IR-Improved Amplitude-Based Resummation in Quantum Field Theory: New Results and New Issues

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5/21/21

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OUTLINE

- Introduction
- Review of Exact Amplitude-Based Resummation Theory
- Precision LHC Physics: New Results and New Issues
- Precision FCC Physics: New Results and New Issues
- Quantum Gravity: New Results and New Issues
- Summary

Introduction

- WHAT IS RESUMMATION(IR,UV,CL)?
  - FAMILIAR SUMMATION: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
  - RESUMMATION:

$$\sum_{n=0}^{\infty} C_n\alpha_s^n \begin{cases} = F_{\mathrm{RES}}(\alpha_s)\sum_{n=0}^{\infty} B_n\alpha_s^n, \text{ EXACT , YFS} \\ \cong G_{\mathrm{RES}}(\alpha_s)\sum_{n=0}^{\mathrm{N}} B'_n\alpha_s^n, \text{ APPROX , J-S} \end{cases}$$

APPROX : THE ALGORITHM FOR $\mathrm{B'_n(G_{RES})}$ TO ALL ORDERS IS UNKNOWN.

Introduction

- 1989: LEP DATA TAKING THAT LED, BY PRECISION PHYSICS, TO THE ‘t HOOFT-VELTMAN (1999) EW AND GROSS-WILCZEK-POLITZER (2004) QCD NOBEL PRIZES IN PHYSICS -- ALREADY IN ICHEP 1988 A KEY QUESTION,

  ’How Accurate Can Exponentiation (RESUMMATION) Really Be?’
- Would It Limit or Enhance Exactness: LO, NLO, NNLO, ... ?

Introduction

- As we indicated, the "two" classes of realizations are: Jackson-Scharre(JS)(APPROX) vs YFS (EXACT)
- JS → 'limit to precision', determined by N
- YFS → 'no limit to precision', algorithmically
- 1989 CERN Yellow Book article: Some were almost convinced, but not completely!
- Today, new paradigms, analogous discussions: precision LHC/FCC physics and quantum gravity

Introduction

- 54 YEARS of $SU_{2L} \times U_1$,S. Weinberg, PRL**19** (1967) 1264[#]; 48 YEARS of QCD, D.J. Gross and F. Wilczek, *ibid.***30** (1973) 1343,H.D. Politzer, *ibid.***30** (1973) 1346
(*SM@50*, B. Lynn *et al.*, Case Western, June, 2018) ⇒

SM@50

The Standard Model at 50 Years:

a celebratory symposium
will take place in the

Physics Department
Case Western Reserve University
Cleveland, Ohio, June 1-4, 2018.
For more information, email SMat50@case.edu

see also PASCOS2018, taking place immediately following SM@50

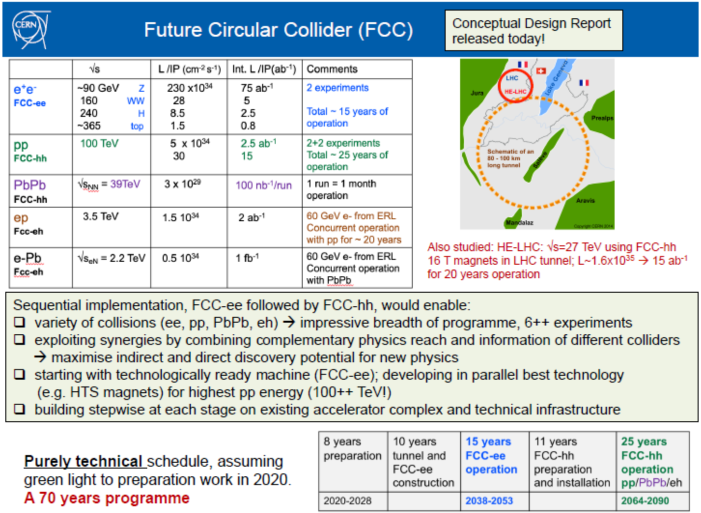
Speakers

Steven Adler
James "BJ" Bjorken
Alain Blondel
Norman Christ
Savas Dimopoulos
Henriette Elvang
Jerome Friedman
Mary K. Gaillard
David Gross
Gerard 't Hooft
Takaaki Kajita
Bryan W. Lynn
Pavel Fileviez Perez
Michael Peskin
Hellen Quinn
Carlo Rubbia
Jurgen Schukraft
George Smoot
Glenn Starkman
Cyrus Taylor
Samuel Ting
Bennie F.L. Ward
Steven Weinberg
Mark Wise
Sau Lan Wu

Must Keep Historical Perspective

[#] S.L. Glashow, NP 22(1961) 579; A. Salam, in 8th Nobel Sym, 1968, p.367.

Introduction



Future Circular Collider (FCC)

Conceptual Design Report released today!

	$\sqrt{s}$	L /IP ($cm^{-2} s^{-1}$)	Int. L /IP(ab^{-1})	Comments
e^+e^- **FCC-ee**	~90 GeV Z 160 WW 240 H ~365 top	230×10^{34} 28 8.5 1.5	75 ab^{-1} 5 2.5 0.8	2 experiments Total ~ 15 years of operation
pp **FCC-hh**	100 TeV	5×10^{34} 30	2.5 ab^{-1} 15	2+2 experiments Total ~ 25 years of operation
PbPb **FCC-hh**	$\sqrt{s_{NN}}$ = 39TeV	3×10^{29}	100 nb^{-1}/run	1 run = 1 month operation
ep **Fcc-eh**	3.5 TeV	1.5 10^{34}	2 ab^{-1}	60 GeV e- from ERL Concurrent operation with pp for ~ 20 years
e-Pb **Fcc-eh**	$\sqrt{s_{eN}}$ = 2.2 TeV	0.5 10^{34}	1 fb^{-1}	60 GeV e- from ERL Concurrent operation with PbPb

Also studied: HE-LHC: $\sqrt{s}$=27 TeV using FCC-hh 16 T magnets in LHC tunnel; L~1.6×10^{35} → 15 ab^{-1} for 20 years operation

Sequential implementation, FCC-ee followed by FCC-hh, would enable:

- ☐ variety of collisions (ee, pp, PbPb, eh) → impressive breadth of programme, 6++ experiments
- ☐ exploiting synergies by combining complementary physics reach and information of different colliders → maximise indirect and direct discovery potential for new physics
- ☐ starting with technologically ready machine (FCC-ee); developing in parallel best technology (e.g. HTS magnets) for highest pp energy (100++ TeV!)
- ☐ building stepwise at each stage on existing accelerator complex and technical infrastructure

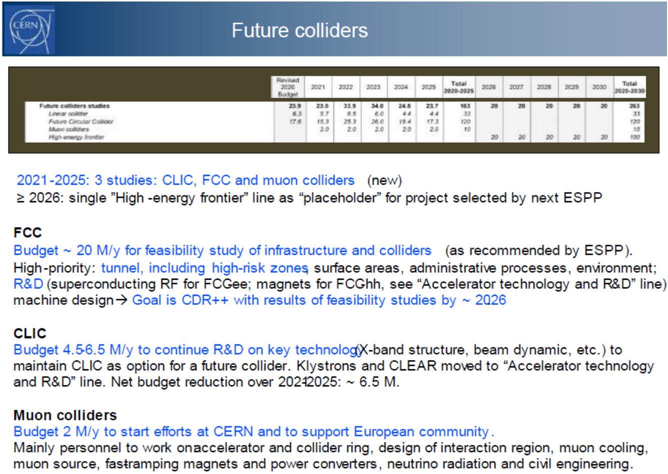
Purely technical schedule, assuming green light to preparation work in 2020.
A 70 years programme

8 years preparation	10 years tunnel and FCC-ee construction	**15 years FCC-ee operation**	11 years FCC-hh preparation and installation	**25 years FCC-hh operation pp/PbPb/eh**
2020-2028		**2038-2053**		**2064-2090**

Must keep historical perspective.

Introduction

- F. Gianotti, 10/8/20



Future colliders

	Revised 2020 Budget	2021	2022	2023	2024	2025	Total 2020-2025	2026	2027	2028	2029	2030	Total 2020-2030
Future colliders studies	**23.9**	**23.0**	**33.9**	**34.0**	**24.8**	**23.7**	**163**	**20**	**20**	**20**	**20**	**20**	**263**
Linear collider	*6.3*	*5.7*	*6.5*	*6.0*	*4.4*	*4.4*	*33*						*33*
Future Circular Collider	*17.6*	*15.3*	*25.3*	*26.0*	*18.4*	*17.3*	*120*						*120*
Muon colliders		*2.0*	*2.0*	*2.0*	*2.0*	*2.0*	*10*						*10*
High-energy frontier								*20*	*20*	*20*	*20*	*20*	*100*

2021-2025: 3 studies: CLIC, FCC and muon colliders (new)
≥ 2026: single "High-energy frontier" line as "placeholder" for project selected by next ESPP

FCC
Budget ~ 20 M/y for feasibility study of infrastructure and colliders (as recommended by ESPP).
High-priority: tunnel, including high-risk zones, surface areas, administrative processes, environment;
R&D (superconducting RF for FCGee; magnets for FCGhh, see "Accelerator technology and R&D" line)
machine design → Goal is CDR++ with results of feasibility studies by ~ 2026

CLIC
Budget 4.5-6.5 M/y to continue R&D on key technology (X-band structure, beam dynamic, etc.) to maintain CLIC as option for a future collider. Klystrons and CLEAR moved to "Accelerator technology and R&D" line. Net budget reduction over 2021-2025: ~ 6.5 M.

Muon colliders
Budget 2 M/y to start efforts at CERN and to support European community.
Mainly personnel to work on accelerator and collider ring, design of interaction region, muon cooling, muon source, fast-ramping magnets and power converters, neutrino radiation and civil engineering.

Must keep historical perspective.

Exact Amplitude-Based Resummation --Review

$$
\begin{aligned}
d\bar{\sigma}_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \\
\prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\
\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3 p_2}{p_2^0}\frac{d^3 q_2}{q_2^0},
\end{aligned} \qquad (1)
$$

where *new* (YFS-style) *non-Abelian* residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons.

$QCD \otimes QED$ $\mathcal{EXACT}$ Resummation Theory

Here,

$$\begin{aligned} \mathrm{SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED} \\ D_{\mathrm{QCED}} &= \int \frac{d^3k}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right) \tilde{S}^{nls}_{QCED} \end{aligned} \tag{2}$$

where K_{max} is “dummy” and

$$\begin{aligned} B^{nls}_{QCED} &\equiv B^{nls}_{QCD} + \frac{\alpha}{\alpha_s} B^{nls}_{QED}, \\ \tilde{B}^{nls}_{QCED} &\equiv \tilde{B}^{nls}_{QCD} + \frac{\alpha}{\alpha_s} \tilde{B}^{nls}_{QED}, \\ \tilde{S}^{nls}_{QCED} &\equiv \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED}. \end{aligned} \tag{3}$$

“nls”≡ DGLAP-CS synthesization.

Shower/ME Matching: $\tilde{\bar{\beta}}_{n,m} \to \hat{\tilde{\bar{\beta}}}_{n,m}$

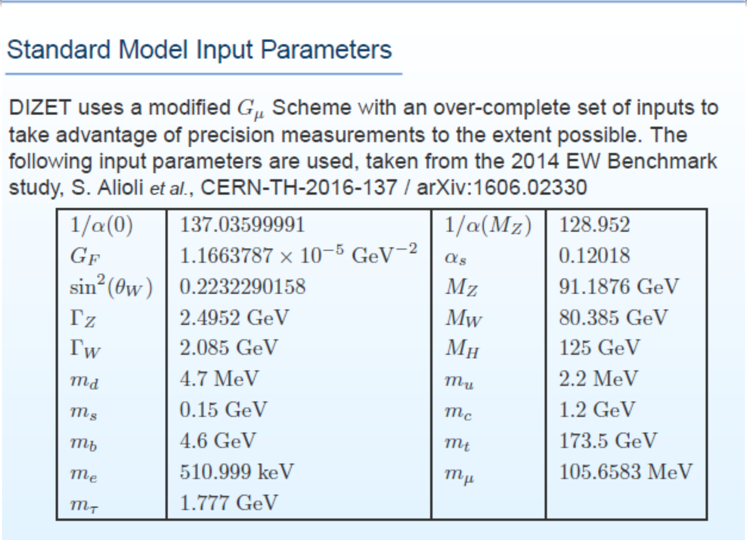
Precision LHC Physics: New Results and New Issues

- IR-Improved DGLAP-CS Theory: Herwiri1.031

  Interfaced to MC@NLO and MG5_aMC@NLO:

  Z and W+jets Production, ...
- *KK*MC-hh: Exact *O*(α^2L) CEEX EW Corrections Interfaced to Herwig6.5 and Herwiri1.031--new, interfaced to MG5_aMC@NLO
- In Z and W+ jets Production, IR-Improvement gives a comparable or better data fit without ad hoc parameters
- In KKMC-hh, IR-improvement allows to quantify role of ISR in precision predictions for Z production observables, as we now illustrate.

Precision LHC Physics: New Results and New Issues

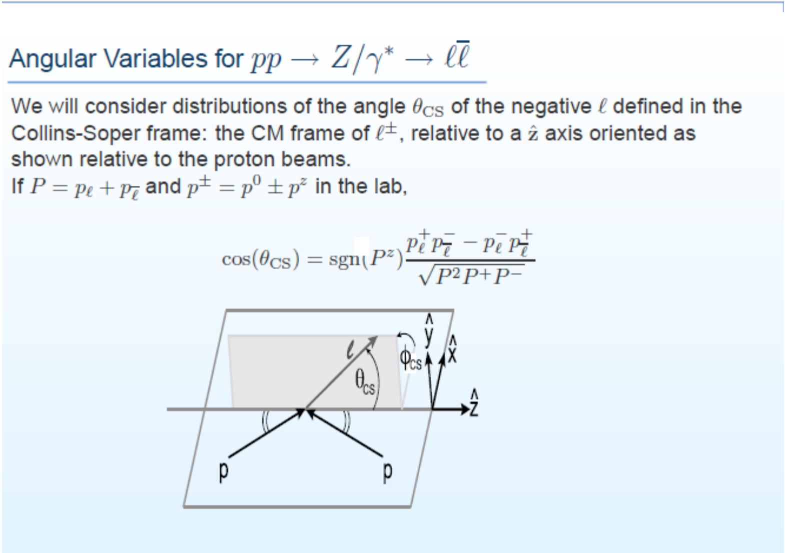


Standard Model Input Parameters

DIZET uses a modified G_μ Scheme with an over-complete set of inputs to take advantage of precision measurements to the extent possible. The following input parameters are used, taken from the 2014 EW Benchmark study, S. Alioli *et al.*, CERN-TH-2016-137 / arXiv:1606.02330

$1/\alpha(0)$	137.03599991	$1/\alpha(M_Z)$	128.952
G_F	1.1663787×10^{-5} GeV^{-2}	α_s	0.12018
$\sin^2(\theta_W)$	0.2232290158	M_Z	91.1876 GeV
Γ_Z	2.4952 GeV	M_W	80.385 GeV
Γ_W	2.085 GeV	M_H	125 GeV
m_d	4.7 MeV	m_u	2.2 MeV
m_s	0.15 GeV	m_c	1.2 GeV
m_b	4.6 GeV	m_t	173.5 GeV
m_e	510.999 keV	m_μ	105.6583 MeV
m_τ	1.777 GeV		

Precision LHC Physics: New Results and New Issues



Angular Variables for $pp \to Z/\gamma^* \to \ell\bar{\ell}$

We will consider distributions of the angle $\theta_{\rm CS}$ of the negative ℓ defined in the Collins-Soper frame: the CM frame of $\ell^\pm$, relative to a $\hat{z}$ axis oriented as shown relative to the proton beams.

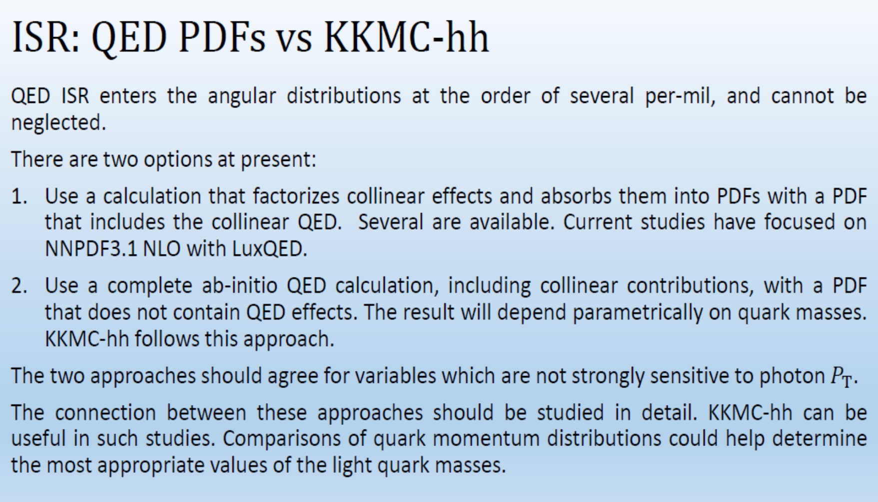
If $P = p_\ell + p_{\bar{\ell}}$ and $p^\pm = p^0 \pm p^z$ in the lab,

$$\cos(\theta_{\rm CS}) = {\rm sgn}(P^z)\frac{p_\ell^+ p_{\bar{\ell}}^- - p_\ell^- p_{\bar{\ell}}^+}{\sqrt{P^2 P^+ P^-}}$$

A_{FB}, A_4 --> $\sin^2\theta_W$

Interplay of (IR-Improved) DGLAP-CS QCD Theory and Exact $\mathcal{O}(\alpha^2 L)$ CEEX EW Corrections

- Consider recent ATLAS measurement of the angular coefficients in Z-boson events at 8 TeV, arXiv: 1606.00689
- Z/γ^* data with electron and muon pairs used; EW treated as 'small'



ISR: QED PDFs vs KKMC-hh

QED ISR enters the angular distributions at the order of several per-mil, and cannot be neglected.

There are two options at present:

1. Use a calculation that factorizes collinear effects and absorbs them into PDFs with a PDF that includes the collinear QED. Several are available. Current studies have focused on NNPDF3.1 NLO with LuxQED.
2. Use a complete ab-initio QED calculation, including collinear contributions, with a PDF that does not contain QED effects. The result will depend parametrically on quark masses. KKMC-hh follows this approach.

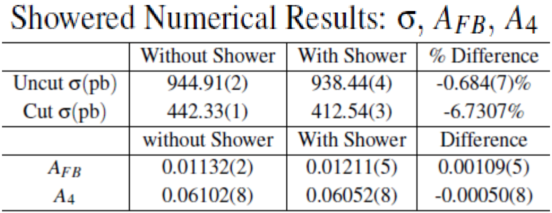
The two approaches should agree for variables which are not strongly sensitive to photon P_T.

The connection between these approaches should be studied in detail. KKMC-hh can be useful in such studies. Comparisons of quark momentum distributions could help determine the most appropriate values of the light quark masses.

Precision LHC Physics: New Results and New Issues

- Results from KKMC-hh: arXiv:2002.11692
- We see clear evidence that the transverse degrees of freedom in the photon radiation for ISR do impact observables in single $Z/\gamma*$ production: $\cos(\theta_{CS})$, M_{ll}, A_4, A_{FB}, Y_{ll}, w/wo shower.

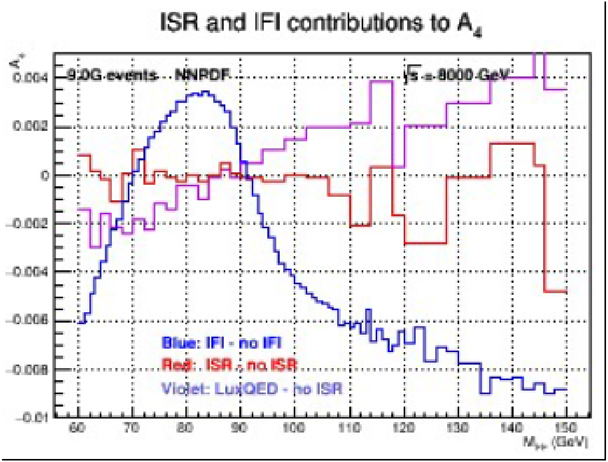
Illustrations: cuts - $60\ \mathrm{GeV} < M_{\ell\ell} < 116\ \mathrm{GeV}$, $P_T^{\ell\ell} < 30\ \mathrm{GeV}$, $P_T^{\ell} > 25\ \mathrm{GeV}$, $|\eta_\ell| < 2.5$.



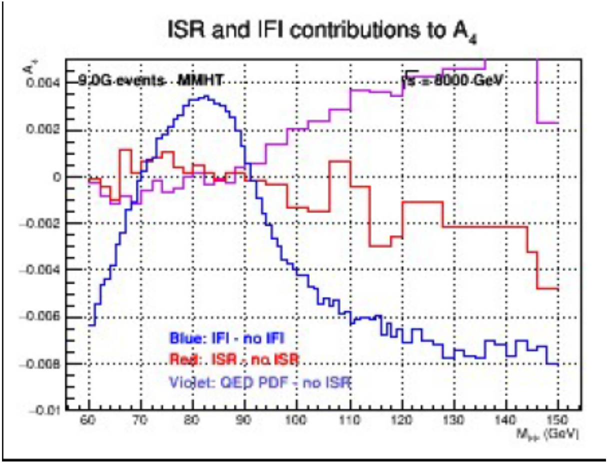
Showered Numerical Results: σ, A_{FB}, A_4

	Without Shower	With Shower	% Difference
Uncut σ(pb)	944.91(2)	938.44(4)	-0.684(7)%
Cut σ(pb)	442.33(1)	412.54(3)	-6.7307%
	without Shower	With Shower	Difference
A_{FB}	0.01132(2)	0.01211(5)	0.00109(5)
A_4	0.06102(8)	0.06052(8)	-0.00050(8)

No cuts:

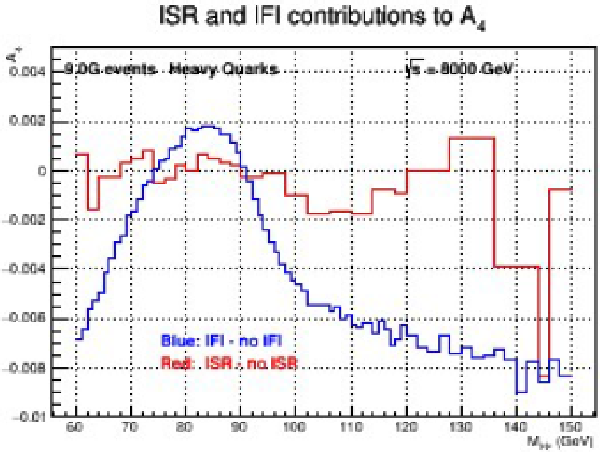

ISR and IFI contributions to A_4
A_4
9.0G events NNPDF
$\sqrt{s}$ = 8000 GeV
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
60
70
80
90
100
110
120
130
140
150
$M_{\mu\mu}$ (GeV)
Blue: IFI - no IFI
Red: ISR - no ISR
Violet: LuxQED - no ISR

No cuts:


ISR and IFI contributions to A_4
A_4
9.0G events MMHT
$\sqrt{s}$ = 8000 GeV
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
60
70
80
90
100
110
120
130
140
150
$M_{\ell\ell}$ (GeV)
Blue: IFI - no IFI
Red: ISR - no ISR
Violet: QED PDF - no ISR

Precision LHC Physics: New Results and New Issues

No cuts:



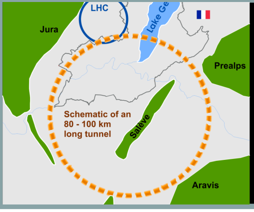
Precision LHC Physics: New Results and New Issues

- New Issues:
  Role of photon transverse degrees of freedom
  Role of quark masses:
  1. Observable parameters? YES
  2. Just unphysical (IR and CL regulators)? NO

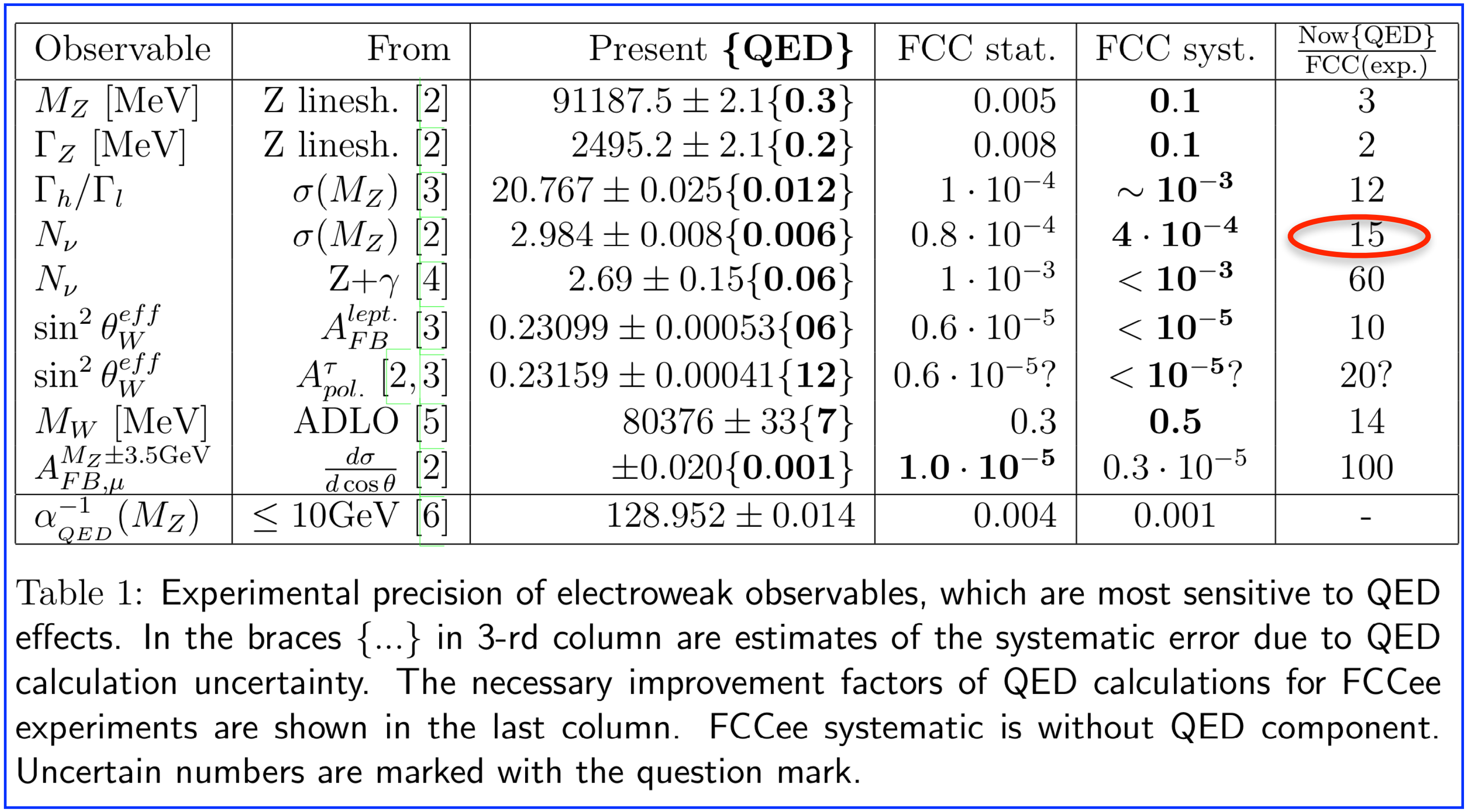
  Input data for non-QED PDFs at $Q_0 \sim 1$ GeV:
  1. Is this double counting if CL singular quark mass effects are not removed?
     QFT: processes at different space-time regimes cannot double count! (Shower!)
  2. Can we get a PDF with them removed
     -- probably YES.

Precision FCC Physics: New Results and New Issues

- FCC <=> FCC-ee + FCC-hh
- IR-Improvement of even the FCC-hh discovery spectra is needed--see arXiv:1801.03303
- For FCC-ee, a key issue is the theoretical precision of the Luminosity.
- Today, for illustration, we address the latter concern.
- We review what is the current state-of-the-art.
- We show the path forward to 0.01%



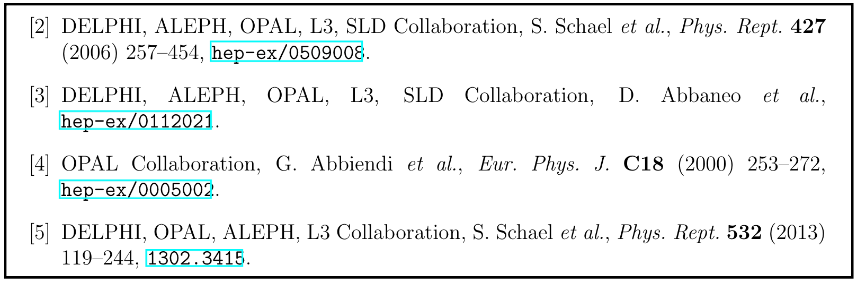
General context:
QED uncertainties in EW observables



Observable	From	Present **{QED}**	FCC stat.	FCC syst.	$\frac{\text{Now\{QED\}}}{\text{FCC(exp.)}}$
M_Z [MeV]	Z linesh. [2]	$91187.5 \pm 2.1\{\mathbf{0.3}\}$	0.005	**0.1**	3
Γ_Z [MeV]	Z linesh. [2]	$2495.2 \pm 2.1\{\mathbf{0.2}\}$	0.008	**0.1**	2
Γ_h/Γ_l	$\sigma(M_Z)$ [3]	$20.767 \pm 0.025\{\mathbf{0.012}\}$	$1 \cdot 10^{-4}$	$\sim \mathbf{10^{-3}}$	12
N_ν	$\sigma(M_Z)$ [2]	$2.984 \pm 0.008\{\mathbf{0.006}\}$	$0.8 \cdot 10^{-4}$	$\mathbf{4 \cdot 10^{-4}}$	15
N_ν	Z+γ [4]	$2.69 \pm 0.15\{\mathbf{0.06}\}$	$1 \cdot 10^{-3}$	$< \mathbf{10^{-3}}$	60
$\sin^2\theta_W^{eff}$	$A_{FB}^{lept.}$ [3]	$0.23099 \pm 0.00053\{\mathbf{06}\}$	$0.6 \cdot 10^{-5}$	$< \mathbf{10^{-5}}$	10
$\sin^2\theta_W^{eff}$	$A_{pol.}^{\tau}$ [2,3]	$0.23159 \pm 0.00041\{\mathbf{12}\}$	$0.6 \cdot 10^{-5}$?	$< \mathbf{10^{-5}}$?	20?
M_W [MeV]	ADLO [5]	$80376 \pm 33\{\mathbf{7}\}$	0.3	**0.5**	14
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [2]	$\pm 0.020\{\mathbf{0.001}\}$	$\mathbf{1.0 \cdot 10^{-5}}$	$0.3 \cdot 10^{-5}$	100
$\alpha_{QED}^{-1}(M_Z)$	$\leq$ 10GeV [6]	128.952 ± 0.014	0.004	0.001	-

Table 1: Experimental precision of electroweak observables, which are most sensitive to QED effects. In the braces {...} in 3-rd column are estimates of the systematic error due to QED calculation uncertainty. The necessary improvement factors of QED calculations for FCCee experiments are shown in the last column. FCCee systematic is without QED component. Uncertain numbers are marked with the question mark.

To be discussed in the following

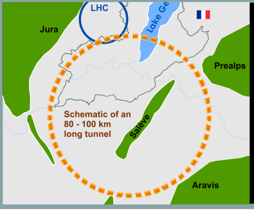


[2] DELPHI, ALEPH, OPAL, L3, SLD Collaboration, S. Schael *et al.*, *Phys. Rept.* **427** (2006) 257–454, `hep-ex/0509008`.

[3] DELPHI, ALEPH, OPAL, L3, SLD Collaboration, D. Abbaneo *et al.*, `hep-ex/0112021`.

[4] OPAL Collaboration, G. Abbiendi *et al.*, *Eur. Phys. J.* **C18** (2000) 253–272, `hep-ex/0005002`.

[5] DELPHI, OPAL, ALEPH, L3 Collaboration, S. Schael *et al.*, *Phys. Rept.* **532** (2013) 119–244, `1302.3415`.



QED challenges at FCCee

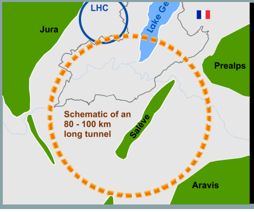
QED challenges at FCCee are of 2-fold type:

A. More higher (fixed) orders, better resummation, more sophisticated Monte Carlo programs

B. Possibly completely new methodology of the QED "deconvolution" and related new definition of the EW pseudo-observables (EWPO's)
--S. Jadach, private communication

An illustrative example:

Low angle Bhabha for luminosity measurement which enters into many observables, notably neutrino counting.



Example of low angle Bhabha (luminosity) at FCCee in IFJPAN-IV-2018-07, BU-EPP-18-03, MPP-2018-91 by S. Jadach, W. Płaczek, M. Skrzypek, B.F.L.W, S.A.Yost (PLB **790** (2019) 314)

- Motivation: better measurement of invisible Z width from Z peak x-section
- LEP legacy:

$$R^0_{inv} = \frac{\Gamma_{inv}}{\Gamma_{\ell\ell}} = \sqrt{\frac{12\pi R^0_\ell}{\sigma^0_{had} m_Z^2}} - R^0_\ell - (3 + \delta_\tau)$$

- assuming lepton universality

$$(R^0_{inv})_{exp} = N_\nu \left(\frac{\Gamma_{\nu\bar\nu}}{\Gamma_{\ell\ell}}\right)_{SM}$$

- from LEP Z-peak measurements

$$N_\nu = 2.9840 \pm 0.0082$$

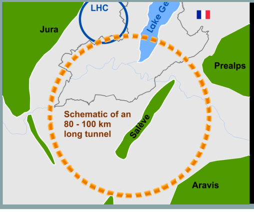
$$\delta N_\nu \simeq 10.5 \frac{\delta n_{had}}{n_{had}} \oplus 3.0 \frac{\delta n_{lept}}{n_{lept}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \Longrightarrow \delta N_\nu = 0.0046$$

ADLO, SLD and LEPEWWG, Phys. Rept. **427** (2008) 257, hep-ex/0509008

- Recently, Janot and Jadach, arXiv:1912.02067, current status for LEP lumi theory error $\Leftrightarrow 0.037\% \cup$ improved measurement error analysis (see also Voutsinas *et al.*, arXiv:1908.01704) $\Longrightarrow N_\nu = 2.9975 \pm 0.0074$ with $\delta N_\nu = 0.0028$ from $\frac{\delta \mathcal{L}}{\mathcal{L}}$

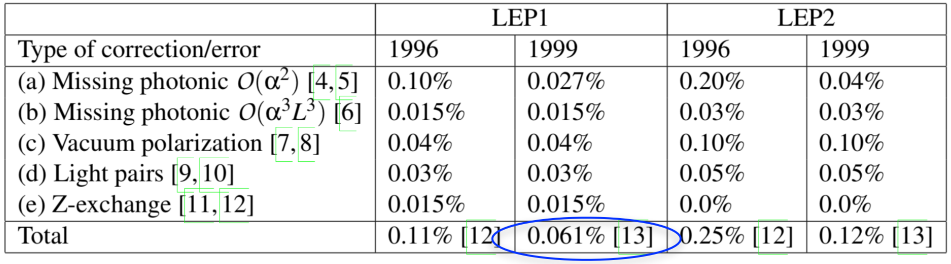
- 7.5x0.061%=0.0046. Shall we do better at FCCee?? YES!
- In 1999 lumi TH error 0.061% was dominated by VP => No motivation tp improve QED components Now, 0.037% (JJ) dominated by photonic correction => motivation already to improve QED error. At FCCee VP error will be reduced by another factor 2 compared to today! New reality!
- Low angle Bhabha luminometer already defined, Mogens Dam, FCC Week 2018, 2019 wkshp



Example of low angle Bhabha (luminosity) at FCCee
Overview of IFJPAN-IV-2018-07, BU-EPP-18-03, MPP-2018-91 by S.Jadach, W. Płaczek, M. Skrzypek, B.F.L.W., S.A.Yost (PLB **790** (2019) 314)

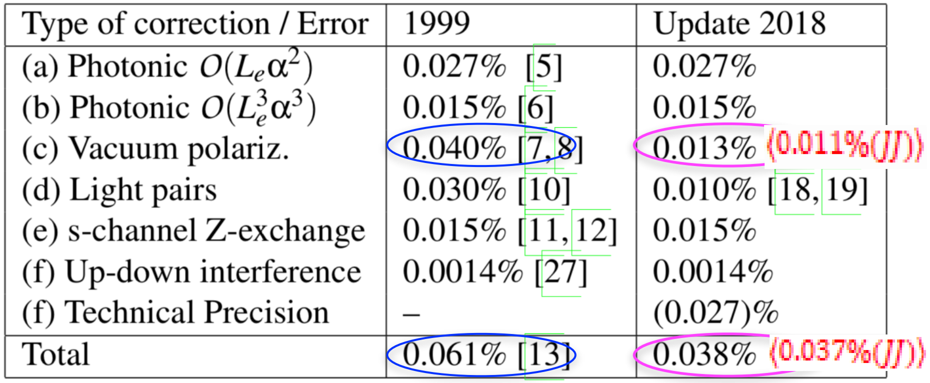
- LEP legacy, lumi TH error budget

LEP update 2018(2019)



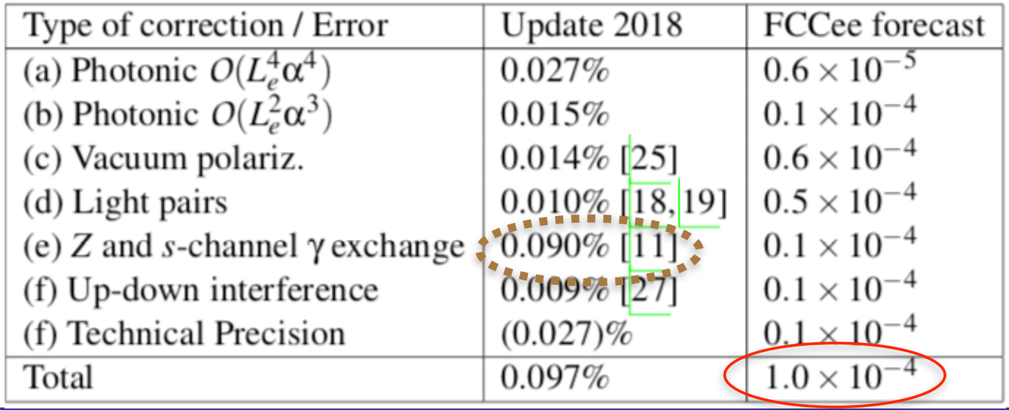
	LEP1		LEP2	
Type of correction/error	1996	1999	1996	1999
(a) Missing photonic $O(\alpha^2)$ [4,5]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $O(\alpha^3 L^3)$ [6]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [7,8]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [9,10]	0.03%	0.03%	0.05%	0.05%
(e) Z-exchange [11,12]	0.015%	0.015%	0.0%	0.0%
Total	0.11% [12]	0.061% [13]	0.25% [12]	0.12% [13]

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within 1°-3° (18-52 mrads), and for LEP2 energies up to 176 GeV and an angular range within 3°-6°. Total uncertainty is taken in quadrature. Technical precision included in (a).



Type of correction / Error	1999	Update 2018
(a) Photonic $O(L_e\alpha^2)$	0.027% [5]	0.027%
(b) Photonic $O(L_e^3\alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7,8]	0.013% (0.011%(JJ))
(d) Light pairs	0.030% [10]	0.010% [18,19]
(e) s-channel Z-exchange	0.015% [11,12]	0.015%
(f) Up-down interference	0.0014% [27]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038% (0.037%(JJ))

- By the time of FCCee VP contribution will be merely 0.006%(F. Jegerlehner)
- QED corrections and Z contrib. come back to front!
- Z contr. easy to master, even if rises at FCCee, because (28-58)->(64-86) mrad.



Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
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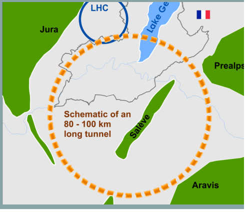
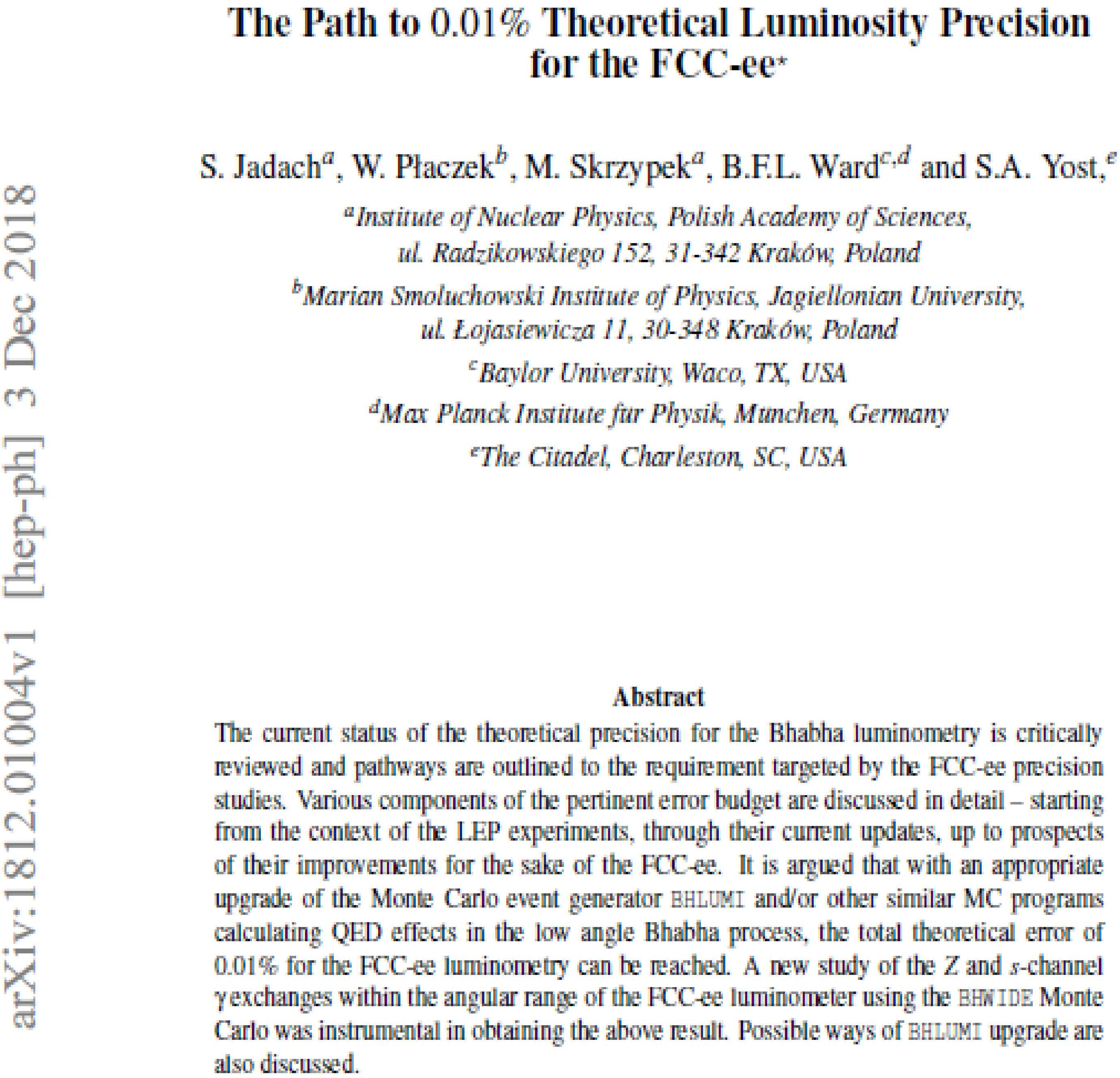
(d) Light pairs	0.010% [18,19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

- Our FCCee forecast is 0.001%, provided QED is improved.

Bibliography in last slides





The Path to 0.01% Theoretical Luminosity Precision for the FCC-ee*

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[d]Max Planck Institute für Physik, München, Germany

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Abstract

The current status of the theoretical precision for the Bhabha luminometry is critically reviewed and pathways are outlined to the requirement targeted by the FCC-ee precision studies. Various components of the pertinent error budget are discussed in detail – starting from the context of the LEP experiments, through their current updates, up to prospects of their improvements for the sake of the FCC-ee. It is argued that with an appropriate upgrade of the Monte Carlo event generator `BHLUMI` and/or other similar MC programs calculating QED effects in the low angle Bhabha process, the total theoretical error of 0.01% for the FCC-ee luminometry can be reached. A new study of the Z and s-channel γ exchanges within the angular range of the FCC-ee luminometer using the `BHWIDE` Monte Carlo was instrumental in obtaining the above result. Possible ways of `BHLUMI` upgrade are also discussed.

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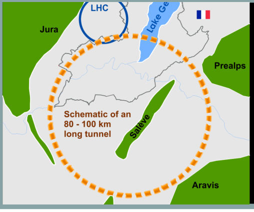
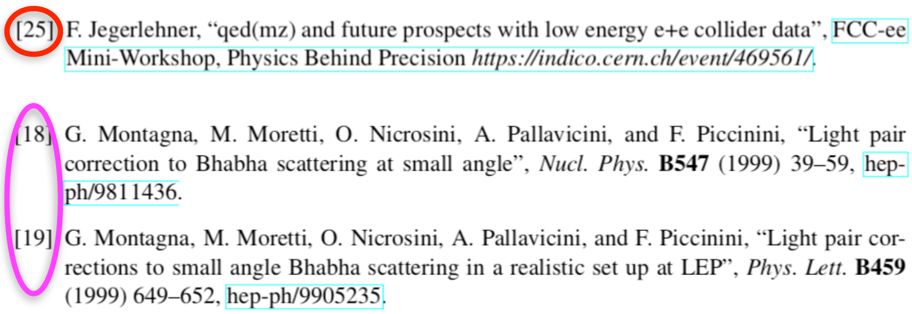
Details follow...



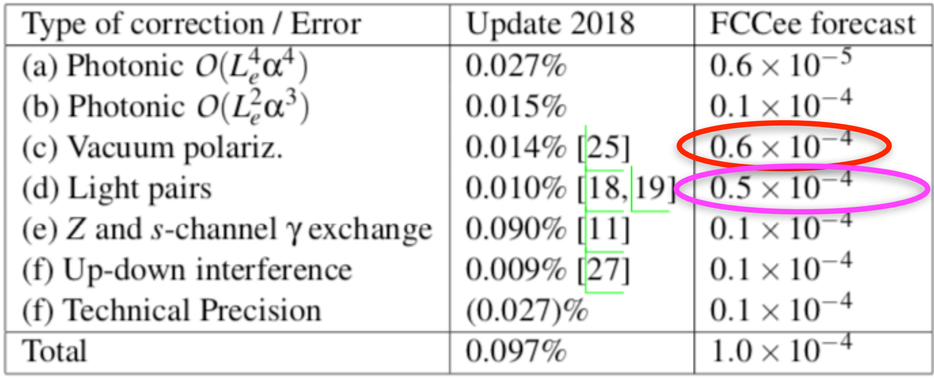
LEP legacy and update 2018



[25] F. Jegerlehner, "qed(mz) and future prospects with low energy e+e collider data", FCC-ee Mini-Workshop, Physics Behind Precision *https://indico.cern.ch/event/469561/*.

[18] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, "Light pair correction to Bhabha scattering at small angle", *Nucl. Phys.* **B547** (1999) 39–59, hep-ph/9811436.

[19] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, "Light pair corrections to small angle Bhabha scattering in a realistic set up at LEP", *Phys. Lett.* **B459** (1999) 649–652, hep-ph/9905235.



Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2\alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
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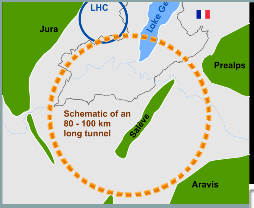
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

- All of LEP/SLD luminosity QED error estimates represent corrections **missing in BHLUMI v.4.04** Monte Carlo, used by all LEP and SLD collaborations.
- BHLUMI features $O(\alpha^1)$ and $O(L_e^2\alpha^2)$ corrections with YFS resumation, neglecting photonics interferences between e$^+$ and e$^-$ lines, where $L_e = \ln(|t|/m_e^2)$.
- Vacuum polarisation and pairs not dominant any more — QED photonic corrections and Z-exchange come back to front line!



QED photonics corrs.

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Total	0.097%	1.0×10^{-4}

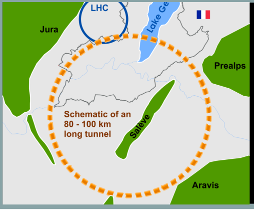
1. Photonic corrections are large, but higher orders contrib. known, hence soft/collinear re-summation is mandatory!
2. M.E. in BHLUMI includes $O(\alpha^1)$ and $O(L_e^2\alpha^2)$ corrections within YFS soft photon re-summation, neglecting photonics interferences between e$^+$ and e$^-$ lines (suppressed by |t|/s factor).
3. Photonics 2nd order NLO $O(L_e\alpha^2)$ and 3rd order LO $O(\alpha^3 L_e^3)$ corrections were calculated long ago [4], [6]. Presently they are not in BHLUMI v4.02 and accounted for in the error budget. Once included, error estimate is done for $O(L_e^0\alpha^2)$, $O(\alpha^4 L_e^4)$ and $O(\alpha^3 L_e^2)$ corrections.
4. Corrections $O(L_e^0\alpha^2)$ ~10^{-5} are not quoted in FCC error budget because are known.
5. Using scaling rules of thumb we estimate $O(\alpha^4 L_e^4)$ as $0.015\%\times\gamma = 0.6\times10^{-5}$ and $O(\alpha^3 L_e^2) \sim \gamma^2\alpha/\pi \simeq 10^{-5}$
6. N.B. BHLUMI with $O(L_e\alpha^2)$ has been already realised but not published because VP was dominant in 1998.

$$\boxed{\gamma = \frac{\alpha}{\pi}\ln\frac{|\bar{t}|}{m_e^2} = 0.042}$$

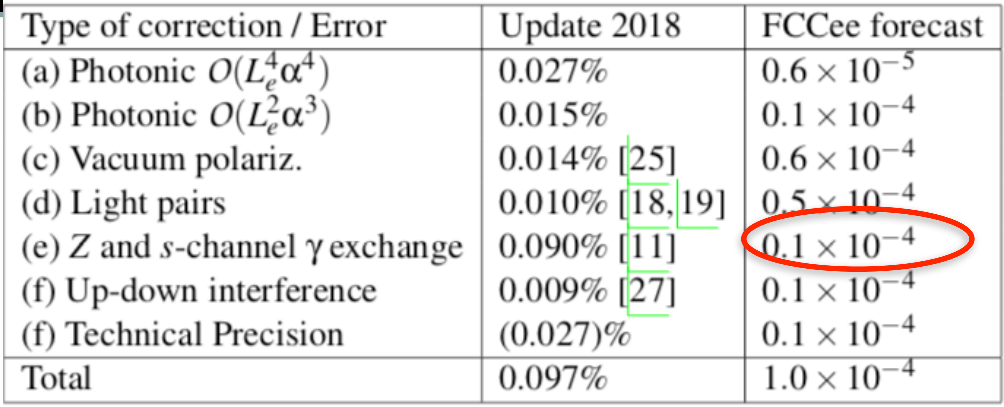
$$\boxed{|\bar{t}|^{1/2} = \langle|t|\rangle^{1/2} \simeq 3.25\ \text{GeV}}$$

[4] S. Jadach, M. Melles, B. F. L. Ward, and S. A. Yost, "Exact results on O (alpha) corrections to the single hard bremsstrahlung process in low angle Bhabha scattering in the SLC / LEP energy regime", *Phys. Lett.* **B377** (1996) 168–176, hep-ph/9603248.

[6] S. Jadach and B. F. L. Ward, "Missing third order leading log corrections in the small angle Bhabha calculation", *Phys. Lett.* **B389** (1996) 129–136.



Z and s-channel gamma exchange for FCCee angular range 64-86mrad



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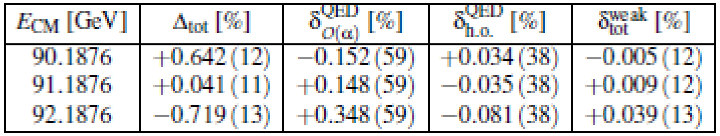
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

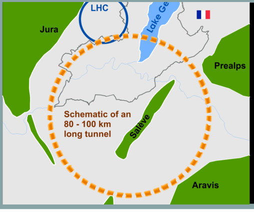
1. With respect to **dominant** t-channel gamma excha $|\gamma_t|^2 = \gamma_t \otimes \gamma_t$, all other contributions are suppressed (near Z) by factor <|t|>/s = 1.3 · 10^{-3} (instead 0.4 · 10^{-3} for LEP!)
2. However, **resonant** Z_s exchange gets enhanced by M_Z/Γ_Z and $\gamma_t \otimes Z_s$ term will be up to 1%.
   It is included in BHLUMI at the complete 1-st order level (with QED running couplings).
   Using results of ref. [11] its uncertainty due to QED corrections is **presently** estimate above as 0.090%
3. **Non-resonant** $\gamma_t \otimes \gamma_s$ ~0.1% is included in BHLUMI, gets small QED cor. with uncertainty 0.01%
4. Other contribution not in BHLUMI are: $|Z_s|^2$ ~0.01%, $\gamma_t \otimes Z_t \sim 3 \cdot 10^{-5}$, $|\gamma_s|^2 \sim 10^{-6}$ and $|Z_t|^2 \sim 10^{-6}$
5. It will be straightforward to reduce the above uncertainties to ~10^{-4} level by means of upgrade of the BHLUMI matrix element to the level of BHWIDE (EEX type).
6. With the implementation of the mat.el. of the CEEX type, as in KKMC, one could get for this group of contributions precision level of ~10^-5.

Study of Z and s-channel γ exchanges using `BHWIDE`

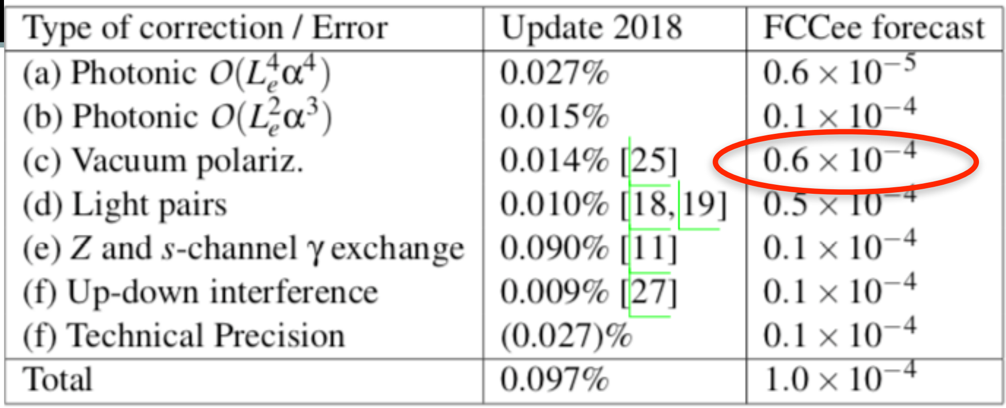


E_{CM} [GeV]	Δ_{tot} [%]	$\delta^{QED}_{O(\alpha)}$ [%]	$\delta^{QED}_{h.o.}$ [%]	δ^{weak}_{tot} [%]
90.1876	+0.642 (12)	−0.152 (59)	+0.034 (38)	−0.005 (12)
91.1876	+0.041 (11)	+0.148 (59)	−0.035 (38)	+0.009 (12)
92.1876	−0.719 (13)	+0.348 (59)	−0.081 (38)	+0.039 (13)

[11] S. Jadach, W. Placzek, and B. F. L. Ward, "Precision calculation of the gamma - Z interference effect in the SLC / LEP luminosity process", *Phys. Lett.* **B353** (1995) 349–361.



Vacuum polarization



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Total	0.097%	1.0×10^{-4}

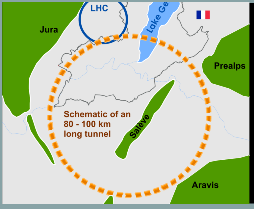
1. The error due to imprecise knowledge of the QED coupling constant for the t-channel exchange is $\frac{\delta_{VP}\sigma}{\sigma} = 2\frac{\delta\alpha_{eff}(\bar{t})}{\alpha_{eff}(\bar{t})}$
2. With $\Delta\alpha^{(5)}(-s_0) = (64.09 \pm 0.63) \times 10^{-4}$, of ref. [26], at s_0=2GeV we get $(\delta_{VP}\sigma)/\sigma = 1.3 \times 10^{-4}$
3. Anticipating improvement of hadronic e^+e^- cross section
   we expect by the FCCee time factor 2 improvement down to $\delta_{VP}\sigma/\sigma = 0.65 \times 10^{-4}$
4. N.B. The above is part of strategy of obtaining $\alpha_{\text{eff}}(M_Z^2)$ in two steps:
   (a) obtaining $\Delta\alpha^{(5)}(-s_0)$ from $\sigma_{had}(s)$, $s^{1/2} \leq 2.5\,\text{GeV}$ using dispersion relations,
   (b) calculating $\Delta\alpha^{(5)}(M_Z^2) - \Delta\alpha^{(5)}(-s_0)$ using perturbative QCD.
   Getting $\Delta\alpha^{(5)}(-s_0)$ for Bhabha luminometry from $\alpha_{\text{eff}}(M_Z^2)$ could be an interesting crosscheck:)

[25] F. Jegerlehner, "qed(mz) and future prospects with low energy e+e collider data", FCC-ee Mini-Workshop, Physics Behind Precision *https://indico.cern.ch/event/469561/*.

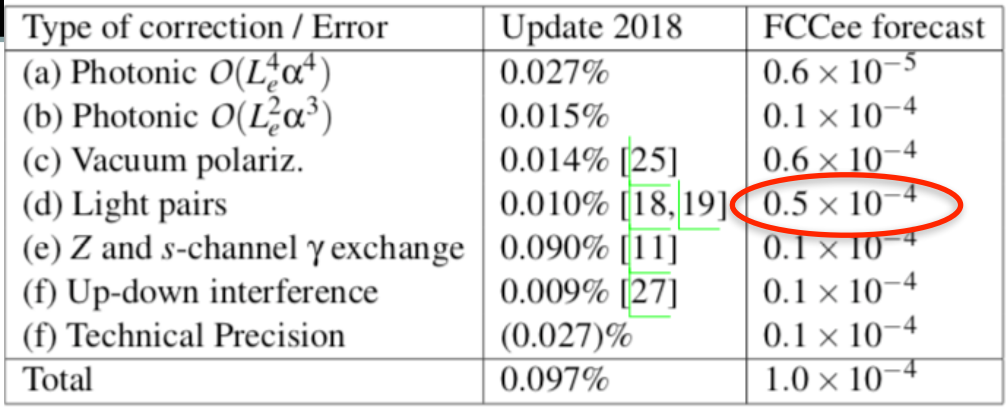
[26] F. Jegerlehner, "Variations on Photon Vacuum Polarization", 1711.06089.

[48] F. Jegerlehner, "Precision measurements of sigma(hadronic) for alpha(eff)(E) at ILC energies and (g-2)(mu)", *Nucl. Phys. Proc. Suppl.* **162** (2006) 22–32, [,22(2006)], hep-ph/0608329.

[49] F. Jegerlehner, "The Anomalous Magnetic Moment of the Muon", *Springer Tracts Mod. Phys.* **274** (2017) pp.1–693.



Light fermion pairs



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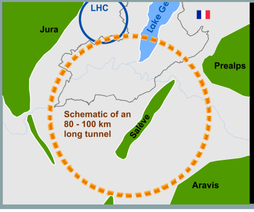
1. Additional light fermion pair production in Bhabha proces $e^-e^+\to e^-e^+\,f\bar{f},\quad f=e,\mu,\tau,u,d,s$ together with the corresponding virtual correction (fermion loop on photon line) is a valid 2nd order correction.

2. Numerically most sizeable is electron pair production subprocess $e^-e^+\to e^-e^+\,\gamma^*,\quad \gamma^*\to e^-e^+$ which very well known [9,10,18,19,53-60] and its precision is usually quoted to be $\sim 0.5\cdot 10^{-4}$.

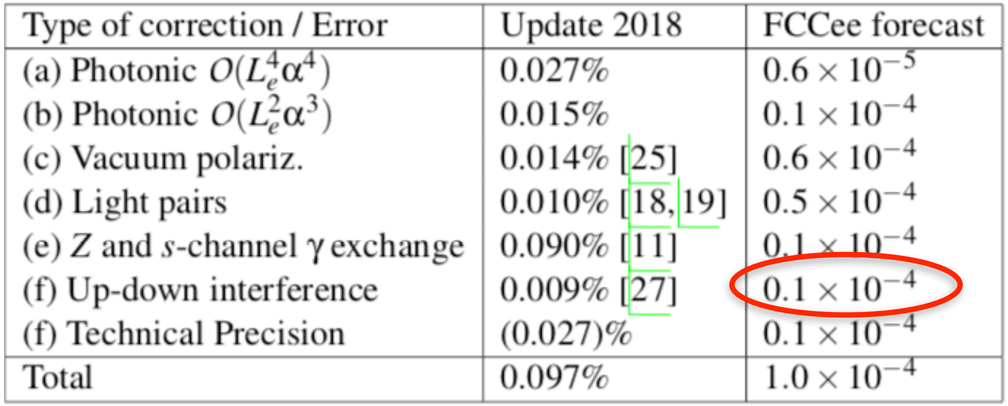
3. Second pair production $e^-e^+\to e^-e^+\,2(e^-e^+)$ and addition photon production $e^-e^+\to e^-e^+e^-e^+\gamma$ are calculable [10,18,54] and quoted to be negligible.

4. Contributions from heavier leptons and light quarks $f=\mu,\tau,u,d,s$ are typically $\sim 0.8\cdot 10^{-4}$ and in LEP context were entirely accounted as part of an error.
   They can be however calculated with the precision $<<\ 0.5\cdot 10^{-4}$.

5. These corrections can be incorporated only partly in BHLUMI (electron pair exponentiation in [10]), most likely auxiliary MC programs will be needed to calculate them.



Up-down interference



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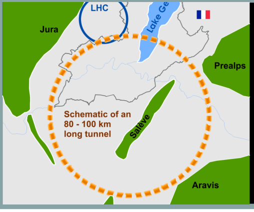
Total	0.097%	1.0×10^{-4}

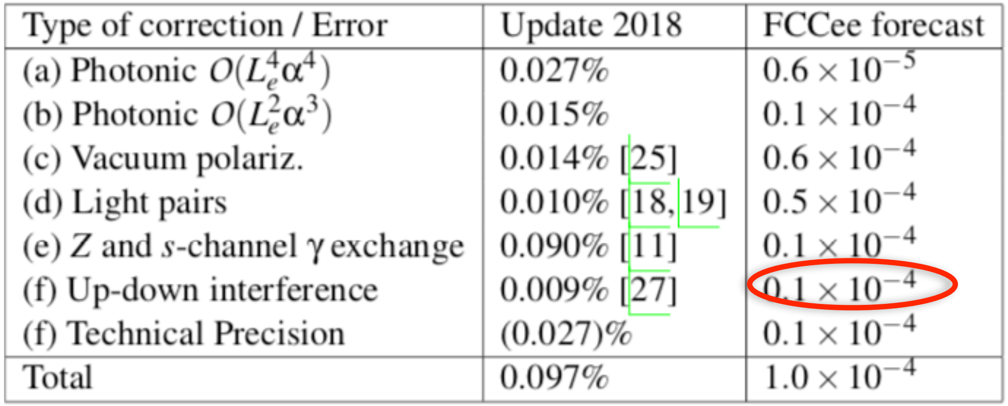
1. From ref. [27] this photonics (1st order correction) is known to be $\delta\sigma/\sigma \simeq 0.07\ |t|/s$ and for the luminometry it was negligible.

2. For FCCee it will come in a natural way in the upgrade M.E. of BHLUMI, to be done either as in BHWIDE or in KKMC.

3. We use conservatively factor $2\gamma \simeq 0.1$ in its precision estimate.



Technical precision



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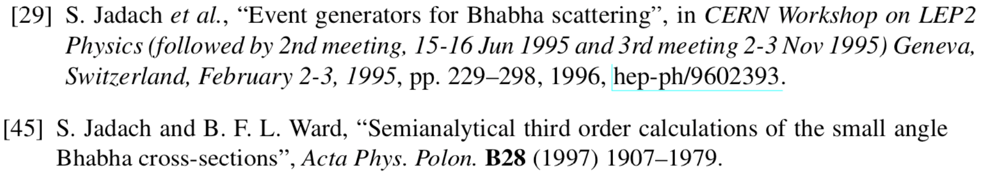
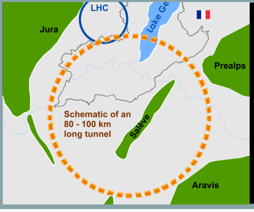
Total	0.097%	1.0×10^{-4}

1. Technical precision is the hardest problem!
2. In LEP workshop ref. [29] (1998) it was based on two pillars: comparison with semi-analytical calculation in ref. [45] and on comparison of BHLUMI with two hybrid MCs, LUMLOG+OLBBIS and SABSPV.
3. It was established to be 0.27%, together with missing photonics corrections.
4. Later on another BabaYaga MC was developed [20-24] based on the parton shower algorithm, and in principle could be used to evaluate technical precision independently.
5. However, once BHLUMI will be upgraded to include complete $O(L_e\alpha^2)$ and $O(\alpha^3L_e^3)$ the problem will come back, because it will be much harder to upgrade BabaYaga to the same NNLO level due to known peculiarities of the parton shower methodology.
6. Alternative solution could/should be worked out. See S. Frixione, 1909.03886, V. Bertone *et al.*, 1911.12040 .



[29] S. Jadach *et al.*, "Event generators for Bhabha scattering", in *CERN Workshop on LEP2 Physics (followed by 2nd meeting, 15-16 Jun 1995 and 3rd meeting 2-3 Nov 1995) Geneva, Switzerland, February 2-3, 1995*, pp. 229–298, 1996, hep-ph/9602393.

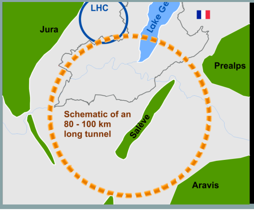
[45] S. Jadach and B. F. L. Ward, "Semianalytical third order calculations of the small angle Bhabha cross-sections", *Acta Phys. Polon.* **B28** (1997) 1907–1979.



SYNERGIES

- Historically, our exact $\mathcal{O}(\alpha^2 L_e)$ corrections were done for BHLUMI 4 precision => Combined via crossing with CEEX => KKMC for state-of-art 2f production => KKMC-hh for Z production in pp
- KKMC-hh => MG5_aMC@NLO/KKMC-hh (to appear) => exact QCD NLO $\otimes$ exact $\mathcal{O}(\alpha^2 L_e)$ EW

- **When we add to BHLUMI QED matrix element corrections of** $O(L_e\alpha^2)$ **and** $O(\alpha^3 L_e^3)$
- => Already reduce δN_ν from $\delta\mathcal{L}/\mathcal{L}$ to 0.0015.
- We now need to take CEEX to BHLUMI (a technical precision solution)
- => For FCCee, take CEEX to all the EEX YFS realizations for LEP:
- YFSWW3 & KORALW (see Skrzypek)
- YFSZZ
- BHWIDE
- We do need sufficient theory resources.



SYNERGIES

- For example, A_{FB} : Jadach & Yost, arXiv: 1801.08611 => use CEEX
  => KKMC for state-of-art 2f production => already have $\Delta\ [A_{FB}]_{IFI} \sim 10^{-4}$
- ΔMW: (Skrzypek(FCCee Workshp,2020)):

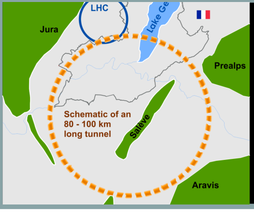
- **Threshold & Reconstruction: Need ~0.3 MeV for FCC-ee**
- CEEX extension of the LEP2 MC YFSWW3&KORALW needed in both cases:
- In progress: Jadach et al., arXiv:1906.09071 -- CEEX formalism applied
- to $e^+e^- \to WW + n\gamma \to 4f + n'\gamma$
- Note: Contact with the usual Kleiss-Stirling spinor product-based photon helicity infrared factors in CEEX via

$$e j_X^{\mu}(k_i) = e Q_X \theta_X \frac{2p_X^{\mu}}{2p_X k_i} \to \underset{\sigma_i}{\mathrm{s}}(k_i) = e Q_X \theta_X \frac{b_{\sigma_i}(k_i, p_X)}{2p_X k_i},$$

with

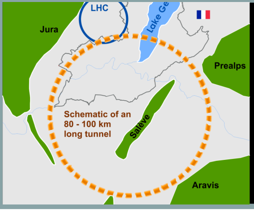
$$b_{\sigma}(k,p) = \sqrt{2}\,\frac{\bar{u}_{\sigma}(k)\ \not{p}\ u_{\sigma}(\zeta)}{\bar{u}_{-\sigma}(k) u_{\sigma}(\zeta)}$$

- The way forward is open.

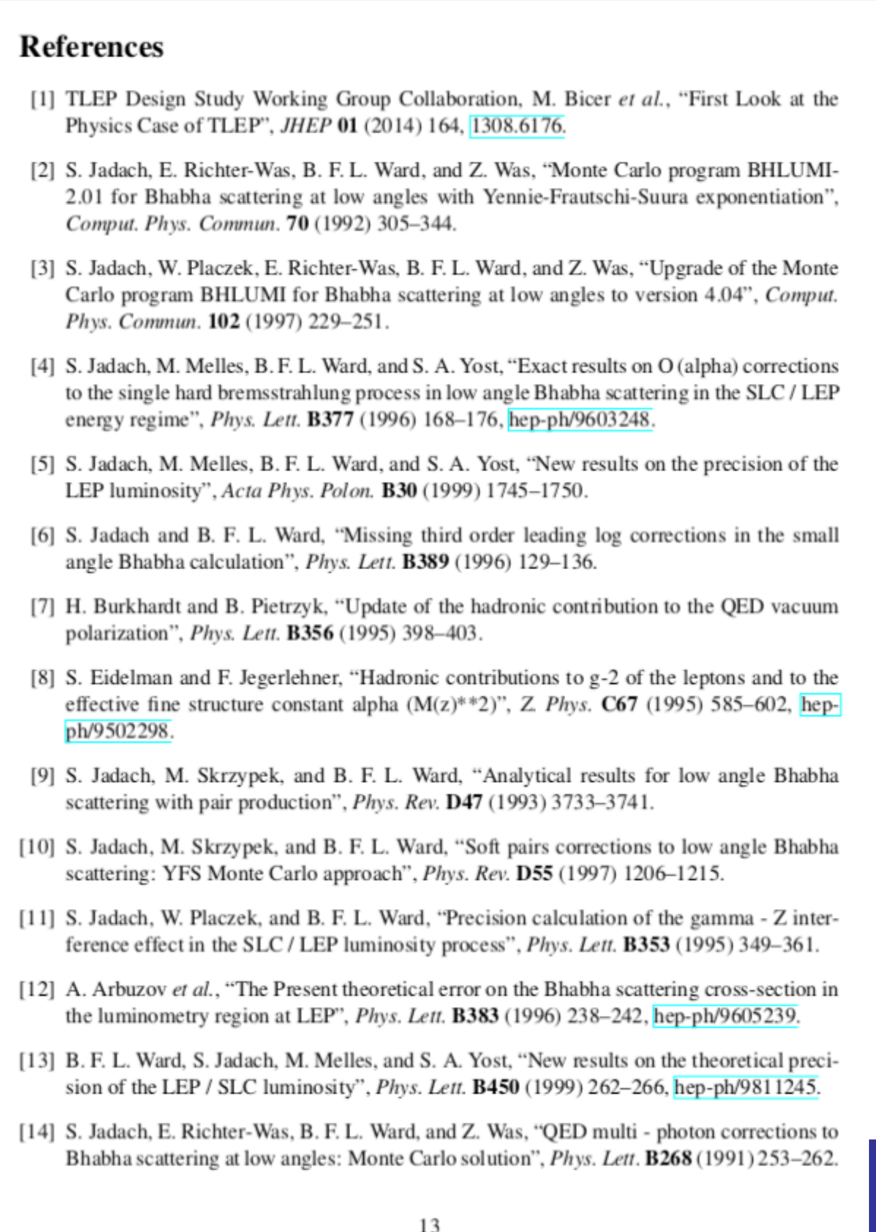


LUMI-SUMMARY

- All of LEP/SLD luminosity QED error estimates represent corrections **missing in BHLUMI v.4.04** Monte Carlo, used by all LEP and SLD collaborations.
- BHLUMI features $O(\alpha^1)$ and $O(L_e^2\alpha^2)$ corrections with YFS resumation, neglecting photonics interferences between e⁺ and e⁻ lines, where $L_e = \ln(|t|/m_e^2)$.
- **One has to add to BHLUMI QED matrix element corrections of** $O(L_e\alpha^2)$ **and** $O(\alpha^3 L_e^3)$
- They were calculated by Cracow-Knoxville collaboration long time ago (1996-99), but there was no strong motivation to publish them in the MC form, because of large VP uncertainty.
- Interferences between e⁺ and e⁻ lines should be added at 1-st order, with resummation.
- This class of corrections are implemented in the KKMC and BHWIDE since 1999.
- Corrections due to Z exchange and s-chanel gamma are big but easy to master (ME upgrade).
- There is (almost) enough auxiliary programs and calculations to control light pair corrections.
- Summarising there is no hard obstacles on the way to 0.01% QED precision on the theory side.
- The sticky issue is that of "technical precision".--The New Issue!!
  If BabaYaga Monte Carlo team makes sufficient progress this problem is solved (Piccinini).
- Alternative solutions are available: comparing CEEX and EEX upgrades of BHLUMI, Frixione *et al*., Sherpa, ...
- We do need sufficient theory resources.



Bibliography



References

[1] TLEP Design Study Working Group Collaboration, M. Bicer *et al.*, "First Look at the Physics Case of TLEP", *JHEP* **01** (2014) 164, 1308.6176.

[2] S. Jadach, E. Richter-Was, B. F. L. Ward, and Z. Was, "Monte Carlo program BHLUMI-2.01 for Bhabha scattering at low angles with Yennie-Frautschi-Suura exponentiation", *Comput. Phys. Commun.* **70** (1992) 305–344.

[3] S. Jadach, W. Placzek, E. Richter-Was, B. F. L. Ward, and Z. Was, "Upgrade of the Monte Carlo program BHLUMI for Bhabha scattering at low angles to version 4.04", *Comput. Phys. Commun.* **102** (1997) 229–251.

[4] S. Jadach, M. Melles, B. F. L. Ward, and S. A. Yost, "Exact results on O (alpha) corrections to the single hard bremsstrahlung process in low angle Bhabha scattering in the SLC / LEP energy regime", *Phys. Lett.* **B377** (1996) 168–176, hep-ph/9603248.

[5] S. Jadach, M. Melles, B. F. L. Ward, and S. A. Yost, "New results on the precision of the LEP luminosity", *Acta Phys. Polon.* **B30** (1999) 1745–1750.

[6] S. Jadach and B. F. L. Ward, "Missing third order leading log corrections in the small angle Bhabha calculation", *Phys. Lett.* **B389** (1996) 129–136.

[7] H. Burkhardt and B. Pietrzyk, "Update of the hadronic contribution to the QED vacuum polarization", *Phys. Lett.* **B356** (1995) 398–403.

[8] S. Eidelman and F. Jegerlehner, "Hadronic contributions to g-2 of the leptons and to the effective fine structure constant alpha (M(z)**2)", *Z. Phys.* **C67** (1995) 585–602, hep-ph/9502298.

[9] S. Jadach, M. Skrzypek, and B. F. L. Ward, "Analytical results for low angle Bhabha scattering with pair production", *Phys. Rev.* **D47** (1993) 3733–3741.

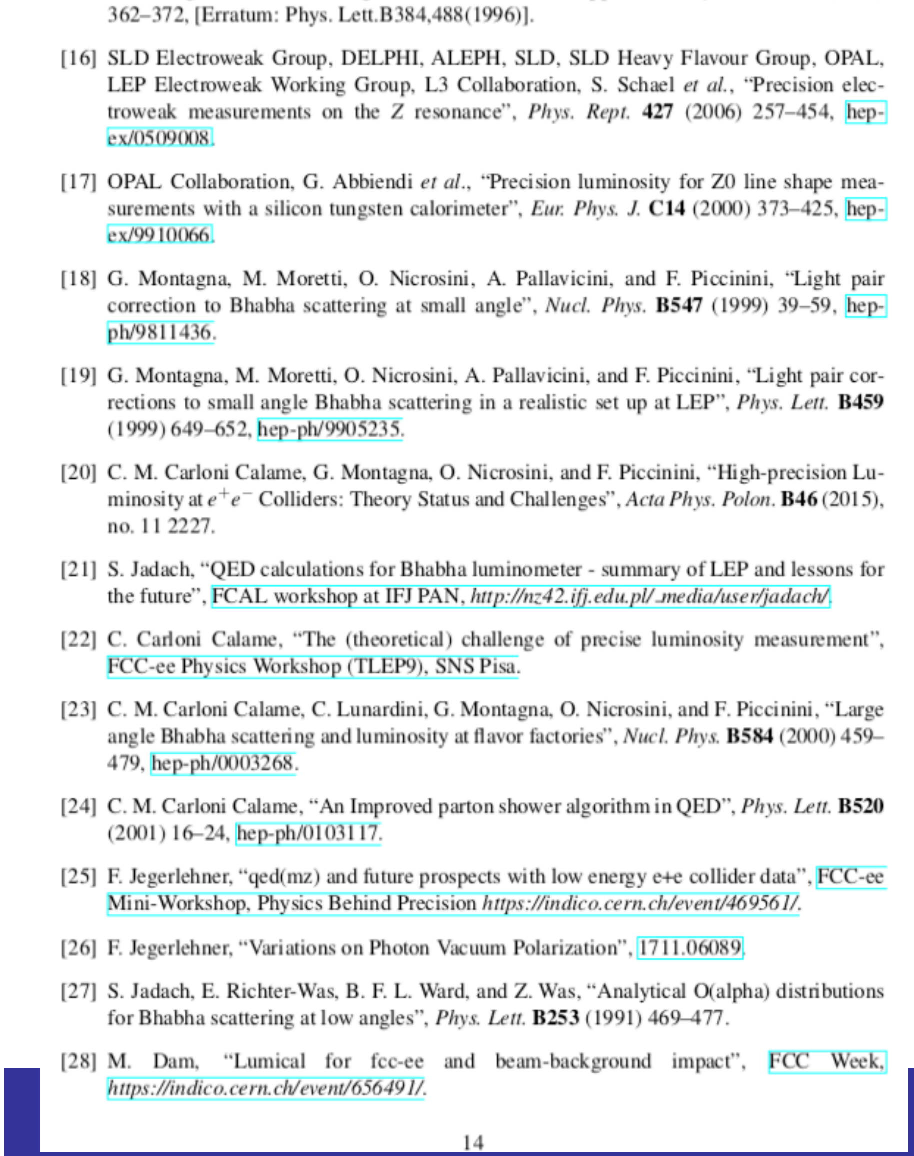
[10] S. Jadach, M. Skrzypek, and B. F. L. Ward, "Soft pairs corrections to low angle Bhabha scattering: YFS Monte Carlo approach", *Phys. Rev.* **D55** (1997) 1206–1215.

[11] S. Jadach, W. Placzek, and B. F. L. Ward, "Precision calculation of the gamma - Z interference effect in the SLC / LEP luminosity process", *Phys. Lett.* **B353** (1995) 349–361.

[12] A. Arbuzov *et al.*, "The Present theoretical error on the Bhabha scattering cross-section in the luminometry region at LEP", *Phys. Lett.* **B383** (1996) 238–242, hep-ph/9605239.

[13] B. F. L. Ward, S. Jadach, M. Melles, and S. A. Yost, "New results on the theoretical precision of the LEP / SLC luminosity", *Phys. Lett.* **B450** (1999) 262–266, hep-ph/9811245.

[14] S. Jadach, E. Richter-Was, B. F. L. Ward, and Z. Was, "QED multi - photon corrections to Bhabha scattering at low angles: Monte Carlo solution", *Phys. Lett.* **B268** (1991) 253–262.



[15] S. Jadach, E. Richter-Was, B. F. L. Ward, and Z. Was, "Higher order radiative corrections to low angle Bhabha scattering: The YFS Monte Carlo approach", *Phys. Lett.* **B353** (1995) 362–372, [Erratum: Phys. Lett.B384,488(1996)].

[16] SLD Electroweak Group, DELPHI, ALEPH, SLD, SLD Heavy Flavour Group, OPAL, LEP Electroweak Working Group, L3 Collaboration, S. Schael *et al.*, "Precision electroweak measurements on the *Z* resonance", *Phys. Rept.* **427** (2006) 257–454, hep-ex/0509008.

[17] OPAL Collaboration, G. Abbiendi *et al.*, "Precision luminosity for Z0 line shape measurements with a silicon tungsten calorimeter", *Eur. Phys. J.* **C14** (2000) 373–425, hep-ex/9910066.

[18] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, "Light pair correction to Bhabha scattering at small angle", *Nucl. Phys.* **B547** (1999) 39–59, hep-ph/9811436.

[19] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, "Light pair corrections to small angle Bhabha scattering in a realistic set up at LEP", *Phys. Lett.* **B459** (1999) 649–652, hep-ph/9905235.

[20] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, "High-precision Luminosity at e^+e^- Colliders: Theory Status and Challenges", *Acta Phys. Polon.* **B46** (2015), no. 11 2227.

[21] S. Jadach, "QED calculations for Bhabha luminometer - summary of LEP and lessons for the future", FCAL workshop at IFJ PAN, *http://nz42.ifj.edu.pl/_media/user/jadach/*.

[22] C. Carloni Calame, "The (theoretical) challenge of precise luminosity measurement", FCC-ee Physics Workshop (TLEP9), SNS Pisa.

[23] C. M. Carloni Calame, C. Lunardini, G. Montagna, O. Nicrosini, and F. Piccinini, "Large angle Bhabha scattering and luminosity at flavor factories", *Nucl. Phys.* **B584** (2000) 459–479, hep-ph/0003268.

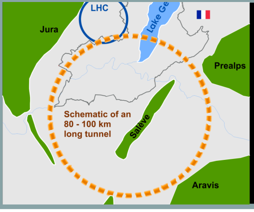
[24] C. M. Carloni Calame, "An Improved parton shower algorithm in QED", *Phys. Lett.* **B520** (2001) 16–24, hep-ph/0103117.

[25] F. Jegerlehner, "qed(mz) and future prospects with low energy e+e collider data", FCC-ee Mini-Workshop, Physics Behind Precision *https://indico.cern.ch/event/469561/*.

[26] F. Jegerlehner, "Variations on Photon Vacuum Polarization", 1711.06089.

[27] S. Jadach, E. Richter-Was, B. F. L. Ward, and Z. Was, "Analytical O(alpha) distributions for Bhabha scattering at low angles", *Phys. Lett.* **B253** (1991) 469–477.

[28] M. Dam, "Lumical for fcc-ee and beam-background impact", FCC Week, *https://indico.cern.ch/event/656491/*.



Bibliography

[29] S. Jadach *et al.*, "Event generators for Bhabha scattering", in *CERN Workshop on LEP2 Physics (followed by 2nd meeting, 15-16 Jun 1995 and 3rd meeting 2-3 Nov 1995) Geneva, Switzerland, February 2-3, 1995*, pp. 229–298, 1996, hep-ph/9602393.

[30] S. Jadach, "MC tools for extracting luminosity spectra", Seminar at SLAC *http://nz42.ifj.edu.pl/_media/user/jadach/*.

[31] A. A. Penin, "Two-loop photonic corrections to massive Bhabha scattering", *Nucl. Phys.* **B734** (2006) 185–202, hep-ph/0508127.

[32] M. Czakon, J. Gluza, and T. Riemann, "On the massive two-loop corrections to Bhabha scattering", *Acta Phys. Polon.* **B36** (2005) 3319–3326, hep-ph/0511187.

[33] R. Kleiss and W. J. Stirling, "Spinor Techniques for Calculating p anti-p —¿ W+- / Z0 + Jets", *Nucl. Phys.* **B262** (1985) 235–262.

[34] CALKUL Collaboration, F. A. Berends, P. De Causmaecker, R. Gastmans, R. Kleiss, W. Troost, and T. T. Wu, "Multiple Bremsstrahlung in Gauge Theories at High-energies. 6. The Process $e^+e^- \to e^+e^-\gamma\gamma$", *Nucl. Phys.* **B264** (1986) 265–276.

[35] S. Jadach, B. F. L. Ward, and S. A. Yost, "Exact results on e+ e- —¿ e+ e- 2 gamma at SLC / LEP energies", *Phys. Rev.* **D47** (1993) 2682–2689, hep-ph/9211252.

[36] S. Jadach, B. F. L. Ward, and S. A. Yost, "Comparisons of exact results for the virtual photon contribution to single hard bremsstrahlung in radiative return for electron-positron annihilation", *Phys. Rev.* **D73** (2006) 073001, hep-ph/0602197.

[37] S. Actis, P. Mastrolia, and G. Ossola, "NLO QED Corrections to Hard-Bremsstrahlung Emission in Bhabha Scattering", *Phys. Lett.* **B682** (2010) 419–427, 0909.1750.

[38] F. A. Berends, W. L. van Neerven, and G. J. H. Burgers, "Higher Order Radiative Corrections at LEP Energies", Nucl. Phys. B297 (1988) 429, [Erratum: Nucl. Phys. B304(1988) 921]; J. Blümlein, A. De Freitas, C. Raab, K. Schönwald, "The O(α^2) Initial State QED Corrections to $e^+e^- \to \gamma^*/Z^{*}_0$", arXiv:2003.14289; J. Ablinger, J. Blümlein, A. De Freitas, K. Schönwald, "Subleading Logarithmic QED Initial State Corrections to $e^+e^- \to \gamma^*/Z^{0*}$ to $O(\alpha^6 L^5)$"; J. Bluemlein, A. De Freitas, K. Schoenwald, "The QED Initial State Corrections to the Forward-Backward Asymmetry of $e^+e^- \to \gamma^*/Z^{0*}$ to Higher Orders", arXiv:2102.12237, and references therein.

[39] Z. Bern, L. J. Dixon, and A. Ghinculov, "Two loop correction to Bhabha scattering", *Phys. Rev.* **D63** (2001) 053007, hep-ph/0010075.

[40] R. Bonciani and A. Ferroglia, "Two-loop Bhabha scattering in QED", *Phys. Rev.* **D72** (2005) 056004, hep-ph/0507047.

[41] S. Actis, M. Czakon, J. Gluza, and T. Riemann, "Two-loop fermionic corrections to massive Bhabha scattering", *Nucl. Phys.* **B786** (2007) 26–51, 0704.2400.

[42] S. Actis, M. Czakon, J. Gluza, and T. Riemann, "Virtual Hadronic and Heavy-Fermion O(alpha**2) Corrections to Bhabha Scattering", *Phys. Rev.* **D78** (2008) 085019, 0807.4691.

[43] R. Bonciani, A. Ferroglia, and A. A. Penin, "Heavy-flavor contribution to Bhabha scattering", *Phys. Rev. Lett.* **100** (2008) 131601, 0710.4775.

[44] J. H. Kuhn and S. Uccirati, "Two-loop QED hadronic corrections to Bhabha scattering", *Nucl. Phys.* **B806** (2009) 300–326, 0807.1284.

[45] S. Jadach and B. F. L. Ward, "Semianalytical third order calculations of the small angle Bhabha cross-sections", *Acta Phys. Polon.* **B28** (1997) 1907–1979.

[46] D. R. Yennie, S. C. Frautschi, and H. Suura, "The infrared divergence phenomena and high-energy processes", *Annals Phys.* **13** (1961) 379–452.

[47] S. Jadach, B. Ward, and Z. Wąs, "Coherent exclusive exponentiation for precision Monte Carlo calculations", *Phys. Rev.* **D63** (2001) 113009, hep-ph/0006359.

[48] F. Jegerlehner, "Precision measurements of sigma(hadronic) for alpha(eff)(E) at ILC energies and (g-2)(mu)", *Nucl. Phys. Proc. Suppl.* **162** (2006) 22–32, [,22(2006)], hep-ph/0608329.

[49] F. Jegerlehner, "The Anomalous Magnetic Moment of the Muon", *Springer Tracts Mod. Phys.* **274** (2017) pp.1–693.

[50] S. Eidelman, F. Jegerlehner, A. L. Kataev, and O. Veretin, "Testing nonperturbative strong interaction effects via the Adler function", *Phys. Lett.* **B454** (1999) 369–380, hep-ph/9812521.

[51] P. Janot, "Direct measurement of $\alpha_{QED}(m_Z^2)$ at the FCC-ee", *JHEP* **02** (2016) 053, [Erratum: JHEP11,164(2017)], 1512.05544.

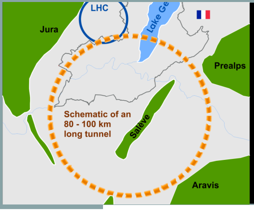
[52] G. Abbiendi *et al.*, "Measuring the leading hadronic contribution to the muon g-2 via μe scattering", *Eur. Phys. J.* **C77** (2017), no. 3 139, 1609.08987.

[53] A. B. Arbuzov, K. I. Gach, V. Yu. Gonchar, E. A. Kuraev, N. P. Merenkov, and L. Trentadue, "Small angle Bhabha scattering at LEP-1. Analytical results for wide - narrow angular acceptance", *Phys. Lett.* **B399** (1997) 312–320, hep-ph/9612201.

[54] A. B. Arbuzov, V. S. Fadin, E. A. Kuraev, L. N. Lipatov, N. P. Merenkov, and L. Trentadue, "Small angle electron - positron scattering with a per mille accuracy", *Nucl. Phys.* **B485** (1997) 457–502, hep-ph/9512344.

[55] A. B. Arbuzov, E. A. Kuraev, N. P. Merenkov, and L. Trentadue, "Pair production in small angle Bhabha scattering", *J. Exp. Theor. Phys.* **81** (1995) 638–646, [Zh. Eksp. Teor. Fiz.108,1164(1995)], hep-ph/9509405.

[56] A. B. Arbuzov, V. S. Fadin, E. A. Kuraev, L. N. Lipatov, N. P. Merenkov, and L. G. Trentadue, "Small angle electron - positron scattering", *Phys. Lett.* **B394** (1997) 218–224, hep-ph/9606425.



Bibliography

[57] N. P. Merenkov, A. B. Arbuzov, V. S. Fadin, E. A. Kuraev, L. N. Lipatov, and L. Trentadue, "Analytical calculation of small angle Bhabha cross-section at LEP-1", *Acta Phys. Polon.* **B28** (1997) 491–507.

[58] F. Caravaglios and M. Moretti, "An algorithm to compute Born scattering amplitudes without Feynman graphs", *Phys. Lett.* **B358** (1995) 332–338, hep-ph/9507237.

[59] R. Barbieri, J. A. Mignaco, and E. Remiddi, "Electron form-factors up to fourth order. 1.", *Nuovo Cim.* **A11** (1972) 824–864.

[60] R. Barbieri, J. A. Mignaco, and E. Remiddi, "Electron form factors up to fourth order. 2.", *Nuovo Cim.* **A11** (1972) 865–916.

[61] G. J. H. Burgers, "On the Two Loop QED Vertex Correction in the High-energy Limit", *Phys. Lett.* **164B** (1985) 167–169.

[62] S. Jadach, M. Skrzypek, and B. F. L. Ward, "Soft pairs real and virtual infrared functions in QED", *Phys. Rev.* **D49** (1994) 1178–1182.

[63] A. Denner, S. Dittmaier, M. Roth, and D. Wackeroth, "RACOONWW1.3: A Monte Carlo program for four fermion production at e+ e- colliders", *Comput. Phys. Commun.* **153** (2003) 462–507, hep-ph/0209330.

[64] F. A. Berends, R. Kleiss, and W. Hollik, "Radiative Corrections to Bhabha Scattering at High-Energies. 2. Hard Photon Corrections and Monte Carlo Treatment", *Nucl. Phys.* **B304** (1988) 712–748.

[65] W. Beenakker, F. A. Berends, and S. C. van der Marck, "Large angle Bhabha scattering", *Nucl. Phys.* **B349** (1991) 323–368.

[66] W. Beenakker, F. A. Berends, and S. C. van der Marck, "Small angle Bhabha scattering", *Nucl. Phys.* **B355** (1991) 281–294.

[67] S. Jadach, W. Placzek, and B. F. L. Ward, "BHWIDE 1.00: O(alpha) YFS exponentiated Monte Carlo for Bhabha scattering at wide angles for LEP-1 / SLC and LEP-2", *Phys. Lett.* **B390** (1997) 298–308, hep-ph/9608412.

[68] M. Battaglia, S. Jadach, and D. Bardin, "Luminosity determination at CLIC", *eConf* **C010630** (2001) E3015, E3015.PDF.

[69] S. Frixione and B. R. Webber, "Matching NLO QCD computations and parton shower simulations", *JHEP* **06** (2002) 029, hep-ph/0204244.

[70] P. Nason, "A new method for combining NLO QCD with shower Monte Carlo algorithms", *JHEP* **11** (2004) 040, hep-ph/0409146.

[71] S. Jadach, W. Paczek, S. Sapeta, A. Sidmok, and M. Skrzypek, "Matching NLO QCD with parton shower in Monte Carlo scheme the KrkNLO method", *JHEP* **10** (2015) 052, 1503.06849.

[72] S. Jadach, B. F. L. Ward, and Z. Was, "The precision monte carlo event generator kk for two- fermion final states in e+ e- collisions", *Comput. Phys. Commun.* **130** (2000) 260–325, Program source available from http://jadach.web.cern.ch/, hep-ph/9912214.

Quantum Gravity: New Results and New Issues

- Preliminary Remarks
- Overview of Resummed Quantum Gravity
- Planck Scale Cosmology
- An Estimate of Λ
- An Open Question?
- Einstein-Heisenberg Consistency Condition
- Constraints on SUSY GUTs

Preliminary Remarks

- IS QUANTUM GRAVITY (Einstein-Hilbert Theory) CALCULABLE IN RELATIVISTIC QFT?
- STRING THEORY: NO. You need superstrings, supersymmetric one-dimensional objects of Planck length size, 1.62×10–33 cm.
- LOOP QUANTUM GRAVITY: NO. You need Planck length size loops that are the fundamental constructs for quantum gravity.
- HORAVA-LIFSHITZ THEORY: NO. You need anisotropic scaling at Planck length scales:

  Time and space differ by a factor of z in scale dimension at Planck length distances with $z = 3$ in the original proposal–this violates local Lorentz invariance.

Preliminary Remarks

- New Approach: Exact Amplitude-Based Resummation of Feynman's Formulation of Einstein's Theory – Resummed Quantum Gravity (RQG)
- RESULT (1): UV Finiteness!
- RESULT (2): Constraints on SUSY GUT's
- RESULT (3): Prediction for the Cosmological Constant Λ with Relatively Small Theoretical Uncertainty.
- RESULT (4): Consistent with Weinberg's Asymptotic Safety Ansatz, as realized by Exact Field Space Renormalization Group Program of Reuter *et al.*

Preliminary Remarks

- RESULT (5): Consistent with Kreimer's Leg Renormalizability Results ...
- Today we give highlights on the status and outlook for this new RQG approach.

Overview of Resummed Quantum Gravity

SM $\Leftrightarrow$ Many Massive Point Particles.
Feynman: spin is an inessential complication – checked. We replace $L^{\mathcal{G}}_{SM}(x)$ with that a free physical Higgs field, $\varphi(x)$, with a rest mass 125 GeV(ATLAS,CMS) $\Rightarrow$ the representative model **{R.P. Feynman, Acta Phys. Pol. 24 (1963) 697;** *Feynman Lectures on Gravitation*, **eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971). }**

$$
\begin{aligned}
\mathcal{L}(x) &= \frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g} \\
&= \quad \frac{1}{2}\left\{ h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'} \right\} \\
&\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right] \\
&\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots
\end{aligned}
\tag{1}
$$

Overview of Resummed Quantum Gravity

where $\varphi_{,\mu} \equiv \partial_\mu \varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$
- $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho\right)$ for any tensor $y_{\mu\nu}$
- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge, $\partial^\mu \bar{h}_{\nu\mu} = 0$

$\Leftrightarrow$ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

Overview of Resummed Quantum Gravity

YFS resum the propagators in the NON-ABELIAN gauge theory of QG:

⇒ from the YFS formula

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'_F(p)}, \tag{2}$$

we find for Quantum Gravity, proceeding as above, the analogue of

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4} \frac{-i\eta^{\mu\nu}}{(\ell^2-\lambda^2+i\epsilon)} \frac{-ie(2ik_\mu)}{(\ell^2-2\ell k+\Delta+i\epsilon)} \frac{-ie(2ik'_\nu)}{(\ell^2-2\ell k'+\Delta'+i\epsilon)}\Big|_{k=k'} \tag{3}$$

as $-B''_g(k)$ with

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2-\lambda^2+i\epsilon} \frac{1}{(\ell^2+2\ell k+\Delta+i\epsilon)^2} \tag{4}$$

for $\Delta = k^2 - m^2$ ⇒ for a scalar field

$$i\Delta'_F(k)|_{YFS-resummed} = \frac{ie^{B''_g(k)}}{(k^2-m^2-\Sigma'_s+i\epsilon)}.$$

Overview of Resummed Quantum Gravity

$\Rightarrow$
Expand theory with the 'improved Born' propagators

$$iP_{\alpha_1\cdots;\alpha_1'\cdots}\Delta_F'(k)|_{YFS-resummed,\Sigma_s'=0} = \frac{iP_{\alpha_1\cdots;\alpha_1'\cdots}e^{B_g''(k)}}{(k^2-m^2+i\epsilon)} \tag{5}$$

where in the DEEP UV we get

$$B_g''(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \tag{6}$$

$\Rightarrow$ ALL PROPAGATORS FALL FASTER THAN ANY POWER OF $|k^2|$ $\Rightarrow$ QG IS FINITE (SEE MPL**A17** (2002) 2371;hep-ph/0607198)!

CONTACT WITH ASYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G(k) = G_N/(1+\frac{k^2}{a^2})$$

$\Rightarrow$ FIXED POINT BEHAVIOR FOR
$k^2 \to \infty$,
IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONANNO & REUTER IN PRD**62**(2000) 043008.

- OUR RESULTS $\Rightarrow$ AN ELEMENTARY PARTICLE HAS NO HORIZON. THIS AGREES WITH BONANNO & REUTER THAT A BLACK HOLE WITH A MASS LESS THAN
$M_{cr} \sim M_{Pl}$
HAS NO HORIZON.
BASIC PHYSICS:
$G(k)$ VANISHES FOR $k^2 \to \infty$.

Planck Scale Cosmology

- Bonanno and Reuter see arXiv.org:0803.2546,and refs. therein – phenomenological approach to Planck scale cosmology: STARTING POINT IS THE EINSTEIN-HILBERT THEORY

  $$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}\,(R - 2\Lambda) \tag{7}$$

  PHENOMENOLOGICAL EXACT RENORMALIZATION GROUP FOR THE WILSONIAN COARSE GRAINED EFFECTIVE AVERAGE ACTION IN FIELD SPACE$\Rightarrow$ RUNNING NEWTON CONSTANT $G_N(k)$ AND COSMOLOGICAL CONSTANT $\Lambda(k)$ APPROACH UV FIXED POINTS AS k GOES TO ∞ IN THE DEEP EUCLIDEAN REGIME – $k^2 G_N(k) \to g_*,\ \Lambda(k) \to \lambda_* k^2$.

  - Due to the thinining of the degrees of freedom in Wilsonian field space renormalization theory, the arguments of Foot et al.(PLB**664**(2008)199) are obviated.– See also MPLA **25**(2010)607;SHAPIRO&SOLA,PLB682(2009)105

Planck Scale Cosmology

CONTACT WITH COSMOLOGY PROCEEDS AS FOLLOWS: PHENOMENOLOGICAL CONNECTION BETWEEN THE MOMENTUM SCALE k CHARACTERIZING THE COARSENESS OF THE WILSONIAN GRAININESS OF THE AVERAGE EFFECTIVE ACTION AND THE COSMOLOGICAL TIME t, B-R SHOW STANDARD COSMOLOGICAL EQUATIONS ADMIT(see also Bonanno et al.,1006.0192) THE FOLLOWING EXTENSION:

$$\begin{aligned}(\frac{\dot{a}}{a})^2+\frac{K}{a^2}&=\frac{1}{3}\Lambda+\frac{8\pi}{3}G_N\rho\\ \dot{\rho}+3(1+\omega)\frac{\dot{a}}{a}\rho&=0\\ \dot{\Lambda}+8\pi\rho\dot{G}_N&=0\\ G_N(t)&=G_N(k(t))\\ \Lambda(t)&=\Lambda(k(t))\end{aligned}\tag{8}$$

FOR DENSITY ρ AND SCALE FACTOR $a(t)$.

Planck Scale Cosmology

WITH ROBERTSON-WALKER METRIC REPRESENTATION

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{9}$$

$K = 0, 1, -1 \Leftrightarrow$ RESPECTIVELY FLAT, SPHERICAL AND PSEUDO-SPHERICAL 3-SPACES FOR CONSTANT TIME t FOR A LINEAR RELATION BETWEEN THE PRESSURE p and ρ (EQN. OF STATE)

$$p(t) = \omega\rho(t). \tag{10}$$

Planck Scale Cosmology

FUNCTIONAL RELATIONSHIP BETWEEN MOMENTUM SCALE k AND COSMOLOGICAL TIME t DETERMINED PHENOMENOLOGICALLY VIA (see also Shapiro and Sola, PLB**475**(2000)236)

$$k(t) = \frac{\xi}{t} \tag{11}$$

WITH POSITIVE CONSTANT ξ.

Using the UV fixed points for $k^2 G_N(k) = g_*$ and $\Lambda(k)/k^2 = \lambda_*$ B-R SHOW THAT (8) ADMITS, FOR $K = 0$, A SOLUTION IN THE PLANCK REGIME ($0 \le t \le t_{\text{class}}$, with t_{class} a few times the Planck time t_{Pl}), WHICH JOINS SMOOTHLY ONTO A SOLUTION IN THE CLASSICAL REGIME ($t > t_{\text{class}}$) which agrees with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems solved by Planck scale quantum physics.

Planck Scale Cosmology

PHENOMENOLOGICAL NATURE OF THE ANALYSIS: THE fixed-point results g_*, λ_* depend on the cut-offs used in the Wilsonian coarse-graining procedure.
KEY PROPERTIES OF g_*, λ_* USED FOR THE B-R ANALYSES: they are both positive and the product $g_*\lambda_*$ is cut-off/threshold function independent.

An Estimate of Λ

- In Phys. Dark Univ. **2** (2013) 97, using (5) and (6) we get rigorous cut-off independent values for the fixed points $g_*,\ \lambda_*$ and the following estimate of Λ:

$$\begin{aligned}\rho_\Lambda(t_0) &\cong \frac{-M_{Pl}^4(1+c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64}\sum_j \frac{(-1)^{F_j}n_j}{\rho_j^2}\\ &\quad\times\frac{t_{tr}^2}{t_{eq}^2}\times(\frac{t_{eq}^{2/3}}{t_0^{2/3}})^3\\ &\cong \frac{-M_{Pl}^2(1.0362)^2(-9.194\times10^{-3})}{64}\frac{(25)^2}{t_0^2}\\ &\cong (2.4\times10^{-3}eV)^4,\end{aligned} \tag{12}$$

  where the age of the universe is $t_0 \cong 13.7\times10^9$ yrs.
- Compare: $\rho_\Lambda(t_0)|_{\text{expt}} \cong ((2.37\pm0.05)\times10^{-3}eV)^4$.

An Open Question

- A MAIN UNCERTAINTY: $t_{\rm tr}$
- B-R: NUMERICAL STUDIES $\Rightarrow t_{\rm tr} \cong 25/M_{Pl}$
- IN GENERAL, A FACTOR of $\mathcal{O}(100)$ IS ALLOWED
- CAN WE DO BETTER - NEW ISSUE?

Einstein-Heisenberg Consistency Condition

- In MPLA**30** (2015)1550206, we use the de Sitter space solutions of Duerr et al. to get the Einstein-Heisenberg consistency condition

$$k \geq \frac{\sqrt{5}}{2w_0} = \frac{\sqrt{5}}{2}\frac{1}{\sqrt{3/\Lambda(k)}} \qquad (13)$$

  from the Heisenberg uncertainty relation $\Delta p \Delta q \geq \frac{1}{2}$, with $\Delta p = k$ and $(w_0 = \sqrt{3/\Lambda})$

$$(\Delta q)^2 \cong \frac{\int_0^{w_0} dw w^2 w^2 < \cos^2\theta >}{\int_0^{w_0} dw w^2} = \frac{1}{5}w_0^2. \qquad (14)$$

- Violation of (13) ends Planck scale inflation: solving for k_{tr} $\Rightarrow k_{tr} \cong M_{Pl}/25.3$, in agreement with what Bonnano and Reuter suggested from numerical studies.
- $\Rightarrow$ uncertainty on our estimate of ρ_Λ is $\mathcal{O}(10)$ -YES!

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Constraints on SUSY GUTS

Note

$$\langle 0|\mathcal{H}|0\rangle \sim \int^{M_{Pl}} \frac{d^3k}{(2\pi)^3}\frac{1}{2}\omega(k) = \int^{M_{Pl}} \frac{d^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2}$$

Raises question of GUTS: Use SO(10) SUSY GUT Approach of Dev & Mohapatra (PRD82(2010)035014):

Intermediate Stage:

$SU_{2L} \times SU_{2R} \times SU(3)^c$

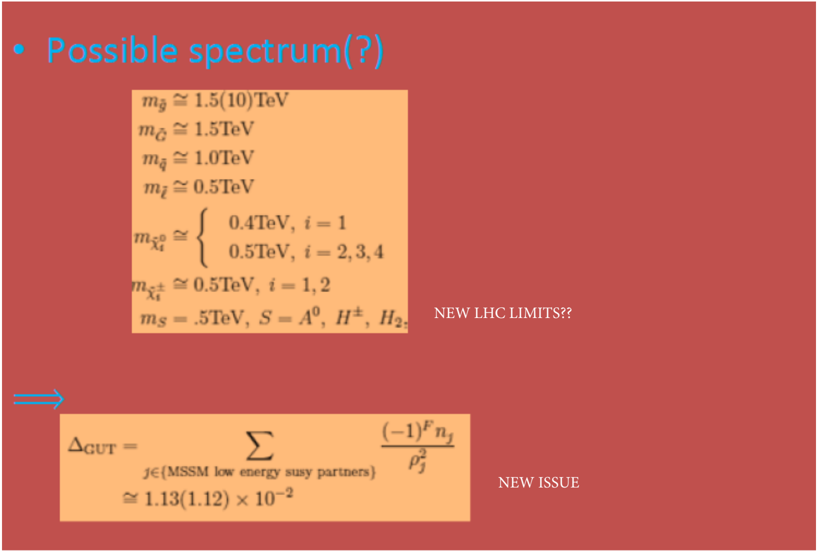
SM Stage at $\sim 2\text{TeV} = M_R$:

$SU_{2L} \times U_1 \times SU(3)^c$

SUSY Breaking at EW scale M_S :

$U_1 \times SU(3)^c$

Constraints on SUSY GUTS



- Possible spectrum(?)

$$m_{\tilde g} \cong 1.5(10)\mathrm{TeV}$$
$$m_{\tilde G} \cong 1.5\mathrm{TeV}$$
$$m_{\tilde q} \cong 1.0\mathrm{TeV}$$
$$m_{\tilde l} \cong 0.5\mathrm{TeV}$$
$$m_{\tilde\chi^0_i} \cong \begin{cases} 0.4\mathrm{TeV},\ i=1 \\ 0.5\mathrm{TeV},\ i=2,3,4 \end{cases}$$
$$m_{\tilde\chi^\pm_i} \cong 0.5\mathrm{TeV},\ i=1,2$$
$$m_S = .5\mathrm{TeV},\ S=A^0,\ H^\pm,\ H_2,$$

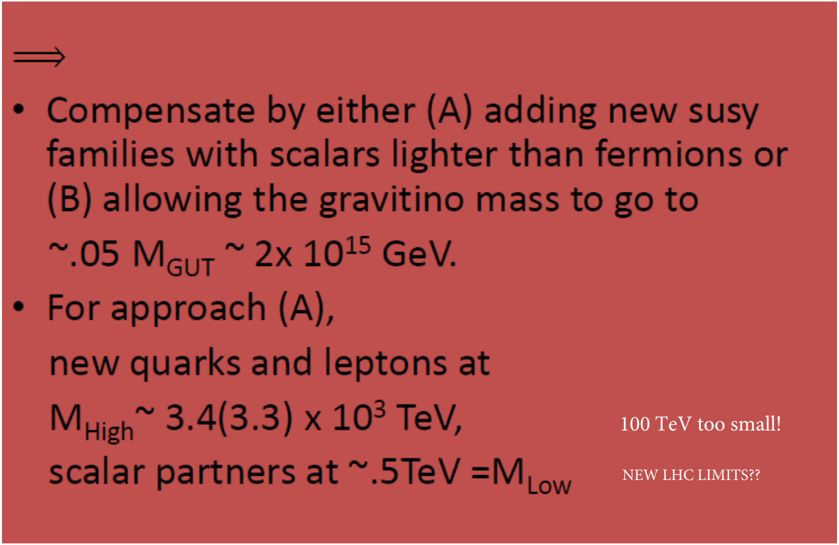
NEW LHC LIMITS??

$\Longrightarrow$

$$\Delta_{\mathrm{GUT}} = \sum_{j\in\{\text{MSSM low energy susy partners}\}} \frac{(-1)^F n_j}{\rho_j^2} \cong 1.13(1.12)\times 10^{-2}$$

NEW ISSUE

Constraints on SUSY GUTS



$\Longrightarrow$

- Compensate by either (A) adding new susy families with scalars lighter than fermions or (B) allowing the gravitino mass to go to ~.05 M_{GUT} ~ 2x 10^{15} GeV.
- For approach (A),

  new quarks and leptons at

  M_{High}~ 3.4(3.3) x 10^{3} TeV,

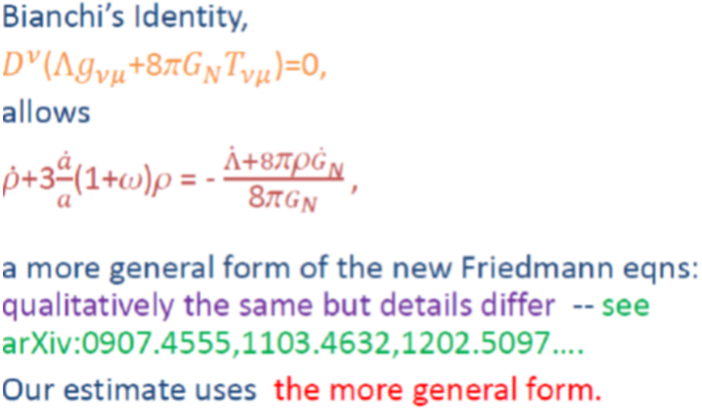
  scalar partners at ~.5TeV =M_{Low}

100 TeV too small!

NEW LHC LIMITS??

Constraints on SUSY GUTS

- GUT and other breaking scales: small compared to $0.01M_{Pl}^4/64$ => drop
- Covariance issues:



Bianchi's Identity,

$D^{\nu}(\Lambda g_{\nu\mu}+8\pi G_N T_{\nu\mu})=0,$

allows

$$\dot{\rho}+3\frac{\dot{a}}{a}(1+\omega)\rho = -\frac{\dot{\Lambda}+8\pi\rho\dot{G}_N}{8\pi G_N},$$

a more general form of the new Friedmann eqns: qualitatively the same but details differ -- see arXiv:0907.4555,1103.4632,1202.5097....

Our estimate uses the more general form.

SUMMARY

- Precision Quantum Field Theory: EW, QCD, QG $\equiv$ Control all limits:

  IR ($z \to 1$)

  and

  Collinear ($p_T \to 0$)

  UV limit

- We now have control over all aspects of the QG corrections.
- Toward quantitative understanding of ρ_Λ along with other precision observables: possible tests in new GWP($d^{UV}{}_H$)??

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