.58

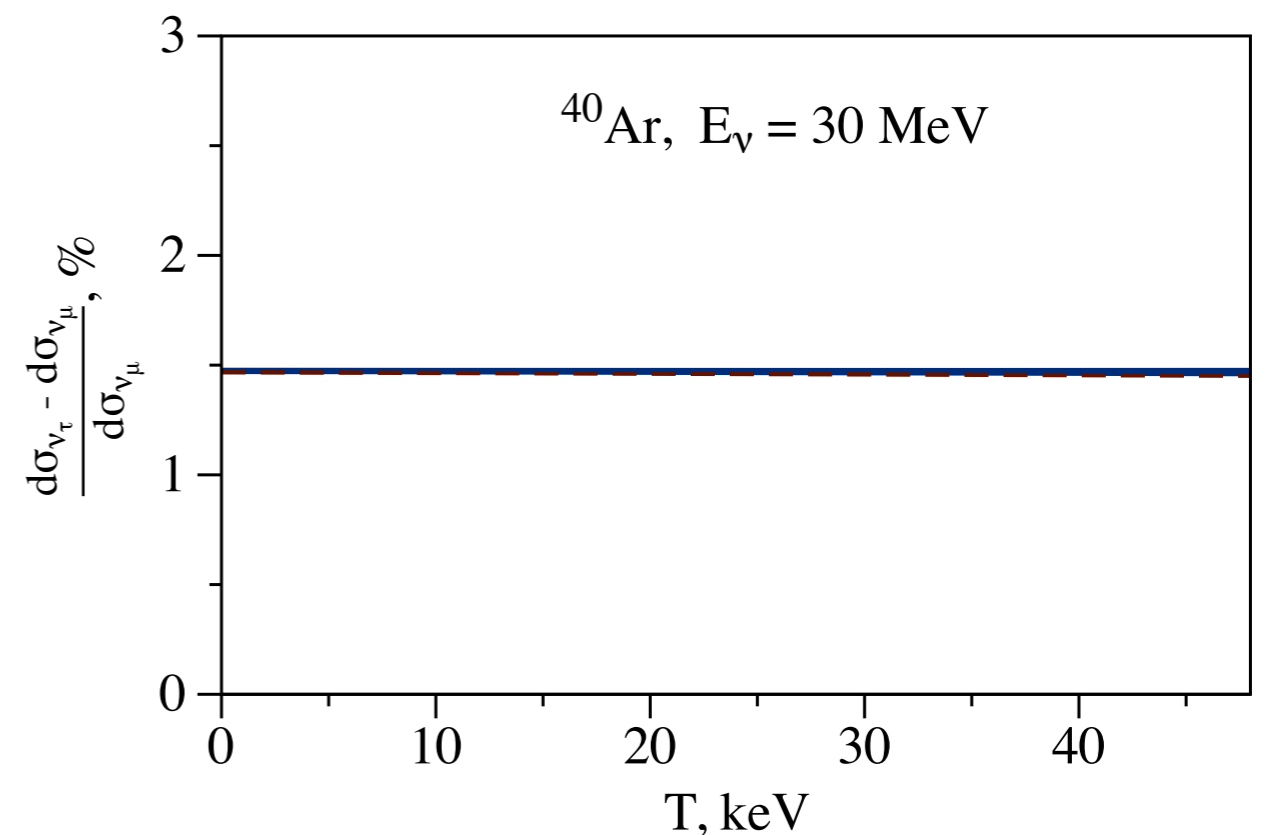
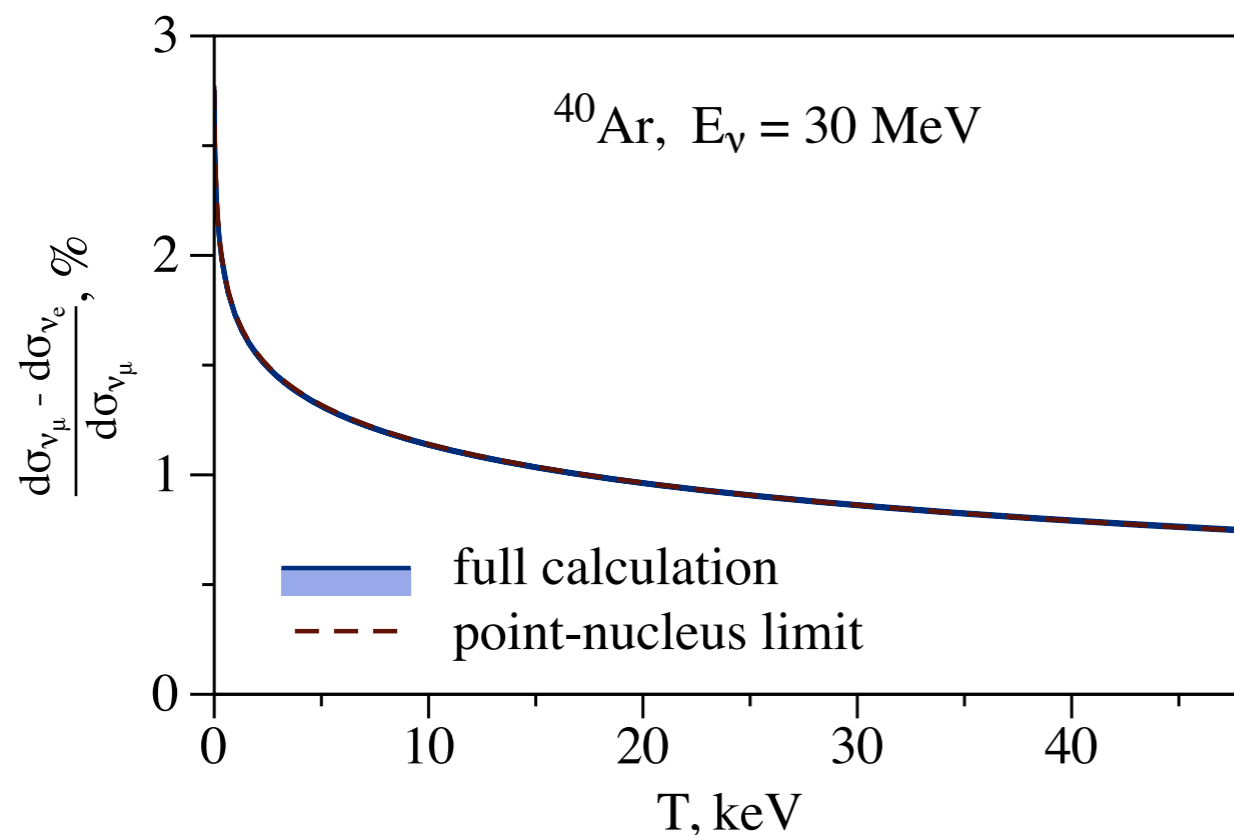
- hadronic error 0.6% at low energy, nuclear error at higher energy

Flavor difference

- well described by point-nucleus limit

$$\lim_{R_p, R_n \rightarrow 0} \frac{d\sigma_{\nu\ell} - d\sigma_{\nu\ell'}}{d\sigma_{\nu\ell}} = 4 \frac{\alpha_0}{\pi} \frac{Z}{Q_W} [\Pi(Q^2, m_\ell) - \Pi(Q^2, m_{\ell'})]$$

- kinematic dependence: full result vs point-nucleus limit



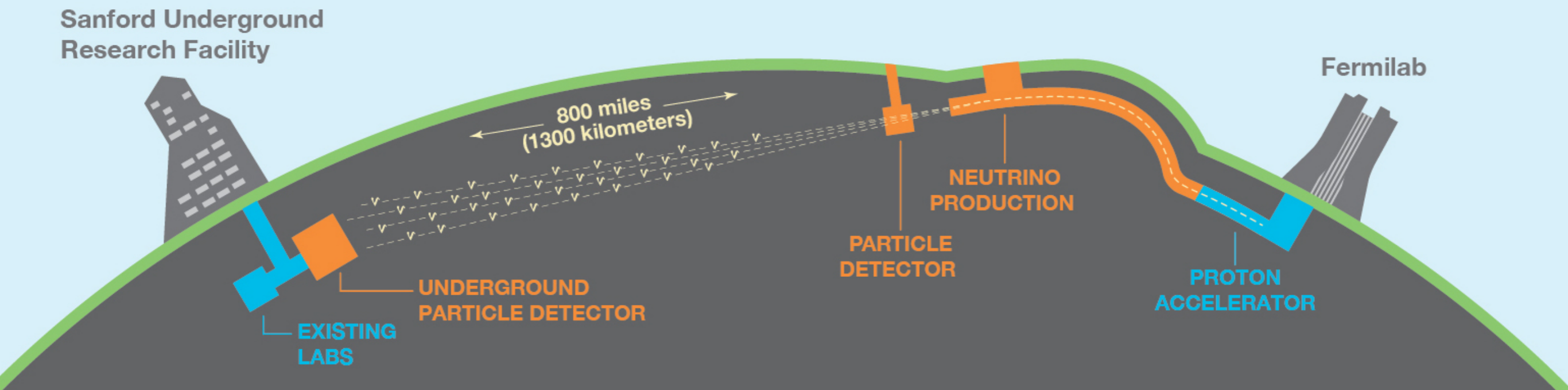
- factor 3-6 change in precisely predicted electron-muon asymmetry

Radiative corrections in CCQE on free nucleons

O.T., Qing Chen, Richard J. Hill and Kevin S. McFarland (arXiv: 2105.07939)

Neutrino experiments

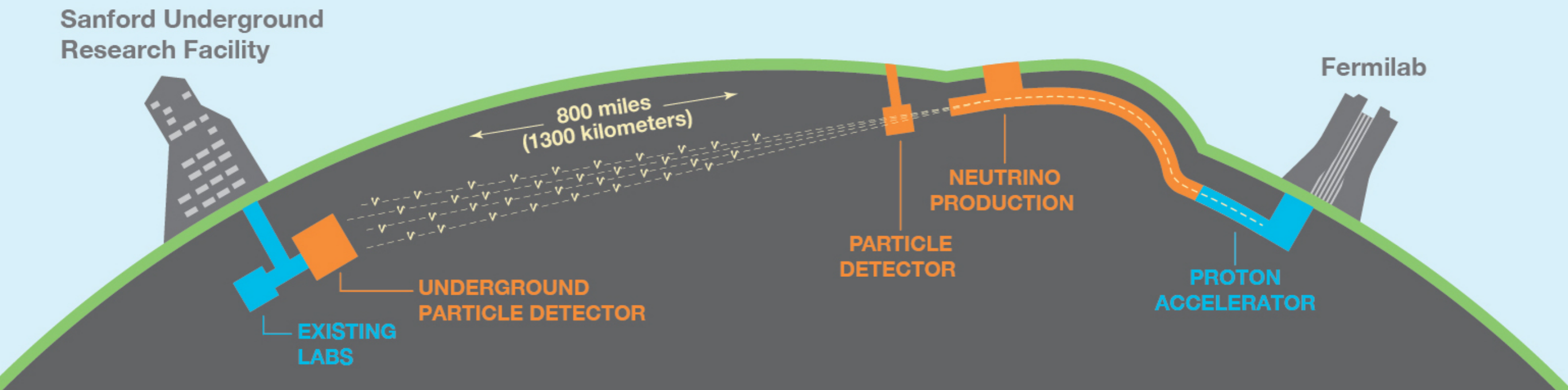
- **DUNE** and Hyper-K: leading-edge ν science experiments



- origin of matter-antimatter asymmetry δ_{CP}
- mass hierarchy and oscillation parameters PMNS matrix, Δm_{31}^2
- Grand Unified Theories proton decay
- dynamics of supernova explosion wait for one;)

Neutrino experiments

- **DUNE** and Hyper-K: leading-edge ν science experiments



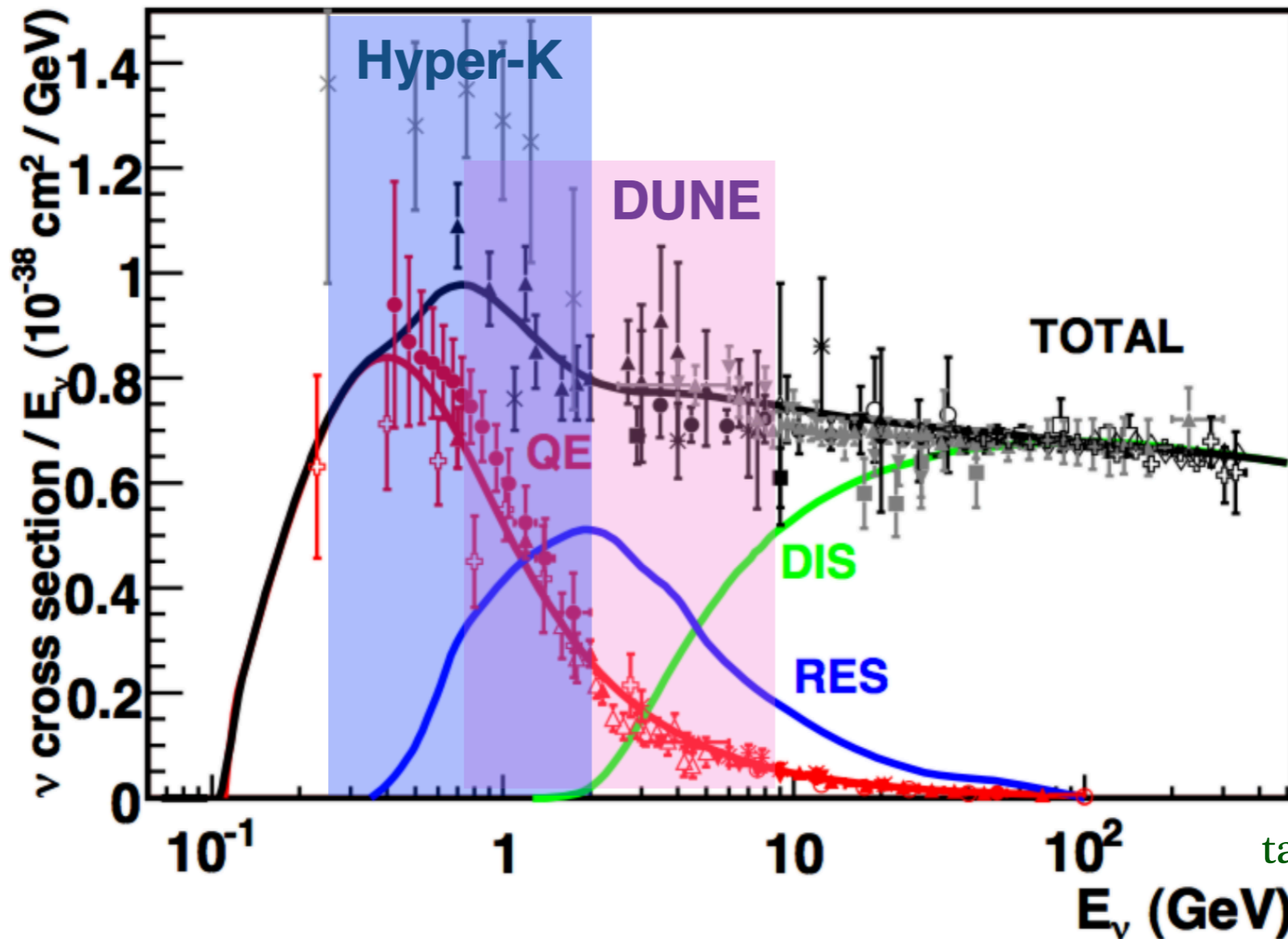
- measurement of ν_μ disappearance and ν_e appearance

$$N_\nu \sim \int dE_\nu \Phi_\nu(E_\nu) \times \sigma(E_\nu) \times R(E_\nu, E_\nu^{\text{rec}})$$

- near detector: determine flux and cross sections

CCQE. Why should we care?

- neutrino-nucleus cross sections and future accelerator-based fluxes

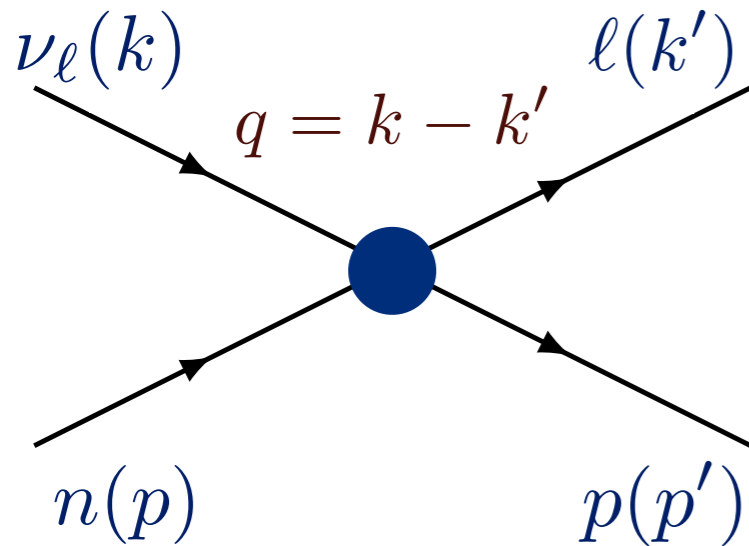


Formaggio
and Zeller
(2013)

Noemi Rocco
talk at Neutrino 2020

- basic process: bulk of events at Hyper-K and DUNE
- best channel for reconstruction of neutrino energy

CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \langle p | \bar{u} (\gamma^\mu - \gamma^\mu \gamma_5) d | n \rangle$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

form factors: isovector Dirac and Pauli axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$

Radiative corrections in CCQE

- large kinematic logarithms enhance radiative corrections

$$\frac{\alpha}{\pi} \sim 0.2 \% \quad \text{multiplied by} \quad \ln \frac{E_\nu}{m_e} \sim 6 - 10$$

- CCQE with electron flavor is subject to large corrections

- phase-space restrictions enhance radiative corrections

$$\frac{\alpha}{\pi} \sim 0.2 \% \quad \text{multiplied by} \quad \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

$$E_\gamma < \Delta E \quad \text{soft photons} \quad 2 \ln \frac{E_\nu}{m_e} \ln \frac{\Delta E}{m_e} \sim 35 - 60$$

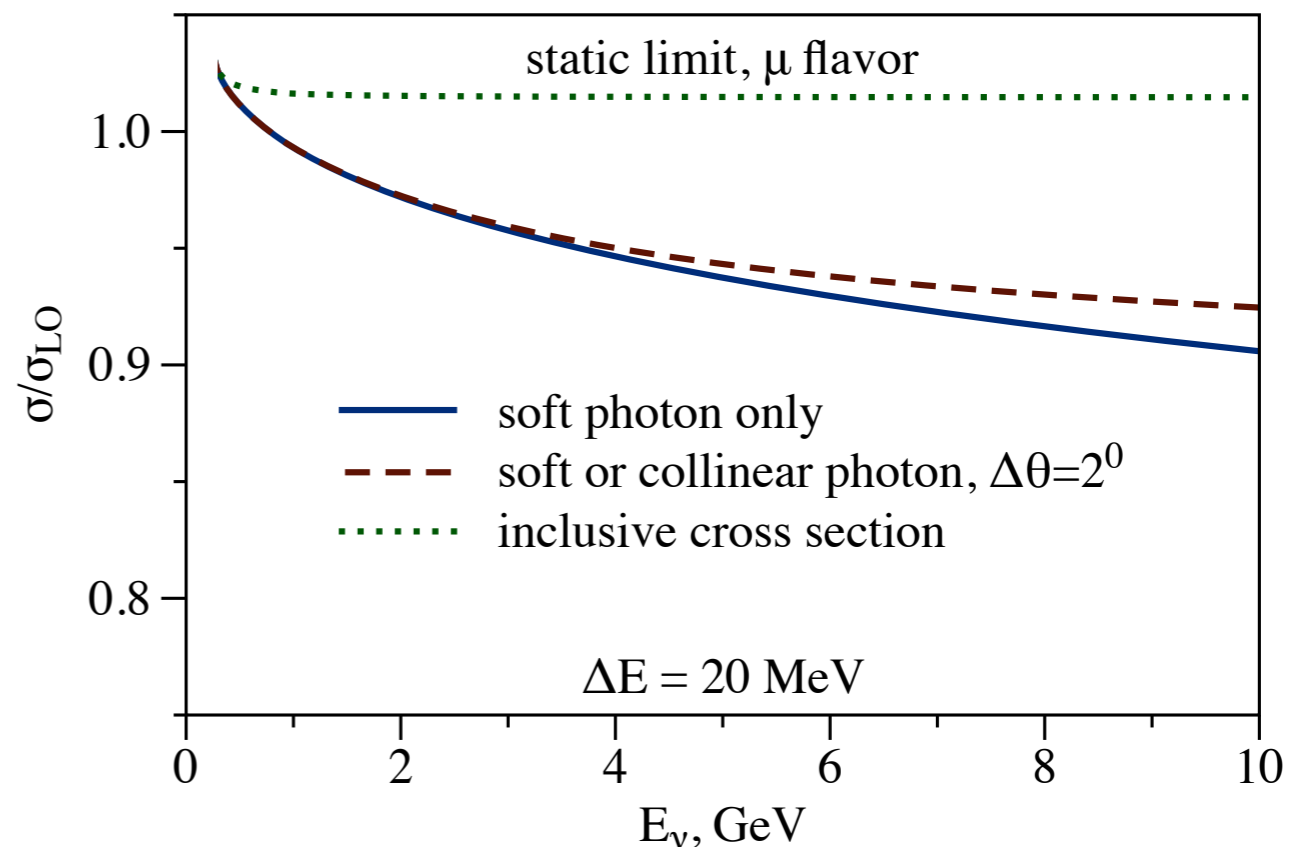
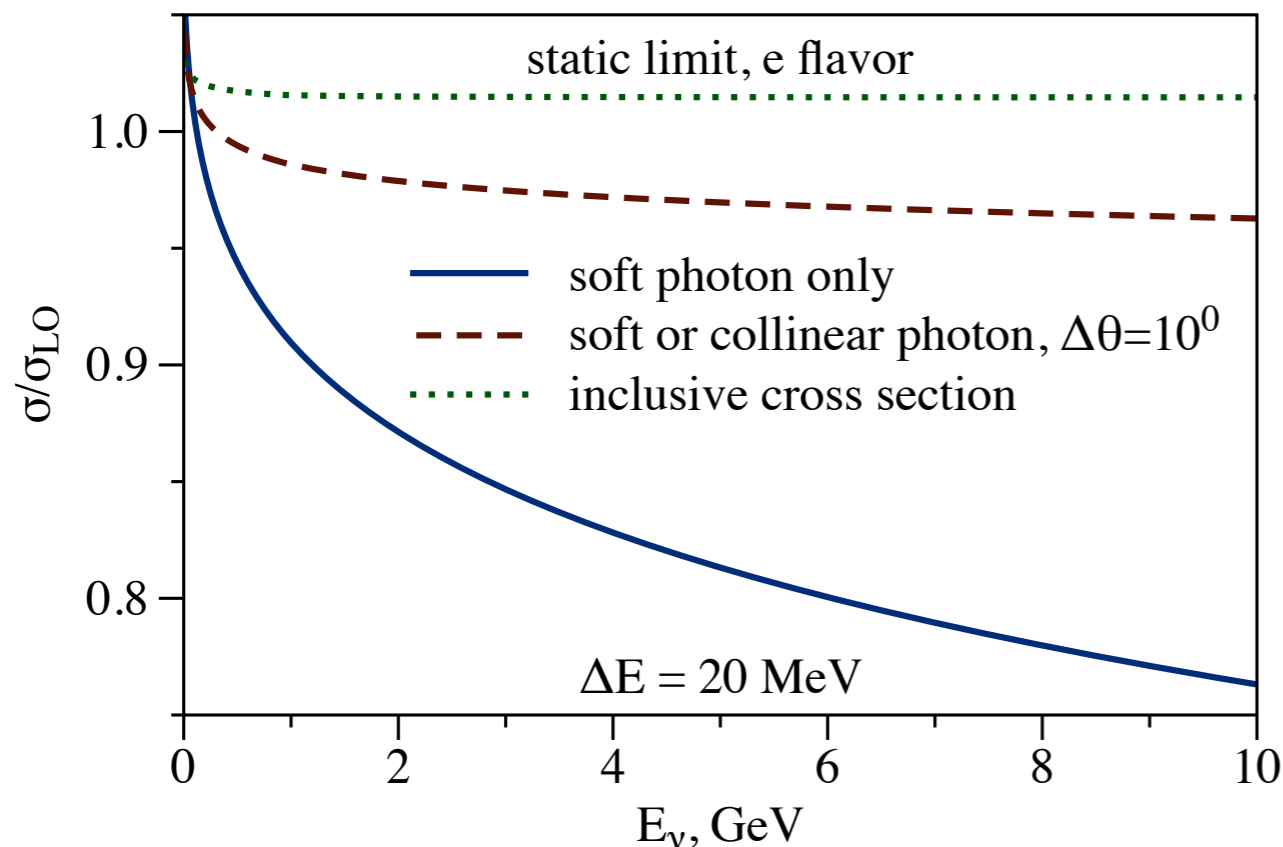
smaller collinear logarithms

- crucial dependence on detector details

- radiative corrections crucial for %-level oscillation program

Static nucleon limit

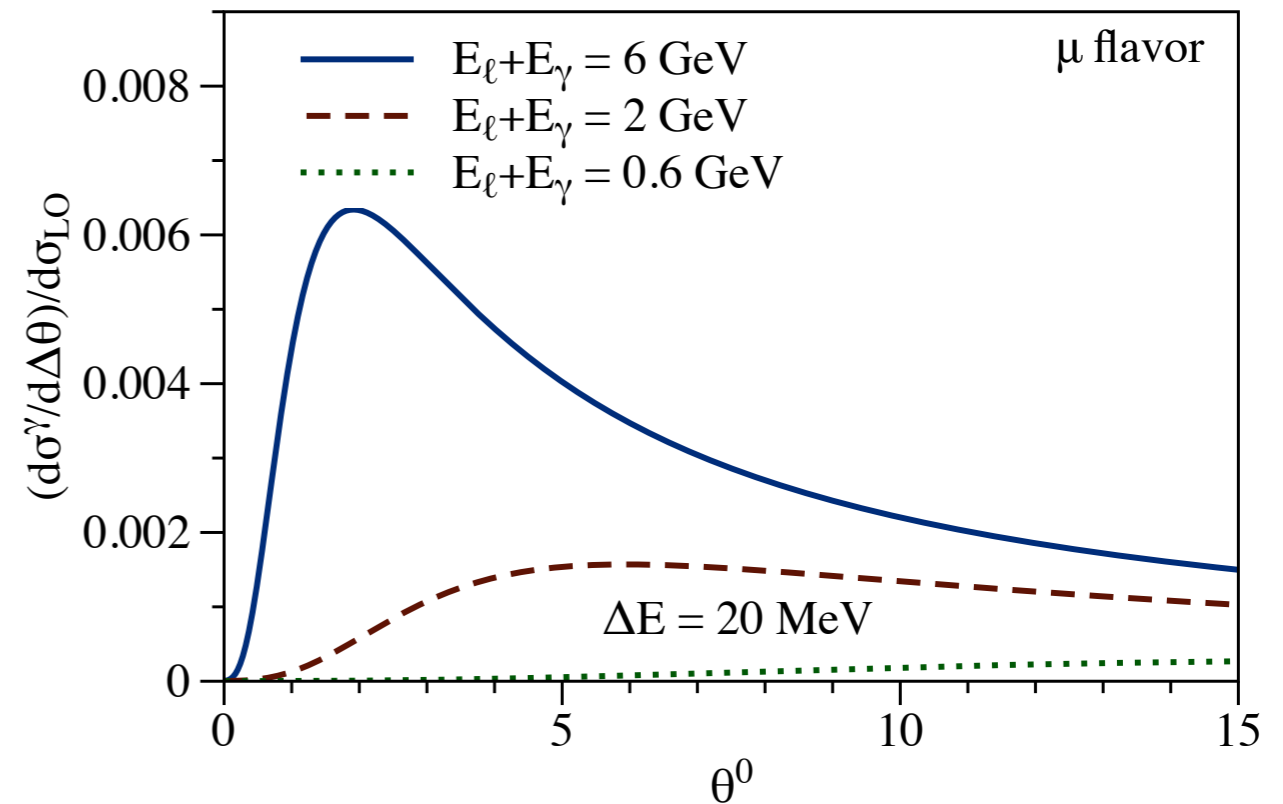
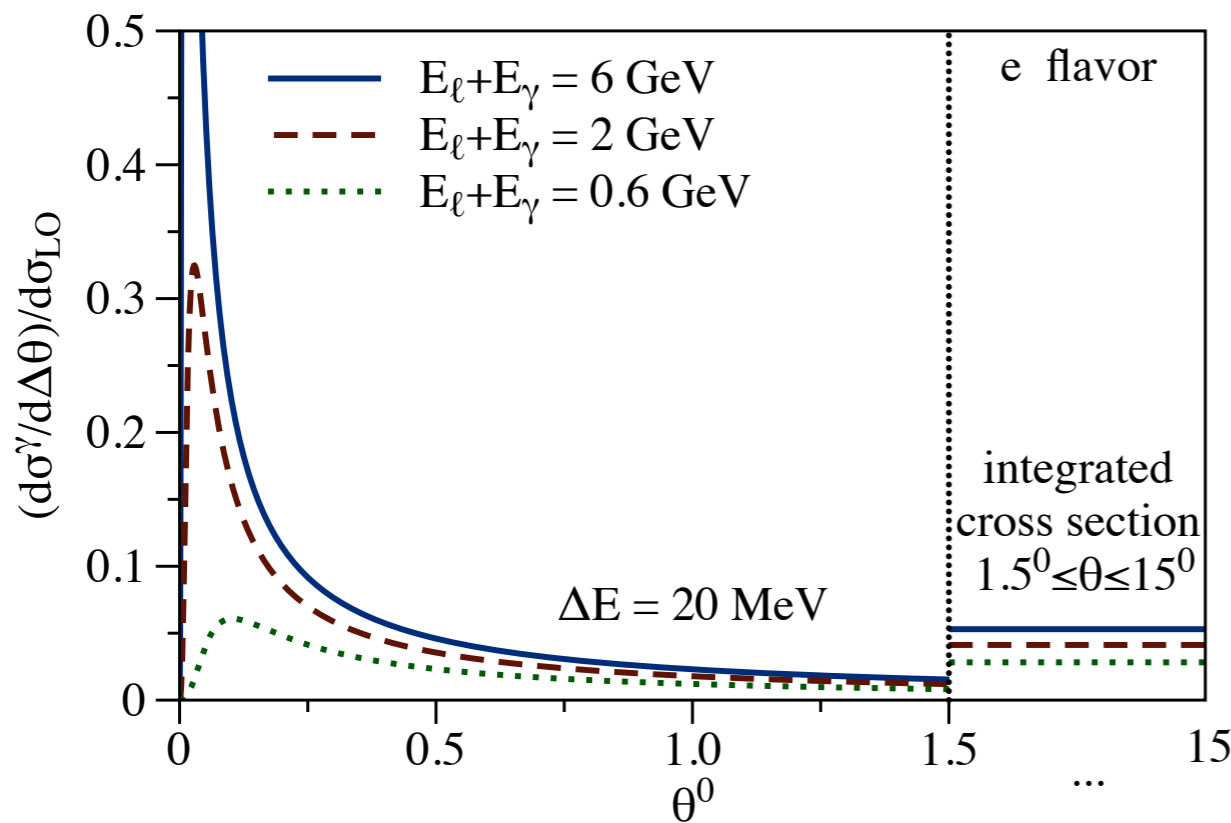
- formal limit of infinitely heavy nucleus $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy < 20 MeV, jet size: 10° for electron and 2° for muon



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables (+ γ): few % level, flavor independent

Electron vs muon jets

- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy > 20 MeV, fixed energy in the cone



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

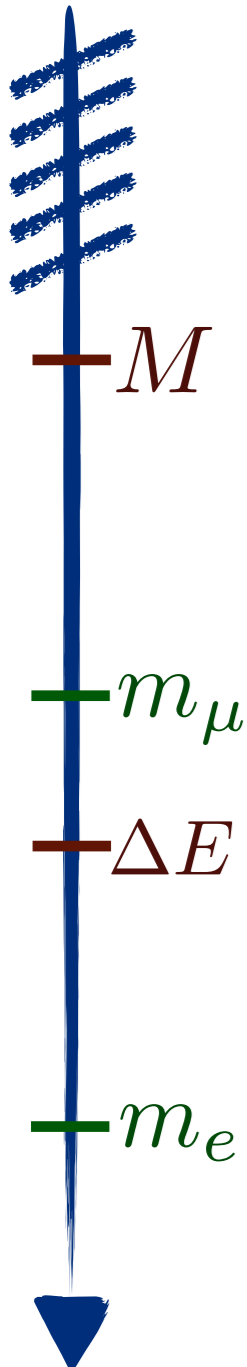
Factorization approach

- cross section is given by **factorization formula**

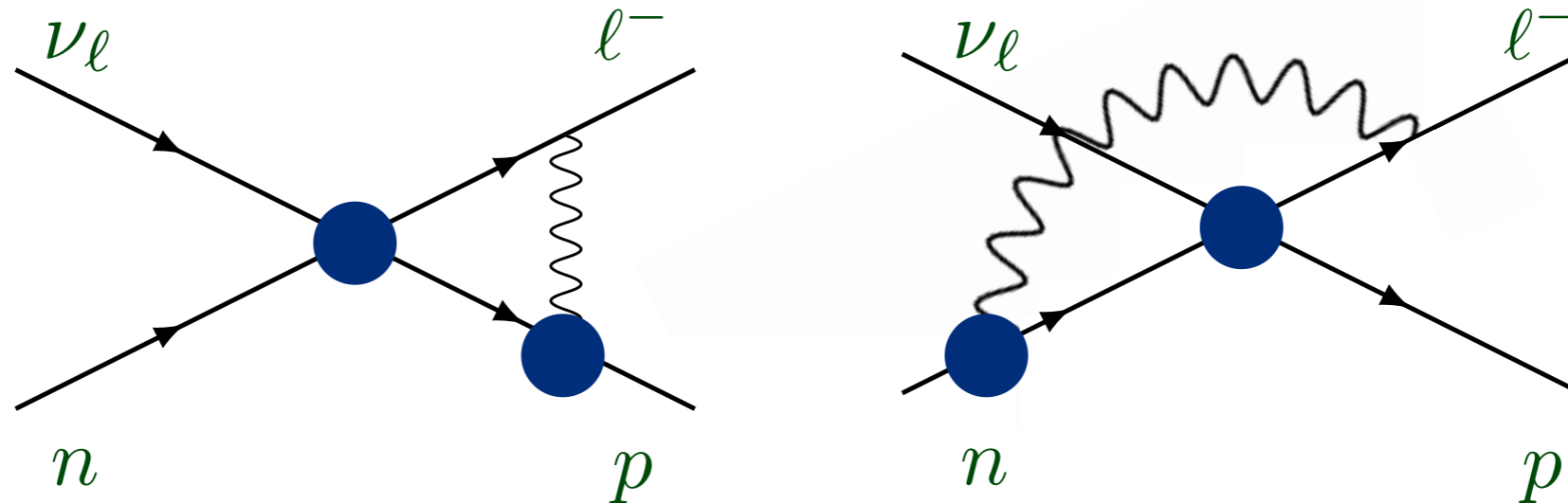
$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- soft and collinear functions are evaluated **perturbatively**



Hadronic model at GeV scale




- exchange of photon between the charged lepton and nucleons
- assume **onshell form** for each interaction with dipole form factors
discussed for CCQE: Graczyk (2013)
- add **self energy** for charged particles
- reproduce soft and collinear regions of SCET

- best determination of hard function

Factorization approach

- cross section is given by **factorization formula**

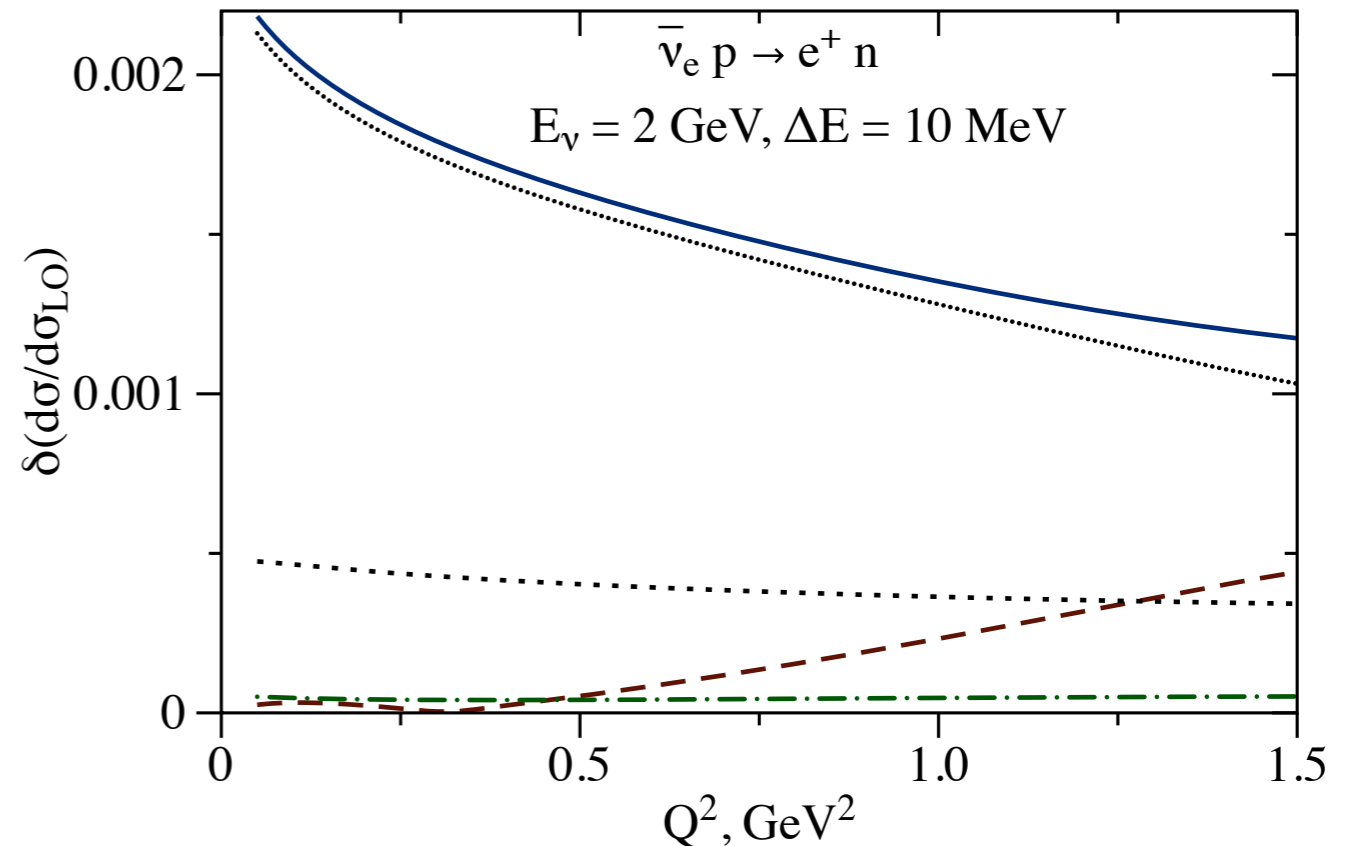
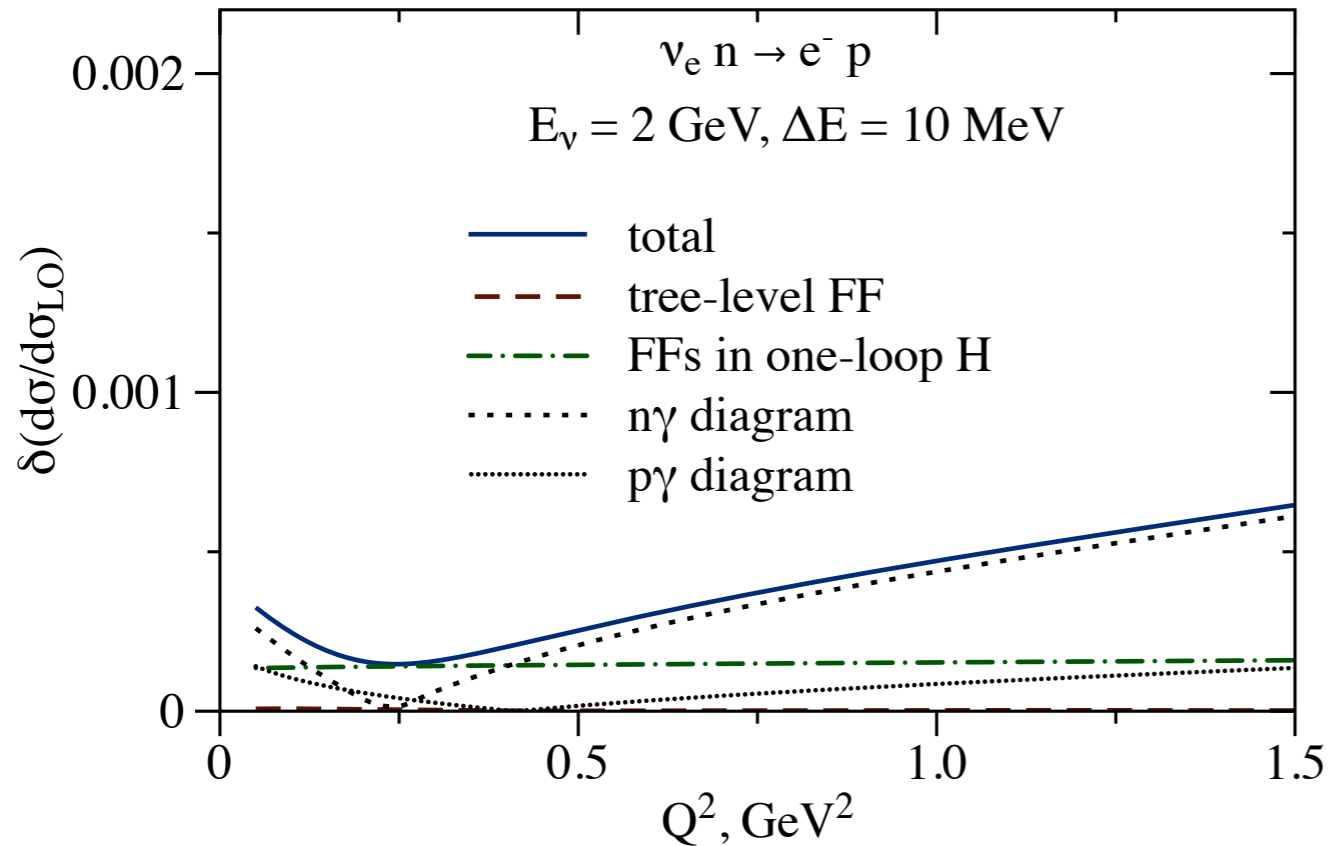
$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- 
- determine **hard function** at hard scale by matching experiment or **hadronic model** to the theory with heavy nucleon
 - **RGE evolution** of the hard function to scales $\Delta E, m_\ell$
 - **soft and collinear functions** are evaluated **perturbatively**
 - calculate cross section at low energies accounting for **all large logs**
ep scattering with soft radiation only: Richard J. Hill (2016)

- **soft and collinear functions** determined **analytically**
- **hard function** describes physics at GeV energies

Assignment of errors

- uncertainties from hard function



Meyer, Betancourt, Gran and Hill (2016)

- nucleon form factors

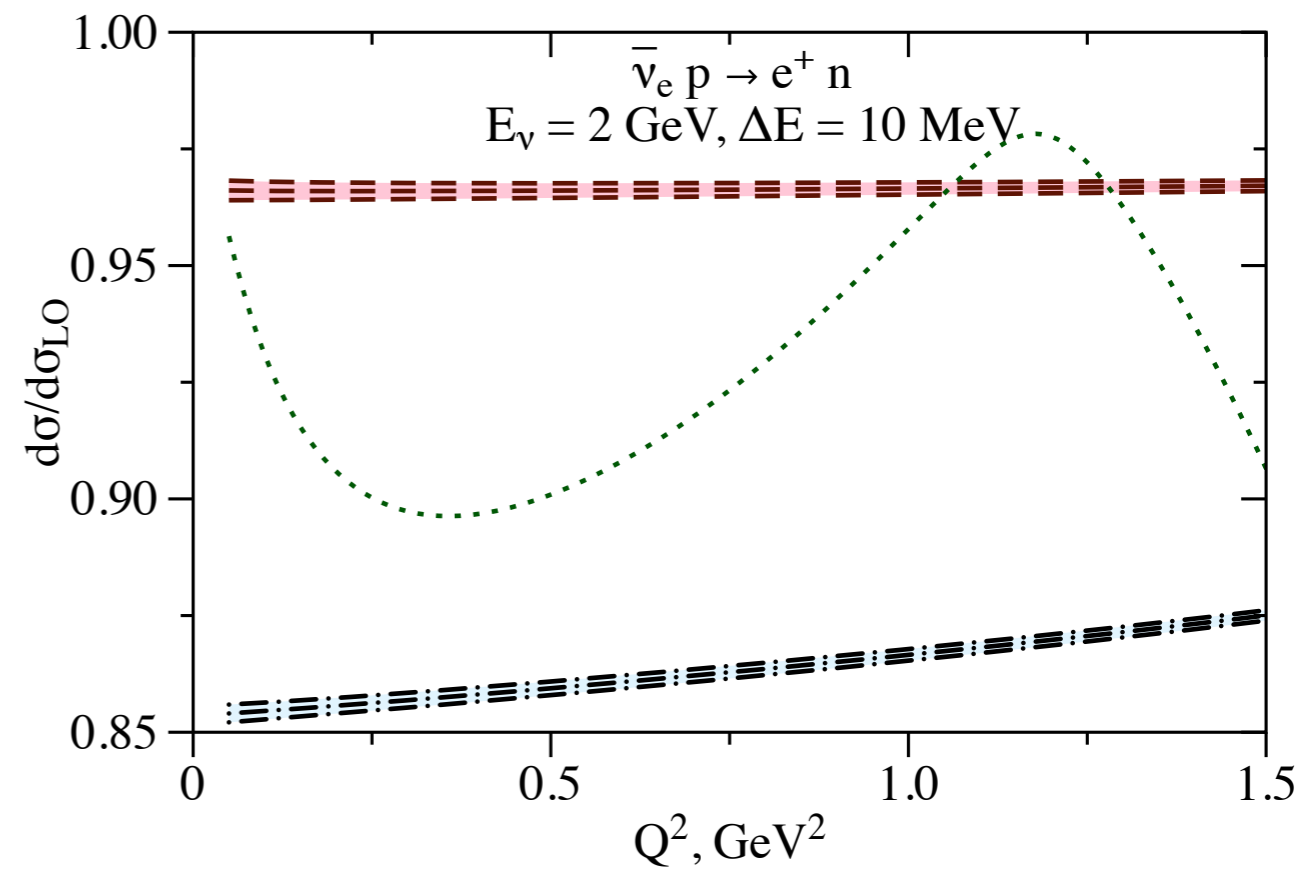
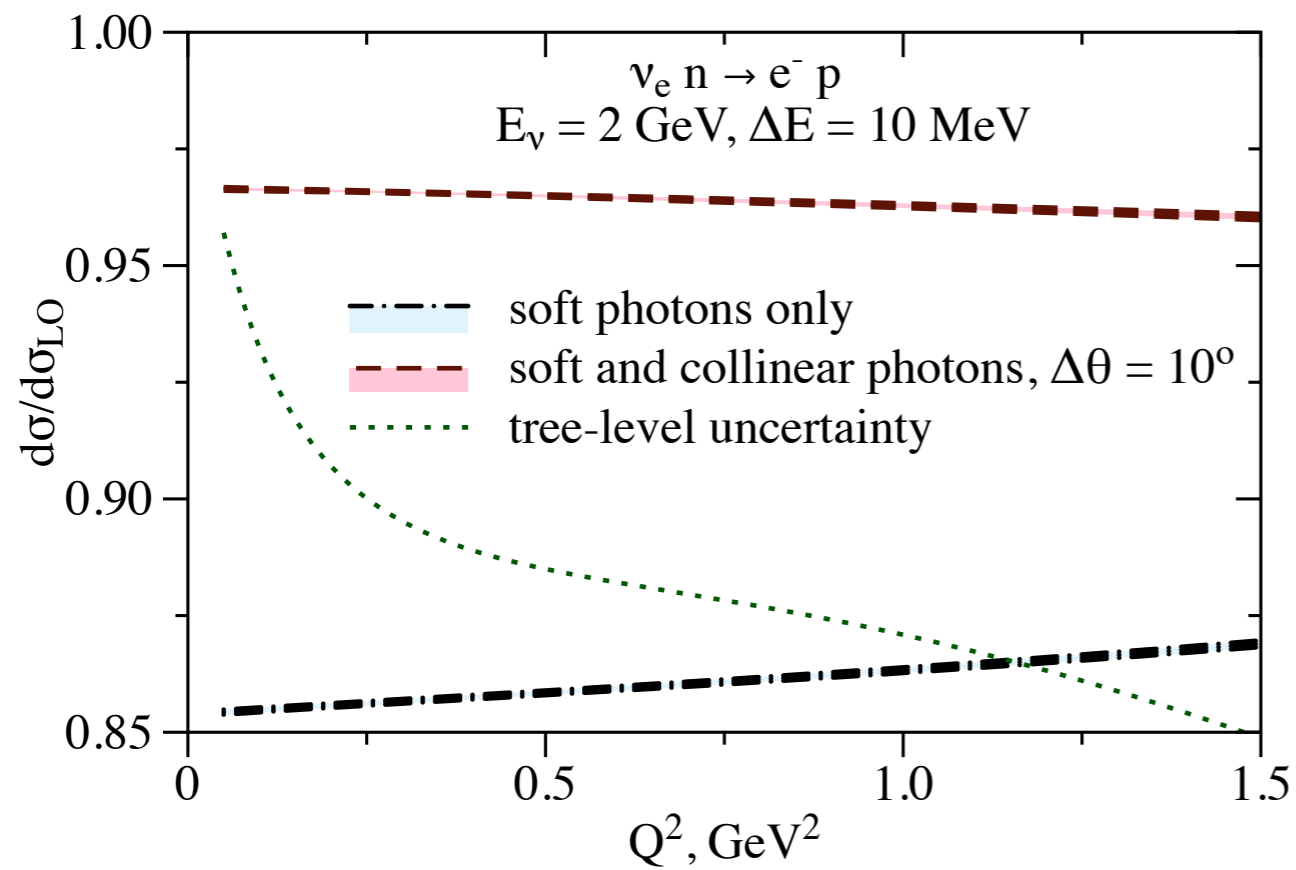
Kaushik Borah, Gabriel Lee, Richard J. Hill and O.T. (2020)

- add perturbative errors by variation of scale

- uncertainty of per mille level for the ratio to LO result

Cross section. Electron flavor

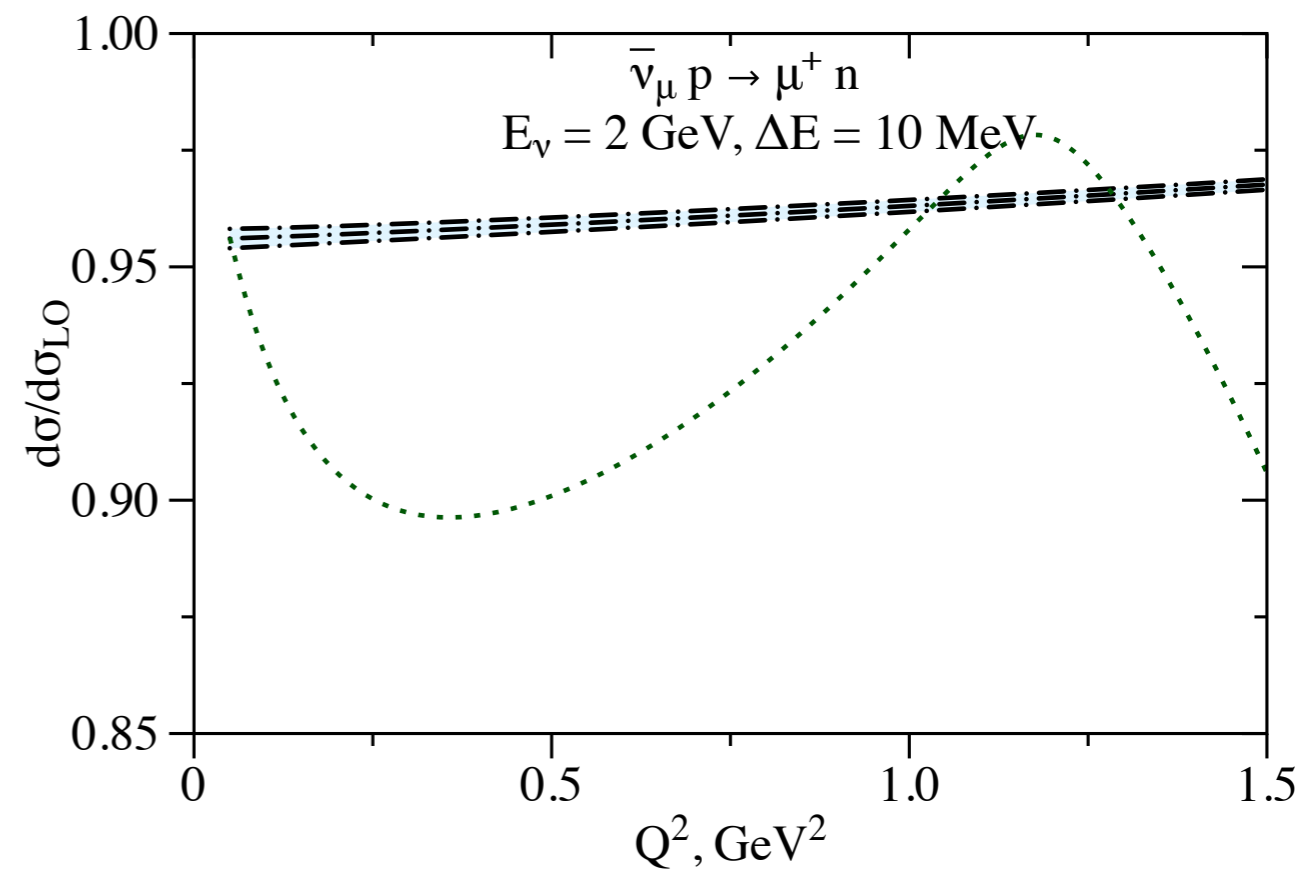
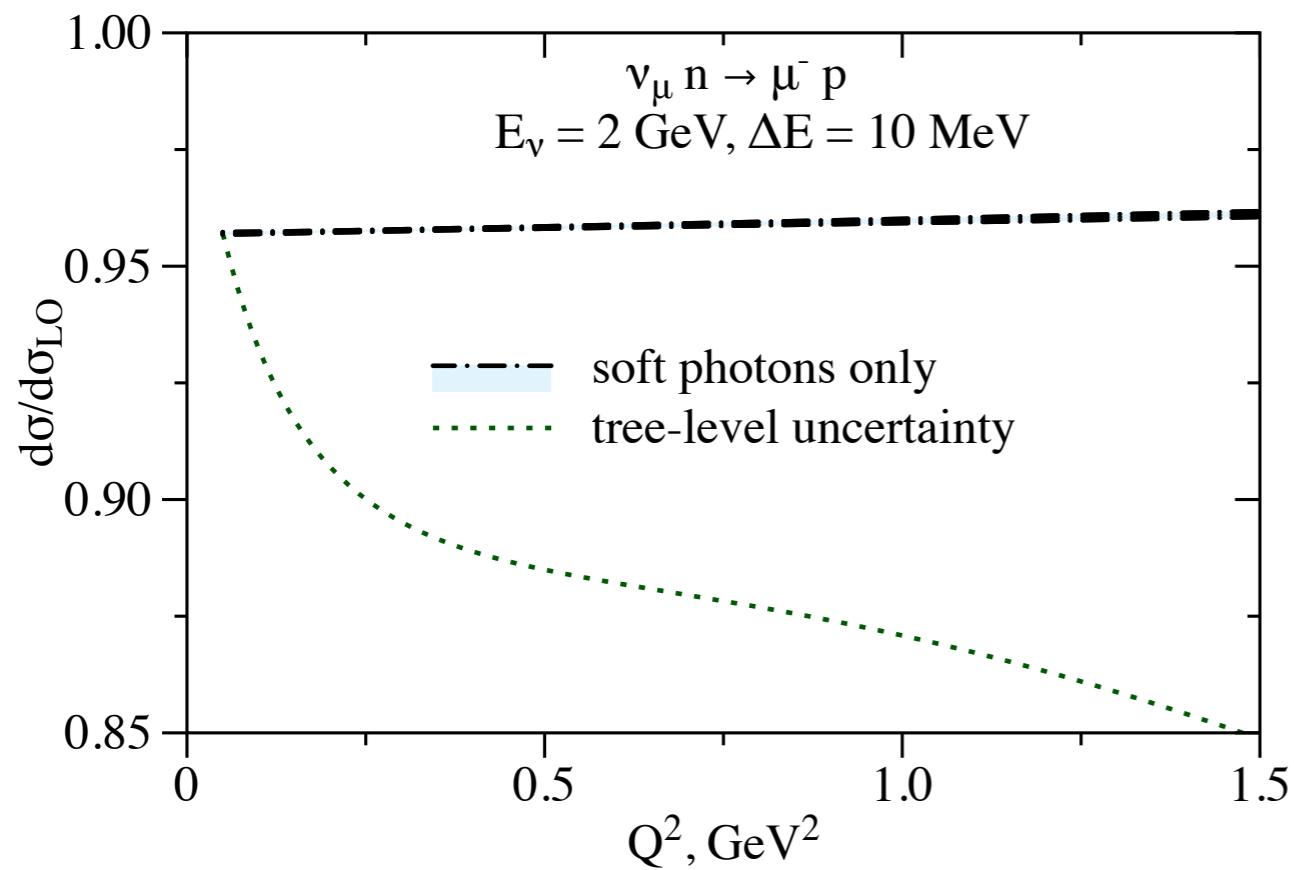
- exclusive observables



- cancellation of error from hard function for ratio to LO

Cross section. Muon flavor

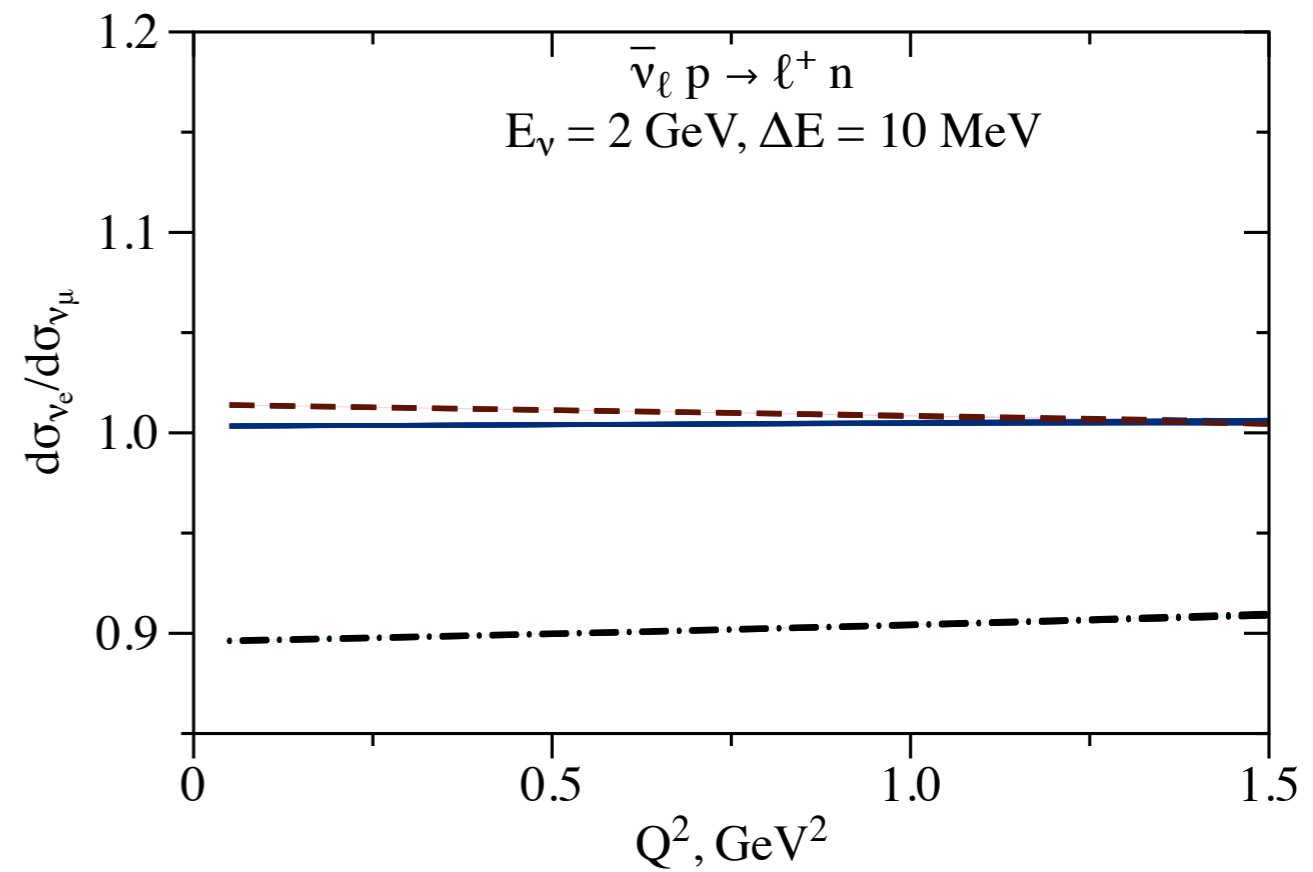
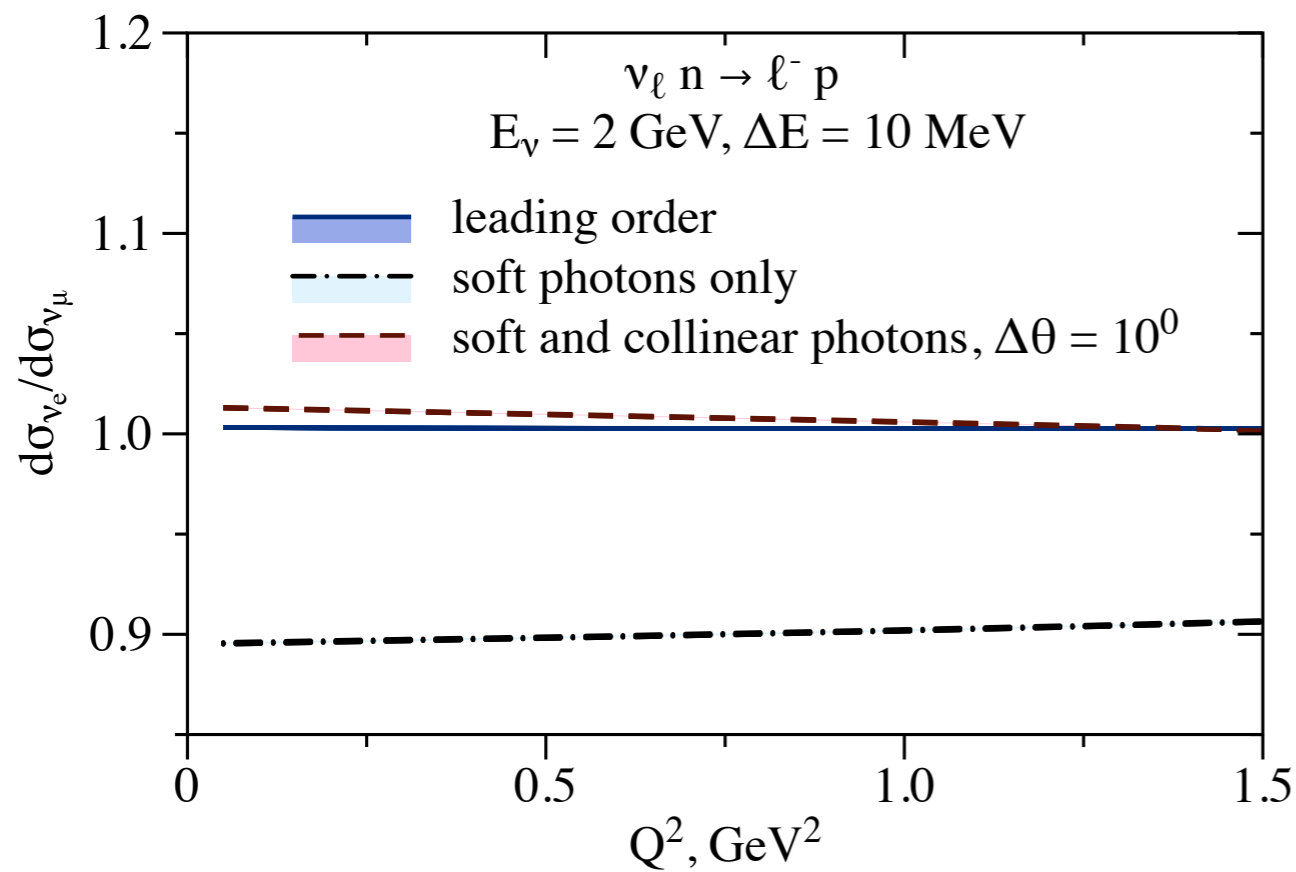
- exclusive observables



- cancellation of error from hard function for ratio to LO

Electron/muon ratio

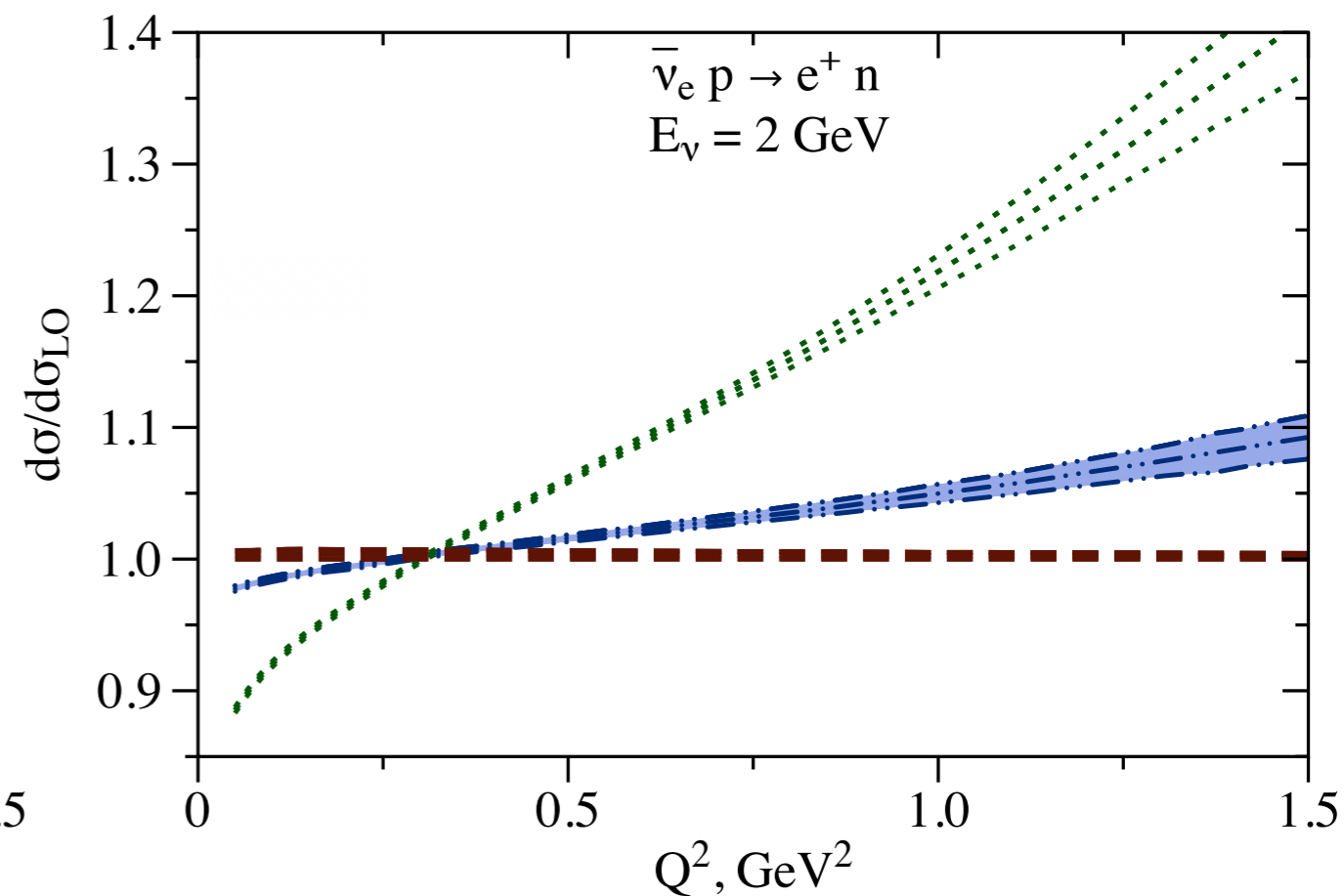
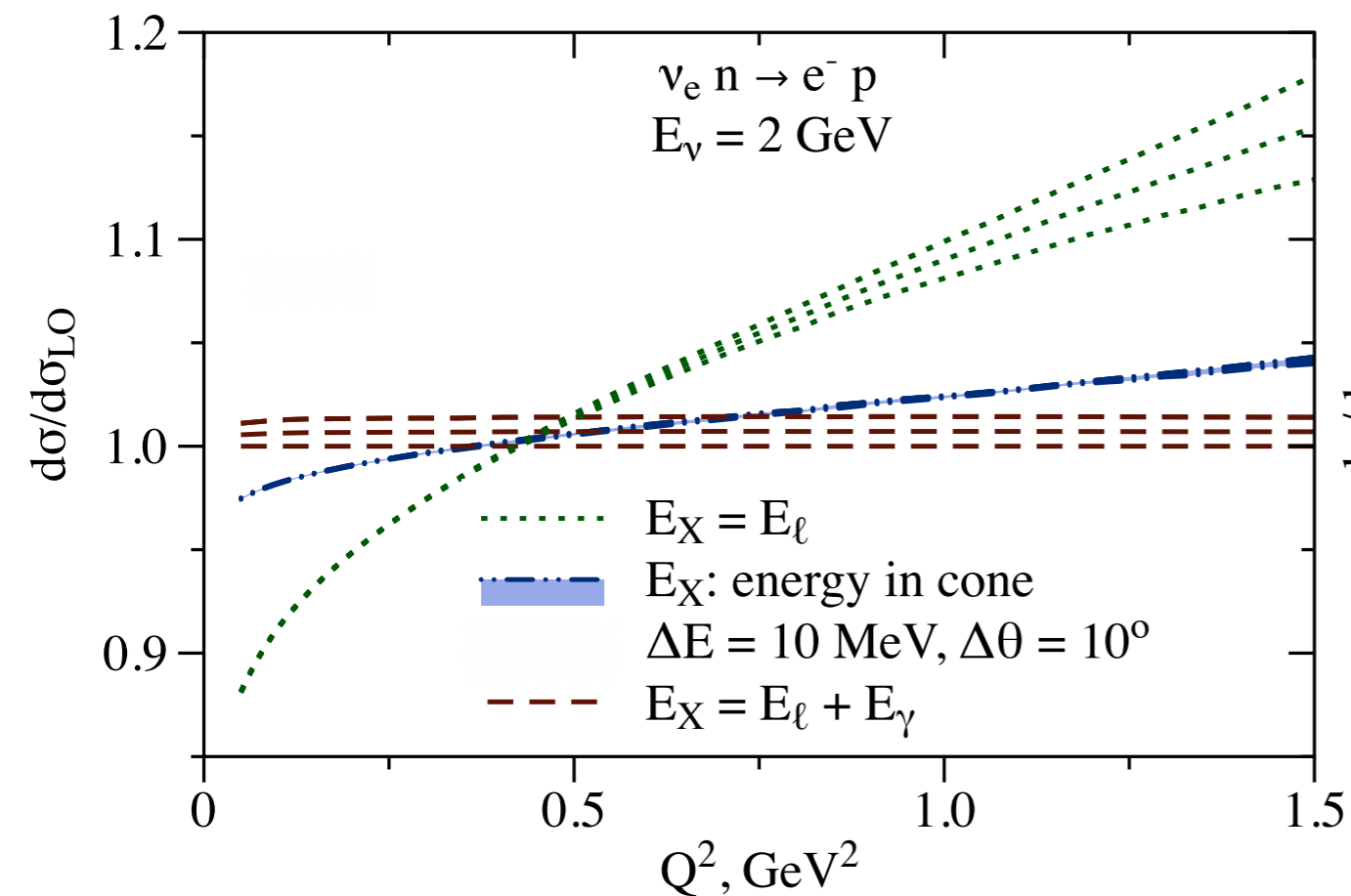
- exclusive observables



- small error: hard function does not depend on mass

Cross section. Electron flavor

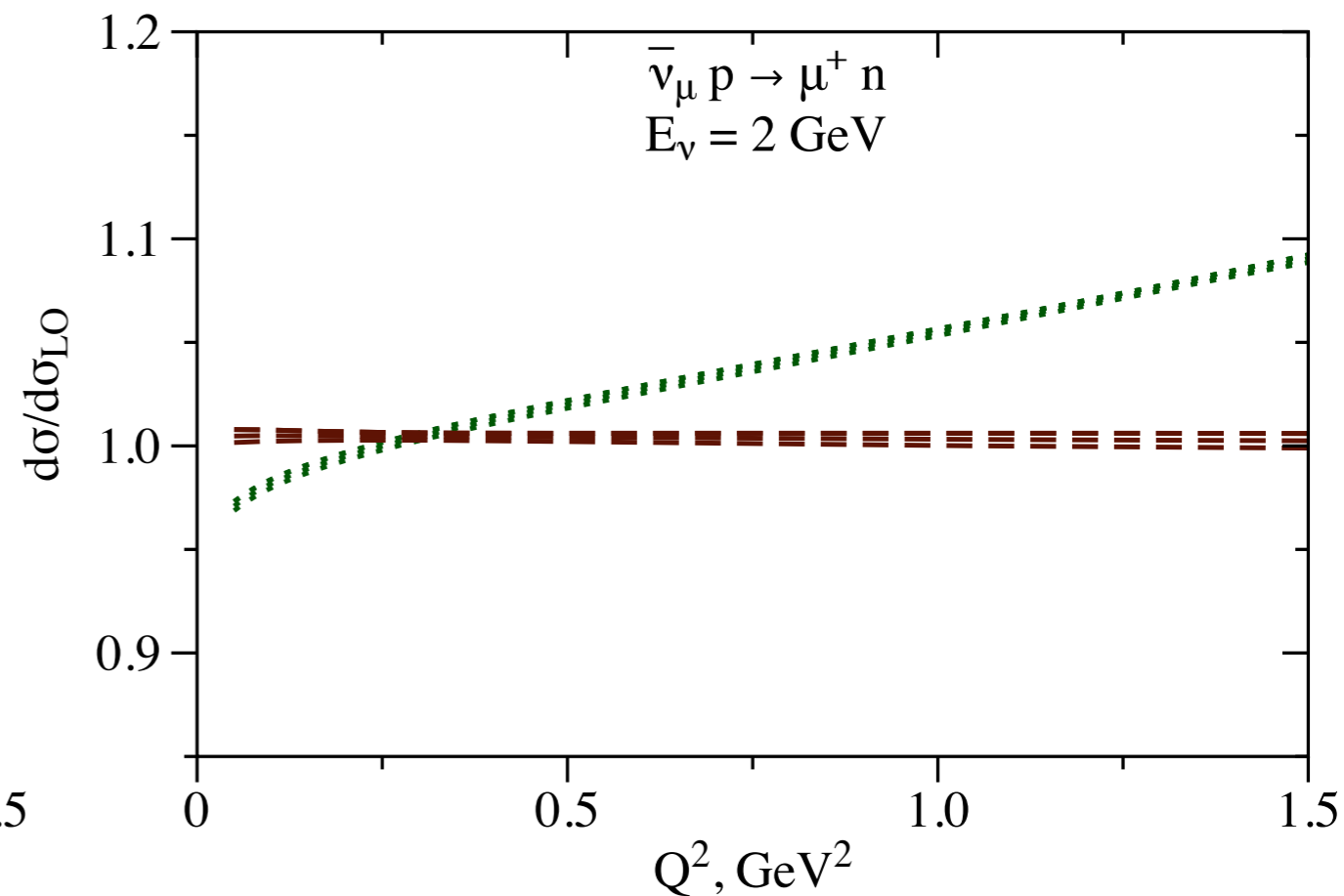
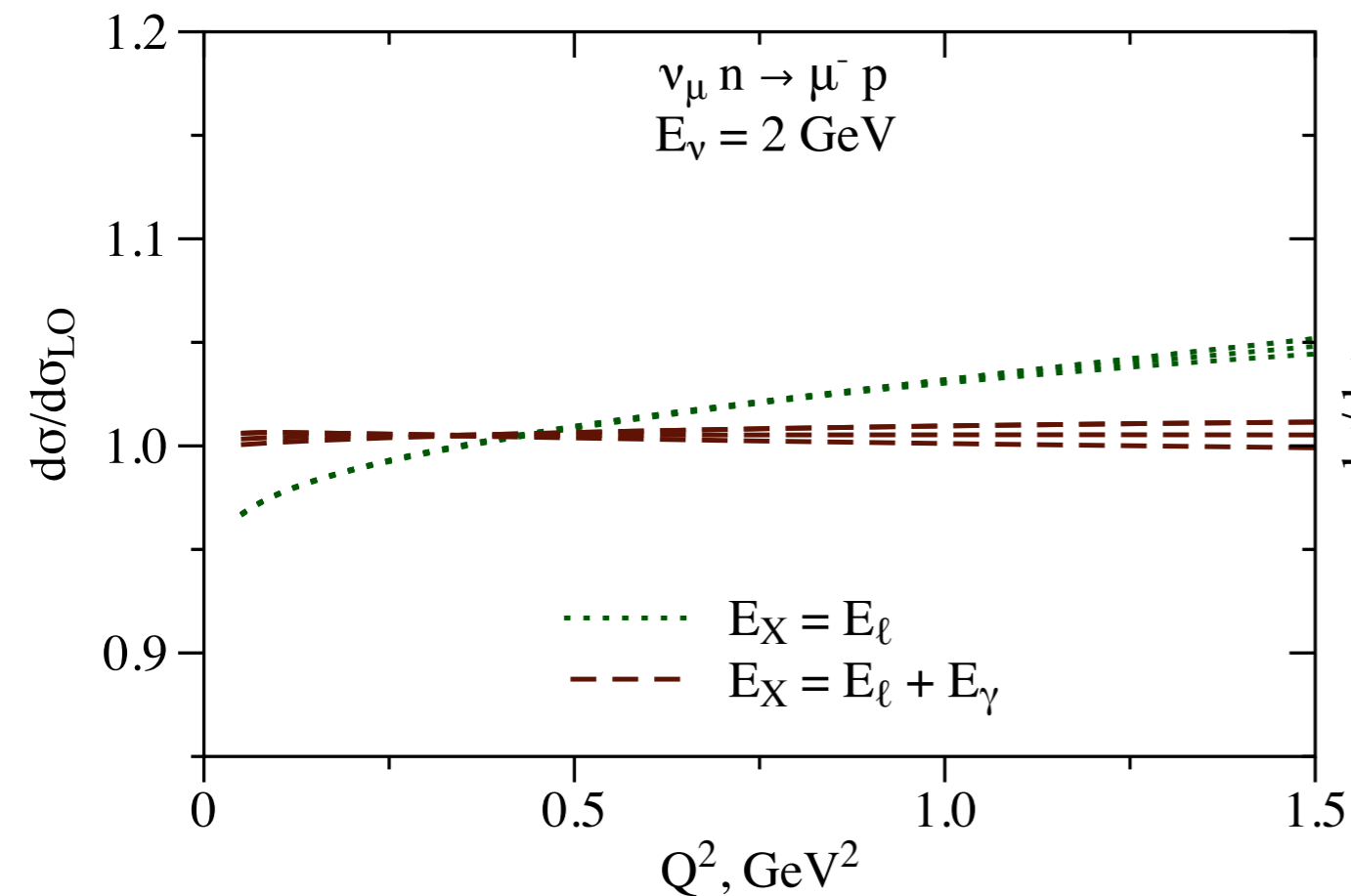
- kinematics $Q^2 = 2M(E_\nu - E_X)$ is reconstructed with 3 different E_X
- inclusive observables



- dependence on reconstruction of kinematics and cuts

Cross section. Muon flavor

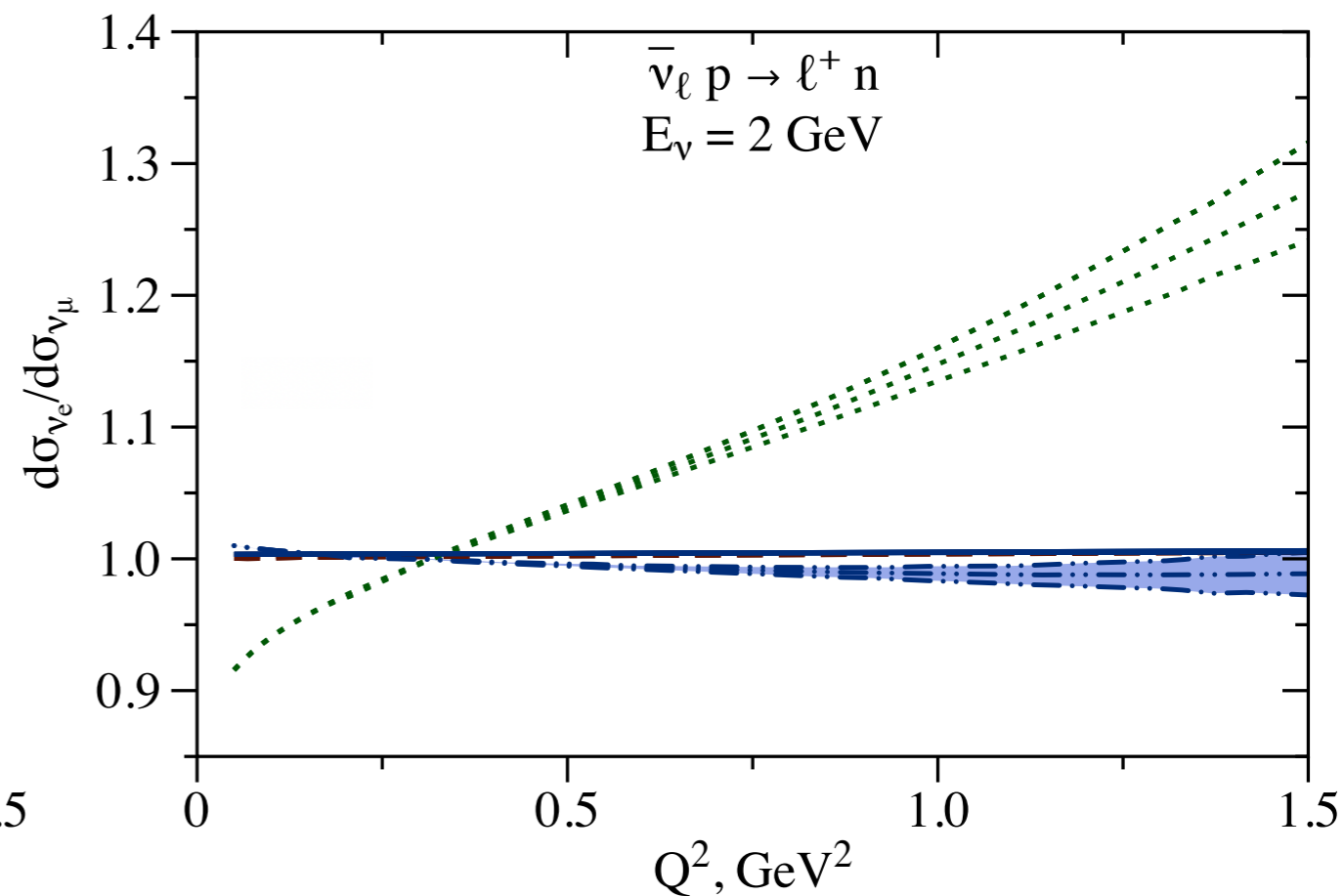
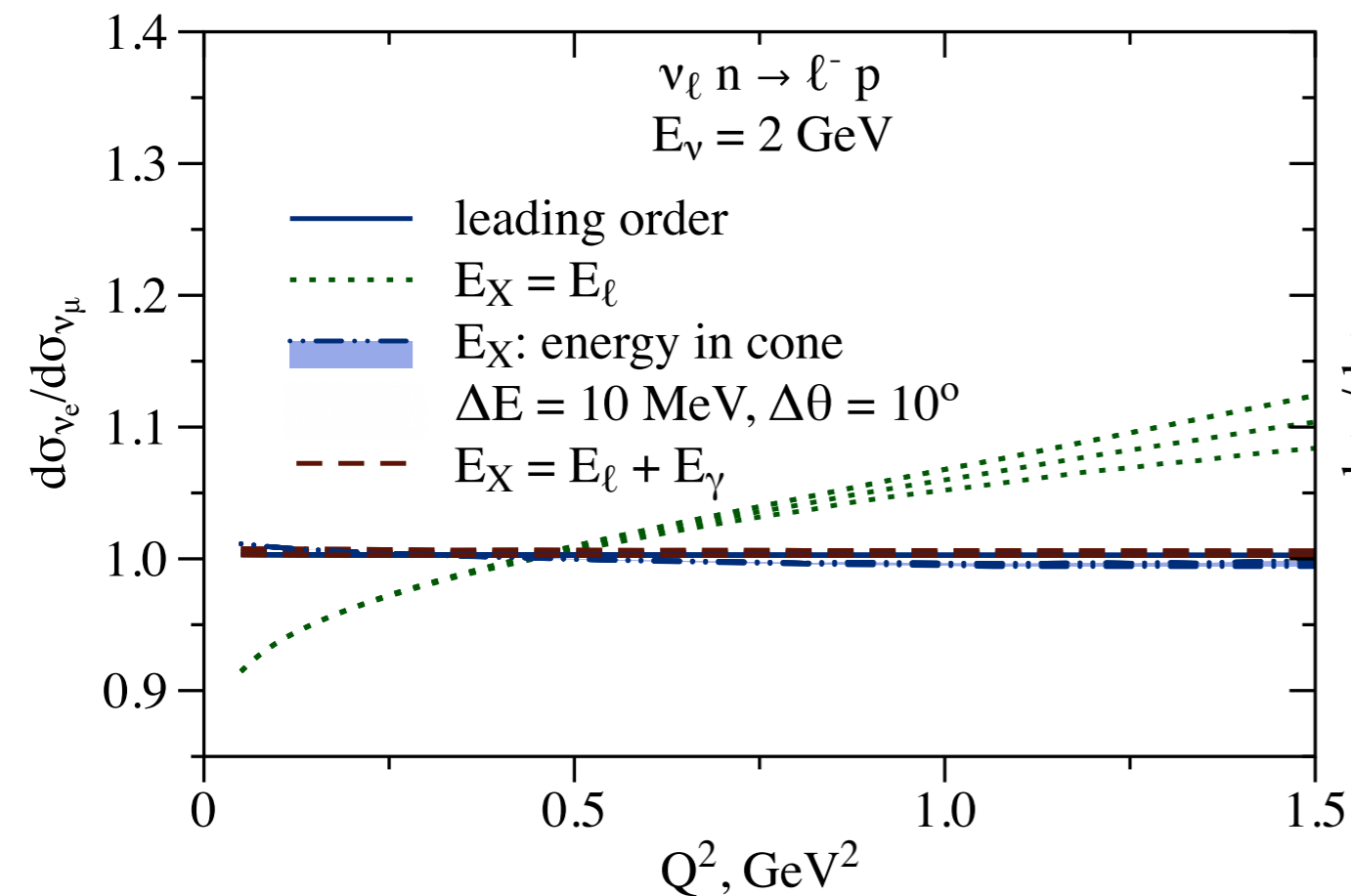
- kinematics $Q^2 = 2M(E_\nu - E_X)$ is reconstructed with 3 different E_X
- inclusive observables



- dependence on reconstruction of kinematics and cuts

Electron/muon ratio

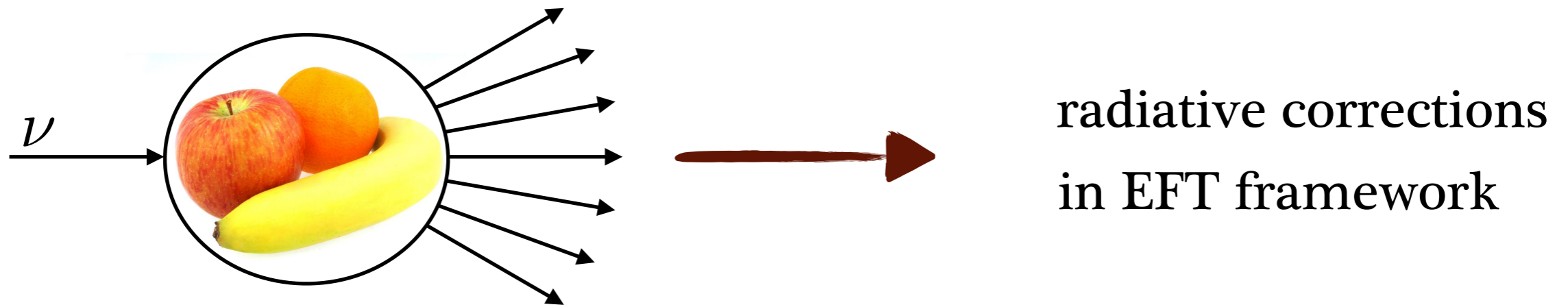
- kinematics $Q^2 = 2M(E_\nu - E_X)$ is reconstructed with 3 different E_X
- inclusive observables



- predict σ_{ν_e} from σ_{ν_μ} measurements with neutrino beam



Conclusions



- radiative corrections to CEvNS starting from four-fermion EFT
- radiative corrections to neutrino-nucleon cross sections in factorization framework
- σ_{ν_e} can be predicted from σ_{ν_μ} measurements with neutrino beam

Thanks for your attention !!!



This presentation is based upon work
that is supported by the Visiting Scholars Award Program
of the Universities Research Association