University of Zurich[UZH]

ZH production in gluon fusion:

NLO virtual corrections with full top quark mass dependence

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in collaboration with
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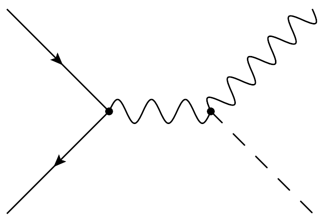
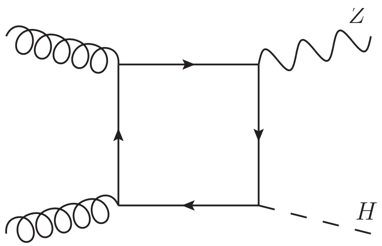
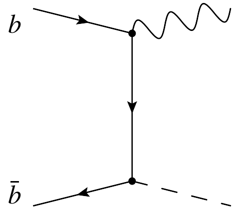
Introduction – ZH Production Modes

ZH production modes



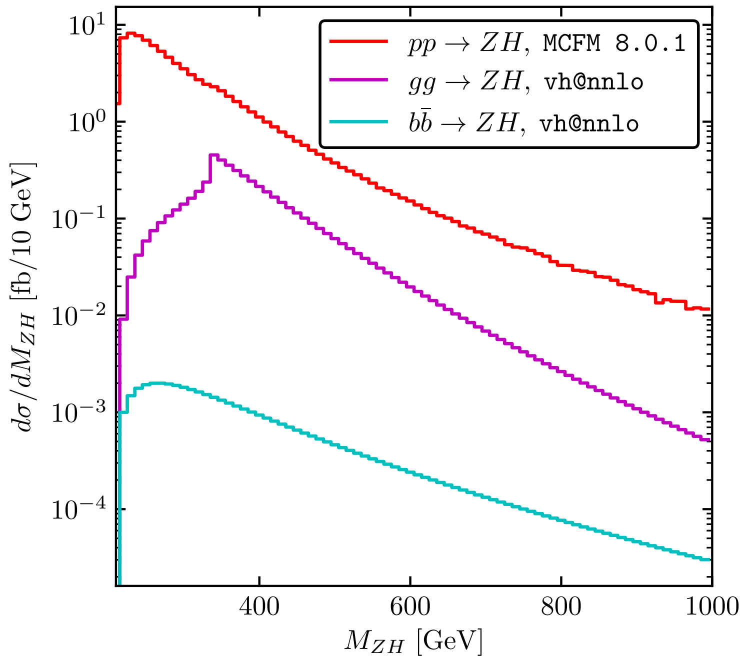
NNLO:
Brein, Djouadi, Harlander 03





NNLO
Ahmed, Ajjath, Chen, Dhani, Mukherjee, Ravindran 19

[Harlander, Klappert, Liebler, Simon 18]



Gluon-induced production:

- Contribution to to total cross section ~10%
- Large scale uncertainties

quark-initiated production known with high accuracy:

NNLO: Brein, Harlander, Wiesemann, Zirke; Ferrera, Grazzini, Somogyi, Tramontano; Campbell, Ellis, Williams; Gauld, Gehrmann-De Rideer, Glover, Huss, Majer

NLO EW(+QCD): Ciccolini, Denner, Dittmaier, Kallweit, Krämer, Mück; Granata, Lindert, Oleari, Pozzorini; Obul, Dulat, Hou, Tursun, Yulkun

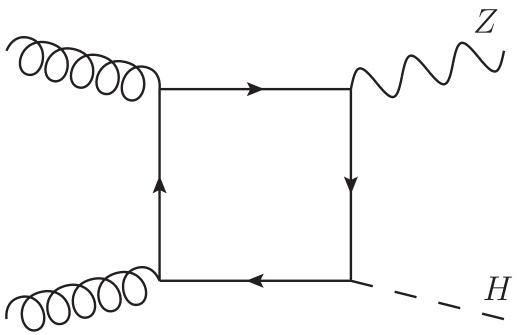
+ parton shower, resummation, ...

Uncertainties in ZH, WH measurements ATLAS 2007.02873

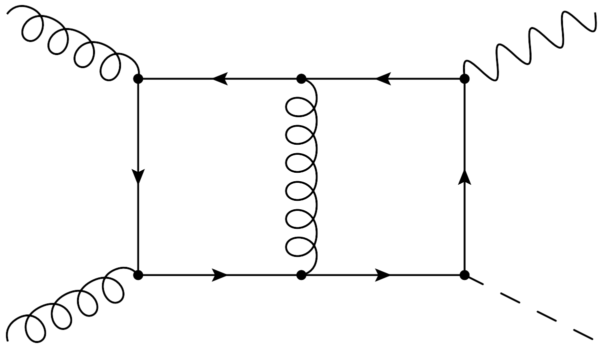
	Signal
Cross-section (scale)	0.7% (qq), 25% (gg)
$H \to b\bar{b}$ branching fraction	1.7%
Scale variations in STXS bins	3.0%–3.9% ($qq \to WH$), 6.7%–12% ($qq \to ZH$), 37%–100% ($gg \to ZH$)
PS/UE variations in STXS bins	1%–5% for $qq \to VH$, 5%–20% for $gg \to ZH$
PDF+α_S variations in STXS bins	1.8%–2.2% ($qq \to WH$), 1.4%–1.7% ($qq \to ZH$), 2.9%–3.3% ($gg \to ZH$)
m_{bb} from scale variations	M+S ($qq \to VH$, $gg \to ZH$)
m_{bb} from PS/UE variations	M+S
m_{bb} from PDF+α_S variations	M+S
p_T^V from NLO EW correction	M+S

Introduction – gg→ZH: LO and approximated NLO

Leading Order
[Dicus, Kao 88; Kniehl 90]



NLO in $m_t \to \infty$ limit
[Altenkamp, Dittmaier, Harlander, H. Rzehak, Zirke 12]



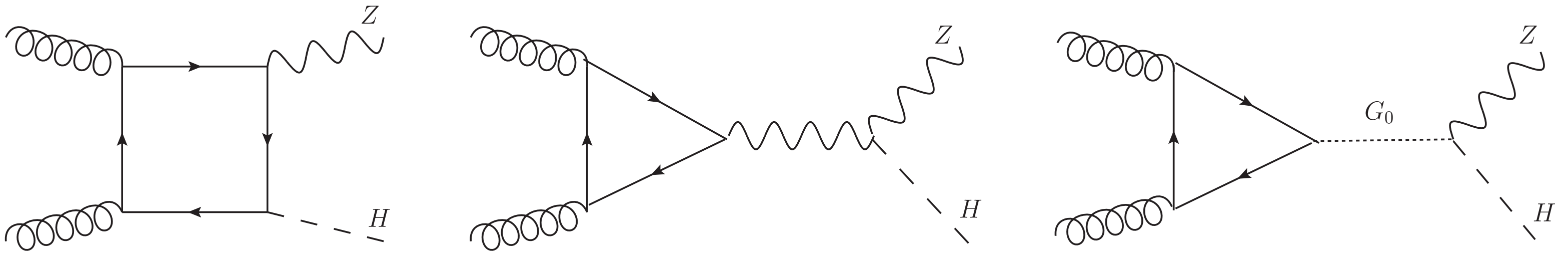
Improved virtual corrections via expansions:

- in large m_t, up to $1/m_t^8$, improved by Padé approx.
  [Hasselhuhn, Luthe, Steinhauser 17]
- in small and large m_t, up to $1/m_t^{10}, m_t^{32}$ + Padé approx.
  [Davies, Mishima, Steinhauser 20]
- in small p_T up to p_T^4
  [Alasfar, Degrassi, Giardino, Gröber, Vitti 21]

→ Talk by Go Mishima

Introduction – ZH in Gluon Fusion

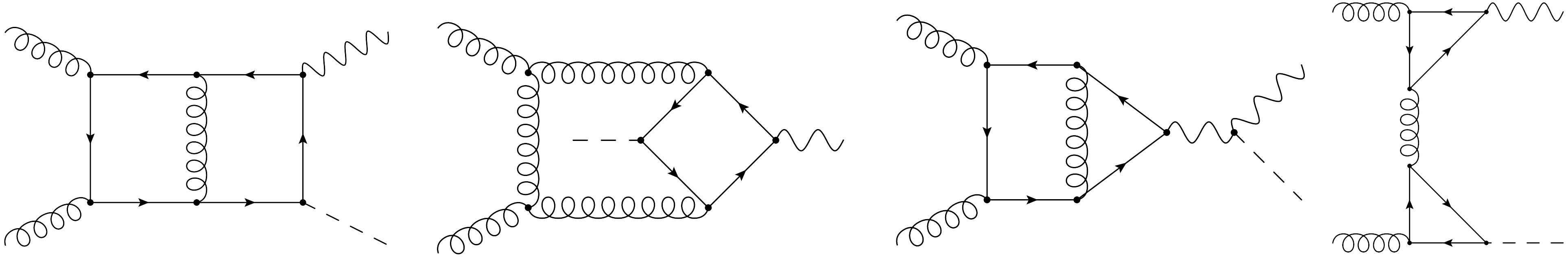
LO



only top-quark contributing

triangle contributions vanish for mass-degenerate quark doublets
→ contributions by b- and t-quark

NLO virtual



We calculated the full 2-loop amplitude

- using numerical evaluation of loop integrals
- treating bottom quark as massless
- with full dependence on m_t
- using Larin's prescription of γ_5

Computation Strategy

Virtual Amplitude

1. Apply Feynman rules, using linear polarizations, map loop integrals to integral families
2. Reduce 2-loop integrals to minimal set of masters
3. Integrate master integrals numerically

Tools used:

QGraf [Nogueira], Reduze [von Manteuffel, Studerus]
GoSam-XLoop [Heinrich, Jahn, Jones, MK, Schlenk et.al.]

Kira [Klappert, Lange, Maierhöfer, Usovitsch]
Firefly [Klappert, Lange]

pySecDec, qmc
[Borowka, Heinrich, Jahn, Jones, MK, Schlenk]

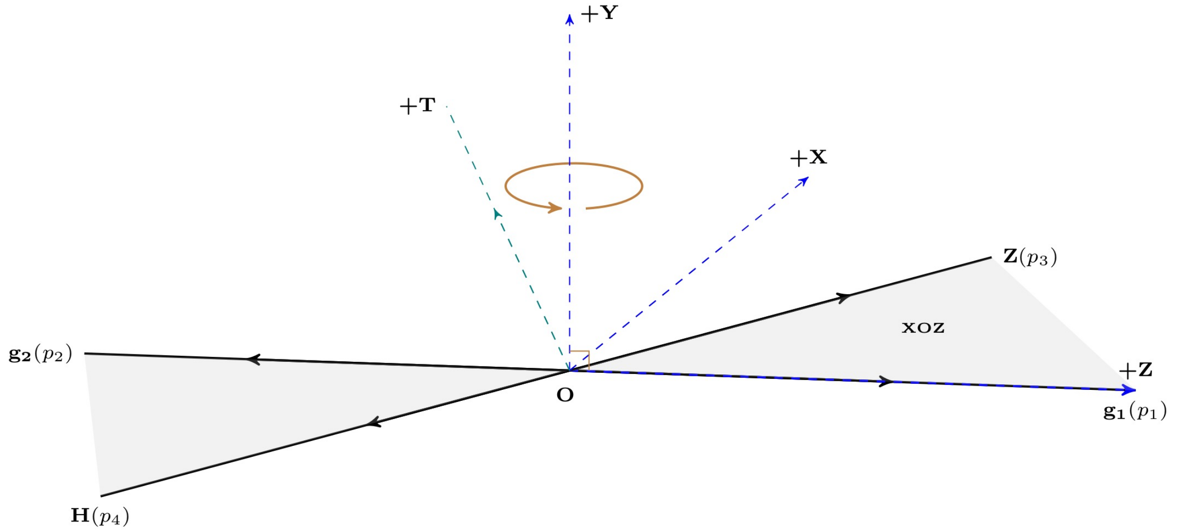
Full NLO results
(work in progress)

- Phase-space sampling of virtual amplitude based on unweighted events (LO)
- Use high-energy expansion for large p_T [Davies, Mishima, Steinhauser 19]
- Add real radiation using matrix elements generated by GoSam [Cullen et.al.]

Polarized Amplitudes

$$\mathcal{A} = \varepsilon^{\mu_1}_{\lambda_1}(p_1)\, \varepsilon^{\mu_2}_{\lambda_2}(p_2)\, (\varepsilon^{\mu_3}_{\lambda_3}(p_3))^\star\, \mathcal{A}_{\mu_1\mu_2\mu_3}$$

$$g(p_1) + g(p_2) \to Z(p_3) + H(p_4)$$



Polarization vectors can be constructed from external momenta [L. Chen 19]

choose

$$\begin{aligned}
\varepsilon^\mu_x &= \mathcal{N}_x\,\left(-s_{23}p_1^\mu - s_{13}p_2^\mu + s_{12}p_3^\mu\right)\,, \\
\varepsilon^\mu_y &= \mathcal{N}_y\,\left(\epsilon^\mu_{\mu_1\,\mu_2\,\mu_3}\, p_1^{\mu_1}\, p_2^{\mu_2}\, p_3^{\mu_3}\right), \\
\varepsilon^\mu_T &= \mathcal{N}_T\,\Big(\left(-s_{23}(s_{13}+s_{23}) + 2m_Z^2 s_{12}\right) p_1^\mu + \left(s_{13}(s_{13}+s_{23}) - 2m_Z^2 s_{12}\right) p_2^\mu \\
&\qquad + s_{12}(-s_{13}+s_{23})\, p_3^\mu\Big)\,, \\
\varepsilon^\mu_l &= \mathcal{N}_l\,\left(-2m_Z^2\,(p_1^\mu + p_2^\mu) + (s_{13}+s_{23})\, p_3^\mu\right)\,,
\end{aligned}$$

such that

$$\{\varepsilon_x,\, \varepsilon_y\}\cdot\{p_1,\, p_2\} = 0, \qquad \{\varepsilon_y,\, \varepsilon_T,\, \varepsilon_l\}\cdot p_3 = 0, \qquad \varepsilon_i^2 = -1$$

Can be used as polarization vectors of gluons and Z, respectively

circular polarizations:

$$\varepsilon^{\mu_1}_\pm(p_1) = \frac{1}{\sqrt{2}}\left(\varepsilon^{\mu_1}_x \pm i\varepsilon^{\mu_1}_y\right) \quad \varepsilon^{\mu_2}_\pm(p_2) = \frac{1}{\sqrt{2}}\left(\varepsilon^{\mu_2}_x \mp i\varepsilon^{\mu_2}_y\right) \quad \varepsilon^{\mu_3}_\pm(p_3) = \frac{1}{\sqrt{2}}\left(\varepsilon^{\mu_3}_T \pm i\varepsilon^{\mu_3}_y\right)$$

6 polarization configurations:

$$\begin{aligned}
\mathcal{P}_1^{\mu_1\mu_2\mu_3} &= \varepsilon_x^{\mu_1}\,\varepsilon_x^{\mu_2}\,\varepsilon_y^{\mu_3}\,, & \mathcal{P}_2^{\mu_1\mu_2\mu_3} &= \varepsilon_x^{\mu_1}\,\varepsilon_y^{\mu_2}\,\varepsilon_T^{\mu_3} \\
\mathcal{P}_3^{\mu_1\mu_2\mu_3} &= \varepsilon_x^{\mu_1}\,\varepsilon_y^{\mu_2}\,\varepsilon_l^{\mu_3}\,, & \mathcal{P}_4^{\mu_1\mu_2\mu_3} &= \varepsilon_y^{\mu_1}\,\varepsilon_x^{\mu_2}\,\varepsilon_T^{\mu_3} \\
\mathcal{P}_5^{\mu_1\mu_2\mu_3} &= \varepsilon_y^{\mu_1}\,\varepsilon_x^{\mu_2}\,\varepsilon_l^{\mu_3}\,, & \mathcal{P}_6^{\mu_1\mu_2\mu_3} &= \varepsilon_y^{\mu_1}\,\varepsilon_y^{\mu_2}\,\varepsilon_y^{\mu_3}
\end{aligned}$$

Integral Reduction

Use Integration-by-Parts Identities [Chetyrkin, Tkachov; Laporta]
to express appearing 2-loop integrals in terms of master integrals.

$$\int \mathrm{d}^d p_i \frac{\partial}{\partial p_i^\mu} \left[q^\mu \, \mathbf{I}'(p_1, \ldots, p_l; k_1, \ldots, k_m) \right] = 0$$

~13.000 unreduced integrals → 452 masters

Reduction is quite challenging, can be simplified by fixing mass ratios $\frac{m_Z^2}{m_t^2} = \frac{23}{83}, \qquad \frac{m_H^2}{m_t^2} = \frac{12}{23}$

→ Eliminates 2 of the 5 mass scales s, t, m_t, m_Z, m_H

Obtained using the programs:

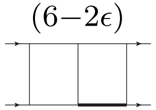
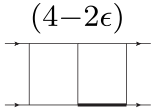
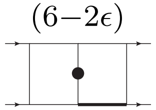
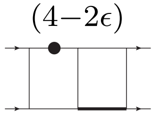
- Kira [Klappert, Lange, Maierhöfer, Usovitsch] → Talk by Johann Usovitsch
- Firefly [Klappert, Klein, Lange]

→ uses finite-field methods to avoid large intermediate expressions

Choice of Master Integrals

- Use a (quasi-)finite basis of master integrals [von Manteuffel, Panzer, Schabinger 14]
  - Simplifies numerical evaluation of integrals
  - Poles in ε only in coefficients
  - Requires integrals in shifted dimensions [Bern, Dixon, Kosower 92; Tarasov 96; Lee 10]

finite integrals

				run time	rel. error
 (s,t)	280 s	1.00×10^{-3}	 (s,t)	214135 s	8.29×10^{-3}
 (s,t)	294 s	1.21×10^{-3}	 (s,t)	3484378 s	30.9

[von Manteuffel, Schabinger 17]

Choice of Master Integrals

- Use a (quasi-)finite basis of master integrals [von Manteuffel, Panzer, Schabinger 14]
- Further improvements of integral basis to achieve:
  (by trying different basis choices for each sector)
  - d-dependence factorizes from kinematic dependence in denominators of reduction coefficients [Smirnov, Smirnov `20; Usovitsch `20]

$$\frac{N(s,t,d)}{D_1(d)D_2(s,t)}$$

  - simple denominator factors D_1, D_2
  - avoid poles in coefficients of integrals in top-level sectors as far as possible
  - small file-size of reductions

→ Some spurious poles & cancellations between integrals can be avoided
→ Reduced File sizes of expressions
  - Amplitude: factor of 5 improvement
  - Largest coefficient (double-tadpole): 150 MB → 5 MB

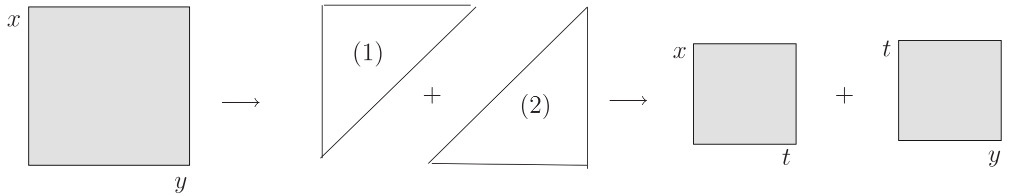
Loop Integrals – Sector Decomposition

Numerical evaluation of loop integrals with pySecDec

[Borowka, Heinrich, Jahn, Jones, MK, Langer, Magerya, Põldaru, Schlenk, Villa]

Available at
github.com/gudrunhe/secdec

- Sector decomposition [Binoth, Heinrich 00]
  factorizes overlapping singularities



- Subtraction of poles & expansion in ε
- Contour deformation [Soper 00; Binoth et.al. 05, Nagy, Soper 06; Borowka et al. 12]

→ Finite integrals at each order in ε

→ Numerical integration possible

New release (coming soon…)
→ Talk by Emilio Villa

- expansion by regions
- evaluation of linear combinations of integrals, with automated optimization of sampling points per sector
- automated reduction of contour-def. parameter
- automatically adjusts FORM settings

pySecDec integral libraries can be directly linked to amplitude code

Numerical Integration I

Quasi-Monte Carlo integration using rank-1 shifted lattice rule

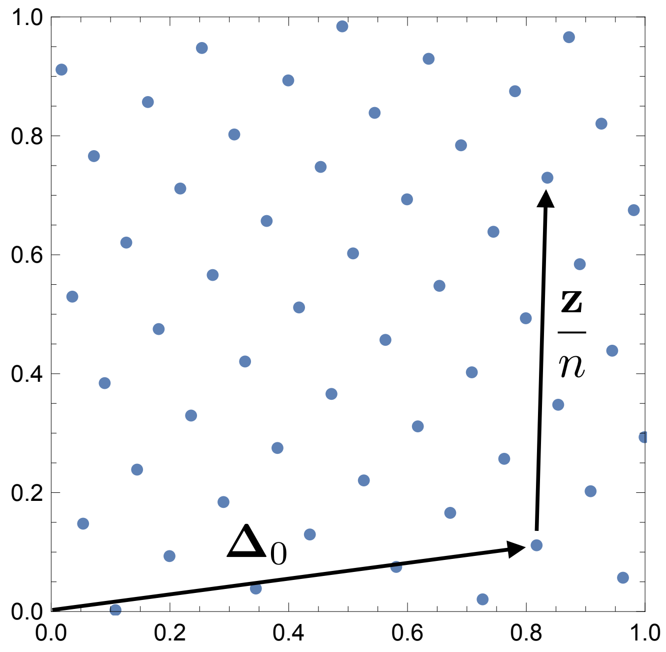
$$I[f] \approx I_k = \frac{1}{N} \cdot \sum_{i=1}^{N} f(\mathbf{x}_{i,k}), \quad \mathbf{x}_{i,k} = \left\{ \frac{i \cdot \mathbf{z}}{N} + \mathbf{\Delta}_k \right\}$$

Review: Dick, Kuo, Sloan 13
First application to loop integrals:
Li, Wang, Yan, Zhao 15

$\{\ldots\} =$ fractional part ($\rightarrow x \in [0;1[$)

$\mathbf{\Delta}_k =$ randomized shifts
- $\rightarrow m$ different estimates of Integral: $I_1, \ldots, I_m$
- $\rightarrow$ error estimate of result

$\mathbf{z} =$ generating vector
constructed component-by-component [Nuyens 07]
minimizing worst-case error ϵ_γ



Implementation with support for CPUs and GPUs:
github.com/mppmu/qmc
[Borowka, Heinrich, Jahn, Jones, MK, Schlenk]

Numerical Integration II

For integrands belonging to a weighted Korobov space of smoothness α: $$||f||^2_\gamma = \sum_{\mathbf{h}\in\mathbb{Z}^d} \frac{\prod_{j\in\mathfrak{u}(\mathbf{h})} |h_j|^{2\alpha}}{\gamma_{\mathfrak{u}(\mathbf{h})}} |\hat{f}(\mathbf{h})|^2$$

Fourier Coefficients

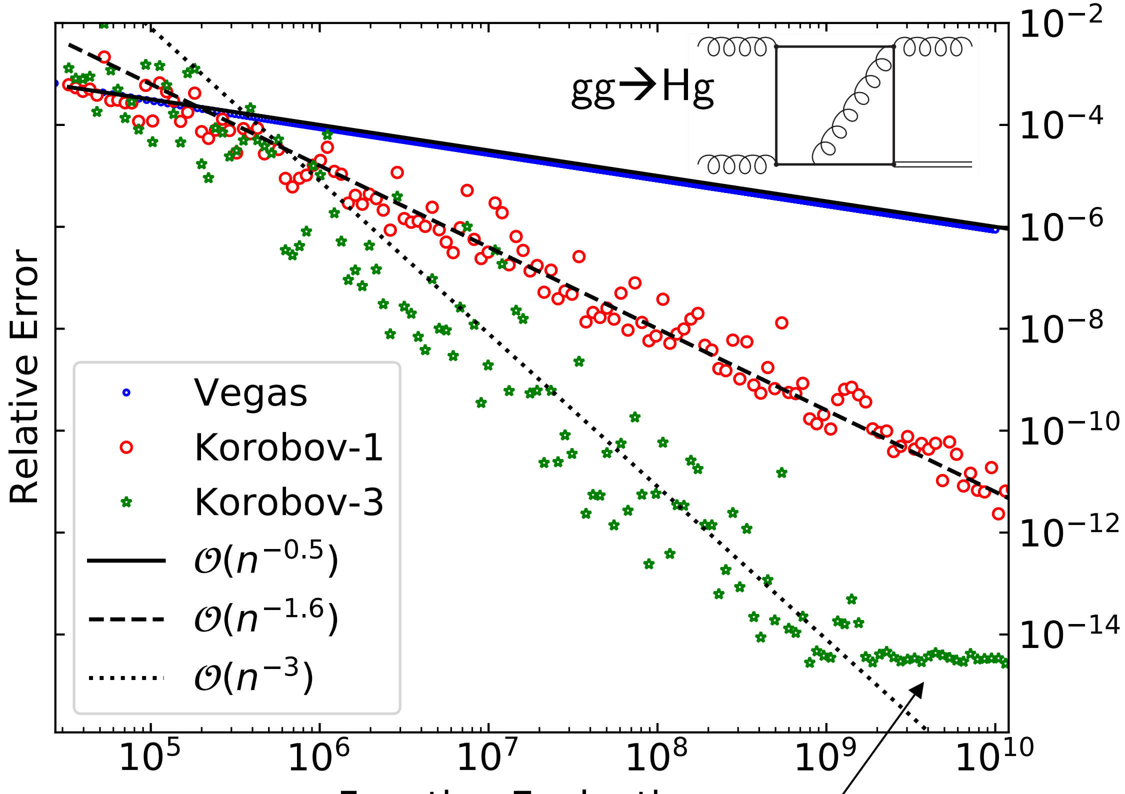
→ worst-case error: $$\epsilon^2_\gamma \leq \left(\frac{1}{\psi(N)} \sum_{\emptyset\neq\mathfrak{u}\subseteq\{1,\ldots,d\}} \gamma^\lambda_\mathfrak{u} \left(2\zeta(2\alpha\lambda)\right)^{|\mathfrak{u}|} \right)^{\frac{1}{\lambda}}$$

$$\rightarrow \varepsilon_\gamma = \mathcal{O}(N^{-\alpha})$$

Can be achieved by periodization of smooth functions, e.g. using Korobov transform

$$I[f] \equiv \int_{[0,1]^d} \mathrm{d}\mathbf{x}\ f(\mathbf{x}) = \int_{[0,1]^d} \mathrm{d}\mathbf{u}\ \omega_d(\mathbf{u}) f(\phi(\mathbf{u})) \qquad \omega(u) = \phi'(u)$$

$$\omega(u) = \frac{u^\alpha(1-u)^\alpha}{\int_0^1 u^\alpha(1-u)^\alpha}$$



Limited by double precision arithmetic

Evaluation of Amplitude

After sector decomposition and expansion in ε → amplitude written in terms of 19.530 finite integrals

Optimizations to reduce run time:

- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral}\, i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

σ_i = error estimate (including coefficients in amplitude)
λ = Lagrange multiplier $\qquad \sigma$ = precision goal

- avoid reevaluation of integrals for different orders in ε and form factors

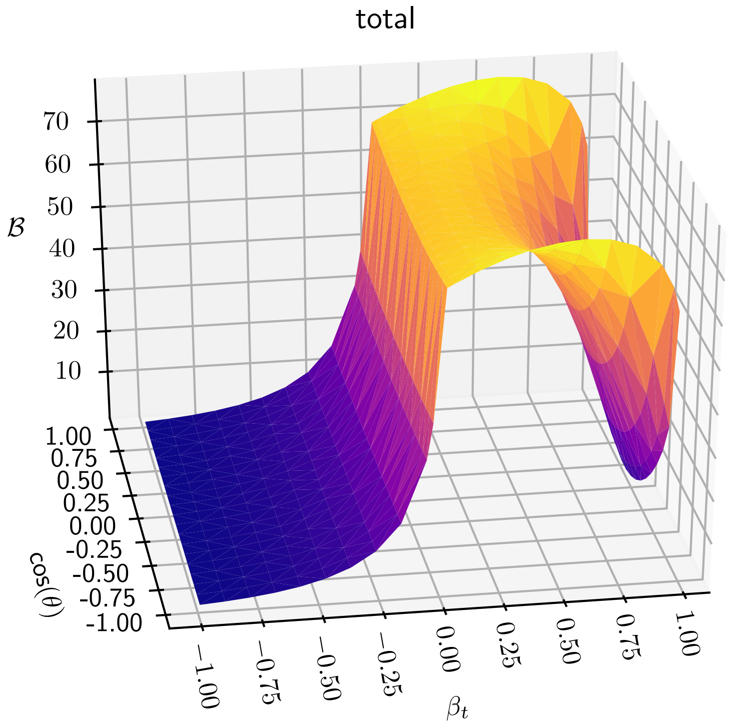
compute once

$$F^a = \sum_i \left[\left(\sum_j C^a_{i,j} \varepsilon^j \right) \cdot \left(\sum_k I_{i,k} \varepsilon^k \right) \right] = \frac{C^a_{1,-2} I_{1,0} + C^a_{1,-1} I_{1,-1} + \dots}{\varepsilon^2} + \frac{C^a_{1,-1} I_{1,0} + \dots}{\varepsilon^1} + \dots$$

- parallelization on GPUs
  typical run-time to obtain virtual amplitude with 0.3% precision:
  2h using 2x Nvidia Tesla V100 GPUs

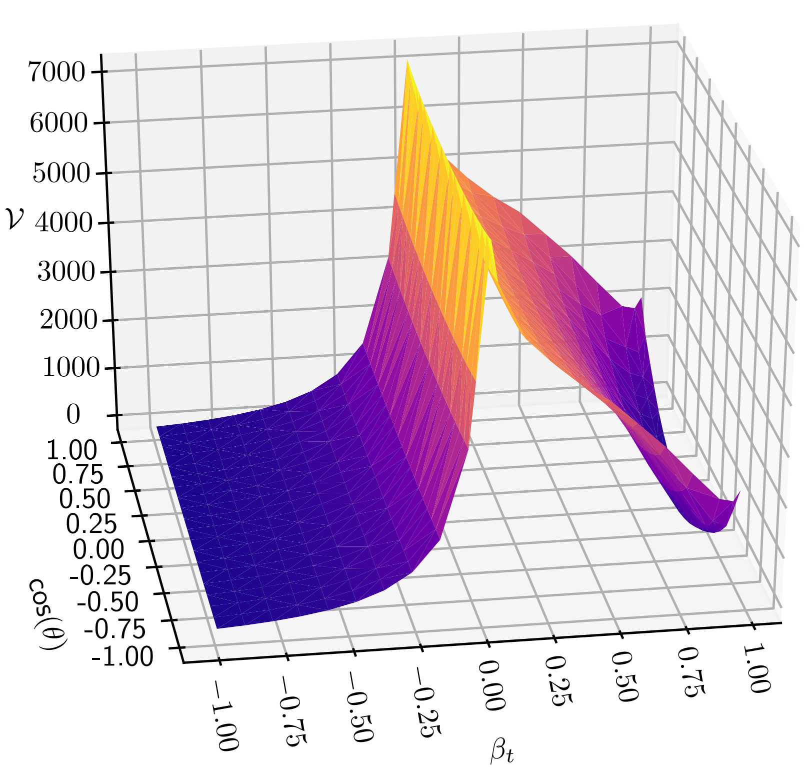
Results – Unpolarized Amplitudes

Born squared amplitude

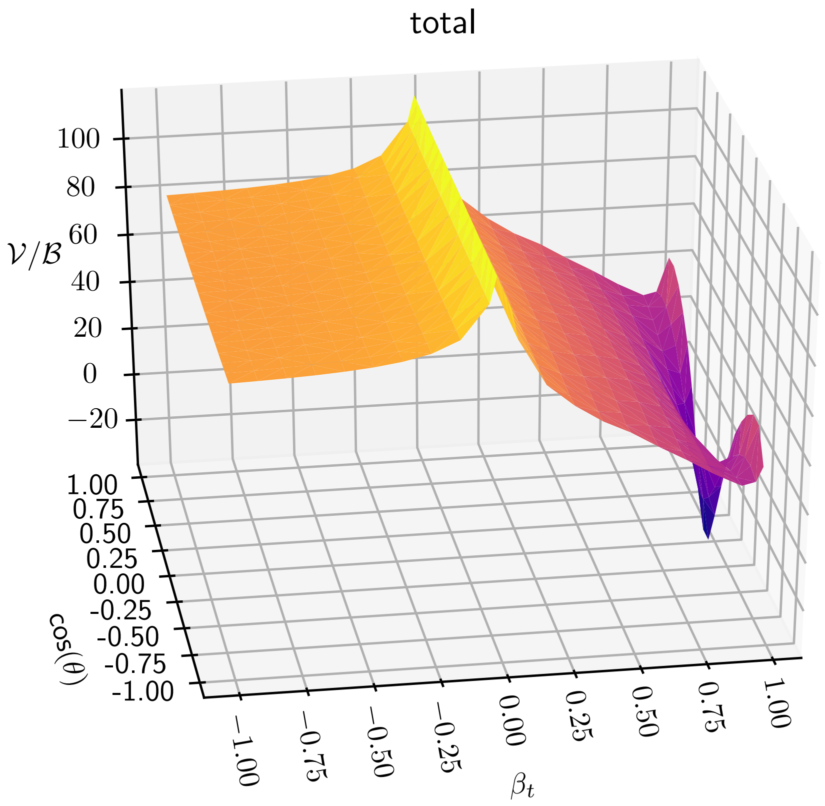


Virtual amplitude

$\mathcal{V} = 2\mathrm{Re}(\mathcal{A}_{1L}^{*}\mathcal{A}_{2L}^{\mathrm{ren.,fin.}})$



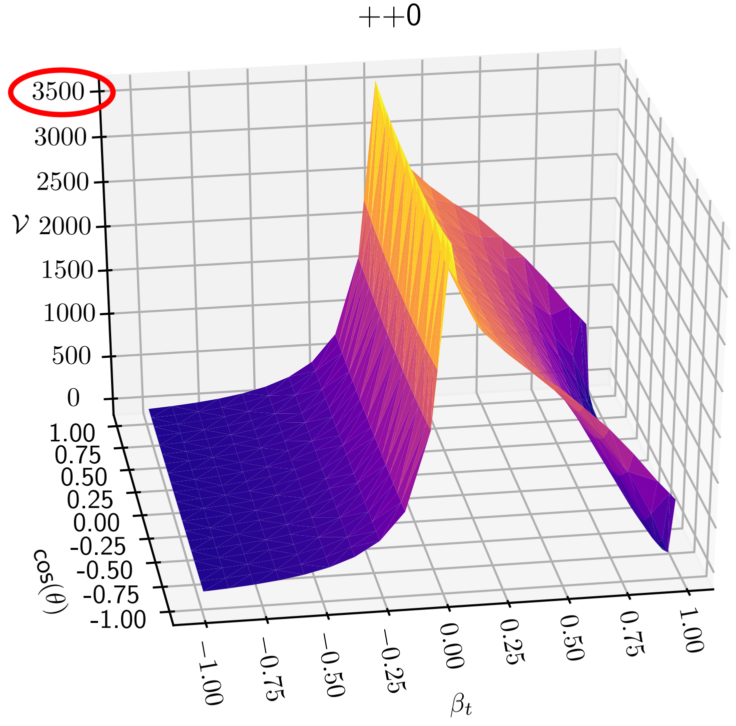
Virtual/Born

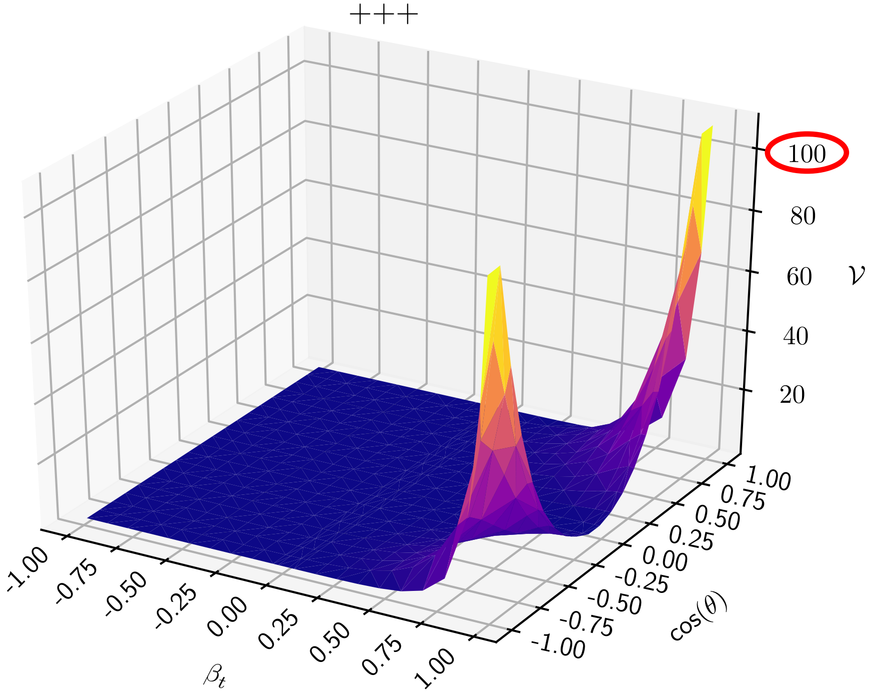


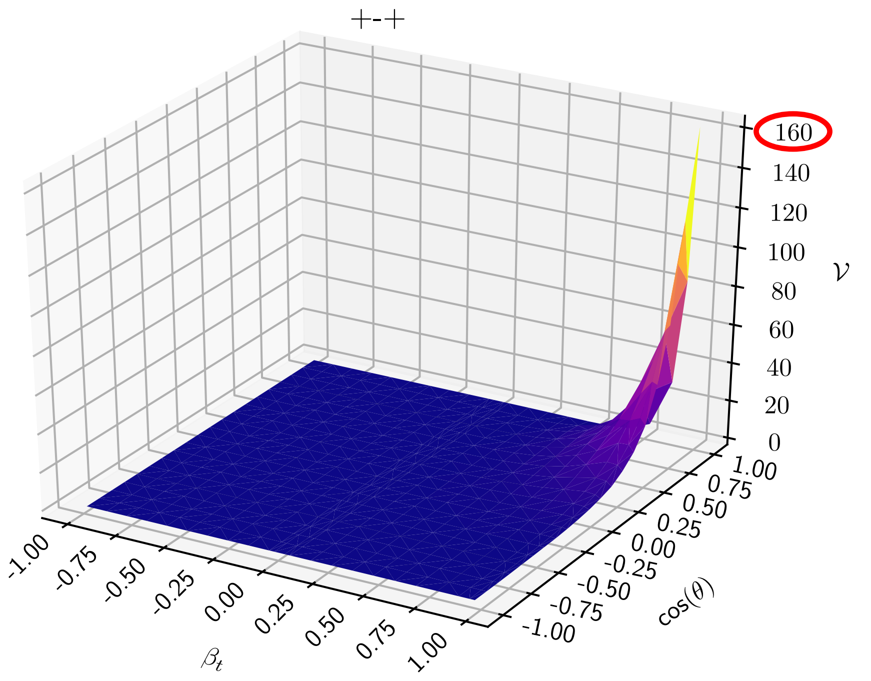
$$\beta_t = \frac{s - 4m_t^2}{s + 4m_t^2 - 2(m_Z + m_H)^2} \qquad \sqrt{s} = \begin{cases} m_Z + m_H \\ 2m_t \\ \infty \end{cases} \rightarrow \beta_t = \begin{cases} -1 \\ 0 \\ 1 \end{cases}$$

- top-quark threshold clearly visible
- strong dependence on $\sqrt{s}$
- $\cos\theta$ dependence important for large $\sqrt{s}$

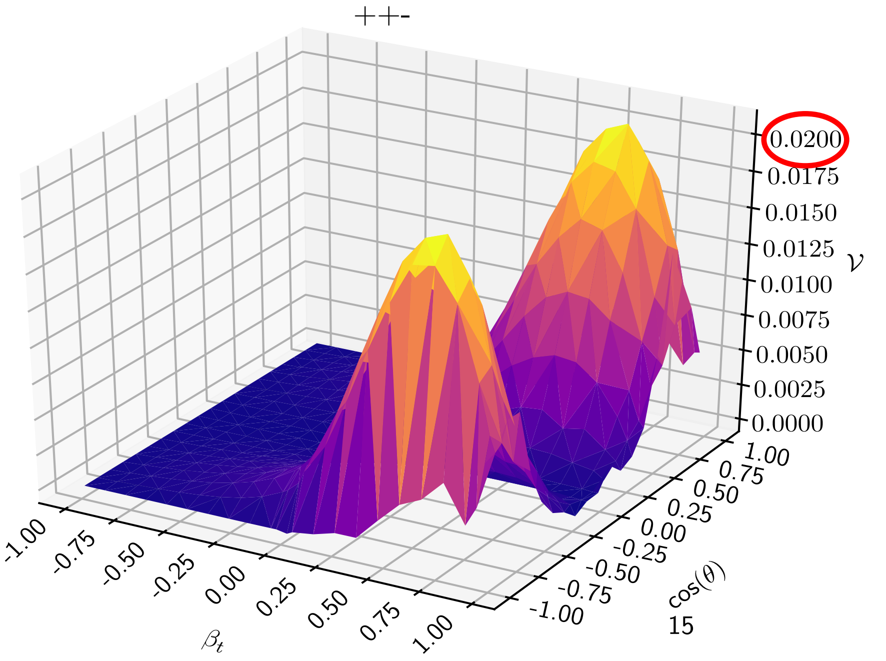
Results – Polarized Virtual Amplitudes





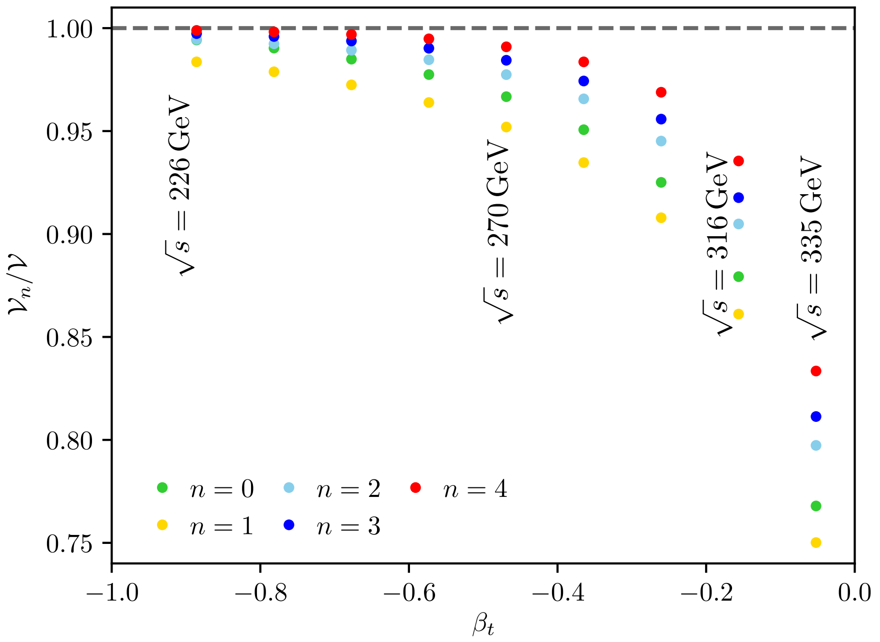


Longitudinal Z polarization is dominating



Comparison to Expansions

large mass expansion

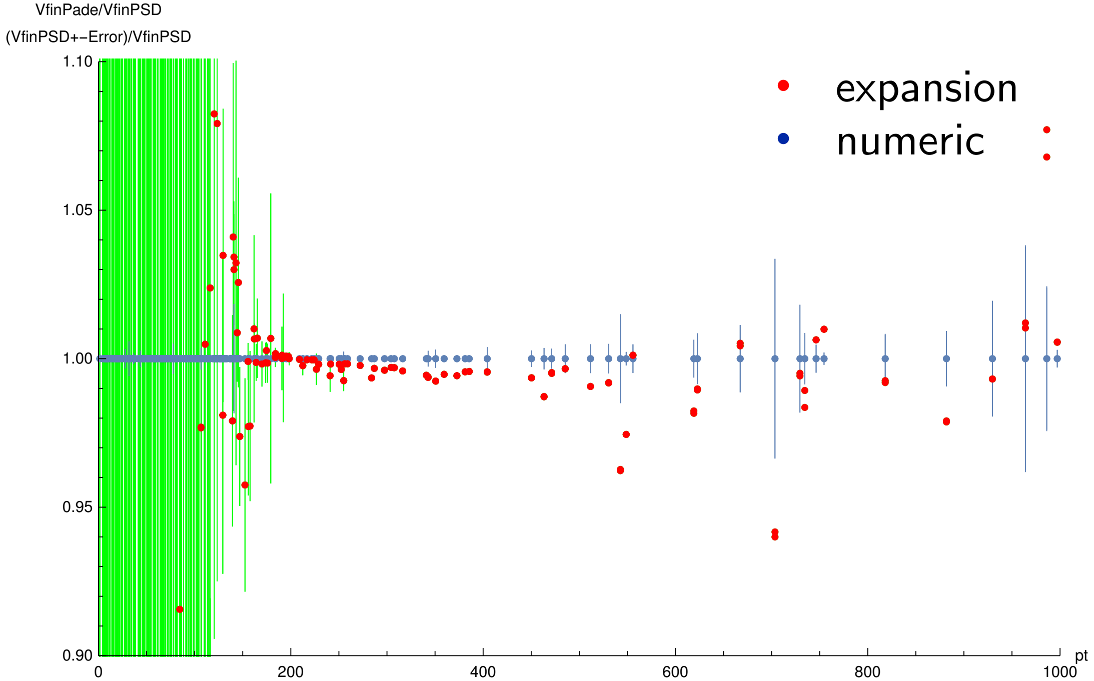


$$\mathcal{V}_n = \frac{\mathcal{B}}{\mathcal{B}_n}\widetilde{\mathcal{V}}_n + \mathcal{V}^{\mathrm{red}}$$

$\mathcal{B}_n, \tilde{\mathcal{V}}_n$: expansions up to $1/m_t^{2n}$
[Hasselhuhn, Luthe, Steinhauser 16]

→ agreement at $\sqrt{s} = 226\,\mathrm{GeV}$: $\sim 0.1\%$

high-energy expansion → talk by Go Mishima
[Davies, Mishima, Steinhauser 20]



Plot by Joshua Davies

for $p_T > 200\,\mathrm{GeV}$:

Agreement typically at 1-2%-level

Conclusion

NLO virtual corrections to ZH production in gluon-fusion

- using numerical integration of loop integrals
- retaining full top-quark mass dependence
- polarized amplitudes obtained without form-factor decomposition
- good agreement with expansions (where expected)

Work in progress: Full NLO results for ZH production in gluon-fusion

- phase-space integration using unweighted LO events
- use m_t expansion in high-energy region
- real-radiation amplitudes generated with GoSam

Thank you for your attention!