Precision studies for Drell-Yan processes at NNLO

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Based on work done in collaboration with:

- *Precision studies for Drell-Yan processes at NNLO*
  Sergey Alekhin, Adam Kardos, S. M. and Zoltan Trócsányi
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W- and Z-boson cross sections

Status (I)

Data

- High precision experimental data from LHC ATLAS, CMS, LHCb and Tevatron D0 useful for determinations of parton distributions
  - statistically significant $NDP = 172$ in ABMP16
- Differential distributions in decay leption pseudo-rapidity extend kinematics to forward region
  - sensitivity to light quark flavors at $x \simeq 10^{-4}$
  - leading order kinematics with:
    $\sigma(W^+) \simeq u(x_2)\bar{d}(x_1)$ and $\sigma(W^-) \simeq d(x_2)\bar{u}(x_1)$;
    $\sigma(Z) \simeq Q_u^2 u(x_2)\bar{u}(x_1) + Q_d^2 d(x_2)\bar{d}(x_1)$
  - cf. DIS: $\sigma(\mathrm{DIS}) \simeq q_u^2 u(x) + q_d^2 d(x)$

Status (II)

Theory

- Complete NNLO QCD corrections with fully differential kinematics to match experimental cuts $\longrightarrow$ solved problem
  - combination of squared matrix elements with three different multiplicities of partons in final state
  - subtraction scheme to cancel of soft and collinear singularities upon integration over their phase space
- Public codes at NNLO including the leptonic decay
  - FEWZ (v3.1) (sector decomp.) Gavin, Li, Petriello, Quackenbush '12
  - DYNNLO (v1.5) (q_T slicing) Catani, Grazzini '07
  - MATRIX (v1.0.4) (q_T slicing) Grazzini, Kallweit, Wiesemann '17
  - MCFM at NNLO (N-jettiness slicing) Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams '16
- Also:
  - (SHERPA-NNLO-FO (q_T slicing) Höche, Li, Prestel '14)
  - (DYTURBO (q_T slicing, based on DYNNLO (v1.5)) Camarda, Boonekamp, Bozzi et al. '19)

Regularization

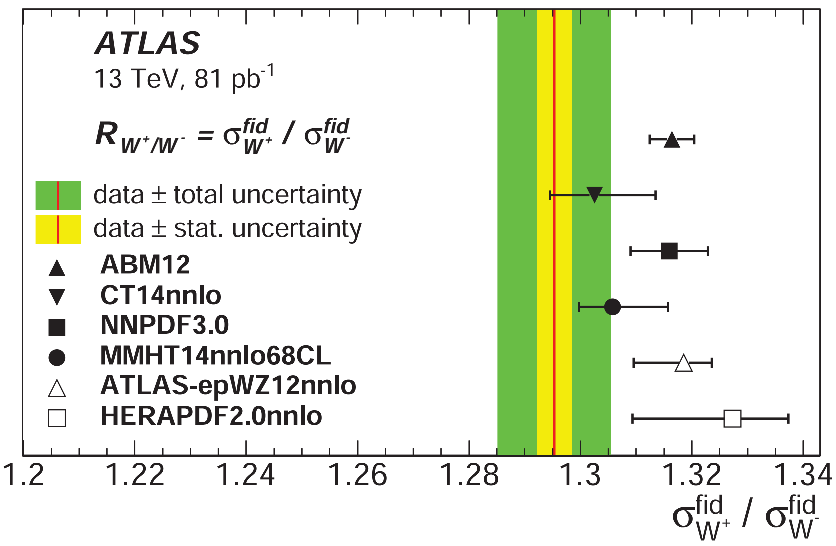
Subtraction

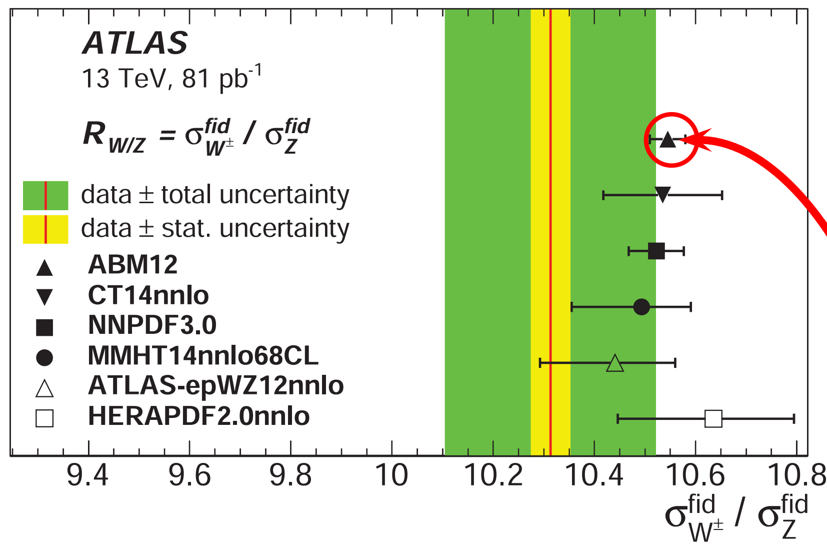
- removes all $\frac{1}{\epsilon^k}$ singularities in $d = 4 - 2\epsilon$ analytically; $k \leq 4$ at NNLO
  - Antenna subtraction — Gehrmann-De Ridder, Gehrmann, Glover '05
  - Colourful subtraction — Del Duca, Somogyi, Trocsanyi '05
  - Sector subtraction — Czakon '10; Boughezal, Melnikov, Petriello '11; Czakon, Heymes '14
  - Projection to Born — Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15
  - Local analytic sector subtraction — Magnea, Maina, Pelliccioli, Signorile-Signorile, Torrielli, Uccirati '18

Slicing

- Cuts imposed in phase space; singularities $\ln^k(\mathrm{some\ cut})$ need to cancel numerically; $k \leq 4$ at NNLO
  - q_T-subtraction — Catani, Grazzini '07
  - N-jettiness subtraction — Boughezal, Focke, Liu, Petriello '15; Gaunt, Stahlhofen, Tackmann, Walsh '15

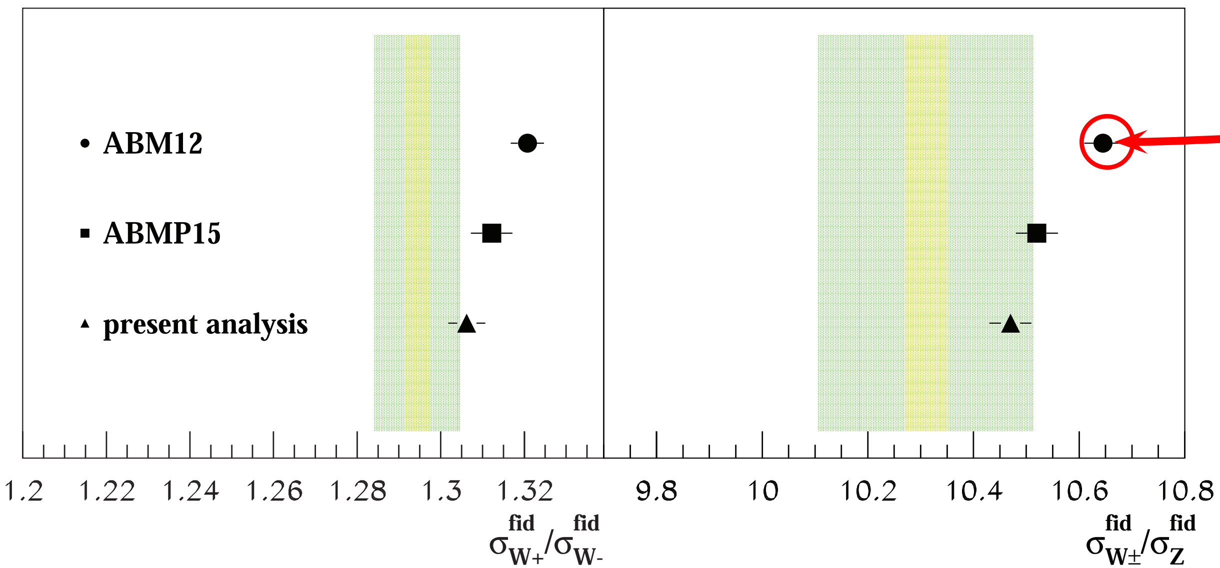
Theory issues





DYNNLO (v1.5)
Catani, Grazzini '07

ATLAS (13 TeV, 81 pb^{-1}) 1603.09222

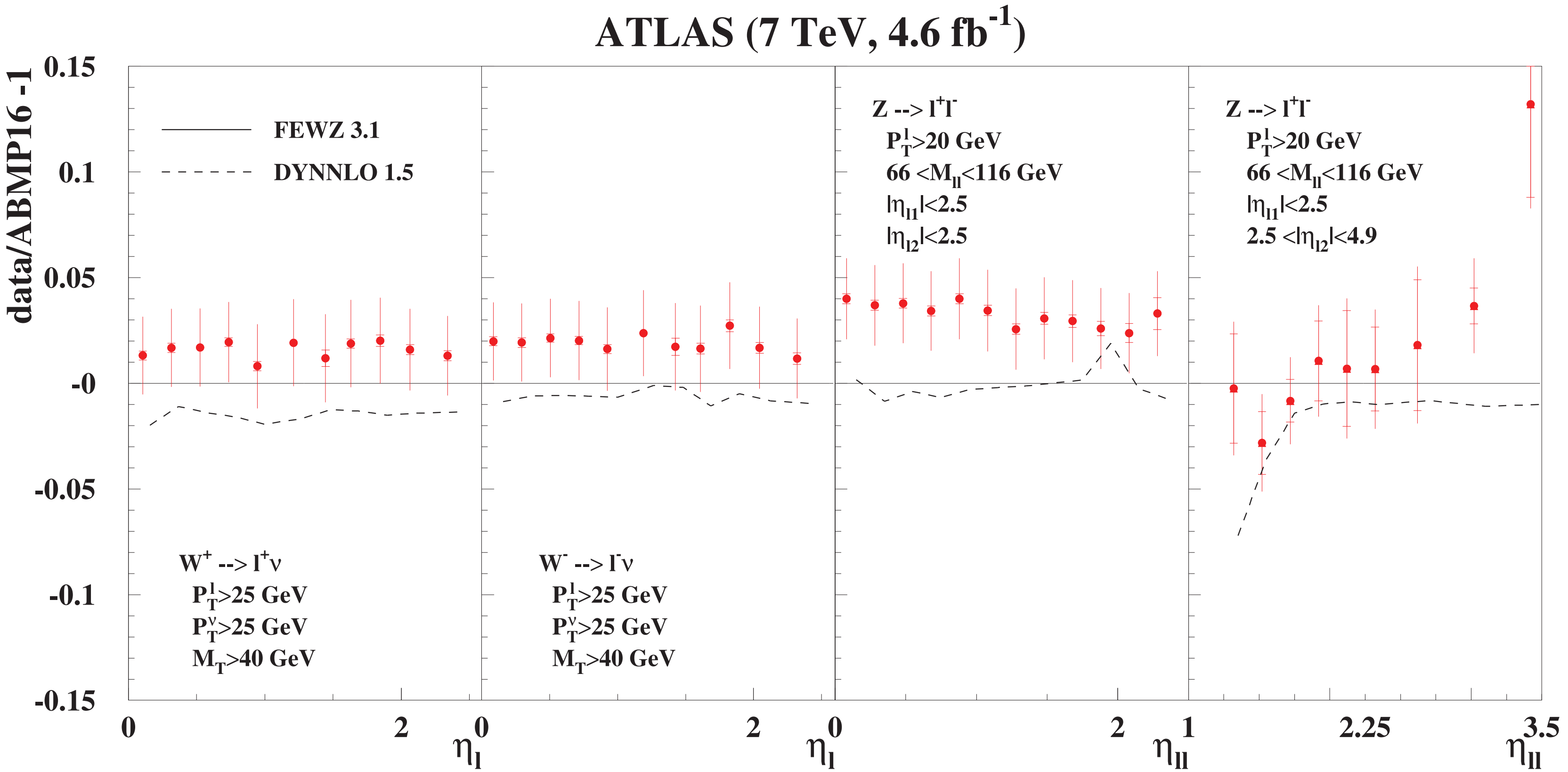


FEWZ (v3.1)
Gavin, Li, Petriello, Quackenbush '12

- Differences at NNLO between DYNNLO and FEWZ up to $\mathcal{O}(1\%)$ or more

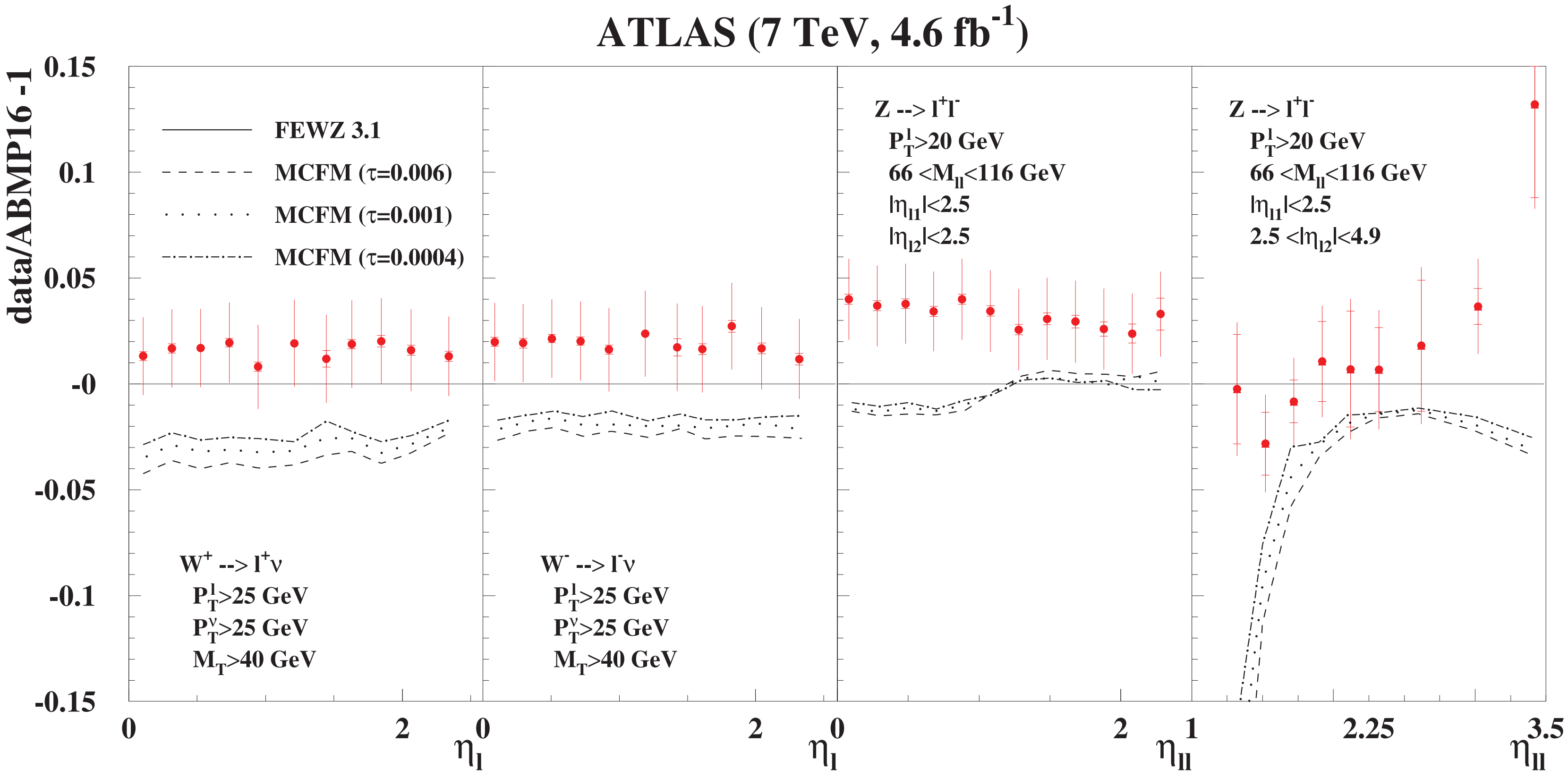
Benchmark computations

Benchmarking DYNNLO



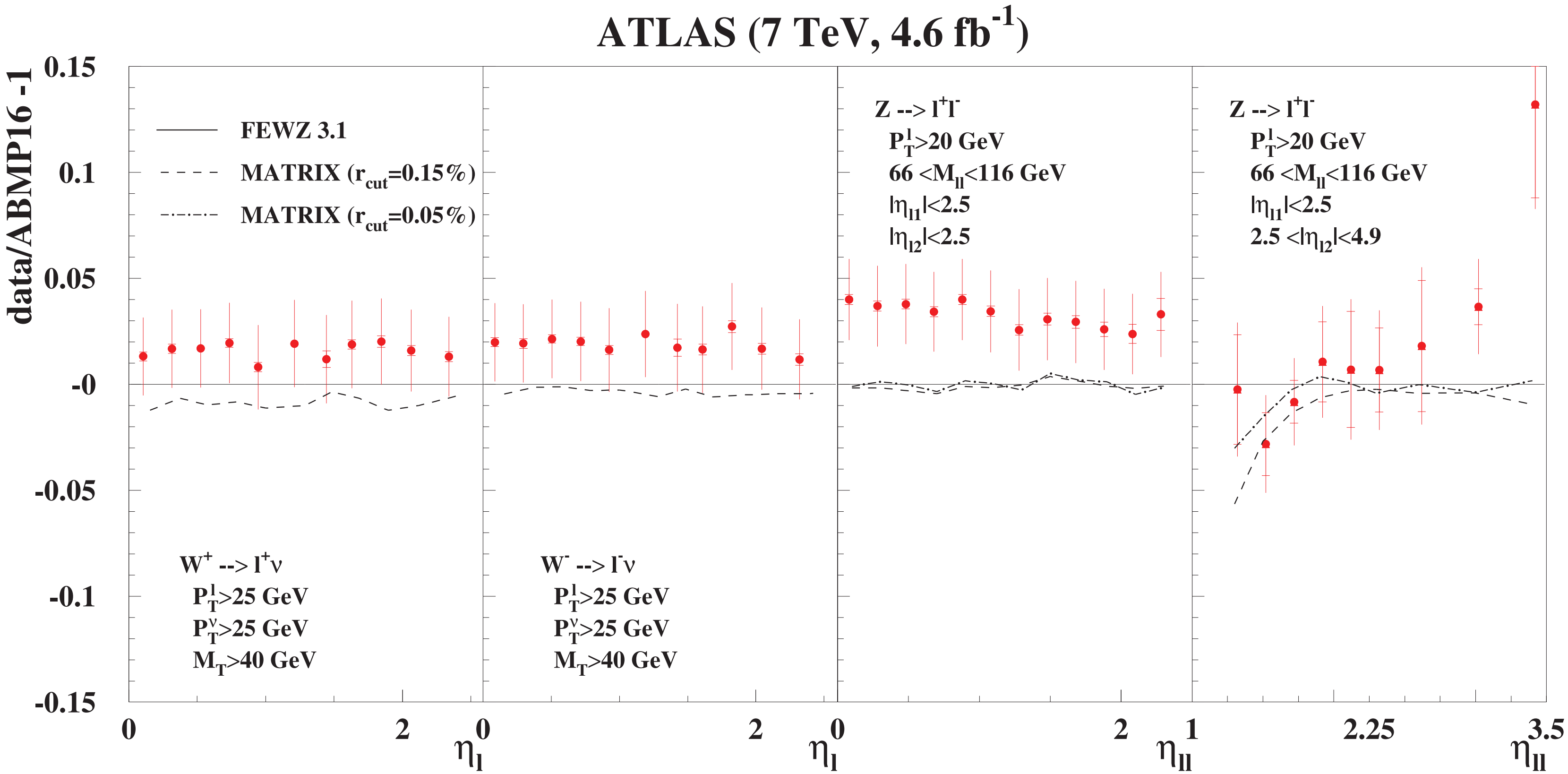
- Deviations of DYNNLO (q_T slicing) at NNLO
  - up to $\mathcal{O}(1\%)$ for W^+-production
  - few per mill for W^-- and Z-production
  - up to $\mathcal{O}(2-3\%)$ for forward Z-production, $\mathcal{O}(10\%)$ in first bin

Benchmarking MCFM



- Substantial deviations of MCFM at NNLO (N-jettiness slicing)
- Variation of $\tau_{\rm cut}$ (smallest value $\tau_{\rm cut} = 4 \cdot 10^{-4}$ at computational limits)
  - up to $\mathcal{O}(3\%)$ for W^+-production
  - up to $\mathcal{O}(2\%)$ for W^-- and Z-production
  - up to $\mathcal{O}(2-3\%)$ for forward Z-production, $\mathcal{O}(20\%)$ in first bin

Benchmarking MATRIX

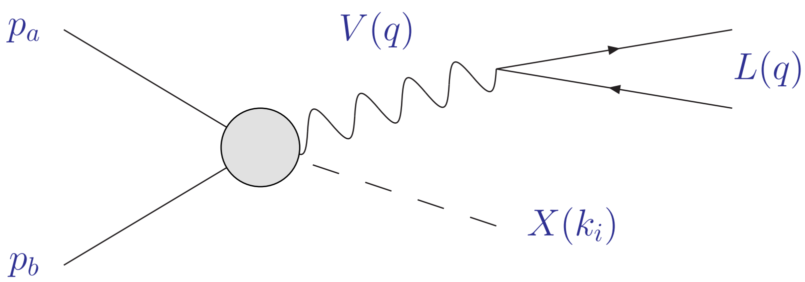


- Findings for MATRIX similar to DYNNLO (both based on q_T slicing)
  - up to $\mathcal{O}(1\%)$ for W^+-production
  - few per mill for W^-- and Z-production
  - up to $\mathcal{O}(1-2\%)$ for forward Z-production, $\mathcal{O}(5\%)$ in first bin
- Variation of $r_{\rm cut}$ has similar effect as smaller $\tau_{\rm cut}$ in MCFM

Slicing methods (I)

- W- and Z-boson production with decay to leptonic final state L
  - gauge boson mass $q^2 = Q^2$

$$a(p_a) + b(p_b) \quad \to V(q) + X(k_i) \quad \to L(q) + X(k_i)$$



- Slicing parameter τ for q_T-subtraction defined with $\vec{k}_{T,i}$ (transverse momenta of hadronic final states) Catani, Grazzini '07

$$\tau \quad = q_T^2/Q^2 \quad = \left(\sum_i \vec{k}_{T,i} \right)^2 /Q^2$$

- Slicing parameter τ for leptonic 0-jettiness $\mathcal{T}_0$

Boughezal, Focke, Liu, Petriello '15; Gaunt, Stahlhofen, Tackmann, Walsh '15

$$\tau \quad = \mathcal{T}_0/Q \quad = \sum_i \min \{2p_a \cdot k_i, 2p_b \cdot k_i\} /Q^2$$

Slicing methods (II)

- Definitions of τ vanish at Born level and resolve additional radiation in infrared-safe manner
- Phase space integration for cross section with cut for slicing $\tau_{\rm cut}$

$$\sigma \;=\; \int d\tau\, \frac{d\sigma}{d\tau} = \int^{\tau_{\rm cut}} d\tau\, \frac{d\sigma}{d\tau} + \int_{\tau_{\rm cut}} d\tau\, \frac{d\sigma}{d\tau} = \sigma(\tau_{\rm cut}) + \int_{\tau_{\rm cut}} d\tau\, \frac{d\sigma}{d\tau}$$

- Dependence of $d\sigma/d\tau$ on τ predicted from universal QCD factorization in soft and collinear limits
  - power corrections proportional to τ^{p-1} with $p > 0$ are integrable

$$\frac{d\sigma}{d\tau} \;\sim\; \delta(\tau) + \sum_i \left[\frac{\ln^i \tau}{\tau}\right]_+ + \sum_j \tau^{p-1} \ln^j \tau + \mathcal{O}(\tau^p)$$

- Analytical result for $\sigma(\tau_{\rm cut})$

$$\sigma(\tau_{\rm cut}) \;\sim\; 1 + \sum_i \ln^{i+1} \tau_{\rm cut} + \sum_j \tau_{\rm cut}^p \ln^j \tau_{\rm cut} + \mathcal{O}(\tau_{\rm cut}^{p+1})$$

- Subtraction scheme implemented via global subtraction term $\sigma^{\rm sub}(\tau_{\rm cut})$

$$\sigma \;=\; \sigma^{\rm sub}(\tau_{\rm cut}) + \int_{\tau_{\rm cut}} d\tau\, \frac{d\sigma}{d\tau} + \Delta\sigma^{\rm sub}(\tau_{\rm cut})$$

Power corrections (I)

- Residual power corrections are numerically sizable
  $\Delta\sigma^{\rm sub}(\tau_{\rm cut}) = \sigma(\tau_{\rm cut}) - \sigma^{\rm sub}(\tau_{\rm cut})$
  - difference between $\sigma(\tau_{\rm cut})$ and global subtraction term $\sigma^{\rm sub}(\tau_{\rm cut})$

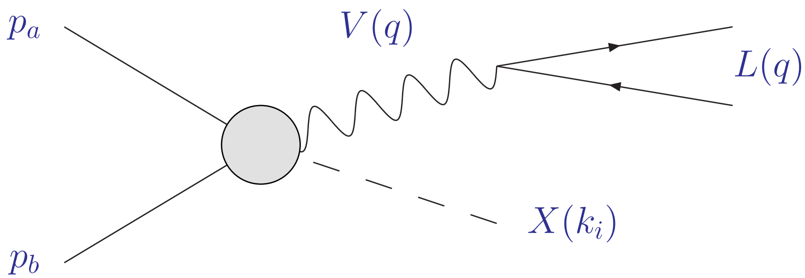
Scaling behavior of power corrections

- Exponent p in power corrections $\tau_{\rm cut}^p$ in $\sigma(\tau_{\rm cut})$
- Stable gauge boson V
  - positive integer values for exponent $p = 1, 2, 3, \dots$
  - power corrections scale as $\tau = q_T^2/Q^2$ or $\mathcal{T}_0/Q$
- Decay of V to leptonic final state L
  - p_T-cuts on leptons in decay change power counting of power corrections — Ebert, Tackmann '19
  - half-integers values $p = 1/2, 1, 3/2, \dots$ for exponent p
  - power corrections scale as $\sqrt{\tau} = q_T/Q$ or $\sqrt{\mathcal{T}_0/Q}$

Cuts on leptons in decay (I)

- Experimental data has cuts on p_T of leptons
  - ATLAS data with $p_T^l \geq 20$GeV for decay of Z-boson (or $p_T^{l/\nu} \geq 25$GeV for W-boson)
- p_T-cuts break azimuthal symmetry in phase space integration
  - recall e.g. Drell-Yan process with $L(q) \to l^-(p_1) + l^+(p_2)$

$$a(p_a) + b(p_b) \quad \to V(q) + X(k_i) \quad \to l^-(p_1) + l^+(p_2) + X(k_i)$$



- Leptonic final state phase space Φ_L

$$\Phi_L(q_T) \; = \; \frac{1}{4\,\pi^2} \int_0^{\pi} d\phi \int_{-\infty}^{\infty} d\Delta y \, \frac{p_{T1}^2}{Q^2}$$

  - ϕ (azimuthal angle) and Δy (difference in rapidity between l^- and l^+)
- Lepton p_T-cuts $p_{T1}, p_{T2} \geq p_T^{\rm min}$ induce power corrections of order $\vec{k}_{T,i}$ for hadronic final state $X(k_i)$

Cuts on leptons in decay (II)

- p_T-cuts restrict phase space to $\min\{p_{T1}, p_{T2}\} \geq p_T^{\min}$
  - $(p_{T1})^2 = p_T^2$
  - $(p_{T2})^2 = (q_T + p_{T1})^2 = p_T^2 - 2p_T q_T \cos\phi + q_T^2$
- Azimuthal integration for small q_T under condition

$$\min\{p_T, p_T^2 - 2p_T q_T \cos\phi\} = \begin{cases} p_T^2\,, & \cos\phi \leq 0 \\ p_T^2 - 2p_T q_T \cos\phi\,, & \cos\phi > 0 \end{cases}$$

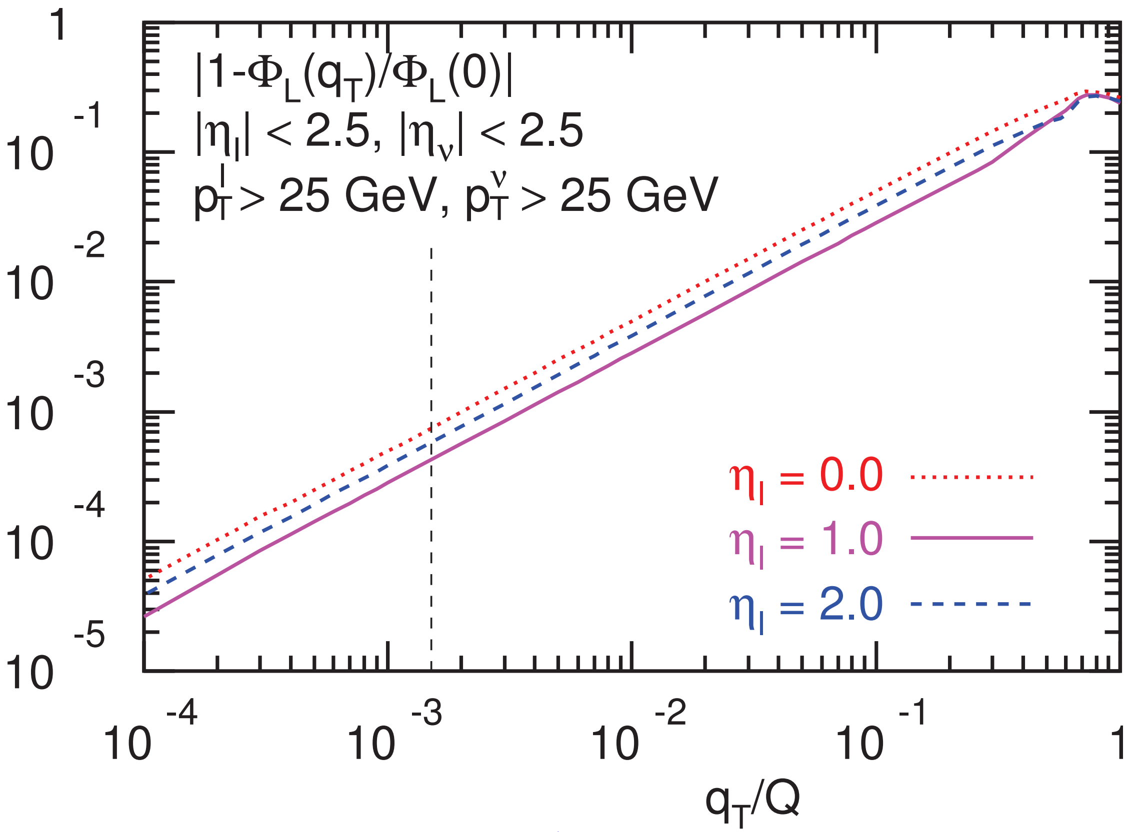
- Leptonic phase space $d\Phi_L$ after integration yields

$$d\Phi_L = \pi\sqrt{1-(2p_T^{\min})^2/Q^2} - \frac{4}{\sqrt{1-(2p_T^{\min})^2/Q^2}}\frac{p_T^{\min}\,q_T}{Q^2}$$

- η_l-cuts impose additional restriction on azimuthal angle ϕ
  - value ϕ^* for boundary of phase space depends on gauge-boson rapdity Y and on $\Delta y = \eta_1 - \eta_2$

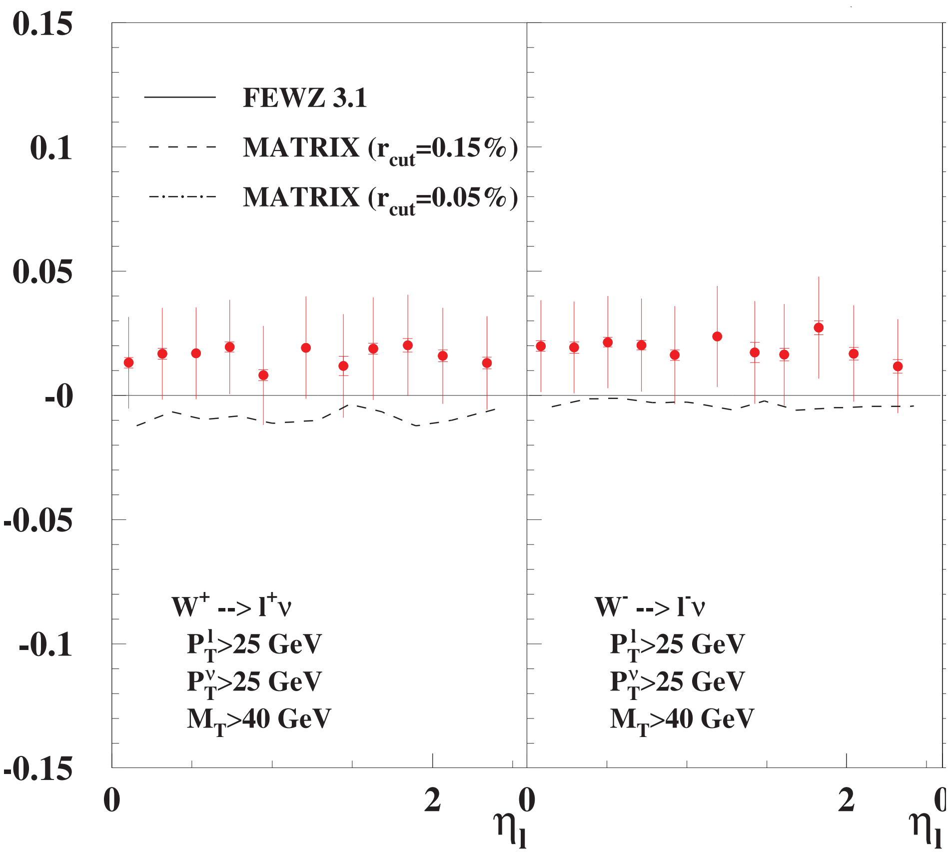
$$\cos\phi^* = \frac{Q}{2q_T}\frac{\sinh(2Y)}{\sinh(2Y+\Delta y)} + \mathcal{O}(q_T/Q)$$

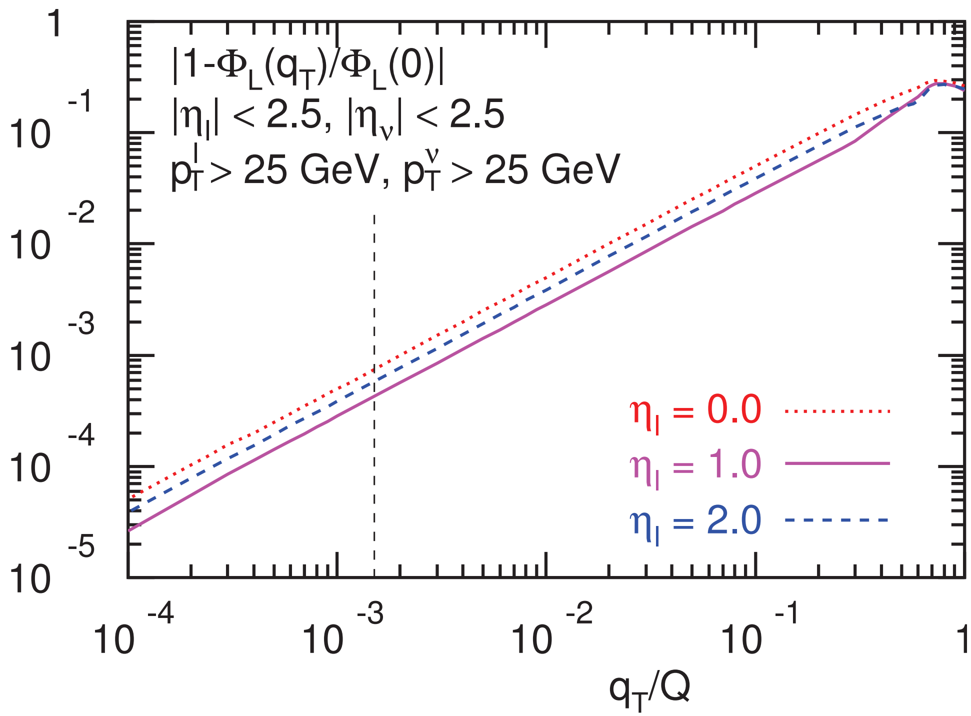
Lepton decay phase space (I)



- Plot of $|1 - \Phi_L(q_T)/\Phi_L(0)|$ for $W^\pm$-production with cuts in ATLAS data
  - deviations of $\Phi_L(q_T)$ from Born level leading power results for $q_T = 0$
- Pseudo-rapidity distribtions for $W^\pm$-production fix η_l
- p_T-cuts lead to linear power corrections in q_T
- MATRIX technical cutoff $r_{\rm cut}^{\rm min} = 0.15\%$ indicated by vertical dashed line

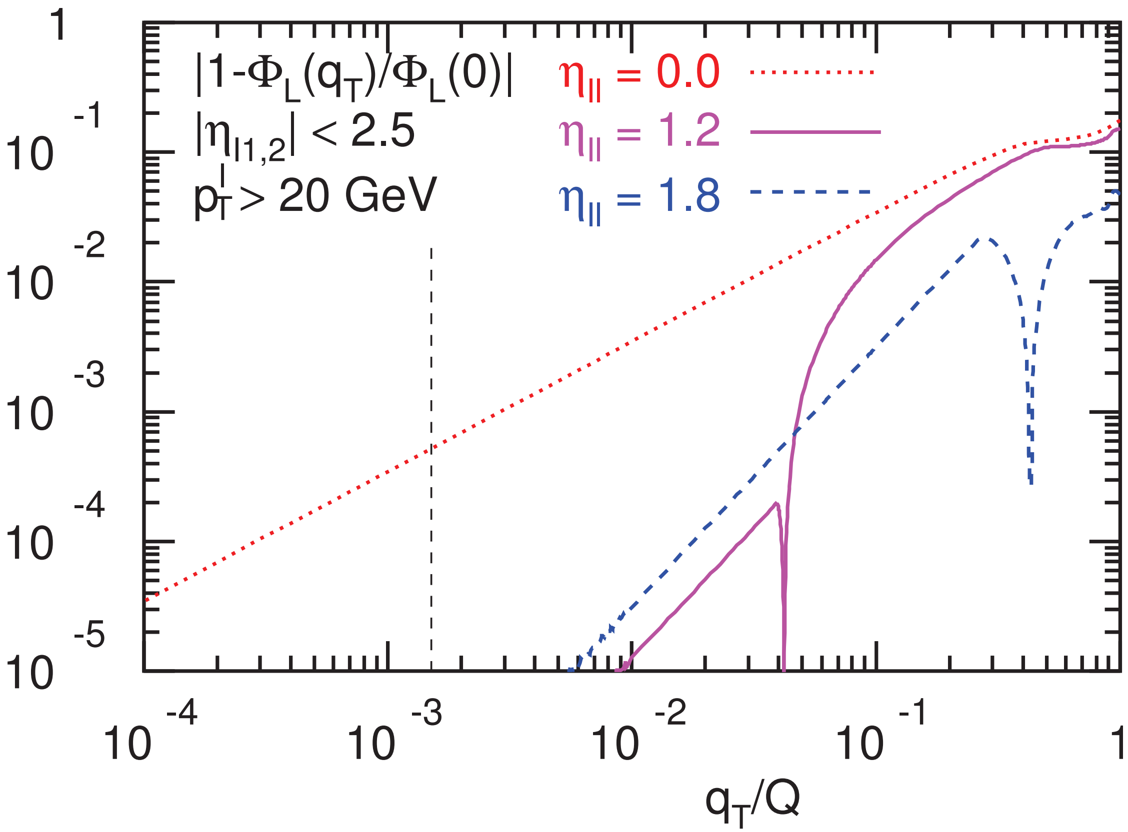
W-production with MATRIX





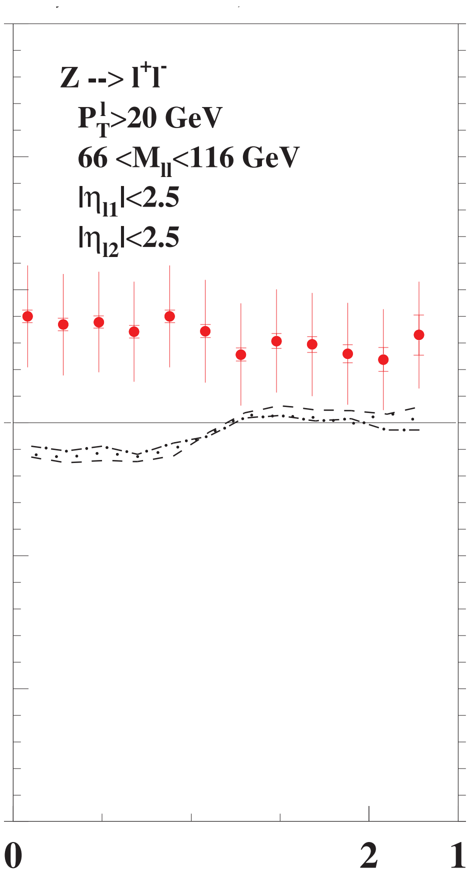
- Linear power corrections in q_T in leptonic phase space $d\Phi_L$ present in all η_l bins
- Deviations of MATRIX from FEWZ uniform across all η_l bins
  - up to $\mathcal{O}(1\%)$ for W^+-production
  - few per mill for W^--production

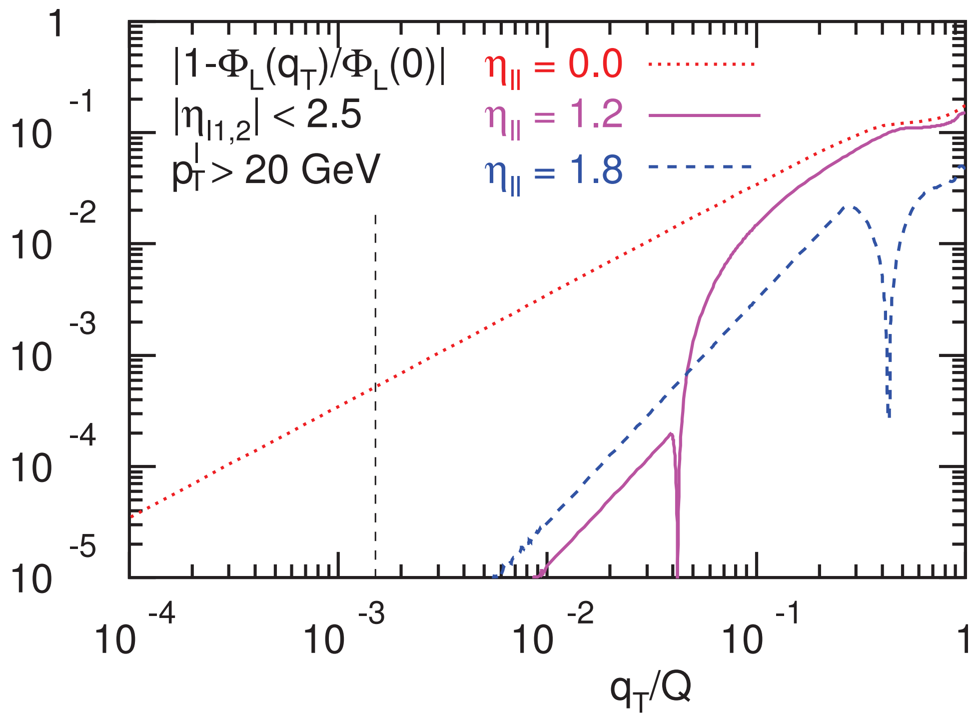
Lepton decay phase space (II)



- Plot of $|1 - \Phi_L(q_T)/\Phi_L(0)|$ for central Z-production with ATLAS cuts
- Power corrections depend on gauge boson pseudo-rapidity η_{ll}
- Small η_{ll}: cuts on p_T dominate and lead to linear power corrections in q_T
- Large η_{ll}: cuts on pseudo-rapidities η_{l_1}, η_{l_2} dominate
  - azimuthal symmetry is restored for small q_T, when $|\cos\phi^*| \geq 1$
  - transition between linear and quadratic behavior around $\eta_{ll} \simeq 1.2$

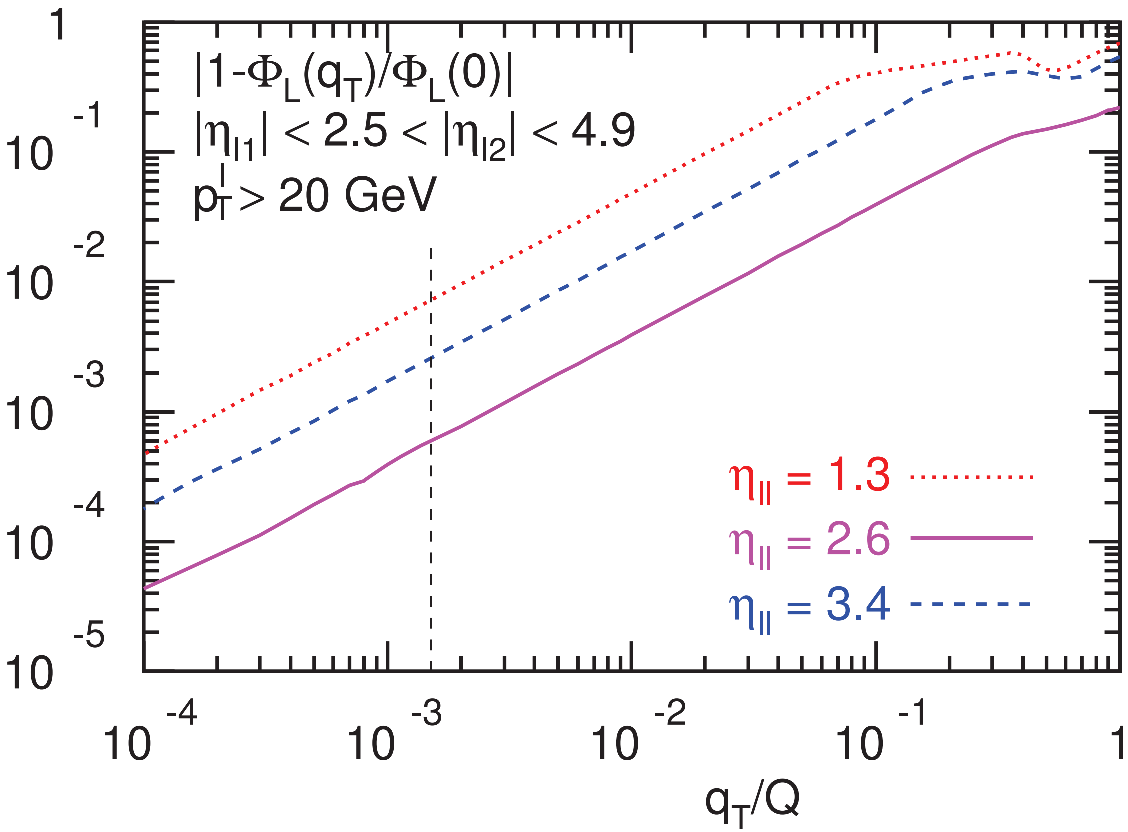
Central Z-production with MCFM





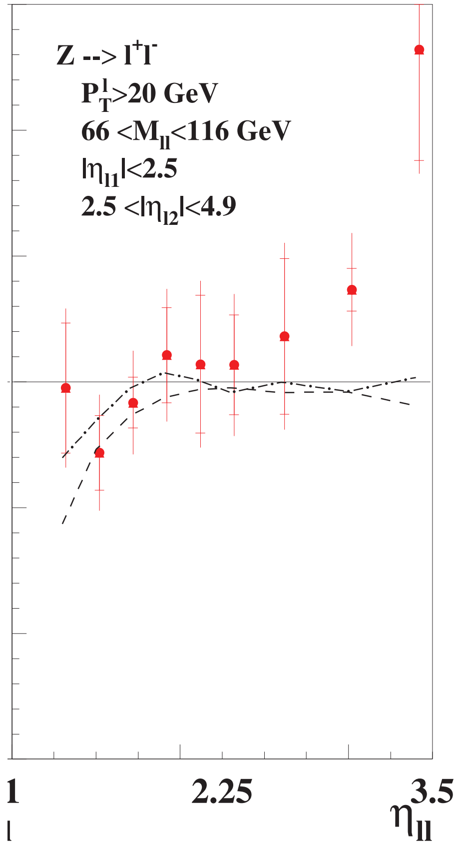
- Deviations of MCFM from FEWZ display particular pattern as function of pseudo-rapidity η_{ll}
  - consistent with appearance/disappearance of linear power corrections in q_T in leptonic phase space $d\Phi_L$

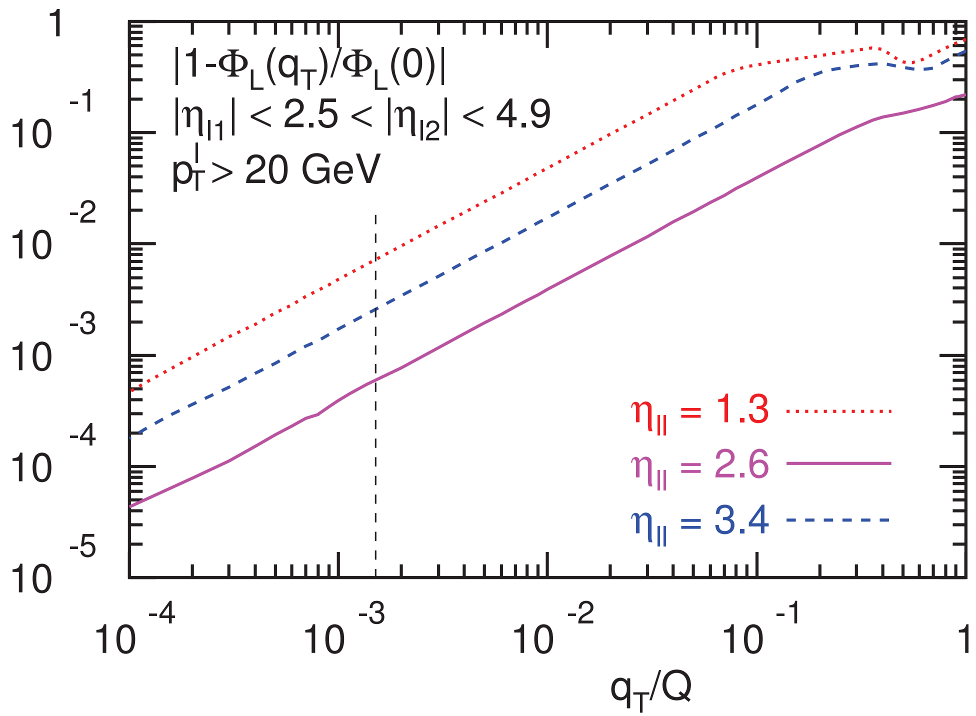
Lepton decay phase space (III)



- Plot of $|1 - \Phi_L(q_T)/\Phi_L(0)|$ for forward Z-production with ATLAS cuts
- Cuts on pseudo-rapidities $|\eta_{l_1}| \leq 2.5 \leq |\eta_{l_2}| \leq 4.9$ do no overlap
- p_T-cuts lead to linear power corrections in q_T
- Size of linear power corrections depend on pseudo-rapidity η_{ll}
  - power corrections for small and large η_{ll} are order of magnitude larger than for $W^{\pm}$-production

Forward Z-production with MATRIX





- Linear power corrections in q_T in leptonic phase space $d\Phi_L$ present in all η_{ll} bins
  - $d\Phi_L(\eta_{ll} = 1.3) \gg d\Phi_L(\eta_{ll} = 3.6) \gg d\Phi_L(\eta_{ll} = 2.4)$
- Size of deviations of MATRIX from FEWZ display same pattern as function of pseudo-rapidity η_{ll}

Summary

$W^\pm$- and Z-boson production at the LHC

- Currently available public codes with short-comings
  - long run times
  - problems with precision (short-comings of slicing methods)
- Sizable differences between public codes
  - differences at least similar, sometimes larger than sizes of NNLO corrections
- Linear power corrections from cuts on final state momenta affect precision of NNLO results

Upshot

- Need for fast and precise public code for differential distribution at NNLO (per mill level accuracy)