RESUMMING NEXT-TO-LEADING POWER THRESHOLD CORRECTIONS IN QCD

Pooja Mukherjee

(in collaboration with: Ajjath A H and V Ravindran)

The Institute Of Mathematical Sciences, India

RADCOR-LoopFest 2021

May, 2021

arXiv : 2006.06726 [hep-ph]

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INTRODUCTION

What is Next-to-Leading power ?

- In QCD improved parton model, Hadron Collider : transformed into "Parton collider" via Parton distribution functions (pdfs) :

$$\sigma(\tau, q^2) = \sigma_0(\mu_R^2) \sum_{ab=q,\bar{q},g} \int dx_1 dx_2 dz f_a(x_1, \mu_F^2)\ \Delta_{ab}(z, q^2, \mu_R^2, \mu_F^2)\ f_b(x_2, \mu_F^2) \delta(\tau - z x_1 x_2)$$

- Definitions :
  - Δ_{ab} : Finite Partonic Coefficient Function (CF), q: scale of the process,
  - $\sqrt{\hat{s}}$: partonic center of mass energy , $z = \dfrac{q^2}{\hat{s}}$: partonic scaling variable.

- **The Partonic Coefficient Function near threshold, $z \to 1$:**

$$\Delta_{ab} \overset{z \to 1}{\simeq} a_i \left[\frac{\ln^i(1-z)}{1-z}\right]_+ + b\ \delta(1-z) + c_i \ln^i(1-z) + d$$

- **Leading power(LP)/Soft-Virtual (SV)**
- **Corrections from Diagonal Channels**
- **Resummation to N³LL accuracy**

- **Next-to-Leading power(NLP) /Next-to-soft virtual (NSV)**
- **Corrections from Diagonal and off-Diagonal Channels**
- **Resummation to LL accuracy**

INTRODUCTION

- **Why Next-to-Leading power ?**
  - **Significant contributions to the hadronic cross-section :**

    Because of large coefficients

[Anastasiou, Duhr, Dulat et al.(`14,`19, `20)]

a_s^3	$\ln^5(1-z)$	$\left[\frac{\ln^5(1-z)}{1-z}\right]_+$	$\ln^4(1-z)$	$\left[\frac{\ln^4(1-z)}{1-z}\right]_+$	TOTAL NLP	TOTAL LP
$gg \to H$	117.95%	96.72%	103.36%	20.648%	25.83%	-2.28%
Drell-Yan	8.59%	5.44%	9.82%	2.62%	1.49%	0.02%

  - **But these logarithms give large contributions in certain kinematic region : Spoils perturbativity of the series**
  - **Resolution : Find a way to resum NLP logarithms beyond Leading logarithms (LL).**

In today's talk I will be using NLP/NSV interchangeably.

PREVIOUS WORKS

- **Early attempts :**
  - Kraemer, Laenen, Spira (98),
  - Akhoury, Sotiropoulos & Sterman (98)

- **Important Results & Predictions using Physical Kernel Approach & explicit computation:**
  - Moch , Vogt et al. (09-20),
  - Anastasiou, Duhr, Dulat et al.(14).

- **Universality of NLP effects and LL Resummation:**
  - Laenen, Magnea, et al. (08-19),
  - Grunberg & Ravindran (09),
  - Ball, Bonvini, Forte, Marzani, Ridolfi (13),
  - Del Duca et al. (17).

- **Subleading Factorisation and LL Resummation at NLP using SCET:**
  - Larkoski, Nelli , Stewart et al. (14) ,
  - Kolodrubetz, Moult, Neill ,Stewart et al. (17),
  - Beneke et al. (19-20).

FORMALISM

- **LP was well understood through the seminal work of Sterman, Catani et.al.**

- **LP formalism was earlier applied for DY, Higgs production and DIS based on the following:** **[Ravindran (`05,`06)]**
    - **Collinear factorisation.**
    - **Renormalization Group Invariance**
    - **Logarithmic structure of perturbative quantities in dimensional regularisation.**

- **Extend the very formalism for NLP logarithms of the Diagonal Channels for color singlet processes.**

- **We start with the mass factorisation formula:** ϵ : **Dimensional Regularization parameter**

$$\frac{1}{z}\hat{\sigma}_{ab}(z,\epsilon) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}(\mu_F^2, z, \epsilon) \otimes \left(\Delta_{a'b'}(\mu_F^2, z, \epsilon) \right) \otimes \Gamma_{b'b}(\mu_F^2, z, \epsilon)$$

Partonic cross section containing only Initial state collinear singularities ($\hat{\sigma}_{ab}$)

Collinear Finite ($\Delta_{a'b'}$)

Altarelli-Parisi Splitting Kernel (Collinear Singular) ($\Gamma_{b'b}$)

FORMALISM

- **Since we want to obtain SV and NSV terms it is sufficient to keep terms which gives SV and/or NSV upon convolutions.**

- **Hence we can safely drop terms like :**

$$\Gamma^{(0)}_{qq} \otimes \Delta^{(1)}_{qg} \otimes \Gamma^{(1)}_{g\bar{q}} \longrightarrow (\mathbf{1-z})^{\alpha}, \forall \alpha > 0 \text{ \textbf{NNSV terms}}$$

- **We find that only Diagonal channel and Diagonal AP kernels contribute and so :**

$$\frac{1}{z}\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \sigma_0 \Gamma_{cc}(\mu_F^2, z, \epsilon) \otimes \left(\Delta_{c\bar{c}}(\mu_F^2, z, \epsilon) \right) \otimes \Gamma_{\bar{c}\bar{c}}(\mu_F^2, z, \epsilon)$$

- **This gives rise to a decomposition Formula :**

$$\Delta_{c\bar{c}}(z, \epsilon, q^2 \mu_R^2, \mu_F^2) = \left(\Gamma^T\right)^{-1}_{cc} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2, \epsilon)|^2 S_c(q^2, z, \epsilon) \right\} \otimes \left(\Gamma\right)^{-1}_{\bar{c}\bar{c}}$$

- **Each building block obeys first order differential equations and additional evolution equations w.r.t factorisation scale** (μ_F, μ_R)

BUILDING BLOCKS

- **DGLAP Kernel, Γ_{cc}:**

$$a_s(\mu_R^2) = \frac{g_s^2(\mu_R^2)}{16\pi^2}$$

  - **Collinear singular terms.**
  - **Functional form :**

$$\ln \Gamma_{cc}(\hat{a}_s, \mu_F^2, \mu^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_F^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \sum_{j=1}^{i} \mathscr{P}_{cc}^{(i,j)}(z) \frac{1}{\epsilon^j}$$

  - **Expressed in terms of :** $\mathscr{P}_{cc}^{(i,j)}(z) = \{A^c, B^c, C^c, D^c\}$
  - **Process-Independent :** $\{A^c, B^c, C^c, D^c\}$

[Moch,Vogt , Vermaseren]

  - z **dependency :**

$$\left[A^c \left(\frac{1}{1-z}\right)_+ + B^c \delta(1-z) + C^c \ln(1-z) + D^c\right] + \mathcal{O}((1-z))\,.$$

  - **Nomenclature:**
    - **Cusp Anomalous Dimension :** A^c
    - **Collinear Anomalous Dimension :** $\{B^c, C^c, D^c\}$
    - **Mass Factorisation scale :** μ_F
  - **All the Anomalous Dimensions have power series expansion in terms of** a_s

BUILDING BLOCKS

- **Form Factor, $\hat{F}_c$:**

[Sterman,Sen, Magnea]

  - **Captures virtual corrections.**
  - **Functional form :**

$$\ln \hat{F}_c(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \sum_{j=-\infty}^{i+1} \mathscr{L}_c^{(i,j)} \frac{1}{\epsilon^j}$$

  - **Expressed in terms of :** $\mathscr{L}_c^{(i,j)} = \{A^c, B^c, f^c, \gamma^c, g^c\}$
  - **Process-Independent :** $\{A^c, B^c, f^c, \gamma^c\}$ $\quad f_c$ **is soft anomalous dimension**
  - **Process-Dependent :** $\{g^c\}$

- **Overall Renormalization constant, $Z_{c,UV}$:**
  - **Functional form :**

$$\ln Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \sum_{j=1}^{i} \mathscr{Z}_c^{(i,j)} \frac{1}{\epsilon^j}$$

  - **Expressed in terms of :** $\mathscr{Z}_c^{(i,j)} = \{\gamma^c\}$
  - **Process-Independent :** $\{\gamma^c\}$ **: UV anomalous dimension**
  - **Renormalization scale :** μ_R

BUILDING BLOCKS

- **Soft-Collinear Function, S_c:**
  - **Born normalized Soft and collinear contributions.**
  - **Functional form :** **[Ajjath,Ravindran et.al(`20)]**

$$\ln S_c(\hat{a}_s, q^2, \mu^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2} \right)^{i\frac{\epsilon}{2}} S_\epsilon^i \, \varphi_c^{(i)}(z, \epsilon)$$

  - **$\{\epsilon\}$ dependency :**

$$\varphi_c^{(1)}(z, \varepsilon) = \frac{1}{\varepsilon} \mathcal{G}^c_{L,1}(z, \varepsilon)\,,$$

$$\varphi_c^{(2)}(z, \varepsilon) = \frac{1}{\varepsilon^2} \left(-\beta_0 \mathcal{G}^c_{L,1}(z, \varepsilon) \right) + \frac{1}{2\varepsilon} \mathcal{G}^c_{L,2}(z, \varepsilon)$$

$$\varphi_c^{(3)}(z, \varepsilon) = \frac{1}{\varepsilon^3} \left(\frac{4}{3} \beta_0^2 \mathcal{G}^c_{L,1}(z, \varepsilon) \right) + \frac{1}{\varepsilon^2} \left(-\frac{1}{3} \beta_1 \mathcal{G}^c_{L,1}(z, \varepsilon) - \frac{4}{3} \beta_0 \mathcal{G}^c_{L,2}(z, \varepsilon) \right) + \frac{1}{3\varepsilon} \mathcal{G}^c_{L,3}(z, \varepsilon)$$

  - **Hence using the RG evolution of strong coupling constant and the energy evolution equation of S_c we derive the functional form till 4-loop.**

BUILDING BLOCKS

- $\{z\}$ **dependency :**

SV	NSV
$\mathcal{G}^c_{L,1}(z,\varepsilon) = \frac{2A_1}{1-z} + \epsilon \frac{\mathcal{G}^{c,(1)}_{sv,1}}{1-z} + \mathcal{O}(\epsilon^2)$	$\mathcal{G}^c_{L,1}(z,\varepsilon) = 2D_1 + 2C_1 \ln(1-z) + \epsilon\ \mathcal{G}^{c,(1)}_{nsv,1}(z) + \mathcal{O}(\epsilon^2)$
$\mathcal{G}^c_{L,2}(z,\varepsilon) = \frac{2A_2}{1-z} - 2\beta_0 \frac{\mathcal{G}^{c,(1)}_{sv,1}}{1-z} + + \mathcal{O}(\epsilon)$	$\mathcal{G}^c_{L,2}(z,\varepsilon) = 2D_2 + 2C_2 \ln(1-z) - 2\beta_0 \mathcal{G}^{c,(1)}_{nsv,1}(z) + \mathcal{O}(\epsilon)$

- **Here the NSV coefficient is parametrised as :**

$$\mathcal{G}^{c,(j)}_{nsv,i}(z) = \sum_{k=0}^{i+j-1} \mathcal{G}^{c,(j,k)}_{nsv,i} \ln^k(1-z)$$

- **The Fixed Order result known till N^3LO demonstrate the above logarithmic structure and hence we propose an ansatz to all orders.**

- **The SV and NSV coefficients are determined from the explicit computations.**

PREDICTIONS

- **With these building blocks we have a structure for $\Delta_{c\bar{c}}$:**

$$\ln \Delta_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \left(\ln\left(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon) \right)^2 + \ln \left| \hat{F}_c(\hat{a}_s, \mu^2, Q^2, \varepsilon) \right|^2 \right) \delta(1-z)$$
$$+ \ln S_c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2C \ln \Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon) \,.$$

- **What do we achieve as a consequence to this decomposition:**

$$L_z^i = \ln^i(1-z)$$

GIVEN	PREDICTIONS					
FO Coefficient	$\Delta_{c\bar{c}}^{(2)}$	$\Delta_{c\bar{c}}^{(3)}$	$\Delta_{c\bar{c}}^{(4)}$	$\Delta_{c\bar{c}}^{(5)}$	$\Delta_{c\bar{c}}^{(6)}$	$\Delta_{c\bar{c}}^{(i)}$
χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

PREDICTIONS

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	FO Coefficient	$\Delta_{c\bar{c}}^{(2)}$	$\Delta_{c\bar{c}}^{(3)}$	$\Delta_{c\bar{c}}^{(4)}$	$\Delta_{c\bar{c}}^{(5)}$	$\Delta_{c\bar{c}}^{(6)}$	$\Delta_{c\bar{c}}^{(i)}$
1-loop	χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
	χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
	χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

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$$\ln \Delta_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \left(\ln \left(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon) \right)^2 + \ln \left| \hat{F}_c(\hat{a}_s, \mu^2, Q^2, \varepsilon) \right|^2 \right) \delta(1-z)$$
$$+ \ln S_c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2C \ln \Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon) \, .$$

- **What do we achieve as a consequence to this decomposition:**

$$L_z^i = \ln^i(1-z)$$

	GIVEN	PREDICTIONS					
	FO Coefficient	$\Delta_{c\bar{c}}^{(2)}$	$\Delta_{c\bar{c}}^{(3)}$	$\Delta_{c\bar{c}}^{(4)}$	$\Delta_{c\bar{c}}^{(5)}$	$\Delta_{c\bar{c}}^{(6)}$	$\Delta_{c\bar{c}}^{(i)}$
1-loop	χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
2-loop	χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
	χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

PREDICTIONS

- **With these building blocks we have a structure for $\Delta_{c\bar{c}}$:**

$$\ln \Delta_{c\bar{c}}\left(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\right) = \left(\ln \left(Z_{UV,c}\left(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon\right)\right)^2 + \ln \left| \hat{F}_c\left(\hat{a}_s, \mu^2, Q^2, \varepsilon\right)\right|^2 \right) \delta(1-z)$$
$$+ \ln S_c\left(\hat{a}_s, \mu^2, q^2, z, \varepsilon\right) - 2C \ln \Gamma_{cc}\left(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon\right) .$$

- **What do we achieve as a consequence to this decomposition:**

$$L_z^i = \ln^i(1-z)$$

	GIVEN	PREDICTIONS					
	FO Coefficient	$\Delta^{(2)}_{c\bar{c}}$	$\Delta^{(3)}_{c\bar{c}}$	$\Delta^{(4)}_{c\bar{c}}$	$\Delta^{(5)}_{c\bar{c}}$	$\Delta^{(6)}_{c\bar{c}}$	$\Delta^{(i)}_{c\bar{c}}$
1-loop	χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
2-loop	χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
3-loop	χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

CHECKS AND PREDICTIONS

- But there are certain logarithms which **we cannot predict completely from previous order informations.**
- For instance : $\ln^3(1-z)$ coefficient at the 3[rd] order.
- Even though 2-loop cannot predict completely, but we find many color factors come from 2-loop.
- So let's see how far we can get it right !

The left column stands for the exact result and the right for the predictions using two loop.

	$gg \to H$			Drell-Yan (DY)		$b\bar{b} \to H$	
C_A^3	$\frac{-111008}{27} + 3584\zeta_2$	$\frac{-110656}{27} + 3584\zeta_2 + \chi_1$	C_F^3	$2272 + 3072\zeta_2$	$2272 + 3072\zeta_2$	$736 + 3072\zeta_2$	$736 + 3072\zeta_2$
$C_A^2 n_f$	$\frac{6560}{9}$	$\frac{19616}{27} + \chi_2$	$C_F^2 n_f$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_3$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_3$
$C_A n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$	$C_A C_F^2$	$\frac{-111904}{27} + 512\zeta_2$	$\frac{-37184}{9} + 512\zeta_2 + \chi_4$	$\frac{-111904}{27} + 512\zeta_2$	$\frac{-37184}{9} + 512\zeta_2 + \chi_4$
			$C_F n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$	$\frac{-256}{27}$	$\frac{-256}{27}$
			$C_A C_F n_f$	$\frac{2816}{27}$	$\frac{2816}{27}$	$\frac{2816}{27}$	$\frac{2816}{27}$
			$C_A^2 C_F$	$\frac{-7744}{27}$	$\frac{-7744}{27}$	$\frac{-7744}{27}$	$\frac{-7744}{27}$

[Anastasiou et al.] [Duhr et al.]

CHECKS AND PREDICTIONS

- **Predictions till 7-loop for the first three logs for DY and $b\bar{b} \to H$, using 3-loop.**

$$
\begin{aligned}
\Delta_q^{NSV} &= a_s \Delta_q^{NSV(1)} + a_s^2 \Delta_q^{NSV(2)} + a_s^3 \Delta_q^{NSV(3)} + a_s^4 \Bigg[\bigg\{-\frac{4096}{3}C_F^4\bigg\}L_z^7 + \bigg\{\frac{39424}{9}C_F^3C_A + \frac{19712}{3}C_F^4 \\
&- \frac{7168}{9}n_f C_F^3\bigg\}L_z^6 + \bigg\{-\frac{123904}{27}C_F^2C_A^2 - \bigg(\frac{805376}{27} - 3072\zeta_2\bigg)C_F^3C_A + \bigg(9088 + 20480\zeta_2\bigg)C_F^4 \\
&+ \frac{45056}{27}n_f C_F^2 C_A + \frac{139520}{27}n_f C_F^3 - \frac{4096}{27}n_f^2 C_F^2\bigg\}L_z^5 + \mathcal{O}(L_z^4)\Bigg] \\
&+ a_s^5\Bigg[\bigg\{-\frac{8192}{3}C_F^5\bigg\}L_z^9 + \bigg\{\frac{51200}{3}C_F^5 - \frac{8192}{3}C_F^4 n_f + \frac{45056}{3}C_F^4 C_A\bigg\}L_z^8 + \bigg\{\bigg(\frac{72704}{3} + \frac{229376}{3}\zeta_2\bigg)C_F^5 \\
&- \bigg(\frac{1120256}{9} - \frac{32768}{3}\zeta_2\bigg)C_F^4C_A - \frac{81920}{81}C_F^3n_f^2 + \frac{194560}{9}C_F^4n_f + \frac{901120}{81}C_F^3C_An_f - \frac{2478080}{81}C_F^3C_A^2\bigg\}L_z^7 \\
&+ \mathcal{O}(L_z^6)\Bigg] + a_s^6\Bigg[\bigg\{-\frac{65536}{15}C_F^6\bigg\}L_z^{11} + \bigg\{\frac{167936}{5}C_F^6 - \frac{180224}{27}C_F^5n_f + \frac{991232}{27}C_F^5C_A\bigg\}L_z^{10} \\
&+ \bigg\{\bigg(\frac{145408}{3} + 196608\zeta_2\bigg)C_F^6 + \frac{5054464}{81}C_F^5n_f - \frac{327680}{81}C_F^4n_f^2 - \bigg(\frac{28997632}{81} - \frac{81920}{3}\zeta_2\bigg)C_F^5C_A \\
&+ \frac{3604480}{81}C_F^4C_An_f - \frac{9912320}{81}C_F^4C_A^2\bigg\}L_z^9 + \mathcal{O}(L_z^8)\Bigg] + a_s^7\Bigg[\bigg\{-\frac{262144}{45}C_F^7\bigg\}L_z^{13} + \bigg\{\frac{2392064}{45}C_F^7 \\
&- \frac{1703936}{135}C_F^6n_f + \frac{9371648}{135}C_F^6C_A\bigg\}L_z^{12} + \bigg\{\bigg(\frac{1163264}{15} + \frac{5767168}{15}\zeta_2\bigg)C_F^7 + \frac{55115776}{405}C_F^6n_f \\
&- \bigg(\frac{315080704}{405} - \frac{262144}{5}\zeta_2\bigg)C_F^6C_A - \frac{917504}{81}C_F^5n_f^2 + \frac{10092544}{81}C_F^5C_An_f - \frac{27754496}{81}C_F^5C_A^2\bigg\}L_z^{11} \\
&+ \mathcal{O}(L_z^{10})\Bigg] + \mathcal{O}(a_s^8)\,, \\
\Delta_b^{NSV} &= a_s\Delta_b^{NSV(1)} + a_s^2\Delta_b^{NSV(2)} + a_s^3\Delta_b^{NSV(3)} + a_s^4\Big[\Delta_q^{NSV(4)} - 6144C_F^4L_z^5 + \mathcal{O}(L_z^4)\Big] \\
&+ a_s^5\Big[\Delta_q^{NSV(5)} - 16384C_F^5L_z^7 + \mathcal{O}(L_z^6)\Big] + a_s^6\Big[\Delta_q^{NSV(6)} - 32768C_F^6L_z^9 + \mathcal{O}(L_z^8)\Big] \\
&+ a_s^7\Big[\Delta_q^{NSV(7)} - \frac{262144}{5}C_F^7L_z^{11} + \mathcal{O}(L_z^{10})\Big] + \mathcal{O}(a_s^8)\;,
\end{aligned}
$$

Till 4-loop
[Vogt, Moch et al.],
[De Florian et al.],
[Das et all]

CHECKS AND PREDICTIONS

- **Predictions till 7-loop for the first three logs for gluon fusion, using 3-loop.**

$$
\begin{aligned}
\Delta_g^{NSV} &= a_s \Delta_g^{NSV(1)} + a_s^2 \Delta_g^{NSV(2)} + a_s^3 \Delta_g^{NSV(3)} \\
&+ a_s^4 \Bigg[\Bigg\{ - \frac{4096}{3} C_A^4 \Bigg\} L_z^7 + \Bigg\{ \frac{98560}{9} C_A^4 - \frac{7168}{9} n_f C_A^3 \Bigg\} L_z^6 + \Bigg\{ \Bigg(- \frac{298240}{9} + 23552 \zeta_2 \Bigg) C_A^4 \\
&+ \frac{174208}{27} n_f C_A^3 - \frac{4096}{27} n_f^2 C_A^2 \Bigg\} L_z^5 + \mathcal{O}(L_z^4) \Bigg] + a_s^5 \Bigg[\Bigg\{ - \frac{8192}{3} C_A^5 \Bigg\} L_z^9 + \Bigg\{ \frac{96256}{3} C_A^5 \\
&- \frac{8192}{3} C_A^4 n_f \Bigg\} L_z^8 + \Bigg\{ \Bigg(- \frac{12283904}{81} + \frac{262144}{3} \zeta_2 \Bigg) C_A^5 + \frac{2569216}{81} C_A^4 n_f - \frac{81920}{81} n_f^2 C_A^3 \Bigg\} L_z^7 \\
&+ \mathcal{O}(L_z^6) \Bigg] + a_s^6 \Bigg[\Bigg\{ - \frac{65536}{15} C_A^6 \Bigg\} L_z^{11} + \Bigg\{ \frac{9490432}{135} C_A^6 - \frac{180224}{27} C_A^5 n_f \Bigg\} L_z^{10} + \Bigg\{ \Bigg(\frac{671744}{3} \zeta_2 \\
&- \frac{4261888}{9} \Bigg) C_A^6 + \frac{8493056}{81} C_A^5 n_f - \frac{327680}{81} n_f^2 C_A^4 \Bigg\} L_z^9 + \mathcal{O}(L_z^8) \Bigg] \\
&+ a_s^7 \Bigg[\Bigg\{ - \frac{262144}{45} C_A^7 \Bigg\} L_z^{13} + \Bigg\{ \frac{3309568}{27} C_A^7 - \frac{1703936}{135} C_A^6 n_f \Bigg\} L_z^{12} + \Bigg\{ \Bigg(- \frac{449429504}{405} \\
&+ \frac{1310720}{3} \zeta_2 \Bigg) C_A^7 + \frac{11583488}{45} C_A^6 n_f - \frac{917504}{81} n_f^2 C_A^5 \Bigg\} L_z^{11} + \mathcal{O}(L_z^{10}) \Bigg] + \mathcal{O}(a_s^8) .
\end{aligned}
$$

Till 4-loop
[Vogt, Moch et al.],
[De Florian et al.],
[Das et all]

- **Predictions till 7-loop for the first three logs for gluon fusion, using 3-loop.**

$$\Delta_g^{NSV} = \ldots$$

$$+ a_s \ldots C_A^4$$

$$+ \frac{1}{\ldots} \ldots$$

$$- \frac{8}{\ldots} \ldots C_A^3 \Big\} L_z^7$$

$$+ \mathcal{O}(L_z^6)\Big] + a_s^6 \Big[\Big\{ - \frac{65536}{15} C_A^6 \Big\} L_z^{11} + \Big\{ \frac{9490432}{135} C_A^6 - \frac{180224}{27} C_A^5 n_f \Big\} L_z^{10} + \Big\{ \Big(\frac{671744}{3} \zeta_2$$

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In General :

$$\ln^k(1-z),\ n+1 \le k \le 2n-1$$

at order a_s^n

Till 4-loop
[Vogt, Moch et al.],
[De Florian et al.],
[Das et all]

INTEGRAL REPRESENTATION

- **Knowing the functional form of each building blocks one can derive the integral form as:**

$$\ln \Delta_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z) = \ln C_0^c(q^2, \mu_R^2, \mu_F^2) + \left\{ \int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} P'_{cc}(a_s(\lambda^2), z) + Q^c(a_s(q^2(1-z)^2), z) \right\}$$

- **Some Details:**
  - C_0^c **captures the** $\delta(1-z)$ **contribution from** $\hat{F}_c$ & S_c
  - **Finite contributions from cancellation between** Γ_{cc} & S_c

$$P'_{cc}(z) \propto \left[A^c \left(\frac{1}{1-z} \right)_+ + C^c \ln(1-z) + D^c \right]$$

  - **Finite contributions coming from** S_c

$$Q^c(a_s(q^2(1-z)^2), z) \propto \left(\frac{1}{1-z} \mathcal{G}_{sv}(a_s(q^2(1-z)^2)) \right)_+ + \mathcal{G}_{nsv}(a_s(q^2(1-z)^2), z)$$

MELLIN SPACE N

- **To study the all-order behaviour we need integral representation for $\Delta_{c\bar{c}}$.**

$$\Delta_N^{c\bar{c}}(q^2) = \int_0^1 dz\ z^{N-1}\ \Delta_{c\bar{c}}(q^2, z)$$

- **Threshold limit $z \to 1$ in z - space translates to $N \to \infty$ in N-space.**
- **Taking till $\dfrac{1}{N}$ corrections from SV and NSV terms :**

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \sim \frac{\ln^2 N}{2} - \frac{\ln N}{2N} + \frac{1}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\ln^k(1-z) \sim \frac{\ln^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

NSV RESUMMATION

- **Hence the inclusion of the NSV logarithms modifies the existing resumed expression as :**

$$\Delta_{c,N}(q^2,\mu_R^2,\mu_F^2) = \left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\tilde{g}_{0,i}(q^2,\mu_R^2,\mu_F^2)\right)\exp\left(\Psi^c_{SV,N}(q^2,\mu_F^2) + \Psi^c_{NSV,N}(q^2,\mu_F^2)\right)$$

- **where,**

$$\Psi^c_{\mathrm{sv,N}} = g_1^c(\omega)\ln(N) + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g^c_{i+2}(\omega)$$

[Sterman et al.]
[Catani et al.]

- **and,**

$$\omega = 2a_s\beta_0 \ln N$$

$$\Psi^c_{\mathrm{NSV,N}} = \frac{1}{N}\left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2) h_i^c(\omega,N)\right)$$

$$h_0^c(\omega,N) = h_{00}^c(\omega) + h_{01}^c(\omega)\ln(N), \quad h_i^c(\omega,N) = \sum_{k=0}^{i} h_{ik}^c(\omega)\ \ln^k(N)$$

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Known Result since 1989

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New Result !!

$$\Psi^c_{\mathrm{NSV,N}} = \frac{1}{N}\left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2) h_i^c(\omega,N)\right)$$

$$h_0^c(\omega,N) = h_{00}^c(\omega) + h_{01}^c(\omega)\ln(N), \quad h_i^c(\omega,N) = \sum_{k=0}^{i} h_{ik}^c(\omega)\ \ln^k(N)$$

$\ln N/N$ TOWERS

- **The towers of $\ln N/N$ that we sum over :**

$\Delta_N^c =$

	Only 1-loop info	Only 2-loop info	…	Only n-loop info
Resumed terms :	$a_s \frac{\ln N}{N}$ $a_s^2 \frac{\ln^3 N}{N}$ $\vdots$ $a_s^i \frac{\ln^{2i-1} N}{N}$	$a_s^2 \frac{\ln^2 N}{N}$ $a_s^3 \frac{\ln^4 N}{N}$ $\vdots$ $a_s^i \frac{\ln^{2i-2} N}{N}$	$\bullet\bullet\bullet$	$a_s^n \frac{\ln^n N}{N}$ $\vdots$ $a_s^i \frac{\ln^{2i-n} N}{N}$
Exponents :	g_1^c, h_0^c	g_2^c, h_1^c		g_{n+1}^c, h_n^c

CHECKS ON RESUMMATION

$$\Delta_{c,N}(q^2,\mu_R^2,\mu_F^2) = \left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\tilde{g}_{0,i}(q^2,\mu_R^2,\mu_F^2)\right)\exp\left(\Psi^c_{SV,N}(q^2,\mu_F^2) + \Psi^c_{NSV,N}(q^2,\mu_F^2)\right)$$

- **Expansion of the resummed result matches with the fixed order till 4-loop.**

- **The leading logarithm for SV+ NSV matches with the existing result :**

$$\Delta_{LL}^{DY} = g_0 \exp\left[\ln N g_1(\omega) + \frac{1}{N} h_0(\omega, N)\right]$$
$$= \exp\left[8 C_F a_s\left(\ln^2 N + \frac{\ln N}{N}\right)\right]$$

[Beneke et al.]
[Laenen et al.]

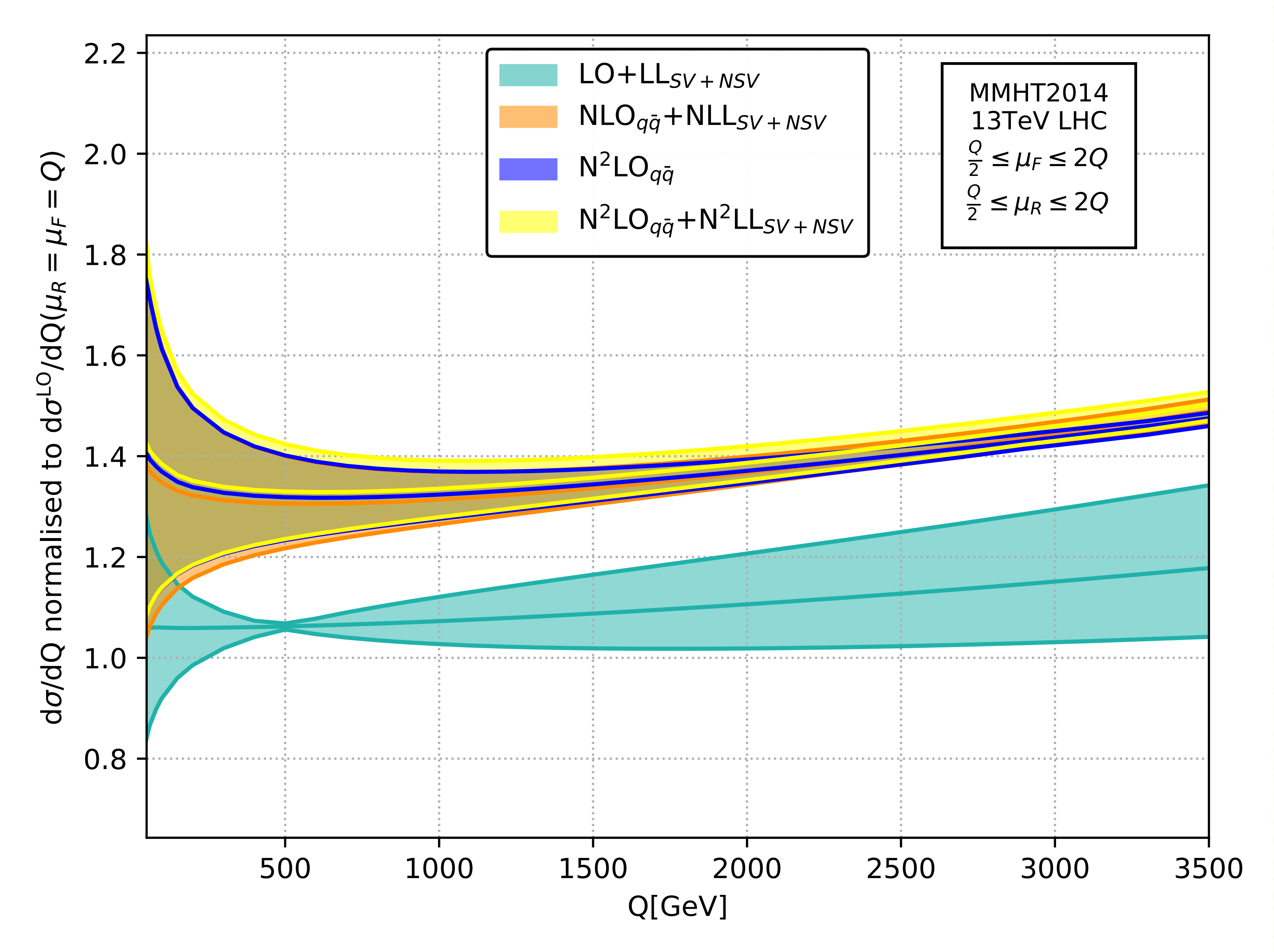
- **Now we perform Mellin Inversion of the resummed result to study the numerical impact.**

DY NSV PHENOMENOLOGY

[Preliminary]

$\mu_R = \mu_F$ =Q (GeV)	LO	LO+LL (sv+nsv)	$NLO_{q\bar{q}}$	$NLO_{q\bar{q}}$+NLL (sv+nsv)	$N^2LO_{q\bar{q}}$	$N^2LO_{q\bar{q}}+N^2LL$ (sv+nsv)
1500	$2.73^{6.61\%}_{-5.96\%}$	$2.98^{7.08\%}_{-6.34\%}$	$3.78^{1.6\%}_{-1.4\%}$	$3.94^{2.96\%}_{-2.42\%}$	$4.00^{2.29\%}_{-2.43\%}$	$4.04^{2.98\%}_{-3.08\%}$

Fixed order and Resummed result for DY (in 10^{-6} pb/GeV)



- **The inclusion of SV resummation increases the $N^2LO_{q\bar{q}}$ by 0.417% for $\mu_R = \mu_F = Q = 1500$ GeV**
- **the inclusion of SV+NSV resummation increases the $N^2LO_{q\bar{q}}$ by 1% for $\mu_R = \mu_F = Q = 1500$ GeV**

DY NSV PHENOMENOLOGY

[Preliminary]

$\mu_R = \mu_F$ =Q (GeV)	LO	LO+LL (sv+nsv)	NLO$_{q\bar{q}}$	NLO$_{q\bar{q}}$+NLL (sv+nsv)	N^{2}LO$_{q\bar{q}}$	N^{2}LO$_{q\bar{q}}$+N^{2}LL (sv+nsv)
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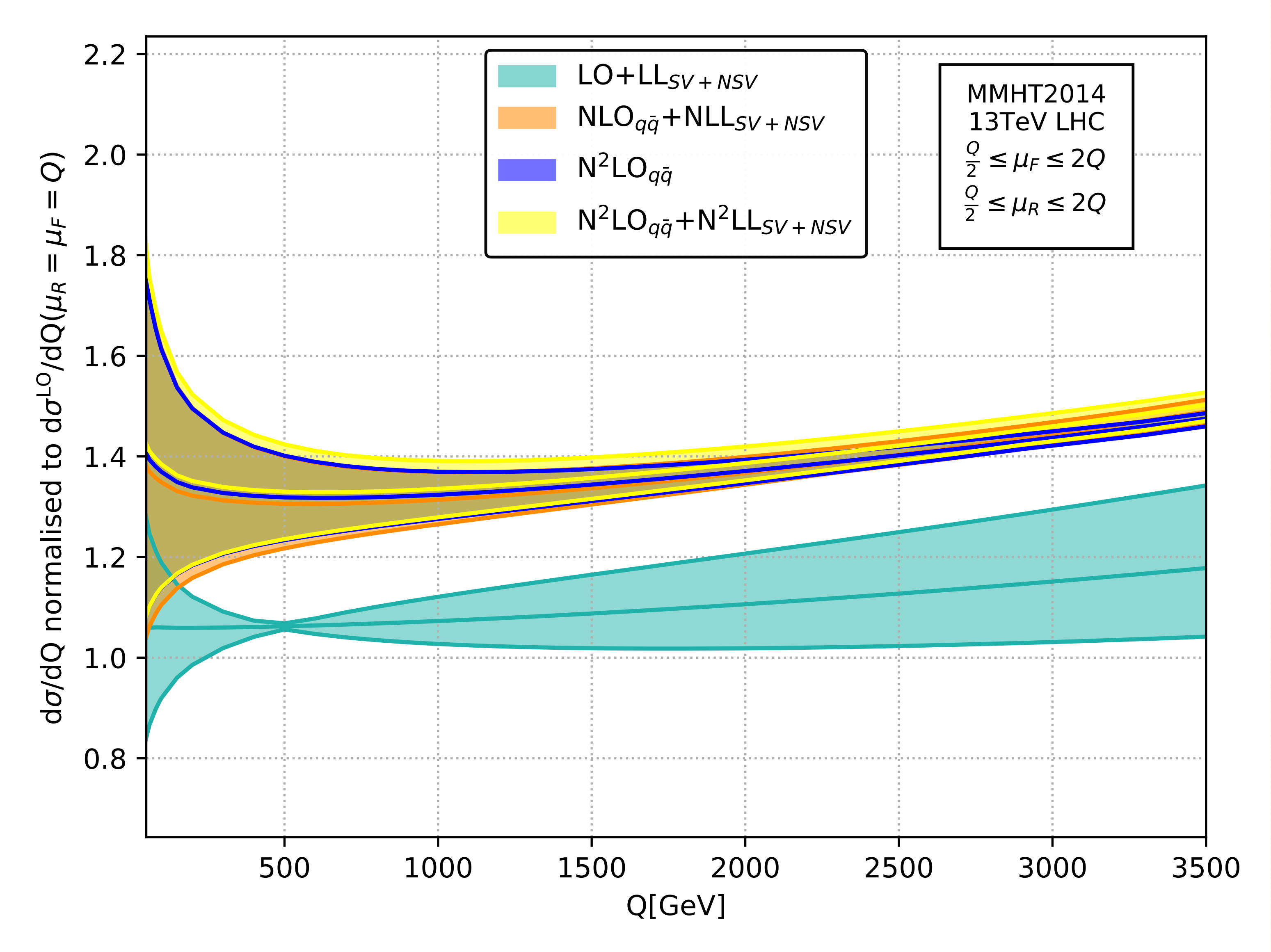

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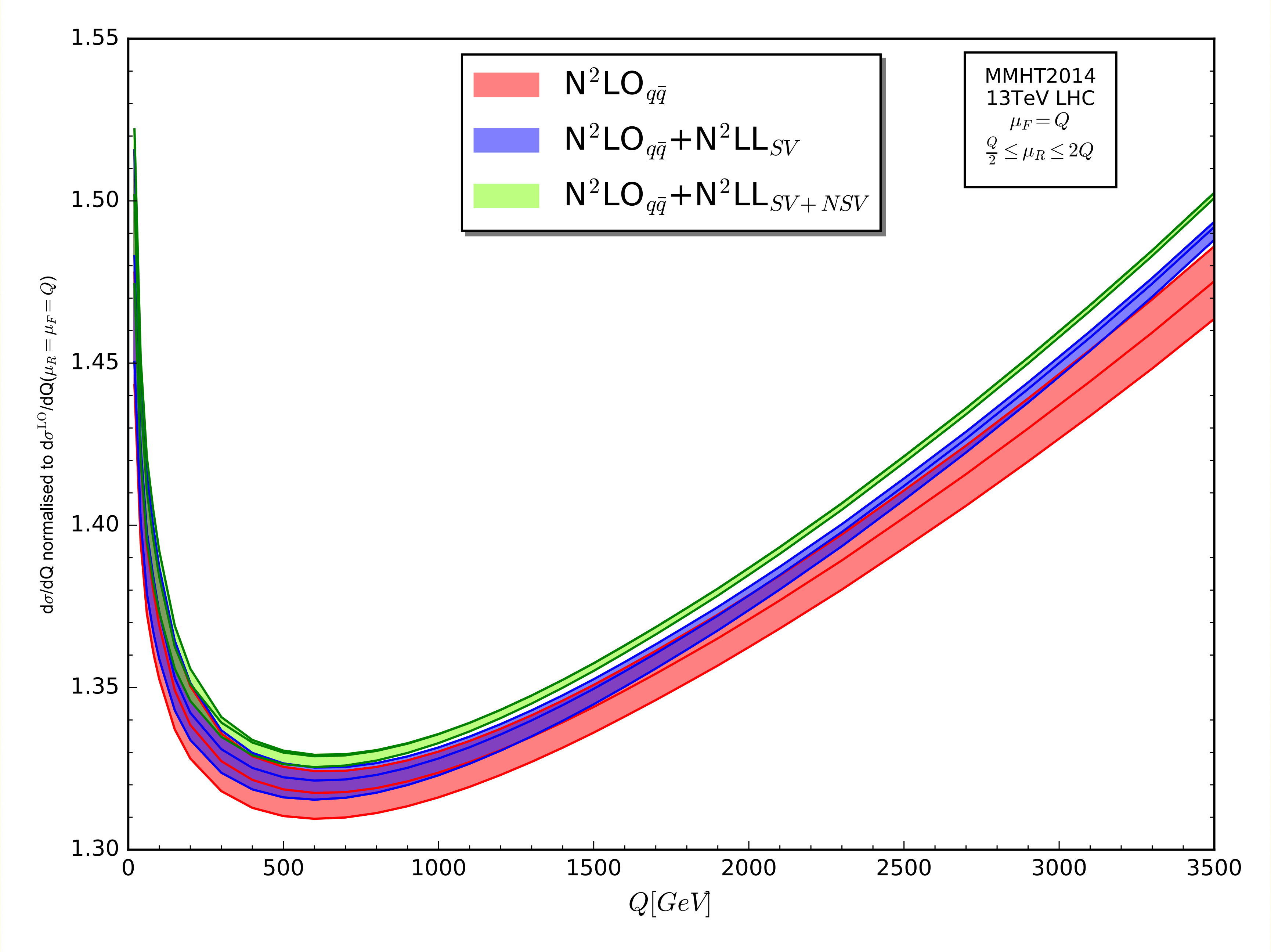
NSV PHENOMENOLOGY

[Preliminary]

For Drell-Yan

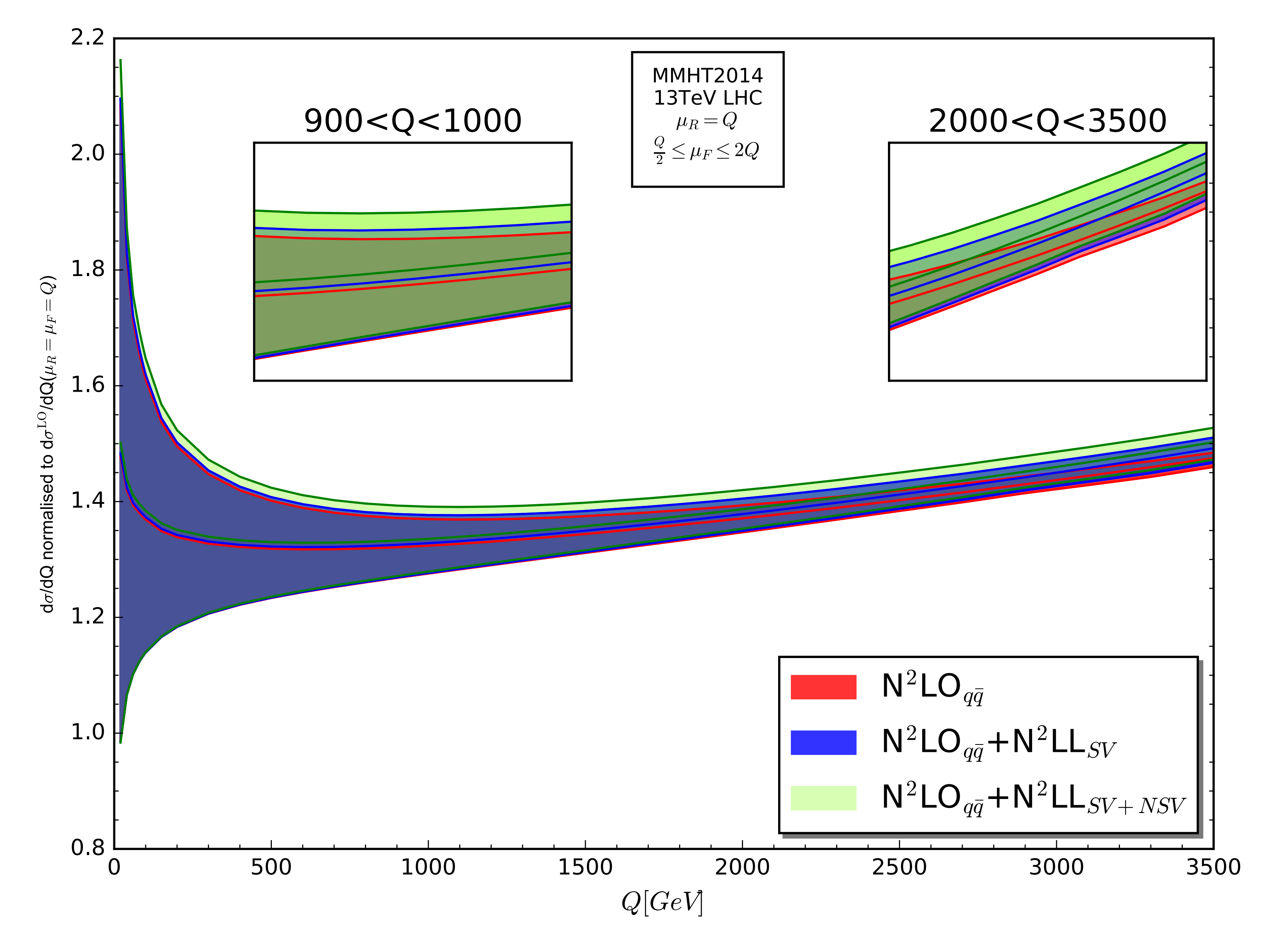


- **Plot shows μ_R dependency keeping μ_F fixed.**
- **We know that each partonic channel is invariant under μ_R and hence inclusion of more corrections within a channel is expected to reduce the uncertainty.**
- **The width of the μ_R band decreases from $N^2LO_{q\bar{q}}$ to resummed results.**
- **The inclusion of the NSV resummation improves the μ_R uncertainty more than the SV resummation.**

NSV PHENOMENOLOGY

[Preliminary]

For Drell-Yan



- **Plot shows μ_F dependency keeping μ_R fixed.**
- **But different partonic channels mix under μ_F variation and hence inclusion of other channels is expected to reduce the uncertainty.**
- **The μ_F uncertainty is most for SV+NSV resummed result.**
- **This only hints to the existing fact that there is a large cancellation b/w $q\bar{q}$ and qg channels.**

CONCLUSIONS

[Preliminary] **Fixed order and Resummed results for DY 13TeV LHC**

$\mu_R = \mu_F$ =Q (GeV)	N^2LO (pb/GeV)	N^2LO+N^2LL (sv+nsv)(pb/GeV)
200	$5.63 \times 10^{-2}{}^{0.41\%}_{-0.45\%}$	$5.69 \times 10^{-2}{}^{2.4\%}_{-1.48\%}$
500	$9.04 \times 10^{-4}{}^{0.21\%}_{-.207\%}$	$9.12 \times 10^{-4}{}^{1.56\%}_{-1.08\%}$
1500	$3.79 \times 10^{-6}{}^{0.29\%}_{-0.43\%}$	$3.83 \times 10^{-6}{}^{0.97\%}_{-0.80\%}$

- **What has been studied so far :**
  - **Using collinear factorisation and RG invariance and exploiting fixed order results, we propose an all order formula.**
  - **We propose an integral representation which can resume both SV and NSV logarithms to all orders.**

CONCLUSIONS

[Preliminary] **Fixed order and Resummed results for DY 13TeV LHC**

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  - **We propose an integral representation which can resume both SV and NSV logarithms to all orders.**

CONCLUSIONS

- Hence we have extended the Resummation of the NLP/NSV logarithms till N^2LL accuracy.
- We find the SV + NSV resummed results give significant contributions owing to the large coefficients of the NSV terms.

- What more to do ?
  - Impact of the Functional form of the soft collinear function on the resummed result.
  - Impact of different prescriptions on the resummed results.
  - Also estimate the corrections from Higgs productions.
  - Modify the existing formalism for off-Diagonal Channels.

THANK YOU