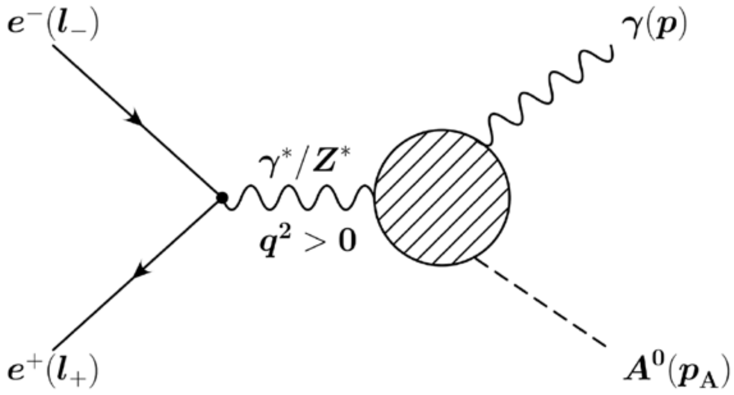
Time-like Transition Form Factor for CP-odd Higgs Boson Production



Tsuneo Uematsu (Kyoto Univ.)

In collaboration with

Ken Sasaki (Yokohama Nat. U.)

RADCOR-LoopFest 2021（FSU)

May 17th-21st 2021

Plan of the talk

1. Introduction and Motivation
2. Space-like and Time-like Transition Form Factor
3. CP-odd Higgs A^0 Production in e^+e^- Collisions
4. Concluding remarks

N.Watanabe, Y. Kurihara, K.Sasaki & T.U. Phys. Lett. B728 (2014) 202

N.Watanabe, Y. Kurihara, K.Sasaki & T.U. Phys. Rev. D90 (2014) 033015

K.Sasaki & T.U. Phys. Lett. B781 (2018) 290 and PoS (RADCOR2019) 021

1. Introduction and Motivation

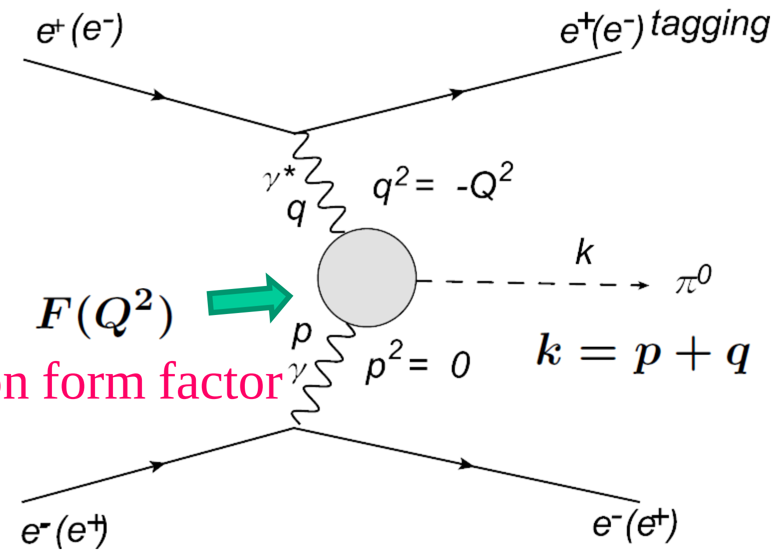
- The Transition Form Factor (TFF) has been studied for pion (π^0) production in e^+e- or $e\gamma$ collisions.
- The coupling of the CP-odd Higgs boson A^0 to fermions is pseudo-scalar, so it is similar to π^0.
- The question is whether the TFF is useful or not, for describing the A^0 production in e^+e- or $e\gamma$ collisions.
- In our previous work, we discussed $e\gamma \to eA^0$ in terms of space-like TFF. Now we investigate the $e^+e^- \to A^0\gamma$ by the time-like TFF which is related to space-like TFF in $e\gamma \to eA^0$ by the analytic continuation.

Pion Transition Form Factor

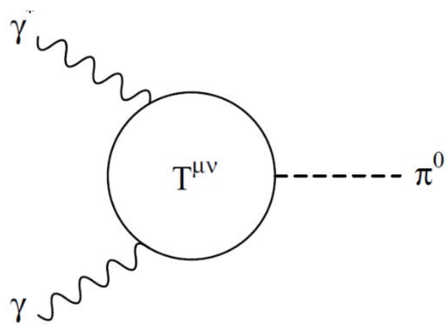
Belle and BaBar data

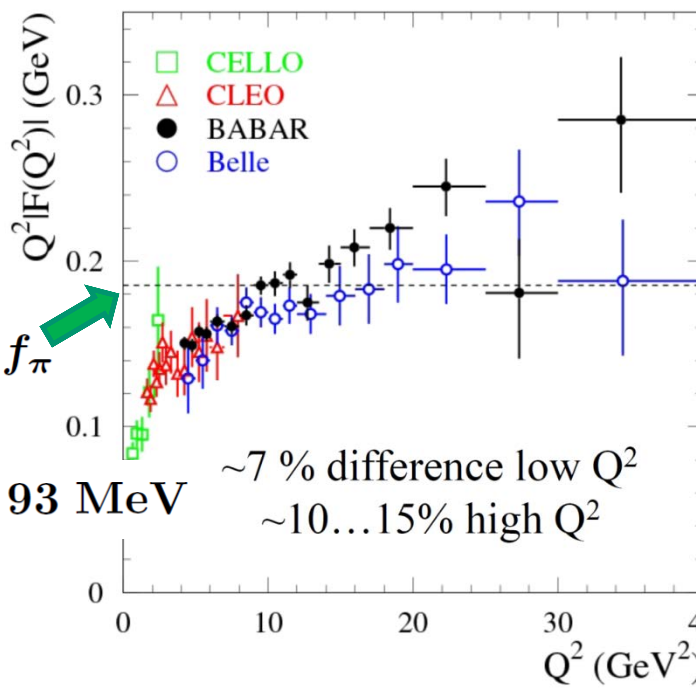
$F(Q^2)$

transition form factor



$\gamma^* \gamma \to \pi^0$ amplitude





$2f_\pi$

$f_\pi \approx 93$ MeV

~7 % difference low Q^2

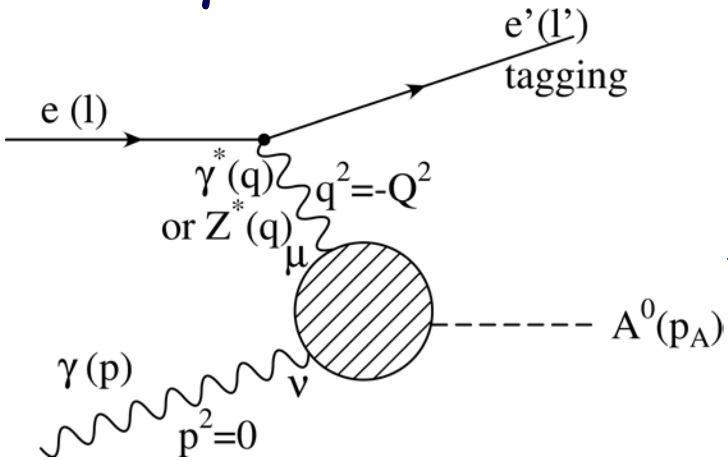
~10…15% high Q^2

$F(Q^2) \sim 2f_\pi / Q^2$ Brodsky et al.

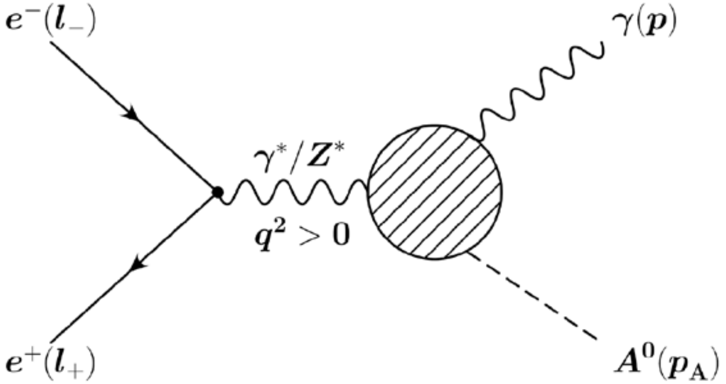
$$\langle \pi^0(k)|T|\gamma(p)\gamma^*(q)\rangle = \epsilon_\mu(p)\epsilon_\nu(q)T^{\mu\nu}(p,q) \qquad T^{\mu\nu} = e^2 F(Q^2)\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta$$

Space-like and Time-like Transition Form Factor

$e\gamma \to eA^0$



$e^+e^- \to \gamma A^0$



Space-like region $q^2<0$

Time-like region $q^2>0$

CP-odd Higgs boson A^0

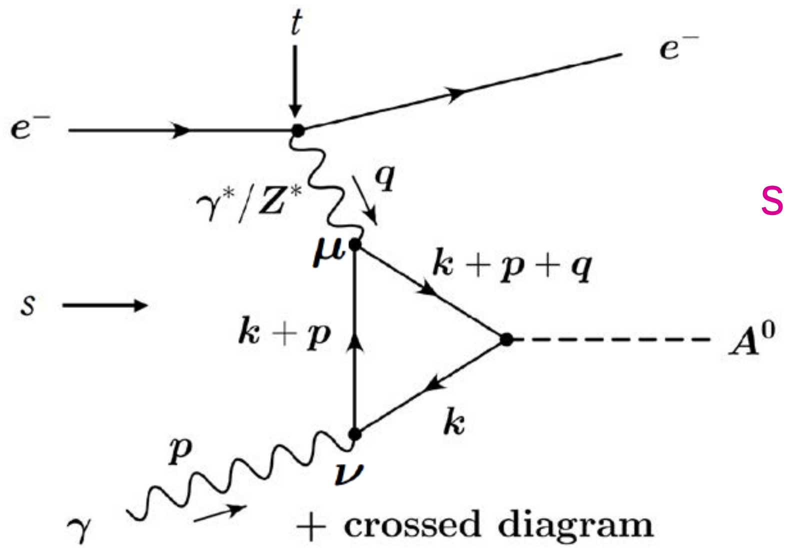
- Minimal extension of Higgs Sector

  Here, we consider 2HDM type II or MSSM

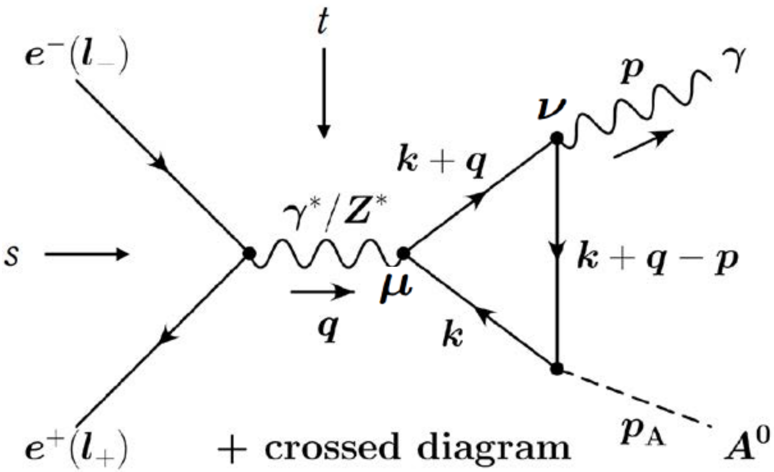
  Charged H^+, H^-; CP-even h^0, H^0; CP-odd A^0
- A^0 does not couple to W^+W^- ZZ pairs at tree level in contrast to CP-even Higgs h^0 or H^0
- A^0 does not couple to 2 physical Higgs bosons in the cubic interaction
- Coupling of A^0 with a fermion is proportional to the fermion mass:

  $-\frac{g m_u \cot\beta}{2 m_W}\gamma_5$ (up-type) $-\frac{g m_d \tan\beta}{2 m_W}\gamma_5$ (down-type)
- we consider the top quark contributions

Two processes related by s-t crossing



s-t crossing



$$A^t_{\mu\nu} = -\frac{e^2 g}{(4\pi)^2} N_C q_t^2 \frac{\cot\beta}{2m_W} F_t(\rho,\tau)\varepsilon_{\mu\nu\rho\sigma} q^\rho p^\sigma$$

Transition form factor

2. Space-like and Time-like Transition Form Factor

- In our previous work on $e\gamma \to eA^0$ process we have introduced the space-like transition form factor as

$$F_t^{\,e\gamma\to eA^0}(\rho,\tau) = \frac{\tau}{1+\rho\tau}[g(\rho)+4f(\tau)]$$

where

$$\rho \equiv \frac{-q^2}{4m_t^2}, \quad \tau \equiv \frac{4m_t^2}{m_A^2} \quad (q^2<0)$$

- In the case of $e^+e^- \to A^0\gamma$ we introduce time-like TFF now as

$$F_t^{\,e^+e^-\to\gamma A^0}(\rho,\tau) = \frac{\tau}{1-\rho\tau}[-g(\rho)+4f(\tau)]$$

$$\rho \equiv \frac{q^2}{4m_t^2}, \quad \tau \equiv \frac{4m_t^2}{m_A^2} \quad (q^2>0)$$

- For the space-like case in eγ collisions, the $g(\rho)$ is

$$g^{e\gamma}(\rho) = \left[\log \frac{\sqrt{\rho+1}+\sqrt{\rho}}{\sqrt{\rho+1}-\sqrt{\rho}}\right]^2$$

- While in the case of $e^+e^- \to A^0\gamma$ $g(\rho)$ given by

$$g^{e^+e^-}(\rho) = -\left[\log \frac{\sqrt{\rho}+\sqrt{\rho-1}}{\sqrt{\rho}-\sqrt{\rho-1}} - i\pi\right]^2$$

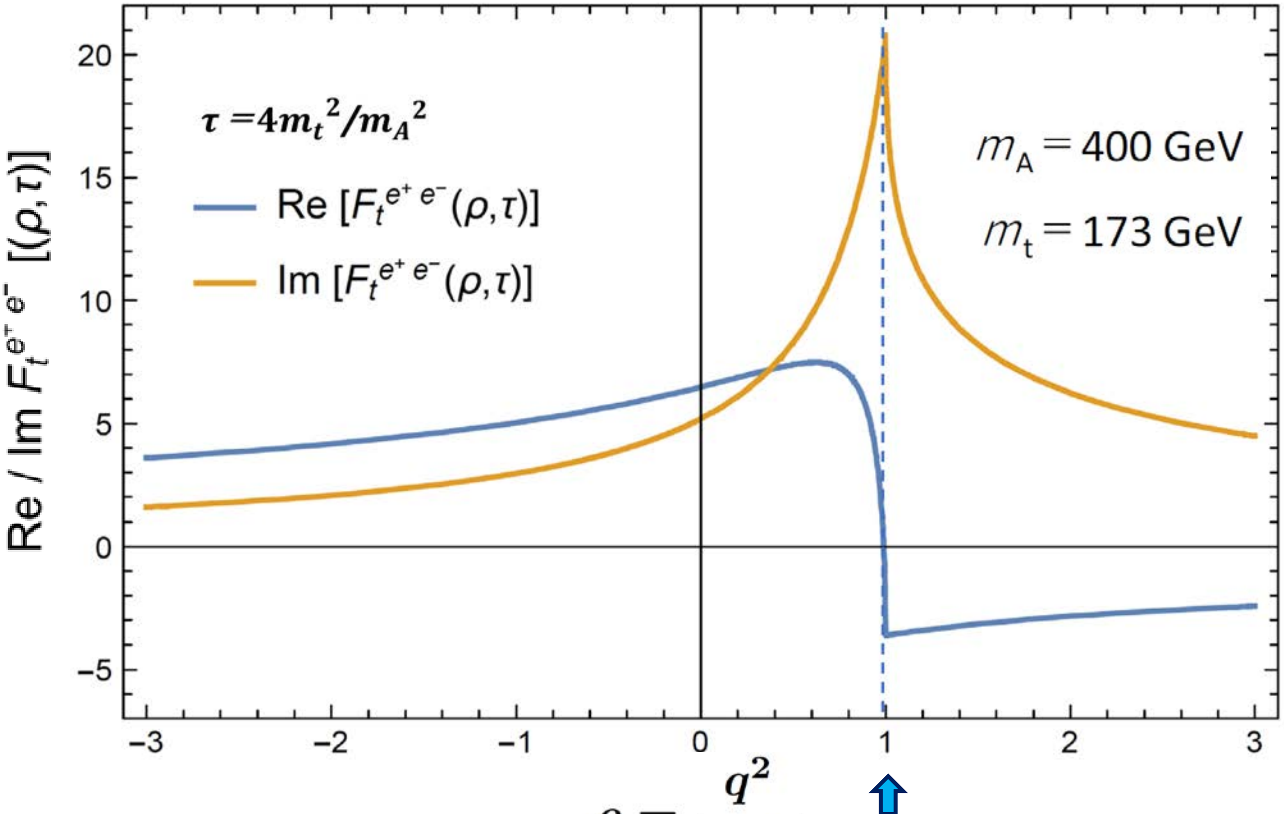
- We have the following analytic continuation : $q^2 \to q^2 + i\epsilon$

$$g^{e\gamma}\left(\frac{-q^2}{4m_t^2} - i\epsilon\right) = -g^{e^+e^-}\left(\frac{q^2}{4m_t^2} + i\epsilon\right)$$ i.e. $$g^{e\gamma}(-\rho) = -g^{e^+e^-}(\rho)$$

$$F_t^{e\gamma\to eA^0}(-\rho,\tau) = \frac{\tau}{1-\rho\tau}[g^{e\gamma}(-\rho) + 4f(\tau)]$$

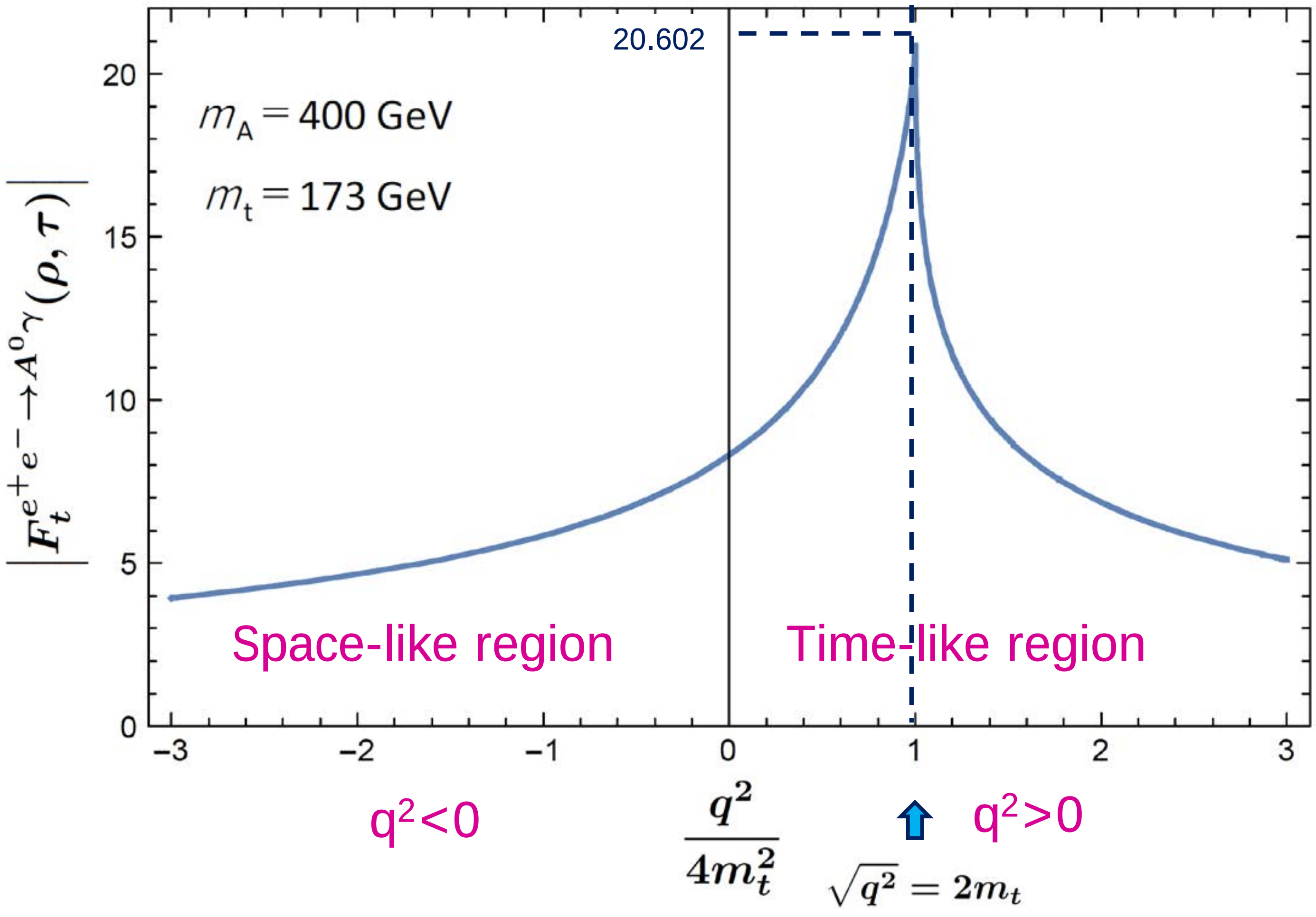
$$= \frac{\tau}{1-\rho\tau}[-g^{e^+e^-}(\rho) + 4f(\tau)] = F_t^{e^+e^-\to\gamma A^0}(\rho,\tau)$$

Transition form factor $e^+e^- \to A^0\gamma$

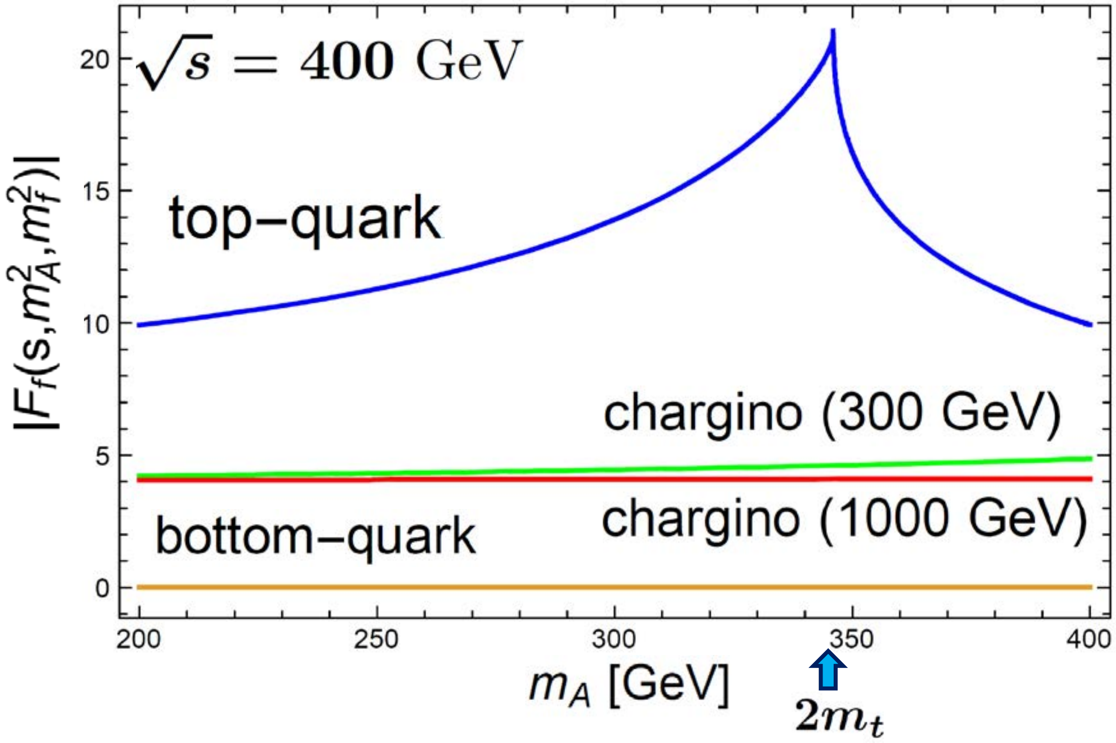


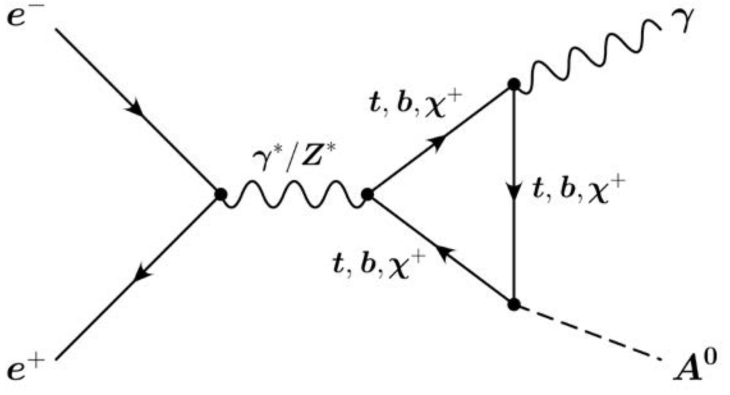
$$\rho = \frac{q^2}{4m_t^2} \qquad \sqrt{q^2} = 2m_t$$

Transition form factor $F_t(\rho,\tau)$ for $e^+e^- \rightarrow A^0\gamma$



Various Contributions to Transition Form Factor





we also note no contribution from squark-loop for A^0 production

3. CP-odd Higgs A^0 Production in e^+e^- Collisions

cf. e.g. Djouadi et al. Nucl. Phys. B491 (1997) 68-102.

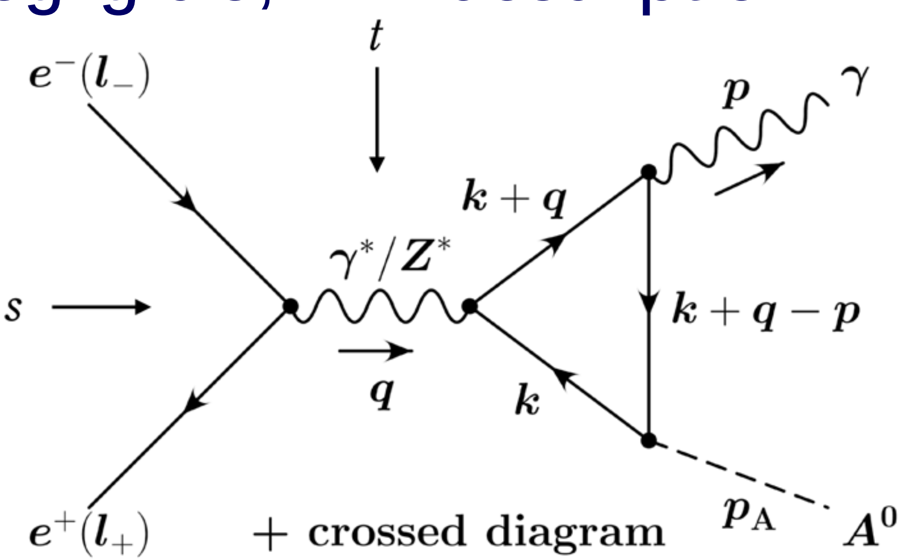
- Top-quark triangle loop diagram through $\gamma*$ & Z*
- No QCD radiative corrections by non-renormalization theorem of anomaly, in the case $m_A << m_t$.
- If extra contributions are negligible, TFF description makes sense.

Kinematical variables

$$s = (l_+ + l_-)^2 = 4E^2 = q^2 > 0$$

$$t = (l_- - p)^2 = -2l_- \cdot p$$

$$u = (l_+ - p)^2 = 2l_+ \cdot p$$



Scattering amplitudes and transition form factor

$$\langle\gamma A^0|T|e^-e^+\rangle^t_{\gamma^*} = [\overline{v}(l_+)(-ie\gamma_\rho)u(l_-)]\frac{-ig^{\rho\mu}}{q^2+i\epsilon}A^t_{\mu\nu}\epsilon^\nu(p)$$

$$\langle\gamma A^0|T|e^-e^+\rangle^t_{Z^*} = \frac{g}{4\cos\theta_W}[\overline{v}(l_+)(i\gamma_\mu)(f_{Ze}+\gamma_5)u(l_-)]\frac{-i}{q^2-m_Z^2}\widetilde{A}^t_{\mu\nu}\epsilon^\nu(p)$$

$$A^t_{\mu\nu} = -\frac{e^2 g}{(4\pi)^2}N_C q_t^2\frac{\cot\beta}{2m_W}F_t(\rho,\tau)\varepsilon_{\mu\nu\rho\sigma}q^\rho p^\sigma$$

$$\widetilde{A}^t_{\mu\nu} = -\frac{e^2 g}{(4\pi)^2}\frac{N_C q_t f_{Zt}}{4\cos\theta_W}\frac{\cot\beta}{2m_W}F_t(\rho,\tau)\varepsilon_{\mu\nu\rho\sigma}q^\rho p^\sigma$$

Transition Form Factor

$$F_t(\rho,\tau) = \frac{\tau}{1-\rho\tau}[-g(\rho)+4f(\tau)], \quad \rho = \frac{q^2}{4m_t^2}, \quad \tau = \frac{4m_t^2}{m_A^2}$$

$$\begin{aligned} f(\tau) &= \left[\sin^{-1}\sqrt{\frac{1}{\tau}}\right]^2 && \tau \geq 1\,, \\ &= -\frac{1}{4}\left[\log\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 && 0<\tau<1 \end{aligned}$$

$$\begin{aligned} g(\rho) &= -\left[\log\frac{\sqrt{\rho}+\sqrt{\rho-1}}{\sqrt{\rho}-\sqrt{\rho-1}} - i\pi\right]^2 && \rho \geq 1 \\ &= 4\left[\sin^{-1}\sqrt{\rho}\right]^2 && 0<\rho<1\,. \end{aligned}$$

Cross sections through γ* and Z*

$$\left(\frac{d\sigma}{dt}\right)_{\gamma^*} = \frac{\alpha_{\rm em}^3}{64\pi}\frac{g^2}{4\pi}\left(\frac{\cot\beta}{2m_W}\right)^2\frac{1}{s}\left(\frac{t^2+u^2}{s^2}\right)\left|N_C^t q_t^2 F_t(q^2, m_A^2, m_t^2)\right|^2$$

$$\left(\frac{d\sigma}{dt}\right)_{Z^*} = \frac{\alpha_{\rm em}}{64\pi}\left(\frac{g^2}{4\pi}\right)^3\left(\frac{\cot\beta}{2m_W}\right)^2\left(\frac{1}{16\cos^2\theta_W}\right)^2\frac{s}{(s-m_Z^2)^2}$$
$$\times f_{Zt}^2(f_{Ze}^2+1)\left(\frac{t^2+u^2}{s^2}\right)\left|N_C^t q_t F_t(q^2, m_A^2, m_t^2)\right|^2,$$

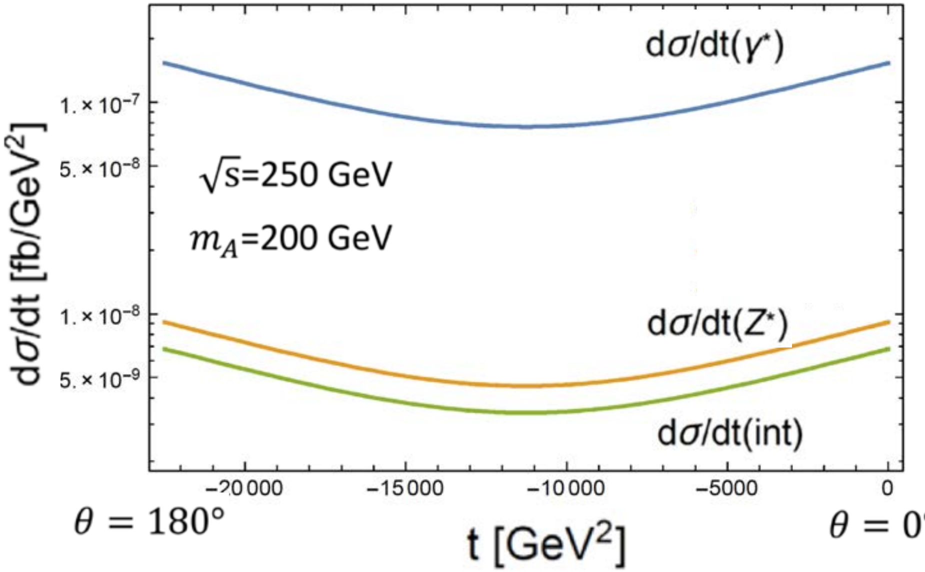
$$\left(\frac{d\sigma}{dt}\right)_{\rm int} = -2\times\frac{\alpha_{\rm em}^2}{64\pi}\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\cot\beta}{2m_W}\right)^2\frac{1}{16\cos^2\theta_W}\frac{1}{s-m_Z^2}$$
$$\times f_{Zt} f_{Ze} q_t\left(\frac{t^2+u^2}{s^2}\right)\left|N_C^t q_t F_t(q^2, m_A^2, m_t^2)\right|^2,$$

$$f_{Ze} = -1 + 4\sin^2\theta_W \qquad f_{Zt} = 1 - \frac{8}{3}\sin^2\theta_W$$

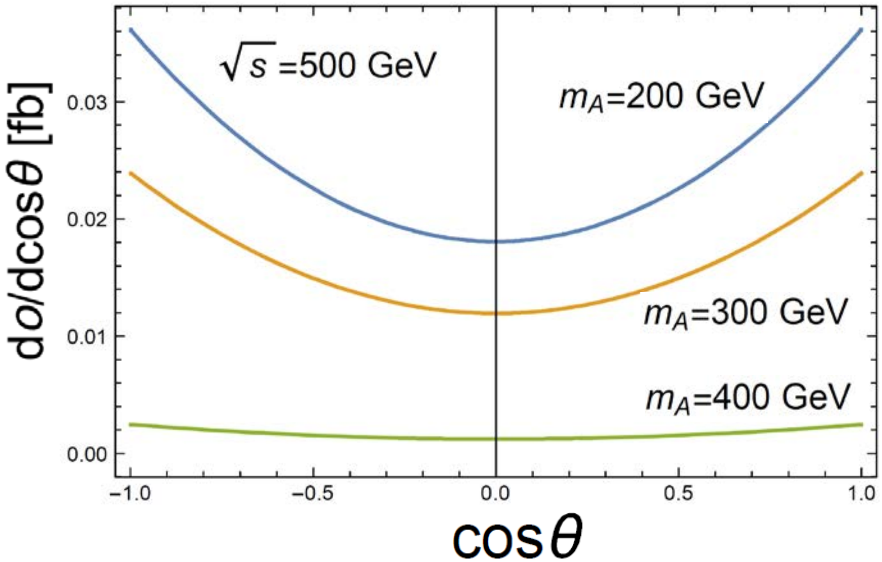
Differential Cross Section for $e^+e^- \rightarrow A^0\gamma$

$$\cot\beta = 1$$

components of cross section



A⁰ mass dependence

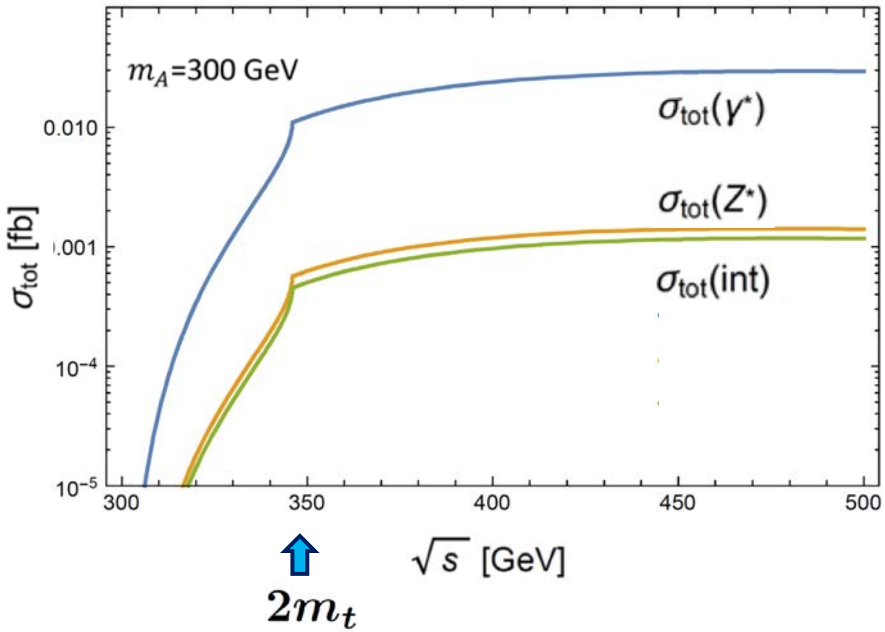


The cross section is proportional to $\cot^2\beta$. Here we consider the case $\tan\beta$ not large and A^0 is rather light $m_A \leq 500$ GeV.

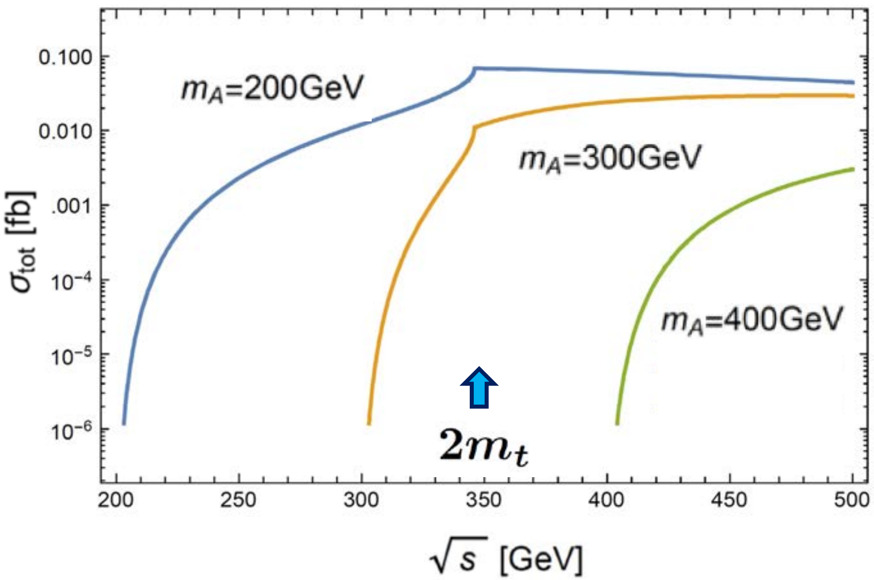
Total Cross Section through γ^*, Z^* and Interference term

$\cot\beta = 1$

components of cross section

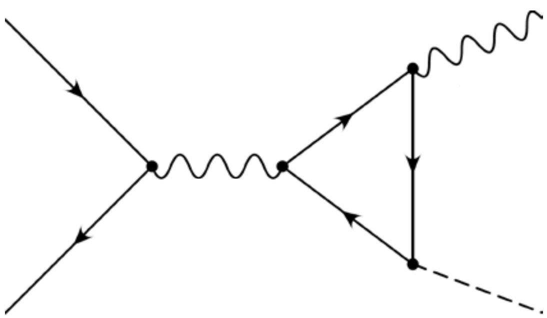


A^0 mass dependence

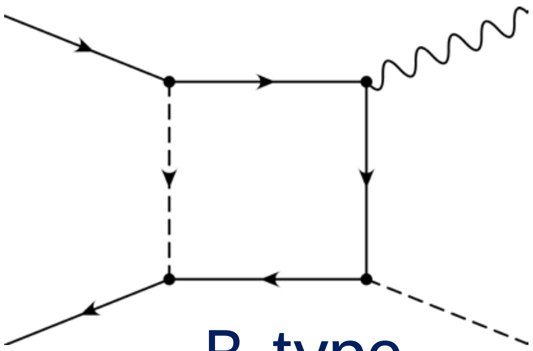


Triangle vs. Box diagrams and Transition FF

Diagrams
- T-type — Triangle diagrams
- B-type — Box diagrams



T-type



B-type

If T-type >>B-type ⇒ $\frac{d\sigma}{dQ^2} \propto |T|^2$

Transition Form Factor interpretation!

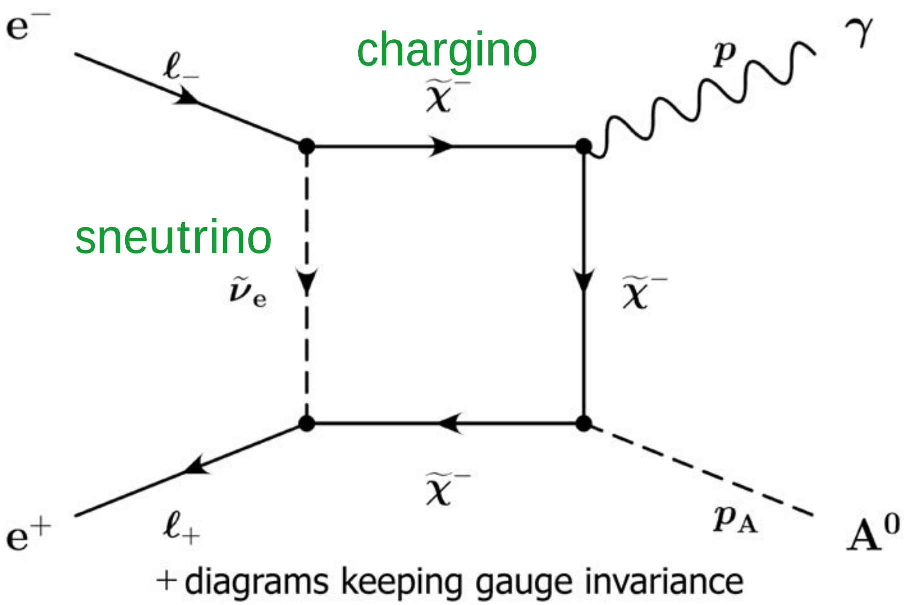
Box-diagram contributions

$W \Longleftrightarrow \tilde{\chi}^-$　　$Z \Longleftrightarrow \tilde{\chi}^0$

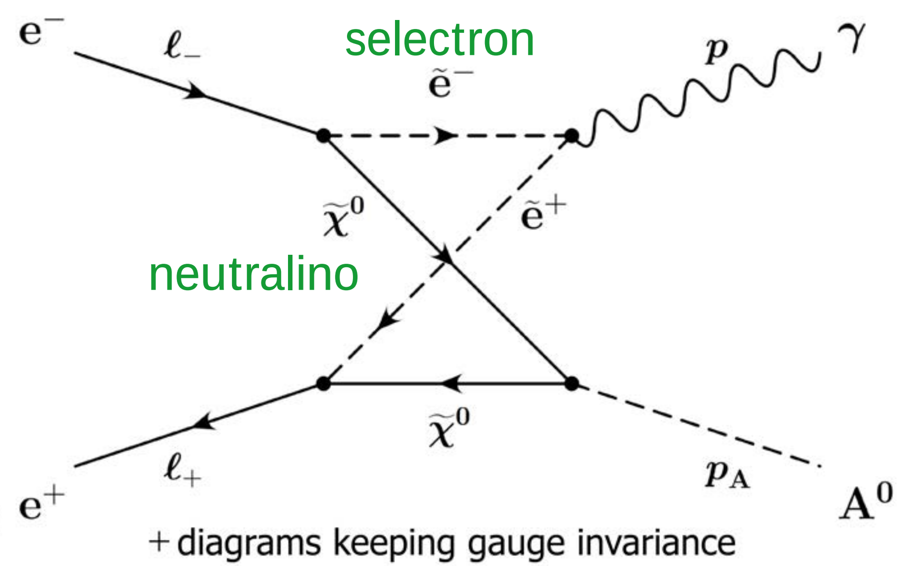
$\nu_e \Longleftrightarrow \tilde{\nu}_e$　　$e^- \Longleftrightarrow \tilde{e}^-$

Contributions violating transition-form-factor interpretations.

Chargino-sneutrino-box



Neutralino-selectron-box



We are only interested in the order of magnitudes of their contributions.

Chargino-sneutrino process

chargino's coupling to A^0

$$\mathcal{L}_{A^0\bar{\tilde{\chi}}\tilde{\chi}} = -ig\kappa_1\bar{\tilde{\chi}}_1\gamma_5\tilde{\chi}_1 A^0$$

Amplitude

$$A^{(\tilde{\chi}\tilde{\nu})}_{e^+e^-\to\gamma A^0} = \left(\frac{eg^3\kappa_1|V_{11}|^2}{16\pi^2}\right)\frac{m_{\tilde{\chi}}}{4}\epsilon^*(p)^\beta\left[\overline{v}(l_+)F_{(\tilde{\chi}\tilde{\nu})\beta}(1-\gamma_5)u(l_-)\right]$$

Gauge invariant decomposition

$$\kappa_1 = \frac{1}{\sqrt{2}}(\sin\beta U_{12}V_{11} + \cos\beta U_{11}V_{12})$$

$$F_{(\tilde{\chi}\tilde{\nu})\beta} = \left(\frac{2l_{-\beta}\not{p}}{t}+\gamma_\beta\right)S_1^{\tilde{\chi}\tilde{\nu}}(s,t,m^2_{A^0},m^2_{\tilde{\chi}},m^2_{\tilde{\nu}}) + \left(\frac{2l_{+\beta}\not{p}}{u}+\gamma_\beta\right)S_2^{\tilde{\chi}\tilde{\nu}}(s,t,m^2_{A^0},m^2_{\tilde{\chi}},m^2_{\tilde{\nu}})$$

$$p^\beta F_{(\tilde{\chi}\tilde{\nu})\beta} = 0$$

Differential cross section

$$\frac{d\sigma_{\tilde{\chi}\tilde{\nu}}}{dt} = \frac{1}{16\pi s^2}\left(\frac{eg^3\kappa_1|V_{11}|^2}{16\pi^2}\right)^2\cdot\frac{m^2_{\tilde{\chi}}}{8}s\times\left\{|S_1^{\tilde{\chi}\tilde{\nu}}|^2+|S_2^{\tilde{\chi}\tilde{\nu}}|^2\right\}$$

$S_1^{\tilde{\chi}\tilde{\nu}}$ and $S_2^{\tilde{\chi}\tilde{\nu}}$ are given in terms of Passarino and Veltman's scalar integrals C_0's and D_0's

we assume $\kappa_1|V_{11}|^2 \sim \mathcal{O}(1)$

Chargino-sneutrino-box -two form factors-

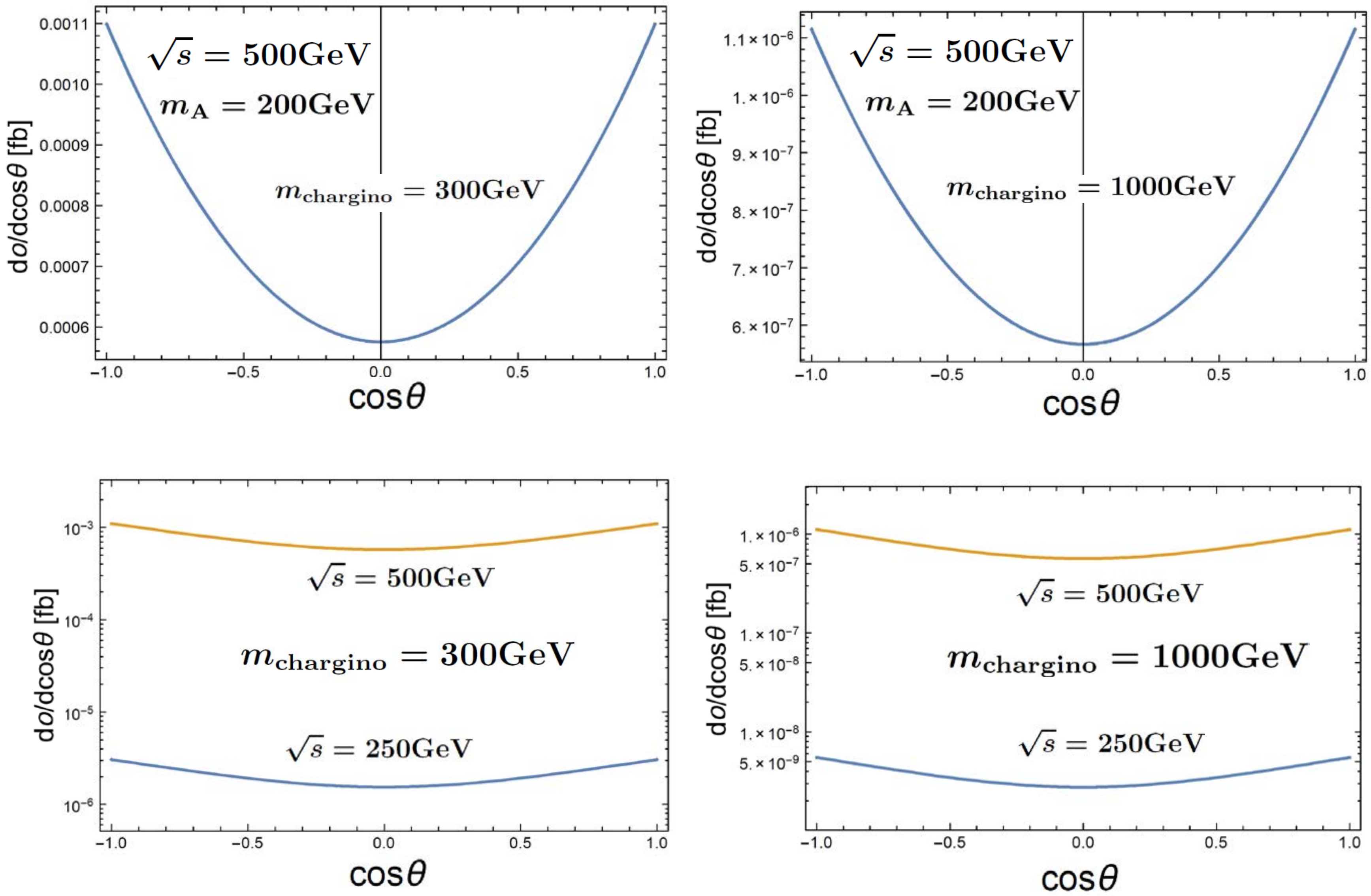
$$
\begin{aligned}
&S_1^{\tilde{\chi}\tilde{\nu}}(s,t,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)\\
&= -\frac{t-s}{s}C_0(0,u,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2) - \frac{u}{s}C_0(0,0,u,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)\\
&+\frac{2(t+u)}{s}C_0(0,s,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2)\\
&-\frac{(m_{\tilde{\chi}}^2(t+u)-m_{\tilde{\nu}}^2(t+u)-su)}{s}D_0(0,0,0,m_{A^0}^2,s,u,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2)\\
&-\frac{s+u}{s}C_0(0,t,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2) - \frac{t}{s}C_0(0,0,t,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)\\
&-\frac{(m_{\tilde{\chi}}^2(t+u)-m_{\tilde{\nu}}^2(t+u)+st)}{s}D_0(0,0,0,m_{A^0}^2,t,s,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2)
\end{aligned}
$$

and

$$
\begin{aligned}
&S_2^{\tilde{\chi}\tilde{\nu}}(s,t,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)\\
&= -\frac{u-s}{s}C_0(0,t,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2) - \frac{t}{s}C_0(0,0,t,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)\\
&+\frac{2(t+u)}{s}C_0(0,s,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2)\\
&-\frac{(m_{\tilde{\chi}}^2(t+u)-m_{\tilde{\nu}}^2(t+u)-st)}{s}D_0(0,0,0,m_{A^0}^2,t,s,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2)\\
&-\frac{t+s}{s}C_0(0,u,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2) - \frac{u}{s}C_0(0,0,u,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)\\
&-\frac{(m_{\tilde{\chi}}^2(t+u)-m_{\tilde{\nu}}^2(t+u)+su)}{s}D_0(0,0,0,m_{A^0}^2,s,u,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2,m_{\tilde{\chi}}^2,m_{\tilde{\chi}}^2)
\end{aligned}
$$

Differential cross section

sneutrino 300 GeV



Neutralino-selectron process

We are only interested in the order of magnitudes, assuming that the diagrams of one mass eigenstate of neutralino $\widetilde{\chi}_1^0$ and one mass eigenstate of selectron $\widetilde{e}_1$ dominantly contribute.

$$A^{(\tilde{\chi}_1^0\tilde{e}_1)}_{e^+e^-\to\gamma A^0} = \frac{ieg^3\eta_1|\tilde{N}_{12}|^2}{16\pi^2}\frac{m_{\tilde{\chi}_1^0}}{4}\epsilon^*(p)^\beta\overline{v}(l_+)F^{(\tilde{\chi}_1^0\tilde{e}_1)}_{2\beta}(1-\gamma_5)u(l_-)$$

Similar gauge-invariant decomposition leads to $S_1^{\tilde{\chi}\tilde{e}}$ and $S_2^{\tilde{\chi}\tilde{e}}$

Differential cross section

$$\frac{d\sigma_{\tilde{\chi}\tilde{e}}}{dt} = \frac{1}{16\pi s^2}\left(\frac{eg^3\eta_1|\tilde{N}_{12}|^2}{16\pi^2}\right)^2\cdot\frac{m^2_{\tilde{\chi}}}{8}s\times\left\{|S_1^{\tilde{\chi}\tilde{e}}|^2+|S_2^{\tilde{\chi}\tilde{e}}|^2\right\}$$

$S_1^{\tilde{\chi}\tilde{e}}$ and $S_2^{\tilde{\chi}\tilde{e}}$ are given in terms of Passarino and Veltman's

scalar integrals C_0's and D_0's we also assume $\eta_1|\tilde{N}_{12}|^2\sim\mathcal{O}(1)$

Neutralino-selectron-box -two form factors-

$$S_1^{\tilde{\chi}\tilde{e}}(s,t,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)$$
$$= \frac{s+u}{2s}C_0(0,t,m_{A^0}^2,m_{\tilde{\chi}_1^0}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2) + \frac{t-s}{2s}C_0(0,u,m_{A^0}^2,m_{\tilde{\chi}_1^0}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2)$$
$$-\frac{t}{2s}C_0(0,0,t,m_{\tilde{e}_1}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2) - \frac{u}{2s}C_0(0,0,u,m_{\tilde{e}_1}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2)$$
$$+\frac{(m_{\tilde{e}_1}^2 - m_{\tilde{\chi}_1^0}^2)(t+u)+tu}{2s}D_0(0,0,0,m_{A^0}^2,t,u,m_{\tilde{\chi}_1^0}^2,m_{\tilde{e}_1}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2)$$

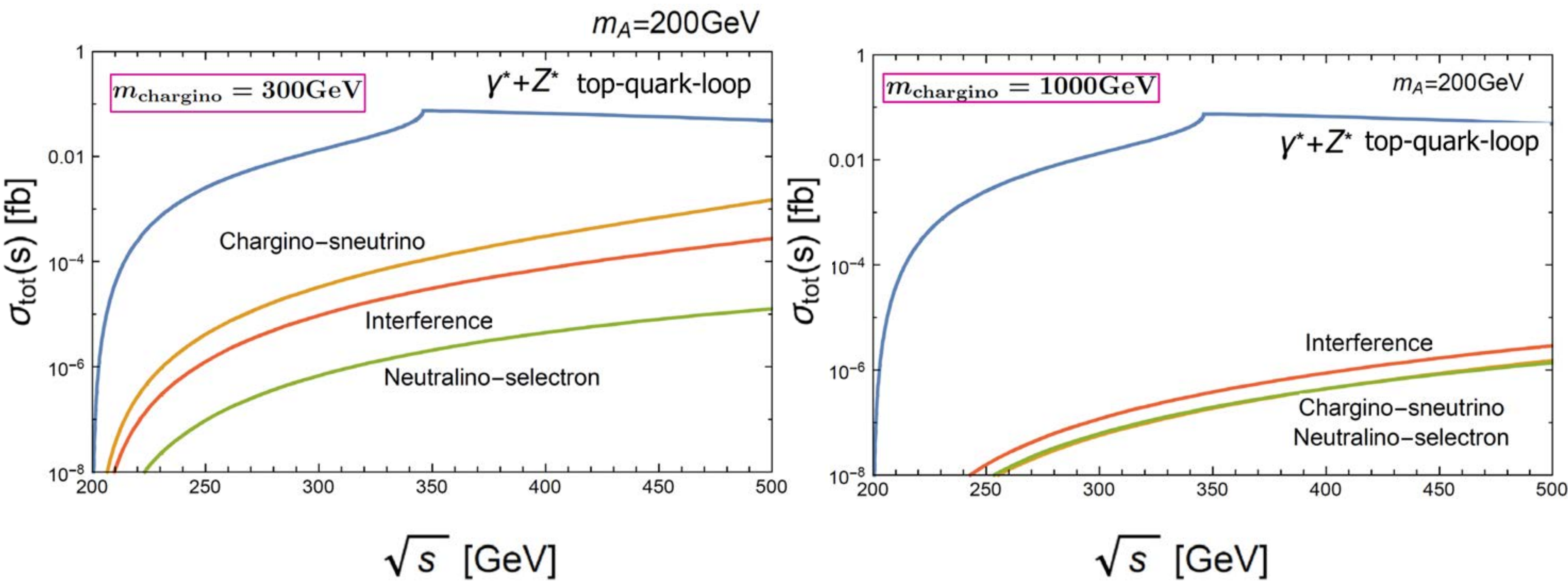
and

$$S_2^{\tilde{\chi}\tilde{e}}(s,t,m_{A^0}^2,m_{\tilde{\chi}}^2,m_{\tilde{\nu}}^2)$$
$$= \frac{u-s}{2s}C_0(0,t,m_{A^0}^2,m_{\tilde{\chi}_1^0}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2) + \frac{t+s}{2s}C_0(0,u,m_{A^0}^2,m_{\tilde{\chi}_1^0}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2)$$
$$-\frac{t}{2s}C_0(0,0,t,m_{\tilde{e}_1}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2) - \frac{u}{2s}C_0(0,0,u,m_{\tilde{e}_1}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2)$$
$$+\frac{(m_{\tilde{e}_1}^2 - m_{\tilde{\chi}_1^0}^2)(t+u)+tu}{2s}D_0(0,0,0,m_{A^0}^2,t,u,m_{\tilde{\chi}_1^0}^2,m_{\tilde{e}_1}^2,m_{\tilde{e}_1}^2,m_{\tilde{\chi}_1^0}^2)$$

Total cross section $e^+e^- \rightarrow A^0\gamma$

sneutrino 300 GeV
neutralino 300 GeV
selectron 300 GeV

Top-quark-loop vs. Box diagram contributions



Interference between top-quark-loop and box diagram is found to be very small.

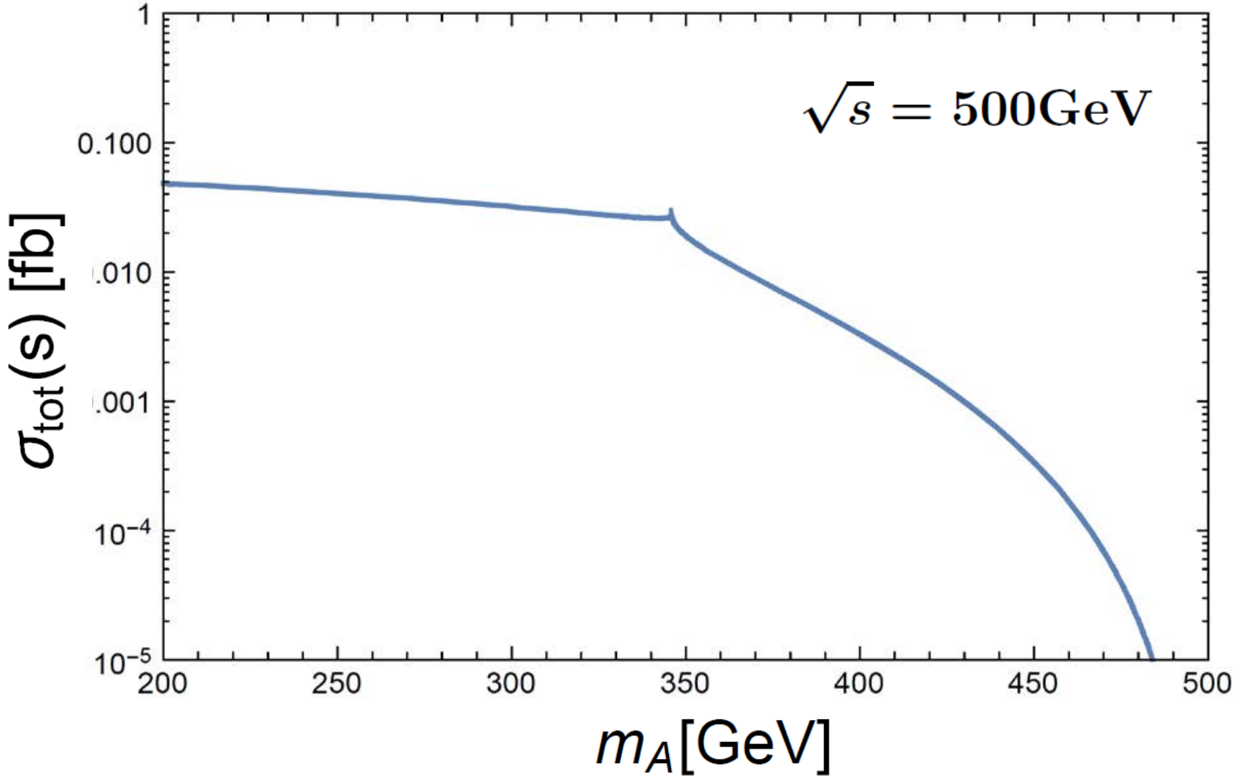
4. Concluding Remarks

- We have investigated production of CP-odd Higgs A^0 associated with a real photon γ in e^+e^- collisions, in terms of time-like transition form factor.
- The dominant contribution coming from top-quark one-loop diagrams. The γ^* process is far more dominant over Z^* process.
- The Box contributions, from chargino-sneutrino and neutralino-selectron related processes do not give sizable effects, in the parameter space we have studied. If $\tan\beta$ is not large and chargino very heavy, their contributions are negligible. Then TFF description makes sense, but more thorough analysis is needed.

Back up slides

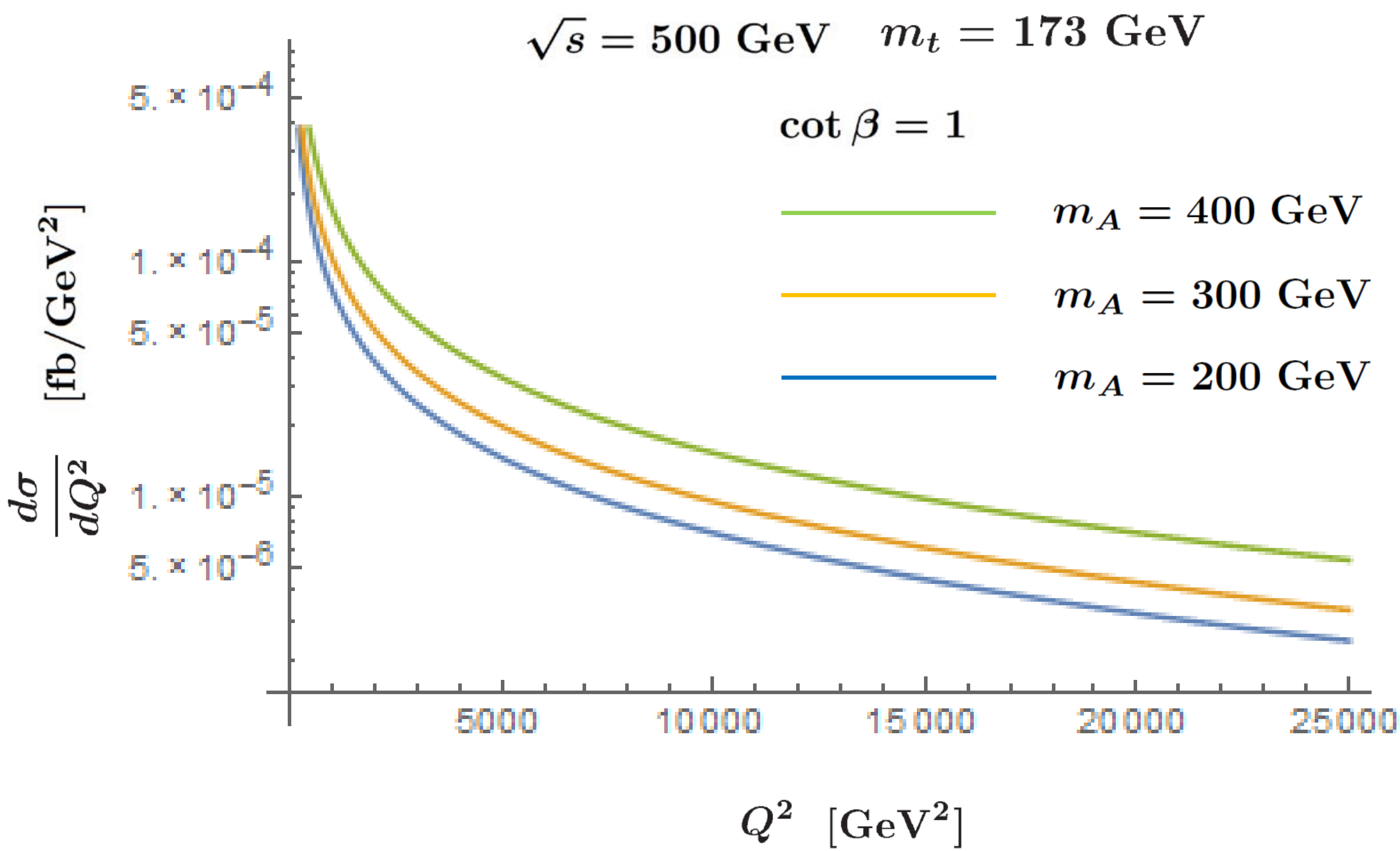
Total cross section $e^+e^- \to A^0\gamma$

mass dependence



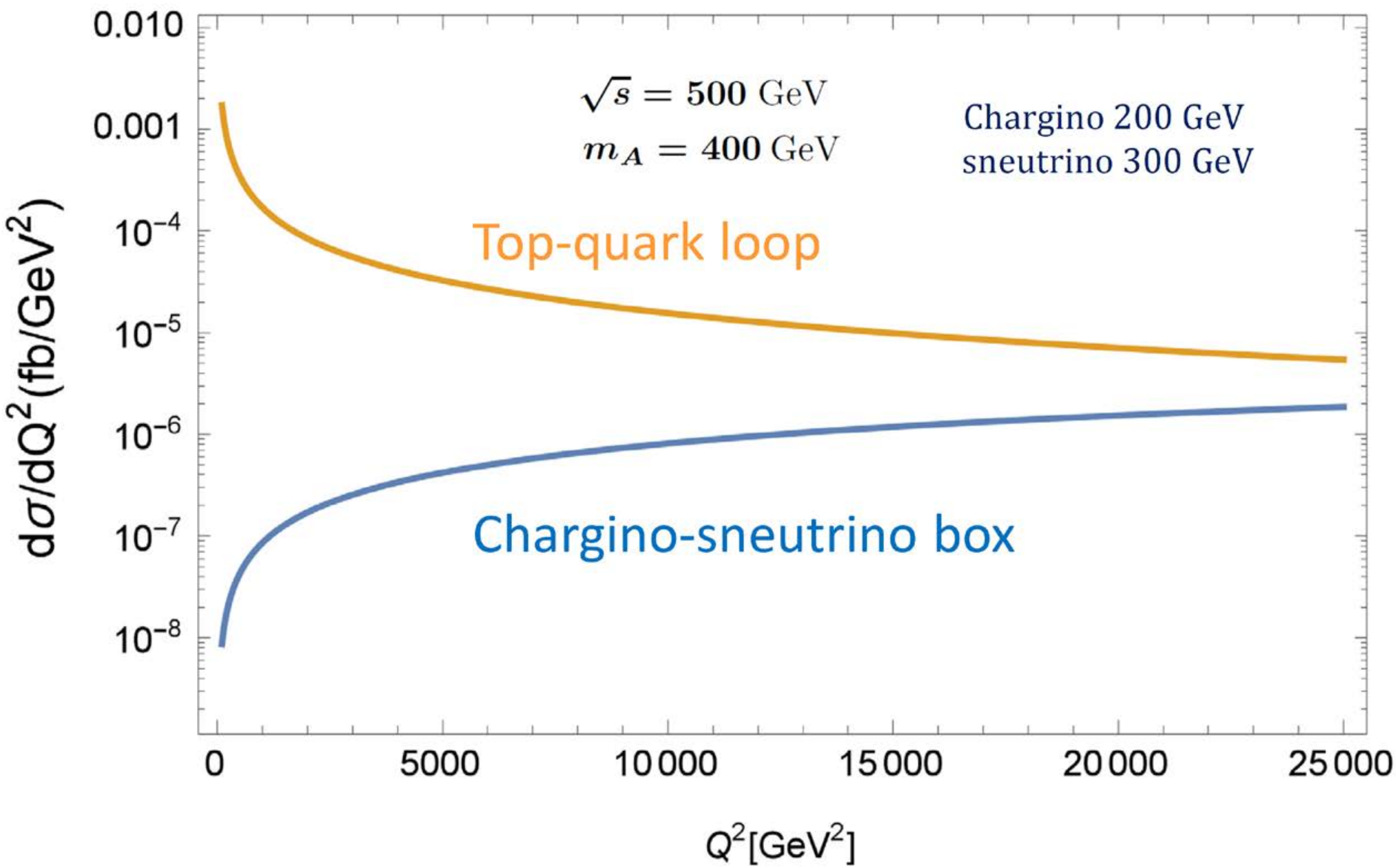
Differential Cross Section

$e\gamma \to eA^0$ process



Cross section : chargino-sneutrino process

$e\gamma \to eA^0$ process



Cross section : Neutralino-selectron process

$e\gamma \to eA^0$ process