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Top quark contribution to two-loop helicity amplitudes for W/Z boson pair production in gluon fusion

Based on *arXiv:2009.03742* & *arXiv:2101.12095* with Christian Brønnum-Hansen.

Chen-Yu Wang | 2021-05-19 | RADCOR-LoopFest 2021

Outline

Motivation

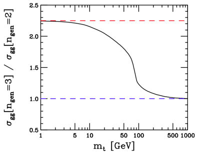
- Higher experimental precision at the LHC requires more accurate theoretical predictions.
  $\Rightarrow$ **more loops**
- As $\sqrt{s}$ goes higher, massive particle in the loop becomes more and more important. $\Rightarrow$ **more masses**
- Precision wish list *Amoroso et al. 2020*

process	known	desired
$\vdots$	$\vdots$	$\vdots$
$pp \to H + t\bar{t}$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$\vdots$	$\vdots$	$\vdots$
$pp \to VV'$	$\mathrm{NNLO_{QCD}} + \mathrm{NNLO_{EW}}$ + $\mathrm{NLO_{QCD}}$ (gg channel)	$\mathrm{NNLO_{QCD}}$ (gg channel, w/ massive loops)
$\vdots$	$\vdots$	$\vdots$
$pp \to t\bar{t}$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{NLO_{QCD}}$ (w/ decays, os) $\mathrm{NLO_{EW}}$ (w/ decays, os)	$\mathrm{NNLO_{QCD}}$ (w/ decays)
$\vdots$	$\vdots$	$\vdots$

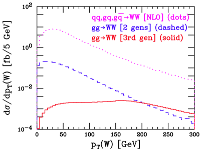
Motivation

- background to $pp \to H^* \to WW/ZZ$, Higgs width and interference effect *Campbell et al. 2011a; Kauer and Passarino 2012; Caola and Melnikov 2013; Campbell et al. 2014; Azatov et al. 2015*
- anomalous gauge couplings

- gg channel: formally enters σ_{ppVV} at NNLO, enhanced by gluon flux & event selection *Binoth et al. 2006*
- massless NLO contribution: $\geq 50\%$ to σ_{ggZZ}, 50% to σ_{ggWW}, $6-8\%$ to σ_{ppZZ}, 2% to σ_{ppWW} *Caola, Melnikov, Röntsch, et al. 2015, 2016*
- massive generation increases LO σ_{ggWW} by 10% *Campbell et al. 2011a*
- dominant contribution for high p_T *Campbell et al. 2011b*



Campbell et al. 2011a



Campbell et al. 2011b

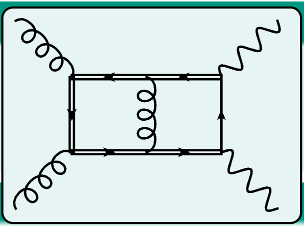
$gg \to WW/ZZ$: Progress

- LO: one-loop *Glover and Bij 1989; Kao and Dicus 1991; Duhrssen et al. 2005; Binoth et al. 2006*
- NLO real: one-loop *Agrawal and Shivaji 2012; Melia et al. 2012; Campanario et al. 2013*
- NLO virtual: two-loop massless: $gg \to VV$ *Caola, Henn, et al. 2015; Manteuffel and Tancredi 2015*
- NLO virtual: two-loop massive:
  - region expansions: $gg \to ZZ$ *Melnikov and Dowling 2015; Gröber et al. 2019; Davies, Mishima, Steinhauser, and Wellmann 2020* $\Rightarrow$ See Go's talk
  - full m_t dependence: $gg \to ZZ$ *Agarwal et al. 2020* $\Rightarrow$ See Bakul's talk

Analytic or numeric?

Analytic

- deeper structure (e.g. Parke-Taylor)
- fast, precise evaluation (e.g. Li_n, ${}_pF_q$, $K(\lambda)$, MPL, eMPL, ...)
- wider applications (e.g. changing parameters)



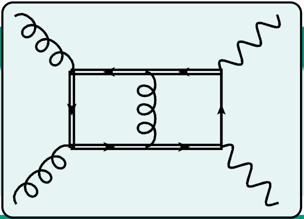
Alternatives

- region expansions: $gg \to ZZ$ *Melnikov and Dowling 2015; Davies, Mishima, Steinhauser, and Wellmann 2020*, $gg \to ZH$ *Davies, Mishima, and Steinhauser 2020* $\Rightarrow$ See Go's talk
- numerical evaluation:
  - sector decomposition: $gg \to HH$ *Borowka et al. 2016*, $gg \to ZZ$ *Agarwal et al. 2020*, $gg \to ZH$ *Chen, Heinrich, et al. 2020* $\Rightarrow$ See Bakul's and Matthias' talk
  - differential equations: $gg \to t\bar{t}$ *Chen, Czakon, et al. 2018*, $gg \to H$ *Czakon and Niggetiedt 2020*
  - dispersion relation: *Song and Freitas 2021* $\Rightarrow$ See Qian's talk
- combined: $gg \to HH$ *Davies, Heinrich, et al. 2019* $\Rightarrow$ See Joshua's talk

Analytic or numeric?

Analytic

- deeper structure (e.g. Parke-Taylor)
- **fast, precise evaluation** (e.g. Li_n, ${}_pF_q$, $K(\lambda)$, MPL, eMPL, ...)
- **wider applications** (e.g. changing parameters)

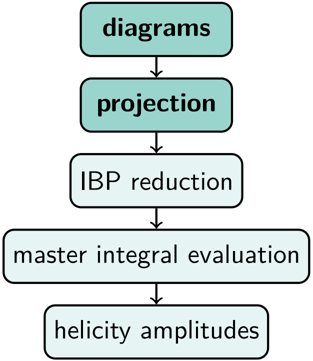


Alternatives

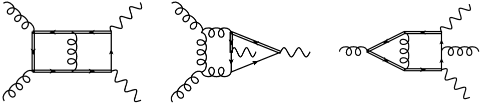
$$\left.\begin{array}{l}\text{fast numerical method that produces \textbf{arbitrary precision} result}\\ \text{good \textbf{interpolation} algorithm}\end{array}\right\} \Rightarrow \begin{array}{l}\text{fast, precise evaluation}\\ \text{in the parameter space}\end{array}$$

- Numeric IBP reduction: set masses (and kinematic variables) to numbers
- Numeric DE evaluation: simple boundary condition + arbitrary precision *Lee et al. 2018; Liu, Ma, and Wang 2018; Abreu et al. 2020; Hidding 2020; Moriello 2020* ⇒ See also Matthias', Martijin's, and Ben's talk

$gg \to WW$: Diagrams & Projection

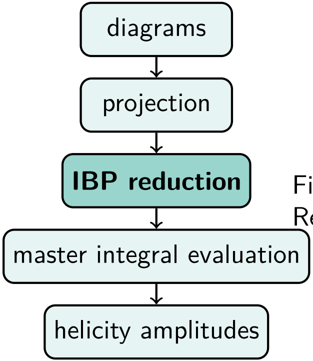


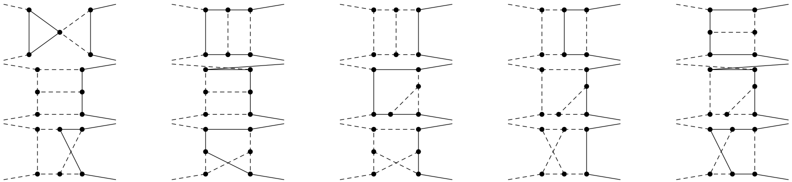
136 diagrams generated in QGRAF *Nogueira 1993* and processed in FORM *Vermaseren 2000; Kuipers et al. 2015; Ruijl et al. 2017*



- No one-loop squared, intermediate Z- or Higgs-bosons diagrams. *Binoth et al. 2006; Aglietti et al. 2007; Anastasiou et al. 2007*
- Decomposition in colour factors: $A = \delta^{AB}\left[N_c A^{[1]} + \frac{1}{N_c} A^{[-1]}\right]$.
- IR structure: $A^{(2)}(\epsilon,\mu) = I^{(1)}(\epsilon,\mu)A^{(1)}(\epsilon,\mu) + F^{(2)}(\epsilon,\mu)$. *Catani 1998*
- γ_5 in d-dimensions through Larin scheme *Larin and Vermaseren 1991; Moch et al. 2015.*
- Projection onto 38 tensor structures T_I and S_I. *Binoth et al. 2006; Manteuffel and Tancredi 2015*

$gg \to WW$: IBP reduction



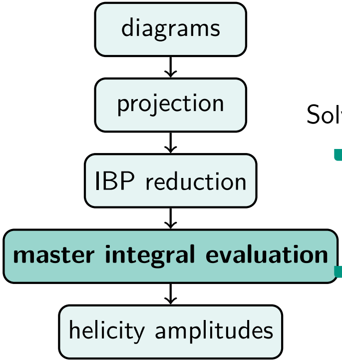


Find symmetry relations with `LiteRed` *Lee 2014* and `REDUZE` 2 *Manteuffel and Studerus 2012.*
Reduction performed with `KIRA` 1.2: *Maierhöfer et al. 2018*

- parametric in d, s, t with $m_t = 173$ GeV and $m_W = 80$ GeV
- projected onto one master at a time, tables $\mathcal{O}(100 \text{ MB})$
- avoid non-factorisable denominators *Smirnov and Smirnov 2020; Usovitsch 2020*
- 334 master integrals in 26 families

$gg \to WW$: Master integral evaluation

Based on the **auxiliary mass flow** method: *Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020*



$$I \propto \lim_{\eta \to 0^+} \int \prod_{i=1}^{2} \mathrm{d}^d l_i \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}}. \tag{1}$$

Solve the differential equation w.r.t mass on the **complex plane**.

- Add an imaginary part to the top quark mass:

$$m_t^2 \to m_t^2 - i\eta.$$

- Solve a system of ordinary differential equations at each phase space point:

$$\partial_x I = MI, \quad m_t^2 - i\eta = m_t^2(1+x). \tag{2}$$

- Boundary condition: $x \to -i\infty \Rightarrow$ Physical point: $x = 0$.

$gg \to WW$: Master integral evaluation

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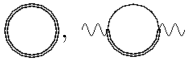
$I = \{$  $\}$

- Expand I around **boundary** in variable $y = x^{-1} = 0$:

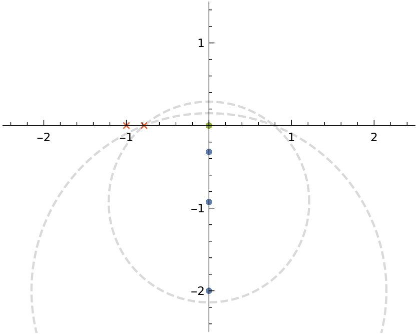
$$I = \sum_j^M \epsilon^j \sum_k^N \sum_l c_{jkl} y^k \ln^l y + \dots \qquad (3)$$

- Evaluate and expand around **regular points**:

$$I = \sum_j^M \epsilon^j \sum_{k=0}^N c_{jk} x'^k + \dots \qquad (4)$$

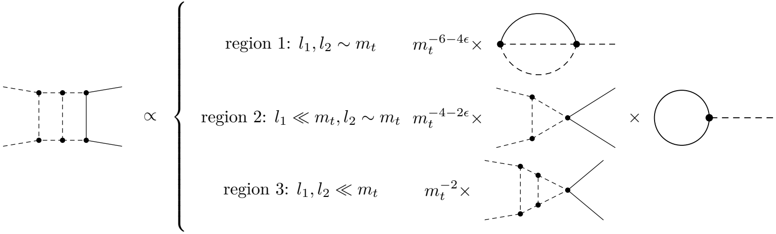
- Evaluate at the **physical point** $x = 0$.
- shorter step / higher order $\Rightarrow$ higher precision

x plane

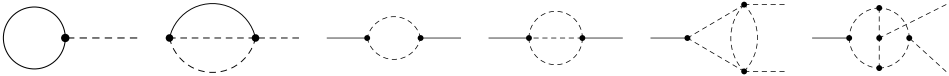


$gg \to WW$: Master integral evaluation

- In our case, several contributions to boundary:



- All boundary conditions are known analytically *'t Hooft and Veltman 1979; Chetyrkin et al. 1980; Scharf and Tausk 1994; Gehrmann and Remiddi 2000; Gehrmann, Huber, et al. 2005*



$gg \to WW$: Master integral evaluation

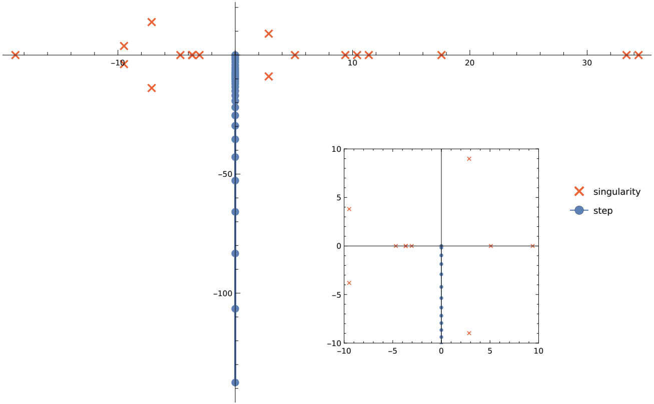


Figure: Steps for a double-box integral with five massive propagators.

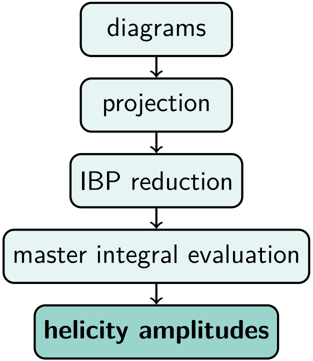
$gg \to WW$: IR structures

Two loops		LLLL	
N_C		ϵ^{-2}	ϵ^{-1}
P1	$A^{(2),[1]}/A^{(1)}$	$-0.9999999989 + 9.7 \cdot 10^{-11} i$	$-1.86131749 - 4.4462006641 i$
	IR pole	-1.0000000000	$-1.86131750 - 4.4462006677 i$
P2	$A^{(2),[1]}/A^{(1)}$	$-1.00000000028 - 3.4 \cdot 10^{-10} i$	$-0.9249605082 - 4.3033199172 i$
	IR pole	-1.00000000000	$-0.9249605067 - 4.3033199148 i$
		LRLL	
N_C		ϵ^{-2}	ϵ^{-1}
P1	$A^{(2),[1]}/A^{(1)}$	$-1.0000000023 - 1.5 \cdot 10^{-9} i$	$-1.502999771 - 5.379923054 i$
	IR pole	-1.0000000000	$-1.502999764 - 5.379923043 i$
P2	$A^{(2),[1]}/A^{(1)}$	$-0.9999999924 + 2.3 \cdot 10^{-9} i$	$1.379867258 - 8.547467432 i$
	IR pole	-1.0000000000	$1.379867207 - 8.547467461 i$

Table: K factor of the renormalised helicity amplitudes at **P1**: $\sqrt{s} \approx 185$ GeV, $\theta \approx 102^\circ$ and **P2**: $\sqrt{s} \approx 367$ GeV, $\theta \approx 37^\circ$, with $\mu = 2m_W$.

- All master integrals to 15 digits: < 1 hour/phase space point on a single CPU.

$gg \to WW$: Helicity amplitudes



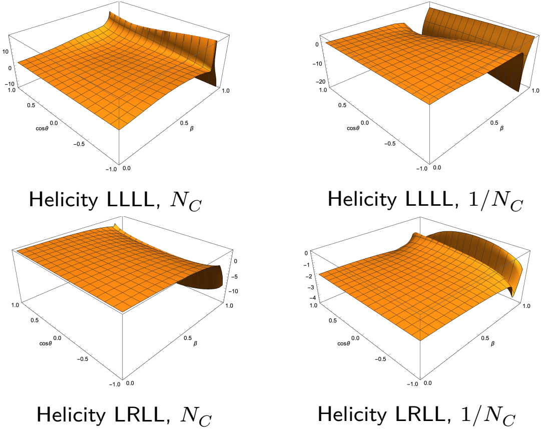


Figure: K factor of the interference term $2\Re\left[F^{(2)}A^{(1)\star}\right]/\left|A^{(1)}\right|^2$ with $\mu = 2m_W$.

$gg \to ZZ$: Diagrams & Projection

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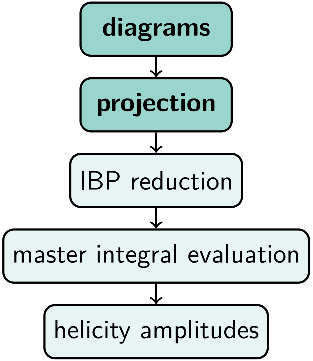
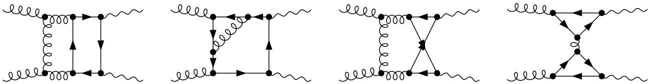
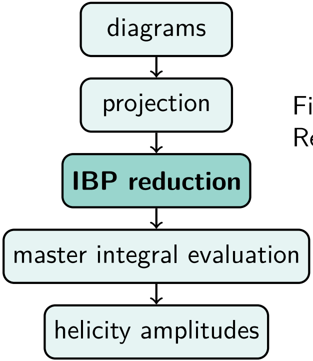


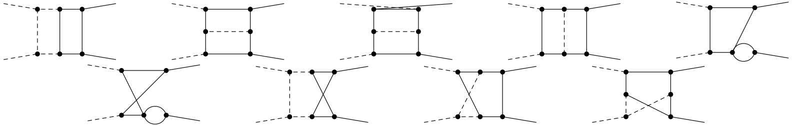
138 diagrams generated in `QGRAF` *Nogueira 1993* and processed in `FORM` *Vermaseren 2000; Kuipers et al. 2015; Ruijl et al. 2017*



- No intermediate Z- or Higgs-bosons. *Spira et al. 1995; Harlander and Kant 2005; Anastasiou et al. 2007*
- Decomposition: $A = \delta^{AB} n_h \left[C_A A^{[C_A]} + C_F A^{[C_F]} + n_h A^{[n_h]} \right]$.
- γ_5 scheme:
  - n_h diagrams: Larin scheme *Larin and Vermaseren 1991; Moch et al. 2015.*
  - all other diagrams: naive scheme, thanks to Furry's theorem.
- IR structure: $A^{(2)}(\epsilon,\mu) = I^{(1)}(\epsilon,\mu) A^{(1)}(\epsilon,\mu) + F^{(2)}(\epsilon,\mu)$. *Catani 1998*
- Projection onto 18 tensor structures T_I. *Binoth et al. 2006*

$gg \to ZZ$: IBP reduction

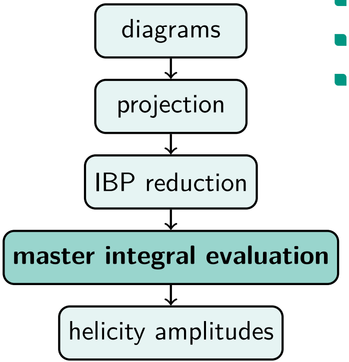




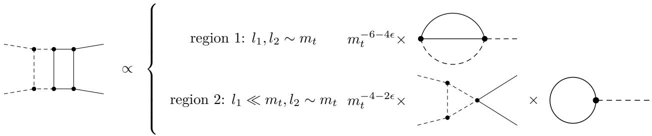
Find symmetry relations with REDUZE 2 *Manteuffel and Studerus 2012.*
Reduction performed **at each phase space point** with KIRA 2.0: *Klappert et al. 2020:*

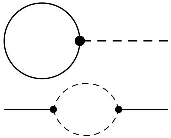
- parametric only in d with $m_t = 173$ GeV, $m_Z = 91$ GeV and rational s, t
- lower the memory/storage consumption significantly
  - get one phase space evaluation out quickly
  - less time to scan the phase space: integral reduction + amplitude evaluation
- avoid non-factorisable denominators *Smirnov and Smirnov 2020; Usovitsch 2020*
- 205 master integrals in 10 families + crossings

NLO $gg \to ZZ$: Master integral evaluation



- Evaluated in the same way as $gg \to WW$.
- More massive propagators ⇒ Simpler boundary condition.
- All master integrals to 20 digits: < 1 hour/phase space point on a single CPU.





't Hooft and Veltman 1979

$gg \to ZZ$: IR structures

C_A		ϵ^{-2}	ϵ^{-1}	ϵ^{0}
LLLL	$A^{(2)}/A^{(1)}$	$1.0000000000008 - 7.6 \cdot 10^{-13} i$	$0.8304916142577 + 3.2298743687703i$	$-3.878332328849 - 3.254364077719i$
	IR pole	1.00000000000000	$0.8304916142539 + 3.229874368771i$	—
LRLL	$A^{(2)}/A^{(1)}$	$1.0000000000009 - 1.42 \cdot 10^{-12} i$	$0.2359507533599 + 2.8851548638498i$	$1.5709899577479 + 0.2619850649223i$
	IR pole	1.00000000000000	$0.2359507533772 + 2.8851548638517i$	—

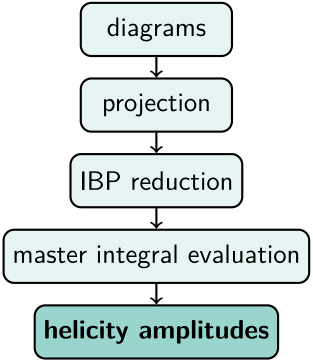
C_F		ϵ^{0}
LLLL	$A^{(2)}/A^{(1)}$	$-5.487100965397 + 0.2839759537883i$
LRLL	$A^{(2)}/A^{(1)}$	$-4.498637043876 + 0.0049840519 42i$

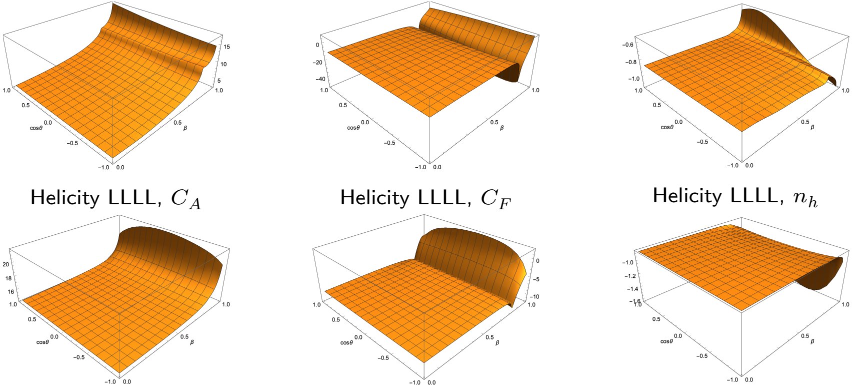
n_h		ϵ^{0}
LLLL	$A^{(2)}/A^{(1)}$	$-0.4472046190541 - 0.00072891867295i$
LRLL	$A^{(2)}/A^{(1)}$	$-0.4401391809472 - 0.0001866854575i$

Table: K factor of the renormalised helicity amplitudes at $\sqrt{s} \approx 210$ GeV and $\theta \approx 114^{\circ}$, with $\mu = m_Z$.

- IR divergence matches Catani's IR operator *Catani 1998.*
- Cross-checked with *Agarwal et al. 2020; Davies, Mishima, Steinhauser, and Wellmann 2020.*
- ~ 5 hours/phase space point on a single CPU: integral reduction + amplitude evaluation.

$gg \to ZZ$: Helicity Amplitudes





Helicity LLLL, C_A | Helicity LLLL, C_F | Helicity LLLL, n_h

Helicity LRLL, C_A | Helicity LRLL, C_F | Helicity LRLL, n_h

Figure: K factor of the interference term $2\Re\left[F^{(2)}A^{(1)\star}\right] / \left|A^{(1)}\right|^2$ with $\mu = m_Z$.

Conclusion

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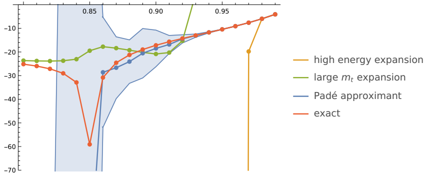
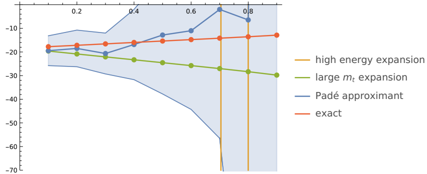
- We calculated two-loop $gg \to WW/ZZ$ with full m_t dependence.
- A fully numeric method is a practical way to evaluate multi-loop processes.
- **Numeric IBP reduction** is efficient in practice for complicated multi-scale processes.
- **Auxiliary mass flow** method provides an efficient and precise way to evaluate multi-loop integrals.
- Use standard multi-loop calculation procedures, benefit from latest development.
- Thank you!

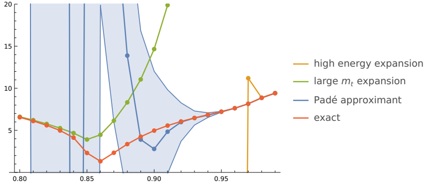
$gg \to ZZ$: comparison to series expansion results



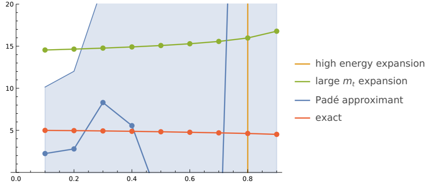
$0.8 \leq \beta \leq 0.99$, $\cos\theta = 0.2$.



$\beta = 0.9$, $0.1 \leq \cos\theta \leq 0.9$.



$0.8 \leq \beta \leq 0.99$, $\cos\theta = 0.2$.



$\beta = 0.9$, $0.1 \leq \cos\theta \leq 0.9$.

Compare with *Davies, Mishima, Steinhauser, and Wellmann 2020*

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