NNLO QCD corrections to the B-meson mixing

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Outline

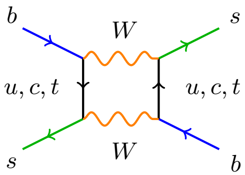
1. **Flavor physics and the precision frontier**
2. **B-meson mixing**
   - Theory
   - Building blocks
   - Technicalities
   - Master integrals
   - Phenomenology
3. **Summary and Outlook**

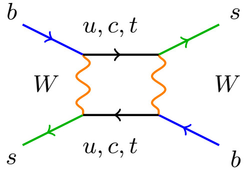
- Ongoing direct and indirect searches for new physics
- A direct discovery of BSM particles within the LHC energy range highly desirable
- No new physics on the high-energy frontier: growing importance of the precision frontier
- An increasing number of anomalies (still below 5σ-level) that challenge the validity of the SM
  $\Rightarrow$ **see M. Passera's talk**
- Necessary conditions for a meaningful discussion:
  - All theoretical uncertainties (perturbative and nonperturbative) must be well under control.
  - All required higher-order perturbative corrections must be worked out
- New physics searches are clearly not the only reason for doing flavor physics
  - QCD corrections to flavor observables: probe our understanding of strong interactions
  - Precise measurements of SM parameters: impossible without taking full account of the EW sector
  - CP-Violation, matter-antimatter asymmetry

- Neutral meson mixing in the SM : Loop-induced FCNC processes.
- $B_s^0 - \bar{B}_s^0$ oscillations

$$\begin{pmatrix} |B_s(t)\rangle \\ |\bar{B}_s(t)\rangle \end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right) \begin{pmatrix} |B_s(t)\rangle \\ |\bar{B}_s(t)\rangle \end{pmatrix},$$

$$\hat{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}$$





- Diagonalizing the matrices leads to the mass eigenstates $|B_L\rangle$ (lighter, CP-even) and $|B_H\rangle$ (heavier, CP-odd) with M_L, Γ_L and M_H, Γ_H.
- Physical observables depend on: $|M_{12}|$, $|\Gamma_{12}|$, ϕ
- ΔM_s: $B_s^0 - \bar{B}_s^0$ oscillation frequency $\Rightarrow |M_{12}|$ from $\mathrm{Re}(A_{\text{mixing}})$ t quark is dominant in SM, sensitivity to NP in the loops
- $\Delta\Gamma_s$: $B_s^0 - \bar{B}_s^0$ width difference $\Rightarrow |\Gamma_{12}|$ from $\mathrm{Im}(A_{\text{mixing}})$ only u and c contribute (linked to $b \to sc\bar{c}$ via optical theorem), precision probe of SM, little room for NP
- ϕ_s: CP-asymmetry in the mixing $a_{\text{fs}} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_s$

$$\Delta M_s = M_H - M_L \approx 2|M_{12}|$$

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi_s$$

$$\phi_s \equiv \arg(-M_{12}/\Gamma_{12}) \approx 0$$

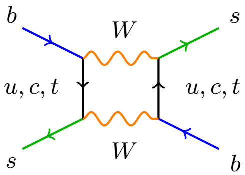
- In this project we are interested in $\Delta\Gamma$ from $B_s^0 - \bar{B}_s^0$
- Experimental value (HFLAV 2020 average on LHCb, ATLAS, CMS and CDF measurements)

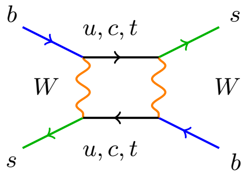
$$\Delta\Gamma^{\text{exp}} = (0.085 \pm 0.004)\ \text{ps}^{-1}$$

- Theory prediction (NLO + n_f-piece of NNLO QCD corrections) [Beneke et al., 1999; Ciuchini et al., 2002, 2003; Lenz & Nierste, 2007; Asatrian et al., 2020, 2017]

$$\Delta\Gamma_{\text{OS}} = (0.077 \pm 0.015_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.0017_{\Lambda_{\text{QCD}}/m_b})\ \text{ps}^{-1}$$
$$\Delta\Gamma_{\overline{\text{MS}}} = (0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.0014_{\Lambda_{\text{QCD}}/m_b})\ \text{ps}^{-1}$$

- Substantial uncertainty from uncalculated NNLO corrections (pert.), much larger than experimental errors
- Theory under pressure, full NNLO corrections highly desirable





$$\Delta M_s = M_H - M_L \approx 2|M_{12}|$$
$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi_s$$
$$\phi_s \equiv \arg(-M_{12}/\Gamma_{12}) \approx 0$$

- Since $m_b \ll m_W, m_t$, we can integrate out W bosons and t quarks to obtain $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$
- Need to evaluate nonlocal products of effective operators $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$

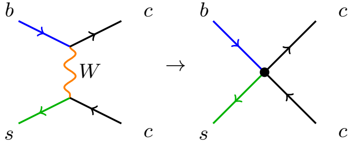
$$\Gamma_{12} = \frac{1}{2M_{B_s}} \text{Im}| \langle B_s^0 | i \int d^4x \, \mathcal{T}\{\mathcal{H}_{\text{eff}}^{|\Delta B|=1}(x) \mathcal{H}_{\text{eff}}^{\Delta F=1}(0)\} | \bar{B}_s^0 \rangle \, |$$

- Using Heavy Quark Expansion [Khoze & Shifman, 1983; Shifman & Voloshin, 1985; Khoze et al., 1987; Chay et al., 1990; Bigi & Uraltsev, 1992; Bigi et al., 1992, 1993; Blok et al., 1994; Manohar & Wise, 1994] (expansion in Λ_{QCD}/m_b) one arrives at

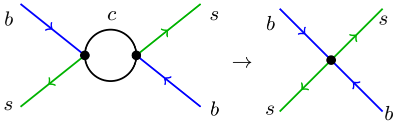
$$\Gamma_{12} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$$

  in the $|\Delta B| = 2$ effective theory.
- $H(z)$ and $\widetilde{H}_S(z)$ are the Wilson coefficients from the perturbative matching of $|\Delta B| = 1$ to $|\Delta B| = 2$.
- Nonperturbative ME $\langle B_s | Q | \bar{B}_s \rangle$ and $\langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle$ from QCD sum rules [Ovchinnikov & Pivovarov, 1988; Reinders & Yazaki, 1988; Korner et al., 2003; Mannel et al., 2011] or lattice QCD [Bazavov et al., 2016; Dowdall et al., 2019].

$|\Delta B = 1|$ effective theory



$|\Delta B = 2|$ effective theory



$$\Gamma_{12} \sim \frac{1}{m_b^3} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_3^{(i)} + \frac{1}{m_b^4} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_4^{(i)} + \ldots$$

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to s c \bar{c}$ decays [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb}^\dagger \left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \right) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

- Current operators

$$\begin{aligned}
Q_1 &= \bar{s}_L \gamma_\mu T^a c_L \; \bar{c}_L \gamma^\mu T^a b_L, \\
Q_2 &= \bar{s}_L \gamma_\mu c_L \; \bar{c}_L \gamma^\mu b_L, \\
Q_1^u &= \bar{s}_L \gamma_\mu T^a u_L \; \bar{u}_L \gamma^\mu T^a b_L, \\
Q_2^u &= \bar{s}_L \gamma_\mu u_L \; \bar{u}_L \gamma^\mu b_L, \\
Q_1^{cu} &= \bar{s}_L \gamma_\mu T^a u_L \; \bar{c}_L \gamma^\mu T^a b_L, \\
Q_2^{cu} &= \bar{s}_L \gamma_\mu u_L \; \bar{c}_L \gamma^\mu b_L, \\
Q_1^{uc} &= \bar{s}_L \gamma_\mu T^a c_L \; \bar{u}_L \gamma^\mu T^a b_L, \\
Q_2^{uc} &= \bar{s}_L \gamma_\mu c_L \; \bar{u}_L \gamma^\mu b_L,
\end{aligned}$$

- Penguin operators

$$\begin{aligned}
Q_3 &= \bar{s}_L \gamma_\mu b_L \textstyle\sum_q \bar{q} \gamma^\mu q\,, \\
Q_4 &= \bar{s}_L \gamma_\mu T^a b_L \textstyle\sum_q \bar{q} \gamma^\mu T^a q\,, \\
Q_5 &= \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q\,, \\
Q_6 &= \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q\,, \\
Q_8 &= \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \sigma^{\mu\nu} T^a b_R \, G^a_{\mu\nu}\,,
\end{aligned}$$

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to s c \bar{c}$ decays [Chetyrkin et al., 1998]

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$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.} \,,$$

- 4-fermion vertices generate Dirac structures with multiple insertions of γ matrices

$$(P_L)_{ij} \times (P_L)_{kl}, \quad (\gamma^\mu P_L)_{ij} \times (\gamma_\mu P_L)_{kl}, \quad (\gamma^\mu \gamma^\nu P_L)_{ij} \times (\gamma_\mu \gamma_\nu P_L)_{kl},$$
$$(\gamma^\mu \gamma^\nu \gamma^\rho P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho P_L)_{kl}, \quad (\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma P_L)_{kl},$$
$$(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_\tau P_L)_{kl}, \ldots$$

- 4-dimensions: Products of γ matrices reducible via the Chisholm identity

$$\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu} \gamma^\rho + g^{\nu\rho} \gamma^\mu - g^{\mu\rho} \gamma^\nu + i \varepsilon^{\mu\nu\rho\sigma} \gamma_\sigma \gamma^5$$

- In d-dimensions such a reduction is not possible (unambiguously).
- Proper way to handle this issue: evanescent operators [Dugan & Grinstein, 1991; Herrlich & Nierste, 1995]

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- LO evanescent operators

$$E_1^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a c_L \; \bar{c}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L - 16 Q_1 \,,$$

$$E_2^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} c_L \; \bar{c}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L - 16 Q_2 \,,$$

$$E_3^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} b_L \sum\nolimits_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} q \; - \; 20 Q_5 \; + \; 64 Q_3 \,,$$

$$E_4^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} T^a b_L \sum\nolimits_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} T^a q \; - \; 20 Q_6 \; + \; 64 Q_4$$

$|\Delta B| = 1$ effective Hamiltonian in the CMM basis for $b \to s c \bar{c}$ decays [Chetyrkin et al., 1998]

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$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Big] + \text{h.c.}\,,$$

- NLO evanescent operators

$$E_1^{(2)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} T^a c_L \; \bar{c} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} T^a b_L - 20 E_1^{(1)} - 256 Q_1\,,$$

$$E_2^{(2)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} c_L \; \bar{c}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} b_L - 20 E_2^{(1)} - 256 Q_2\,,$$

$$E_3^{(2)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} \gamma^{\mu_6} \gamma^{\mu_7} b_L \sum_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} \gamma_{\mu_6} \gamma_{\mu_7} q - 336 Q_5 + 1280 Q_3\,,$$

$$E_4^{(2)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} T^a b_L \sum_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} T^a q - 336 Q_6 + 1280 Q_4$$

$|\Delta B| = 1$ contributions needed for NNLO

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

Important scale: $z \equiv m_c^2/m_b^2$

- LO contributions to $\Delta\Gamma_s$
  - 1-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Hagelin, 1981; Franco et al., 1982; Chau, 1983; Buras et al., 1984; Khoze et al., 1987; Datta et al., 1987, 1988]
- NLO contributions to $\Delta\Gamma_s$ (z-exact)
  - 2-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_8$ correlators (z-exact) [Beneke et al., 1999]

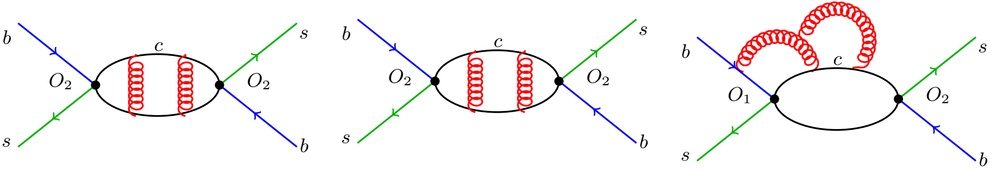
$|\Delta B| = 1$ contributions needed for NNLO

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

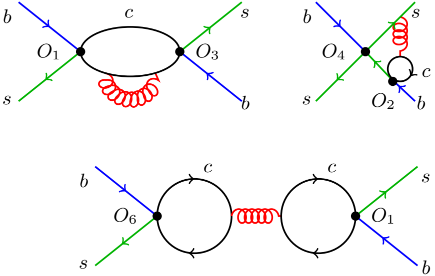
Important scale: $z \equiv m_c^2/m_b^2$

- NNLO contributions to $\Delta\Gamma_s$ (n_f pieces calculated in [Asatrian et al., 2017, 2020])
  - 3-loop $O_{1-2} \times O_{1-2}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, $\mathcal{O}(z^3)$)
  - 2-loop $O_{1-2} \times O_{3-6}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 2-loop $O_{1-2} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_{3-6} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1996]
  - 1-loop $O_{3-6} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_8 \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
- This work
  - Full (n_f + non-n_f) results for all 2-loop NNLO correlators at $\mathcal{O}(z)$
  - Full (n_f + non-n_f) results for the 3-loop $O_{1-2} \times O_{1-2}$ at $\mathcal{O}(z^0)$
  - Full (n_f + non-n_f) results for some $\mathrm{N}^3\mathrm{LO}$ correlators e. g. the 2-loop $O_{3-6} \times O_{3-6}$ at $\mathcal{O}(z^0)$
  - WIP: Final checks for the 3-loop result, higher order expansions in z, possibly z-exact results for selected correlators, computation of some (2-loop) $\mathrm{N}^3\mathrm{LO}$ correlators

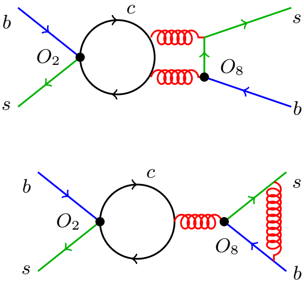
Representative diagrams for 3-loop $O_{1,2} \times O_{1,2}$ correlators



Representative diagrams for 2-loop $O_{1,2} \times O_{3-6}$ correlators



Representative diagrams for 2-loop $O_{1,2} \times O_8$ correlators



$\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke et al., 1999; Lenz & Nierste, 2007; Asatrian et al., 2017]

$$\Gamma_{12} = -(\lambda_c^q)^2 \Gamma_{12}^{cc} - 2\lambda_c^q \lambda_u^q \Gamma_{12}^{uc} - (\lambda_u^q)^2 \Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}/m_b\right)$$

- Matching coefficients from various $|\Delta B| = 1$ correlators

$$H^{ab}(z) = H^{(c)ab}(z) + H^{(cp)ab}(z) + H^{(p)ab}(z)$$

$$\tilde{H}_S^{ab}(z) = \tilde{H}_S^{(c)ab}(z) + \tilde{H}_S^{(cp)ab}(z) + \tilde{H}_S^{(p)ab}(z)$$

$$H^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{ab}(z),$$

$$\tilde{H}_S^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{S,ab}(z),$$

$$H^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{ab}(z) + C_2 p_{2i}^{ab}(z) \right],$$

$$\tilde{H}_S^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{S,ab}(z) + C_2 p_{2i}^{S,ab}(z) \right],$$

$$H^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{ab}(z)$$

$$\tilde{H}_S^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{S,ab}(z)$$

- Physical $|\Delta B| = 2$ operators

$$Q = \bar{s}_i \gamma^\mu \left(1 - \gamma^5\right) b_i \; \bar{s}_j \gamma_\mu \left(1 - \gamma^5\right) b_j \,,$$

$$\widetilde{Q}_S = \bar{s}_i \left(1 - \gamma^5\right) b_j \; \bar{s}_j \left(1 - \gamma^5\right) b_i$$

- Additional operators needed at intermediate stages (e. g. basis changes, def. of ev. operators)

$$\widetilde{Q} = \bar{s}_i \gamma^\mu \left(1 - \gamma^5\right) b_j \; \bar{s}_j \gamma_\mu \left(1 - \gamma^5\right) b_i \,,$$

$$Q_S = \bar{s}_i \left(1 - \gamma^5\right) b_i \; \bar{s}_j \left(1 - \gamma^5\right) b_j \,,$$

$\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke et al., 1999; Lenz & Nierste, 2007; Asatrian et al., 2017]

$$\Gamma_{12} = -(\lambda_c^q)^2 \Gamma_{12}^{cc} - 2\lambda_c^q \lambda_u^q \Gamma_{12}^{uc} - (\lambda_u^q)^2 \Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}/m_b\right)$$

- Matching coefficients from various $|\Delta B| = 1$ correlators

$$H^{ab}(z) = H^{(c)ab}(z) + H^{(cp)ab}(z) + H^{(p)ab}(z)$$
$$\tilde{H}_S^{ab}(z) = \tilde{H}_S^{(c)ab}(z) + \tilde{H}_S^{(cp)ab}(z) + \tilde{H}_S^{(p)ab}(z)$$
$$H^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{ab}(z),$$
$$\tilde{H}_S^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{S,ab}(z),$$
$$H^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{ab}(z) + C_2 p_{2i}^{ab}(z) \right],$$
$$\tilde{H}_S^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{S,ab}(z) + C_2 p_{2i}^{S,ab}(z) \right],$$
$$H^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{ab}(z)$$
$$\tilde{H}_S^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{S,ab}(z)$$

- LO evanescent operators

$$E_1^{(1)} = \widetilde{Q} - Q,$$
$$E_2^{(1)} = \bar{b}_i \gamma^\mu \gamma^\nu \gamma^\rho P_L s_j \bar{b}_j \gamma_\mu \gamma_\nu \gamma_\rho P_L s_i - (16 - 4\epsilon)\widetilde{Q},$$
$$E_3^{(1)} = \bar{b}_i \gamma^\mu \gamma^\nu \gamma^\rho P_L s_i \bar{b}_j \gamma_\mu \gamma_\nu \gamma_\rho P_L s_j - (16 - 4\epsilon) Q,$$
$$E_4^{(1)} = \bar{b}_i \gamma^\mu \gamma^\nu P_L s_j \bar{b}_j \gamma_\nu \gamma_\mu P_L s_i + (8 - 8\epsilon) Q_s,$$
$$E_5^{(1)} = \bar{b}_i \gamma^\mu \gamma^\nu P_L s_i \bar{b}_j \gamma_\nu \gamma_\mu P_L s_j + (8 - 8\epsilon) \widetilde{Q}_s,$$

- Higher order evanescent operators (not shown here) needed for the NNLO calculation but also for the renormalization of LO evanescent operators

$\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke et al., 1999; Lenz & Nierste, 2007; Asatrian et al., 2017]

$$\Gamma_{12} = -(\lambda_c^q)^2 \Gamma_{12}^{cc} - 2\lambda_c^q \lambda_u^q \Gamma_{12}^{uc} - (\lambda_u^q)^2 \Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$$

- Matching coefficients from various $|\Delta B| = 1$ correlators

$$H^{ab}(z) = H^{(c)ab}(z) + H^{(cp)ab}(z) + H^{(p)ab}(z)$$

$$\tilde{H}_S^{ab}(z) = \tilde{H}_S^{(c)ab}(z) + \tilde{H}_S^{(cp)ab}(z) + \tilde{H}_S^{(p)ab}(z)$$

$$H^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{ab}(z),$$

$$\tilde{H}_S^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{S,ab}(z),$$

$$H^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{ab}(z) + C_2 p_{2i}^{ab}(z) \right],$$

$$\tilde{H}_S^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{S,ab}(z) + C_2 p_{2i}^{S,ab}(z) \right],$$

$$H^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{ab}(z)$$

$$\tilde{H}_S^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{S,ab}(z)$$

- Another subtlety: $1/m_b$ suppressed operator R_0

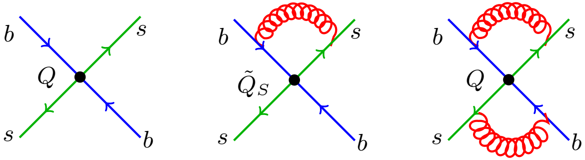
$$R_0 = Q_S + \alpha_1 \tilde{Q}_S + \alpha_2 \frac{1}{2} Q$$

- $\alpha_{1,2} = 1$ at LO
- $\alpha_{1,2}$ receive $\mathcal{O}(\alpha_s)$ corrections at higher loop orders [Beneke et al., 1999]
- One must take into account that the combination

$$Q_S + \alpha_1 \tilde{Q}_S + \alpha_2 \frac{1}{2} Q$$

is $1/m_b$ suppressed. Ignoring it leads to incorrect matching coefficients!

Representative diagrams in the $|\Delta B| = 2$ theory (tree-level, 1-loop, 2-loop)



Wilson coefficients of the $|\Delta B| = 2$ theory determined in the matching to $|\Delta B| = 1$

- Conceptional overview of the NNLO calculation
  - Matching between two EFTs, large number of operators contributing on both sides
  - Mixing of operators (on both sides) under renormalization (also between physical and evanescent operators)

$$(\vec{C}^{\text{bare}}, \vec{C}_E^{\text{bare}}) = (\vec{C}^{\text{ren}}, \vec{C}_E^{\text{ren}}) Z \equiv (\vec{C}^{\text{ren}}, \vec{C}_E^{\text{ren}}) \begin{pmatrix} Z_{QQ} & Z_{QE} \\ Z_{EQ} & Z_{EE} \end{pmatrix},$$

  - Example: The 3-loop $O_{1,2} \times O_{1,2}$ correlator receives contributions from $O_{1,2} \times O_{3-6}$ (1- and 2-loops), $O_{3-6} \times O_{3-6}$ (1-loop) and $O_{1-2} \times O_8$ (1-loop)
  - Comparisons to the existing literature results requires switching between different bases
    - "historical" [Buras et al., 1993] $\Leftrightarrow$ CMM bases for $|\Delta B| = 1$
    - $(Q, Q_S, \tilde{Q}_S) \Leftrightarrow (Q, \tilde{Q}_S, R_0)$ for $|\Delta B| = 2$
  - Extra care needed when IR-divergences are regularized dimensionally

- Calculational strategy
  - Matching done on-shell: $p_b^2 = m_b^2$
  - The s-quark mass is neglected $\Rightarrow p_s = 0$
  - Asymptotic expansion in $z \equiv m_c^2/m_b^2$ (at first up to $\mathcal{O}(z)$ for 2-loop and $\mathcal{O}(z^0)$ for 3-loop)
  - Only the imaginary part of the $|\Delta B| = 1$ diagrams enters the matching
- Regularization
  - Dimensional regularization used both for UV- and IR-divergences
  - Additionally, massive gluons in IR-divergent diagrams at 2-loops as a cross-check
  - When IR-divergences are regularized with m_g, the renormalized amplitudes are manifestly finite $\Rightarrow$ the limit $d \to 4$ is safe
  - Not so when the amplitudes are UV-finite but IR-divergent $\Rightarrow$ every product of $1/\varepsilon_{\text{IR}}$ and an evanescent matrix element is $\mathcal{O}(\varepsilon^0)$, the amplitude must be kept d-dimensional

NLO matching with $\varepsilon_{\text{IR}} = \varepsilon_{\text{UV}} = \varepsilon$ (no gluon mass) [Ciuchini et al., 2002]

- Normally, only the matching coefficients of physical $|\Delta B| = 2$ operators are relevant
- Here matching coefficients of evanescent operators are also needed (at intermediate stages)
- $|\Delta B| = 2$ matching coefficients obtain $\mathcal{O}(\varepsilon)$ pieces

$$C = f_0^{(0)} + \varepsilon\, f_1^{(0)} + \frac{\alpha_s}{4\pi} f_0^{(1)}, \qquad C_E = f_{E,0}^{(0)} + \varepsilon\, f_{E,1}^{(0)} + \frac{\alpha_s}{4\pi} f_{E,0}^{(1)}$$

- LO matching must be carried out up to $\mathcal{O}(\varepsilon)$: fixes $f_0^{(0)}, f_1^{(0)}, f_{E,0}^{(0)}, f_{E,1}^{(0)}$
- At NLO we only need $\mathcal{O}(\varepsilon^0)$
- Upon inserting $f_0^{(0)}, f_1^{(0)}, f_{E,0}^{(0)}, f_{E,1}^{(0)}$ at NLO all $1/\varepsilon_{\text{IR}}$ poles must cancel.
- Finally, the difference

$$A_{\text{ren}}^{|\Delta B|=1} - A_{\text{ren}}^{|\Delta B|=2}$$

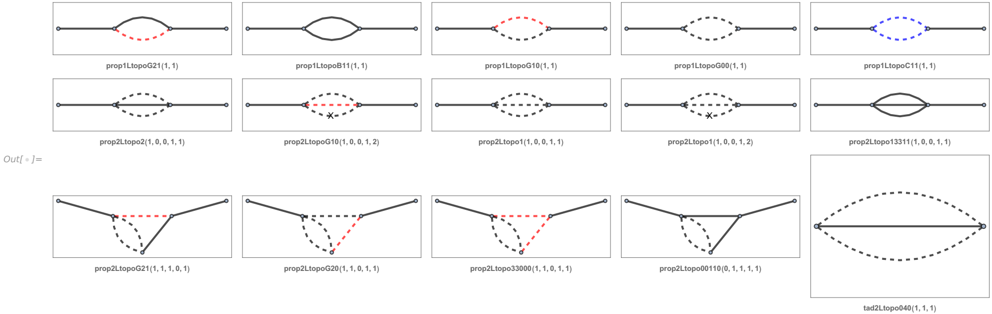
  is manifestly finite $\Rightarrow$ determine $f_0^{(1)}$
- Only $f_0^{(0)}$ and $f_0^{(1)}$ enter the physical matching coefficient
- What about $f_{E,1}^{(0)}$? Not needed at NLO, must be determined for the NNLO calculation!
- At NNLO, the LO matching must be done up $\mathcal{O}(\varepsilon^2)$, the NLO matching up to $\mathcal{O}(\varepsilon)$
- The explicit cancellation of IR poles (and of ξ) is a highly nontrivial cross-check of the whole calculation

- All computations done using our well-tested automatic setup
  - Diagram generation with QGRAF [Nogueira, 1993]
  - Insertion of Feynman rules and topology identification using Q2E/EXP [Seidensticker, 1999; Harlander et al., 1998] or TAPIR [Gerlach, Herren, 2021]
  - Feynman amplitude evaluation: in-house CALC setup written in FORM [Ruijl et al., 2017]
  - IBP-reduction: FIRE 6 [A. V. Smirnov & Chuharev, 2020]
  - Analytic computation of master integrals: HyperInt [Panzer, 2015], HyperlogProceedings [Schnetz], PolyLogTools [Duhr & Dulat, 2019]
  - All master integrals checked numerically using FIESTA [A. V. Smirnov, 2016] and pySecDec [Borowka et al., 2018]
- Cross-checks of selected intermediate results using FeynArts [Hahn, 2001], FeynRules [Christensen & Duhr, 2009; Alloul et al., 2014] and FeynCalc [VS et al., 2020]
- Two complementary approaches to the treatment of tensor integrals in FORM
  - Explicit decomposition formulas (1 ext. momentum, max. rank 10), calculated using FeynCalc and FERMAT [Lewis]
  - Projections to the occurring 4-fermion Dirac structures

$$\{(P_L)_{ij}, (\gamma^\mu P_L)_{ij}, (\gamma^\mu \gamma^\nu P_L)_{ij}, \ldots\} \otimes \{(P_L)_{kl}, (\gamma_\mu P_L)_{kl}, (\gamma_\mu \gamma_\nu P_L)_{kl}, \ldots\}$$

  - Both methods lead to the same results!

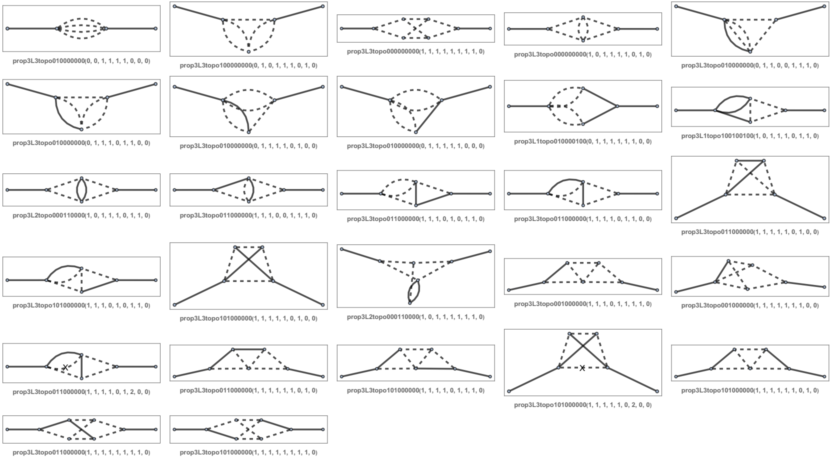
Master integrals encountered up to 2-loops are all well known [Fleischer & Kalmykov, 2000; V. A. Smirnov, 2006]



- solid black line - m_b
- dashed black line - massless
- dashed red line - m_g
- dashed blue line - m_c

3-loop master integrals (modulo factorizing ones) are more interesting

Out[∘]=

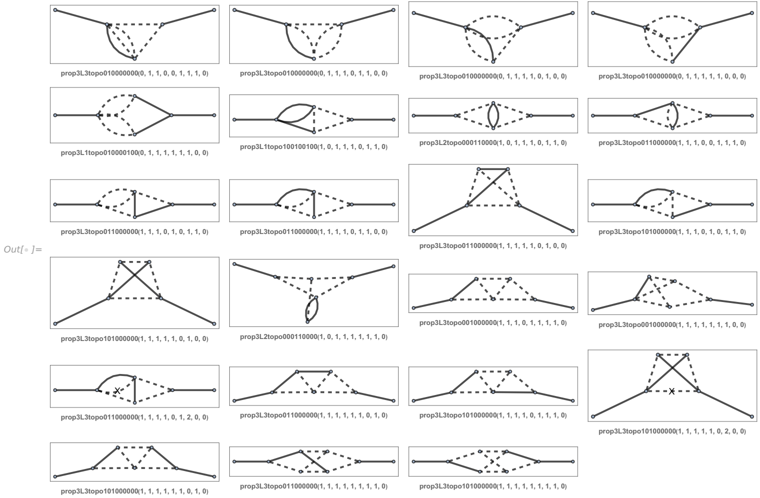


- 27 integrals, first 4 contain only massless (dashed) internal lines ⇒ **Mincer** [Gorishnii et al., 1989; Larin et al., 1991]
- of the 23 remaining integrals only few available in the literature [Asatrian et al., 2017]

- All 23 integrals with massive (solid) lines can be calculated analytically
- We can directly integrate the Feynman parameter integrals using **HyperInt** [Panzer, 2015]
- In most cases complicated intermediate expressions with GPLs containing 6^{th} root of unity [Kalmykov & Kniehl, 2010; Ablinger et al., 2011; Bloch & Vanhove, 2015; Bloch et al., 2015; Henn et al., 2017; Panzer & Schnetz, 2017]

$$\frac{1 \pm i\sqrt{3}}{2}, \frac{-1 \pm i\sqrt{3}}{2}, e^{i\pi/3}$$

- Use **HyperlogProceedings** [Schnetz] and **PolyLogTools** [Duhr & Dulat, 2019] to simplify the results

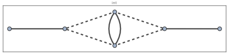


- The imaginary parts of the integrals turn out to be very simple
- Appearing constants

$$\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Cl}_2(\pi/3), \sqrt{3},$$
$$\mathrm{Li}_4(1/2), \ln\left((1+\sqrt{5})/2\right)$$

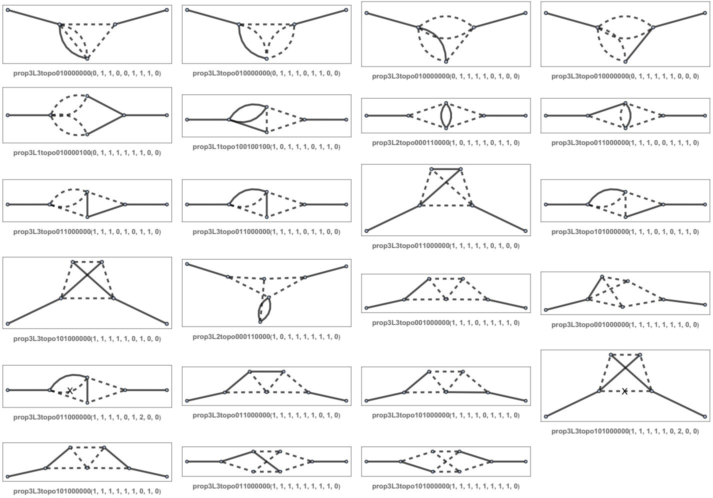
$$\mathrm{Cl}_2(x) = \frac{i}{2}\left(\mathrm{Li}_2(e^{-ix}) - \mathrm{Li}_2(e^{ix})\right)$$

- The golden ratio appears in

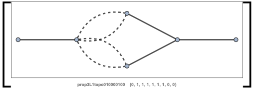


- analytic result already available in [Asatrian et al., 2017]

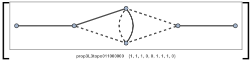
- Real parts more complicated but irrelevant for $\Delta\Gamma_s$



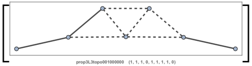
Results for selected master integrals (only imaginary parts are shown)

$\frac{1}{\pi}\mathrm{Im}$ []

$$= -2\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) + \zeta_2 + 2$$

$\frac{1}{\pi}\mathrm{Im}$ []

$$= \frac{1}{2\varepsilon^2} + \frac{7}{2\varepsilon} + \frac{33}{2} - \frac{5\zeta_2}{4} - 2\zeta_3 + \varepsilon\left(\frac{131}{2} - \frac{35\zeta_2}{4} - \frac{21\zeta_3}{2} - \frac{7\zeta_4}{2}\right)$$

$\frac{1}{\pi}\mathrm{Im}$ []

$$= \frac{\zeta_2}{\varepsilon} + 2\zeta_2 + 3\zeta_2\ln(2) + \frac{39\zeta_3}{4}$$

$$+ \varepsilon\left(4\zeta_2 + \frac{39\zeta_3}{2} + 6\zeta_2\ln(2) - 40\mathrm{Li}_4\left(1/2\right) + \frac{629\zeta_4}{8} + 10\zeta_2\ln^2(2) - \frac{5\ln^4(2)}{3}\right)$$

A glimpse of the new matching coefficients

$n_c = 3$, OS scheme; N_H,N_V,N_L are flags to mark the contributions of b, c and the light quarks

- A contribution to the matching coefficient from the 2-loop $O_{1,2} \times O_{3-6}$ correlator

$$p_{24}^{cc,(1)}(z) = -\frac{1}{9}\ln\left(\frac{\mu_1^2}{m_b^2}\right)$$
$$+\left(\frac{10}{9} - \frac{5N_H}{9} - \frac{20N_L}{9} - \frac{10N_V}{9} + 26z\right)\ln\left(\frac{\mu_2^2}{m_b^2}\right)$$
$$+\left(\frac{1729}{18} - \frac{20N_L}{3} - \frac{20N_V}{3} - \frac{10\pi^2}{3}\right)z$$
$$+\frac{137}{27} - \frac{70N_L}{27} - \frac{35N_V}{27} - \frac{5\pi}{18\sqrt{3}} - \frac{5\pi^2}{3}$$
$$+N_H\left(-\frac{85}{27} + \frac{5\pi}{3\sqrt{3}}\right),$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\Big[H^{ab}(z)\langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{ab}(z)\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle\Big] + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}/m_b\right)$$

$$H^{ab}(z) = H^{(c)ab}(z) + H^{(cp)ab}(z) + H^{(p)ab}(z)$$
$$\tilde{H}_S^{ab}(z) = \tilde{H}_S^{(c)ab}(z) + \tilde{H}_S^{(cp)ab}(z) + \tilde{H}_S^{(p)ab}(z)$$

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$$H^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j\, p_{ij}^{ab}(z),$$
$$\tilde{H}_S^{(c)ab}(z) = \sum_{i,j=1}^{2} C_i C_j\, p_{ij}^{S,ab}(z),$$
$$H^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i\left[C_1 p_{1i}^{ab}(z) + C_2 p_{2i}^{ab}(z)\right],$$
$$\tilde{H}_S^{(cp)ab}(z) = \sum_{i=3,\ldots 6,8} C_i\left[C_1 p_{1i}^{S,ab}(z) + C_2 p_{2i}^{S,ab}(z)\right],$$
$$H^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j\, p_{ij}^{ab}(z)$$
$$\tilde{H}_S^{(p)ab}(z) = \sum_{i,j=3,\ldots 6,8} C_i C_j\, p_{ij}^{S,ab}(z)$$

A glimpse of the new matching coefficients

$n_c = 3$, OS scheme; N_H,N_V,N_L are flags to mark the contributions of b, c and the light quarks

- Contribution to the matching coefficient from the 2-loop $O_{1,2} \times O_8$ correlator

$$p_{18}^{cc,(1)}(z) = -\frac{1}{27} \ln\left(\frac{\mu_1^2}{m_b^2}\right) + \left(\frac{343}{81} - \frac{5N_H}{27} - \frac{20N_L}{27} - \frac{10N_V}{27}\right) \ln\left(\frac{\mu_2^2}{m_b^2}\right) + \left(\frac{2915}{54} - \frac{20N_L}{9} - \frac{20N_V}{9} - \frac{10\pi^2}{9}\right) z + \left(\frac{685}{243} - \frac{70N_L}{81} - \frac{35N_V}{81} - \frac{5\pi}{54\sqrt{3}} - \frac{5\pi^2}{9} + N_H\left(-\frac{85}{81} + \frac{5\pi}{9\sqrt{3}}\right)\right)$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \Big[H^{ab}(z)\langle B_s|Q|\bar{B}_s\rangle + \tilde{H}_S^{ab}(z)\langle B_s|\tilde{Q}_S|\bar{B}_s\rangle \Big] + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}/m_b\right)$$

$$H^{ab}(z) = H^{(c)ab}(z) + H^{(cp)ab}(z) + H^{(p)ab}(z)$$
$$\tilde{H}_S^{ab}(z) = \tilde{H}_S^{(c)ab}(z) + \tilde{H}_S^{(cp)ab}(z) + \tilde{H}_S^{(p)ab}(z)$$

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$$H^{(c)ab}(z) = \textstyle\sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{ab}(z),$$
$$\tilde{H}_S^{(c)ab}(z) = \textstyle\sum_{i,j=1}^{2} C_i C_j \, p_{ij}^{S,ab}(z),$$
$$H^{(cp)ab}(z) = \textstyle\sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{ab}(z) + C_2 p_{2i}^{ab}(z)\right],$$
$$\tilde{H}_S^{(cp)ab}(z) = \textstyle\sum_{i=3,\ldots 6,8} C_i \left[C_1 p_{1i}^{S,ab}(z) + C_2 p_{2i}^{S,ab}(z)\right],$$
$$H^{(p)ab}(z) = \textstyle\sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{ab}(z)$$
$$\tilde{H}_S^{(p)ab}(z) = \textstyle\sum_{i,j=3,\ldots 6,8} C_i C_j \, p_{ij}^{S,ab}(z)$$

- All the 2-loop contributions to the NNLO correction already computed and cross-checked
- The 3-loop contribution is already computed, currently being checked
- **Preliminary** numerical estimate on the impact of the new 2-loop $O_{1,2} \times O_{3-6}$ contribution

$$\text{1-loop (already known):} \quad \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s^0}}{\Delta\Gamma_s} = 7.0\% \quad (\text{pole}) \qquad \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s^0}}{\Delta\Gamma_s} = 6.1\% \quad (\overline{\text{MS}}),$$

$$\text{full 2-loops (new):} \quad \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s}}{\Delta\Gamma_s} = 0.2\% \quad (\text{pole}), \qquad \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s}}{\Delta\Gamma_s} = 1.4\% \quad (\overline{\text{MS}}),$$

- OS scheme: m_c^{OS}, m_b^{OS}
- $\overline{\text{MS}}$ scheme: $m_c^{\text{MS}}(m_b)$, $m_b^{\text{MS}}(m_b)$
- Numerical input [Tanabashi et al., 2018; Dowdall et al., 2019; Bazavov et al., 2018]

$$M_{B_s} = 5366.88 \text{ MeV} \quad f_{B_s} = (0.2307 \pm 0.0013) \text{ GeV},$$

$$B_{B_s} = 0.813 \pm 0.034, \quad \widetilde{B}'_{S,B_s} = 1.31 \pm 0.09,$$

$$\frac{\lambda_u^d}{\lambda_t^d} = (0.0122 \pm 0.0097) - (0.4203 \pm 0.0090)i,$$

$$\frac{\lambda_u^s}{\lambda_t^s} = -(0.00865 \pm 0.00042) + (0.01832 \pm 0.00039)i.$$

Summary

- We calculated all building blocks needed to obtain the full NNLO correction to $B_s^0 - \bar{B}_s^0$ mixing
- All the occurring 3-loop master integrals (for $m_c = 0$) can be calculated analytically

Outlook

- These results should be published soon $\Rightarrow$ new theory prediction for $\Delta\Gamma_s$
- Higher order expansions in $z \equiv m_c^2/m_b^2$, ideally z-exact results at least for 2-loop contributions
- NNLO corrections to the $B_d^0 - \bar{B}_d^0$ meson mixing as the next step