Local Unitarity and IR safety

A locally IR-safe representation of differential cross-sections

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ETH Zürich

RADCOR-Loopfest 2021
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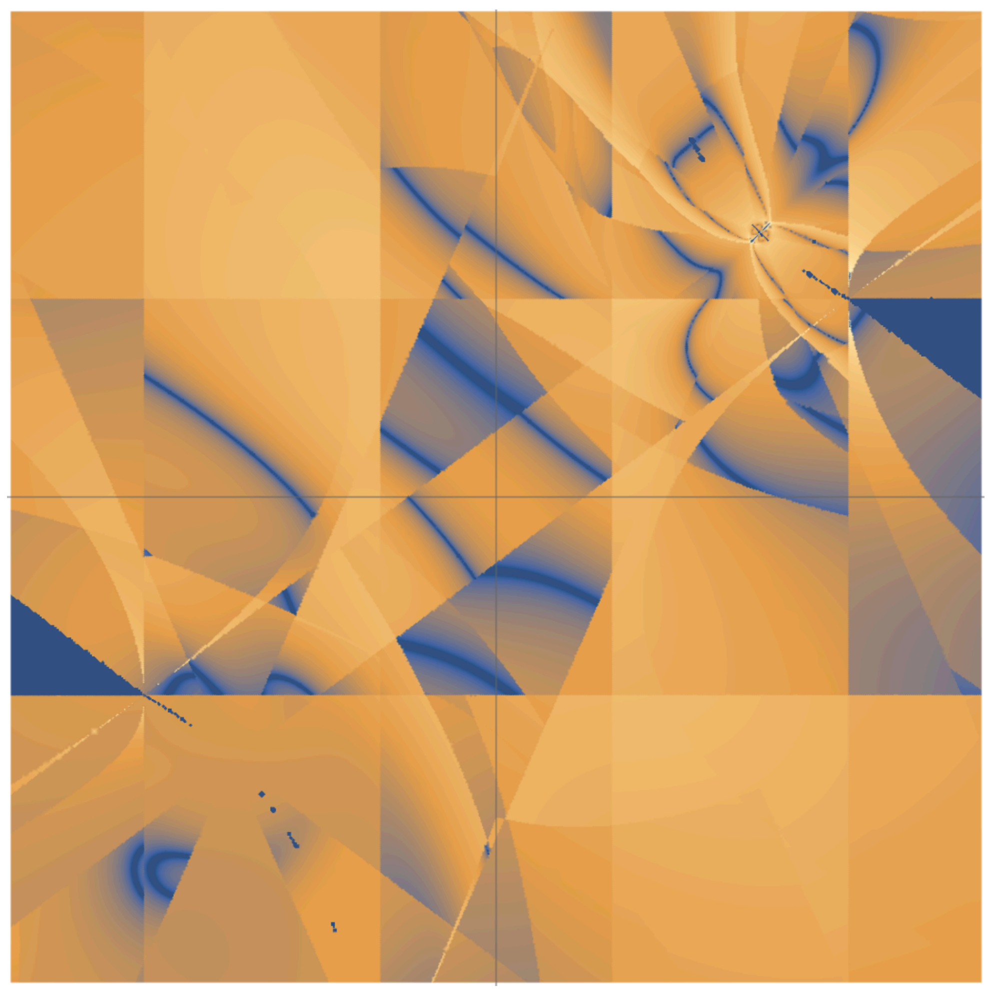

Pictorial representation of the $e^+e^- \to 2j$ @ NLO *differential x-section*

2010.01068: in collaboration with V. Hirschi, A. Pelloni and B. Ruijl
1906.06138, 1912.09291, 2009.05509: in collaboration with V. Hirschi, D. Kermanschah, A. Pelloni and B. Ruijl

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Our objective

Computing cross-sections fully numerically by locally combining real and virtual contributions

That is: Find a representation of perturbative cross-sections in the form

$$\sigma = \sum_{L=1}^{\infty} \alpha^L \int \mathrm{d}\Pi_L \sigma_{\mathrm{d}}^{(L)}$$

where $\sigma_{\mathrm{d}}^{(L)}$ is an **integrable** function, can be MonteCarlo integrated.

Want method to be **generic (scattering process and perturbative order) and competitive**, yield new results in **reasonable time** with **limited resources**

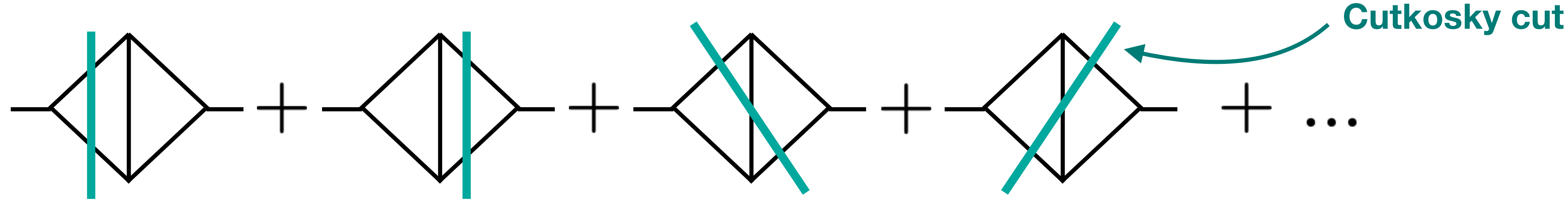
All pieces needed to do this are now available

Local Unitarity: framing the problem

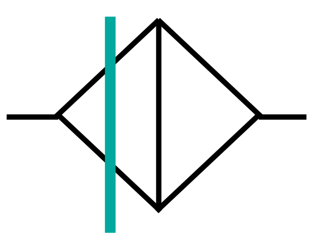
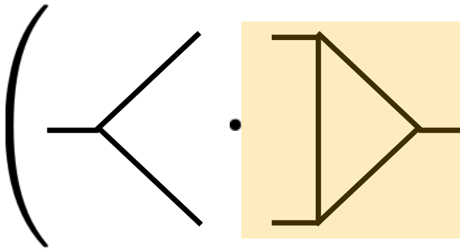
A cross-section admits a perturbative expansion when $\alpha < 1$

$$\sigma = \sum_{L=1}^{\infty} \alpha^L \sigma^{(L)}$$

The coefficients can be represented as a sum of interference diagrams

$\sigma^{(2)} =$ 

Interference diagrams themselves can be represented as integrals of amplitudes

 $= \int \mathrm{d}^3\vec{p}\, \delta(E_4 + E_5 - Q_0) \Big($  $\Big)$

Phase space integral

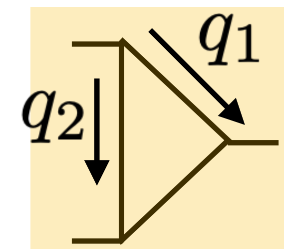
Problem: both integrals are divergent!

- Collinear divergences $q_1 // q_2$
- Soft divergences $q_1 = 0$

} **Non-integrable**

- Thresholds

} **Integrable**



$= \int \frac{\mathrm{d}^4 k}{D_1 D_2 D_3}$

Loop integral

Loop integrals $\mathrm{d}^4 k$

LTD/cLTD/TOPT
Causal flow

$\mathrm{d}^3\vec{p}$ **Phase space integrals**

Infrared singularities

- **Final state singularities (FSS)**
- **Initial state singularities (ISS)**

Integrable singularities

Loops

Infrared singularities

- **Final state radiation (FSR)**
- **Initial state radiation (ISR)**

Trees

KLN theorem (FSS ↔ FSR)

This subdivision **hides an inherent simplicity**

Integrals

ISS + ISR

Integrable singularities

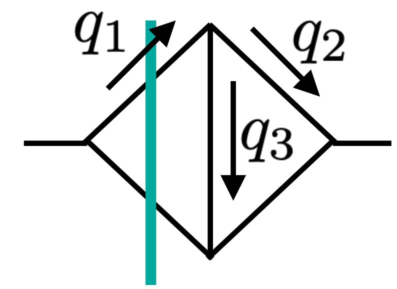
Trees

IR singularities appear in **separate pieces** of the computation of LHC observables, but **not in the final result** (IR-safety)

Forcing IR-safety to be realised **locally** loosens the distinction between phase space and loop integrals

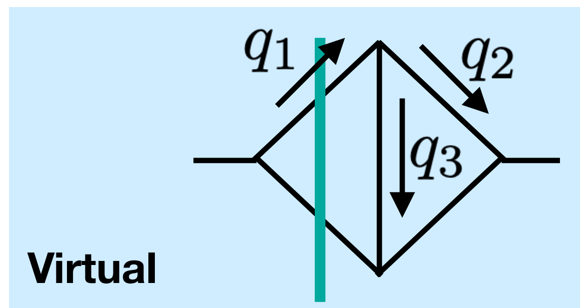
Real and virtual contributions

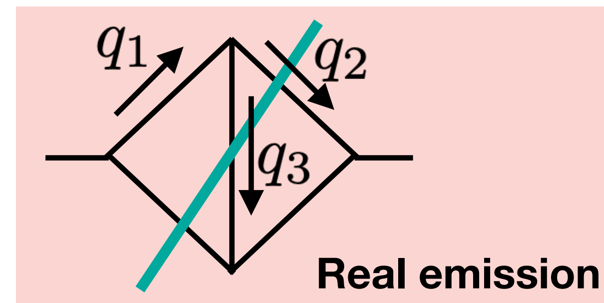
Interference diagram may have a collinear singularity, e.g.



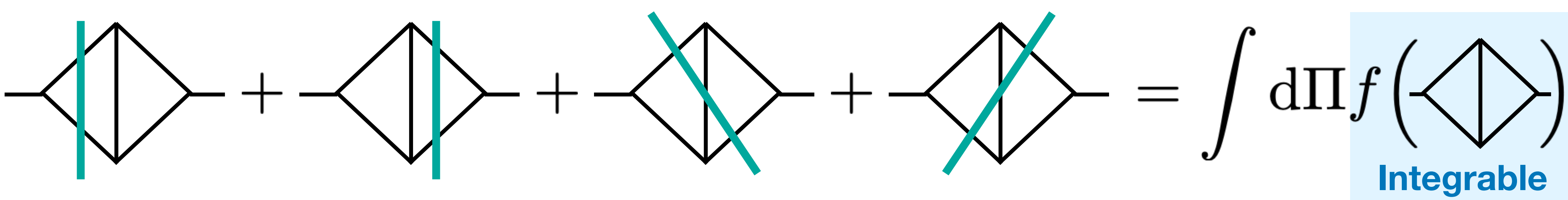
$$\begin{cases} q_2 = x q_1 \\ q_3 = (1-x) q_1 \end{cases}$$

This sum of diagrams is finite in this collinear limit (KLN theorem)

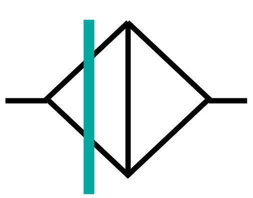




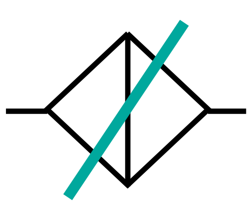
Sum over all the Cutkosky cuts of the double triangle is finite in any IR limit



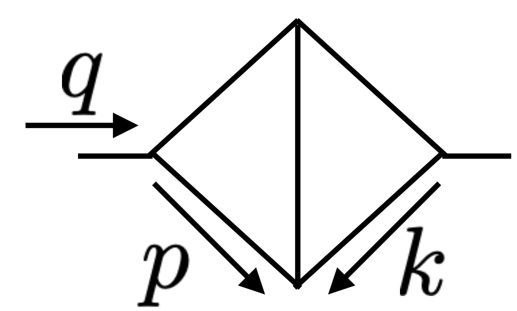
Problem: there is a difference in dimensionality between phase space and loop integrals



$$\frac{d^3\vec{p}}{2|\vec{p}|} d^4k\, \delta(|\vec{p}| + |\vec{p} - \vec{q}| - Q_0)$$



$$\frac{d^3\vec{p}}{2|\vec{p}|}\frac{d^3\vec{k}}{2|\vec{k}|}\delta(|\vec{p}| + |\vec{p} - \vec{k}| + |\vec{k} - \vec{q}| - Q_0)$$



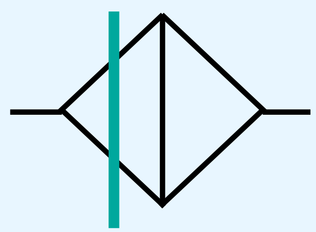
Loop Tree Duality

The **LTD representation** allows for explicit integration of the energy components using residue theorem

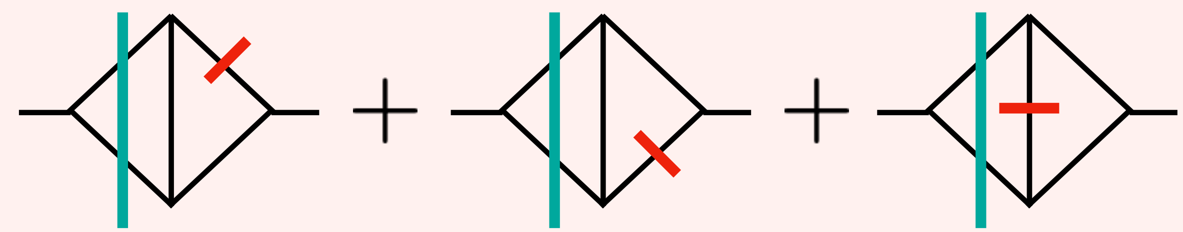
$$\int \left[\prod_{m=1}^{M} \mathrm{d}^4 k_m\right] \frac{N}{\prod_i D_i} = \int \left[\prod_{m=1}^{M} \mathrm{d}^3 \vec{k}_m\right] f_{\mathrm{ltd}}$$

With this result, both loop and phase space integrals are defined over 3D space

$$\frac{\mathrm{d}^3\vec{p}}{2|\vec{p}|}\mathrm{d}^4 k\,\delta(|\vec{p}|+|\vec{p}-\vec{q}|-Q_0) \rightarrow \frac{\mathrm{d}^3\vec{p}}{2|\vec{p}|}\frac{\mathrm{d}^3\vec{k}}{2|\vec{k}|}\delta(|\vec{p}|+|\vec{p}-\vec{q}|-Q_0)$$



$=$



Catani, Gleisberg, Krauss, Rodrigo, Winter
arXiv: 0804.3170 (2008)

Bierenbaum, Catani, Draggiotis, Rodrigo
arXiv: 1007.0194 (2010)

Runkel, Ször, Vesga, Weinzierl
arXiv: 1902.02135 (2019)

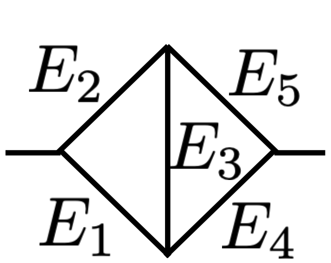
Verdugo, Driencout-Mangin, et al.
arXiv: 2001.03564 (2020)

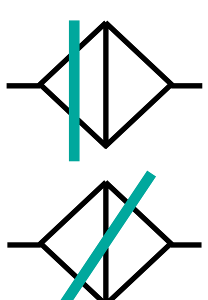
ZC, Hirschi, Kermanshah, Ruijl
arXiv: 1906.06138 (2019)

ZC, Hirschi, Kermanshah, Pelloni, Ruijl
arXiv: 2009.05509 (2020)

Automation of LTD and cLTD (arbitrary loops, topologies, numerators)

Applying LTD to the interference diagrams, we can bring them under the same integral sign





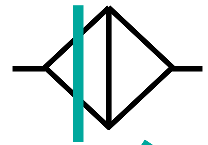
$$= \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\,\left(\delta(E_1+E_2-Q_0)f_{\mathrm{virt}} + \delta(E_1+E_3+E_5-Q_0)f_{\mathrm{real}}\right)$$

Causal flow

$(\vec{p},\vec{k}) \to \vec{\phi}(t,(\vec{p},\vec{k}))$ $\begin{cases}\partial_t\vec{\phi} = \vec{\kappa}\circ\vec{\phi} \\ \vec{\phi}(0,(\vec{k},\vec{l})) = (\vec{k},\vec{l})\end{cases}$

Why "causal flow"?

The measure now differs only in the **delta enforcing on shell energy conservation**

 $\sim \delta(E_1 + E_2 - Q_0)$

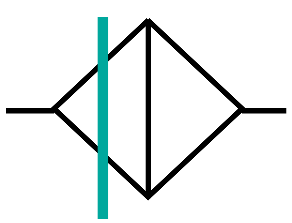
 $\sim \delta(E_1 + E_3 + E_5 - Q_0)$

Find a variable to solve both deltas. Here the first energy works, in general there is not a unique energy that allows that.

Solution: introduce a fictitious variable in which to solve the delta

$$\delta(|\vec{k}| - Q_0) \underset{\vec{k}\to t\vec{k}}{\longrightarrow} \delta(t|\vec{k}| - Q_0) \longrightarrow t = \frac{Q_0}{|\vec{k}|}$$

Then

 $$= \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\delta(E_1 + E_2 - Q_0) f_{\text{virt}} = \int \mathrm{d}^3\vec{k}\mathrm{d}^3\vec{p}\, g_{\text{v}}(t^\star_{\text{v}})$$

where $t^\star_{\text{v}} = t^\star_{\text{v}}(\vec{k},\vec{p}) = \dfrac{Q_0}{E_1 + E_2}$

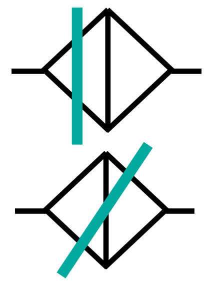
Soper, arXiv: 9804454 (1998)

Soper, arXiv: 0102031 (2001 @ RADCOR)

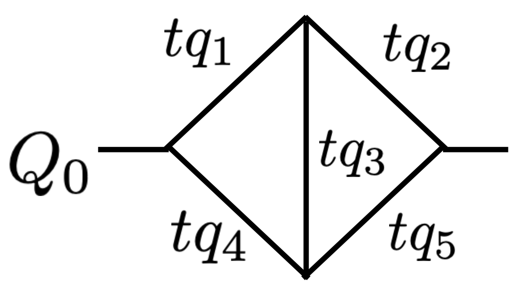
ZC, Hirschi, Pelloni, Ruijl arXiv: 2010.01068 (2020)

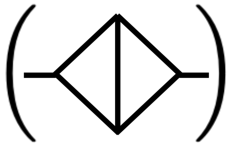
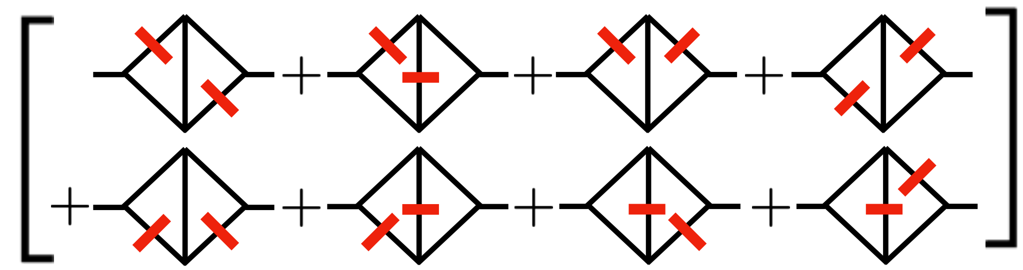
General FSR cancellations For N to M NkLO processes

Apply same procedure to real…

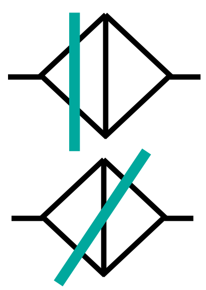
Then:  $= \int \mathrm{d}^3\vec{p}\,\mathrm{d}^3\vec{k}\,\big(g_{\rm v}(t^\star_{\rm v}) + g_{\rm r}(t^\star_{\rm r})\big)$ **We have aligned the measure!**

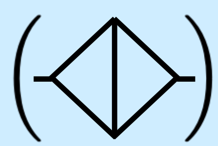
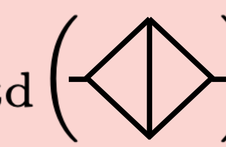
The LTD representation of the double triangle with rescaled momenta is



$f_{\rm ltd}\Big($  $\Big)\Big|_{tq_i} =$  $_{q_i \to tq_i}$

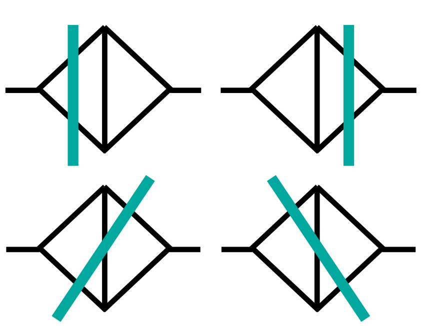
Then

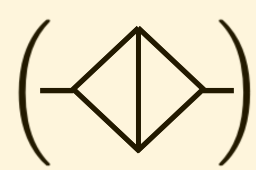


$= \int \mathrm{d}^3\vec{p}\,\mathrm{d}^3\vec{k}\Big[\lim_{t\to t^\star_{\rm v}} (t - t^\star_{\rm v}) f_{\rm ltd}\Big($  $\Big)\Big|_{tq_i} + \lim_{t\to t^\star_{\rm r}} (t - t^\star_{\rm r}) f_{\rm ltd}\Big($  $\Big)\Big|_{tq_i}\Big]$

$g_{\rm v}$, $g_{\rm r}$ can be written as different limits of the same function!

Solving delta in the scaling variable $\Longrightarrow$ 1d residue theorem along the line $\gamma(t) = (t\vec{k}, t\vec{p})$



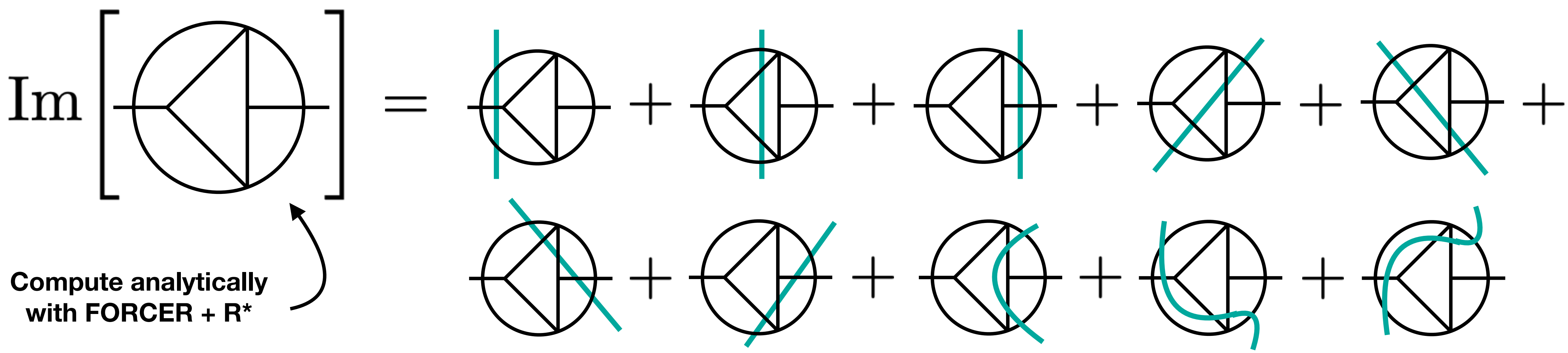
$= \int \mathrm{d}^3\vec{p}\,\mathrm{d}^3\vec{k}\,\Big[\sum_{i=1}^{4} \lim_{t\to t^\star_i} (t - t^\star_i) f_{\rm ltd}\Big($  $\Big)\Big|_{tq_i}\Big] = \sigma_{\rm d}$

LU representation

Cutkosky, but at the local level!

Local IR cancellations: 5-loop example

We proved cancellations rigorously for FSR singularities. Here we use an example

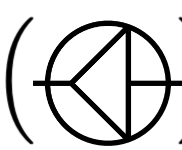


Compute analytically with FORCER + R*

B. Ruijl, T. Ueda, J. Vermaseren
arXiv: 1704.06650 (2017)

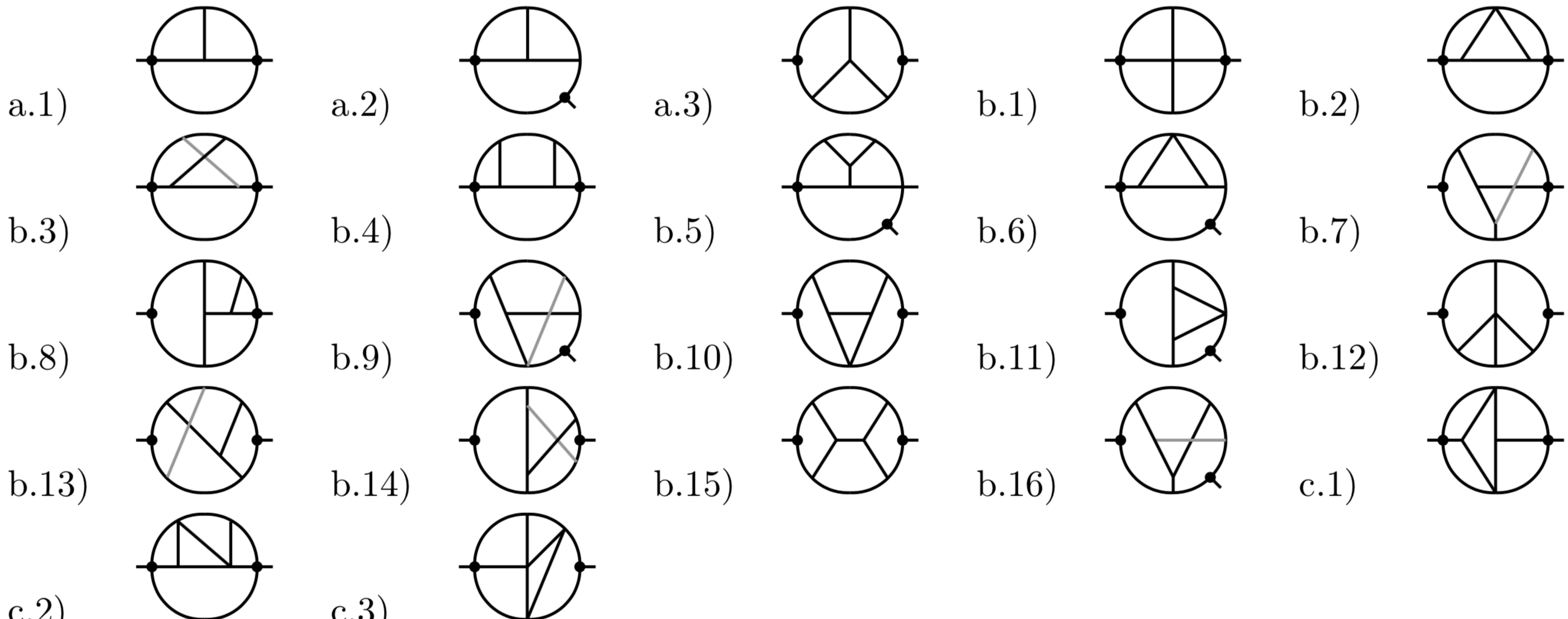
F. Herzog, B. Ruijl
arXiv: 1703.03776 (2017)

$$= \int \left[\prod_{j=1}^{5} \mathrm{d}^3 \vec{k}_j \right] \sum_{i=1}^{10} \lim_{t \to t_i} (t - t_i) f_{\mathrm{ltd}} \Big(\Big)$$



Monte Carlo Integration

$\mathtt{N_p}$ $[10^6]$	t/p [μs] min	t/p [μs] avg	$\mathtt{N_{ch}}$	FORCER [GeV^2]	αLoop [GeV^2]	exp.	Δ [σ]	Δ [%]
Inclusive cross-section per supergraph								
1	1100	49000	128	1.66419	1.6691(79)	-9	0.62	0.0029

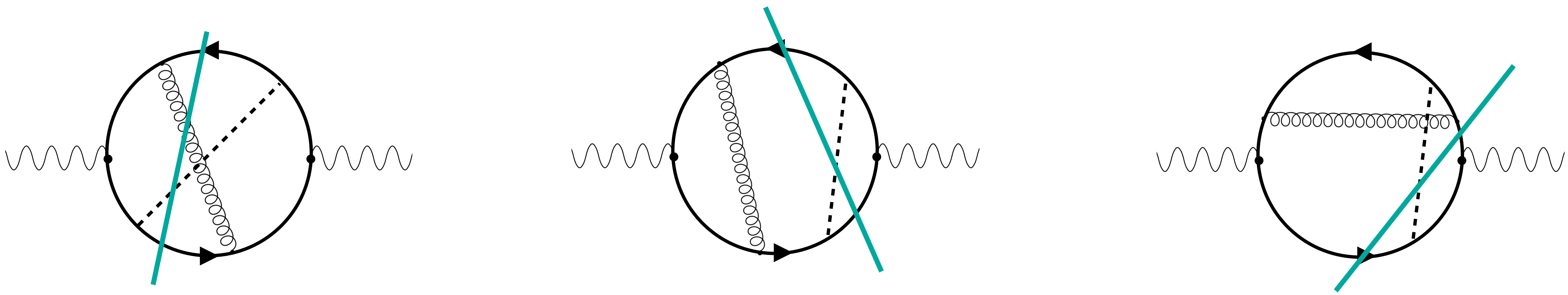


We did the same for all 3-4-5 loop two-point functions that are finite in scalar theory

Inclusive $e^+e^- \to t\bar{t}H$ @ NLO

Same procedure is applied to physical case.

This has many forward-scattering diagrams and Cutkosky cuts, e.g.



	name	multiplicity	neval	real	real_err	eval_time
0	SG_QG0	2.000000e+00	5850000	6.689035e-05	9.240544e-08	0 days 00:15:35.553646000
1	SG_QG2	2.000000e+00	2080000	2.349607e-05	4.541978e-08	0 days 00:00:41.443805000
2	SG_QG6	1.000000e+00	2080000	-8.346356e-05	9.293410e-08	0 days 00:00:53.087342000
			...			
14	SG_QG46	2.000000e+00	2080000	3.534058e-05	3.903003e-08	0 days 00:00:59.076110000
15	SG_QG47	2.000000e+00	2080000	-1.618672e-06	1.686635e-09	0 days 00:00:09.248204000

15 forward-scattering diagrams

O(50) interference diagrams

```
Pure NLO correction:
MG res:          -1.38400e-04 +/- 1.4e-07
aL real res:     -1.38320e-04 +/- 5.9e-07
|(MG-aL)/MG|:     5.75e-04
```

Matches benchmark
From MG5_aMC@NLO

Only analytic integration for UV counter-terms

No IR counter-terms

Alwall, Frederix, Frixione, Hirschi, Maltoni
arXiv: 1405.0301 (2014)

So how is it different from the optical theorem?

- We can be **differential**!
- The exact same formalism will take care of FSR for **generic 2-to-N processes.**

Choose **any** IR-safe observable

$$\sigma_{\mathrm{d}} = \sum_{i=1}^{N} \lim_{t \to t_i} (t - t_i) f_{\mathrm{ltd}}$$

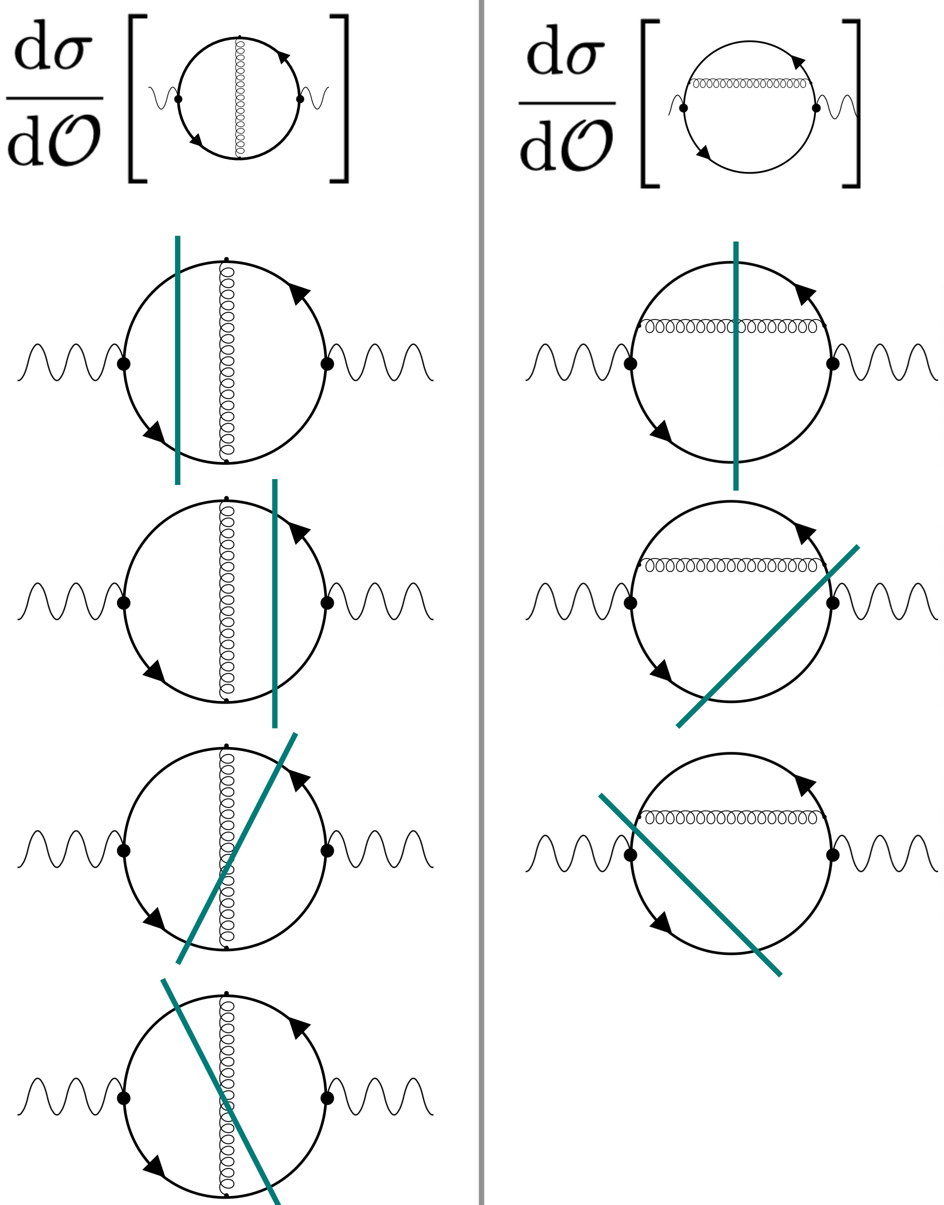
$$\downarrow$$

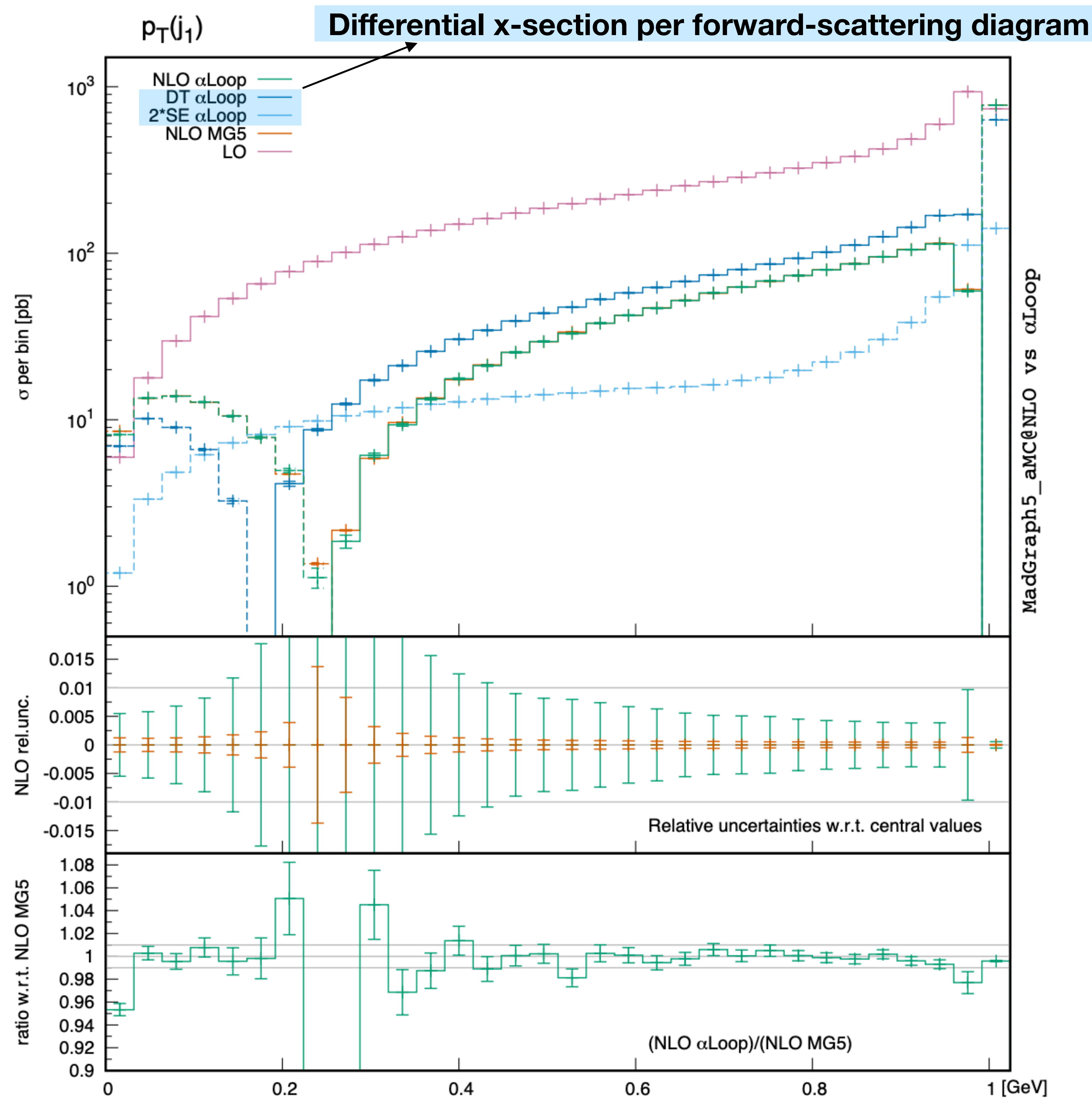
$$\sigma_{\mathrm{d}} = \sum_{i=1}^{N} \lim_{t \to t_i} (t - t_i) f_{\mathrm{ltd}} \mathcal{O}_i$$

For example, we can apply this to

$$e^+ e^- \to 2j \ @ \ \mathrm{NLO}$$

and be differential in the transverse momentum of the leading jet





Differential case also **requires** threshold regularisation

Construct a contour deformation that is causal

$$\vec{k} \to \vec{k} - \mathrm{i}\kappa \text{ with } \kappa \cdot \nabla \eta_i > 0 \text{ if } \eta_i = 0$$

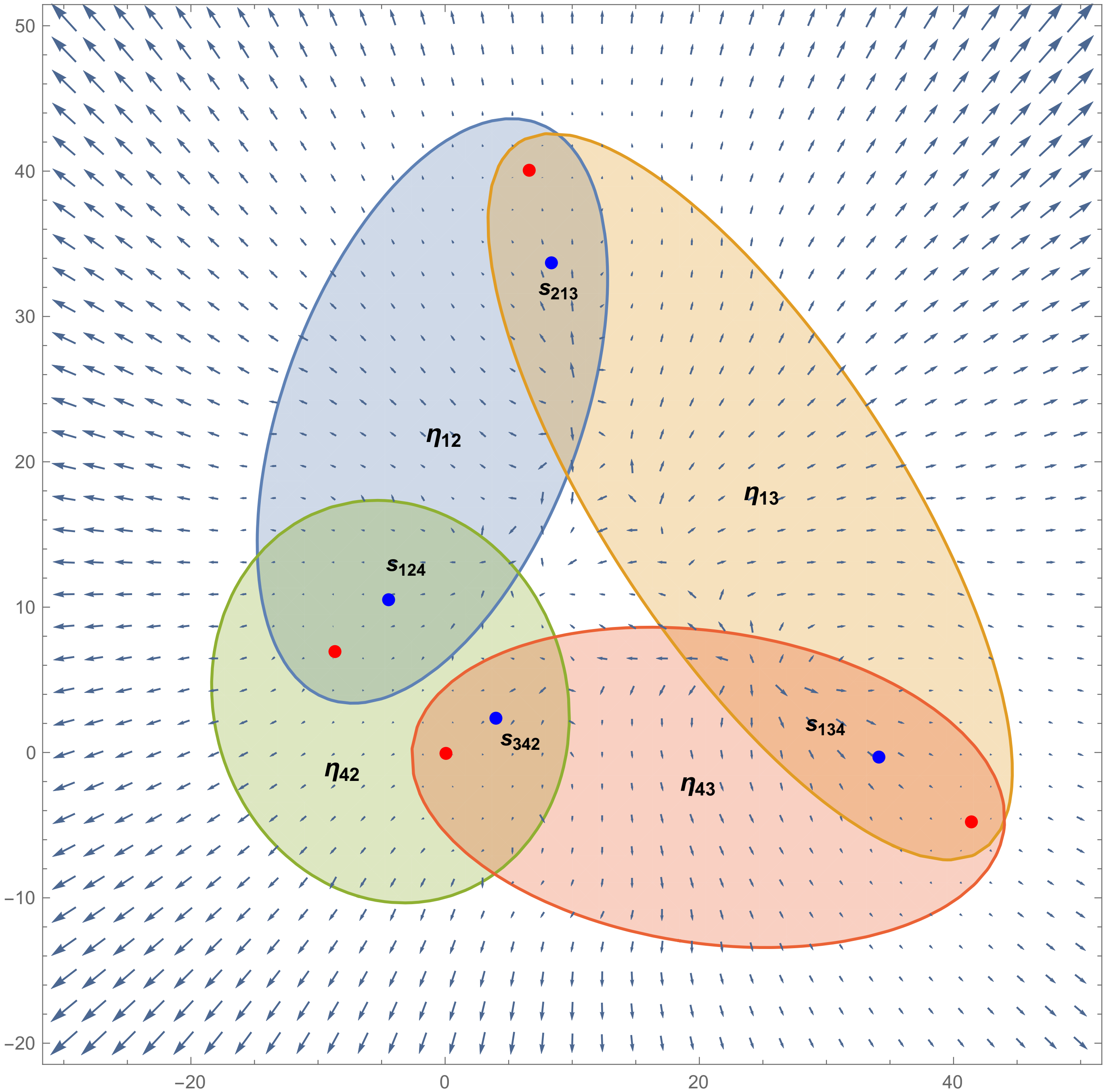
(plus some magnitude constraints)

Gong, Nagy, Soper arXiv:0812.3686 (2009)

Becker, Reuschle, Weinzierl arXiv:1010.4187 (2010)

Buchta, Chachamis, et al. arXiv:1510.00187 (2017)

ZC, Hirschi, Kermanschah, Pelloni, Ruijl arXiv: 1510.00187 (2019)



The deformation has to be **reliable**, **generic** and **efficient**

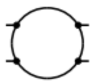
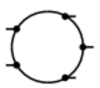
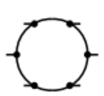
The result must be **hyper-parameter independent**

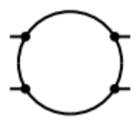
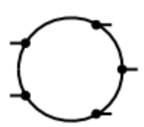
This is complicated, both theoretically and practically, we solved this problem in

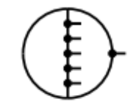
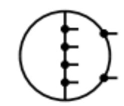
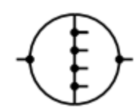
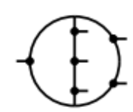
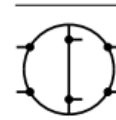
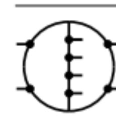
ZC, Hirschi, Kermanschah, Pelloni, Ruijl arXiv: 1510.00187 (2019)

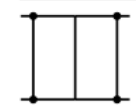
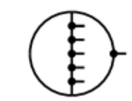
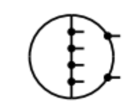
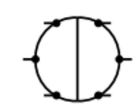
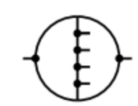
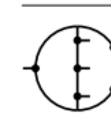
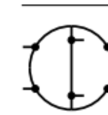
How to prove efficiency and automation: **test your theory on the most disparate topologies!**

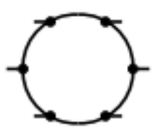
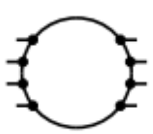
Testing the deformation on finite topologies

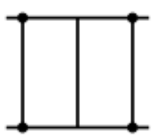
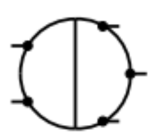
Topology	Numerical LTD
 Box4E	-6.57637 +/- 0.00122
	-7.43805 +/- 0.00121
 1L5P	-3.44317 +/- 0.00045
	-2.56505 +/- 0.00046
	-0.00036 +/- 0.00029
	5.97143 +/- 0.00029
	-0.83888 +/- 0.00016
	-1.71325 +/- 0.00017
	-3.49044 +/- 0.00054
	-3.89965 +/- 0.00054
	0.90036 +/- 0.00076
	4.17823 +/- 0.00080
	0.04227 +/- 0.00068
	-2.18118 +/- 0.00068
 1L6P	0.03046 +/- 0.00006
	-1.17691 +/- 0.00008
	-2.07392 +/- 0.00188
	0.42593 +/- 0.00161
	1.36950 +/- 0.00052
	-2.25957 +/- 0.00053
	1.29802 +/- 0.00038
	-2.16555 +/- 0.00037
	-0.27225 +/- 0.00010
	-1.20895 +/- 0.00011
	2.83777 +/- 0.00040
	0.83144 +/- 0.00040
	-3.01976 +/- 0.00040
	-7.73280 +/- 0.00047
	2.13487 +/- 0.03230
	0.65770 +/- 0.03145
	0.00804 +/- 0.00014
	-1.15278 +/- 0.00014
	-2.81583 +/- 0.00060
	2.47308 +/- 0.00061

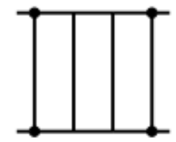
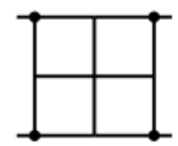
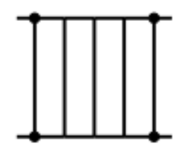
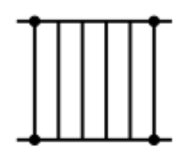
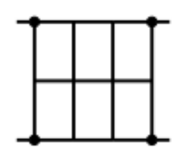
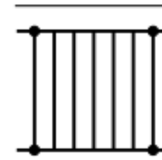
Topology	Numerical LTD
 1L4P	1.13123 +/- 0.00006
	-0.55486 +/- 0.00005
	5.71929 +/- 0.00055
	-7.24055 +/- 0.00053
	1.55376 +/- 0.00012
	-2.07005 +/- 0.00012
	1.85214 +/- 0.00012
	-2.18397 +/- 0.00012
	0.30272 +/- 0.00004
	-1.08130 +/- 0.00004
	-0.17991 +/- 0.00005
	-2.27593 +/- 0.00008
 1L5P	-1.90856 +/- 0.00074
	-6.45306 +/- 0.00077
	-0.15137 +/- 0.00032
	-1.80672 +/- 0.00033
	-0.66271 +/- 0.00032
	-1.23567 +/- 0.00032
	2.60394 +/- 0.00072
	-7.95017 +/- 0.00076
	-0.48305 +/- 0.00059
	-3.27664 +/- 0.00061
	-1.21508 +/- 0.00020
	-1.53126 +/- 0.00020

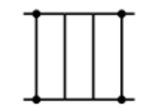
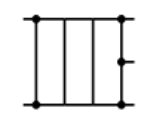
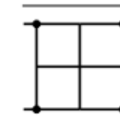
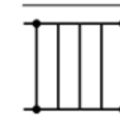
Topology	Numerical LTD
 2L6P.a	4.58688 +/- 0.05132
	5.04144 +/- 0.05075
	-1.04316 +/- 0.35247
	-4.42468 +/- 0.35421
 2L6P.b	1.17336 +/- 0.00888
	3.99809 +/- 0.00896
	5.35217 +/- 0.00153
	3.81579 +/- 0.00150
 2L6P.c	4.90974 +/- 0.01407
	-2.13974 +/- 0.01434
	1.05934 +/- 0.15850
	1.03698 +/- 0.15312
 2L6P.d	1.90487 +/- 0.05753
	-3.55267 +/- 0.05746
	-2.97419 +/- 0.00961
	-2.18847 +/- 0.00957
 2L6P.e	2.87833 +/- 0.00951
	1.99937 +/- 0.00961
	1.67332 +/- 0.00578
	-0.21788 +/- 0.00571
 2L6P.f	-0.95486 +/- 0.00890
	3.28530 +/- 0.00889
	2.55104 +/- 0.00208
	-1.63019 +/- 0.00205
 2L8P	-5.15438 +/- 0.03310
	6.78546 +/- 0.03243

Topology	Numerical LTD
 2L4P.a	3.82875 +/- 0.00015
	-4.66843 +/- 0.00017
	2.83742 +/- 0.00072
	3.38163 +/- 0.00066
 2L4P.b	-5.89794 +/- 0.00099
	0.00112 +/- 0.00095
 2L6P.a	-8.64045 +/- 0.00392
	-0.00220 +/- 0.00393
 2L6P.b	-1.19040 +/- 0.00092
	0.00147 +/- 0.00092
 2L6P.c	-7.62856 +/- 0.00716
	-0.00052 +/- 0.00724
 2L6P.d	-1.83639 +/- 0.00075
	-0.00042 +/- 0.00075
 2L6P.e	-4.61094 +/- 0.00423
	0.00404 +/- 0.00430
 2L6P.f	-1.02723 +/- 0.00111
	0.00165 +/- 0.00112

Topology	Numerical LTD
 1L6P	0.51018 +/- 0.00031
	-1.54768 +/- 0.00032
	0.60407 +/- 0.00216
	-6.96436 +/- 0.00213
	0.40655 +/- 0.00152
	-2.51588 +/- 0.00157
	1.30529 +/- 0.00289
	-2.27744 +/- 0.00284
	-2.20131 +/- 0.00241
	-6.37841 +/- 0.00254
	-1.28057 +/- 0.00088
	-2.21602 +/- 0.00088
 1L8P	5.10300 +/- 0.00400
	-1.62544 +/- 0.00373
	4.21309 +/- 0.00421
	-1.95771 +/- 0.00394
	1.26931 +/- 0.00486
	-0.84023 +/- 0.00503
	-0.35626 +/- 0.00057
	-1.46911 +/- 0.00058
	-1.16905 +/- 0.00794
	-2.72569 +/- 0.00967
	-0.57605 +/- 0.00196
	-4.04047 +/- 0.00202

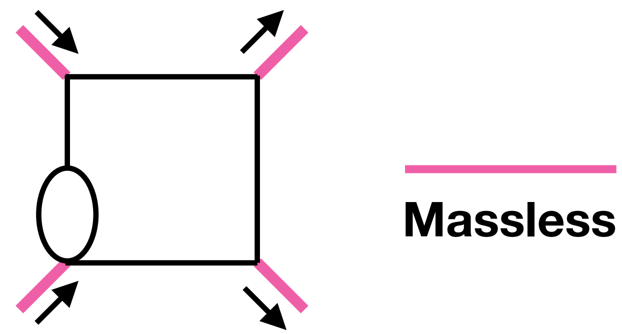
Topology	Numerical LTD
 2L4P.b	-1.08656 +/- 0.00127
	2.86702 +/- 0.00125
	3.09646 +/- 0.00696
	9.53952 +/- 0.00706
	1.70253 +/- 0.00285
	4.56488 +/- 0.00291
	2.80094 +/- 0.00023
	3.34866 +/- 0.00025
	8.15559 +/- 0.00123
	6.10277 +/- 0.00124
	3.10306 +/- 0.00021
	0.09376 +/- 0.00020
 2L5P	0.27368 +/- 0.00131
	1.44760 +/- 0.00129
	1.08568 +/- 0.00342
	1.78725 +/- 0.00339
	2.09848 +/- 0.00648
	2.04022 +/- 0.00648
	1.51586 +/- 0.00027
	1.31451 +/- 0.00027
	1.97798 +/- 0.01394
	1.13209 +/- 0.01173
	2.00638 +/- 0.00061
	-0.08277 +/- 0.00060

Topology	Numerical LTD
 3L4P	0.00796 +/- 0.00877
	-6.73786 +/- 0.00856
 4L4P.a	8.38828 +/- 0.07772
	-0.01028 +/- 0.07754
 4L4P.b	7.96654 +/- 0.11281
	0.07617 +/- 0.11858
 5L4P	3.28900 +/- 0.01964
 6L4P.a	8.36493 +/- 0.02167
 6L4P.b	1.09968 +/- 0.41729

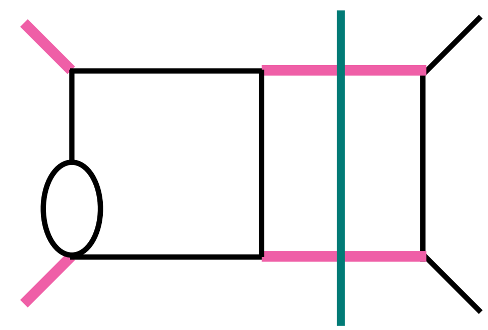
Topology	Numerical LTD
 3L4P	-2.43299 +/- 0.03927
	-3.41797 +/- 0.03956
	-5.36759 +/- 0.14110
	-1.05826 +/- 0.13399
	-4.46226 +/- 0.10022
	-0.72941 +/- 0.09918
	-3.89588 +/- 0.00173
	3.89127 +/- 0.00165
	-3.15581 +/- 0.00639
	2.97368 +/- 0.00633
	-0.10876 +/- 0.00096
	1.86939 +/- 0.00095
 3L5P	-1.06298 +/- 0.02843
	-0.88557 +/- 0.02875
	-3.28794 +/- 0.07308
	-0.29022 +/- 0.07635
	-1.61475 +/- 0.14277
	0.25654 +/- 0.13621
	-1.26220 +/- 0.00124
	1.06124 +/- 0.00123
	4.58640 +/- 0.00609
	1.80523 +/- 0.00645
	-1.05359 +/- 0.01706
	5.92117 +/- 0.01660
 4L4P.a	1.28725 +/- 0.00637
	2.95568 +/- 0.00642
 4L4P.b	-4.34119 +/- 0.01166
	-2.77244 +/- 0.01160

How to regulate Initial state singularities?

An example: $\mathcal{I} =$



Ultimately embedded in interference diagram



One way to deal with **ISS** uses **local counter-terms**

Becker, Reuschle, Weinzierl arXiv:1010.4187 (2010)

Anastasiou, Sterman arXiv:1812.03753 (2018)

Yao Ma arXiv:1910.11304 (2019)

Anastasiou, Haindl, Sterman, Yang, Zeng arXiv:2008.12293 (2020)

Diagram be subtracted with counter-terms, which are then computed analytically

$$I = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{1}{k^2 (k+p_1-p_3)^2 (k+p_1)^2 (k-p_2)^2} \left(1 - \frac{k^2 + (k+p_1-p_3)^2}{t} - \frac{(k+p_1)^2 + (k-p_2)^2}{s}\right) B$$

Counter-terms

$$B = \left(\frac{1}{l^2 (k+l)^2} - \frac{1}{l^2 (l+p_1+p_2)^2}\right)$$

Anastasiou, Sterman arXiv: 1812.03753 (2018)

In the physical regime (**s>0**), **still has thresholds**. We **use a contour deformation**

Numerical integration $I = -(4.110(13) + \mathrm{i}2.0084(87)) \cdot 10^{-5}\ \mathrm{GeV}^{-4}$

Analytic benchmark

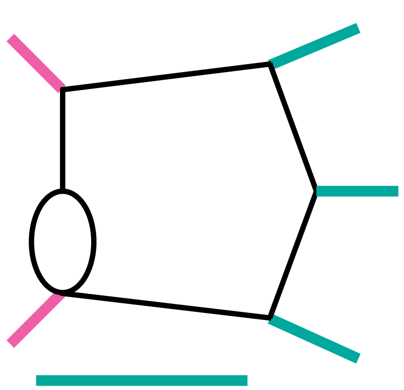
$s = 4$

$t = -0.12305$

$$I = \frac{1}{(16\pi^2)^2}\left[\frac{\ln(-t/s)}{3st}(\ln(-t/s) + \mathrm{i}\pi)(\ln(-t/s) + 2\mathrm{i}\pi)\right]$$
$$= -(4.10310 + 2.01997\mathrm{i})10^{-5}\ \mathrm{GeV}^{-4}$$

We can up the difficulty to add many more scales

e.g.



Massive $(p_i^2 > 0)$

$$q_1^2 = 0,\ q_2^2 = 0,\ p_1^2 = 6.46094 \cdot 10^3\mathrm{GeV}^2,\ p_2^2 = 8.31562 \cdot 10^4\mathrm{GeV}^2,\ p_3^2 = 6.46094 \cdot 10^3\mathrm{GeV}^2,$$
$$(q_1 - p_1)^2 = -3.47368 \cdot 10^5\mathrm{GeV}^2,\ (q_1 - p_1 - p_2)^2 = -5.56970 \cdot 10^5\mathrm{GeV}^2,\ (q_1 + q_2)^2 = 10^6\mathrm{GeV}^2,$$
$$(p_1 + p_2)^2 = 2.54722 \cdot 10^5\mathrm{GeV}^2,\ (p_2 + p_3)^2 = 3.06933 \cdot 10^5\mathrm{GeV}^2$$

$$I = -(2.29234(81) + \mathrm{i}0.20921(77)) \cdot 10^{-19}\mathrm{GeV}^{-5}$$

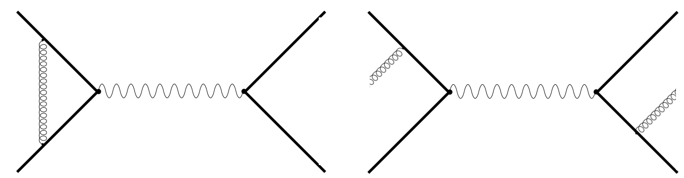
Finite part

Local Unitarity is a fully **local** and **generic** paradigm

- **FSR** regulated by realising **KLN** locally
- **Thresholds** regulated using **local deformation**
- **ISS** regulated with **counter-terms**
- **Local UV** and **renormalisation fully automated**

} **Tested in multi-loop, multi-scale examples**

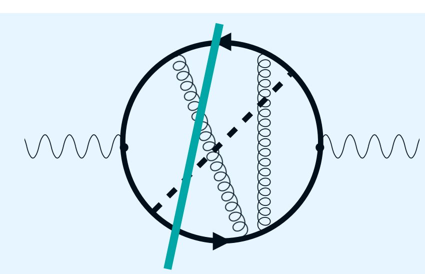
- **New theory: KLN for ISS/ISR**



It is particularly suitable to numerical integration!

- Take advantage of the **robustness** of **MonteCarlo methods**
- **Full automation underway**
- On the other hand, **evaluation speed** and **convergence** constitute **important challenges**

Future application: $e^+e^- \to t\bar{t}H$ $\mathrm{N^2LO}$ **cross-section**



Selected reads

LTD/cLTD

Catani, Gleisberg, Krauss, Rodrigo, Winter
arXiv: 0804.3170 (2008)

Bierenbaum, Catani, Draggiotis, Rodrigo
arXiv: 1007.0194 (2010)

Hernandez-Pinto, Sborlini, Rodrigo
arXiv: 2001.03564 (2015)

Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo
arXiv: 1604.06699 (2016)

Runkel, Ször, Vesga, Weinzierl
arXiv: 1902.02135 (2019)

ZC, Hirschi, Kermanshah, Ruijl
arXiv: 1906.06138 (2019)

Verdugo, Driencourt-Mangin, Hernandez-Pinto et al.
arXiv: 2001.03564 (2020)

ZC, Hirschi, Kermanshah, Pelloni, Ruijl
arXiv: 2009.05509 (2020)

Verdugo, Hernandez-Pinto, Rodrigo, Sborlini et al.
arXiv: 2010.12971 (2020)

Sborlini
arXiv: 2102.05062 (2021)

Torres-Bobadilla
arXiv: 2103.09237 (2021)

Local Unitarity

Soper,
arXiv: 9804454 (1998)

Soper,
Beowulf (pages.uoregon.edu/soper/beowulf/)

ZC, Hirschi, Pelloni, Ruijl
arXiv: 2010.01068 (2020)

Contour Deformation

Gong, Nagy, Soper
arXiv:0812.3686 (2009)

Becker, Reuschle, Weinzierl
arXiv:1010.4187 (2010)

Becker, Götz, Reuschle, Schwan, Weinzierl
arXiv:1111.1733 (2011)

Buchta, Chachamis, Draggiotis, Rodrigo
arXiv:1510.00187 (2017)

ZC, Hirschi, Kermanschah, Pelloni, Ruijl
arXiv: 1510.00187 (2019)

Subtraction of loop integrals

Becker, Reuschle, Weinzierl
arXiv:1010.4187 (2010)

Yao Ma
arXiv:1910.11304 (2019)

Anastasiou, Sterman
arXiv:1812.03753 (2018)

Anastasiou, Haindl, et al.
arXiv:2008.12293 (2020)