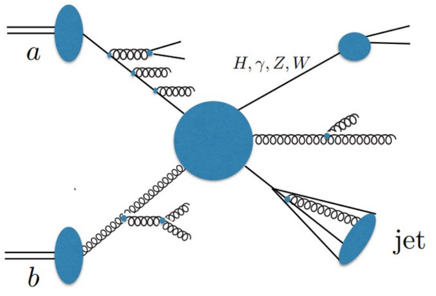
A consistent framework for the regularization of chiral theories in 4D: a two-loop study

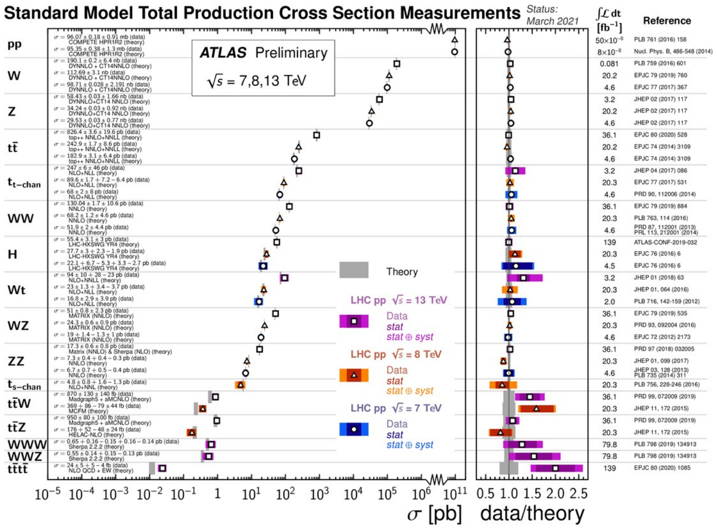
Adriano Cherchiglia

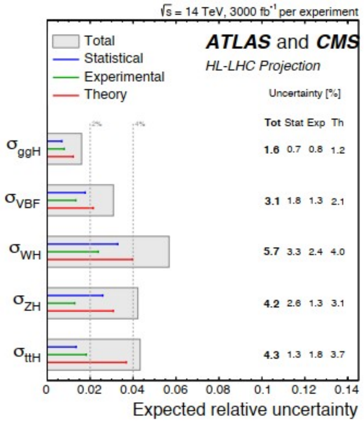
UFABC

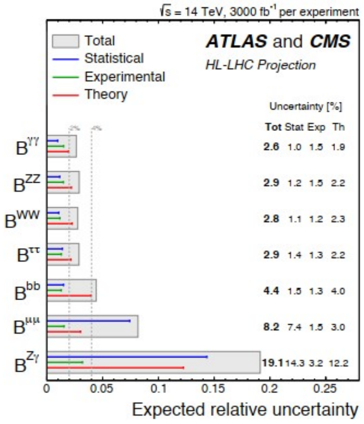
RADCOR-LoopFest 2021

Motivation









Higgs Physics at the HL-LHC and HE-LHC, arXiv:1902.00134v2

$$d\sigma = \sum_{ab} \int dx_a \int dx_b \; f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \times d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_s(\mu_R^2))$$

Partonic higher loop corrections

Motivation

Eur. Phys. J. C (2017) 77:471
DOI 10.1140/epjc/s10052-017-5023-2

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

To d, or not to d: recent developments and comparisons of regularization schemes

C. Gnendiger[1,a], A. Signer[1,2], D. Stöckinger[3], A. Broggio[4], A. L. Cherchiglia[5], F. Driencourt-Mangin[6], A. R. Fazio[7], B. Hiller[8], P. Mastrolia[9,10], T. Peraro[11], R. Pittau[12], G. M. Pruna[1], G. Rodrigo[6], M. Sampaio[13], G. Sborlini[6,14,15], W. J. Torres Bobadilla[6,9,10], F. Tramontano[16,17], Y. Ulrich[1,2], A. Visconti[1,2]

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Eur. Phys. J. C (2021) 81:250
https://doi.org/10.1140/epjc/s10052-021-08996-y

THE EUROPEAN PHYSICAL JOURNAL C

Check for updates

Review

May the four be with you: novel IR-subtraction methods to tackle NNLO calculations

W. J. Torres Bobadilla[1,2,a], G. F. R. Sborlini[3], P. Banerjee[4], S. Catani[5], A. L. Cherchiglia[6], L. Cieri[5], P. K. Dhani[5,7], F. Driencourt-Mangin[2], T. Engel[4,8], G. Ferrera[9], C. Gnendiger[4], R. J. Hernández-Pinto[10], B. Hiller[11], G. Pelliccioli[12], J. Pires[13], R. Pittau[14], M. Rocco[15], G. Rodrigo[2], M. Sampaio[6], A. Signer[4,8], C. Signorile-Signorile[16,17], D. Stöckinger[18], F. Tramontano[19], Y. Ulrich[4,8,20]

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Regularization methods in 4D

Implicit Regularization

Mod. Phys. Lett. A 13, 1597 (1998)

Four-Dimensional Regularization

JHEP 1211, 151(2012)

Four-Dimensional Unsubtraction

JHEP 1608 (2016) 160

- tailored to extract UV divergences
- complies with BPHZ (unitarity, locality, Lorentz invariance)

  A. C, Sampaio, Nemes (2011)
- complies with abellian gauge invariance to all-orders

  Ferreira, **A.C,** Nemes, Hiller, Sampaio (2012)

  Vieira, **A.C,** Sampaio(2016)
- non-abelian gauge invariance working examples

  A. C, Arias-Perdomo, Vieira, Sampaio, Hiller (2020)
- IR divergences under study (1 and 2 loop)

  Eur. Phys. J. C (2017) 77:471

  Eur. Phys. J. C (2021) 81:250

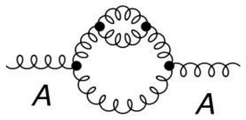
Implicit Regularization - non-abelian

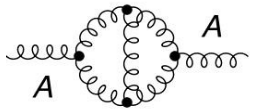
A. C, Arias-Perdomo, Vieira, Sampaio, Hiller (2020)

- tailored to extract UV divergences → β - function

$$Z_g = Z_A^{-1/2}$$

Background field method





$$\int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4} G(k,q,p)$$

→

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\lambda^2)^2} \quad (k \to q)$$

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\lambda^2)^2} \ln\left(-\frac{k^2-\lambda^2}{\lambda^2}\right) \quad (k \to q)$$

$$\int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{(k^2-\lambda^2)^2}\frac{1}{(q^2-\lambda^2)^2}$$

Implicit Regularization - non-abelian

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$$Z_g = Z_A^{-1/2}$$

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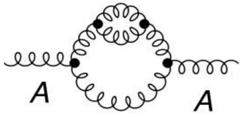

$$\int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4} G(k,q,p)$$

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\lambda^2)^2} \quad (k \to q)$$

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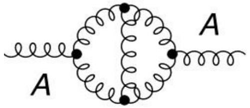
$$\int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{(k^2-\lambda^2)^2}\frac{1}{(q^2-\lambda^2)^2}$$



$$\beta = -g\lambda \frac{\partial \ln Z_g}{\partial \lambda}$$

RGE scale

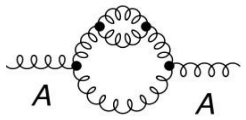
Implicit Regularization - non-abelian

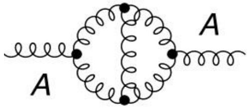
A. C, Arias-Perdomo, Vieira, Sampaio, Hiller (2020)

- tailored to extract UV divergences → β - function

Background field method

$$\beta = -g_s\left[\left(11-\frac{2}{3}n_f\right)\left(\frac{g_s}{4\pi}\right)^2+\left(102-\frac{38}{3}n_f\right)\left(\frac{g_s}{4\pi}\right)^4\right]$$





- (UV part) comply with non-abelian gauge invariance
- Connection with dimensional methods (own subtraction scheme)

Implicit Regularization - chiral

Viglioni, **A.C**, Vieira, Hiller, Sampaio (2016)

Bruque, **A.C**, Pérez-Victoria (2018)

$$\{\gamma_\mu, \gamma_5\} = 0$$

Example – 2D (euclidean space)

$$\mathrm{tr}(\{\gamma_5, \gamma_\mu\}\gamma_\nu\gamma_\rho\gamma_\sigma) = \mathrm{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) + \mathrm{tr}(\gamma_\mu\gamma_5\gamma_\nu\gamma_\rho\gamma_\sigma) = -4\,(g_{\mu\nu}\epsilon_{\rho\sigma} - g_{\mu\rho}\epsilon_{\nu\sigma} + g_{\mu\sigma}\epsilon_{\nu\rho})$$

$$[(\mathrm{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) + \mathrm{tr}(\gamma_\mu\gamma_5\gamma_\nu\gamma_\rho\gamma_\sigma))I^{\mu\sigma}]_R = 4\pi\epsilon_{\rho\nu} \neq 0$$

Implicit Regularization - chiral

Viglioni, **A.C**, Vieira, Hiller, Sampaio (2016)

Bruque, **A.C**, Pérez-Victoria (2018)

$$\{\gamma_\mu, \gamma_5\} = 0$$

Even in 4D methods, chiral theories must be dealt with care!

Example – 2D (euclidean space)

$$[(\mathrm{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) + \mathrm{tr}(\gamma_\mu\gamma_5\gamma_\nu\gamma_\rho\gamma_\sigma))I^{\mu\sigma}]_R = -4\,[(g_{\mu\nu}\epsilon_{\rho\sigma} - g_{\mu\rho}\epsilon_{\nu\sigma} + g_{\mu\sigma}\epsilon_{\nu\rho})\,I^{\mu\sigma}]_R$$

$$g_{\mu\sigma}[I^{\mu\sigma}]_R = g_{\mu\sigma}\left[\int d^2k \frac{k^\mu k^\sigma}{(k^2+m^2)^2}\right] \neq \left[\int d^2k \frac{k^2}{(k^2+m^2)^2}\right] = [g_{\mu\sigma}I^{\mu\sigma}]_R$$

Implicit Regularization - chiral

Viglioni, **A.C**, Vieira, Hiller, Sampaio (2016)

Bruque, **A.C**, Pérez-Victoria (2018)

$$\gamma_5 \in GnS$$

$$QnS = GnS \oplus X$$

IReg

$$QdS = GnS \oplus Q(-2\epsilon)S$$

DReg

$$g_{\mu\sigma}[I^{\mu\sigma}]_R \neq [g_{\mu\sigma}I^{\mu\sigma}]_R$$

$$\bar{g}_{\mu\sigma}[I^{\mu\sigma}]_R = [\bar{g}_{\mu\sigma}I^{\mu\sigma}]_R$$

$$QnS = QdS \oplus Q(2\epsilon)S = GnS \oplus Q(-2\epsilon)S \oplus Q(2\epsilon)S$$

DRed

- One-loop examples – need symmetry-restoring conterterms

Bélusca-Maïto, Ilakovac, Mađor-Božinović, Stöckinger (2020)

Implicit Regularization - chiral

A.C, To appear

- Toward two-loop level
  - abelian left-model

$$GnS \longrightarrow \mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}_L \not{\partial}\psi_L + e\bar{\psi}_L \not{A}\psi_L$$

$$QnS = GnS \oplus X \longrightarrow \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{\partial}\psi + e\bar{\psi}_L \not{A}\psi_L$$

gauge breaking term $\quad i\left(\bar{\psi}_L \hat{\not{\partial}}\psi_R + \bar{\psi}_R \hat{\not{\partial}}\psi_L\right)$

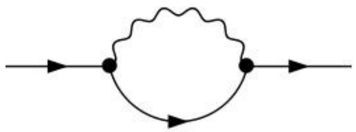
Implicit Regularization - chiral

A.C, To appear

$$\Sigma(p)|_{\text{div}} = ie^2 \left[\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-\lambda^2)^2} \right] \not{p} P_L$$

$$\Sigma(p)|_{\text{fin}} = \frac{e^2}{(4\pi)^2} \left[\log\left(-\frac{p^2}{\lambda^2} \right) - 2 \right] \not{p} P_L$$

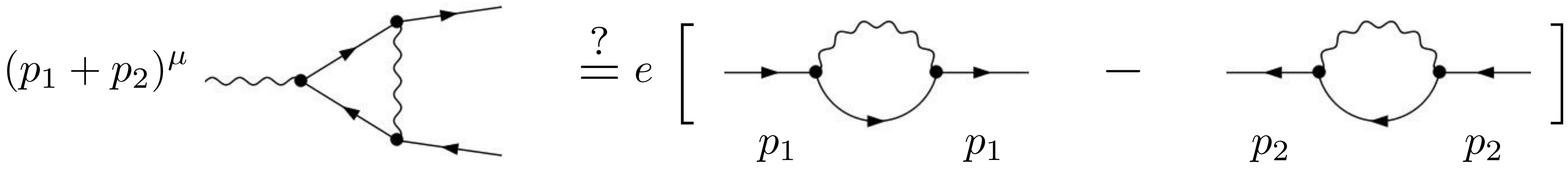
$$\rightarrow GnS$$



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{\partial}\psi + e\bar{\psi}_L \not{A} \psi_L$$

Implicit Regularization - chiral

A.C, To appear



$$(p_1 + p_2)_\mu \Gamma^\mu(p_1, p_2) = e\left[\Sigma(p_1) - \Sigma(-p_2)\right] - \frac{e^3}{(4\pi)^2}(p_1 + p_2)_\mu \bar{\gamma}^\mu P_L$$

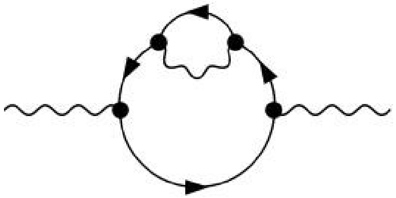
GnS

Implicit Regularization - chiral

A.C, To appear

- tailored to extract UV divergences

naive scheme $\{\gamma_\mu, \gamma_5\} = 0$

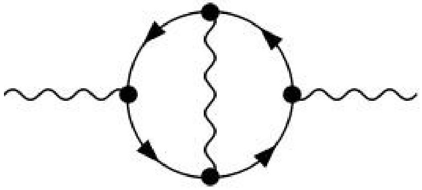


$$\mathcal{A}_{\mu\nu}|_{\rm div} = \frac{ie^4}{(4\pi)^4}\left[g_{\mu\nu}p^2 - p_\mu p_\nu\right]\mathcal{F}$$

transverse

QED

$$\beta = e\left[\frac{4}{3}\left(\frac{e}{4\pi}\right)^2 + 4\left(\frac{e}{4\pi}\right)^4\right]$$



$$\beta = e\left[\frac{2}{3}\left(\frac{e}{4\pi}\right)^2 + 2\left(\frac{e}{4\pi}\right)^4\right]$$

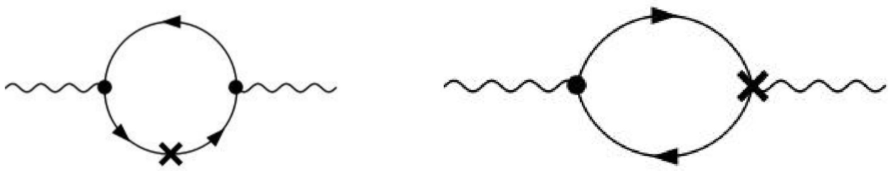
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\partial\!\!\!/\psi - e\bar{\psi}_L A\!\!\!/\psi_L$$

$$\frac{e}{2}\bar{\psi}\bar{\gamma}_\mu\left(1-\gamma_5\right)\psi\, A^\mu$$

Implicit Regularization - chiral

A.C, To appear

naive scheme $\{\gamma_\mu, \gamma_5\} = 0$



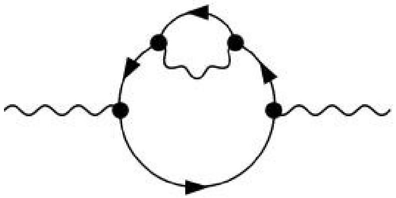
UV countertems cancel
(subdivergences)

$$\beta = e\left[\frac{2}{3}\left(\frac{e}{4\pi}\right)^2 + 2\left(\frac{e}{4\pi}\right)^4\right]$$

Implicit Regularization - chiral

A.C, To appear

$$QnS = GnS \oplus X$$

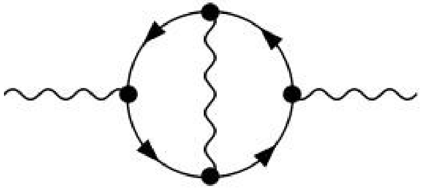


transverse

$$\mathcal{A}'_{\mu\nu}|_{\rm div} = \frac{ie^4}{(4\pi)^4}\left[\bar{g}_{\mu\nu}\bar{p}^2 - \bar{p}_\mu\bar{p}_\nu\right]\mathcal{F}'$$

QED

$$\beta = e\left[\frac{4}{3}\left(\frac{e}{4\pi}\right)^2 + 4\left(\frac{e}{4\pi}\right)^4\right]$$

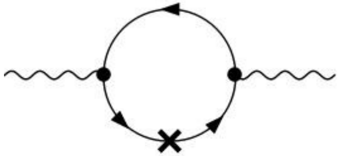


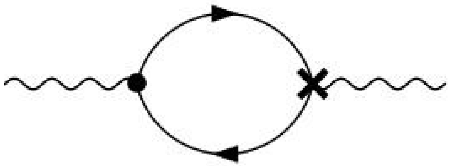
$$\beta = e\left[\frac{2}{3}\left(\frac{e}{4\pi}\right)^2 + \frac{10}{3}\left(\frac{e}{4\pi}\right)^4\right]$$

Implicit Regularization - chiral

A.C, To appear

$$QnS = GnS \oplus X$$

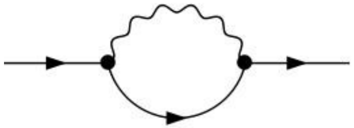




- UV countertems DO NOT cancel (subdivergences)

$$\Sigma(p)|_{\text{div}} = ie^2\left[\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\lambda^2)^2}\right]\not{p}P_L$$

GnS



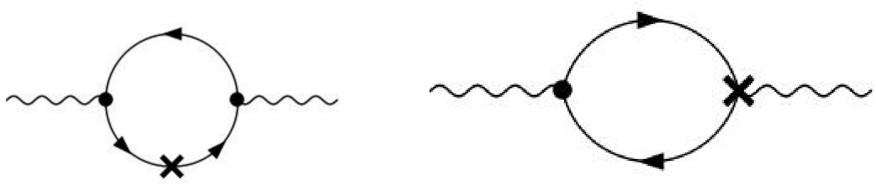
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{\partial}\psi + e\bar{\psi}_L\not{A}\psi_L$$

QnS

Implicit Regularization - chiral

A.C, To appear

$$QnS = GnS \oplus X$$



- Finite restoring-symmetry countertems

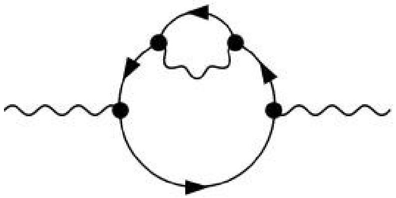
$$(p_1 + p_2)_\mu \Gamma^\mu(p_1, p_2) = e\left[\Sigma(p_1) - \Sigma(-p_2)\right] - \frac{e^3}{(4\pi)^2}(p_1 + p_2)_\mu \bar{\gamma}^\mu P_L$$

GnS

Implicit Regularization - chiral

A.C, To appear

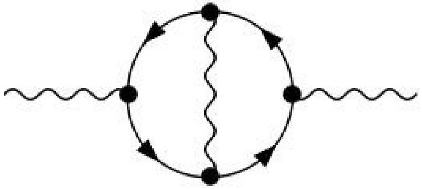
$$QnS = GnS \oplus X$$



$$\beta = e\left[\frac{2}{3}\left(\frac{e}{4\pi}\right)^2 + 2\left(\frac{e}{4\pi}\right)^4\right]$$

QED

$$\beta = e\left[\frac{4}{3}\left(\frac{e}{4\pi}\right)^2 + 4\left(\frac{e}{4\pi}\right)^4\right]$$



- Same result of naive scheme

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\partial\!\!\!/\psi + e\bar{\psi}_L A\!\!\!/\psi_L$$

$$\frac{e}{2}\bar{\psi}A\!\!\!/\left(1-\gamma_5\right)\psi$$

Implicit Regularization – chiral + non-abelian

A.C, To appear

naive scheme $\{\gamma_\mu, \gamma_5\} = 0$

- Non-abelian Left-Model analysis ✔
- SM gauge coupling β - function up to two-loop order ✔

$$QnS = GnS \oplus X$$

- Non-abelian Left-Model analysis
- SM gauge coupling β - function up to two-loop order

Conclusions

- Given the prospects for future years, it is a necessity to increase precision of electroweak radiative corrections;

- It is well-known that the treatment of chiral theories is tricky in dimensional methods;

- Regularization methods in 4D share similar problems. However, the setup may be simpler;

- Step forward 2-loop analysis:

  - simple abelian chiral model as a working example
  - non-abelian chiral theories:
    - analysis in the naive scheme completed
    - amplitude obtained, counterterms ongoing

Backup

Backup

$$\frac{1}{(k-p_i)^2-\mu^2} = \sum_{j=0}^{n_i^{(k)}-1} \frac{(-1)^j (p_i^2 - 2p_i\cdot k)^j}{(k^2-\mu^2)^{j+1}} + \frac{(-1)^{n_i^{(k)}} (p_i^2 - 2p_i\cdot k)^{n_i^{(k)}}}{(k^2-\mu^2)^{n_i^{(k)}} \left[(k-p_i)^2-\mu^2\right]},$$

$$I_{\log}^{\nu_1\cdots\nu_{2r}}(\mu^2) \equiv \int_k \frac{k^{\nu_1}\cdots k^{\nu_{2r}}}{(k^2-\mu^2)^{r+2}}$$

$$\int_k \frac{\partial}{\partial k_\mu} \frac{k^\nu}{(k^2-\mu^2)^n} = 4\left[\frac{g_{\mu\nu}}{4} I_{\log}(\mu^2) - I_{\log}^{\mu\nu}(\mu^2)\right] = 0,$$

$$I_{\log}(\mu^2) = I_{\log}(\lambda^2) + \frac{i}{(4\pi)^2} \ln\frac{\lambda^2}{\mu^2}$$

Backup

$$QnS \qquad g_{\mu\sigma}[I^{\mu\sigma}]_R \neq [g_{\mu\sigma}I^{\mu\sigma}]_R$$

$$GnS \qquad \bar{g}_{\mu\sigma}[I^{\mu\sigma}]_R = [\bar{g}_{\mu\sigma}I^{\mu\sigma}]_R$$

$$i\Pi_{\mu\nu}(p) = (-)(-ie)^2 \int_k \mathrm{Tr}\left\{\gamma_\mu \frac{i}{(\not{k})}\gamma_\nu \frac{i}{(\not{k}-\not{p})}\right\}.$$

$$i\Pi_{\mu\nu}(p) = (-e^2)\mathrm{Tr}\left\{\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta(I_{\alpha\beta} - I_\alpha p_\beta)\right\} \quad \text{where} \quad I_{\alpha_1\,\cdots\,\alpha_n} = \int_k \frac{k_{\alpha_1}\cdots k_{\alpha_n}}{k^2(k-p)^2}.$$

$$i\frac{\Pi_{\mu\nu}}{(-e^2)} = \frac{4}{3}\left[I_{\log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + \frac{5}{3}b\right](g_{\mu\nu}p^2 - p_\mu p_\nu) + \frac{2b}{3}g_{\mu\nu}$$

$$\int_k \frac{k^2}{k^2(k-p)^2} = \int_k \frac{1}{(k-p)^2} = 0 \neq g^{\alpha\beta}\int_k \frac{k_\alpha k_\beta}{k^2(k-p)^2} = -\frac{bp^2}{6}$$

Backup

$$f_{\mu\nu} = \int d^2k \, \frac{\partial}{\partial k_\mu} \frac{k_\nu}{k^2+m^2} = \int d^2k \left(\frac{\delta_{\mu\nu}}{k^2+m^2} - 2\frac{k_\mu k_\nu}{(k^2+m^2)^2} \right)$$

$$[I_{\mu\nu}]^R = \frac{1}{2}\delta_{\mu\nu} \left[\int d^2k \, \frac{1}{k^2+m^2} \right]^R = \frac{1}{2}\delta_{\mu\nu} \left(\left[\int d^2k \, \frac{k^2}{(k^2+m^2)^2} \right]^R + \left[\int d^2k \, \frac{m^2}{(k^2+m^2)^2} \right]^R \right) = \frac{1}{2}\delta_{\mu\nu} \left([I_{\alpha\alpha}]^R + \pi \right)$$

$$[I_{\mu\nu}]^R = \left[\int d^dk \, \frac{k_\mu k_\nu}{(k^2+m^2)^2} \right]^S = \left[\int d^dk \, \frac{1}{d}\delta_{\mu\nu} \frac{k^2}{(k^2+m^2)^2} \right]^S = \left[\int d^dk \left(\frac{1}{2} + \frac{\varepsilon}{4} + O(\varepsilon^2) \right) \delta_{\mu\nu} \frac{k^2}{(k^2+m^2)^2} \right]^S$$

$$= \left[\frac{1}{2}\delta_{\mu\nu} \int d^dk \, \frac{k^2}{(k^2+m^2)^2} + \left(\frac{\varepsilon}{4} + O(\varepsilon^2) \right) \delta_{\mu\nu} \left(2\pi\frac{1}{\varepsilon} + O(\varepsilon^0) \right) \right]^S = \frac{1}{2}\delta_{\mu\nu} \left([I_{\alpha\alpha}]^R + \pi \right),$$