

# NNLO QCD predictions for 2 to 3 processes

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**European Research Council**

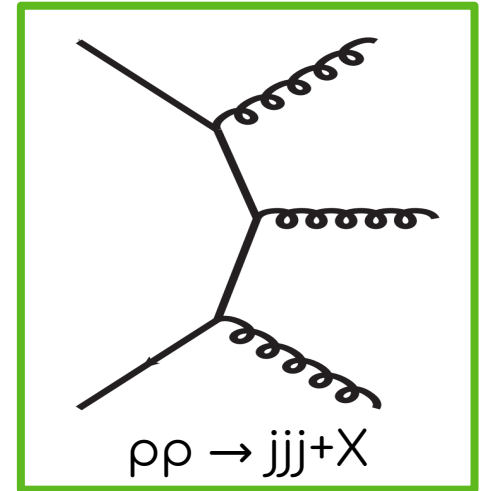
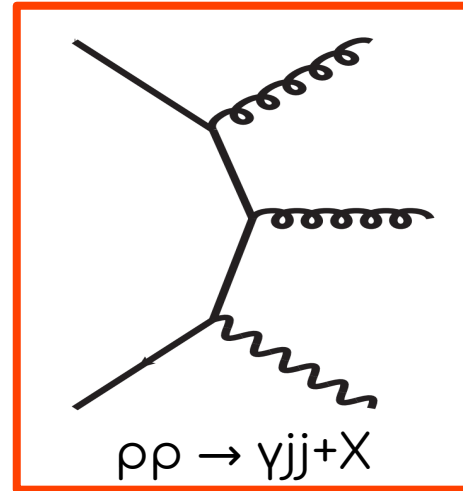
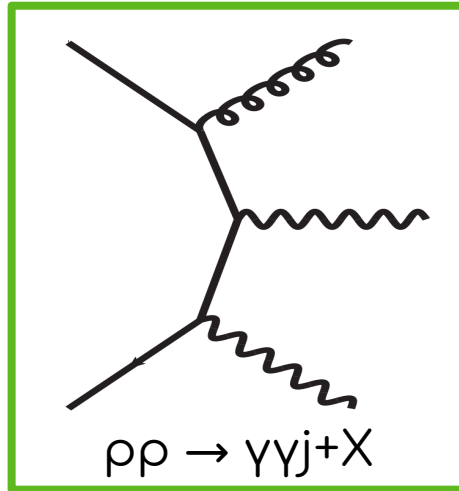
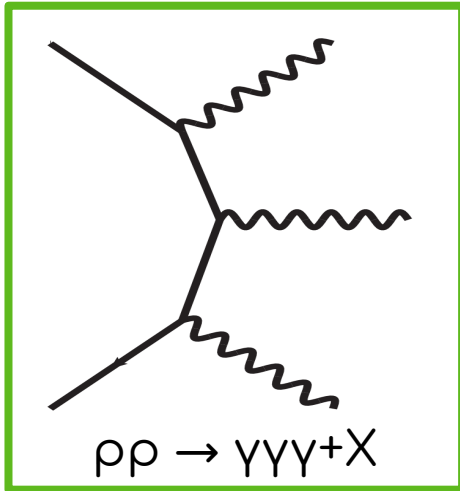
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# Outline

- Precision vs. Multiplicity @ the LHC
- NNLO QCD for 2 to 3 processes without masses:



- Summary

# Precision vs. Multiplicity @ the LHC

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Why are we interested in NNLO QCD for  $2 \rightarrow 3$  processes? (8 talks this week)

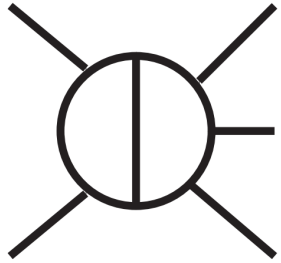
Phenomenological aspects:

- For  $2 \rightarrow 2$  NNLO QCD (+NLO EW) huge success for many measurements!  
In some cases N3LO on the wish list.
- Next phase of LHC  $\rightarrow$  enough statistics to actually resolve  $2 \rightarrow 3$  NNLO?!
  - Massless processes a clear case!
  - But also heavy processes  $H/V+2j, ttH, ttV, VVV, \dots$  call for NNLO predictions!

Theory aspects:

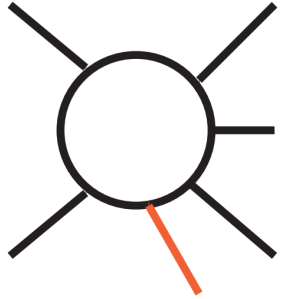
- Development of NNLO QCD technology (amplitudes&subtraction) crucial work on the road towards NNLO event simulation.
- Crucial ingredient for differential  $2 \rightarrow 2$  N3LO QCD

# NNLO QCD prediction beyond $2 \rightarrow 2$

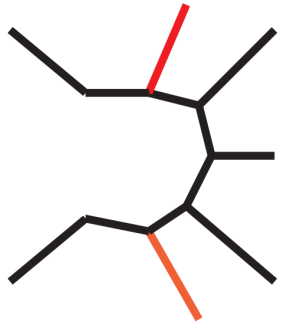


$2 \rightarrow 3$  Two-loop amplitudes:

- (Non-) planar 5 point massless  $\rightarrow$  talk by [Vasily, Herschel, Federico](#)  
fast progress in the last half of year  
 $\rightarrow$  triggered by efficient MI representation [[Chicherin, Sotnikov'20](#)]
- 5 point with one external mass  
 $\rightarrow$  talk by [Nikolaos, Ben, Konstantinos, Bayu](#)



Many leg, IR stable one-loop amplitudes  $\rightarrow$  OpenLoops [[Buccioni'19](#)]

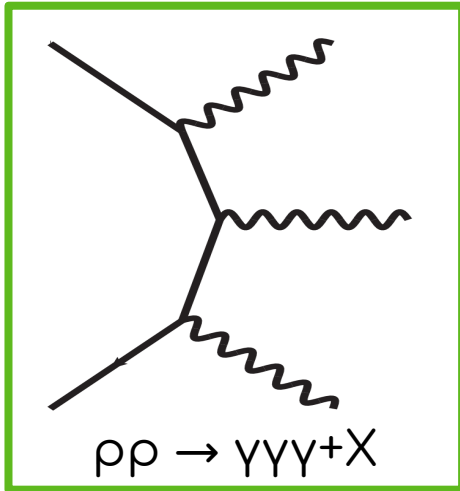


Cross sections  $\rightarrow$  Combination with real radiation

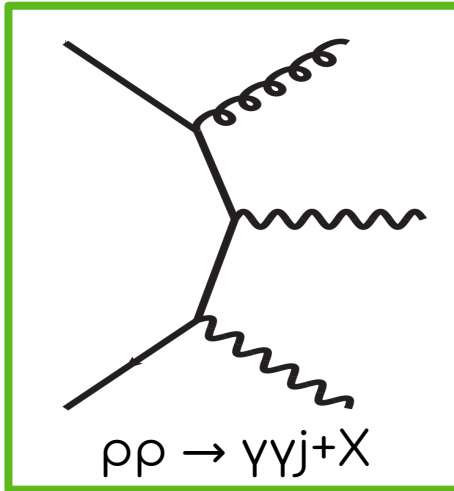
- Various NNLO subtraction schemes are available:  
qT-slicing [[Catani'07](#)], N-jettiness slicing [[Gaunt'15/Boughezal'15](#)], Antenna [[Gehrmann'05-'08](#)], Colorful [[DelDuca'05-'15](#)], Projection [[Cacciari'15](#)], Geometric [[Herzog'18](#)], Unsubtraction [[Aguilera-Verdugo'19](#)], Nested collinear [[Caola'17](#)], Sector-improved residue subtraction [[Czakon'10-'14](#)]

# Phenomenological applications

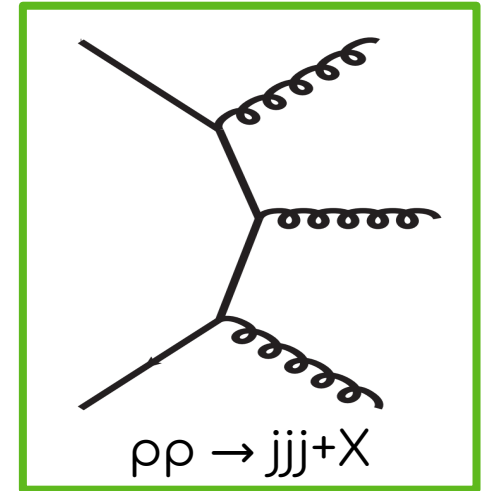
Three photons



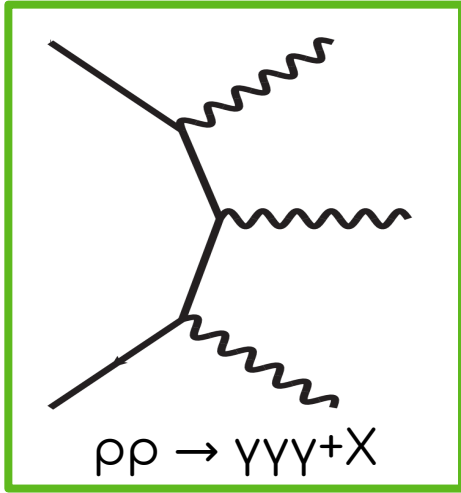
Two photons plus jet



Three jets

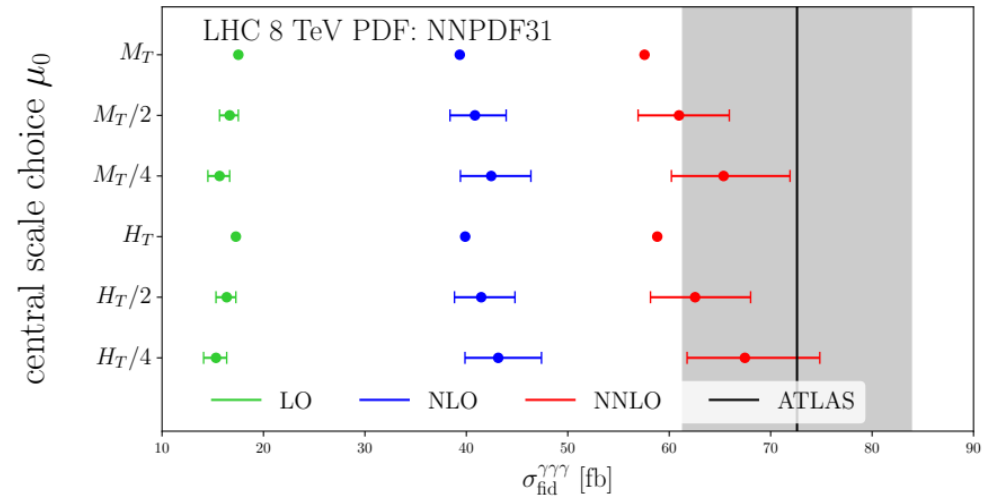


# Three photon production

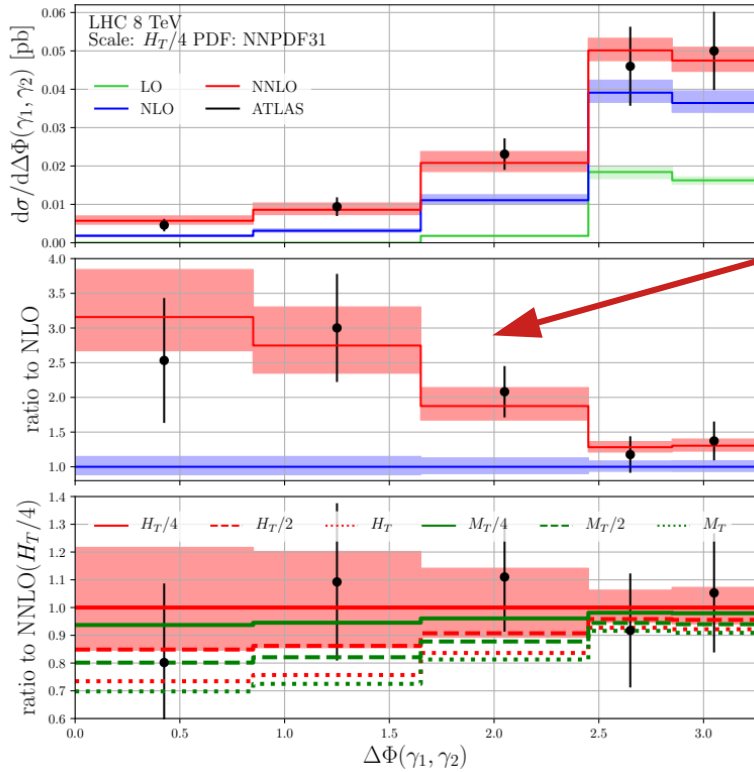


- First NNLO QCD  $2 \rightarrow 3$  cross sections: [Chowdhry'19],[Kallweit'20]
- Simplest among the  $2 \rightarrow 3$  massless cases: colour singlet
- Planar Two-loop virtuals:  
 $2 \cdot \text{Re}(M_0 \cdot F_2)$  with 'original' pentagon functions [Henn'18]  
→ Fast helicity amplitudes: [Abreu'20],[Chowdhry'20]

- Large NNLO/NLO K-factors
- Similar behaviour as  $pp \rightarrow \gamma\gamma$
- NNLO QCD corrections essential for theory/data comparison
- Contribution of 2l amps small  $\approx 1\%$

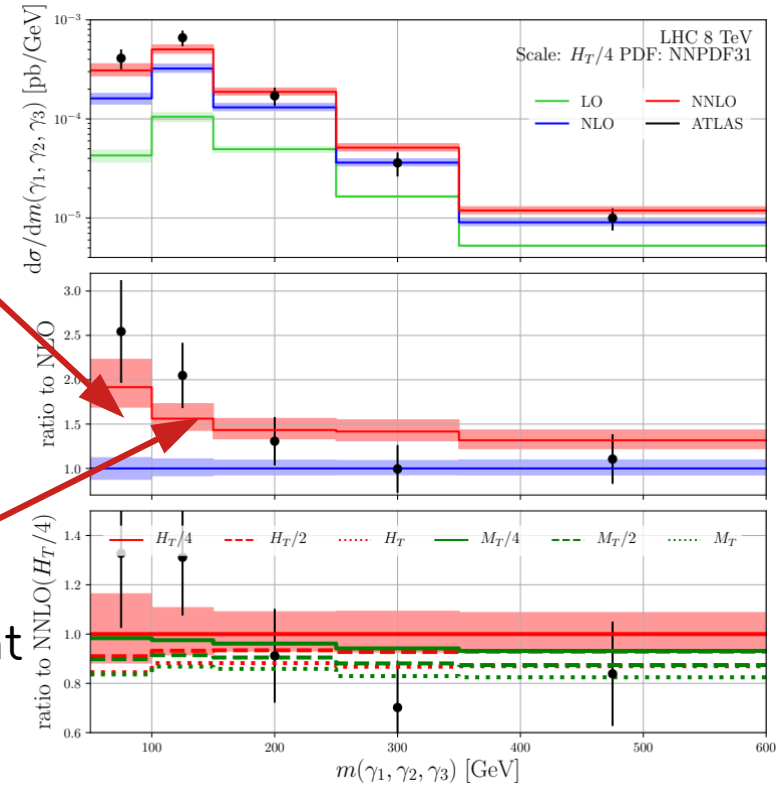


# Three photon production



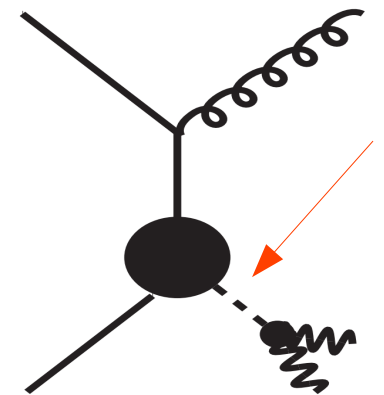
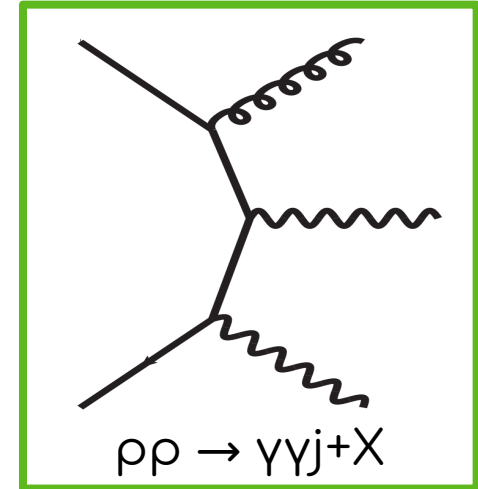
Corrections to shape and normalization

Typical for colour singlets: Scale uncertainty stays large. Very different for  $pp \rightarrow \gamma\gamma j$ ,  $pp \rightarrow jjj$



# Diphoton plus jet production

- Photon pair production @ LHC is of particular interest:
  - Main background to cleanest Higgs decay channel
- Inclusive diphoton shows large NNLO QCD corrections
  - Perturbative convergence @ N3LO?  
First steps: Talks by Xuan, Lorenzo
  - Diphoton plus jet @ NNLO QCD ( $p_T(AA) \rightarrow 0$  limit)
- $p_T(\gamma\gamma)$  spectrum itself interesting for Higgs  $\rightarrow \gamma\gamma$  :
  - Higgs -  $p_T$  measurements at large  $p_T$  resolves local Higgs couplings  $\rightarrow$  BSM searches
  - Angular diphoton observables  $\rightarrow$  spin measurements





# Diphoton plus jet - setup

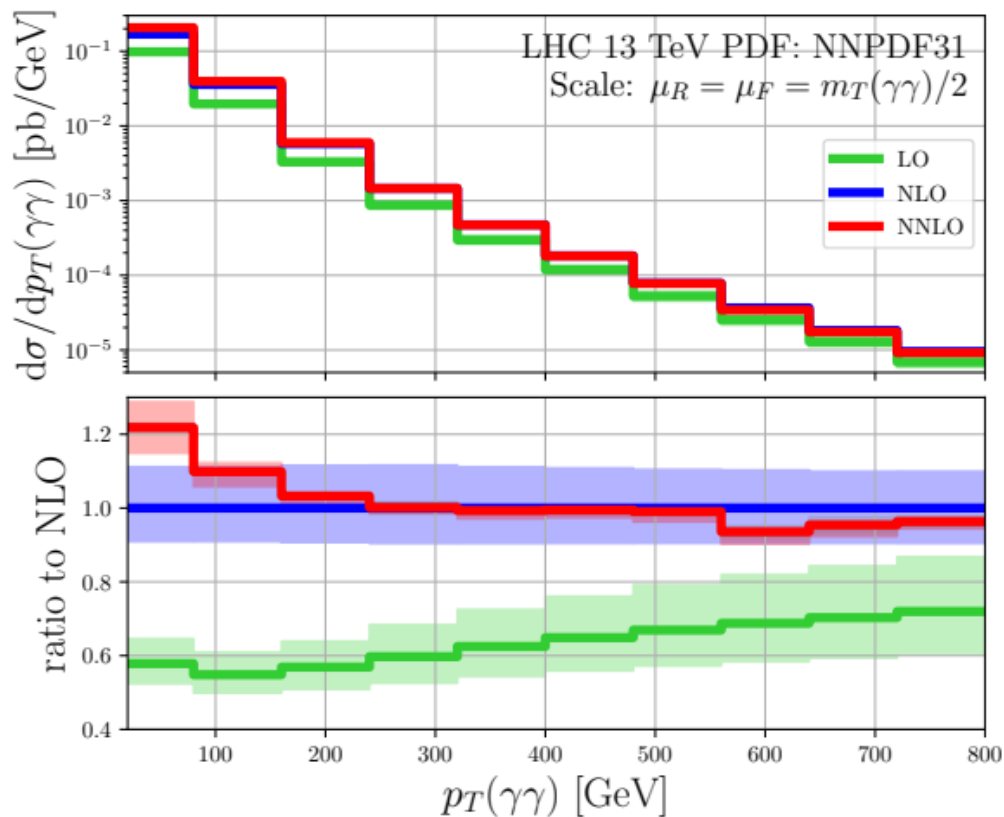
2105.06940: Inspired by Higgs  $\rightarrow \gamma\gamma$  measurement phase spaces

- Smooth photon isolation criteria ( $E_T = 10\text{ GeV}$ ,  $R = 0.4$ ),  $dR(\gamma\gamma) > 0.4\text{ GeV}$
- $p_T(\gamma_1) > 30\text{ GeV}$ ,  $p_T(\gamma_2) > 18\text{ GeV}$  and  $|\eta(\gamma)| < 2.4$
- $m(\gamma\gamma) > 90\text{ GeV}$  and  $p_T(\gamma\gamma) > 20\text{ GeV}$ , below resummation important
- No further restrictions on jets (IR safety from  $p_T(\gamma\gamma)$  cut)

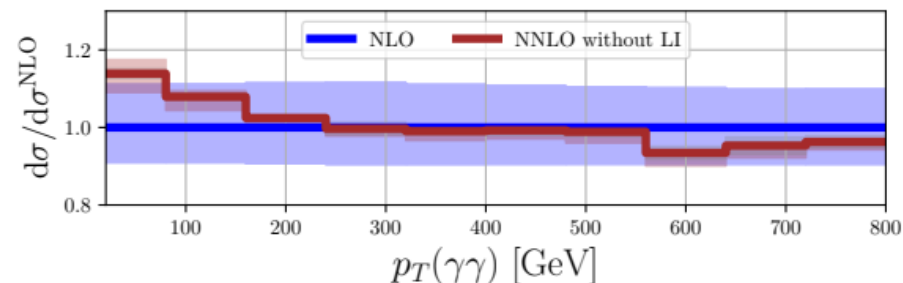
Technicalities:

- LHC 13 TeV, PDF: NNPDF31, Scale:  $\mu_R^2 = \mu_F^2 = \frac{1}{4}(m(\gamma\gamma)^2 + p_T(\gamma\gamma)^2)$
- 5 massless flavours and top-quarks (in all one-loop amps)
- Approximation of two-loop amps:  
2Re(M0\*F2) + F1\*F1 without top-quark loops  
and 2Re(M0\*F2) in leading colour limit [Chawdhry'21]  
 $\rightarrow$  Update to full colour planed [Agarwal'21]

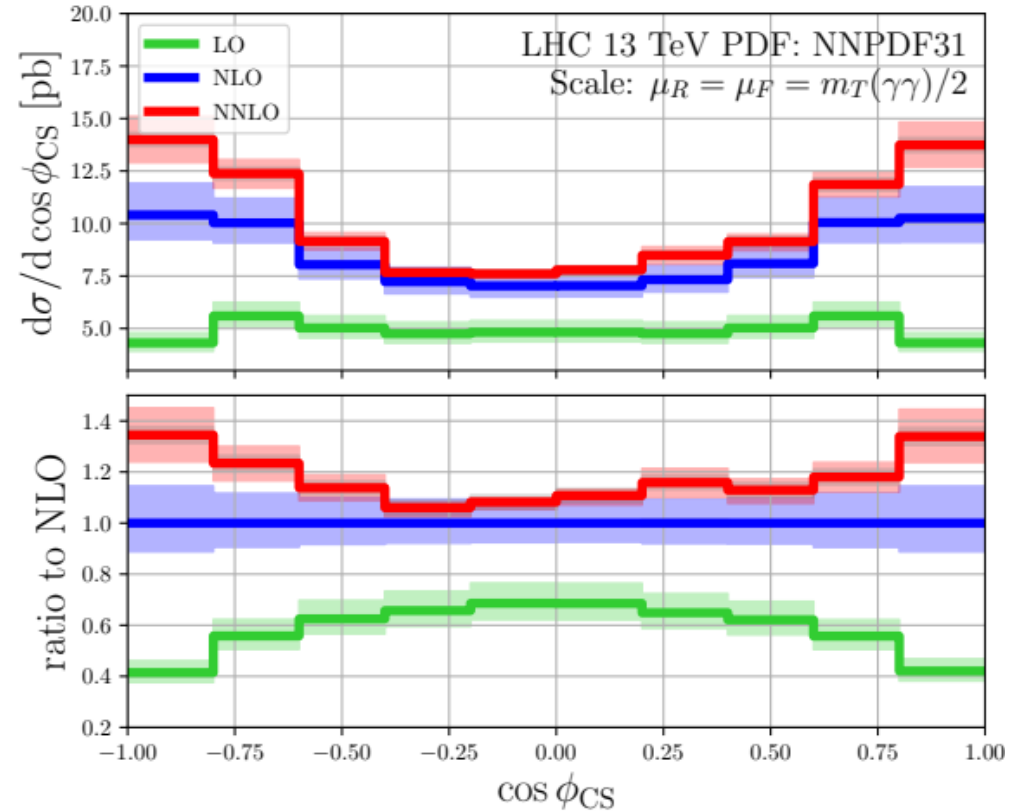
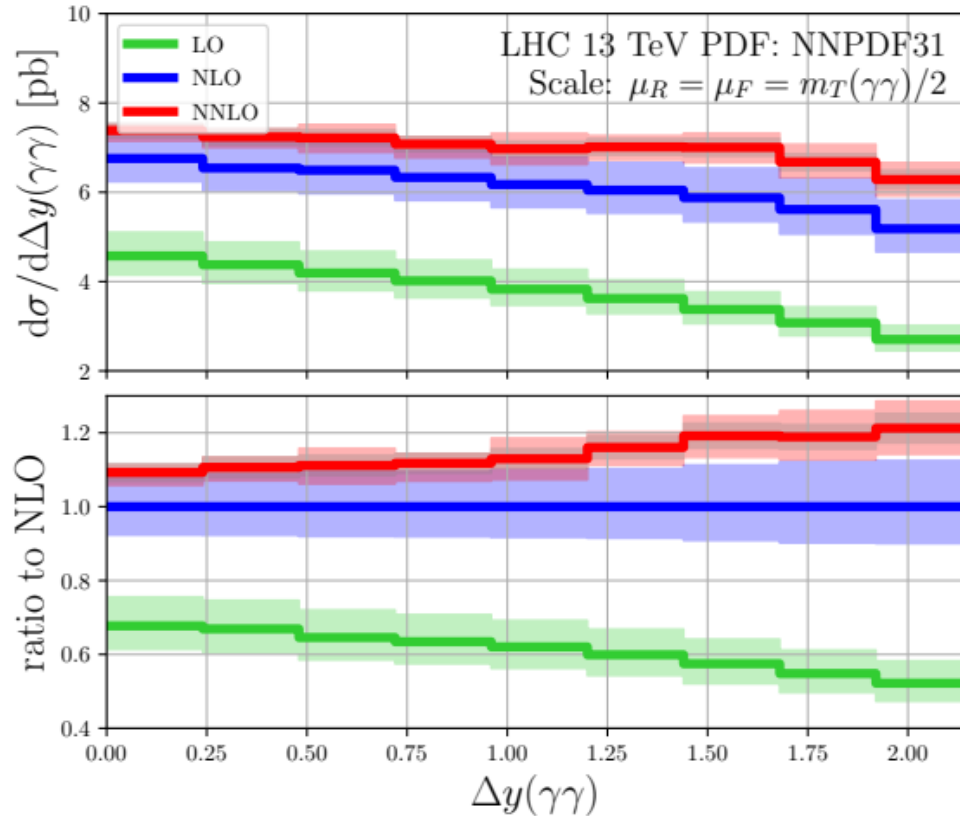
# Diphoton plus jet – $p_T$ spectrum



- Beautiful perturbative convergence
- Scale dependence:  
NLO: ~10%  
NNLO: ~1-2%
- Low  $p_T$  region:
  - ? Resummation for  $p_T(\gamma\gamma)/m(\gamma\gamma) \ll 1$
  - Strong effect from the loop induced!

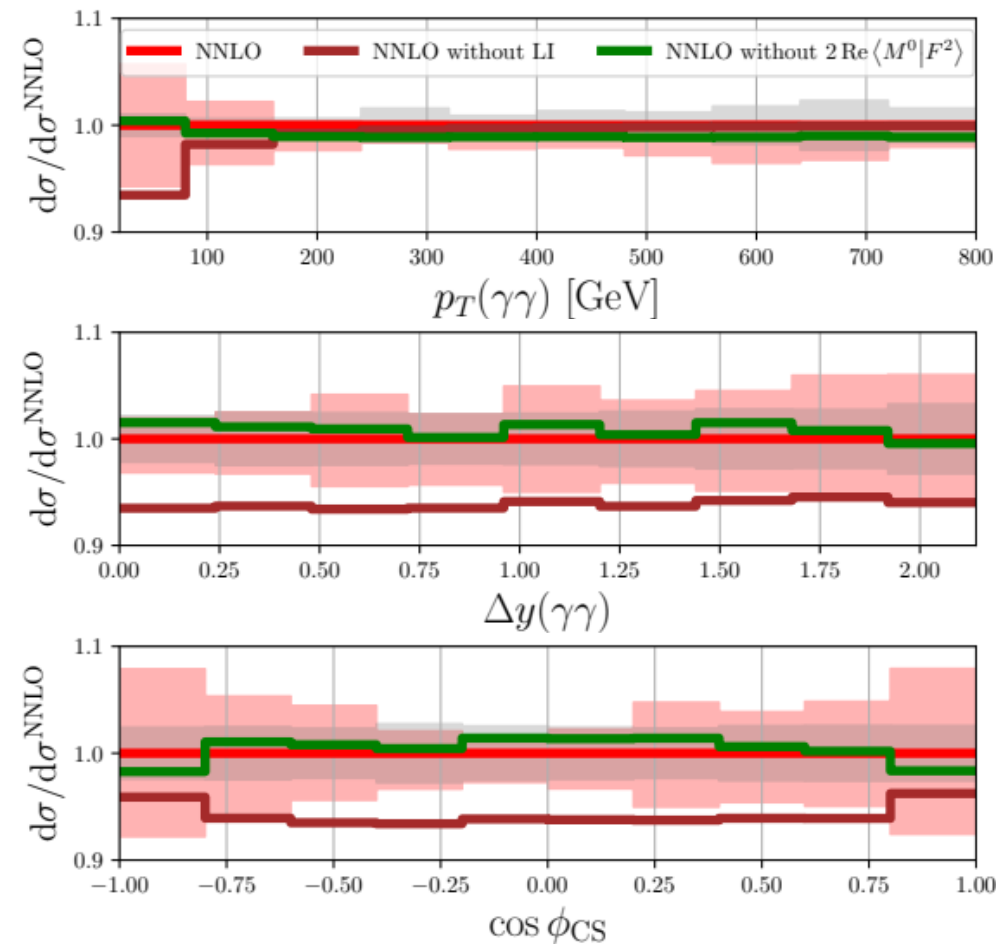


# Diphoton plus jet – Angular observables



Note: Normalization effected by low  $p_T$  behaviour

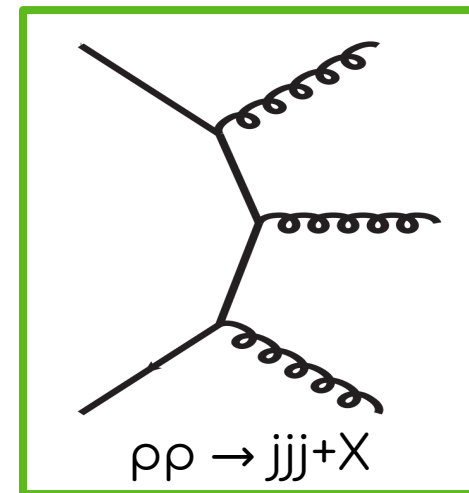
# Diphoton plus jet – two-loop contribution



- Two-loop contribution (green line)  $< \sim 1\%$ ,
- **Loop induced contribution:**
  - sizeable effects for low  $p_T$ , vanishes for high  $p_T$
  - flat effect in ‘bulk’ observables
  - Dominant source of scale dependence
  - NLO QCD correction (formally N3LO) relevant, missing piece:  $gg \rightarrow \gamma\gamma g$  two-loop

# Three jet production

- Multi-jet rates provide an unique possibility to test (perturbative) QCD at the LHC
- Measurements of  $\alpha_S$  from event shapes and jet rate ratios ( $\sim \alpha_S$ )
- Test of  $\alpha_S$  running
- Multi-jet signatures are background for many LHC signatures.
- Allow to probe broad ranges of energy scales for heavy new physics
- Large cross sections  $\rightarrow$  large statistics, in practice only limited by systematics!



# Three jet production

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Advances in perturbative QCD allow precision predictions for multi-jet rates

Here: NNLO QCD predictions for two and three jet rates

- NNLO QCD di-jet production known:
  - Gluons only [Gehrmann-De Ridder'13], partially leading colour [Currie'16]
  - Complete [Czakon'18] → sub-leading colour effects < 1-2%
- NNLO QCD tri-jet production:
  - Bottleneck double virtual amplitudes: recently published in leading colour approximation [Abreu'21]
  - Handling of real radiation:
    - Sector-improved residue subtraction conceptually capable  
→ Tour-de-force (~4000 sectors for RR) → preliminary results

# Three jet production - Setup

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Setup:

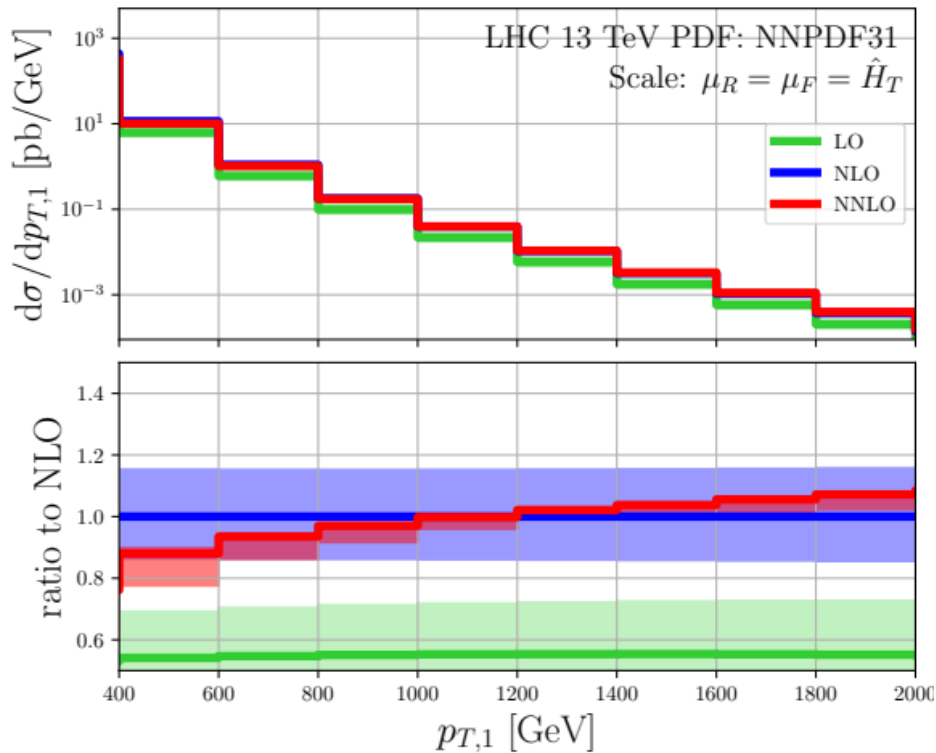
- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets with:
  - $p_T > 60$  GeV,  $|y| < 4.4$
  - $HT2 = p_{T1} + p_{T2} > 250$  GeV
- Scales:  $\mu_R = \mu_F = \hat{H} = \sum p_T$  partons

R32 ratios:

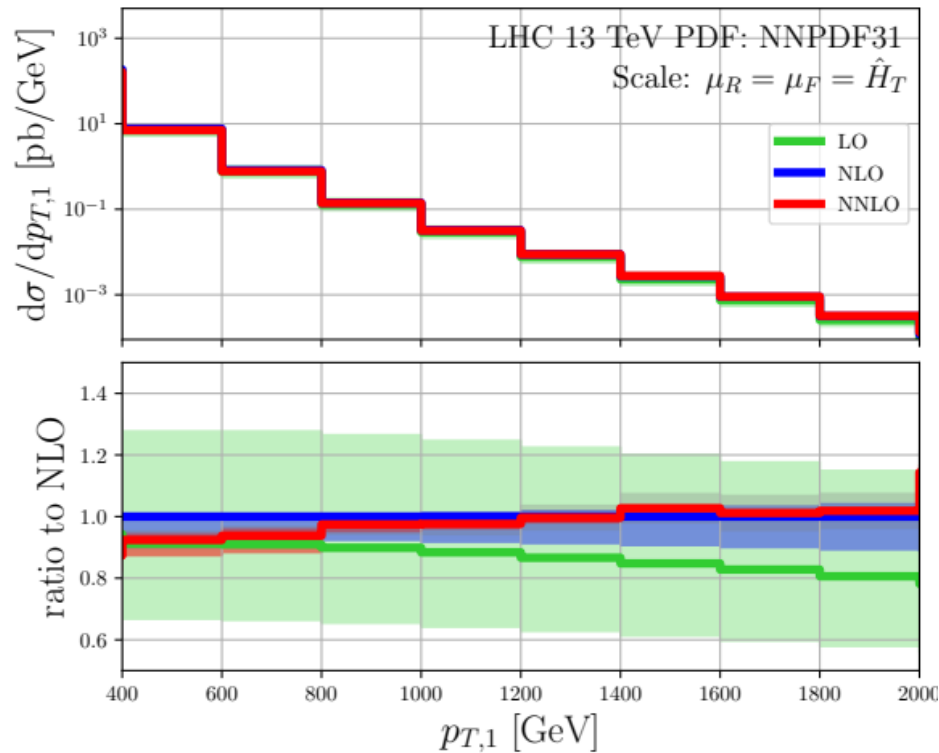
- Two jet rate =  $\sigma_2$   
Three jet rate =  $\sigma_3$
- $R_{32} = \sigma_3 / \sigma_2$
- Differentially in X:  
 $R_{32}(X) = (d\sigma_3/dX) / (d\sigma_2/dX)$
- Scale dependence of  $R_{32}(X)$  is determined by correlated variation in  $\sigma_3$  and  $\sigma_2$

# Three jet production – leading $p_T$

Two jets:

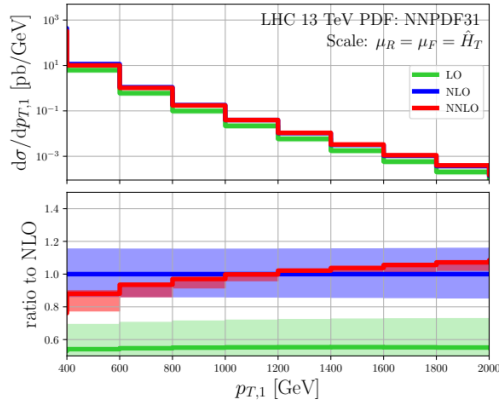
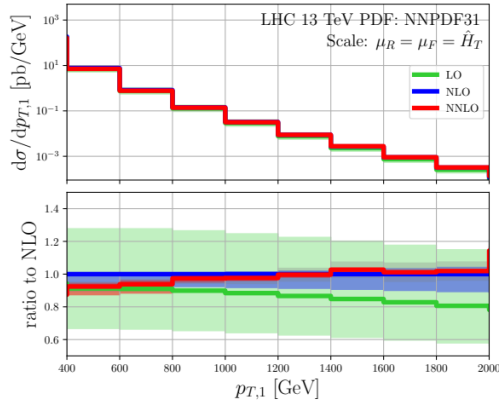


Three jets:

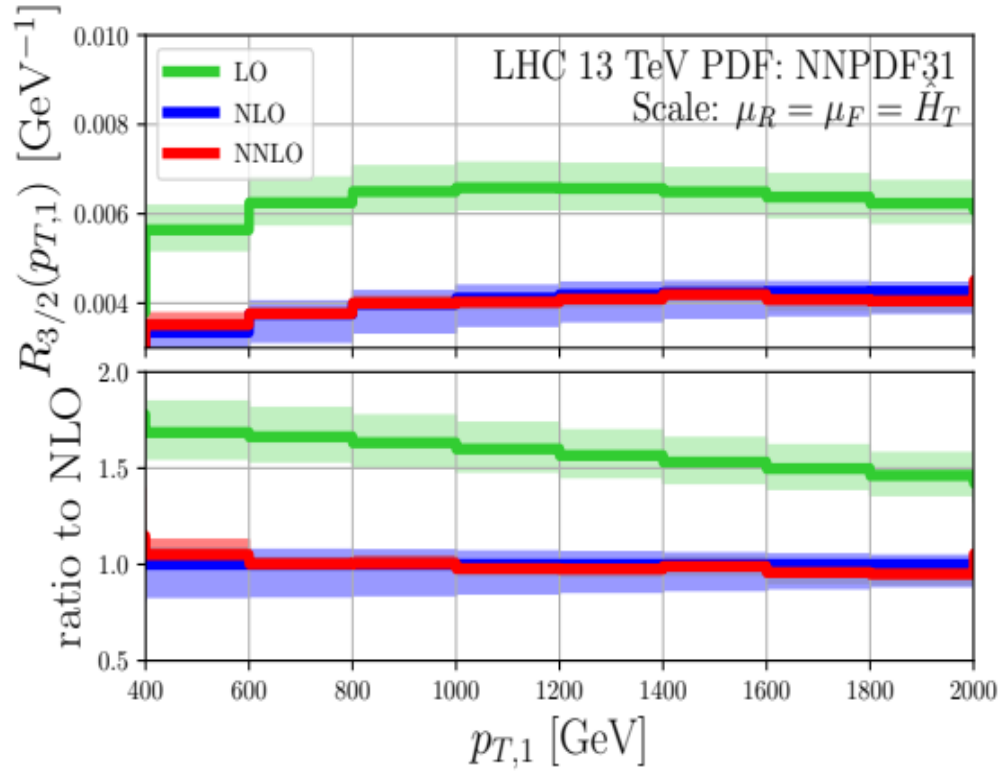




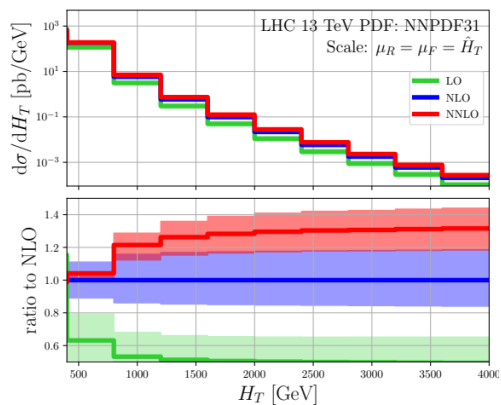
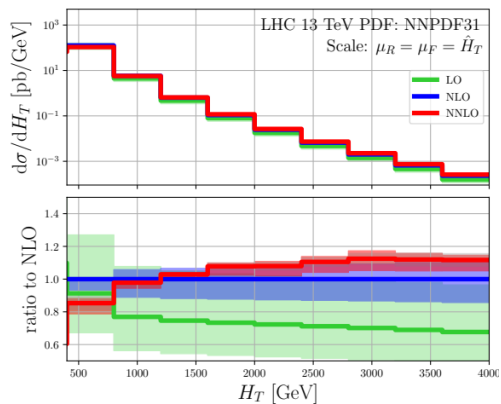
# Three jet production - R32( $p_{T1}$ )



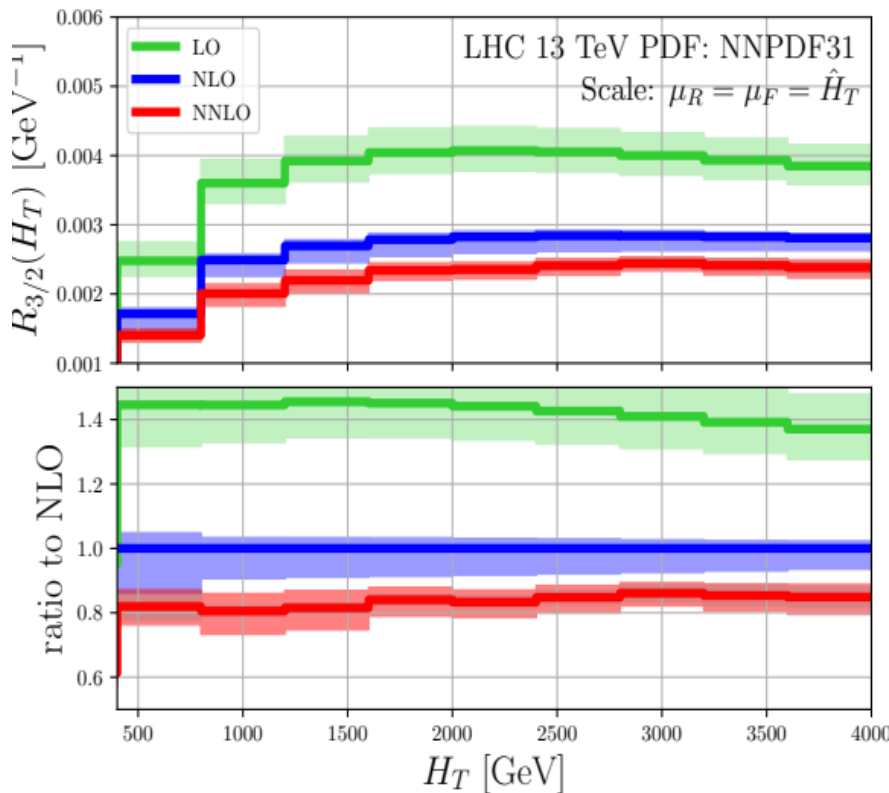
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# Three jet production - R32(HT)



$$HT = \sum \rho T(\text{jet})$$



Scale dependence correlated in ratio

→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap

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# Three jet production – azimuthal decorrelation

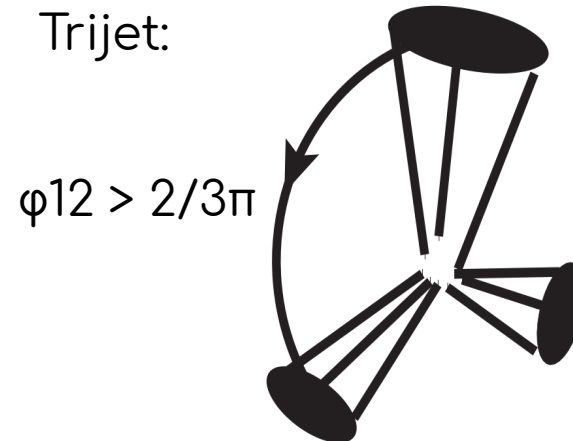
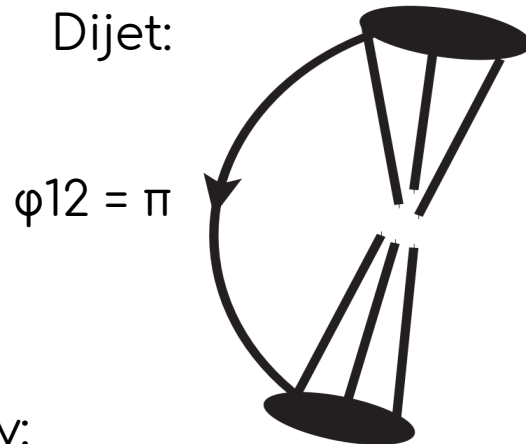
Kinematic constraints on the azimuthal separation between the two leading jets ( $\phi_{12}$ )

$\phi_{12}$  sensitive to the jet multiplicity:

2j:  $\phi_{12} = \pi$

3j:  $\phi_{12} > 2/3\pi$

4j: unconstrained



Study of the ratio

$$R_{32}(HT, y^*, \phi_{\text{Max}}) = \frac{(d\sigma_3(\phi < \phi_{\text{Max}})/dHT/dy^*)}{(d\sigma_2/dHT/dy^*)}$$

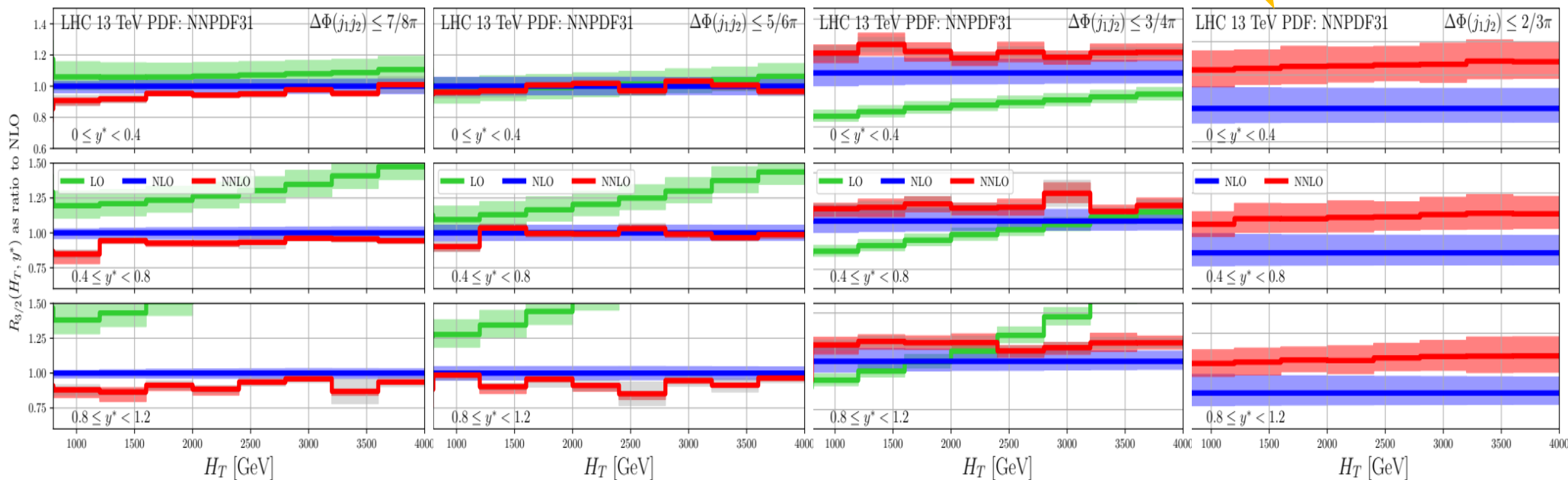
With  $y^* = |y_1 - y_2|/2$

# Three jet production – R32(HT,y\*,φMax)

NNLO/NLO K-factor smaller than NLO/LO  
Scale dependence is reduced

← φMax  
↓ y\*

NLO 4-jet



# Summary and Outlook

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- NNLO QCD predictions for 2 to 3 processes will be essential part of precision phenomenology at the LHC
- Results for:
  - Three photons
  - Diphoton plus jet
  - Three jet production
- Virtual matrix elements with high multiplicity and many scales are the bottleneck!
- Real radiation for  $2 \rightarrow 3$  can be handled.  
But efficiency is a concern and needs some attention!
- Many interesting applications ahead! Stay tuned

Thank you for your attention!