MIXED EW-QCD HELICITY AMPLITUDES FOR DRELL-YAN LEPTON PAIR PRODUCTION

Andreas von Manteuffel

Michigan State University

In collaboration with Matthias Heller, Robert M. Schabinger, Hubert Spiesberger
[1907.00491 + 2012.05918]

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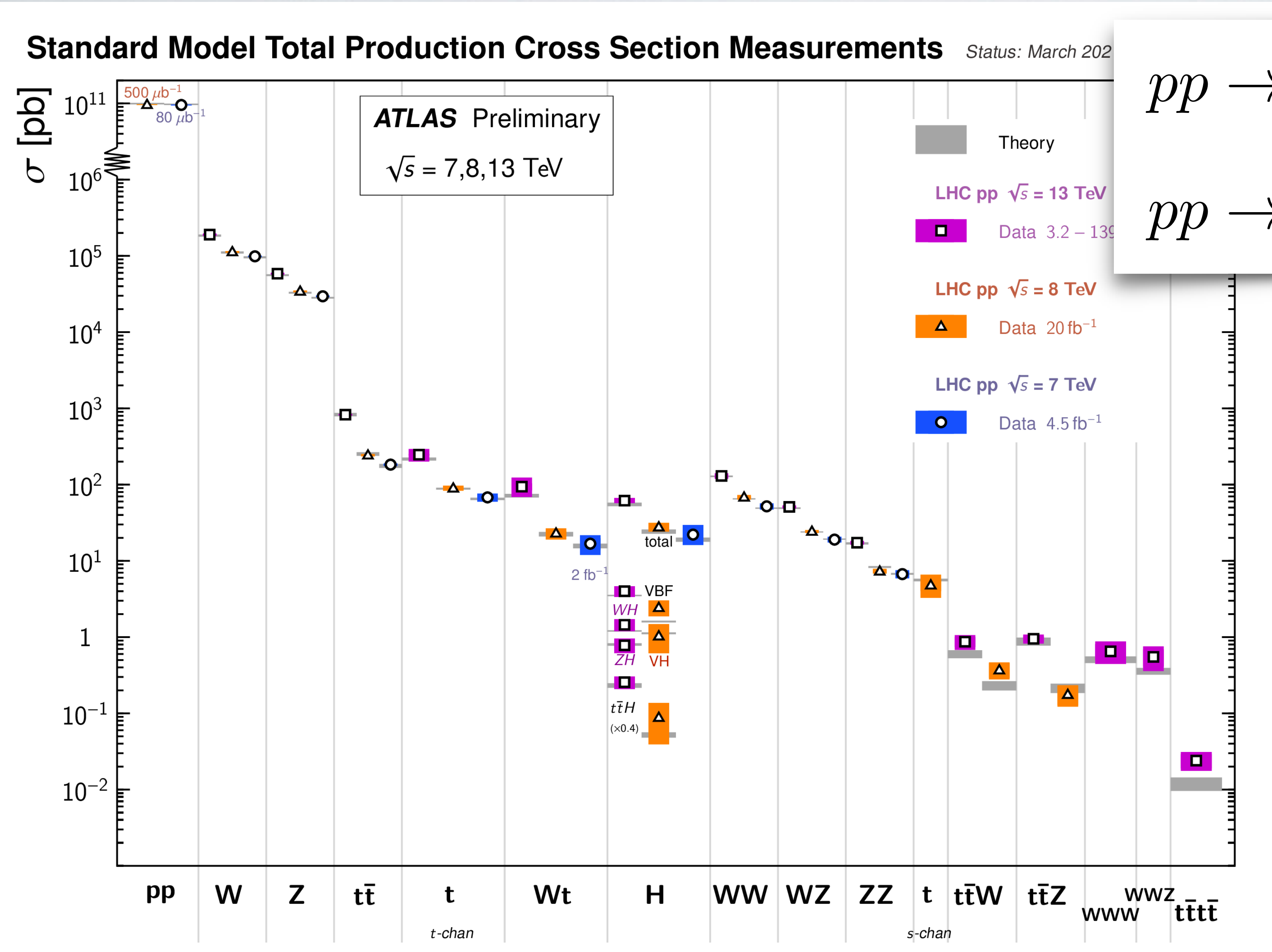
Radcor & Loopfest 2021, Zoom @ Florida State University

DRELL-YAN PROCESS @ LHC

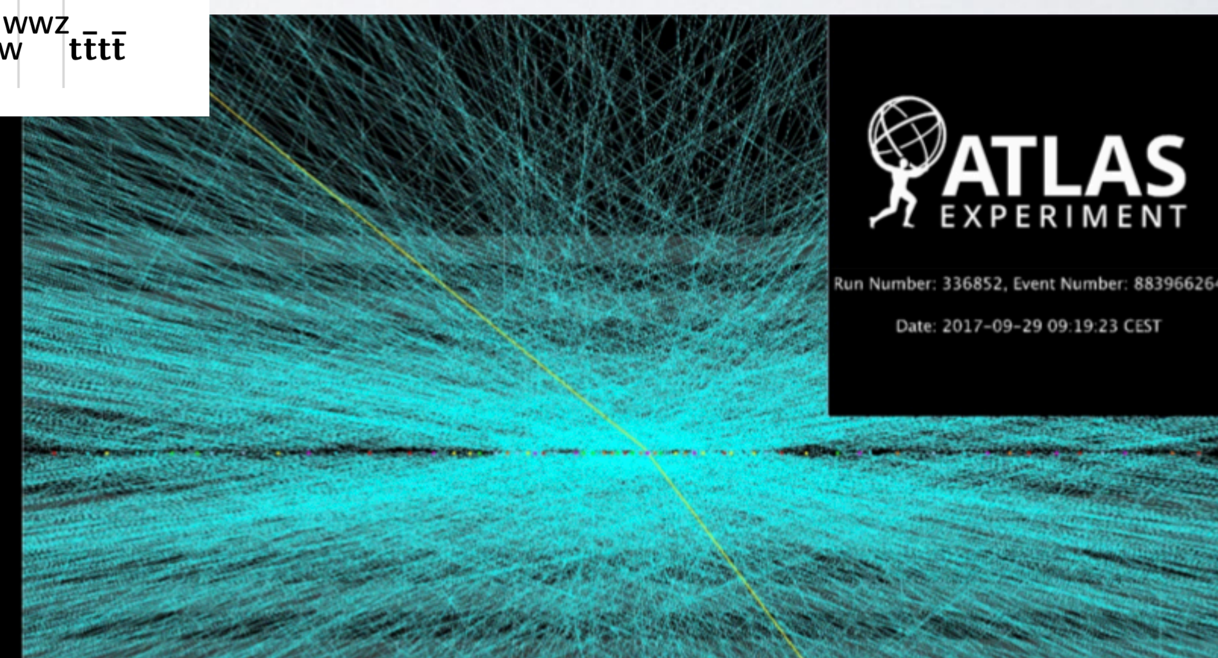


$$pp \to l^- l^+ + X, \quad l = e, \mu \quad \text{(NC)}$$

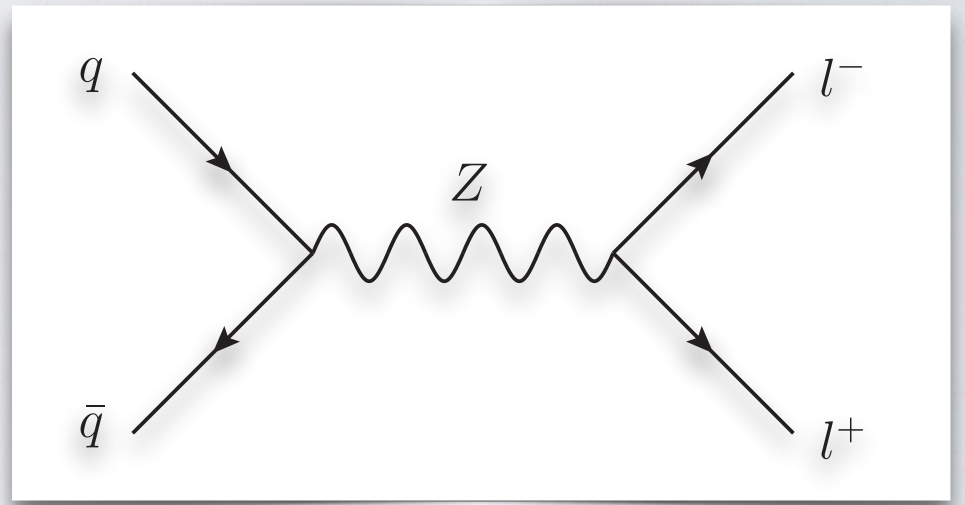
$$pp \to l^{\mp} \overset{(-)}{\nu_l} + X, \quad l = e, \mu \quad \text{(CC)}$$

- W/Z mass, $\sin\theta_W$, new resonances, PDFs
- “easy” signature (but see below)

Z ⟶ μμ candidate event, with 65 **additional** reconstructed primary vertices.



PERTURBATIVE CORRECTIONS



- LO: quark-pair initiated s-channel
- QCD on-shell: NNLO + W, γ^* N3LO
  [Hamberg, van Neerven, Matsuura '91, Harlander, Kilgore '02, Anastasiou, Dixon, Melnikov, Petriello '03, Melnikov, Petriello '06, Duhr, Dulat, Mistlberger '20,'20]
- QED on-shell NC: NNLO
  [Berends, van Neerven, Burgers '88, Blümlein, De Freitas, Raab, Schönwald '19]
- EW: NLO
  [Baur, Brein, Holllik, Schappacher, Wackeroth '01, Dittmaier, Krämer '01, Baur, Wackeroth '04], …
- Mixed QED-QCD:
  off-shell (no Z) *[Kilgore, Sturm '11];* on-shell Z *[de Florian, Der, Fabre '18, Delto, Jaquier, Melnikov, Röntsch '19]*
- Partial mixed EW-QCD in resonance region:
  [Dittmaier, Huss, Schwinn '14,'15, Carloni Calame et al '16]

EW-QCD ON-SHELL Z/W

- N_f for on-shell Z/W, complex mass *[Dittmaier, Schmidt, Schwarz '20]*
- Total Z+X cross section *[Bonciani, Buccioni, Rana, Vicini '20]*

talk N. Rana (Wed)

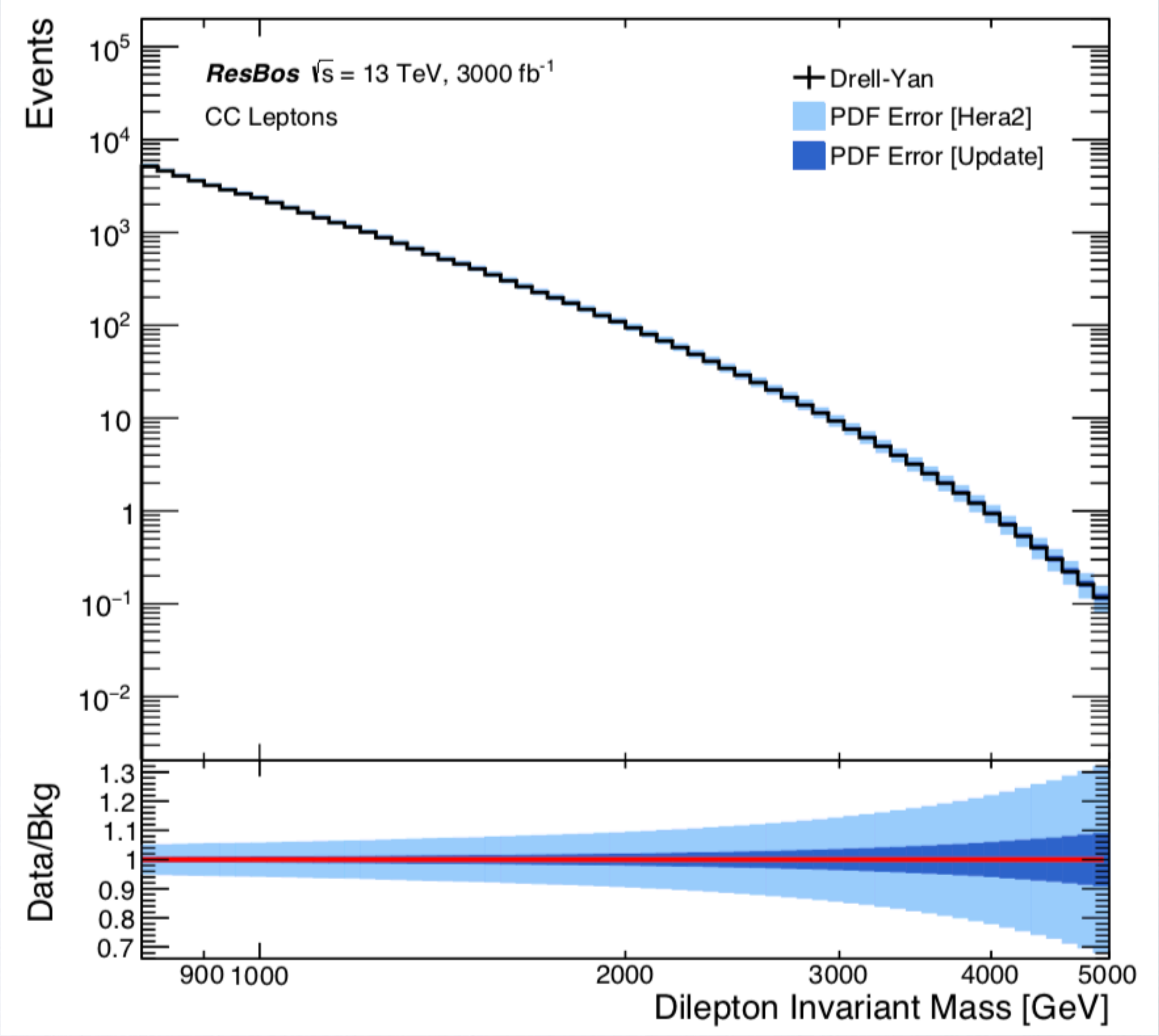
- Differential W+X cross section (nested subtraction) *[Behring, Buccioni, Caola, Delta, Jaquier, Melnikov, Röntsch '20, '21]*

talk A. Behring (Mon)

- α and $\alpha\alpha_s$ can be of similar order
- Cancellations between partonic channels (also for N3LO)

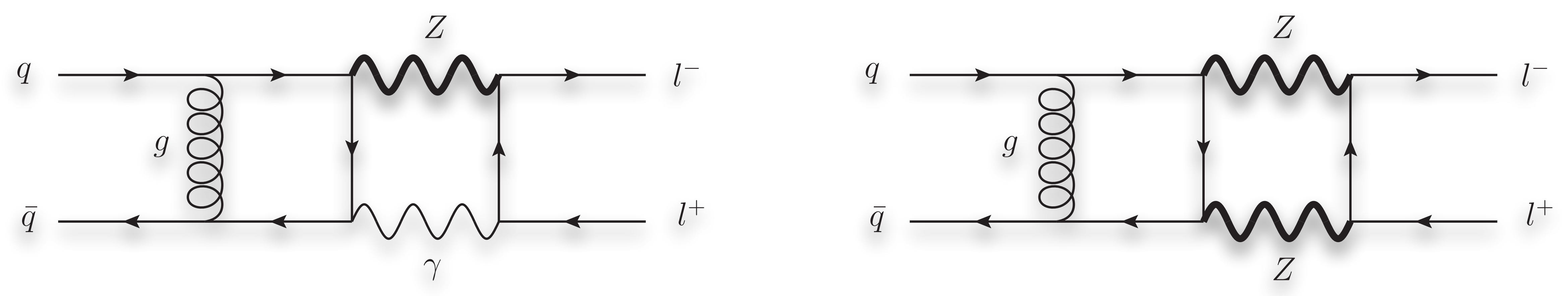
HIGH INVARIANT MASSES

- High invariant mass region important for new physics
- Uncertainties: EW + PDF
- PDF:
  use control region < 1 TeV
  to improve multi-TeV region
  [Willis, Brock, Hayden, Hou, Isaacson, Schmidt, Yuan '19]

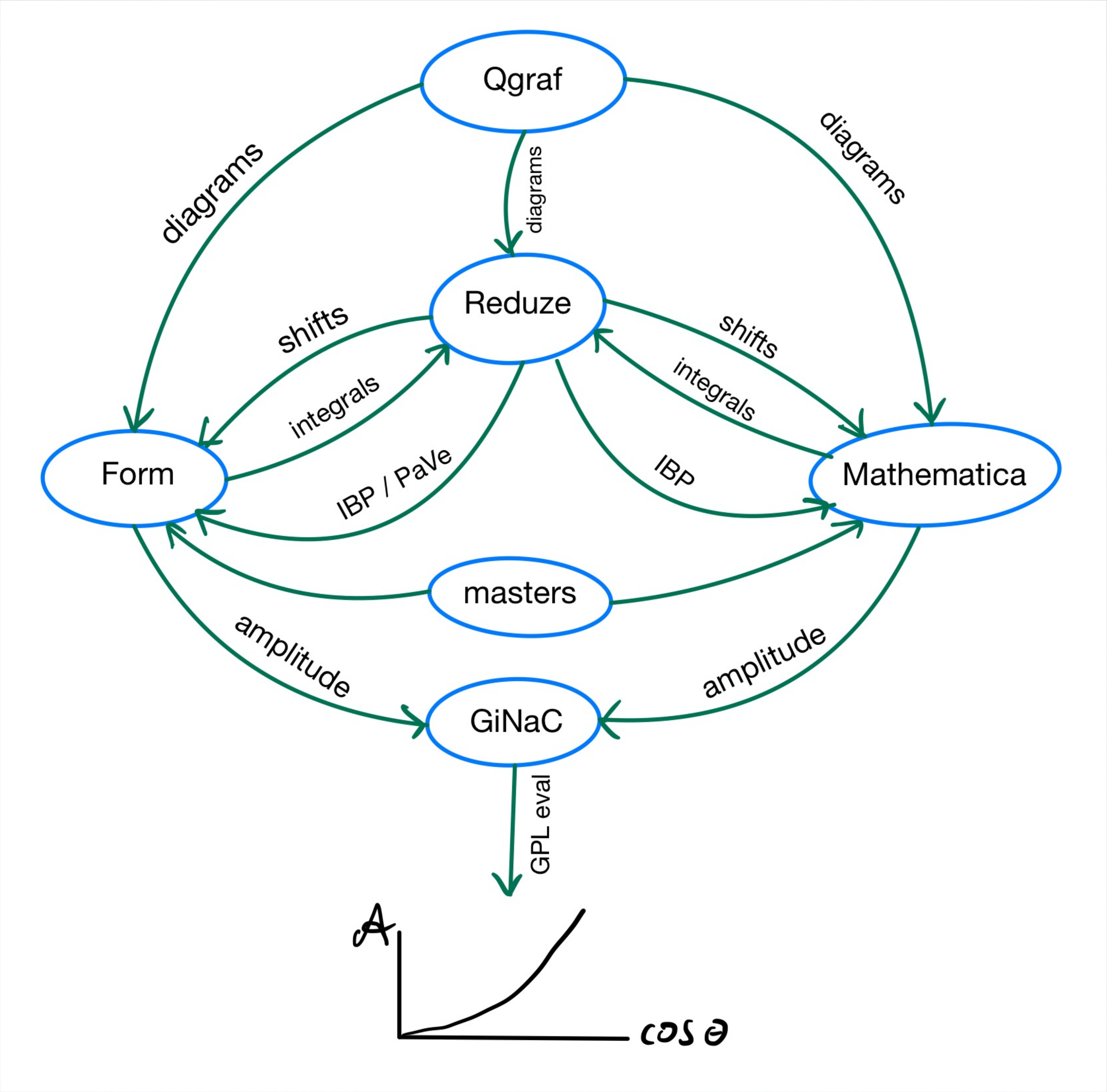


EW-QCD AT HIGH INVARIANT MASS

- Consider dilepton final state instead of V production
- $\mathcal{O}(\alpha\alpha_s)$ corrections O(-20%) for $l\nu$ at p_T=500 GeV
  [Buonocore, Grazzini, Kallweit, Savoini, Tramontano '21]
  talk L. Buonocore (Thu)
- Missing: two-loop virtuals CC/NC
- ***This talk: l^+l^- two-loop amplitudes***



TOOLS



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γ_5 AND D DIMENSIONS

- Problem: γ_5 really a ***4-dimensional*** object
  $\mathrm{tr}\,\{\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5\} = -4i\,\varepsilon_{\mu\nu\rho\sigma}$

- What to do in ***dimensional regularization*** ?

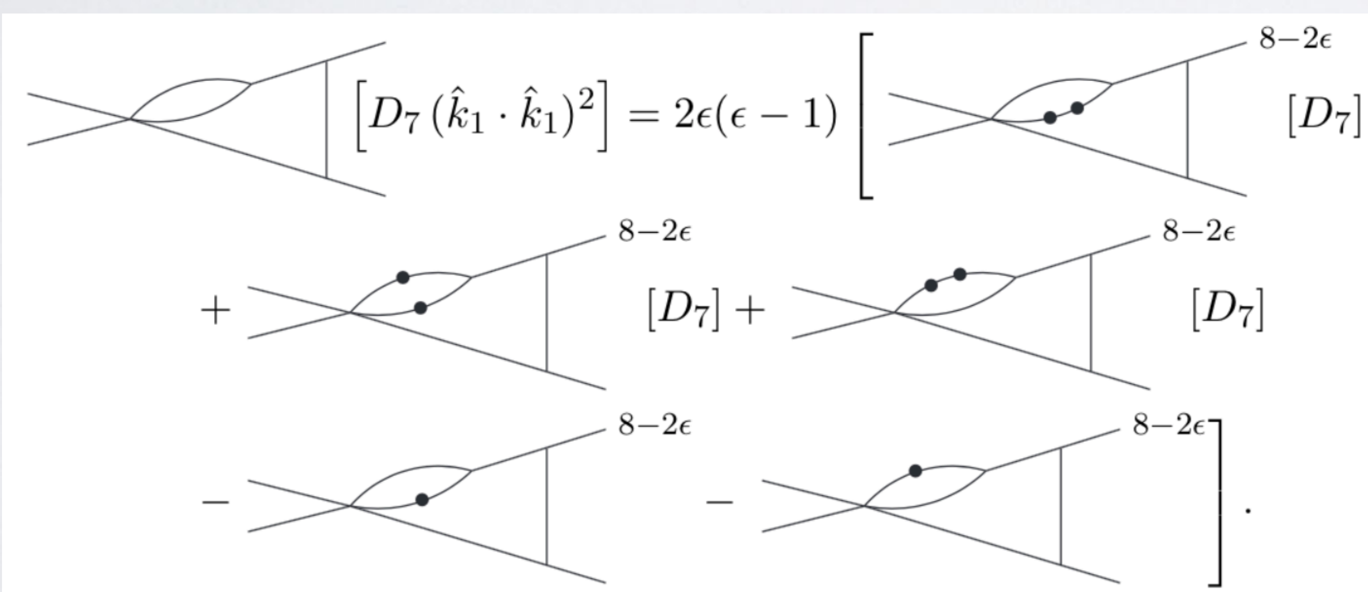
- ***Split*** d dimensional space → 4 dim (bar) + ϵ dim (hat)
  $k^\mu = \bar{k}^\mu + \hat{k}^\mu$
  $\gamma^\mu = \bar{\gamma}^\mu + \hat{\gamma}^\mu$, keep $\epsilon_{\mu\nu\rho\sigma}$ 4-dimensional: $\varepsilon^{\mu\nu\rho}{}_\alpha\,\varepsilon_{\mu\nu\rho\beta} = -6\,\bar{g}_{\alpha\beta}$ etc

- Option 1: *'t Hooft, Veltman ['72], Breitenlohner, Maison ['77]* (HVBM)
  $\{\bar{\gamma}_\mu, \gamma_5\} = 0$
  $[\hat{\gamma}_\mu, \gamma_5] = 0$
  give up ***anti-commutativity*** (violates Ward identities)

- Option 2: *Kreimer ['90]*
  $\{\gamma_\mu, \gamma_5\} = 0$
  give up ***cyclicity*** of Dirac trace (requires reading point)

SETUP AND μ TERMS

- ***3 different setups*** for our calculation:
  1. HVBM scheme + projectors + mu-terms
  2. Kreimer's scheme + projectors + mu-terms
  3. Kreimer's scheme + tensor reduction of integrals / spin-summed interference
- HVBM + Kreimer: short Dirac chains
- HVBM: split indices before Dirac trace, Kreimer: no split for Dirac trace
- Tensor integrals with ϵ ***dim loop momenta*** (μ ***terms***) from parametric representation,



+ standard reduction techniques

SYMMETRY RESTORATION

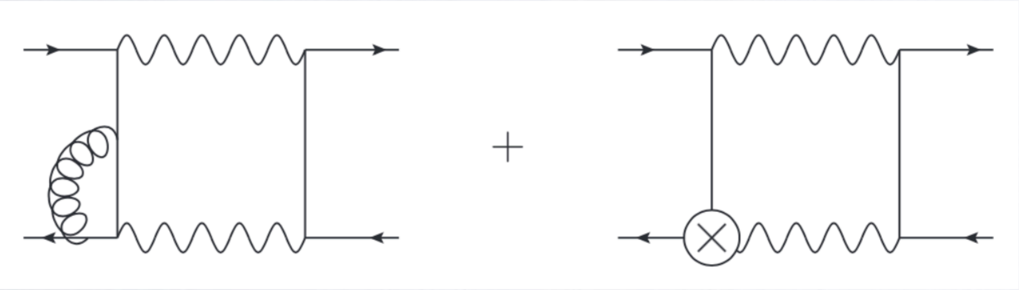
- In HVBM, corrections to ***vector and axial-vector currents*** differ
- Restore ***Ward identities*** by adding counter terms
- For vertex, ***require***

$$\bar{\mathcal{A}}^{(0,1)}_{Z\bar{q}q}(s) = -\frac{a_q}{v_q}\,\bar{\mathcal{V}}^{(0,1)}_{Z\bar{q}q}(s)$$

- Implement by adding ***counter terms***

$$\begin{aligned}\delta Z^{(0,1)}_{Z\bar{q}q} &= \bar{\mathcal{A}}^{(0,1)}_{Z\bar{q}q}(s) - \mathcal{A}^{(0,1)}_{Z\bar{q}q}(s)\\ &= 2\,a_q\frac{(2-\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)e^{\gamma_E\epsilon}}{(1-\epsilon)\Gamma(2-2\epsilon)}C_F e^{i\pi\epsilon}\left(\frac{\mu^2}{s}\right)^{\epsilon}\end{aligned}$$

- Remark 1: need also symmetry restoring counter terms in ***boxes***



- Remark 2: we keep also the ***higher order ϵ terms*** for the counter term

FORM FACTORS

- Computation of projectors to extract form factors, start in d dimensions:

$$i\mathcal{A}_{\rm DY} = i\sum_{\alpha=1}^{16} \mathbf{C}_\alpha T_\alpha, \qquad M_{\alpha\beta} = \sum_{\rm spin,color} T_\alpha^\dagger T_\beta \qquad \mathbf{C}_\alpha = \sum_{\rm spin,color} \mathcal{P}_\alpha i\mathcal{A}_{\rm DY} \qquad \mathcal{P}_\alpha = -i\sum_{\beta=1}^{16} M_{\alpha\beta}^{-1} T_\beta^\dagger$$

- Out of 16, only ***4 tensors independent*** in $d = 4$, equal to number of ***helicity amplitudes***.
- How to ignore remaining 12 tensors ? Note: M^{-1} ***diverges*** for $d \to 4$!
- ***Change basis*** of tensors

$$T_1' = T_1, \qquad T_2' = T_2, \qquad T_\alpha' = T_\alpha + \sum_{\beta=1}^{2} R_{\alpha\beta} T_\beta \quad \text{for } \alpha = 3\ldots 8$$

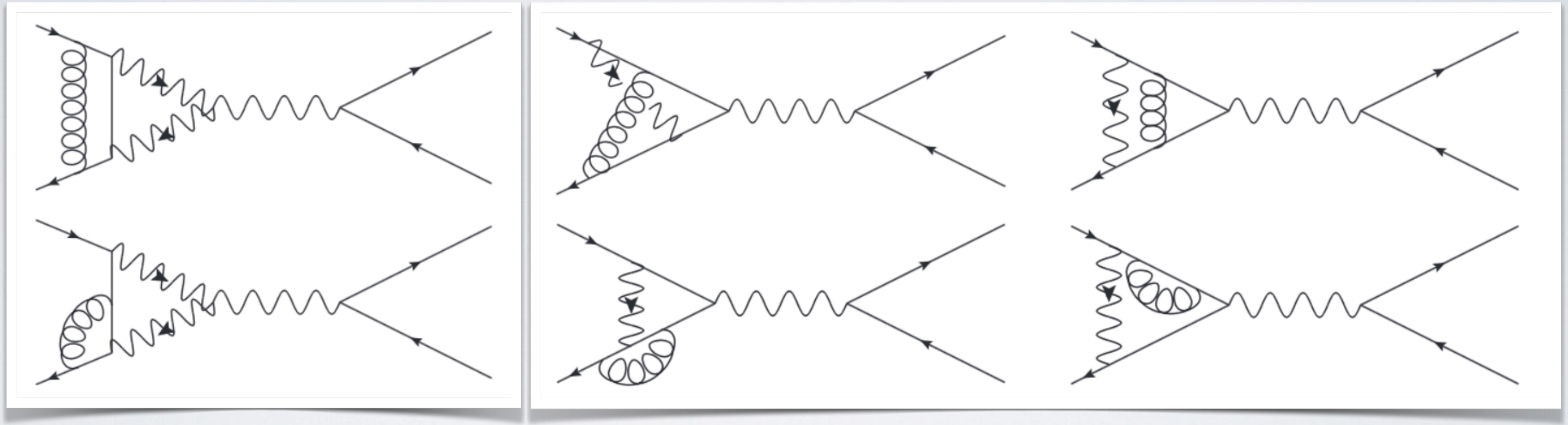
  such that irrelevant directions ***decouple*** exactly in d dimensions *[Peraro, Tancredi '19, '20]*

$$(M_{\alpha\beta}') = \sum_{\rm spin,color} (T_\alpha'^\dagger T_\beta') = (RMR^\dagger) = \begin{pmatrix} M_{2\times2}' & 0 \\ 0 & M_{6\times6}' \end{pmatrix}$$

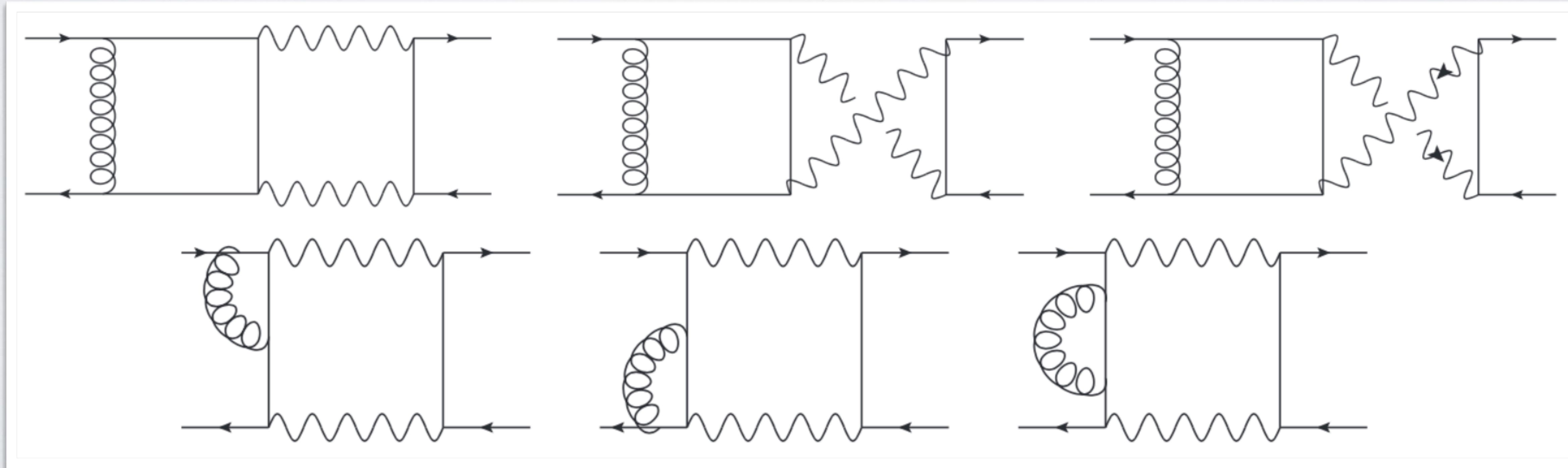
- M_{2x2}^{-1} is ***regular*** for $d \to 4$, irrelevant directions contribute only at order $d-4$!
- **Result:** exact d dim. projectors for relevant form factors and subtraction terms (γ_5 scheme dep.), irrelevant ones not needed for finite remainder
- *[see also: Chen '19, Ahmed et al '19]*

FEYNMAN DIAGRAMS

Examples for vertex corrections at two loops:

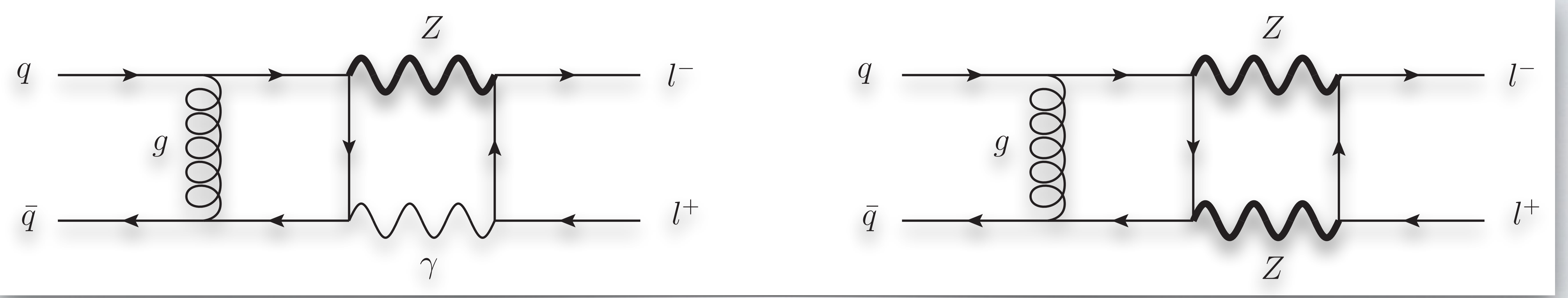


Examples for box corrections at two-loops:



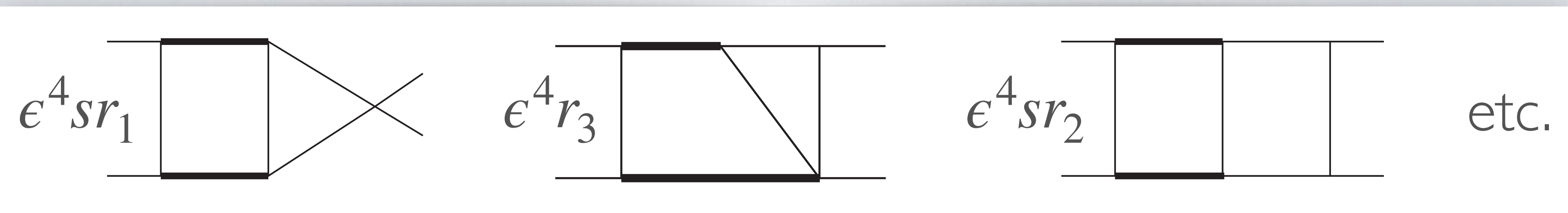
MASTER INTEGRALS

- ***Feynman diagrams*** with one and two masses:



- Master integrals:
  - 1-fold integral over polylogarithms (Euclidean region)
    [Bonciani, Di Vita, Mastrolia, Schubert '16]
  - Multiple polylogarithms (physical region)
    [AvM, Schabinger '17, Heller, AvM, Schabinger '19]
  - $\log(m_e)$ terms for single Z exchange boxes
    [Hasan, Schubert '20]

NON RATIONALIZABLE ROOTS



- Use ***differential equations***, integrals allow for ϵ basis *[Henn '13]*

$$d\vec{m} = \epsilon \, \text{dlog}(l_a) \, \hat{A}^{(a)} \, \vec{m}$$

- Leading singularities introduce 3 roots into the letters:

$$r_1 = \sqrt{s(s-4m^2)}, \;\; r_2 = \sqrt{-st\left(4m^2(t+m^2)-st\right)}, \;\; r_3 = \sqrt{s\left(t^2(s-4m^2)+sm^2(m^2-2t)\right)}$$

- Reparametrization $s = -m^2(1-w)^2/w, \;\; t = -m^2 w(1+z)^2/(z(1+w)^2)$ rationalizes 2 out of 3 roots,

$$r = \sqrt{4(1-w)^2 wz^2 + (w+z)^2(1+wz)^2}$$

is ***not rationalizable*** *[van Straten '14, Besier, Festi, Harrison, Naskrecki '19]*

- Integrals with explicit square roots in terms of Chen-iterated integrals, polylogs, … *[Caron-Huot, Henn '14, Ablinger, Blümlein, Raab, Schneider '14, Papadopoulos, Tommasini, Wever '15, Bonciani et al '16, AvM, Tancredi '17, Abreu et al '20, Chicherin, Sotnikov '20, Syrrakos '20, Kreel, Weinzierl '21] and many more*

DIFFERENTIAL EQUATION

- ***Given a dlog form with non-rationalizable roots***

  $d\vec{m} = \epsilon\, \text{dlog}(l_a)\, \hat{A}^{(a)}\, \vec{m}$

  e.g. $l_{13} = -(1-w)(z-w)(1-w\,z) + (1+w)\sqrt{4(1-w)^2 wz^2 + (w+z)^2(1+wz)^2}$

- ***Question: integrable via multiple polylogarithms w/o spurious letters ?***
  - Can't integrate in terms of GPLs in filtration basis
  - Match against ansatz ? *[Duhr, Gangl, Rhodes '11]*
  - Not all dlog forms describe multiple polylogs ! *[Brown, Duhr '20]*
  - $r = \sqrt{4(1-w)^2 wz^2 + (w+z)^2(1+wz)^2}$ elliptic curve ?

AMBIGUITIES IN LETTERS

talk B. Page (Tue)

- ***Rational letters:***

$$\mathcal{L}_R = \{1-w, -w, 1+w, 1-w+w^2, 1-z, -z, 1+z, \\ 1-wz, 1+w^2 z, -z-w^2, z-w\}$$

- ***Initial algebraic letters:***

$$\begin{aligned}\mathcal{L}_A = \{r, &-(1-w)(z-w)(1-wz)+r(1+w), \\ &-(1-w)\big(4wz+(w+z)(1+wz)\big)-r(1+w), \\ &r^2-2wz^2(1-w)^2+r(w+z)(1+wz), \\ &r^2(1-z)^2+2z^2(z+w^2)(1+w^2z)+r(1-z)(1+z)\big(2wz-(w+z)(1+wz)\big)\end{aligned}$$

$$r = \sqrt{4(1-w)^2 wz^2 + (w+z)^2(1+wz)^2}$$

combinatorial complexity prevents construction of solution based on these letters

- ***New algorithm to construct improved algebraic letters:***

$$\mathcal{L}_{\tilde{A}} = \Big\{r, \tfrac{1}{2}\big(2+z-w+wz(w+z)+r\big), \tfrac{1}{2}\big(2w^2+z-w+wz(w+z)+r\big), \\ \tfrac{1}{2}\big(-(w+z)(1-wz)+r\big), \tfrac{1}{2}\big(-(z-w)(1+wz)+r\big)\Big\}$$

SUCCESS THROUGH SIMPLICITY

- ***Algorithm*** to construct improved letters:
    - Make an ansatz $l = x + \sqrt{\ }$ and require $l\bar{l}$ factorizes over the rational part of the alphabet.
    - Pick simple letters. Make $\sqrt{\ }$ a letter. For several roots, consider also products of roots.
- ***Factorizations*** either using polynomial rings or (cheaper) using numerical sample and integer relation algorithm:
    - $g = l_1^{a_1} l_2^{a_2} \cdots$ implies integer relation between logs:
      $\log(g) - a_1 \log(l_1) - a_2 \log(l_2) - \ldots = 0$
- ***Construct*** multiple polylogs w/o spurious letters, **successfully match deq !**

ANALYTIC RESULT

- Solve diff. eqs. by ***expansions***, fit precise numerics to transport boundary constants analytically *[Lee, Smirnov, Smirnov '18, Heller, AvM, Schabinger '19, Moriello '19, …, Hidding '20, …]*

talk M. Hidding (Tue)

- ***Analytic result in terms of standard multiple polylogs possible*** despite provably non-rationalizable roots:

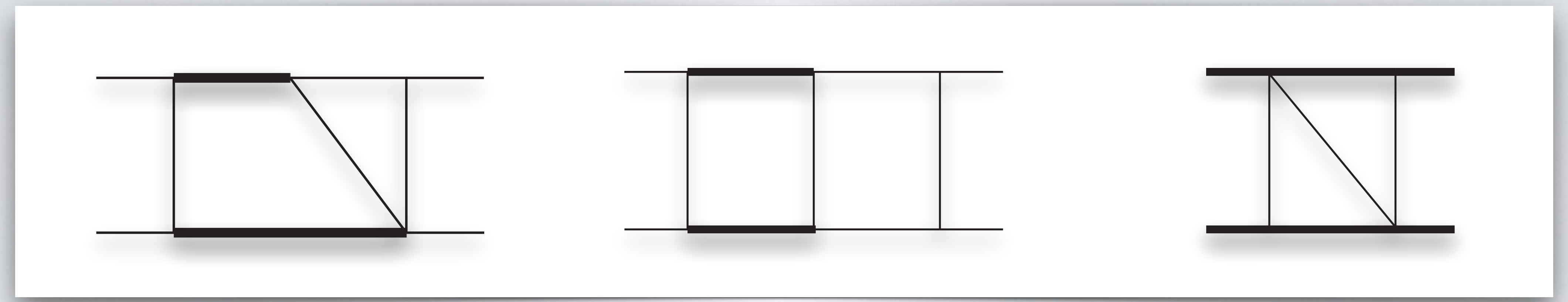
$$m_{32} = \epsilon^3\left[4\,\mathrm{Li}_3\left(\frac{l_1 l_2 l_6 l_7 l_{10} l_{13}}{l_{14} l_{15} l_{16}}\right) - 2\,\mathrm{Li}_3\left(\frac{l_2^3 l_6 l_7^2}{l_{15} l_{16}}\right) + \ldots + 4\,\mathrm{Li}_2\left(\frac{l_6 l_{14} l_{16}}{l_7 l_9 l_{15}}\right)\ln(l_3) + \ldots\right]$$
$$+\epsilon^4\left[-\mathrm{Li}_{2,2}\left(-\frac{l_1^2 l_3 l_{15}}{l_2^2 l_7 l_{14}}, \frac{l_2^2 l_7 l_{15}}{l_1 l_3 l_6 l_1 4}\right) + \ldots + \frac{701}{4}\,\mathrm{Li}_4\left(\frac{l_1 l_3^2 l_6^2 l_9 l_{14}}{l_2 l_7 l_{13} l_{15} l_{16}}\right) + \ldots\right] + O(\epsilon^5)$$

[Heller, AvM, Schabinger 2019]

- No spurious letters, simple constants

- $\mathcal{O}(1s)$ **all integrals** for generic point, double precision
  [Vollinga, Weinzierl '04]

MATHEMATICAL SCOPE



- We also considered DY integrals and planar Bhabha *[Henn, Smirnov ’13]* integrals in ***direct integration*** approach *[Brown ‘08, Panzer ’14]*

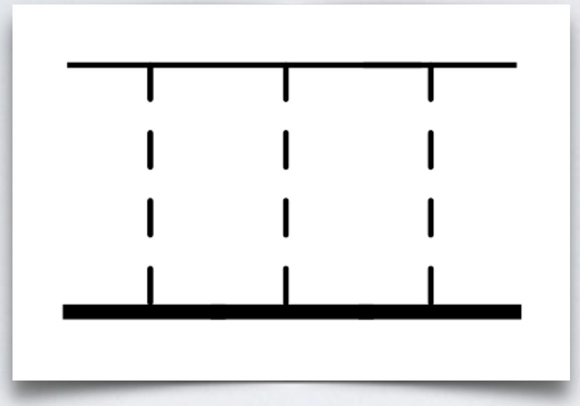
- Found ***variable changes*** to prove polylogs to ***all orders in*** ϵ *[Heller, AvM, Schabinger 2019]*

- Obtained explicit results, but rather useless in practice

- DY and Bhabha K3s not isomorphic *[Besier, Festi, Harrison, Naskrecki ’19]*

talk V. Smirnov (Tue)

$e\mu$ SCATTERING: 4 ROOTS



[Heller '21]

talk S. Weinzierl (Thu)

- Leading singularities introduce four roots [$x, y = (m_1^2 \pm m_2^2)/(2s)$, $z = t/s$]

$$r_1 = \sqrt{1-4x+4y^2}, \quad r_2 = \sqrt{-4xz+4yz+z^2}, \quad r_3 = \sqrt{-4xz-4yz+z^2},$$
$$r_4 = \sqrt{1-4x+4y^2+2z-4xz+z^2}.$$

- ϵ dlog basis, initially 30 complicated letters, deg 4 in x,y,z
- Algorithm gives 20 simplified letters (no rationalization at all)

$$\mathcal{L}_R = \left\{x-y, x+y, -1+4x-4y^2-z, z\right\}$$

$$\mathcal{L}_A = \Big\{r_1, r_2, r_3, r_4, \frac{1}{2}(-1+2y+r_1), \frac{1}{2}(1+2y+r_1), \frac{1}{2}(r_2+z), \frac{1}{2}(r_3+z),$$
$$\frac{1}{2}(-1+2x-z+r_4), \frac{1}{2}(r_2+r_1+r_4), \frac{1}{2}(r_3+r_1+r_4), \frac{1}{2}(r_2-r_1-r_4),$$
$$\frac{1}{2}(r_3-r_1-r_4), \frac{1}{2}(r_1-r_3-r_4), \frac{1}{2}(r_1-r_2-r_4), \frac{1}{2}(z(-2+4x-z)+r_2r_3)\Big\}$$

- Construct G functions of power products of letters, obtain analytic solution

BACK TO DY: RATIONAL COEFFICIENTS

multivariateapart.nb

Univariate partial fractions separate terms with different poles:

In[1]:= Apart$\left[\frac{1}{x(1+x)}, x\right]$

Out[1]= $\frac{1}{x} - \frac{1}{1+x}$

talks B. Hartanto, H. Chawdhry, B. Agarwal, F. Buccioni (Tue)

Let's consider a multivariate example:

In[2]:= multi = $\frac{2y - x}{y(x+y)(y-x)}$;

Naive iteration introduces spurious poles (here 1/x) for multivariate case:

In[3]:= Apart[multi, y]

Out[3]= $\frac{1}{xy} + \frac{1}{2x(-x+y)} - \frac{3}{2x(x+y)}$

Solution: multivariate partial fractions using methods from polynomial ideal theory:

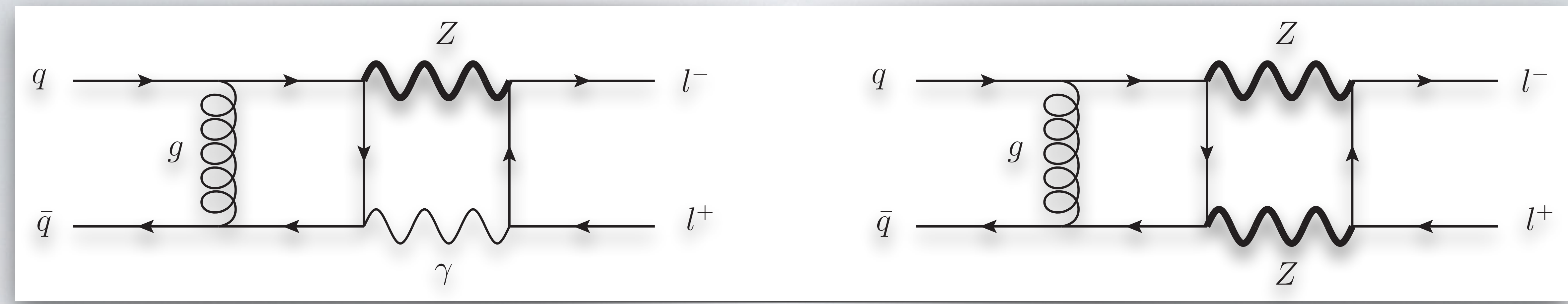
```
In[4]:= << MultivariateApart`
MultivariateApart -- Multivariate partial fractions. By Matthias Heller and Andreas von Manteuffel.
In[5]:= MultivariateApart[multi]
```

Out[5]= $-\frac{1}{2(x-y)y} + \frac{3}{2y(x+y)}$

Note: minimize denominator degrees ($\neq$ Leinartas)

[Pak '11, Abreu et al '19, Boehm et al '20, Heller et al '21, Bendle et al '21]

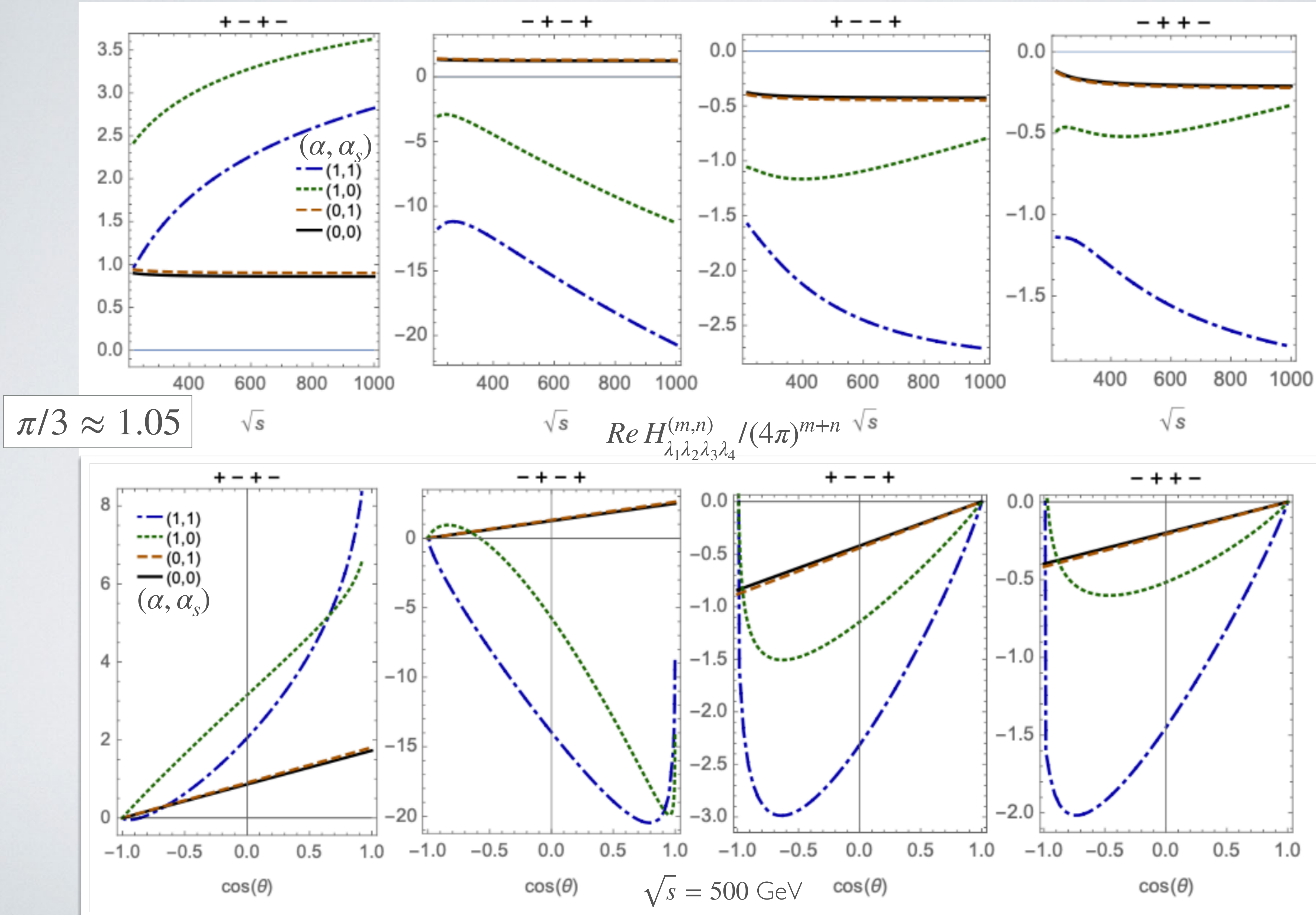
RESULTS FOR AMPLITUDES



- Calculated $O(\alpha_s)$, $O(\alpha)$, $O(\alpha_s\alpha)$ helicity amplitudes
- Analytic UV and IR cancellation
- γ^5 ***scheme dependence*** (after adding symmetry restoring counter terms):
  - $\mathcal{O}(\epsilon^1)$, $\mathcal{O}(\epsilon^2)$ one-loop remainders and $\mathcal{O}(1/\epsilon)$, $\mathcal{O}(\epsilon^0)$ bare two-loop: **scheme and reading point dependent**
  - Finite remainders: **coincide between schemes**

HELICITY AMPLITUDES

[Heller, AvM, Schabinger, Spiesberger 2020]



CONCLUSIONS

- ***$\mathcal{O}(\alpha\alpha_s)$ corrections to l^+l^- production at high energies:***
  - ✓ *analytic two-loop amplitudes*
- ***Algorithm for multiple polylogs of algebraic arguments:***
  - ✓ *Monte-Carlo friendly master integrals*
- ***New MultivariateApart decomposition:***
  - ✓ *optimized denominator powers*
- ***HVBM and Kreimer's γ_5 schemes:***
  - ✓ *viable options for two-loop EW boxes*