# Two-loop QCD corrections to $Wb\bar{b}$ production at hadron colliders

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based on **arXiv:2102.02516**
with Simon Badger and Simone Zoia

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# Precise prediction for the LHC

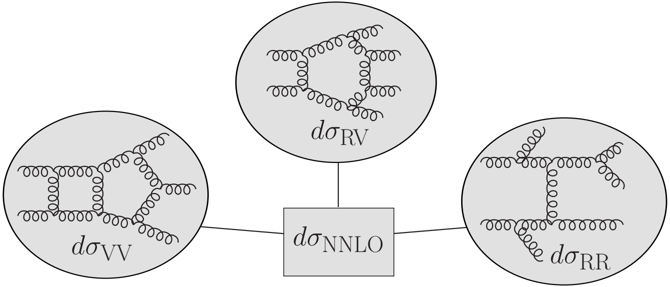
⇒ QCD corrections are important at the LHC

$$d\sigma = d\sigma^{\mathrm{LO}} + \underbrace{d\sigma^{\mathrm{NLO}}}_{10\text{–}30\%} + \underbrace{d\sigma^{\mathrm{NNLO}}}_{1\text{–}10\%} + \ldots$$

## NNLO frontier: 2 to 3 scattering

- $pp \to jjj$: $R_{3/2}$, $m_{jjj} \Rightarrow \alpha_s$ determination at multi-TeV range
- $pp \to \gamma\gamma j$: background to Higgs $p_T$, signal/background interference effects
- $pp \to Hjj$: Higgs $p_T$, background to VBF (probes Higgs coupling)
- $pp \to Vjj$: Vector boson $p_T$, $W^+/W^-$ ratios, multiplicity scaling
- $pp \to VVj$: background for new physics

# NNLO cross sections for 2 → 3 processes

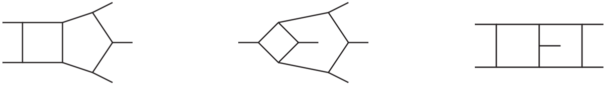


$$\text{loop amplitude} = \sum(\text{rational coefficients}) \times (\text{integral/special functions})$$

$$\text{finite remainder} = \text{loop amplitude} - \text{poles}$$

# Massive progress in massless 2-loop 5-particle scattering

- All 2-loop 5-particle integrals are known talk by Vasily



[Papadopoulos,Tommasini,Wever(2015)] [Gehrmann,Henn,Lo Presti(2015,2018)] [Abreu,Page,Zeng(2018)]
[Abreu,Dixon,Herrmann,Page,Zeng(2018,2019)] [Chicherin,Gehrmann,Henn,Wasser,Zhang,Zoia(2018,2019)][Chicherin,Sotnikov(2020)]

- Many 2-loop 5-particle QCD amplitudes known analytically talks by Federico, Herschel, Vasily

Leading colour $\Rightarrow$ $5g$, $2q3g$, $4q1g$, $2q3\gamma$, $2q1g2\gamma$
[Abreu,Agarwal,Badger,Brønnum-Hansen,Buccioni,Chawdhry,Czakon,Dormans,Febres Cordero, Gehrmann, HBH,Henn,Ita,Lo Presti,Mitov,Page,Peraro,Poncelet,Sotnikov,Tancredi,von Manteuffel,Zeng(2015-2021)]

Full colour $\Rightarrow$ $5g$ all-plus, $2q1g2\gamma$

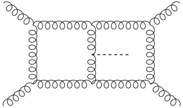
[Badger,Chicherin,Gehrmann,Heinrich,Henn,Peraro,Wasser,Zhang,Zoia(2019)] [Agarwal,Buccioni,Tancredi,von Manteuffel(2021)]

- NNLO QCD calculations for $2 \rightarrow 3$ processes talk by Rene

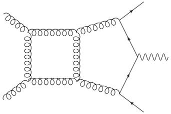
$pp \rightarrow \gamma\gamma\gamma$ [Chawdhry,Czakon,Mitov,Poncelet(2019)][Kallweit,Sotnikov,Wiesemann(2020)]
$pp \rightarrow \gamma\gamma j$ [Chawdhry,Czakon,Mitov,Poncelet(2021)]

# Scattering with an off-shell leg

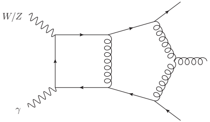
$pp \to H + 2j$



$pp \to W/Z + 2j$



$pp \to W/Z + \gamma j$

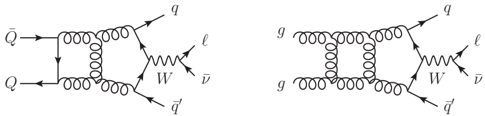


- rich potential phenomenology
- massless internal particles, focus on QCD corrections
- high algebraic and analytic complexity
  $\Rightarrow$ six independent variables
  $\Rightarrow$ 3 square roots

# A first look: two-loop $W+4$ parton amplitudes

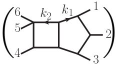
[Badger,Brønnum-Hansen,HBH,Peraro(2019)]

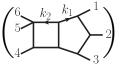
Numerical evaluation of leading colour $q\bar{Q}Q\bar{q}'\bar{\nu}\ell$ and $qgg\bar{q}'\bar{\nu}\ell$ helicity amplitudes at two loops

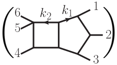


Feynman diagrams | integrand reduction | IBP reduction | finite-field sampling

- Full solutions of the master integrals were not available back then
- unknown MIs are evaluated numerically using `pySecDec`/`Fiesta`

$I$  $\left[\langle 4|k_2|p_{56}|4\rangle\mu_{11}\right] \sim \mathcal{O}(\epsilon)$

$I$  $\left[[4|k_2|p_{56}|4]\mu_{11}\right] \sim \mathcal{O}(\epsilon)$

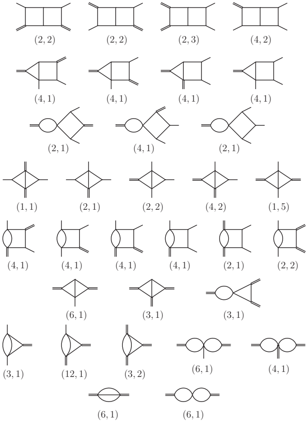
$I$  $\left[\mathrm{tr}_-(1(k_1-p_1)(k_1-p_{12})3)\langle 4|k_2|p_{56}|4\rangle\right] \sim \mathcal{O}(1)$

- Use momentum twistor parametrisation for $2 \to 4$ massless scattering: 8 variables
- Coefficient of master integrals are computed numerically over finite fields

# Two-loop master integrals for 5-point 1-mass process

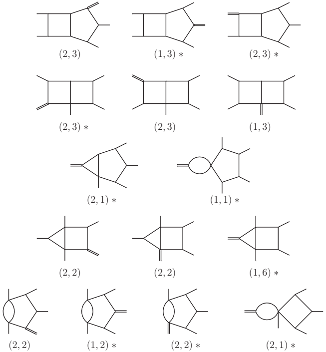
4-point sub-topologies known from $pp \to V_1^* V_2^*$

[Gehrman,Remiddi(2000)] [Henn,Melnikov,Smirnov(2014)]
[Gehrmann,von Manteuffel,Tancredi(2015)]



All planar 2-loop integrals are available

[Papadopoulos,Tomassini,Wever(2015)][Papadopulos,Wever(2019)]
[Abreu,Ita,Moriello,Page,Tschernow,Zeng(2020)]
[Canko,Papadopoulos,Syrrakos(2020)][Syrrakos(2020)]



Non-planar integrals in progress talks by Ben & Costas

# Leading colour $Wb\bar{b}$ amplitude

$$\bar{d}(p_1) + u(p_2) \to b(p_3) + \bar{b}(p_4) + W^+(p_5)$$

- colour decomposition at leading colour → only planar contribution

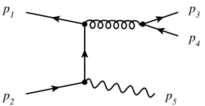
$$\mathcal{A}^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W) \sim g_s^6 g_W \, N_c^2 \, \delta_{i_1}^{\bar{i}_4} \delta_{i_3}^{\bar{i}_2} \, A^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W)$$

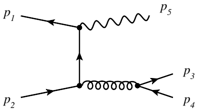
- massless $b$ quarks, $p_3^2 = p_4^2 = 0$
- onshell $W$ boson

$$p_5^2 = m_W^2, \qquad \sum_\lambda \varepsilon_W^{\mu *}(p_5,\lambda)\varepsilon_W^{\nu}(p_5,\lambda) = -g^{\mu\nu} + \frac{p_5^\mu p_5^\nu}{m_W^2}$$

Invariants:

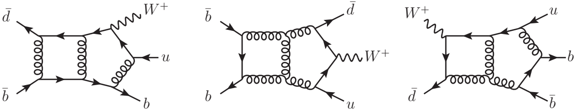
$$s_{12} = (p_1+p_2)^2\,, \quad s_{23} = (p_2-p_3)^2\,, \quad s_{34} = (p_3+p_4)^2\,,$$
$$s_{45} = (p_4+p_5)^2\,, \quad s_{15} = (p_1-p_5)^2\,, \quad s_5 = p_5^2\,,$$
$$\mathrm{tr}_5 = 4i\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma\,.$$





# Integrand construction

Feynman diagrams generated using QGRAF [Nogueira(1993)]



MATHEMATICA+FORM to process the numerator topologies and interfere with tree level

$$M^{(2)} = \sum_{\text{spin}} A^{(0)*} A^{(2)} = M^{(2)}_{\text{even}} + \text{tr}_5\, M^{(2)}_{\text{odd}}$$

Numerators containing: $\text{tr}(\cdots)$ and $\text{tr}(\cdots\gamma_5\cdots\gamma_5\cdots)$ ⇒ anti-commuting $\gamma_5$ prescription

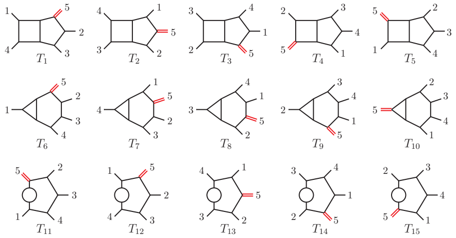
$\text{tr}(\cdots\gamma_5\cdots)$ ⇒ Larin's prescription [Larin(1993)]

Amplitudes in terms of scalar integrals

$$M^{(2)}_k(\{p\}) = \sum_i c_{k,i}(\epsilon,\{p\})\,\mathcal{I}_{k,i}(\epsilon,\{p\}), \qquad k \in \{\text{even},\text{odd}\}$$

# Integration-by-parts (IBP) reduction to master integrals

- Map each topology to a set of maximal topologies
- IBP systems for $T_1 - T_{10}$



Entire workflow on finite fields

$$M_k^{(2)}(\{p\},\epsilon) = \sum_i c_{k,i}(\{p\},\epsilon)\,\mathcal{I}_{k,i}(\{p\},\epsilon)$$

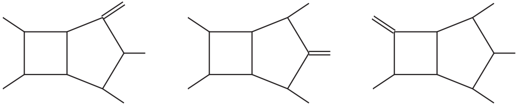
$$\downarrow$$

$$M_k^{(2)}(\{p\},\epsilon) = \sum_i d_{k,i}(\{p\},\epsilon)\,\mathrm{MI}_{k,i}(\{p\},\epsilon)$$

$$k \in \{\mathrm{even}, \mathrm{odd}\}$$

- IBP reduction directly to canonical MIs[Abreu,etal(2020)]
- IBP systems generated with LiteRed[Lee(2012)], solved with FiniteFlow[Peraro(2019)] using Laporta algorithm [Laporta(2000)]
- IBP tables known numerically at each value of $(\{p\},\epsilon)$
- Numerically compute $d_{k,i}$ over finite fields

# Plugging in the master integrals



① [Abreu,Ita,Moriello,Page,Tschernow,Zeng(2020)] talk by Ben

- planar alphabet identified (58 letters, 3 square-roots), canonical DEs derived
- Integrate DEs numerically using generalised series expansions [Moriello(2019)]

② [Canko,Papadopoulos,Syrrakos(2020)][Syrrakos(2020)] talks by Costas, Nikolaos

- Construct Simplified Differential Equations (SDEs) using known canonical basis
- Analytic solutions in term of Goncharov PolyLogarithms (GPLs)

① $\Rightarrow$ analytically reconstructing MI coefficients is still too complicated

② $\Rightarrow$ GPLs not linearly independent: no analytic pole cancellations

# A basis of special functions

- use the components of the $\epsilon$-expansion of the MIs as special functions

$$\mathrm{MI}_i(s) = \sum_{w \geq 0} \epsilon^w \mathrm{MI}_i^{(w)}(s)$$

- starting from canonical DEs[Abreu,etal(2020)] write MIs in terms of Chen's iterated integrals[Chen(1977)] for example:

$$\mathrm{MI}_i^{(2)} = [w_1, w_2]_{s_0} + [w_1, w_3]_{s_0} + \cdots + \mathrm{tc}_j^{(2)}(s_0)$$

  where

$$[w_{i_1}, \ldots, w_{i_n}]_{s_0}(s) = \int_\gamma d \log w_{i_n}(s')[w_{i_1}, \ldots, w_{i_{n-1}}]_{s_0}(s')$$

- Use GPL expressions[Canko,etal(2020)][Syrrakos(2020)] + PSLQ algorithm to prepare the boundary values
- Shuffle algebra to remove products of lower-weight functions + linear algebra to extract linearly independent functions talk by Vasily

$$\left\{\mathrm{MI}_i^{(w)}(s)\right\} \Longrightarrow \left\{f_i^{(w)}(s)\right\}$$

# Reconstructing the finite remainders

$$M_k^{(2)}(\{p\},\epsilon) = \sum_i c_{k,i}(\{p\},\epsilon)\,\mathcal{I}_{k,i}(\{p\},\epsilon)$$

$$\Big\downarrow \quad \text{IBP reduction}$$

$$M_k^{(2)}(\{p\},\epsilon) = \sum_i d_{k,i}(\{p\},\epsilon)\,\mathrm{MI}_{k,i}(\{p\},\epsilon)$$

$$\Big\downarrow \quad \text{map to special function basis}$$

$$\Big\downarrow \quad \text{subtract UV/IR poles}$$

$$\Big\downarrow \quad \epsilon \text{ expansion}$$

$$F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\})\; m_{k,i}(f) + \mathcal{O}(\epsilon)$$

$$k \in \{\text{even}, \text{odd}\}$$

- Finite remainders

$$F_k^{(2)} = M_k^{(2)} - \sum_{j=1}^{2} I^{(j)} M_k^{(2-j)}$$

$I^{(L)}$ → $L$-loop universal UV/IR poles

[Catani(1998)][Becher,Neubert(2009)][Magnea,Gardi(2009)]

- Numerically compute $e_{k,i}$ over finite fields
- Analytic pole cancellation
- Drop in polynomial complexities for $e_{k,i}$
- Reconstruct analytic expressions of $e_{k,i}$ from several numerical evaluations [Peraro(2016)]

# Reconstructing the finite remainders

$$F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\})\, m_{k,i}(f) + \mathcal{O}(\epsilon), \qquad k \in \{\mathrm{even}, \mathrm{odd}\}$$

- set $s_{12} = 1$
- Not all $e_{k,i}$ coefficients independent
  $\Rightarrow$ find linear relations between coefficients and reconstruct the simpler ones

$$\sum_i y_i e_i = 0, \qquad y_i \in \mathbb{Q}$$

$\Rightarrow$ allow to supply known/candidate coefficients $\tilde{e}_j$

$$\sum_i y_i e_i + \sum_j \tilde{y}_j \tilde{e}_j = 0, \qquad y_i, \tilde{y}_j \in \mathbb{Q}$$

- guess the denominator → from letters [Abreu,etal(2019)][Abreu,etal(2020)]
- partial fraction in one variable ($s_{23}$) and reconstruct in the remaining variables ($s_{34}, s_{45}, s_{15}, s_5$)

$\Rightarrow \sim 4$ times speed up $\qquad \Rightarrow$ 2 prime fields needed

- Reconstructed analytic expressions are simplified using MultivariateApart[Heller,von Manteuffel(2021)]

# Numerical evaluation

- Only 19 linear combinations of $f_i^{(4)}$ appear in the two-loop finite remainder

  $\Rightarrow$ define a new basis $g_i^{(w)}$

$$\left\{ f_i^{(w)}(s) \right\} \Longrightarrow \left\{ g_i^{(w)}(s) \right\}$$

- Apply generalised series expansion method directly to the $g_i^{(w)}$ basis

$$\vec{g} = \begin{pmatrix} \epsilon^4 g_i^{(4)} \\ \epsilon^3 g_i^{(3)} \\ \epsilon^2 g_i^{(2)} \\ \epsilon g_i^{(1)} \\ 1 \end{pmatrix}$$

$$d\vec{g} = \epsilon d\tilde{B} \cdot \vec{g}$$

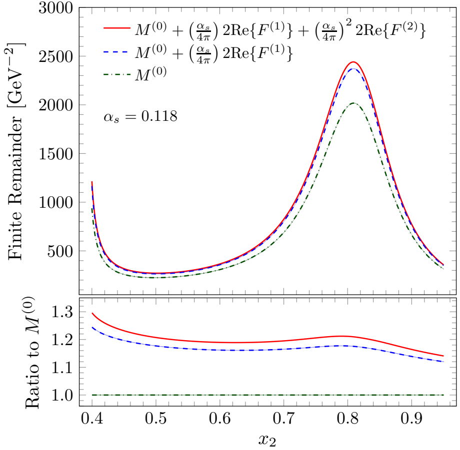
- Much simpler than the DEs for the master integrals
- Use generalised series expansion approach [Moriello(2019)] as implemented in DiffExp [Hidding(2020)] talk by Martijn

# Numerical evaluation

Evaluation on a univariate slice of the physical phase space

$$\begin{aligned}
p_1 &= \frac{\sqrt{s}}{2}(1,0,0,1), \\
p_2 &= \frac{\sqrt{s}}{2}(1,0,0,-1), \\
p_3 &= \tfrac{x_1\sqrt{s}}{2}\left(1,1,0,0\right), \\
p_4 &= \tfrac{x_2\sqrt{s}}{2}\left(1,\cos\theta,-\sin\phi\sin\theta,-\cos\phi\sin\theta\right), \\
p_5 &= \sqrt{s}\,(1,0,0,0) - p_3 - p_4
\end{aligned}$$

$$s = 1,\ m_W^2 = 0.1,\ \phi = 0.1,\ x_1 = 0.6$$



- 1100 points → average 260 s/point
- Reasonable evaluation time with basic DiffExp setup
- further optimisation is possible

[Abreu,etal(2020)][Becchetti,etal(2020)]

# Summary

- ✓ First analytic result for 2-loop 5-point amplitude with one massive leg
  $\Rightarrow$ leading colour $u\bar{d} \to W^+ b\bar{b}$
- ✓ Basis of special functions for leading colour 5-particle amplitudes with 1 off-shell leg up to 2 loops
- ✗ Include $W$-boson decay
- ✗ Application to other processes
- ✗ Full colour (need non-planar integrals)

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THANK YOU!!!