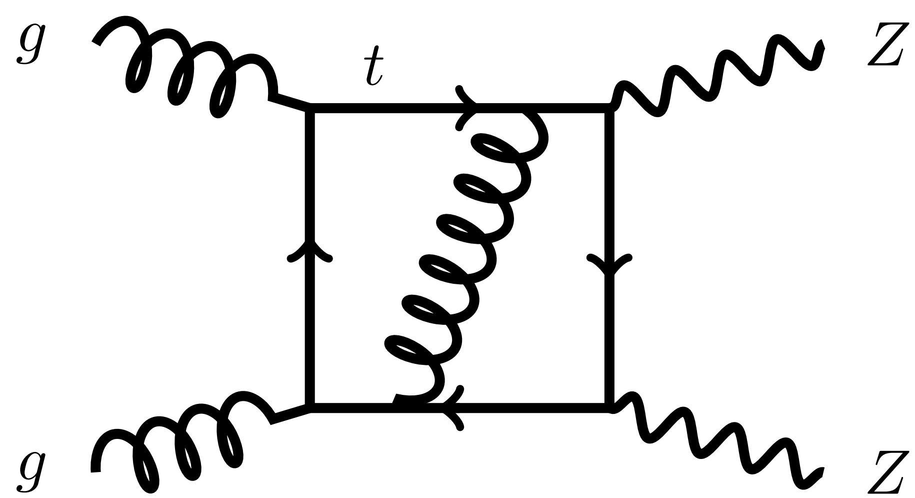
Gluon-induced ZZ and ZH production at NLO

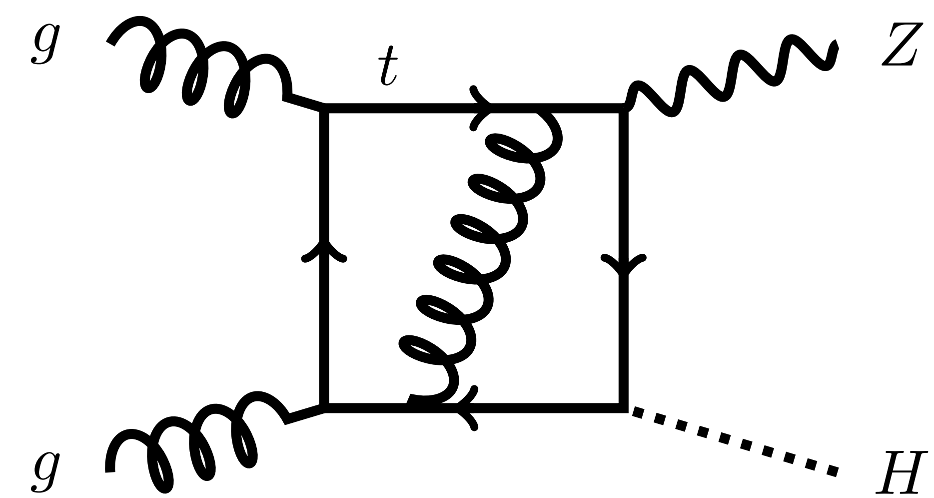
Go Mishima (Tohoku Univ.)

ZZ: JHEP 04(2020)024, 2002.05558
with Joshua Davies, Matthias Steinhauser, David Wellmann

ZH: JHEP 03(2021)034, 2011.12314
with Joshua Davies, Matthias Steinhauser

We focus on two-loop virtual top quark corrections





4 external vector bosons
→ 138 tensor structures
→ 18 physically relevant form factors

$$g^{\mu\sigma} p_1^{\rho} p_3^{\nu} \ , \ldots$$

3 external vector bosons
→ 60 tensor structures
→ 6 physically relevant form factors

$$\epsilon^{\mu\nu\rho\sigma} p_{1\sigma} \ , \ldots$$

High-energy approximation

$$m_Z^2, m_H^2 < m_t^2 \ll s, t, u$$

Mandelstam variables

Note:

$$t = -p_T^2 + \mathcal{O}\left(\frac{m_Z^2}{s}, \frac{m_H^2}{s}, \frac{p_T^2}{s}\right)$$

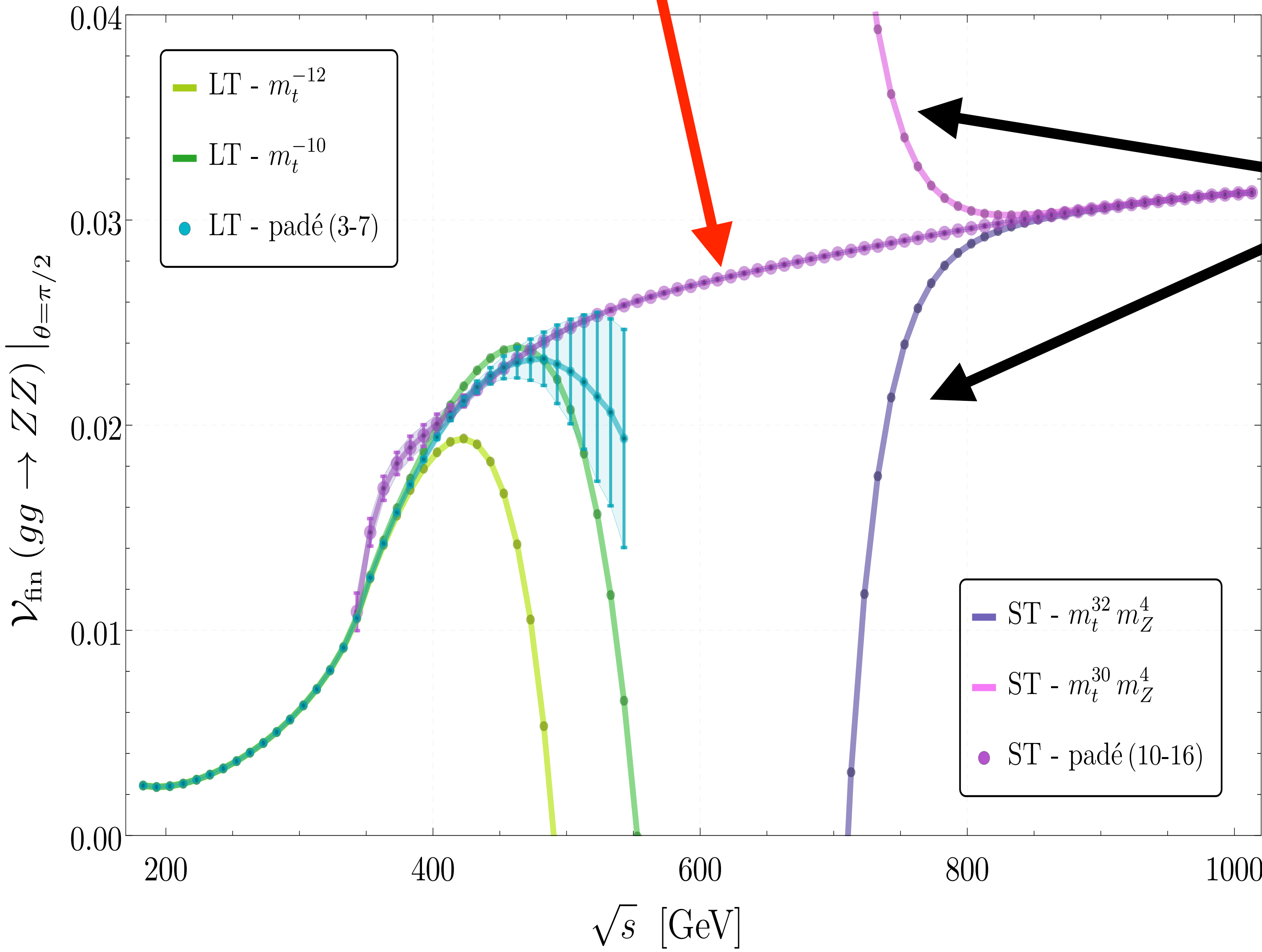
Therefore high-energy means large-pT.

Result (form factor, helicity amplitude, cross section) is expressed as

$$\mathscr{A} = \sum_{n=-1}^{32} (m_t)^n A_n^{(0,0)} + \frac{m_Z^2}{m_t^2} \sum_{n=-1}^{32} (m_t)^n A_n^{(1,0)} + \frac{m_H^2}{m_t^2} \sum_{n=-1}^{32} (m_t)^n A_n^{(0,1)} + \cdots$$

Drastic improvement by Padé approx.

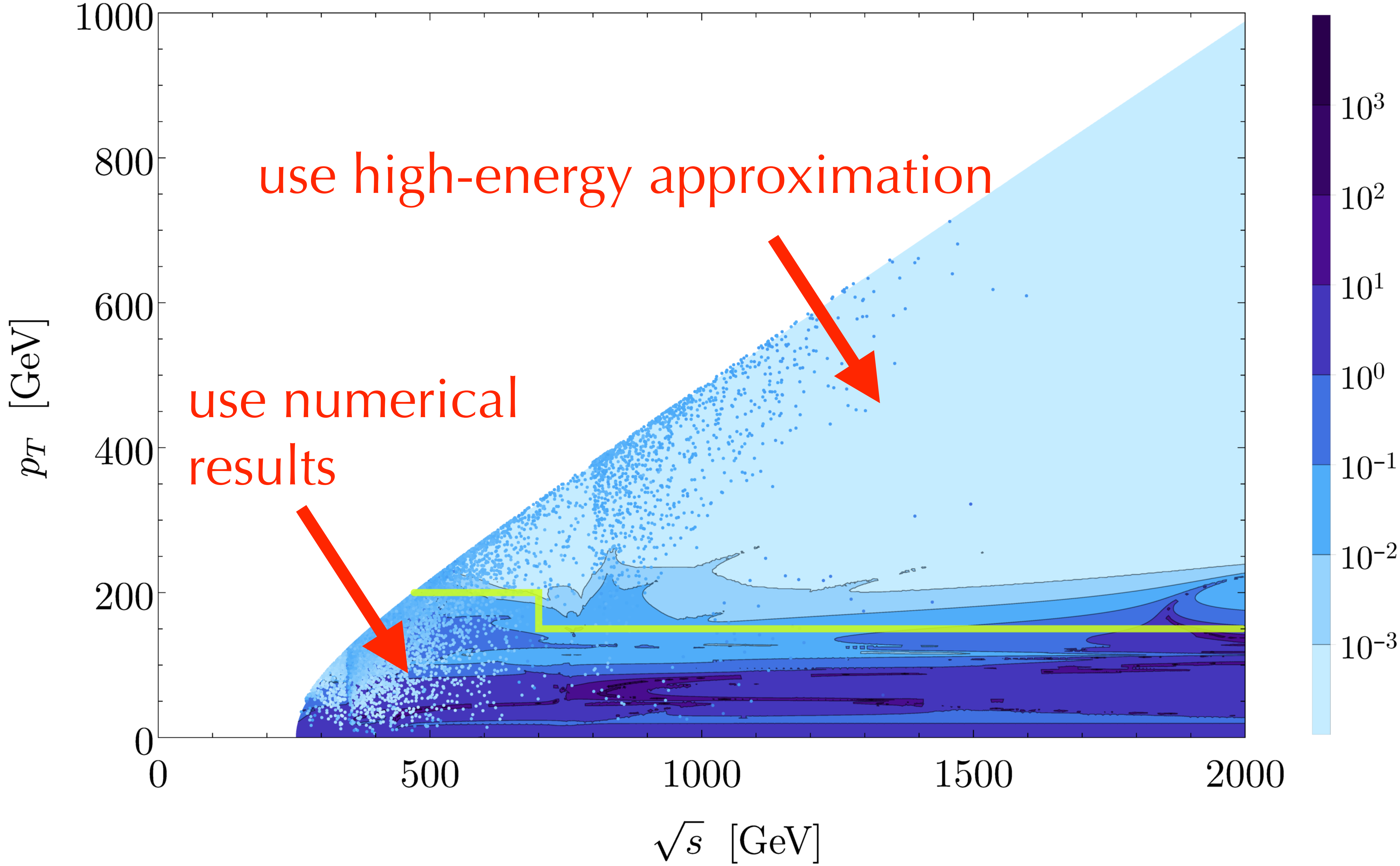
our actual prediction



what audience imagine as "high-energy approx."

[D. Wellmann, Ph. D. thesis]

Complementarity with exact numerical evaluation



from study of gg->HH **[J. Davies, G. Heinrich, S. P. Jones, M. Kerner, GM, M. Steinhauser, D. Wellmann]**

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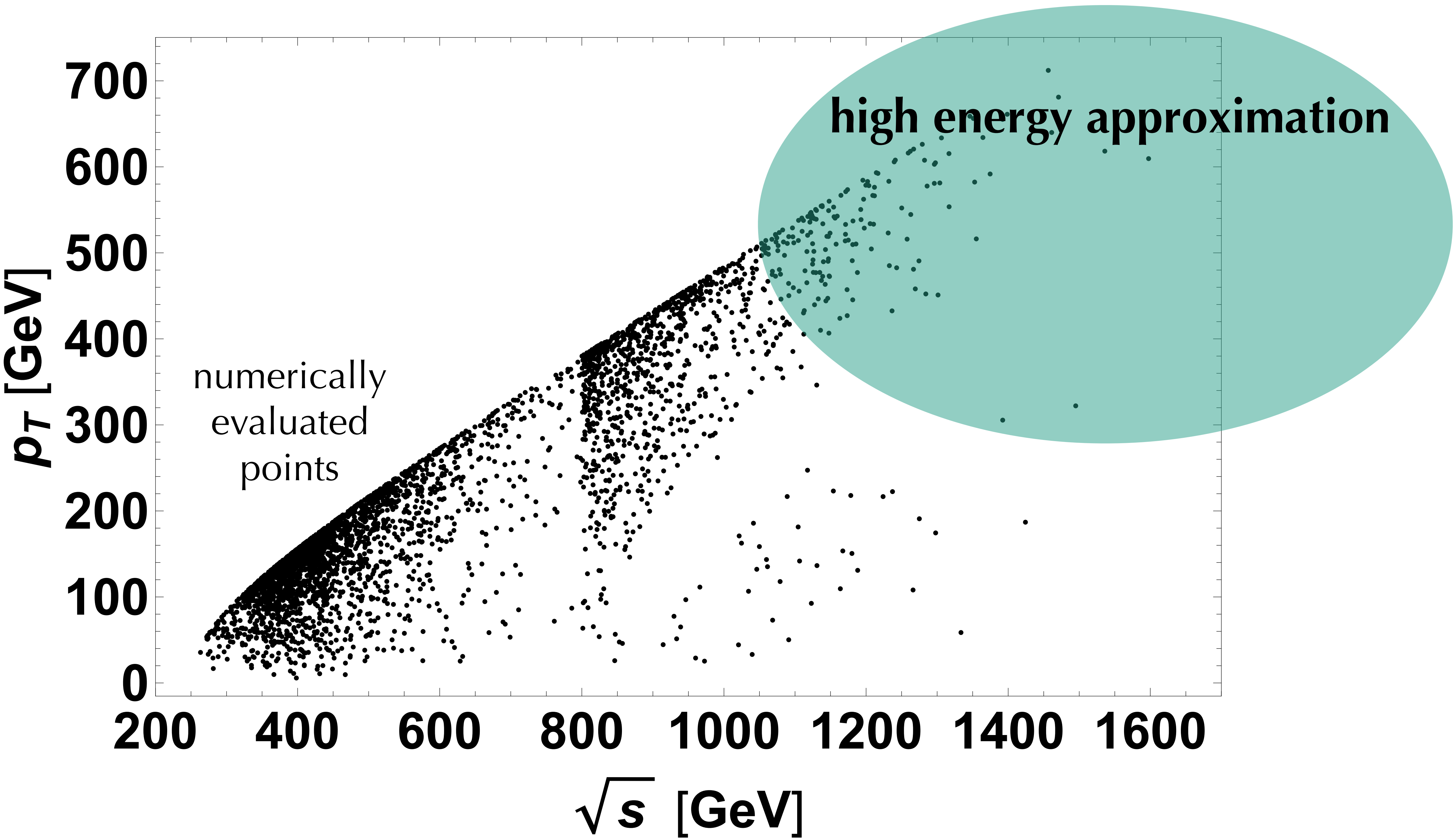

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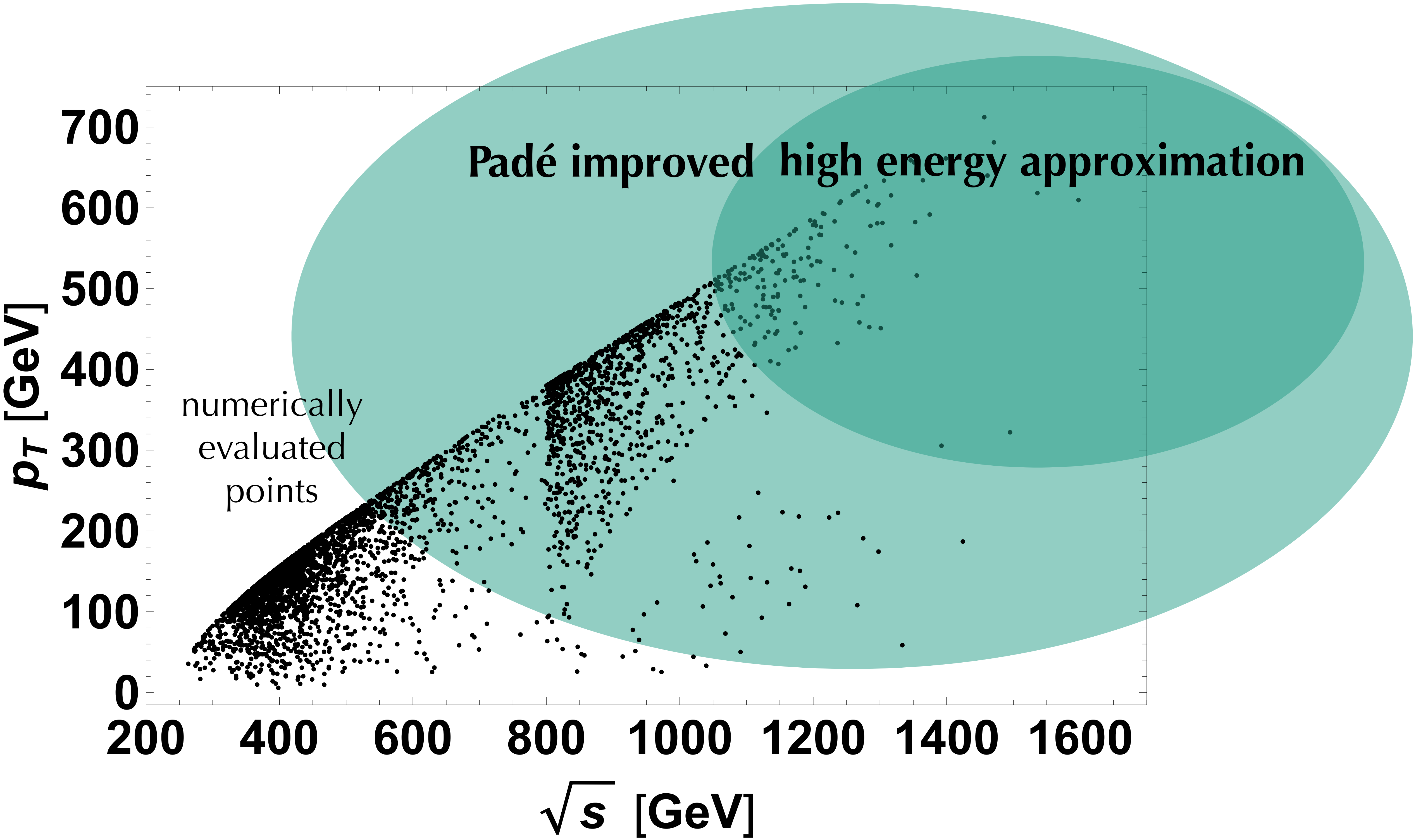
Complementarity with exact numerical evaluation



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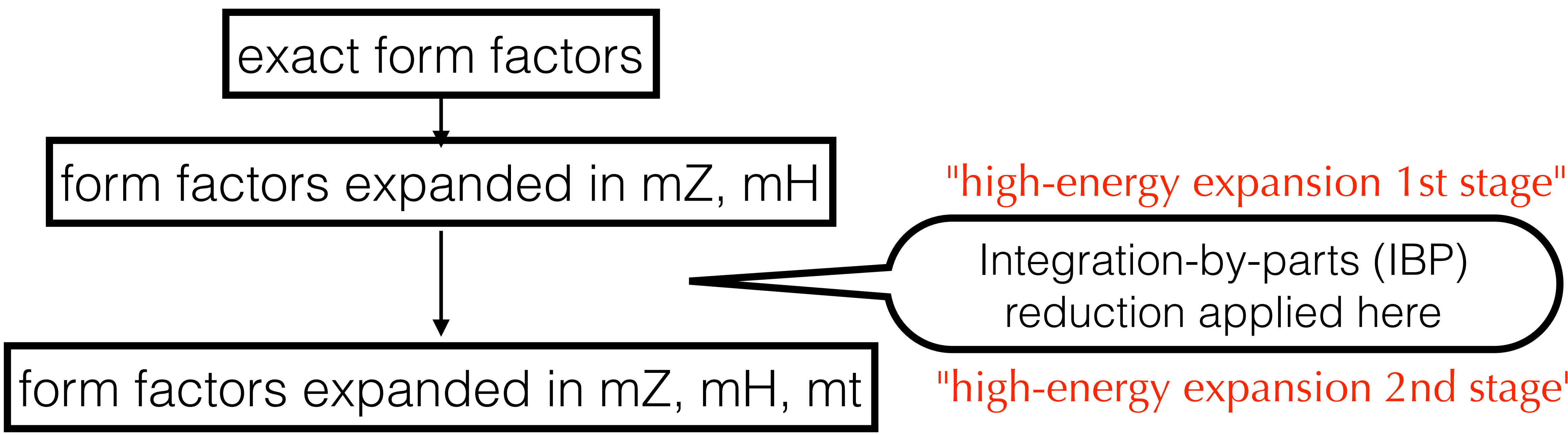
Our setup to calculate the two-loop amplitude

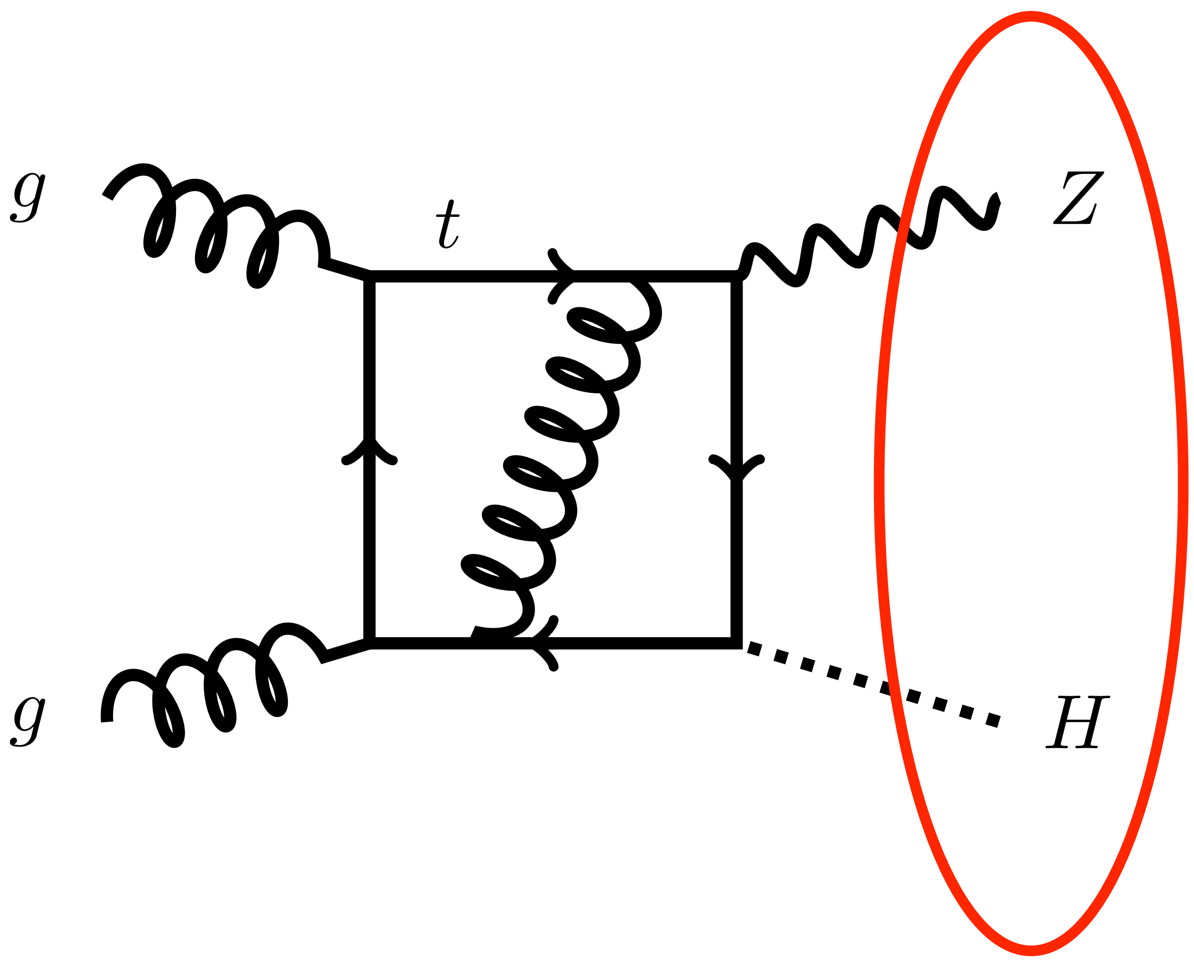
qgraf [Nogueira, '93]	: generate amplitudes
q2e/exp [Harlander, Seidensticker, Steinhauser, '98, Seidensticker, '99]	: rewrite output to FORM notation
TFORM 4.2 [Rujil, Ueda, Vermaseren '17]	: projection to the form factors
LiteRed [Lee, '13]	: mH, mZ expansion
FIRE [Smirnov, '14] (with LiteRed rules [Lee, '13])	: IBP reduction to master integrals
tsort [Smirnov, Pak]	: minimization of master integrals



$$\mathscr{A} = \sum_{n=-1}^{32} (m_t)^n A_n^{(0,0)} + \frac{m_Z^2}{m_t^2} \sum_{n=-1}^{32} (m_t)^n A_n^{(1,0)} + \frac{m_H^2}{m_t^2} \sum_{n=-1}^{32} (m_t)^n A_n^{(0,1)} + \cdots$$

ZZ, ZH amplitudes share the same Master Integrals at high-energy

$$m_Z^2, m_H^2 \ll m_t^2, s, t, u$$

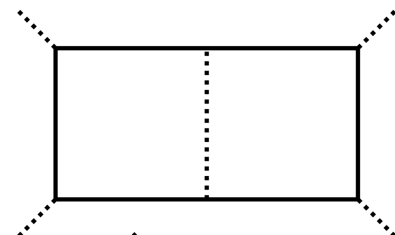


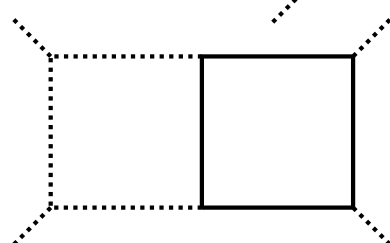
After the "1st stage" expansion,
we obtain massless-legs
for general Feynman integrals.

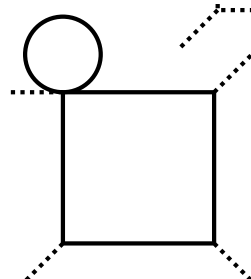
(corrections in mZ and mH
can be expressed in terms of
the massless-leg diagrams, too)

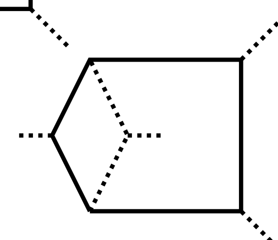
Master integrals at two-loop

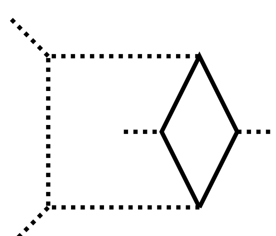
$161 = 131$ (planar&crossing) $+ 30$ (nonplanar&crossing)

$= 28 + (20+15+19+11+9)$  +crossing

$+15 + (8+3)$  +crossing

$+1 + (2)$  +crossing

$+11 + (6+6)$  +crossing

$+7$ 

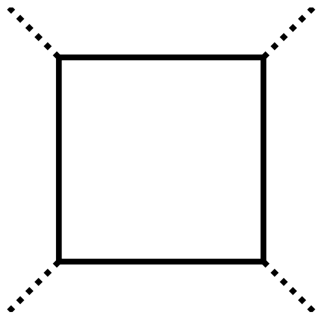
JHEP 1803 (2018) 048 [arXiv:1801.09696]

JHEP 1901 (2019) 176 [arXiv:1811.05489]

"high-energy expansion 2nd stage"

Expansion in mt

"high-energy expansion 2nd stage"



$$= \int Dk \frac{1}{k^2 - m_t^2} \frac{1}{(k+p_1)^2 - m_t^2} \frac{1}{(k+p_1+p_2)^2 - m_t^2} \frac{1}{(k+p_3)^2 - m_t^2}$$

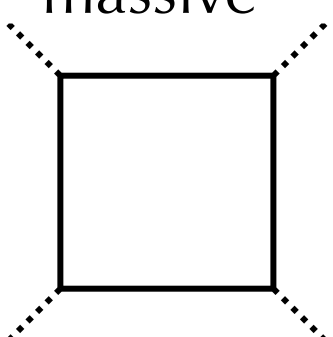
$$= \sum_{n=0}^{\infty} (m_t^2)^n f_n(S, T, \log m_t)$$

Naive expansion of the integrand like

$$\frac{1}{k^2 - m_t^2} = \frac{1}{k^2} + \frac{m_t^2}{(k^2)^2} + \cdots$$

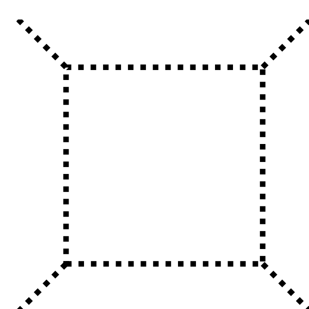
gives wrong result.

massive



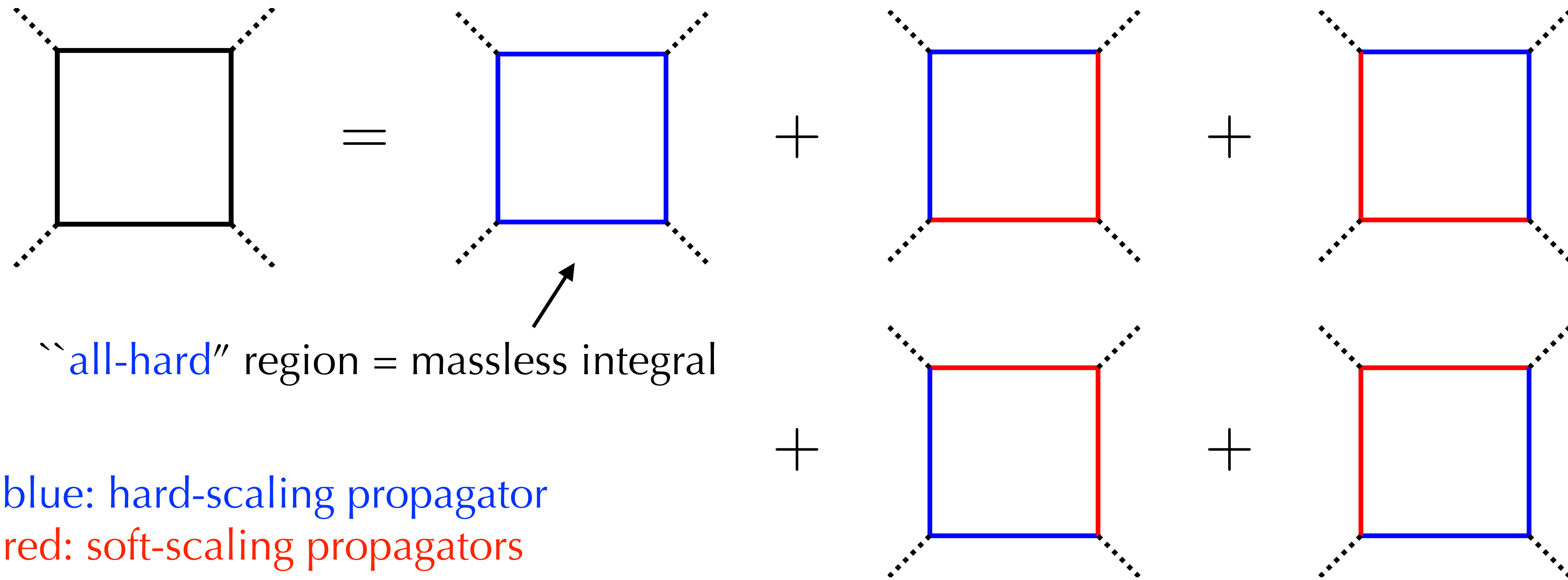
is finite.

massless



$$= \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right) + \mathcal{O}(\epsilon)$$

Method of region [Beneke, Smirnov '97, Smirnov `02, Jantzen `11]

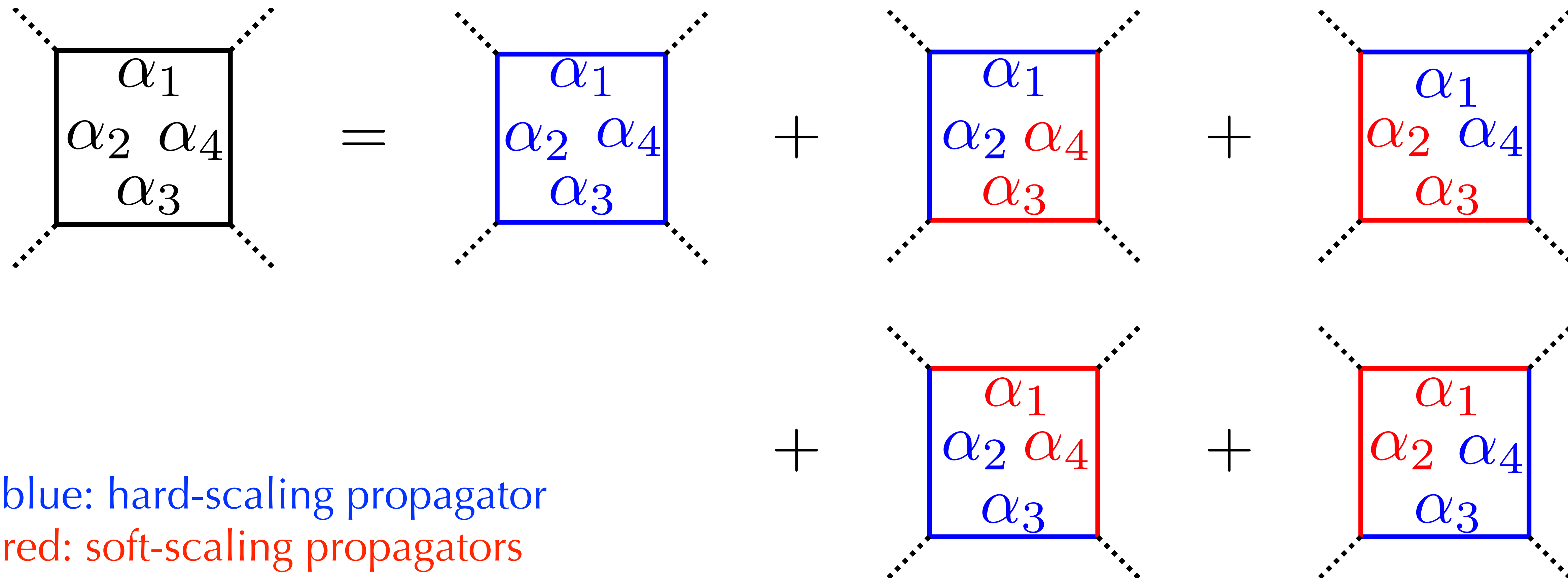


the scaling of propagators in terms of alpha-parameter representation

$$\int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n \right) \alpha_{1234}^{-d/2} \, \mathrm{e}^{-m^2 \alpha_{1234} - (s\alpha_1\alpha_3 + t\alpha_2\alpha_4)/\alpha_{1234}}$$

$$\alpha_{1234} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

Method of region [Beneke, Smirnov ’97, Smirnov `02, Jantzen `11]



blue: hard-scaling propagator
red: soft-scaling propagators

the scaling of propagators in terms of alpha-parameter representation

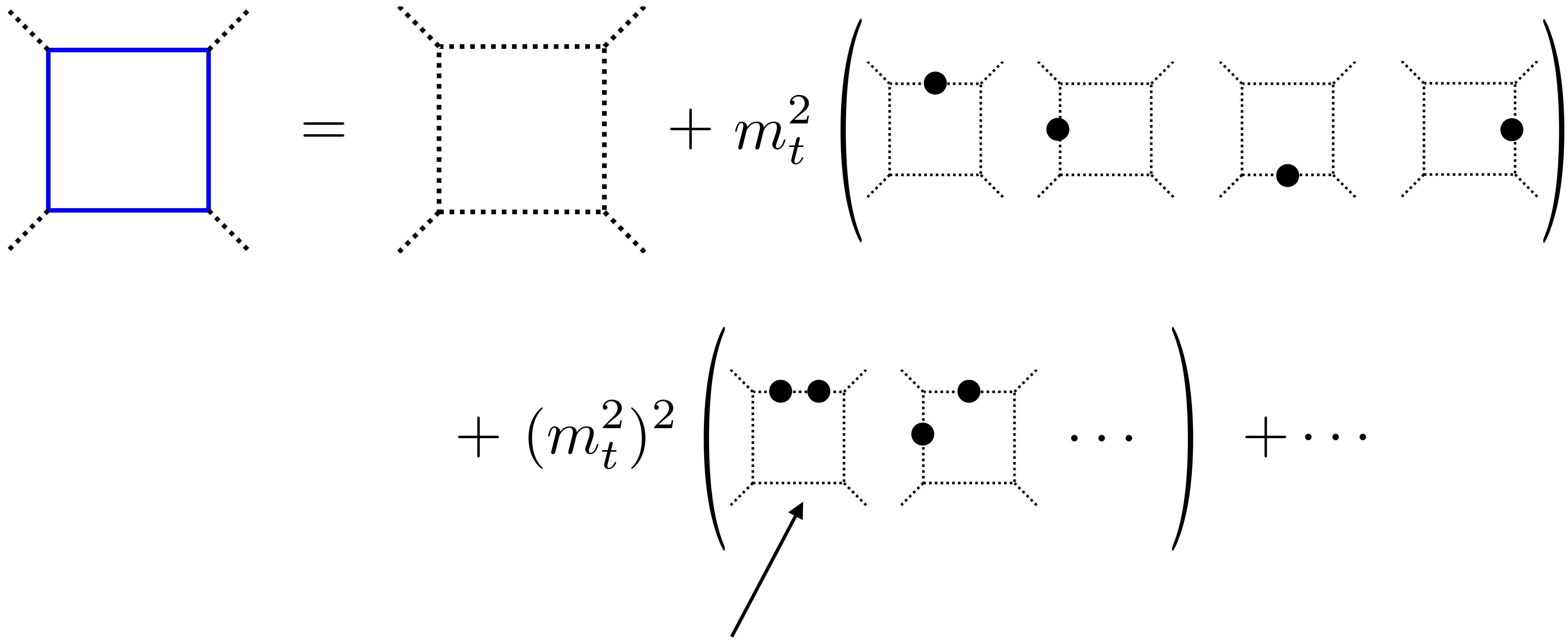
$$\int_0^\infty \left(\prod_{n=1}^4 d\alpha_n\right) \alpha_{1234}^{-d/2}\ \mathrm{e}^{-m^2\alpha_{1234}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{1234}}$$

$$\alpha_{i_1\dots i_n} \equiv \alpha_{i_1} + \cdots + \alpha_{i_n}$$

$$\alpha_{1234} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

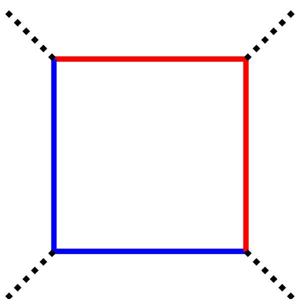
Method of region: "all-hard" region

In our case, the expansion in this region corresponds to the **naive Taylor expansion**.
The right hand side consists of massless diagrams with dots.



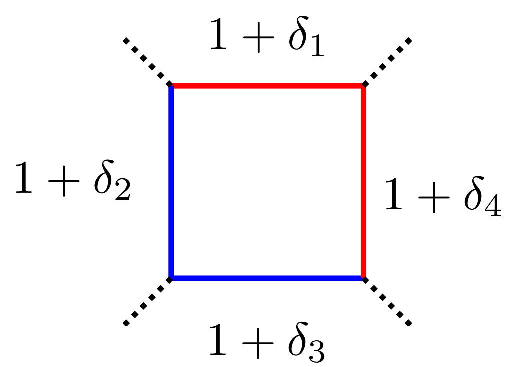
We can apply the integration by parts (IBP) reduction.

Method of region: soft-collinear regions



$$= \int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n\right) \begin{aligned} &\alpha_{12}^{-d/2}\, \mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}} \\ &-\alpha_{12}^{-d/2-2}(\alpha_3+\alpha_4)((d/2)\alpha_{12}+m^2(\alpha_{12})^2-s\alpha_1\alpha_3-t\alpha_2\alpha_4) \\ &\times \mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}} \\ &+\cdots \end{aligned}$$

Usual momentum representation is not always possible…



The integrals are ill-defined,
so we have to introduce
analytic regularization
of the exponent of propagators.

$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log st\right)\right]$$

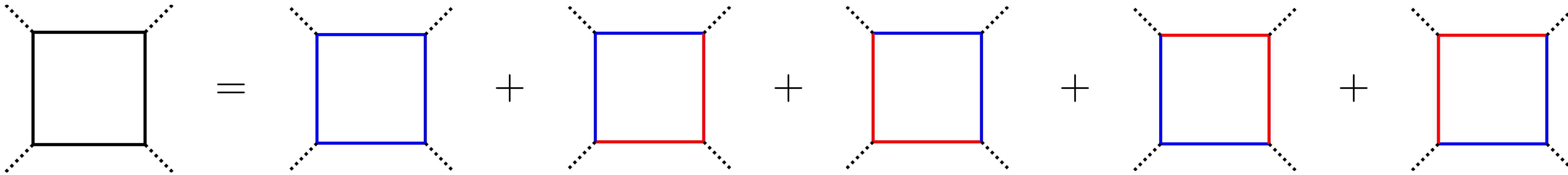
$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log t/m^2\right)+\frac{\pi^2}{12}\right]$$

$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}+\frac{1}{\delta_4}+\log s/m^2\right)+\frac{\pi^2}{12}\right]$$

$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}+\frac{1}{\delta_4}-2\log m^2\right)+\frac{\pi^2}{6}\right]$$

Cancellation of auxiliary parameters between soft regions occurs.

Method of region: total



$$f_0^{(1)} = \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right)$$

$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log st\right)\right]$$

$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log t/m^2\right) + \frac{\pi^2}{12}\right]$$

$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} + \frac{1}{\delta_4} + \log s/m^2\right) + \frac{\pi^2}{12}\right]$$

$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} + \frac{1}{\delta_4} - 2\log m^2\right) + \frac{\pi^2}{6}\right]$$

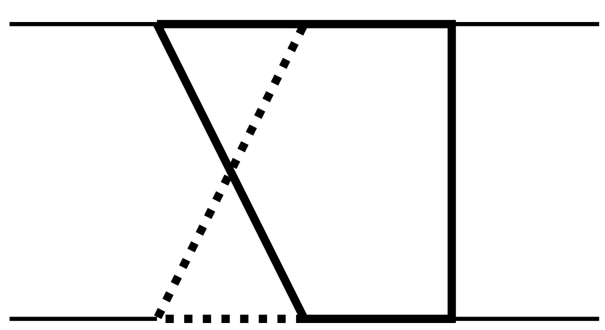
$$I = \sum_{n=0}^{\infty}(m^2)^n f_n$$

$$f_0 = \frac{1}{st}\left(2\log\frac{s}{m^2}\log\frac{t}{m^2} - \pi^2\right)$$

Cancellation of auxiliary parameters between soft regions occurs.

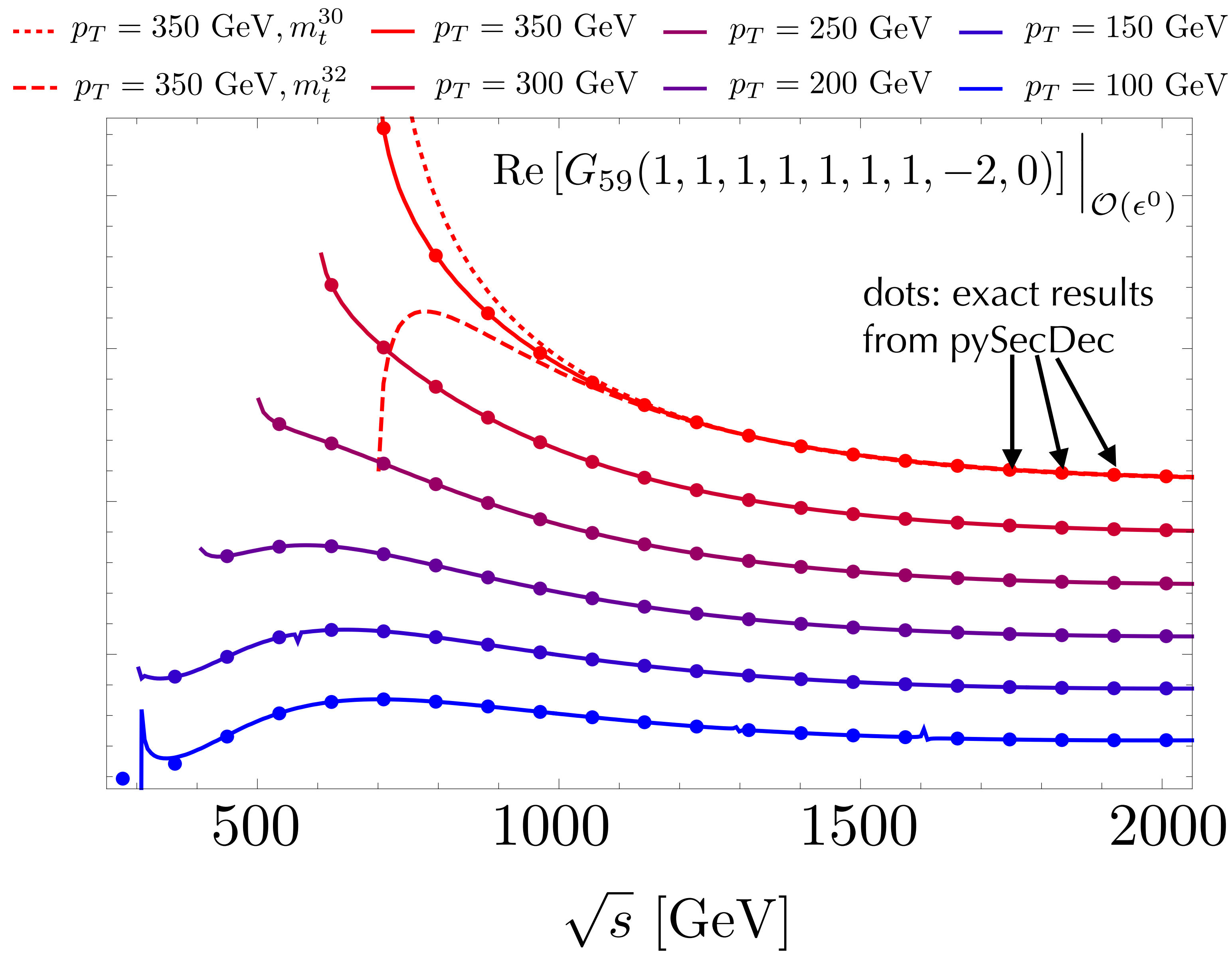
Applying Padé to the resulting series of mt

example: one of the non-planar MIs



Padé approx. of {n/m}:

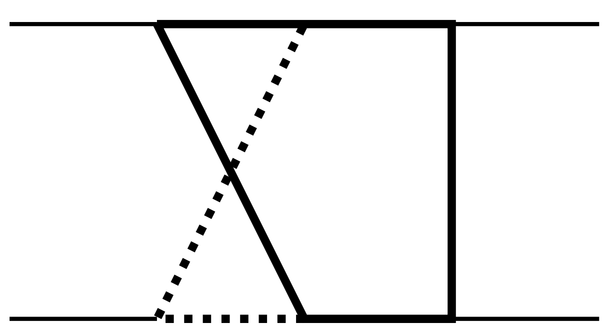
$$f_0 + f_1 x + \cdots + f_{n+m} x^{n+m} \\ \to \frac{a_0 + a_1 x + \cdots + a_n x^n}{1 + b_1 x + \cdots + b_m x^m}$$



room for improvement:
1. sometimes it generates unphysical singularity
2. we want to estimate the uncertainty
(3. form factors, i.e. combinations of MIs, converge not as good as single MI)

Applying Padé to the resulting series of mt

example: one of the non-planar MIs



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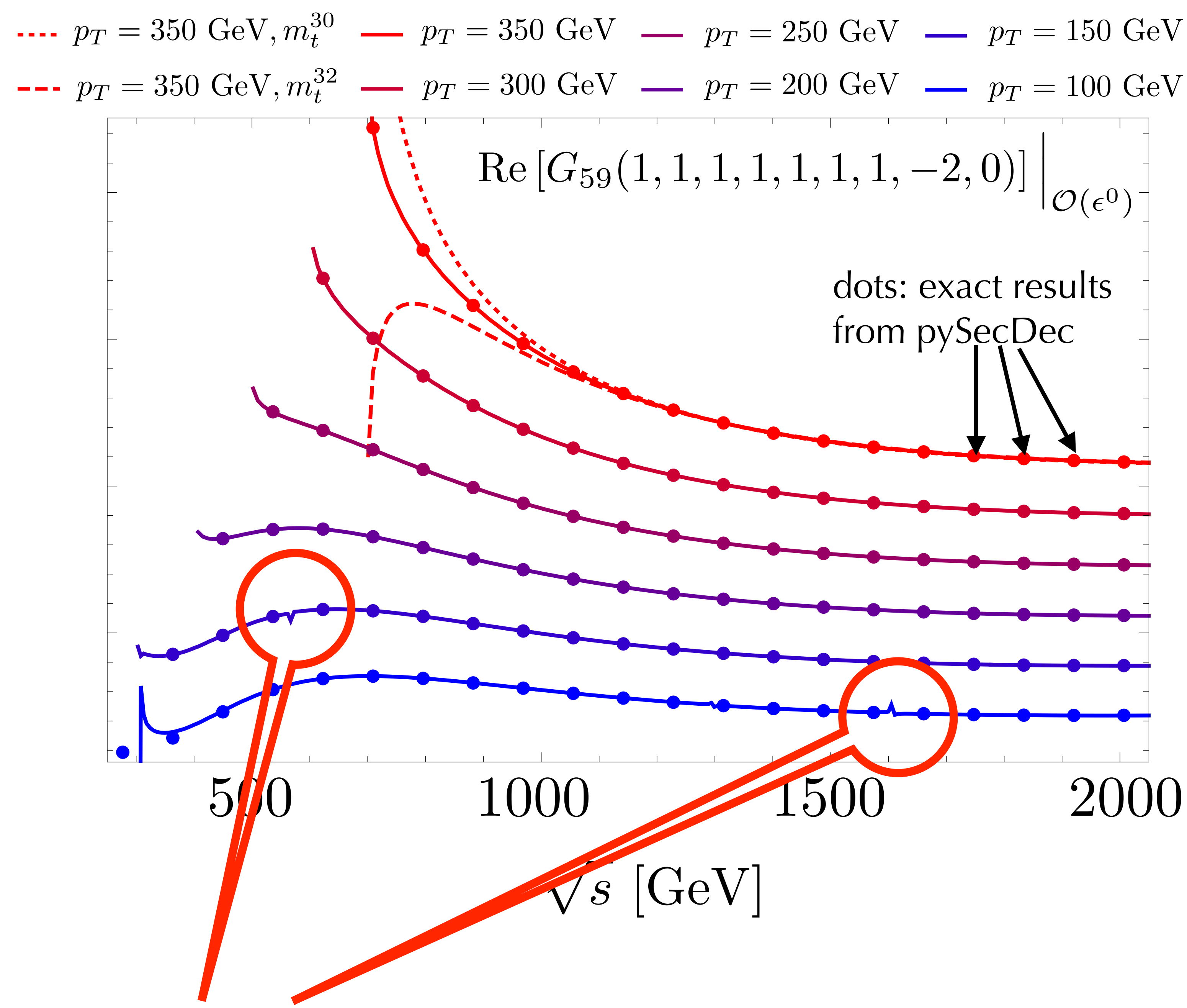


room for improvement:
1. sometimes it generates unphysical singularity
2. we want to estimate the uncertainty
(3. form factors, i.e. combinations of MIs, converge not as good as single MI)

Pole distance re-weighted (PDR) Padé method (slightly simplified version)

For each fixed numerical values of s and t, we re-arrange the series in the form

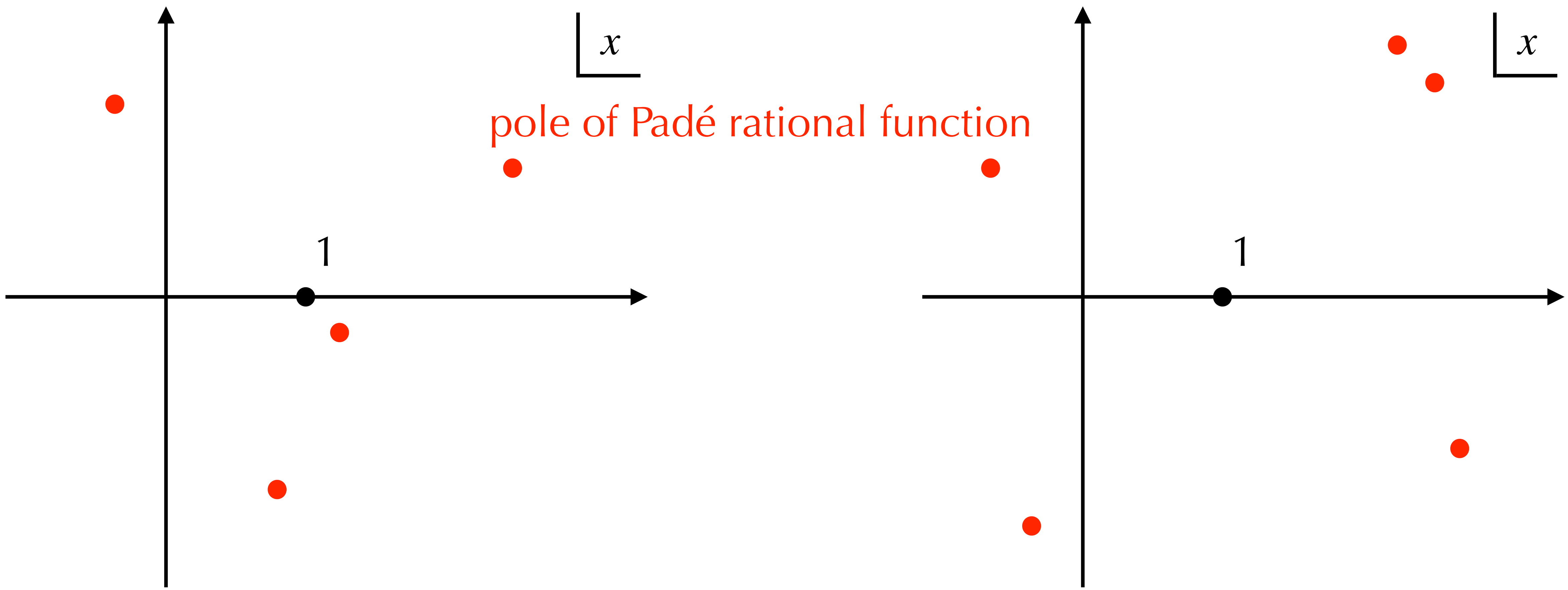
$$R = \sum_{n=1}^{16} c_n x^n$$

The original quantity is reproduced when x=1.

There are several choices for the Padé approximation, {8/8}, {9/7}, {7/9},...

Choice 1

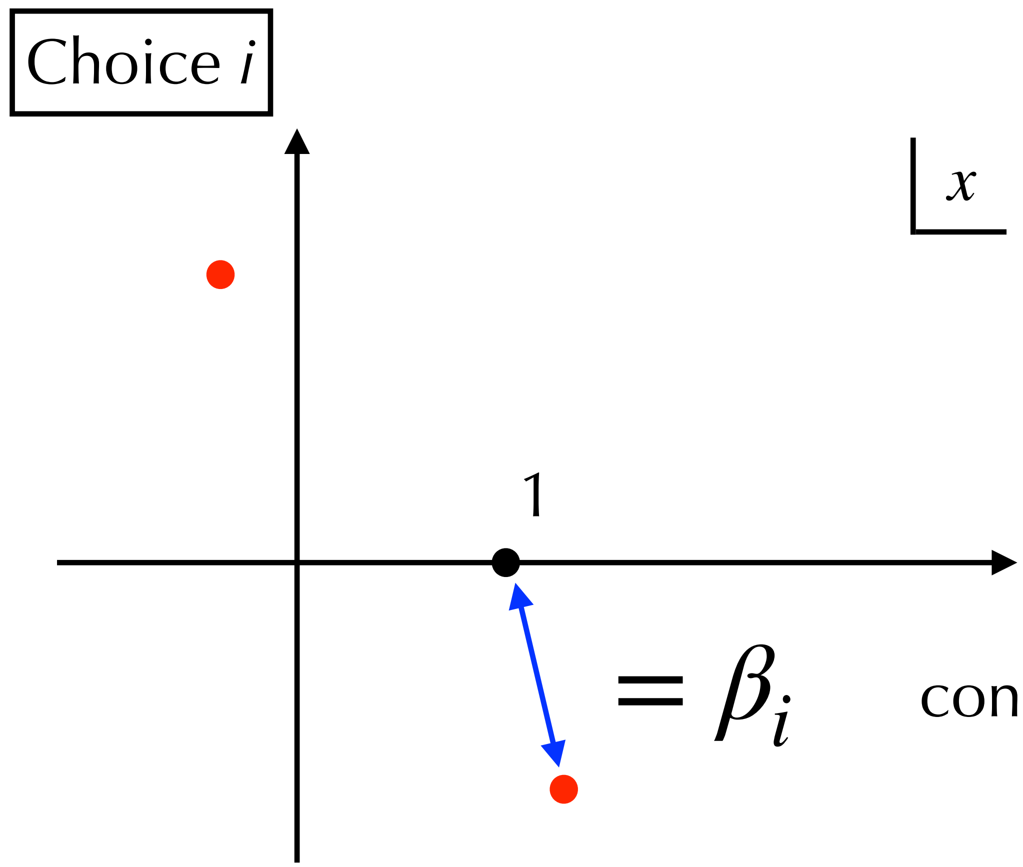
Choice 2



not nice behaviour
tend to be away from the exact value

better behaviour
tend to be close to the exact value

Pole distance re-weighted (PDR) Padé method (slightly simplified version)



α_i : result of "Choice i" Padé
(value at x=1)

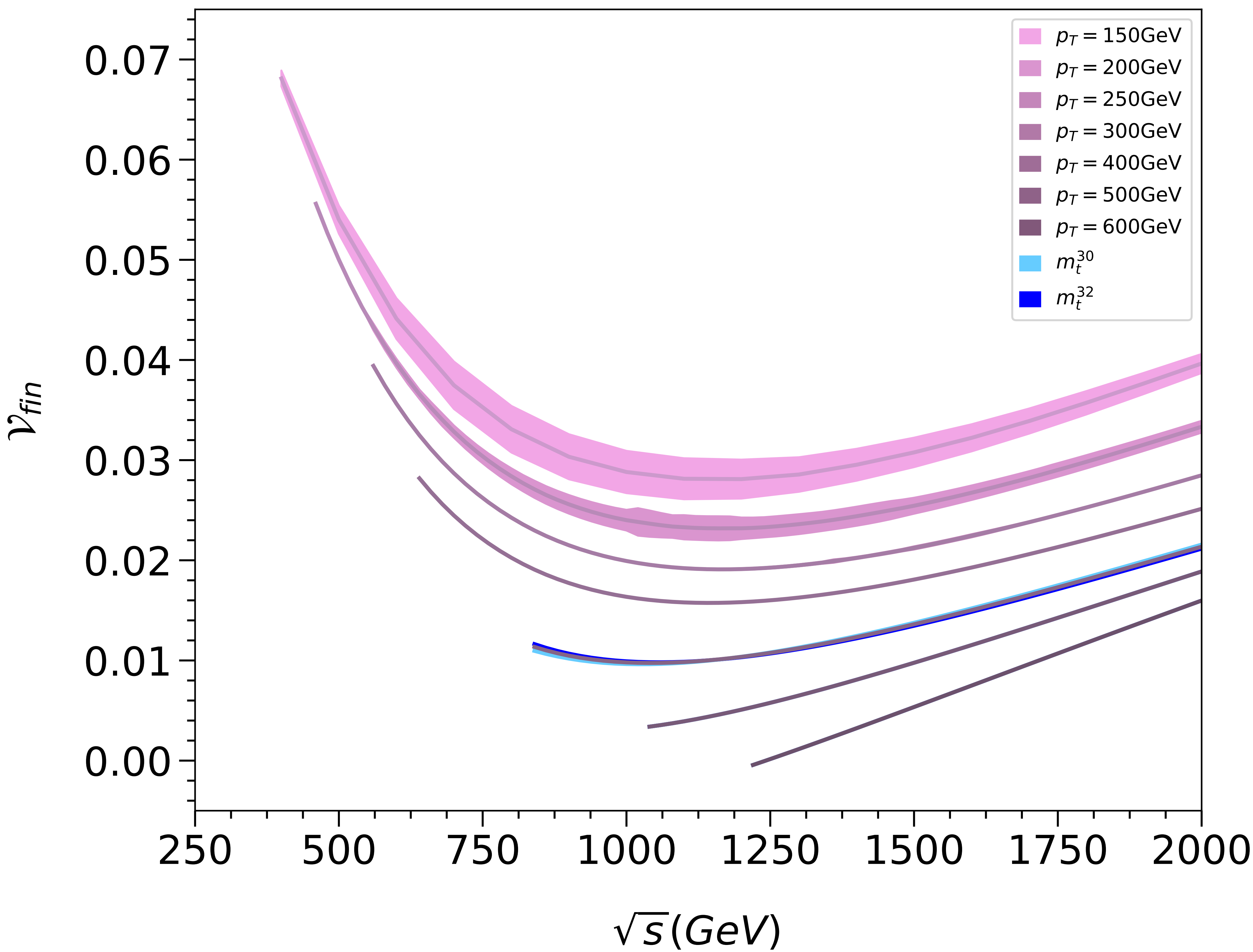
consider the distance between x=1 and the nearest pole

We introduce a weighting function $w_i = \dfrac{\beta_i^2}{\sum_j \beta_j^2}$

Prediction from our method $\alpha = \sum_i w_i \alpha_i$

Estimated uncertainty $\sqrt{\dfrac{\sum_i w_i(\alpha_i - \alpha)^2}{1 - \sum_i w_i^2}}$

Result (ZH case)



Conclusion

- We consider the two-loop virtual top quark correction
to the gluon fusion production of ZZ and ZH.

- In the high-energy approximation, MIs are the same as HH production.

- Padé approximation improves the prediction drastically,
and we further refine the method to obtain reliable estimation.

- Our results can serve
1. cross check for other works (mainly exact numerical calculation)
2. super fast evaluation (without Padé) in high-energy region
3. input for combined grid data base ← work in progress