RADCOR - LoopFest 2021

A PITFALL IN APPLYING A NON-ANTICOMMUTING γ_5 IN $q\bar{q} \to ZH$ AMPLITUDES

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with
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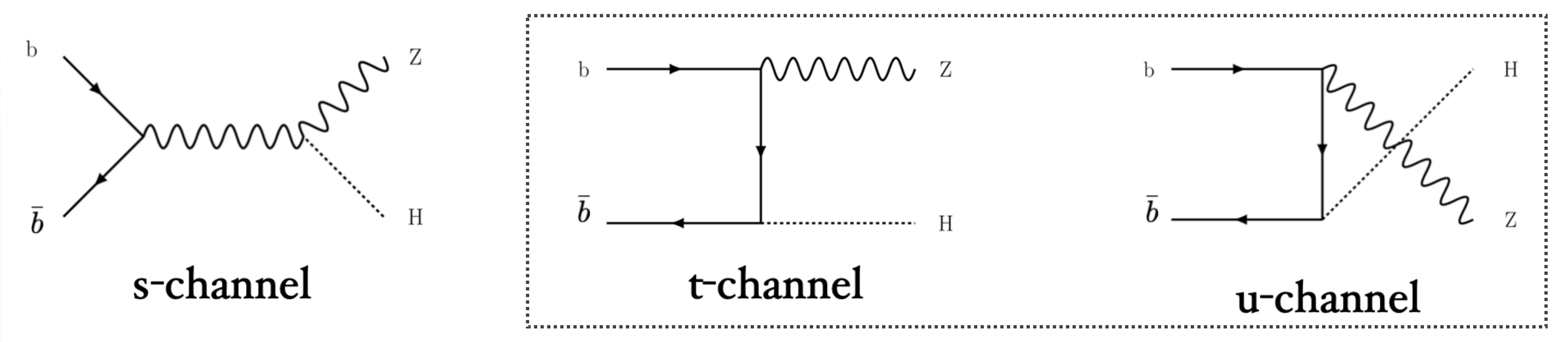
A lot of progress in VH

- $H \to b\bar{b}$ finally observed at the LHC in VH-events! [ATLAS, CMS '18]
- Much work done on ZH production at LHC: **P** + **P** → Z (ll') + H ($b\bar{b}$)
- $q\bar{q} \to ZH$
  - Higgs-bremsstrahlung (Drell-Yan type): NNLO & $\text{N}^3\text{LO}_{\text{approx}}$ in massless QCD [Ferrera, Somogyi, Tramontano '18]
    [Campbell, Ellis, Williams '16] [Ferrera, Grazzini, Tramontano '15] [Kumar, Mandal and Ravindran '15] [Brein, Hartader, Wesemann, Zirke '12] [Brein, Djouadi, Hartader, '04]
  - Bottom quark initiated with Higgs Yukawa (non Drell-Yan type): NNLO in massless QCD
    [TA, Chen, Dhani, Ajjath, Mukherjee, Ravindran '19]
  - Top-loop induced NNLO (non-Drell-Yan type) QCD corrections in the heavy-top limit
    [Brein, Djouadi, Harlander '04] [Brein, Harlander, Wiesemann, Zirke '12]
  - NLO Electroweak corrections [Ciccolini, Dittmaier, Kramer '03] [Denner, Dittmaier, Kallweit, Mueck '11]
- $gg \to ZH$
  - LO: exact with full m_t dependence [Kniehl '90] [Dicus, Kao '88]
  - NLO QCD: - in the heavy-top & high-energy limit [Altenkamp, Dittmaier, Harlander, Rzehak, Zirke '13] [Hasselhuhn, Luthe, Steinhauser '17] [Davies, Mishima, Steinhauser '20] [Brein, Hartader, Wesemann, Zirke '12]
    - with full m_t dependence [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk '21] [Alasfar, Degrassi, Giardino, Groeber, Vitti '21]

[Talk by G. Mishima & M. Kerner]

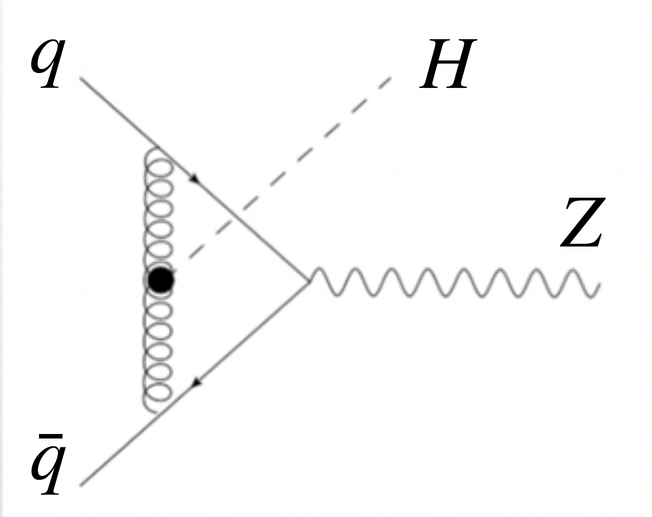
Our Focus

- $b\bar{b} \to ZH$ in massless QCD with non-zero Yukawa coupling



$\mathcal{O}(\lambda_b)$

- $q\bar{q} \to ZH$ in Higgs effective field theory

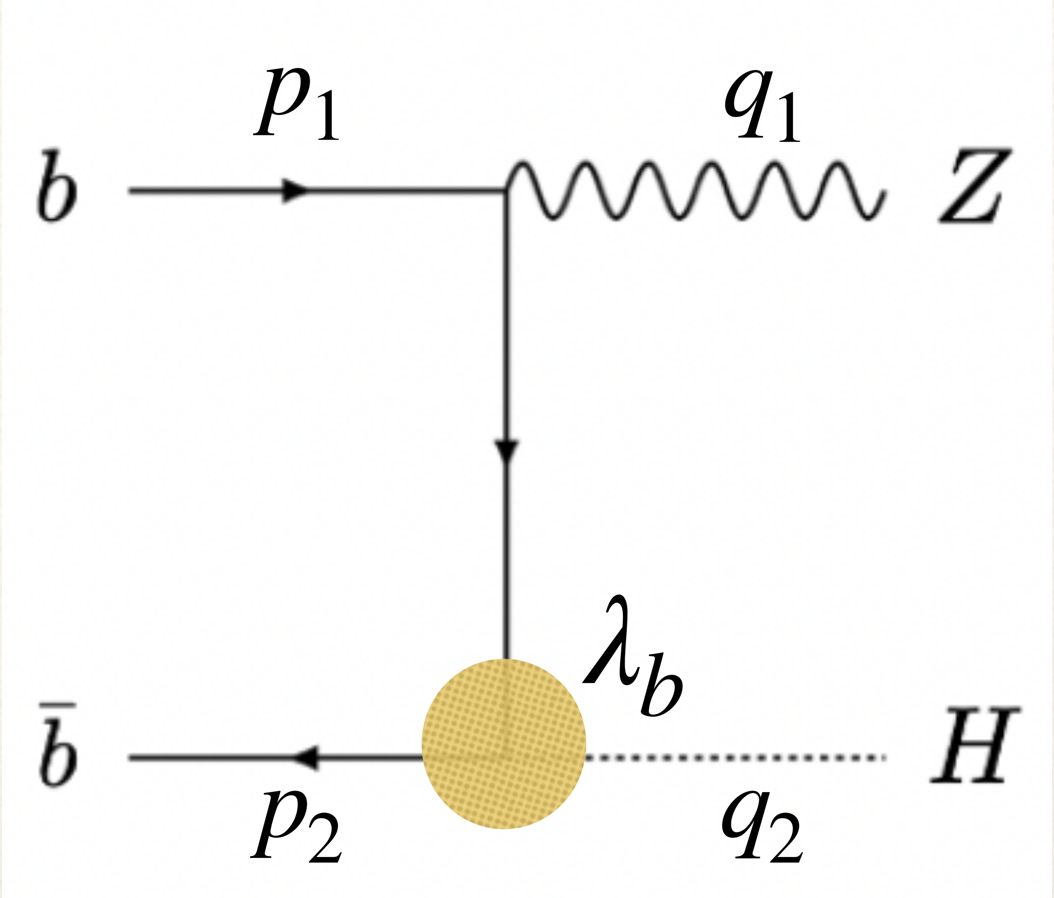


$\mathcal{O}(\lambda_t \alpha_s^2)$

Goal

- 2-loop QCD corrections in analytic form: $\mathcal{O}(\lambda_b \alpha_s^2)$ & $\mathcal{O}(\alpha_s^3)$
- Addressing a subtle issue in FF decomposition for axial current in D-dimensions
- How axial FF can be obtained from vector counterpart without dealing γ_5 issues for $b\bar{b} \to ZH$
- A surprising phenomenon upon applying non-anticommuting γ_5 for $q\bar{q} \to ZH$ in HEFT

Form Factor Decomposition: Vector



Lorentz covariance

$$\mathcal{M}_{vec} = \bar{v}(p_2)\Gamma^{\mu}_{vec}u(p_1)\varepsilon^*_{\mu}(q_1)$$

$$\Gamma^{\mu}_{X} = F_{1,X}p_1^{\mu} + F_{2,X}p_2^{\mu} + F_{3,X}q_1^{\mu} + F_{4,X}\gamma^{\mu}\not{q}_1$$

- Reflects the chirality flipping along massless b-quark line & parity even
- Linearly independent & complete in **D-dimensions** $D = 4 - 2\epsilon$
- Projectors:

$$\mathcal{M} = \sum F_n T_n \longrightarrow \text{Gram Matrix} \longrightarrow \mathbf{P}_n = G^{-1}_{nj}T^{\dagger}_j$$

$$G_{ij} = \langle T^{\dagger}_i, T_j \rangle$$

D-dimensional projectors

- FF in CDR / HV scheme

Form Factor Decomposition: Axial & Issues

- DR preserves Lorentz & gauge invariance but not chiral invariance: γ_5 is inherently 4-dimensional

- 't Hooft–Veltman prescription & Breitenlohner - Maison (HVBM): $\gamma_5 = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$ $\quad \{\gamma^\mu, \gamma_5\} \neq 0$

[Breitenlohner-Maison '77] [Hooft, Veltman '72]

- Basis

$$\bar{v}(p_2)\left\{\gamma_5 p_1^\mu\,, \gamma_5 p_2^\mu\,, \gamma_5 q_1^\mu\,, \gamma^\mu \gamma_5 \not{q}_1\right\}u(p_1)$$

Linearly complete in 4-dim but not in D dimensions (unless fully anticommuting γ_5 is used)

Issues with NAC γ_5

- Not easy to construct full D-dim linearly complete basis for **all loop** orders
- Even with an incomplete basis, keeping full D-dependence leads complicated expressions
- Setting D=4 simplify but not known whether it is legitimate, in general

Form Factor Decomposition: Axial & Issues

$$\mathcal{M}_{axi} = \bar{v}(p_2)\Gamma^{\mu}_{axi}u(p_1)\varepsilon^*_{\mu}(q_1)$$

$$F_{1,axi} \equiv \mathbb{P}^{[4],\mu}_{1,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$
$$F_{2,axi} \equiv \mathbb{P}^{[4],\mu}_{2,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$
$$F_{3,axi} \equiv \mathbb{P}^{[4],\mu}_{3,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$
$$F_{4,axi} \equiv \mathbb{P}^{[4],\mu}_{4,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$

Acrobatic form: set D = 4 in projectors and define 'axial FF'

Intermediate axial amp $\tilde{\Gamma}^{\mu}_{axi} \equiv F_{1,axi}\gamma_5 p_1^{\mu} + F_{2,axi}\gamma_5 p_2^{\mu} + F_{3,axi}\gamma_5 q_1^{\mu} + F_{4,axi}\gamma^{\mu}\gamma_5 \not{q}_1$

is not algebraically identical to original Feynman amplitude

Leads to correct finite remainder in 4-dim

- Ward identity
- Polarised amplitudes using physical projectors that does not rely on FF decomposition [Chen '19]

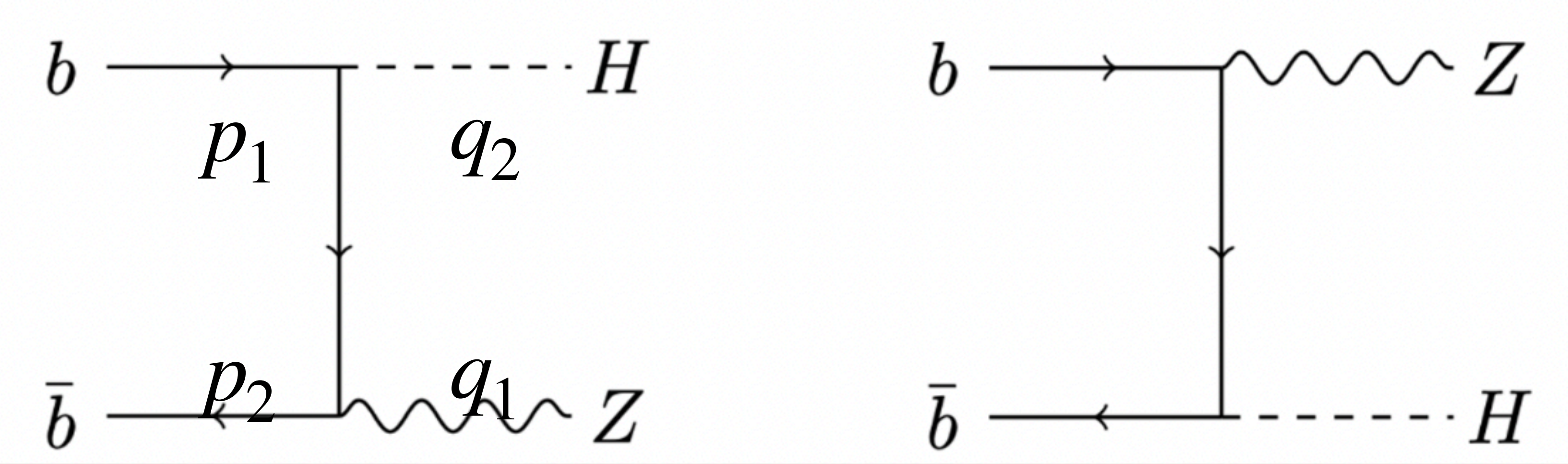
Message 1 **Projectors derived in 4-dim can be used in D-dimensional calculations and lead to correct physical results**

[TA, Chen, Dhani, Ajjath, Mukherjee, Ravindran '19]

[Peraro, Tancredi '19, '20]

Axial Form Factors restored from Vector

class ZH + class HZ



Similarly for higher loop orders

Non-vanishing **non-anomalous** diagrams

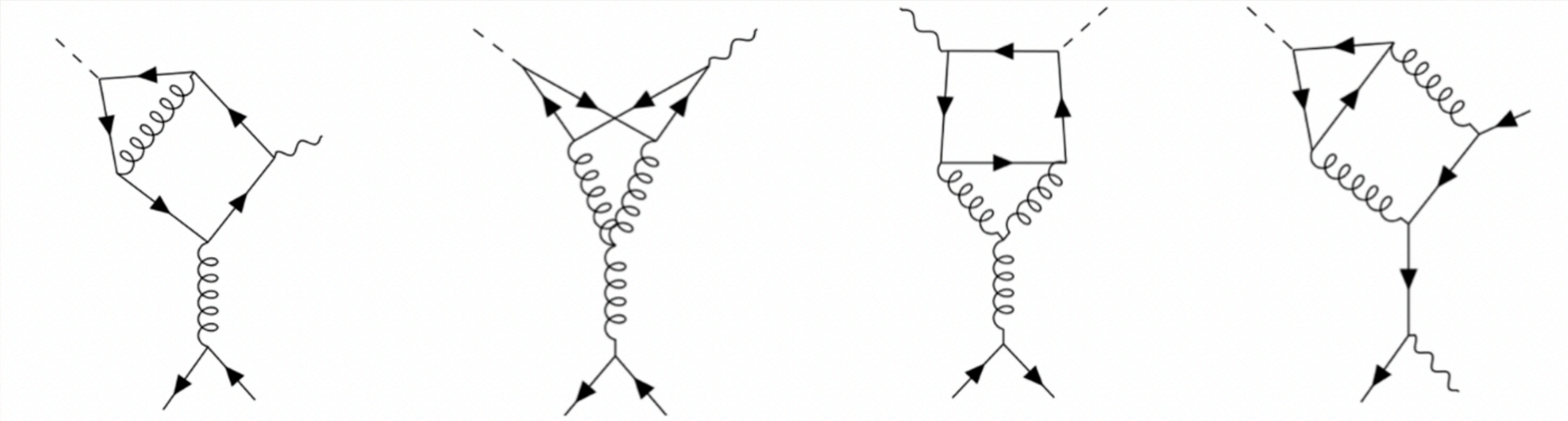
Turning off axial coupling of Z-boson completely

$$\mathcal{M}_{vec} = \bar{v}(p_2)\left(\boldsymbol{\Gamma}^{\mu}_{ZH} + \boldsymbol{\Gamma}^{\mu}_{HZ}\right) u(p_1)\, \varepsilon^*_{\mu}(q_1)$$

$$\boldsymbol{\Gamma}^{\mu}_{X} = F_{1,X}\, p_1^{\mu} + F_{2,X}\, p_2^{\mu} + F_{3,X}\, q_1^{\mu} + F_{4,X}\, \gamma^{\mu} \not{q}_1$$

Linearly independent & complete in **D-dimensions**

All 2-loop diagrams with Higgs or Higgs + Z radiated from a closed fermion loop vanish



$= 0$

Due to odd number of γ matrices

Axial Form Factors restored from Vector

Vector FFs

$$F_{i,vec} = F_{i,HZ} + F_{i,ZH}$$

Axial FFs

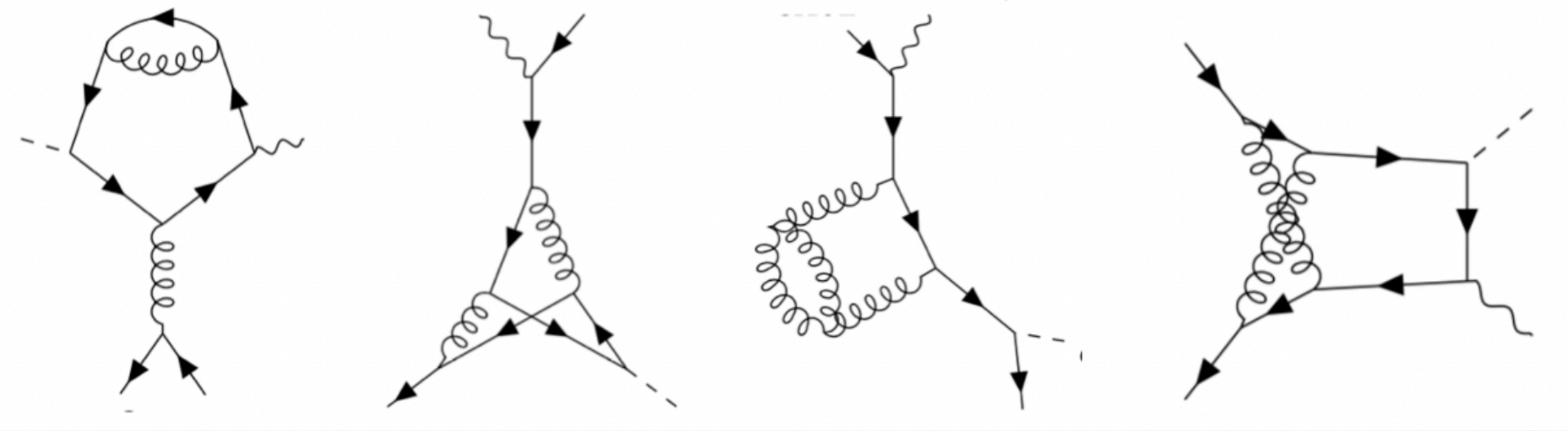
$$\mathcal{M}_{axi} = \mathcal{M}_{axi(ns)} + \mathcal{M}_{axi(s)}$$

$$\mathcal{M}^{\mu}_{axi(ns)} = \bar{v}(p_2)\Big[F_{1,axi(ns)}p_1^{\mu} + F_{2,axi(ns)}p_2^{\mu} + F_{3,axi(ns)}q_1^{\mu} + F_{4,axi(ns)}\gamma^{\mu}\not{q}_1\Big]\gamma_5 u(p_1)$$

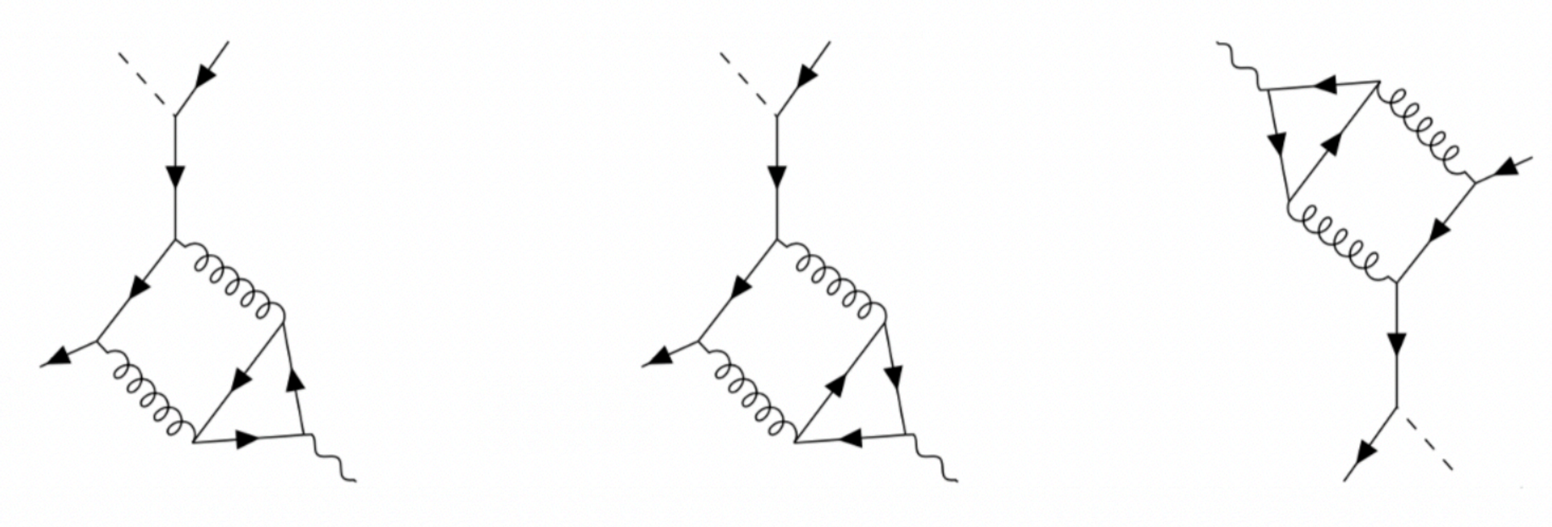
- Basis: Linearly independent & complete in **4-dimensions**

$$F_{i,axi(ns)} = F_{i,HZ} - F_{i,ZH}$$

Flavour non-singlet: non-anomalous



Flavour singlet: Anomalous

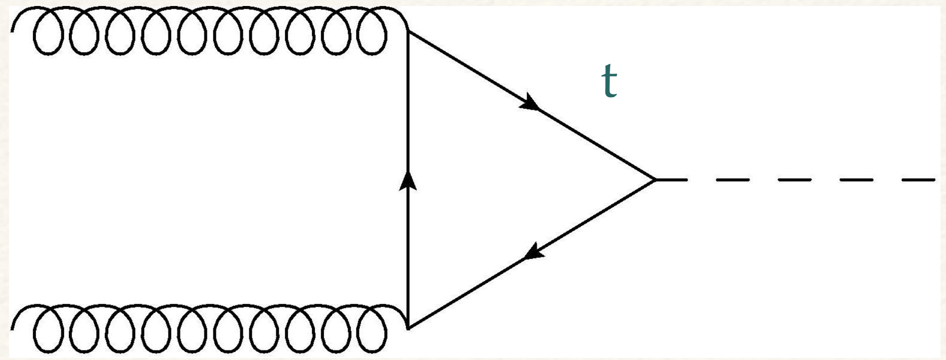


- Chirality-flipping Yukawa interaction on b-quark line generates relative -ve sign: $F_{i,vec} \neq F_{i,axi(ns)}$
- UV renormalisation procedure is identical to vector counterpart

Message 2 **Even for non Drell-Yan type diagrams, axial & vector FFs can be related**

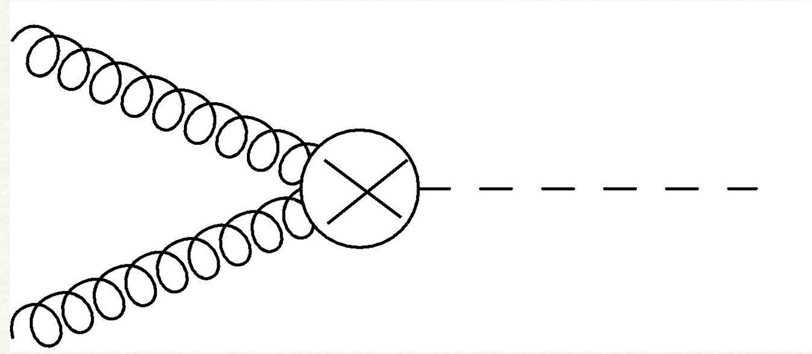
$q\bar{q} \to ZH$ in Higgs Effective Field Theory

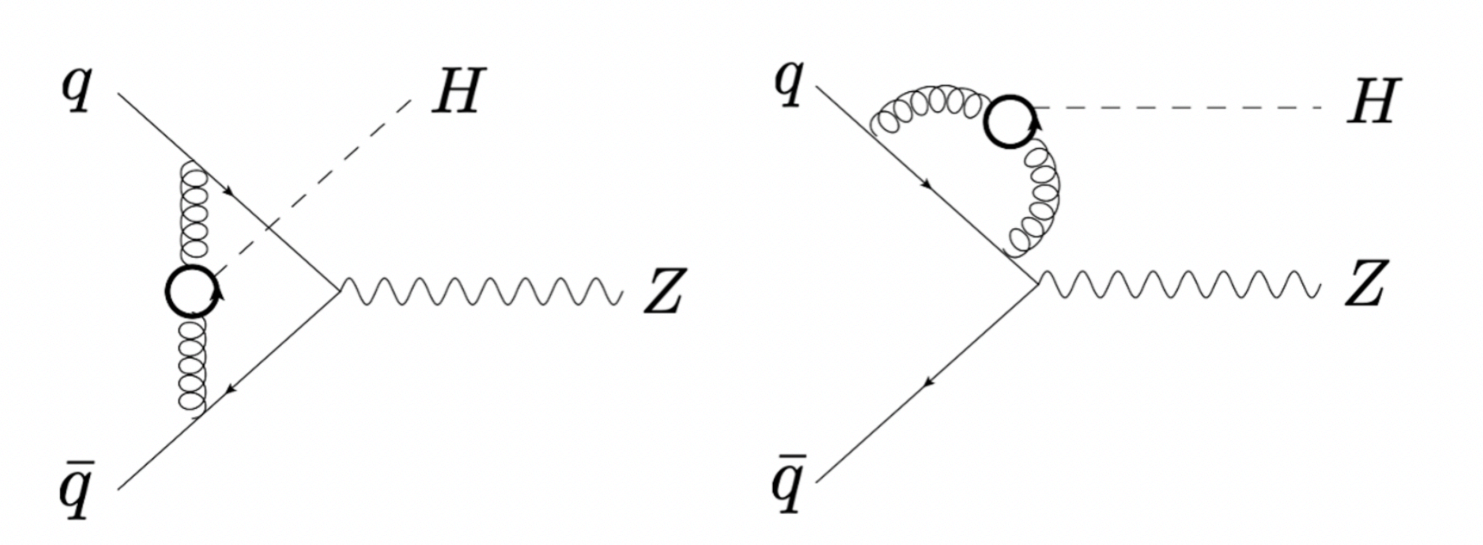
Top quark loop is integrated out

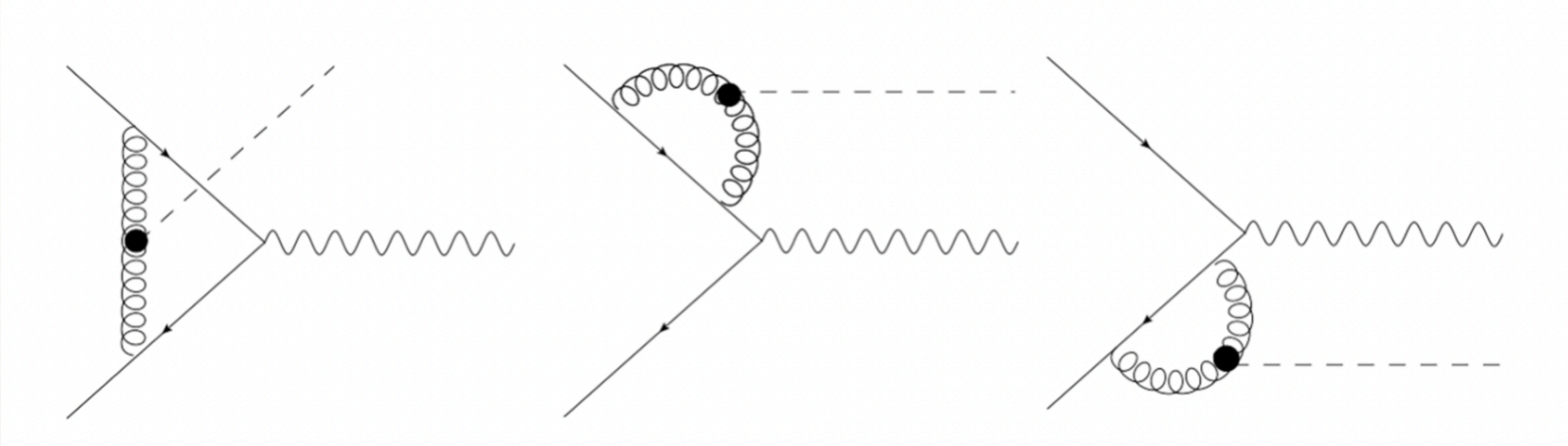


$m_t \to \infty$

C_H







Leading order

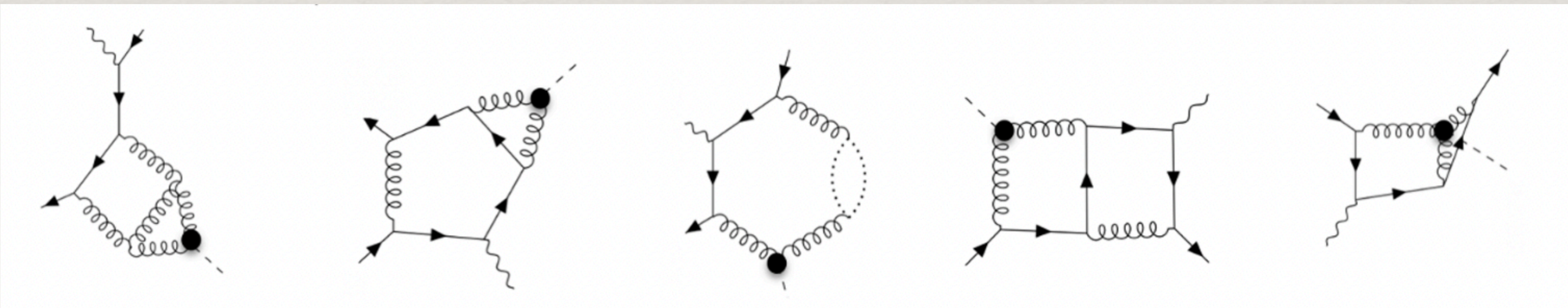
$\mathcal{O}(\alpha_s^2)$

$\propto \lambda_t$

[Brein, Hartader, Wesemann, Zirke '11]

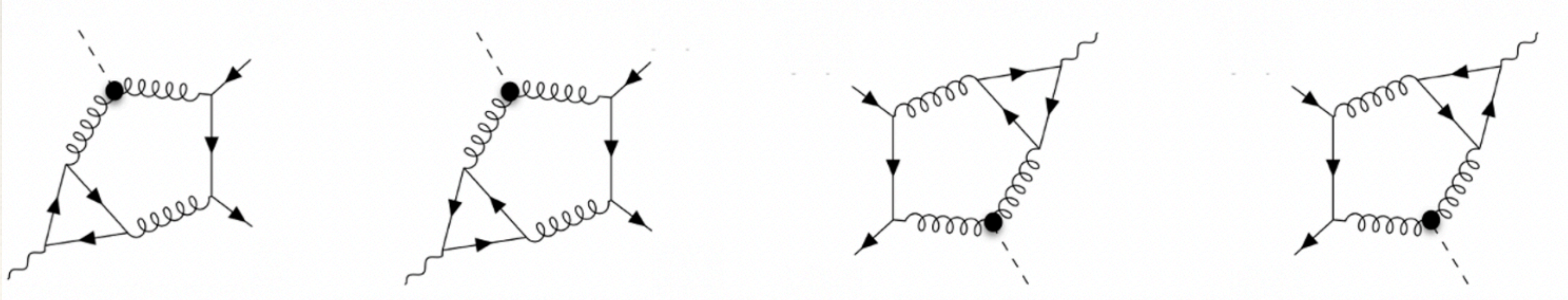
LO: asymptotic expansion in heavy-top limit

Non-anomalous



$\mathcal{O}(\alpha_s^3)$

anomalous



Anticommuting γ_5 in HEFT

Vector FFs $$\mathcal{A}^{\mu}_{vec} = \bar{v}(p_2)\left[\mathcal{F}_{1,vec}\not{q}_1 p_1^{\mu} + \mathcal{F}_{2,vec}\not{q}_1 p_2^{\mu} + \mathcal{F}_{3,vec}\not{q}_1 q_1^{\mu} + \mathcal{F}_{4,vec}\gamma^{\mu}\right]u(p_1)$$

- Linearly independent & complete in **D-dimensions**
- Reflects chiral conservation along massless quark line

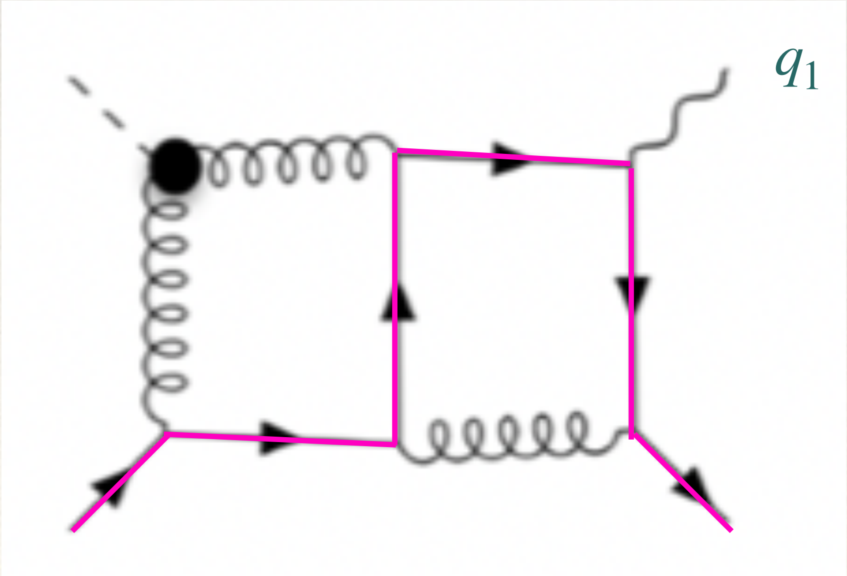
Axial FFs $$\mathscr{A}_{axi} = g_{A,q}\,\mathscr{A}_{axi(ns)} + g_{A,b}\,\mathscr{A}_{axi(s)}$$

$$\mathcal{A}^{\mu}_{axi} = \bar{v}(p_2)\left[\mathcal{F}_{1,axi}\not{q}_1 p_1^{\mu} + \mathcal{F}_{2,axi}\not{q}_1 p_2^{\mu} + \mathcal{F}_{3,axi}\not{q}_1 q_1^{\mu} + \mathcal{F}_{4,axi}\gamma^{\mu}\right]\gamma_5 u(p_1)$$

Anticommuting $\gamma_5^{\mathrm{AC}} : \{\gamma^{\mu}, \gamma_5\} = 0$ $\quad \mathscr{F}_{i,axi(ns)} = \mathscr{F}_{i,vec}$ Since respects chiral invariance

Ward Identity $$q_{1,\mu}\mathscr{A}^{\mu}_{vec} = 0$$

$$q_{1,\mu}\mathscr{A}^{\mu}_{axi(ns)} = 0$$



Non-anticommuting γ_5 in HEFT

Non anticommuting $\gamma_5^{\mathrm{NAC}} : \{\gamma^\mu, \gamma_5\} \neq 0$ $\qquad \gamma_5 = -\frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$

- Vector FFs: do not change, as expected $\qquad q_{1,\mu}\,\mathscr{A}^{\mu,\mathrm{NAC}}_{vec} = 0$
- Axial FFs: $\mathcal{O}(\epsilon^0)$ terms

$$\mathscr{F}^{\mathrm{NAC}}_{i,axi(ns)} = \mathscr{F}_{i,vec}\,, \qquad i = 1,2,3$$

$$\mathscr{F}^{\mathrm{NAC}}_{4,axi(ns)} \neq \mathscr{F}_{4,vec}$$

$q_{1,\mu}\,\mathscr{A}^{\mu,\mathrm{NAC}}_{axi(ns)} \neq 0$

- Violation of Ward identity
- Even at leading order!

s u r p r i s i n g

Cure

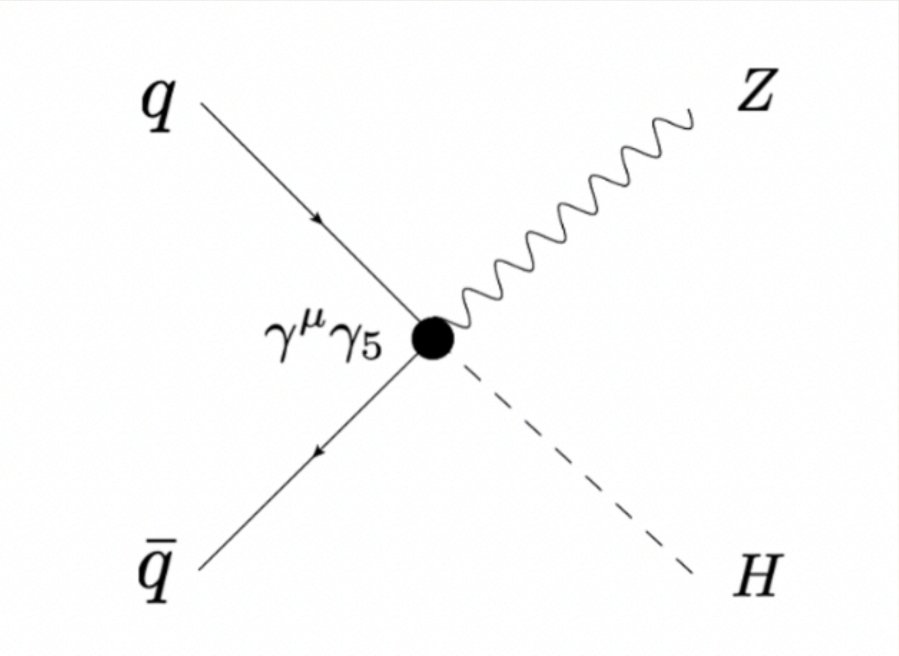
Introduce an amendment

$$\boxed{\mathcal{J}^{\mu,\mathrm{NAC}} \equiv \mathrm{Z}_5^h(a_s)\,\mathbf{C}\left(\bar{v}(p_2)\,[\gamma^\mu\gamma_5]_L\,u(p_1)\right)}$$

$$\mathbf{C} \equiv a_s\left(-4C_F\right)\frac{C_H}{v} \qquad \mathrm{Z}_5^h(a_s) = 1 + \mathcal{O}(a_s)$$

$[\gamma^\mu\gamma_5]_L$ renormalized according to Larin's prescription

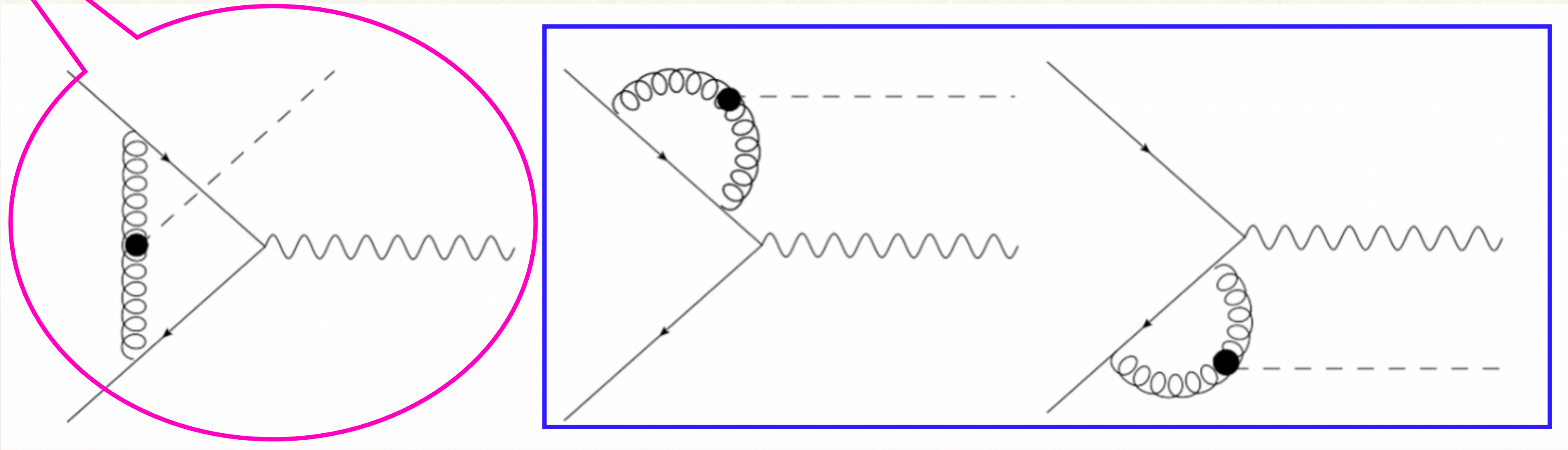
4-point local composite operator



Non-anticommuting γ_5 in HEFT

Source: 1-loop box

Leading order
$\mathcal{O}(\alpha_s^2)$



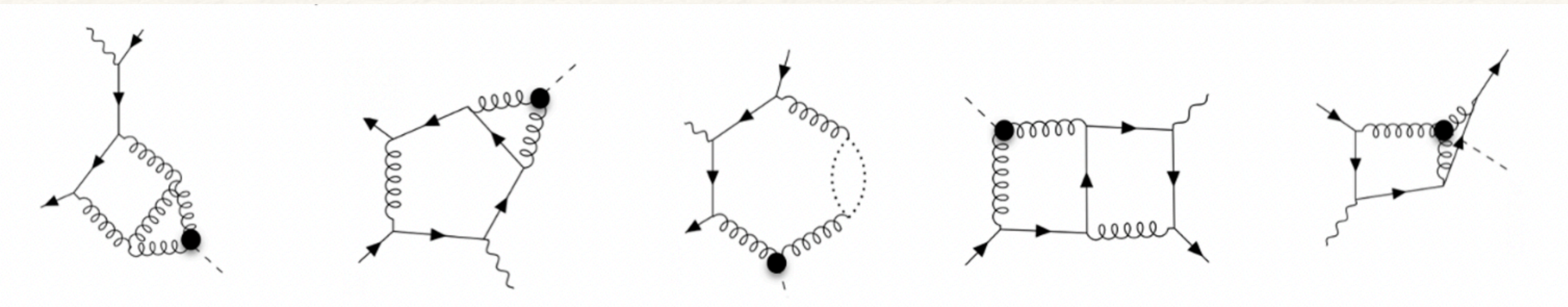
$$\mathcal{F}^{\text{NAC}}_{i,axi(ns)} = \mathcal{F}_{i,vec} \qquad \text{for } i = 1,2,3,4 \quad \text{in 4D limit}$$

- Contains terms which are separately divergent
- NAC & AC γ_5 lead to different D-dependent coefficients in front of divergent terms
- Differences are suppressed by at least one power in (D-4)
- Crucially (D-4) difference is **not** an overall prefactor
- Non-vanishing evanescent anti-commutators are generated upon shifting γ_5^{NAC} from inside the loop to the outside

observed discrepancy

NLO in HEFT: Non-singlet

"History is to repeat itself"

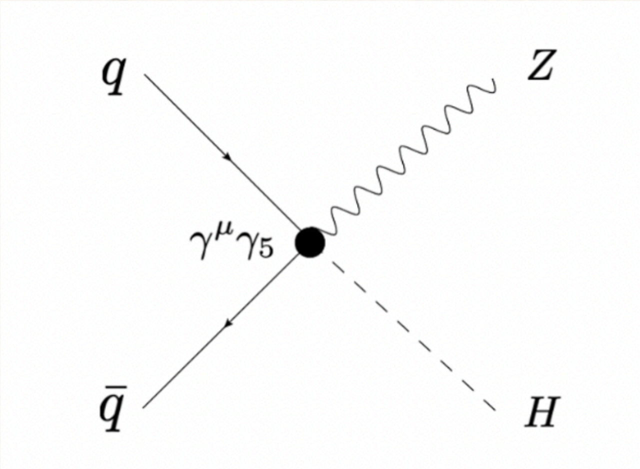
Non-anomalous  $\mathcal{O}(\alpha_s^3)$

Vector FFs $\quad q_{1,\mu}\,\mathcal{A}^{\mu,\mathrm{NAC}}_{vec} = 0$

Axial FFs

- $\left[\mathcal{F}^{\mathrm{NAC}}_{i,axi(ns)}\right]_L$ do not exhibit correct infrared structure: $\frac{1}{\epsilon}$ is wrong! [Catani '98]
- $\left[\mathcal{F}^{\mathrm{NAC}}_{4,axi(ns)}\right]_L \neq \mathcal{F}_{4,vec}$ $\quad\mathcal{O}(\epsilon^0)$ terms $\longrightarrow$ $q_{1,\mu}\left[\mathcal{A}^{\mu,\mathrm{NAC}}_{axi(ns)}\right]_L \neq 0$ ~~Ward~~

NLO in HEFT: Non-singlet



Cure

Incorporate quantum corrections to the amendment $\mathcal{J}^{\mu,\mathrm{NAC}} \equiv \mathrm{Z}_5^h(a_s)\,\mathbf{C}\left(\bar{v}(p_2)\,[\gamma^\mu\gamma_5]_L\;u(p_1)\right)$

- Perturbative expansion of $\mathrm{Z}_5^h(a_s)$
- One-loop correction

demanding correct IR pole

$$\mathrm{Z}^h_{5,ns}(a_s) = 1 + a_s\left(\frac{-\beta_0}{\epsilon} + \underbrace{\frac{107}{18}C_A - 7C_F - \frac{1}{9}n_f}\right) + \mathcal{O}(a_s^2)$$

demanding $\mathcal{O}(\epsilon^0)$ terms $\left[\mathcal{F}^{\mathrm{NAC}}_{4,axi(ns)}\right]_L = \mathcal{F}_{4,vec}$

Complete UV Renormalisation

$$\mathcal{A}^{\mu,\mathrm{NAC}}_{axi(ns)}(a_s) = Z^{ns}_{5,L}(a_s)Z^{ns}_{A,L}(a_s)Z_H(a_s)\hat{\mathcal{A}}^{\mu,\mathrm{NAC}}_{axi(ns)}(\hat{a}_s) + \mathcal{J}^{\mu,\mathrm{NAC}}_{ns}$$

$$\mathcal{J}^{\mu,\mathrm{NAC}}_{ns} = \mathrm{Z}^h_{5,ns}(a_s)Z^{ns}_{5,L}(a_s)Z^{ns}_{A,L}(a_s)\,\mathbf{C}\left(\bar{v}(p_2)\,\gamma^\mu\gamma_5\,u(p_1)\right)$$

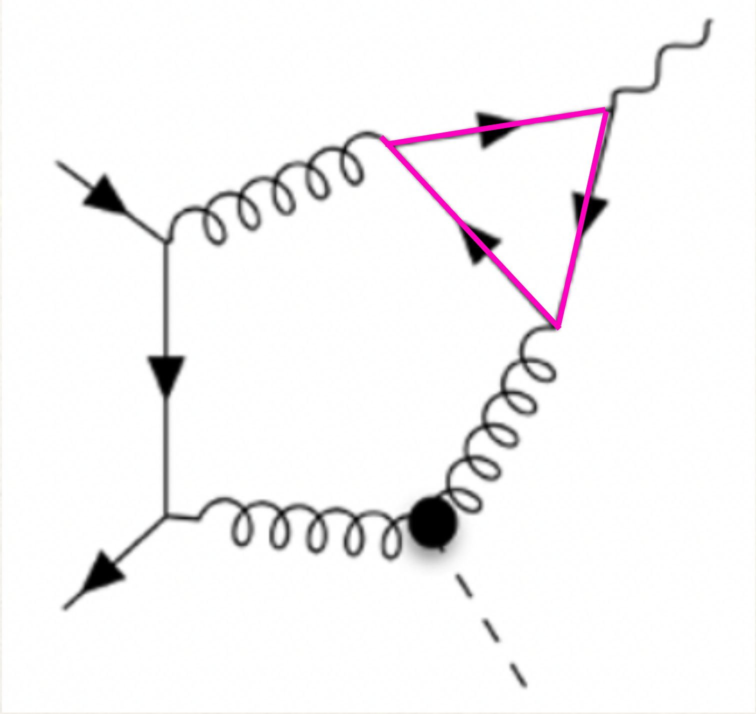
[Larin, Vermaseren '91]

[Larin '93]

NLO in HEFT: Singlet

Visit talk by L. Chen

Anomalous



- Only massless b-quark loop survives ($n_f = 5$)
- Exhibits anomalous Ward identity

[Adler '69, '70]
[Bell, Jackiw '69]

$$\left[\partial^\mu J_{5\mu}\right]_R = a_s \frac{1}{2}\left[G\tilde{G}\right]_R$$

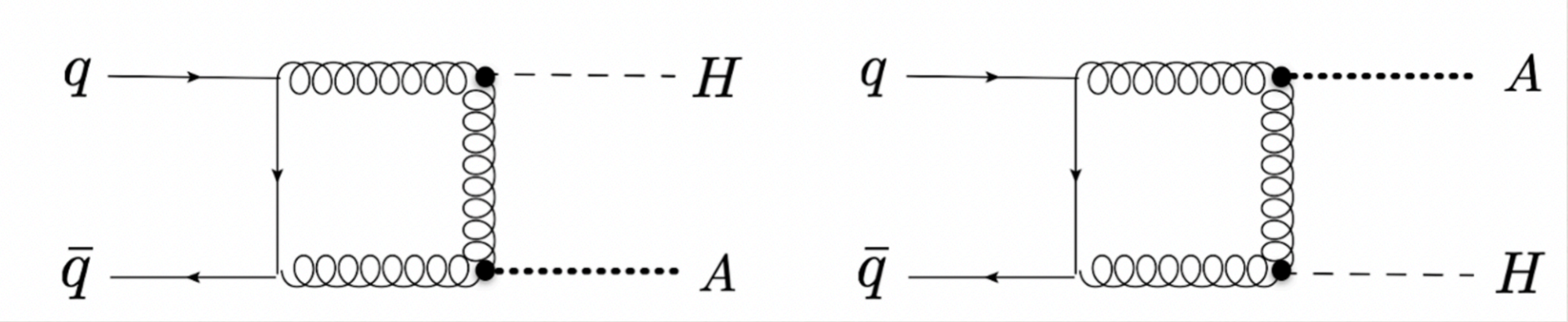
$$J_{5\mu} = \bar{b}\gamma_\mu\gamma_5 b$$

$$G\tilde{G} = -\,\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$$

$$\mathscr{A}_{axi(s)} = \bar{v}(p_2)\,\boldsymbol{\Gamma}^\mu_{(s)}\,u(p_1)\,\varepsilon^*_\mu$$

$$\varepsilon^*_\mu \to q_{1,\mu}$$

$$\bar{v}(p_2)\,\boldsymbol{\Gamma}^\mu_{(s)}\,u(p_1)\,q_{1,\mu} = \frac{a_s}{2}\boxed{\left\langle H(q_2)\middle|\left[G\tilde{G}\right]_R\middle|q(p_1)\bar{q}(p_2)\right\rangle}$$

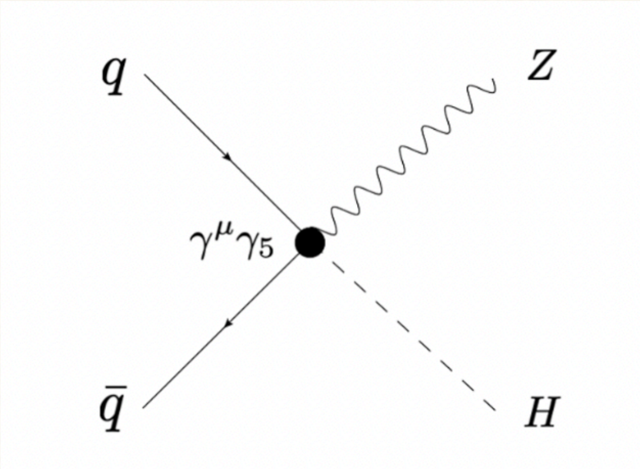


Momentum insertion q_1^μ by the composite $[G\tilde{G}]_R$

anomalous Ward fails!

Note: Larin's Z_5 do not contribute: starts at $\mathcal{O}(a_s^2)$ w.r.t. LO

NLO in HEFT: Singlet



Cure

Incorporate quantum corrections to the amendment

$$\mathcal{J}^{\mu,\mathrm{NAC}} \equiv Z_5^h(a_s)\, \mathbf{C} \left(\bar{v}(p_2)\ [\gamma^\mu \gamma_5]_L\ u(p_1) \right)$$

Demand: finite anomaly obeying anomalous Ward identity

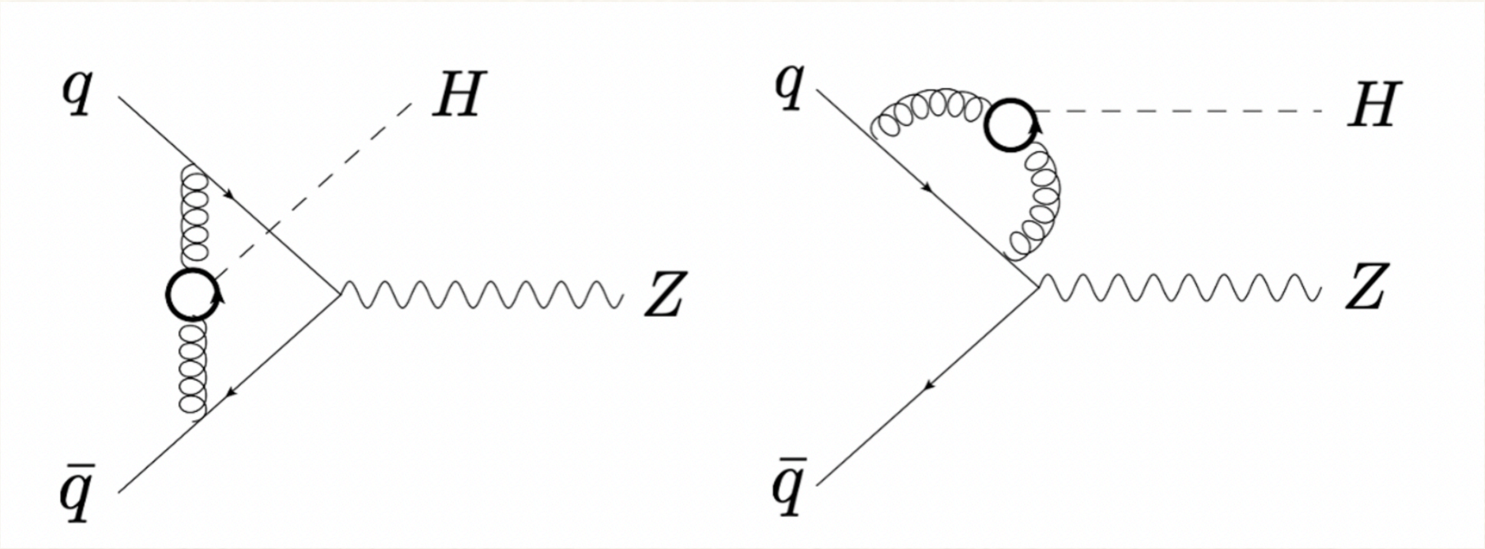
$$Z_{5,s}^h(a_s) = 1 + a_s \left(-\frac{3}{2}\frac{1}{\epsilon} - \frac{3}{4} \right) + \mathcal{O}(a_s^2)$$

At a glance

$\{\gamma_5, \gamma^\mu\} = 0$	$\{\gamma_5, \gamma^\mu\} \neq 0$
Everything is as expected in HEFT vector + non-singlet	A pitfall! Additional local composite operators need to be introduced

What happens in Exact theory?

$n_f = 6$



$\propto \lambda_t$

Primary question with $\{\gamma_5, \gamma^\mu\} \neq 0$

?

- $\mathcal{F}^{\text{NAC}}_{i,axi(ns)} = \mathcal{F}_{i,vec}$
- Ward identity

without any amendment

[Chetyrkin, Tkachov '81]
[Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '18]
[von Manteuffel, Panzer, Schabinger '15]

- 6 diagrams at 2-loop $\xrightarrow{\text{IBP}}$ 55 master integrals
  - 3 different bases
    - No irreducible numerators
    - Irreducible numerators are favoured ⚠
    - Quasi-finite★
  - Numerical evaluation: pySecDec

Kira: denominators' D-dependence factories

[Maierhofer, Usovitsch '18]
[Smirnov, Smirnov '20]
[Usovitsch '20]

$\longrightarrow$ Finite FFs with $\mathcal{F}^{\text{NAC}}_{i,axi(ns)} = \mathcal{F}_{i,vec}$ to $\mathcal{O}(\epsilon^0)$

Message 3

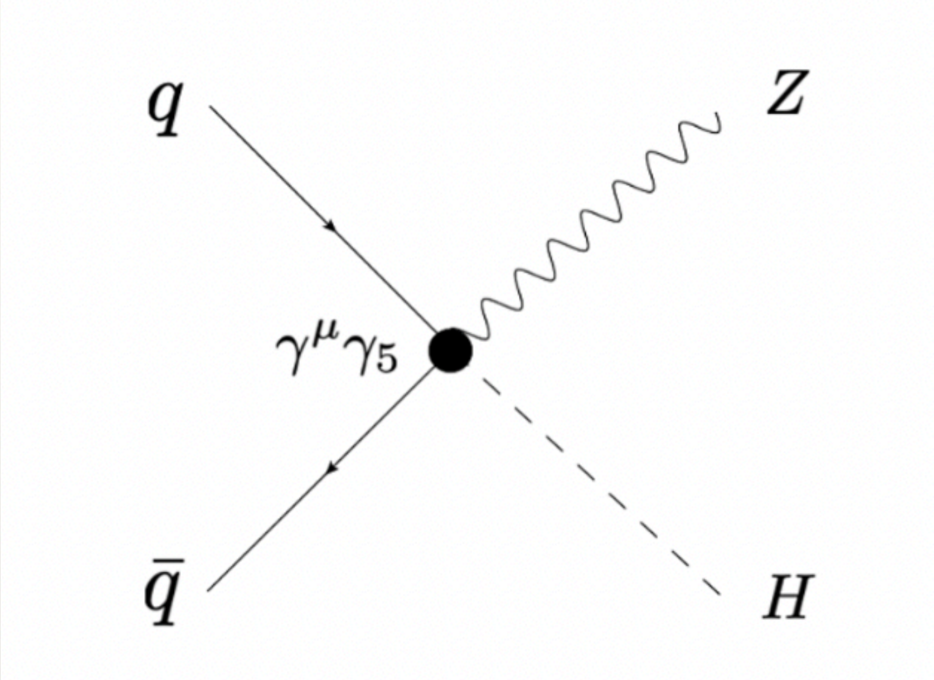
The presence/absence of effective $q\gamma^\mu\gamma_5\bar{q}Z_\mu H$ vertex in heavy top-mass expansion depends on γ_5 prescription

Take-Home Messages

- Projectors derived in 4-dimensions can be used in D-dimensional calculations and lead to correct results for physical observables

- Revealed a relation between axial and vector FFs for non Drell-Yan $b\bar{b} \to ZH$

- $q\bar{q} \to ZH$ in HEFT



  - Non-anticommuting γ_5 requires additional local composite operators
  - Strengthening common lore: one should be careful with taking claims or assuming conditions with an AC γ_5 in a computation where a NAC γ_5 is employed

- Whether *"history is to repeat itself"* for $gg \to ZH$ in HEFT requires investigation

Thank you!