Amplitude Evolution beyond Leading Order

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At
Loopfest/RADCOR
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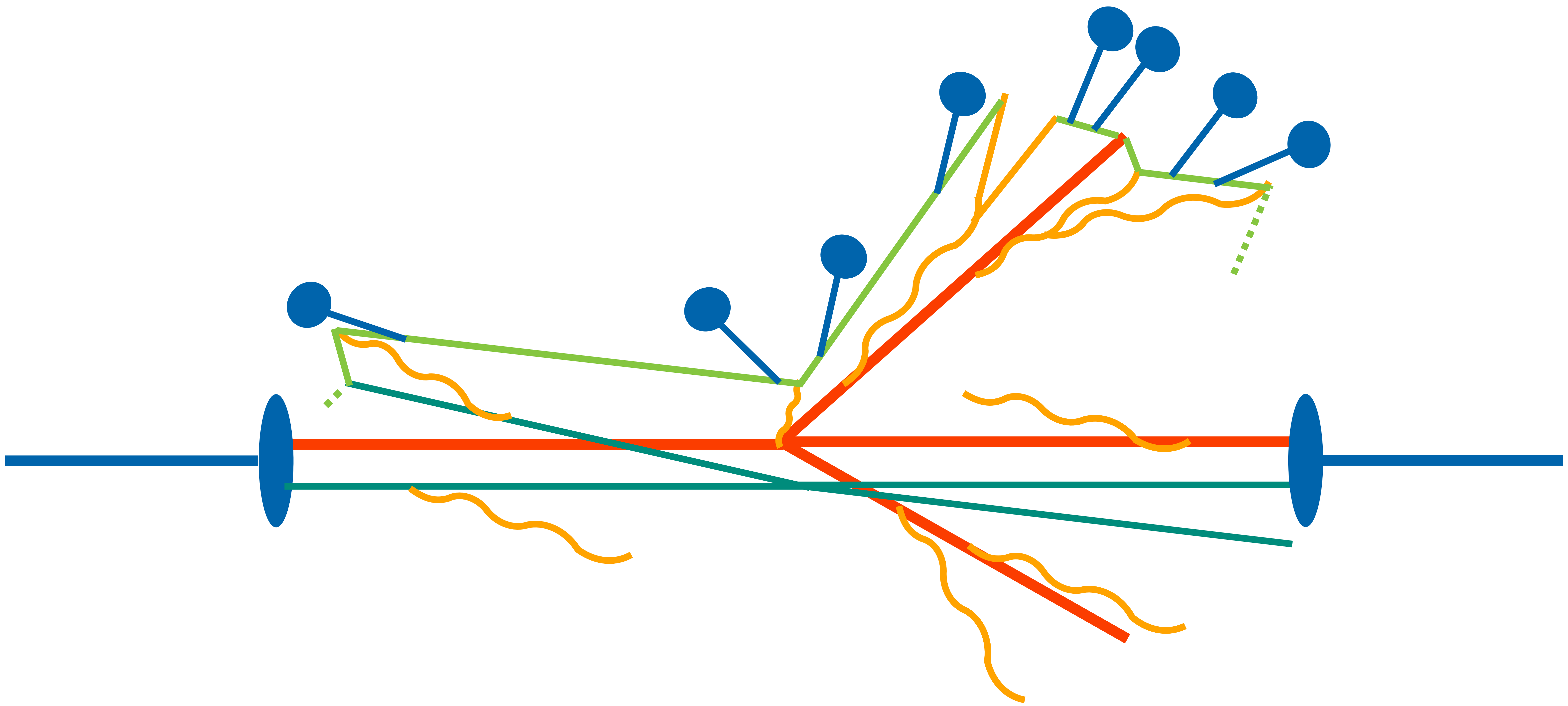
QCD cross sections

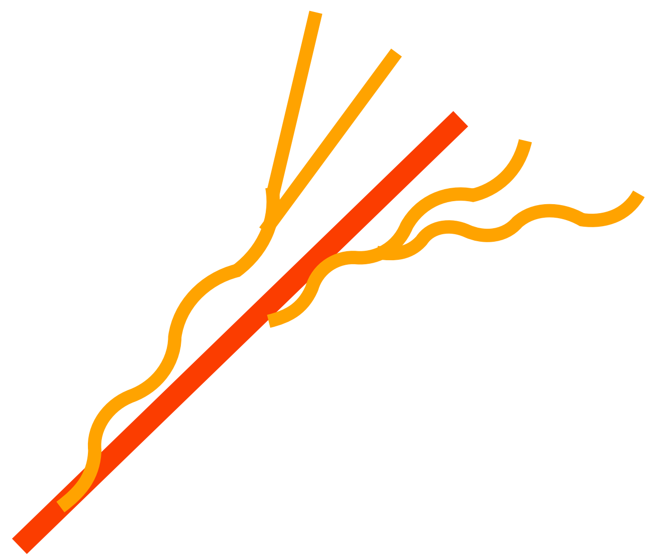


$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Setting the scene

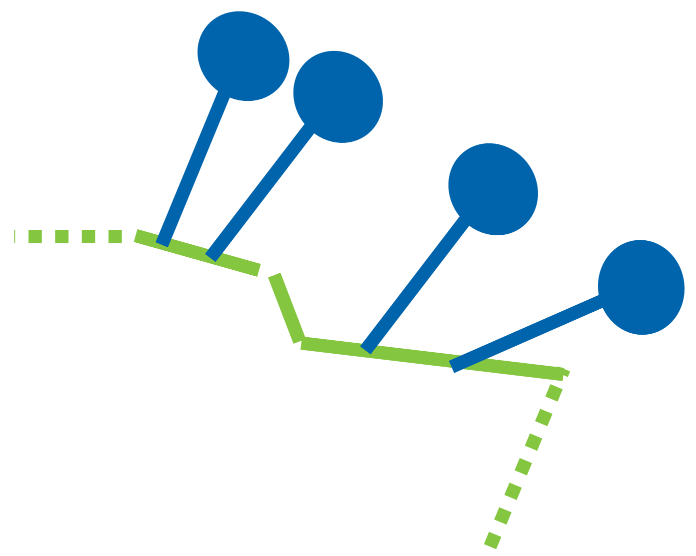
QCD description of collider reactions:
Complexity challenges precision.

Hard partonic scattering:
NLO QCD routinely



Jet evolution — parton branching:
NLL sometimes, mostly unclear

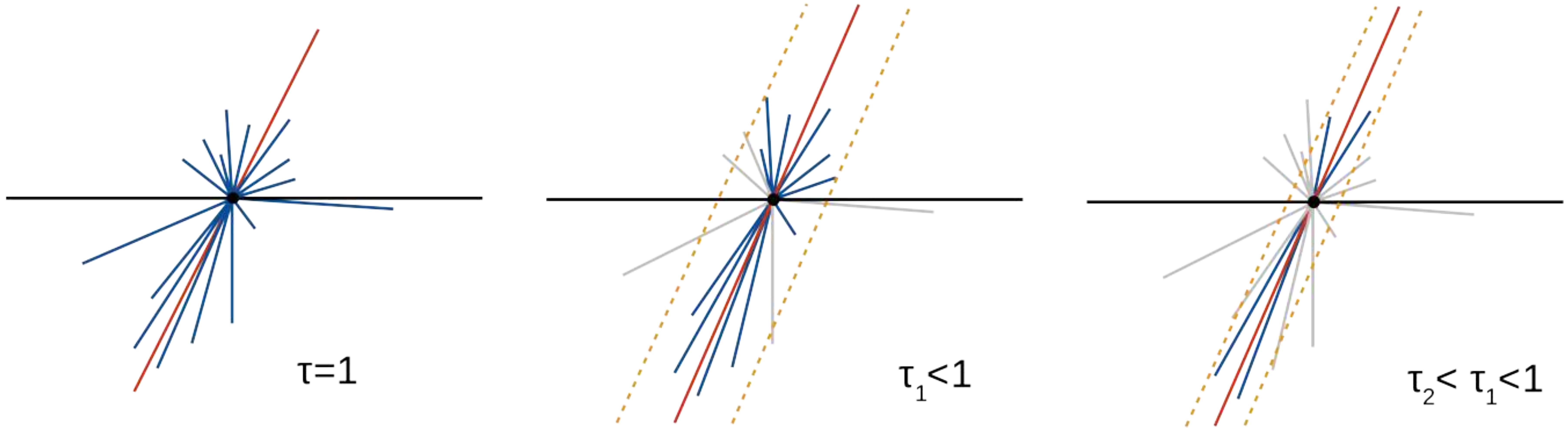
[Salam et al. — JHEP 09 (2018) 033]
[Forshaw, Holguin, Plätzer – JHEP 09 (2020) 014]



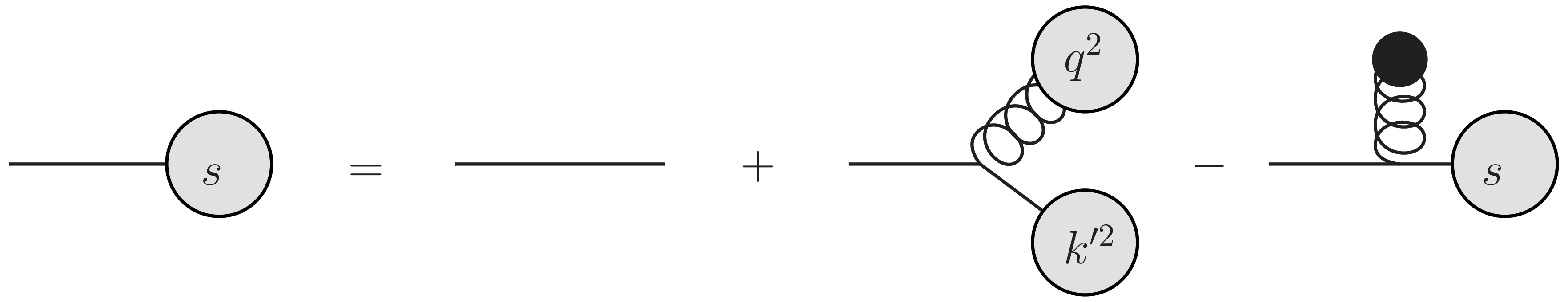
Multi-parton interactions
Hadronization

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

QCD Coherence

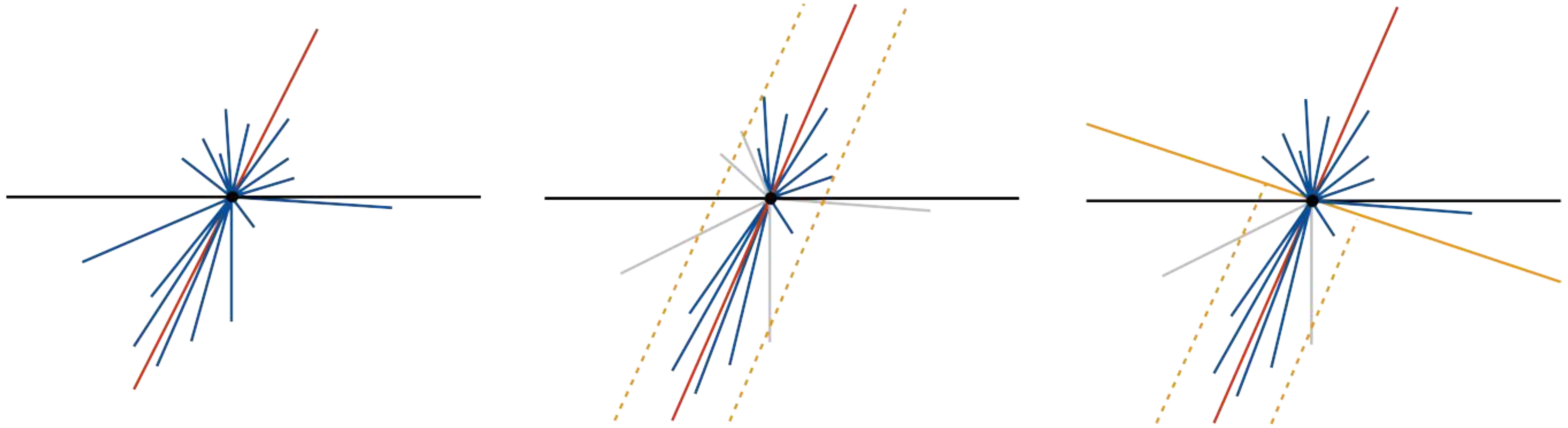


Resummation of observables which globally measure deviations from n-jet limit.
Basis of angular ordered parton showers — highest level of analytic control.



[Catani, Marchesini, Webber — Nucl.Phys.B 407 (1991) 654]
[Hoang, Plätzer, Samitz — JHEP 1810 (2018) 200]

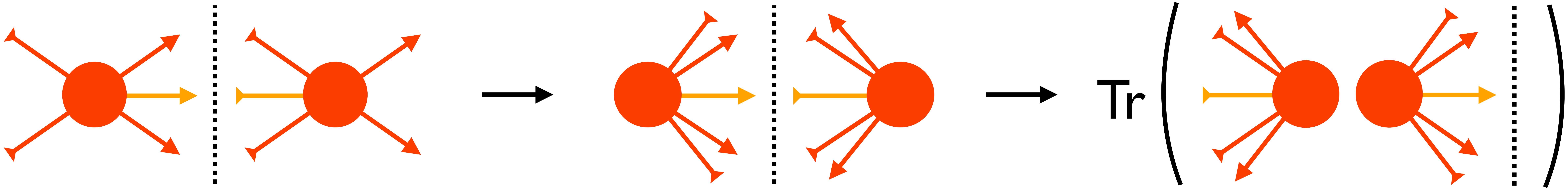
Non-global Observables



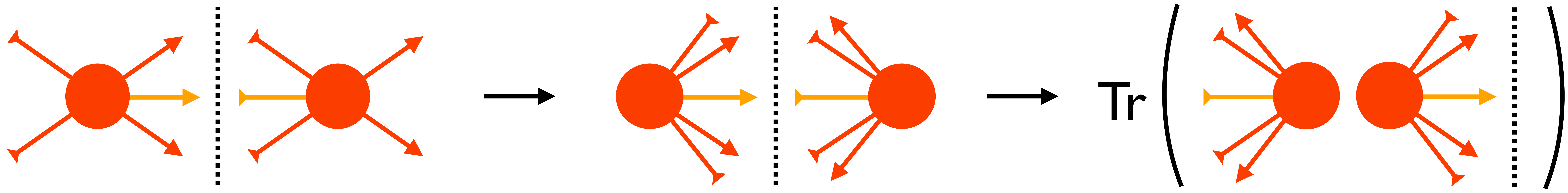
Measure deviation from jet topology only in patch of phase space.
Coherent branching breaks down, full complexity of QCD amplitudes strikes back.
If non-global bit is isolated can use dipole cascades to resum.

[Dasgupta, Salam, Banfi, Marchesini, Smye, Becher et al. …]

Cross Sections and Amplitudes



Cross Sections and Amplitudes



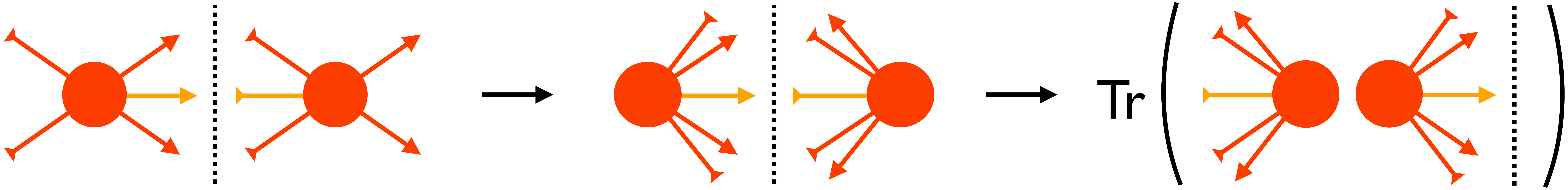
$$\sigma[u] = \sum_n \int \mathrm{Tr}\left[\mathbf{A}_n\right] u(q_1, \ldots, q_n) \mathrm{d}\phi(q_1, \ldots, q_n)$$

sum over emissions

'density operator' ~ amplitude amplitude^{+}

observable and phase space

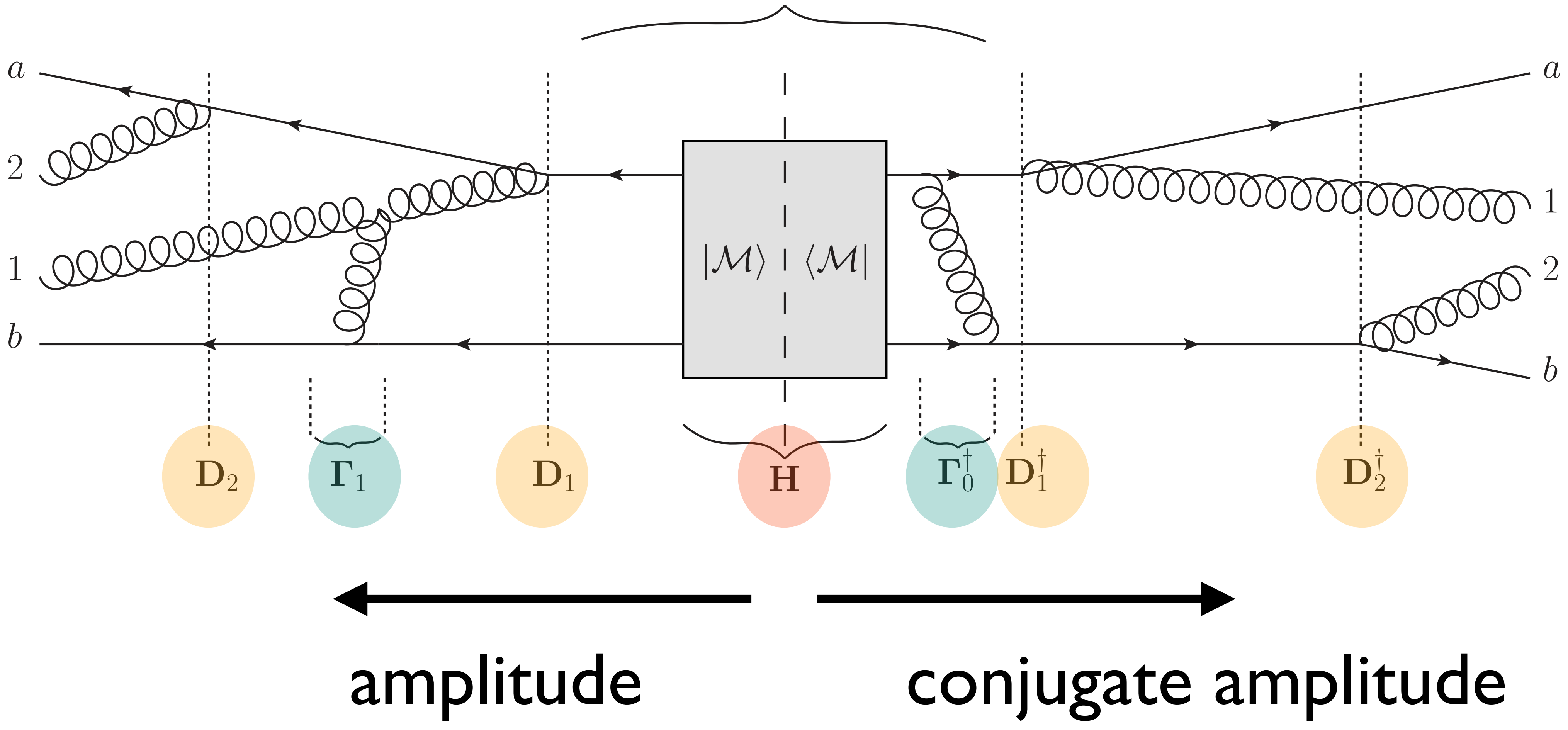
Cross Sections and Amplitudes



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathbf{D}_n(k)\; \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\; \mathbf{A}_{n-1}(k)\; \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}\; \mathbf{D}_n^\dagger(k)$$

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

Different histories in amplitude and conjugate amplitude needed to include interference.

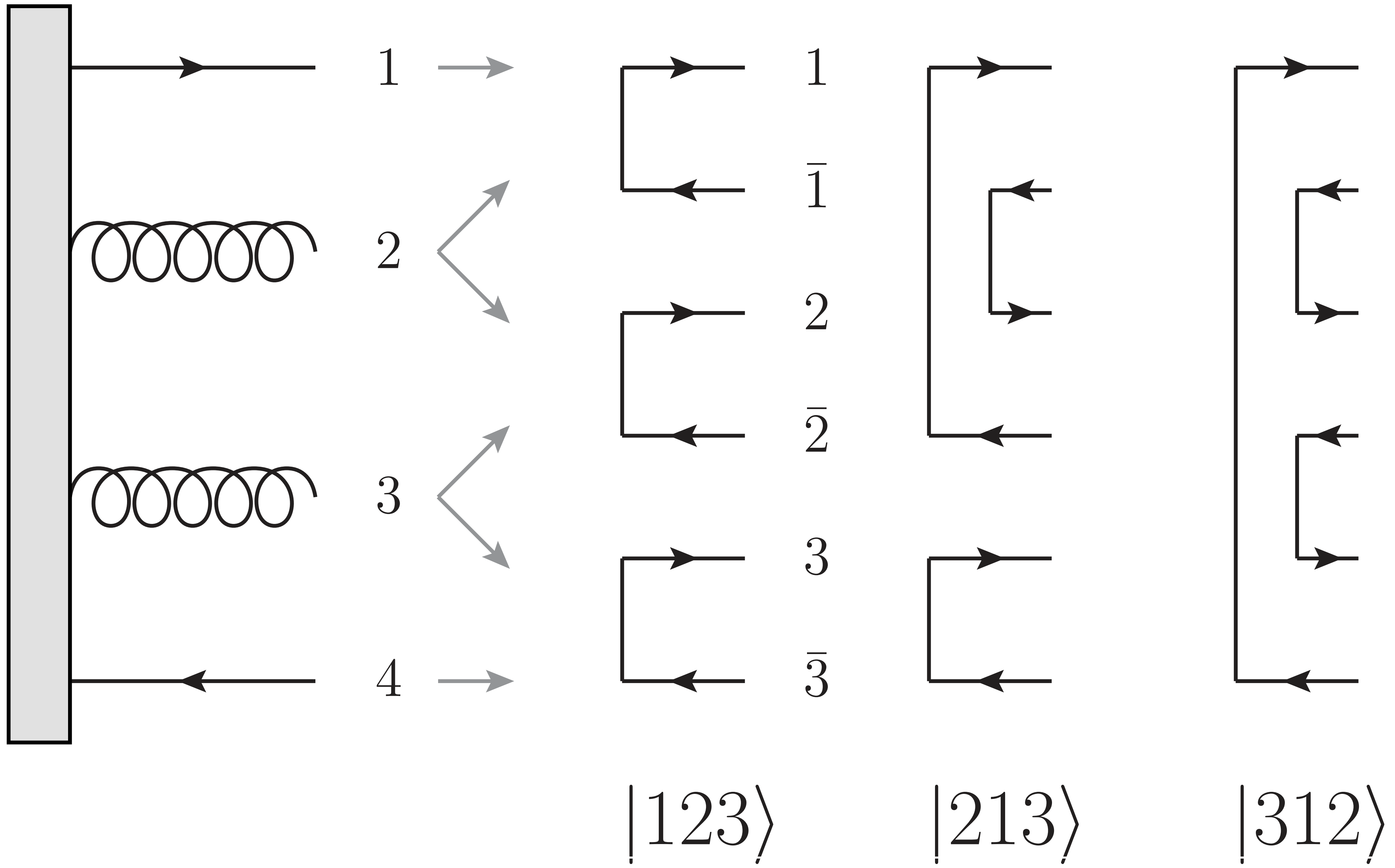


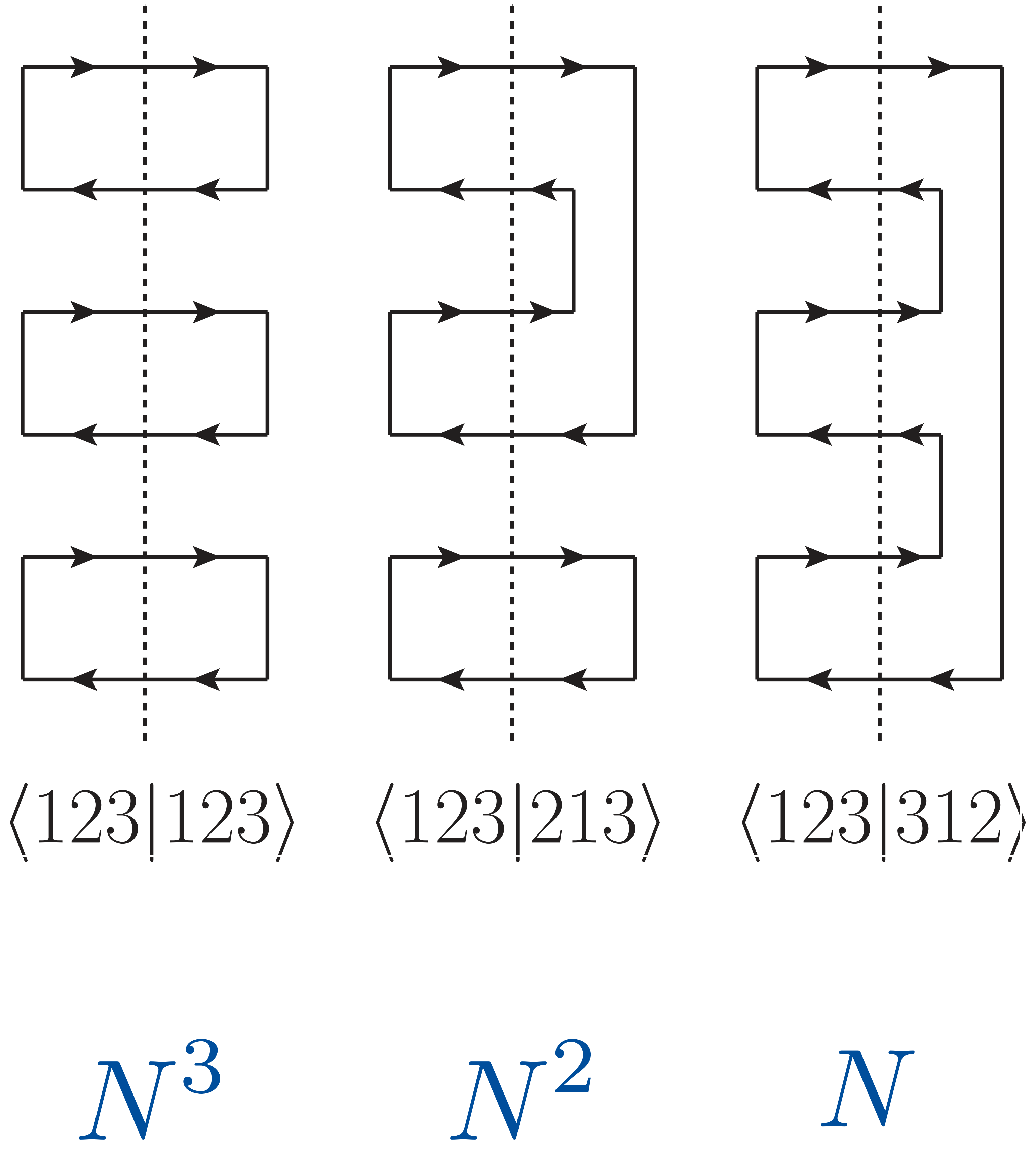
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

Tracking colour

Decompose amplitudes in flow of colour charge.

$$\mathrm{Tr}\left[\mathbf{A}_n\right] = \sum_{\sigma,\tau} A_{\tau\sigma} \langle\sigma|\tau\rangle$$



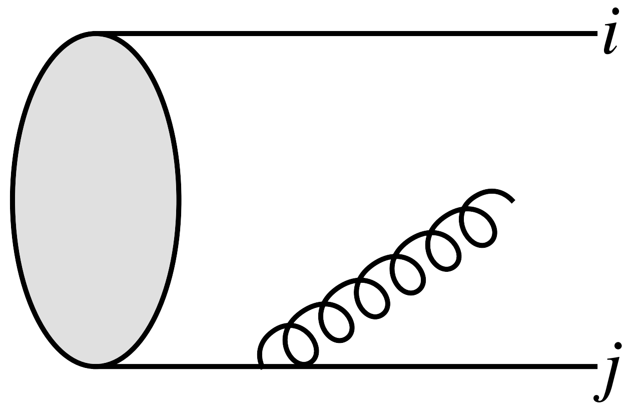


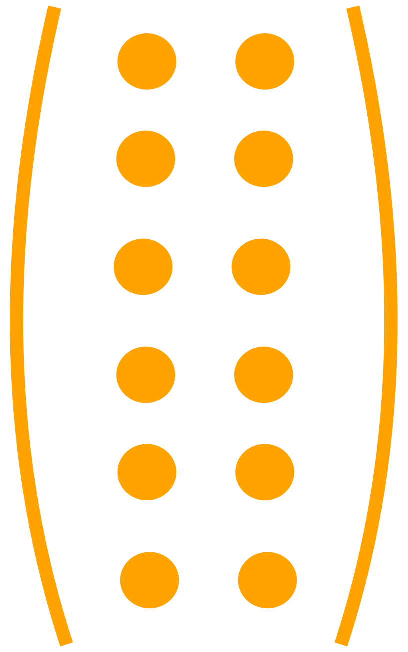
[Plätzer – EPJ C 74 (2014) 2907]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

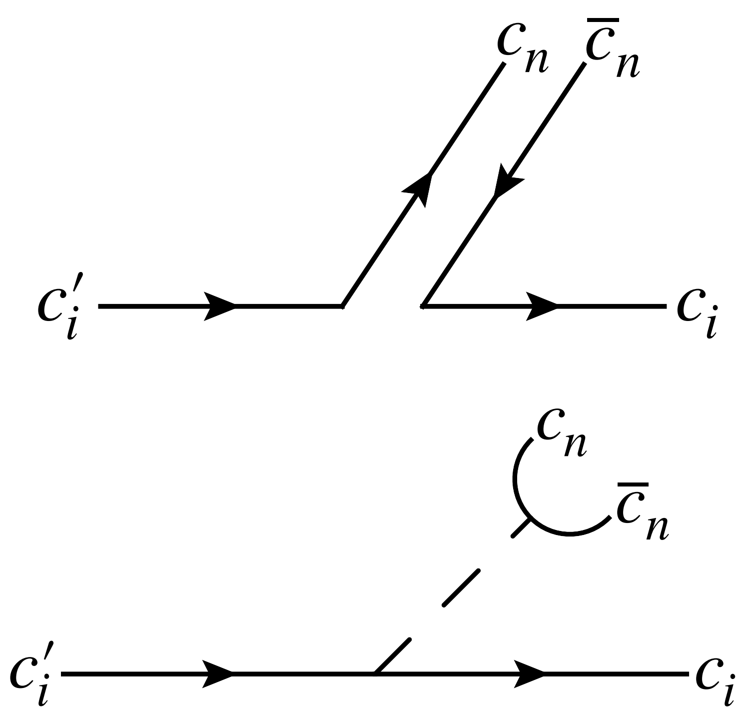
Tracking colour

Gluon emission

$$\mathbf{D}_n(k)$$



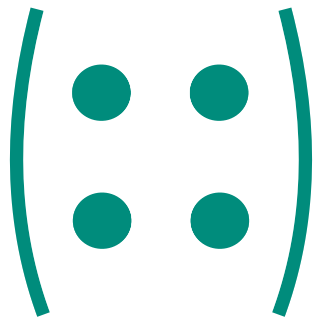


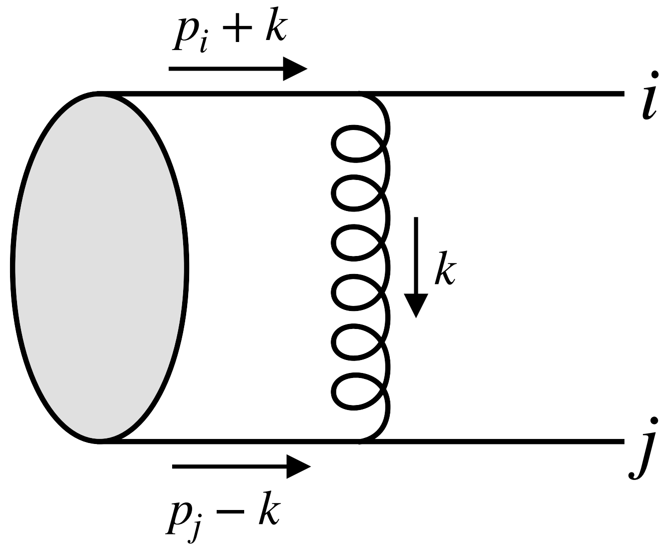


Explicit suppression in 1/N

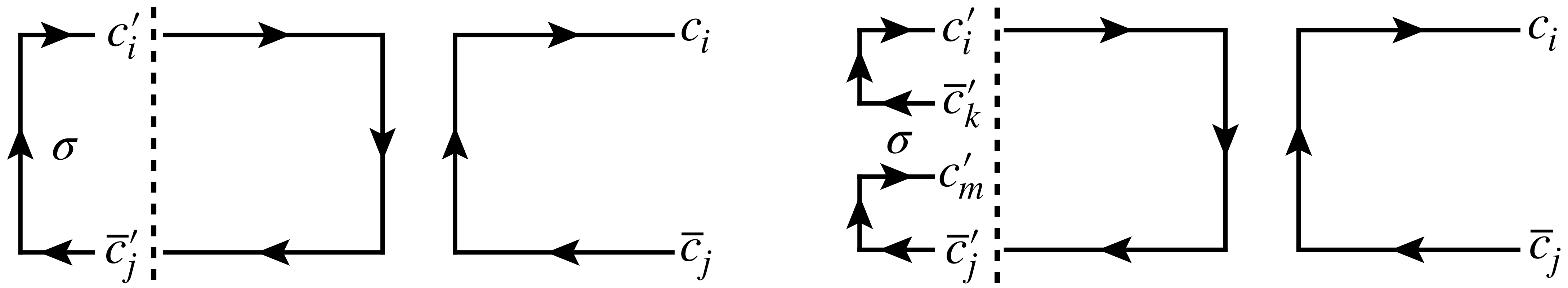
Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$





$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$



$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

[Plätzer – EPJ C 74 (2014) 2907] — diagrams from [Ruffa, MSc thesis 2020]

Non-global Observables and Large-N

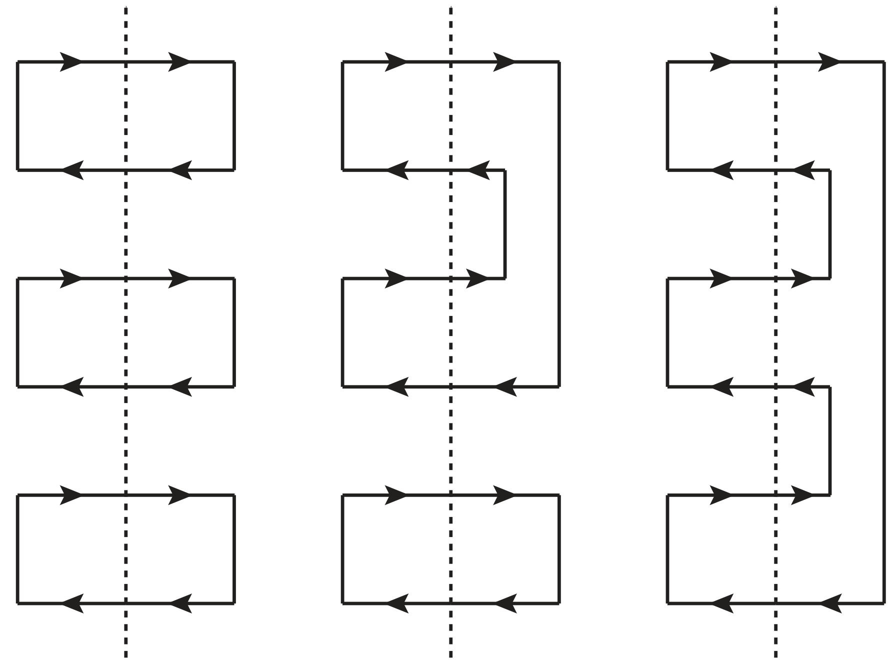
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Primary application: Non-global observables

$$E\frac{\partial \mathbf{G}_n(E)}{\partial E} = -\mathbf{\Gamma}\mathbf{G}_n(E) - \mathbf{G}_n(E)\mathbf{\Gamma}^\dagger + \mathbf{D}_n^\mu\, \mathbf{G}_{n-1}(E)\, \mathbf{D}_{n\mu}^\dagger\, u(E,\hat{k}_n)$$

Utilise colour flow basis, and expand around large-N:

$$\text{Leading}^{(l)}_{\tau\sigma}[\mathbf{A}] = \sum_{k=0}^{l} \mathcal{A}_{\tau\sigma}\Big|_{1/N^k}\ \delta_{\#\text{transpositions}(\tau,\sigma),l-k}$$



Re-derive BMS equation: Prototype of constructing a dipole shower

$$\text{Leading}^{(0)}_{\tau\sigma}\left[\mathbf{V}_n\mathbf{A}_n\mathbf{V}_n^\dagger\right] = \delta_{\tau\sigma}\left|V_\sigma^{(n)}\right|^2\ \text{Leading}^{(0)}_{\tau\sigma}[\mathbf{A}_n]$$

$$V_\sigma^{(n)} = \exp\left(-N\sum_{i,j\ \text{c.c. in}\ \sigma}\lambda_i\bar{\lambda}_j W_{ij}^{(n)}\right)$$

$$\text{Leading}^{(0)}_{\tau\sigma}\left[\mathbf{D}_n\mathbf{A}_{n-1}\mathbf{D}_n^\dagger\right] = \delta_{\tau\sigma}\sum_{i,j\ \text{c.c. in}\ \sigma\backslash n}\lambda_i\bar{\lambda}_j R_{ij}^{(n)}\ \text{Leading}^{(0)}_{\tau\backslash n,\sigma\backslash n}[\mathbf{A}_{n-1}]$$

colour connected dipoles

Colour (Flow) Evolution Beyond Leading Order

[Plätzer, Ruffa — arXiv:2012.15215 (to appear in JHEP)]

Include simultaneously unresolved emissions and higher loop structures

$$E\frac{\partial}{\partial E}\mathbf{A}_n(E) = \mathbf{\Gamma}_n(E)\mathbf{A}_n(E) + \mathbf{A}_n(E)\mathbf{\Gamma}_n^\dagger(E) - \sum_k \mathbf{R}_n^{(k)}(E)\mathbf{A}_{n-k}(E)\mathbf{R}_n^{(k),\dagger}(E)$$

combination of purely virtual and unresolved real corrections, point-by-point in phase space

resolved real emissions and virtual/ unresolved corrections to emissions

Similar in origin to a fixed-order calculation with the subtraction method:

Subtract (unresolved) real emissions — cast virtual corrections into phase-space type integrals instead of integrating subtraction terms.

Leading Order Evolution

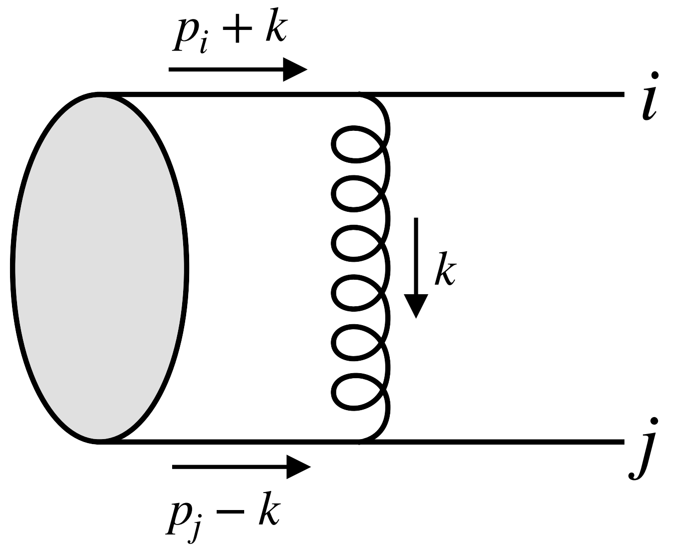
CF vs CA/2

dipole flips

$$\mathbf{\Gamma}^{(1)} = \frac{1}{2}\sum_{i,j}\Omega^{(1)}_{ij}\ \frac{1}{N}\mathbf{T}_i\cdot\mathbf{T}_j \qquad [\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma^{(1)}_\sigma + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma^{(1)}_{\sigma\tau}$$

Expand around colour diagonal limit

Cast e.g. into form of energy ordered observables by doing a contour integral



$$\Omega^{(1)}_{ij} = i\mu^{2\epsilon}\int \frac{\text{đ}^d k}{i\pi^{d/2}}\frac{p_i\cdot p_j}{(k^2+i0)(p_i\cdot k+i0)(p_j\cdot k - i0)} = \int_0^\infty \frac{\mathrm{d}E}{E}\left(\frac{\mu^2}{E^2}\right)^\epsilon \omega^{(ij)}$$

$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\ n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

Beyond Leading Colour

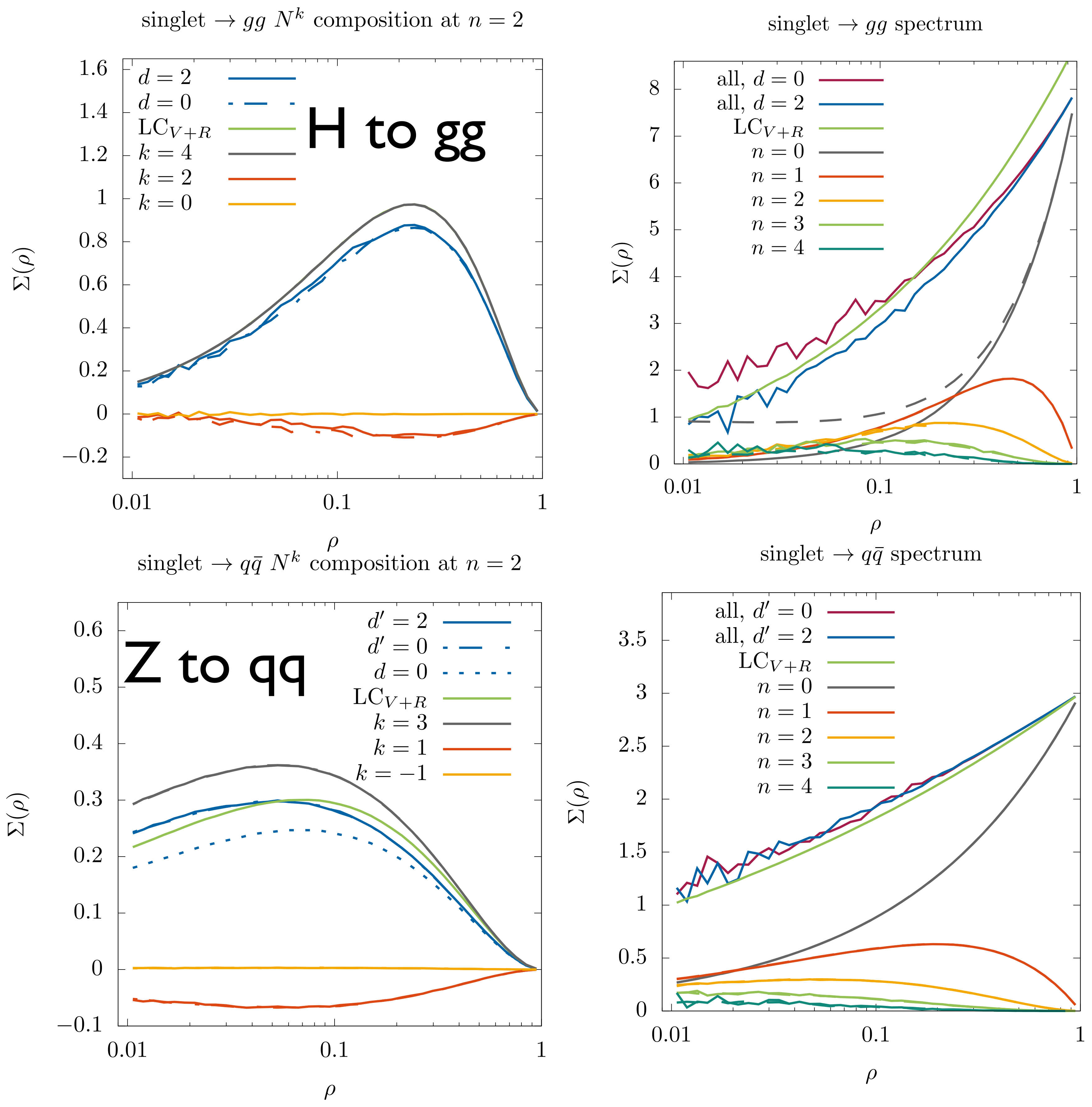
universität wien

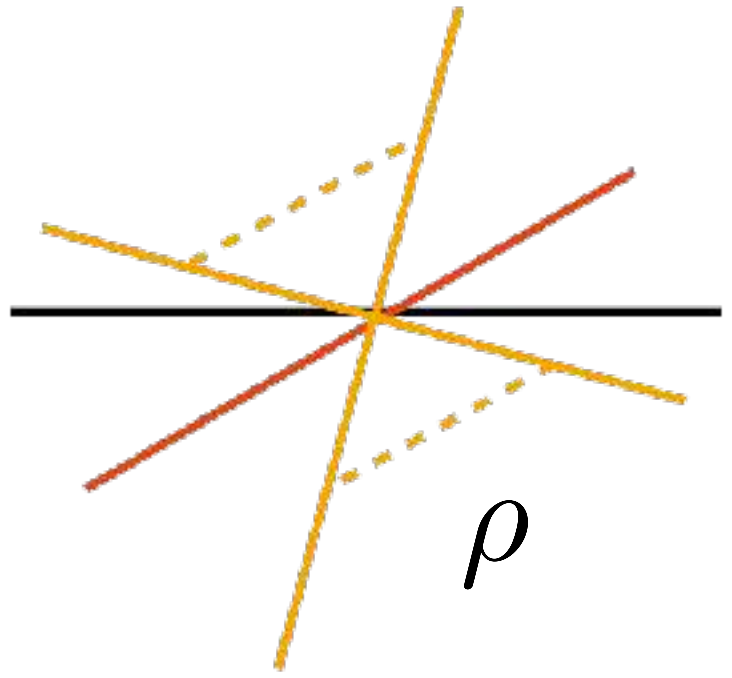
[De Angelis, Forshaw, Plätzer — PRL 126 (2021) 11]

CVolver library implements numerical evolution in colour space.

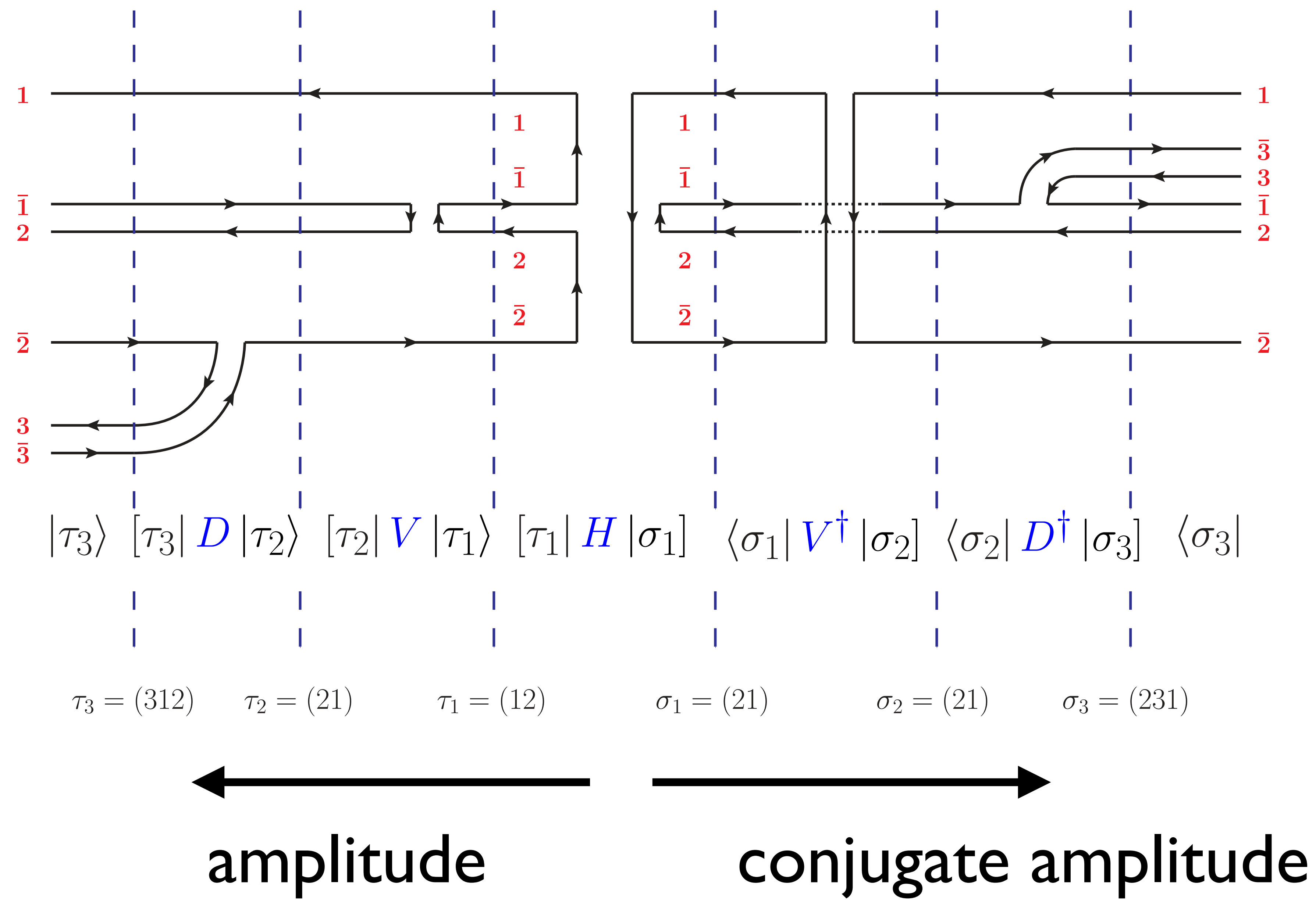
origins in [Plätzer – EPJ C 74 (2014) 2907]

Resummation of non-global logarithms at full colour:





$$\Sigma(\rho) = \sum_n \int \mathrm{d}\sigma(\{p_i\}) \prod_i \theta_{\mathrm{in}}(\rho - E_i)$$

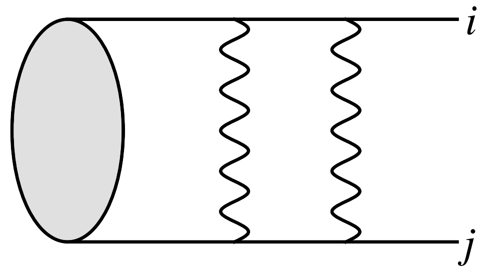
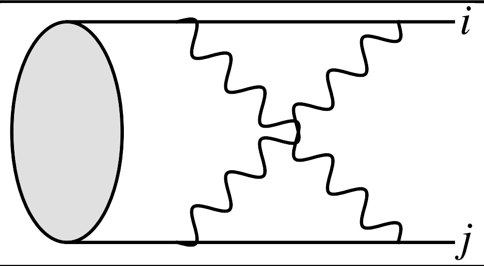
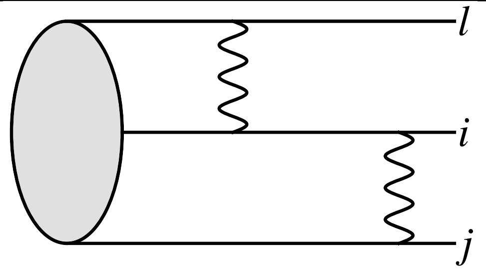
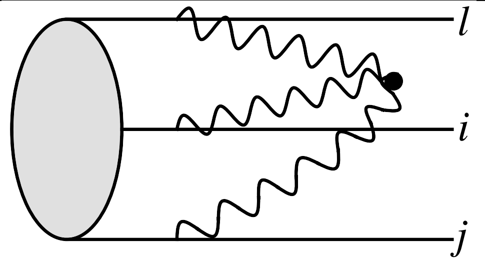
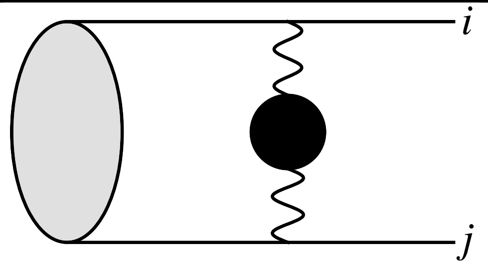
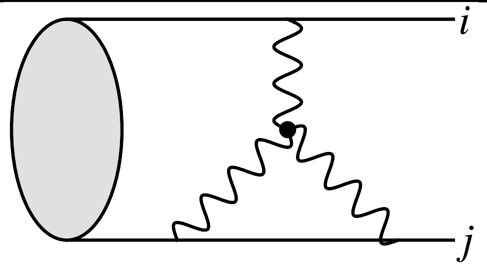
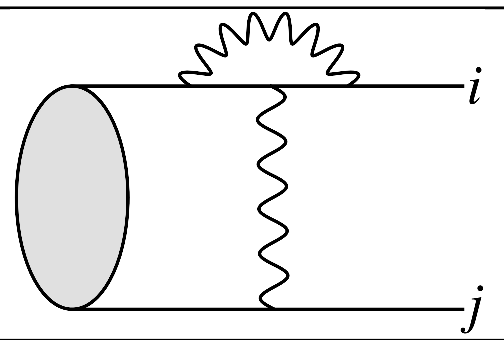


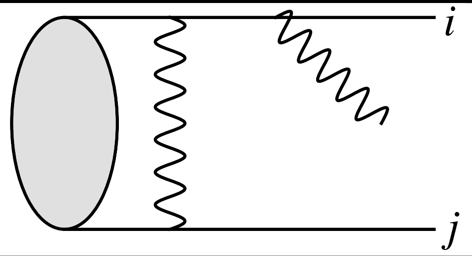
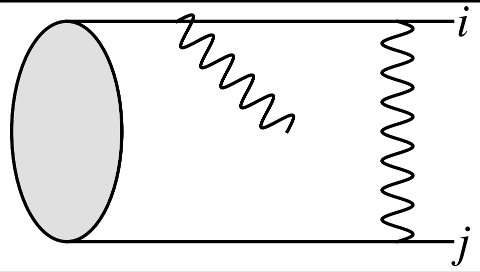
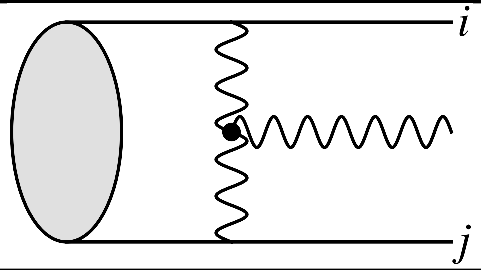
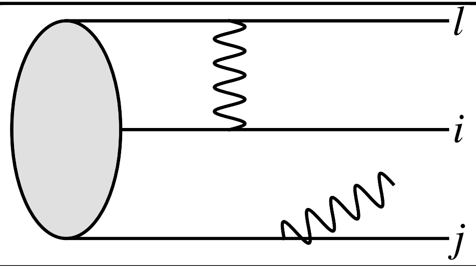
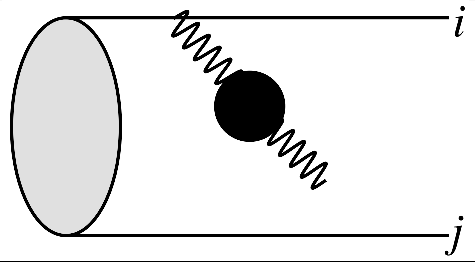
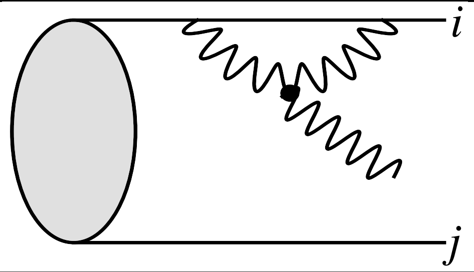
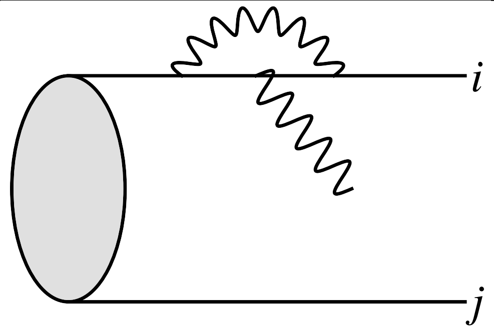
Avoid complexity which grows with colour space dimensionality:

- Monte Carlo over colour flows,
- events at intermediate steps carry complex weights.

Evolution at the Next Order

[Plätzer, Ruffa — arXiv:2012.15215 (to appear in JHEP)]

Coefficient	Diagram	Colour-factor
$\Omega^{(2)}_{ij}$		$(\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\tilde{\Omega}^{(2)}_{ij}$		$\mathbf{T}^a_i\mathbf{T}^b_i\mathbf{T}^b_j\mathbf{T}^a_j$
$\Omega^{(2)}_{ijl}$		$(\mathbf{T}_i \cdot \mathbf{T}_l)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}^{(2)}_{ijl}$		$if^{abc}\mathbf{T}^a_i\mathbf{T}^b_j\mathbf{T}^c_l$
$\Omega^{(2)}_{ij,\mathrm{self-en.}}$		$T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\Omega^{(2)}_{ij,\mathrm{vertex-corr.}}$		$T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}^{(2)}_{ij}$		$\mathbf{T}^b_i\mathbf{T}^a_i\mathbf{T}^b_i\mathbf{T}^a_j$

Coefficient	Diagram	Colour-factor
$\Omega^{(1,1)}_{ij}$		$\mathbf{T}^a_i(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\tilde{\Omega}^{(1,1)}_{ij}$		$(\mathbf{T}_i \cdot \mathbf{T}_j)\mathbf{T}^a_i$
$\overline{\Omega}^{(1,1)}_{ij}$		$if^{abc}\mathbf{T}^b_i\mathbf{T}^c_j$
$\Omega^{(1,1)}_{ijl}$		$\mathbf{T}^a_l(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\Omega^{(1,1)}_{i,\mathrm{self-en.}}$		$T_R\mathbf{T}^a_i$
$\Omega^{(1,1)}_{i,\mathrm{vertex-corr.}}$		$T_R\mathbf{T}^a_i$
$\hat{\Omega}^{(1,1)}_{ij}$		$\mathbf{T}^b_i\mathbf{T}^a_i\mathbf{T}^b_i$

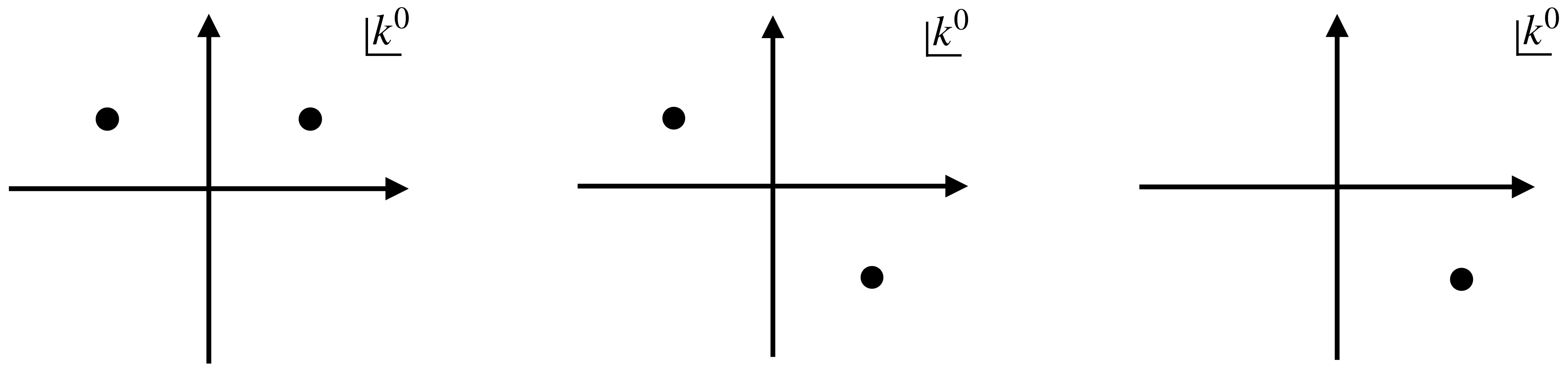
Cutting rules

Algorithmic treatment of virtual corrections needed

[Plätzer, Ruffa — arXiv:2012.15215 (to appear in JHEP)]

$$\frac{1}{k^2 + i0(T\cdot k)^2} = \frac{1}{(k^0)^2 - \vec{k}^2 + 2i0(k^0)^2}$$

$(T^\mu) = (\sqrt{2}, \vec{0})$



Advanced Feynman

Feynman tree theorem:

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]} = \frac{1}{[q^2 + i0(T\cdot q)^2]} + 2\pi i\delta(q^2)\theta(T\cdot q)$$

Cutting rules

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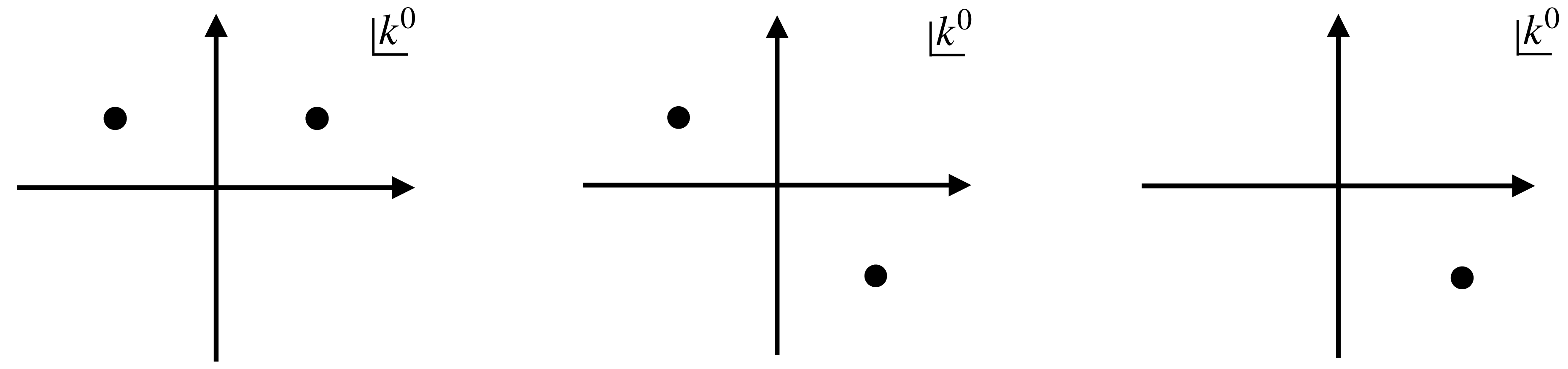
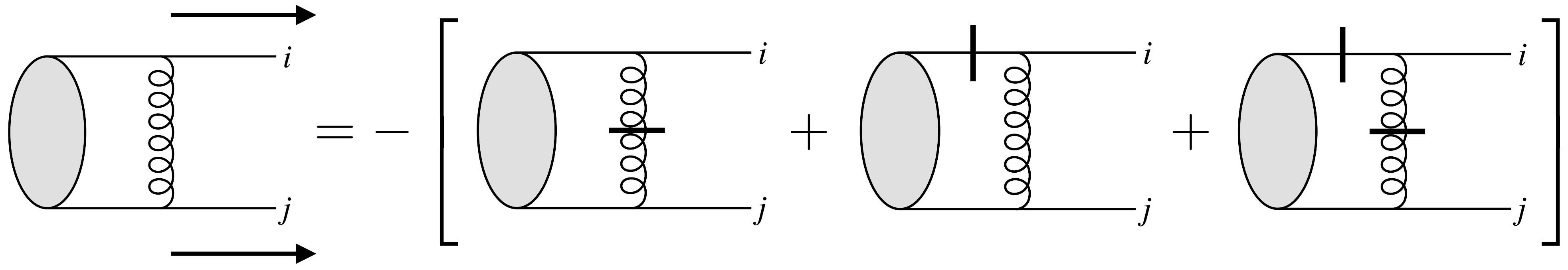


Advanced   Feynman   Eikonal

Feynman tree theorem:

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]} = \frac{1}{[q^2 + i0(T\cdot q)^2]} + 2\pi i \delta(q^2)\theta(T\cdot q)$$



$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

Extend to Eikonal and higher-power propagators:

$$\frac{1}{2p_i\cdot k - i0(T\cdot p_i)^2} = \frac{1}{2p_i\cdot k + i0(T\cdot p_i)^2} + 2\pi i\ \delta(2p_i\cdot k)$$

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]^2} - \frac{1}{[q^2 + i0(T\cdot q)^2]^2} = -2i\pi\theta(T\cdot q)\delta'(q^2)$$

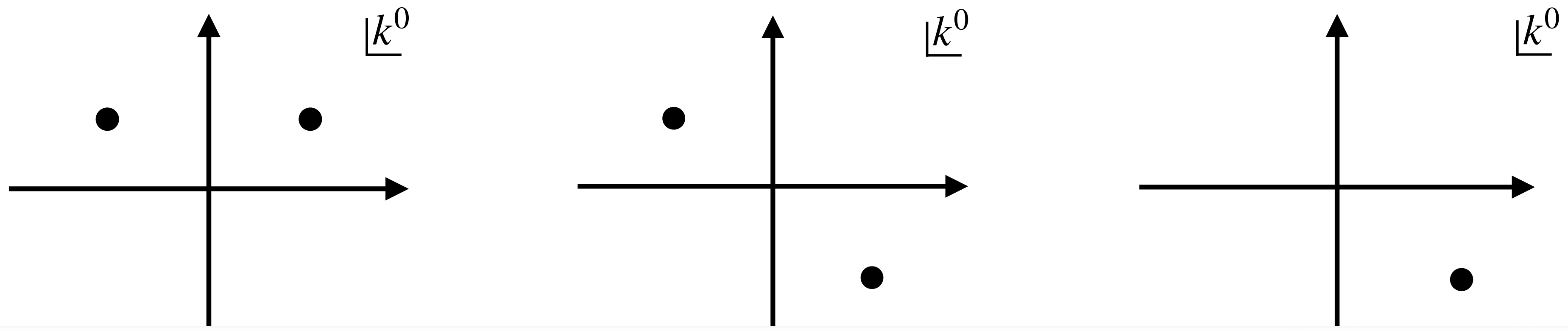
Cutting rules

[Plätzer, Ruffa — arXiv:2012.15215 (to appear in JHEP)]

Algorithmic treatment of virtual corrections needed

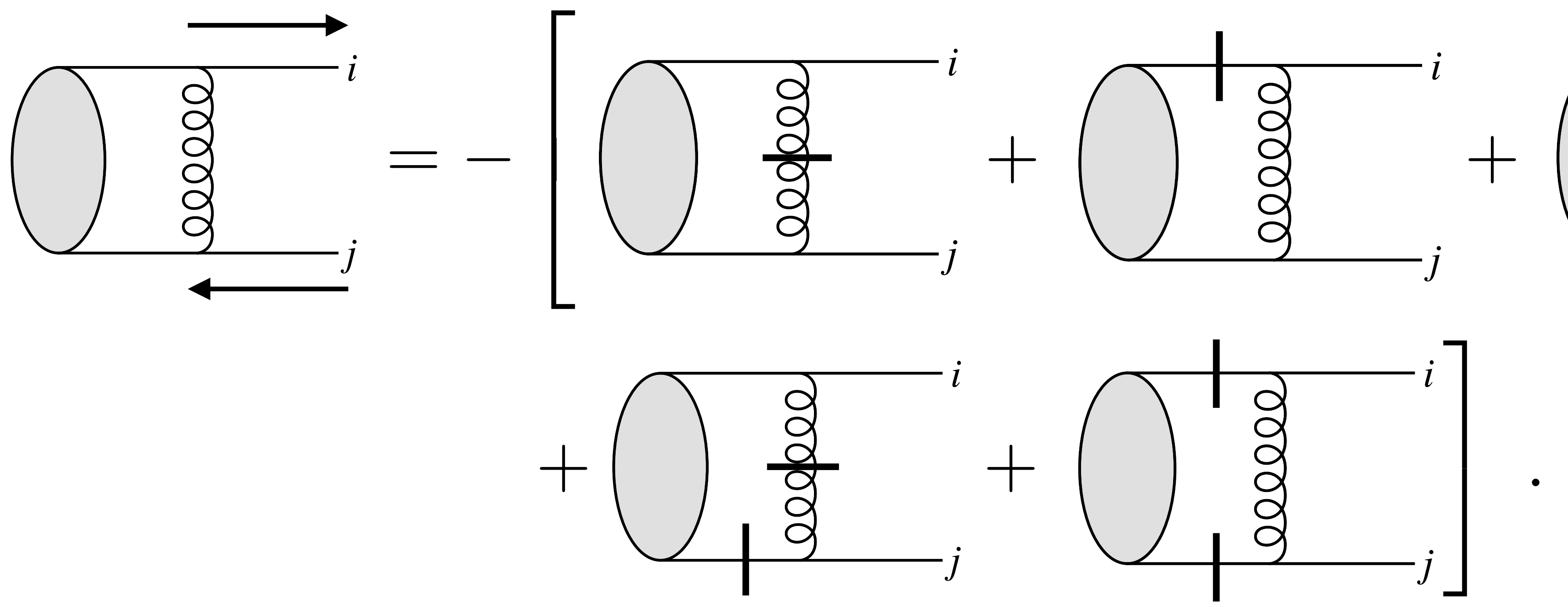
$$\frac{1}{k^2 + i0(T\cdot k)^2} = \frac{1}{(k^0)^2 - \vec{k}^2 + 2i0(k^0)^2}$$

$(T^\mu) = (\sqrt{2}, \vec{0})$



Feynman tree theorem:

$$\frac{1}{[q^2 \ldots i0(T \ldots q)|T\cdot q|]} = \frac{1}{[q^2 + i0(T\cdot q)^2]} + 2\pi i\delta(q^2)\theta(T\cdot q)$$



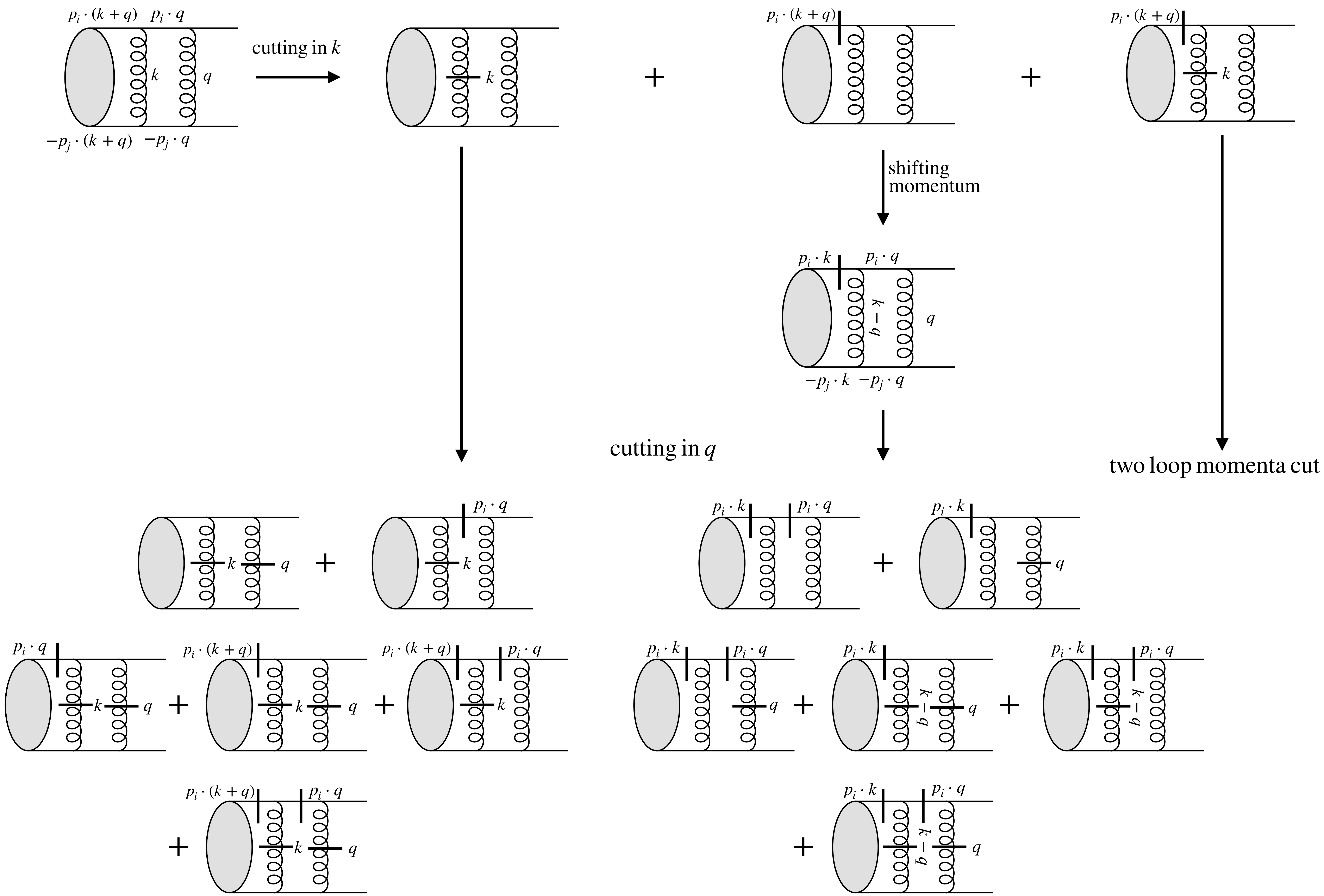
$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} - i\pi \int \frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

l higher-power propagators:

$$\ldots p_i)^2} = \frac{1}{2p_i\cdot k + i0(T\cdot p_i)^2} + 2\pi i\;\delta(2p_i\cdot k)$$

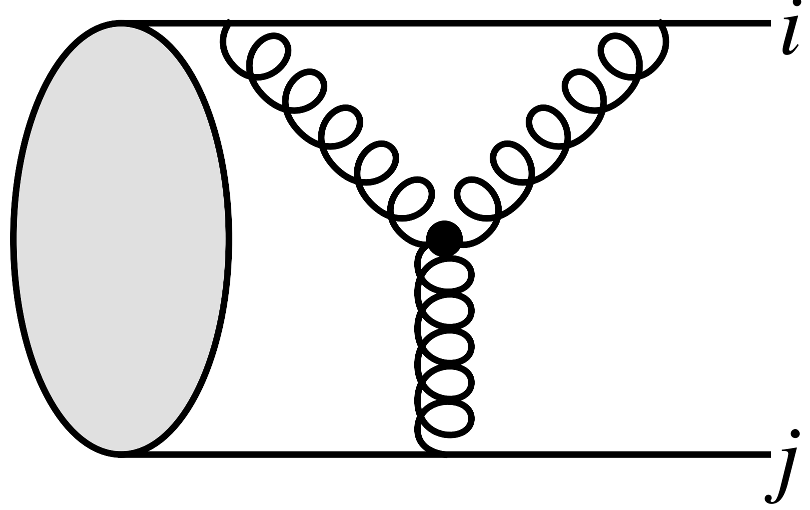
$$\ldots \cdot q|]^2} - \frac{1}{[q^2 + i0(T\cdot q)^2]^2} = -2i\pi\theta(T\cdot q)\delta'(q^2)$$

Cutting rules



Cutting rules

$$\mu^{4\varepsilon}\int \frac{\text{đ}^d k}{i\pi^{d/2}}\frac{\text{đ}^d q}{i\pi^{d/2}}\frac{1}{[p_i\cdot k+i0][-p_j\cdot(k+q)+i0][k^2+i0][q^2+i0][(k+q)^2+i0]}$$



$$\mu^{4\varepsilon}\int \frac{\text{đ}^d k}{i\pi^{d/2}}\frac{\text{đ}^d q}{i\pi^{d/2}}\Bigg\{\frac{(2\pi i)^2\tilde{\delta}(k)\tilde{\delta}(q)}{[p_i\cdot k+i0][(k+q)^2+i0][-p_j\cdot(k+q)+i0]}$$
$$+\frac{(2\pi i)^2\tilde{\delta}(q)\delta(p_i\cdot k)}{[k^2+i0][(k+q)^2+i0][-p_j\cdot(k+q)+i0]}$$
$$+\frac{(2\pi i)^2\tilde{\delta}(k)\tilde{\delta}(q)}{[p_i\cdot k+i0][(q-k)^2+i0][-p_j\cdot q+i0]}$$
$$+\frac{(2\pi i)^2\tilde{\delta}(q)\delta(p_i\cdot k)}{[k^2+i0][(q-k)^2+i0][-p_j\cdot q+i0]}$$
$$-\frac{(2\pi i)^2\tilde{\delta}(q)\tilde{\delta}(k)}{[p_i\cdot(k-q)+i0][(k-q)^2+i0][-p_j\cdot k+i0]}$$
$$+\frac{(2\pi i)^3\tilde{\delta}(k)\tilde{\delta}(q)\tilde{\delta}(k+q)}{[p_i\cdot k+i0][-p_j\cdot(k+q)+i0]}$$
$$+\frac{(2\pi i)^3\tilde{\delta}(q)\tilde{\delta}(k+q)\delta(p_i\cdot k)}{[k^2+i0][-p_j\cdot(k+q)+i0]}+\frac{(2\pi i)^3\tilde{\delta}(q)\tilde{\delta}(k)\tilde{\delta}(q-k)}{[p_i\cdot k+i0][-p_j\cdot q+i0]}$$
$$+\frac{(2\pi i)^3\tilde{\delta}(q-k)\tilde{\delta}(q)\delta(p_i\cdot k)}{[k^2+i0][-p_j\cdot q+i0]}\Bigg\}\,.$$

Eikonal coupling only to hard lines!

see also [Angeles, Forshaw, Seymour – JHEP 12 (2015) 091]

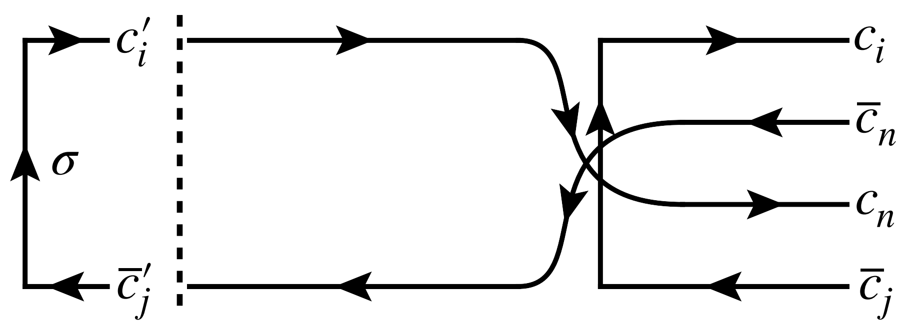
Colour structures at two loops

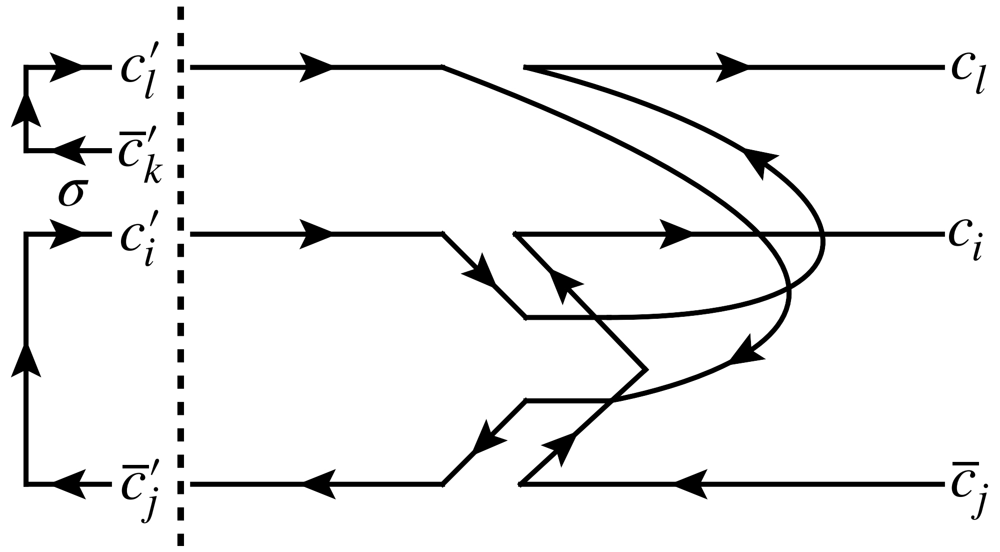
[Plätzer, Ruffa — arXiv:2012.15215 (to appear in JHEP)]

Analyze two-loop and one-loop/one-emission structures in colour flow basis.

$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$

$$[\tau|\mathbf{\Gamma}^{(1,1)}|\sigma\rangle = \left(\frac{1}{N^2}\rho_{\sigma\tau} + \frac{1}{N^4}\rho_\tau\right)\delta_{\sigma\tau\backslash n} + \frac{1}{N}\hat{\Sigma}^{(1,1)}_{\sigma\tau} + \frac{1}{N^3}\left(\tilde{\Sigma}^{(1,1)}_{\sigma\tau} + \hat{\tilde{\Sigma}}_{\sigma\tau}\right) + \frac{1}{N^2}\left(\Sigma'^{(1,1)}_{\sigma\tau} + \Sigma''^{(1,1)}_{\sigma\tau}\right)$$





$$[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle = \left(\Gamma^{(2)}_\sigma + \frac{1}{N^2}(\rho_\sigma + \tilde{\rho}) + \frac{1}{N^4}\rho^{(2)}\right)\delta_{\sigma\tau}$$

no swaps

$$+ \frac{1}{N}\left(\Sigma^{(2)}_{\sigma\tau} + \hat{\Sigma}^{(2)}_{\sigma\tau}\right) + \frac{1}{N^3}\tilde{\Sigma}^{(2)}_{\sigma\tau} + \frac{1}{N^2}\left(\Sigma'^{(2)}_{\sigma\tau} + \Sigma''^{(2)}_{\sigma\tau}\right)$$

single dipole swaps

double dipole swaps

Colour structures imply colour-diagonal **three parton correlations:** Dipoles are not enough. New approaches need to take this into account.

Summary & Outlook

Amplitude level evolution sets level of complexity to understand and design parton shower and resummation algorithms.

Crucial to address effects of Coulomb/Glauber phases, factorisation violation, super-leading logarithms, … — otherwise out of reach.

Investigate soft gluon effects at two loops (and related):

- Understand colour structures for many external legs and systematically expand around large-N limit.
- Design resummation of non-global observables beyond leading log and leading-N, investigate phases.
- Key ingredient for decisive statements about most flexible parton showers beyond leading order.

Thank you!