Next-to-Soft Virtual Corrections to Differential Cross-sections at LHC

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Based on AAH, P. *Mukherjee, V Ravindran, A. Sankar, S. Tiwari,* ***2010.00079*** *(accepted in PRD)*

Threshold Expansion

- Threshold effects arises when all the real emissions are uniformly soft : corresponds to the limit when $\tau = \dfrac{Q^2}{S} \to 1$.

$Q^2 \longrightarrow$ Invariant mass
$S \longrightarrow$ Hadronic center of mass energy
$y \longrightarrow$ Rapidity distribution

- At partonic level, this limit is when the partonic scaling variables $z_1, z_2 \to 1$ simultaneously.

$$z_i = \frac{x_{i0}}{x_i}$$
$$x_{(1,2)0} = \sqrt{\tau} e^{\pm y}$$

- In general the cross section at the threshold region can be expressed in terms of their singular behaviour.

- Consider the differential rapidity distribution of a colorless state in hadron collisions :

$\mu_R \to$ Renormalization scale
$\mu_F \to$ Factorization scale
$\sigma_B^c \to$ Born cross section

$$\frac{d\sigma^c}{dy} = \sigma_{\rm B}^c(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_{x_1^0}^1 \frac{dz_1}{z_1} \int_{x_2^0}^1 \frac{dz_2}{z_2} f_a\left(\frac{x_1^0}{z_1}, \mu_F^2\right) f_b\left(\frac{x_2^0}{z_2}, \mu_F^2\right) \Delta_{d,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2)$$

Parton distribution functions (the f_a, f_b factors)

Finite partonic coefficient function (the $\Delta_{d,ab}$ factor)

Threshold Expansion

- **Perturbative structure of coefficient function at threshold :** $z_{1,2} \to 1$

$$\Delta_d^c \to \Delta_d^{c,\text{virt}}\delta(1-z_1)\delta(1-z_2) \;+\; \sum_{k,l} D_k(z_i)\; D_l(z_j)\; \Delta^c_{d,(k,l)}(z_1,z_2) + \mathcal{O}(1-z)$$

Finite — *beyond NSV*

$$\delta(1-z_i)\;\delta(1-z_j)$$

$$\delta(1-z_i)\left(\frac{\ln^k(1-z_j)}{1-z_j}\right)_+$$

$$\left(\frac{\ln^k(1-z_i)}{1-z_i}\right)_+\left(\frac{\ln^k(1-z_j)}{1-z_j}\right)_+$$

Soft-Virtual

Most singular

$$\delta(1-z_i)\;\ln^k(1-z_j)$$

$$\left(\frac{\ln^k(1-z_j)}{1-z_j}\right)_+\;\ln^k(1-z_j)$$

Next to Soft-Virtual (NSV)

Singular, but suppressed to SV

- **Only diagonal channels**
- **Diagonal and off-diagonal channels**

Threshold Expansion

- Though NSV logarithms are less singular than SV ones, their contributions are numerically sizeable : often comparable or beyond the leading SV corrections.

- For instance, for inclusive $gg \to H$ at a_s^3 :

SV :	-1.38 % of born
NSV :	25 % of born

[Anastasiou, Duhr, Dulat et al]

- Similar findings can be seen for rapidity distributions too! [Dulat, Mistlberger, Pelloni]

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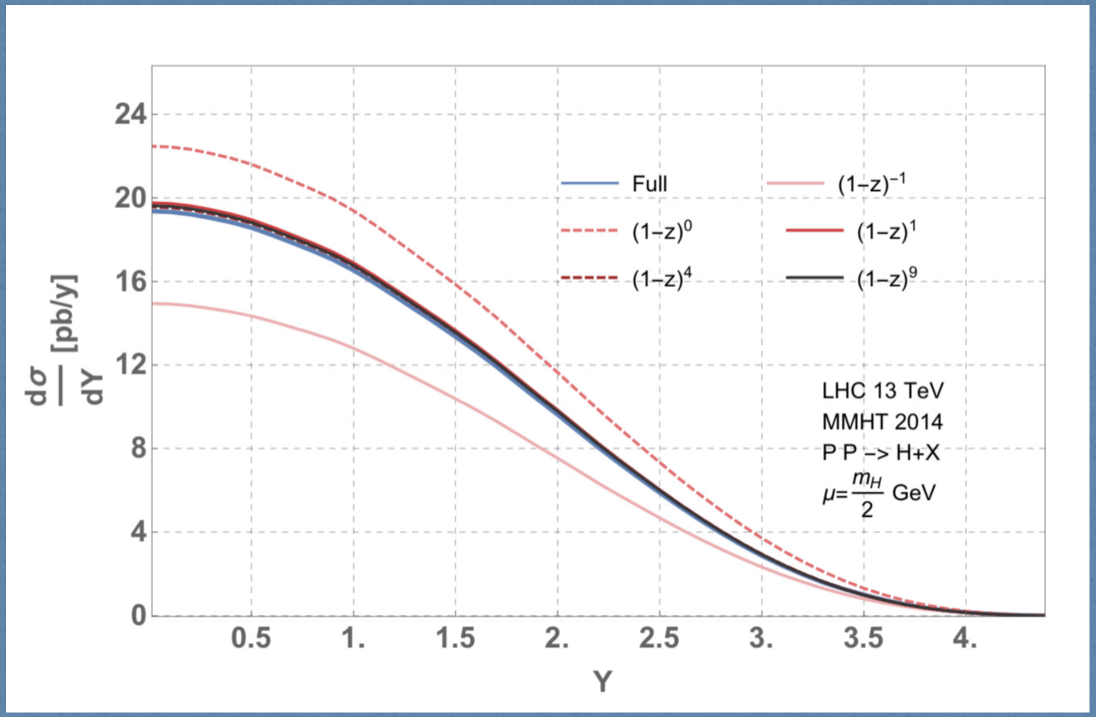
NNLO rapidity distribution for Higgs boson



Threshold Expansion

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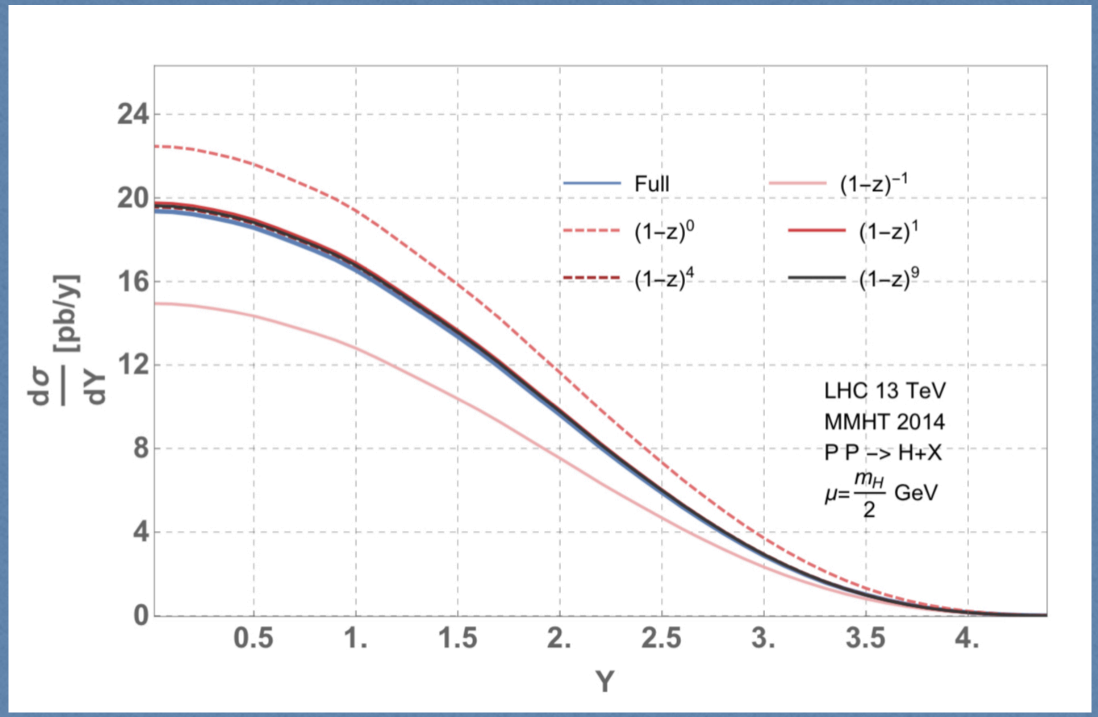

- Hence, including higher terms in the threshold expansion is essential

- Aim : study the structure of NSV logarithms for differential rapidity distribution and construct a formalism to resum these logarithms to all orders.

Previous Works on Rapidity

- **Rapidity distribution at NNLO :**
  - [Anastasiou, Dixon, Melnikov, Petriello]
  - [Bhler, Herzog, Lazopoulos, Mller]
- **Rapidity distribution at N^3LO :**
  - [Ravindran, Smith, Neervan] - threshold
  - [Ravindran, Ahmed, Mandal, Rana]- threshold
  - [Gehrmann, Glover, Husset al] - Full
- **Threshold Resummation for rapidity distribution**
  - [Catani, Trentadue]
  - [Laenen, Sterman], [Sterman, Vogelsang]
  - [Mukherjee, Vogelsang], [Bolzoni]
  - [Becher,Neubert]
  - [Bonvini, Forte, Ridolfi, Rottoli]
  - [Ebert, Michel, Tackmann]
  - [Ravindran, Banerjee, Das, Dhani]
- **Resummation beyond threshold for rapidity distribution**
  - [Dulat, Mistlberger, Pelloni et al]
  - [Tackmann, Michel et al]

Taking forward....

QCD threshold corrections to di-lepton and Higgs rapidity distributions beyond N^2LO

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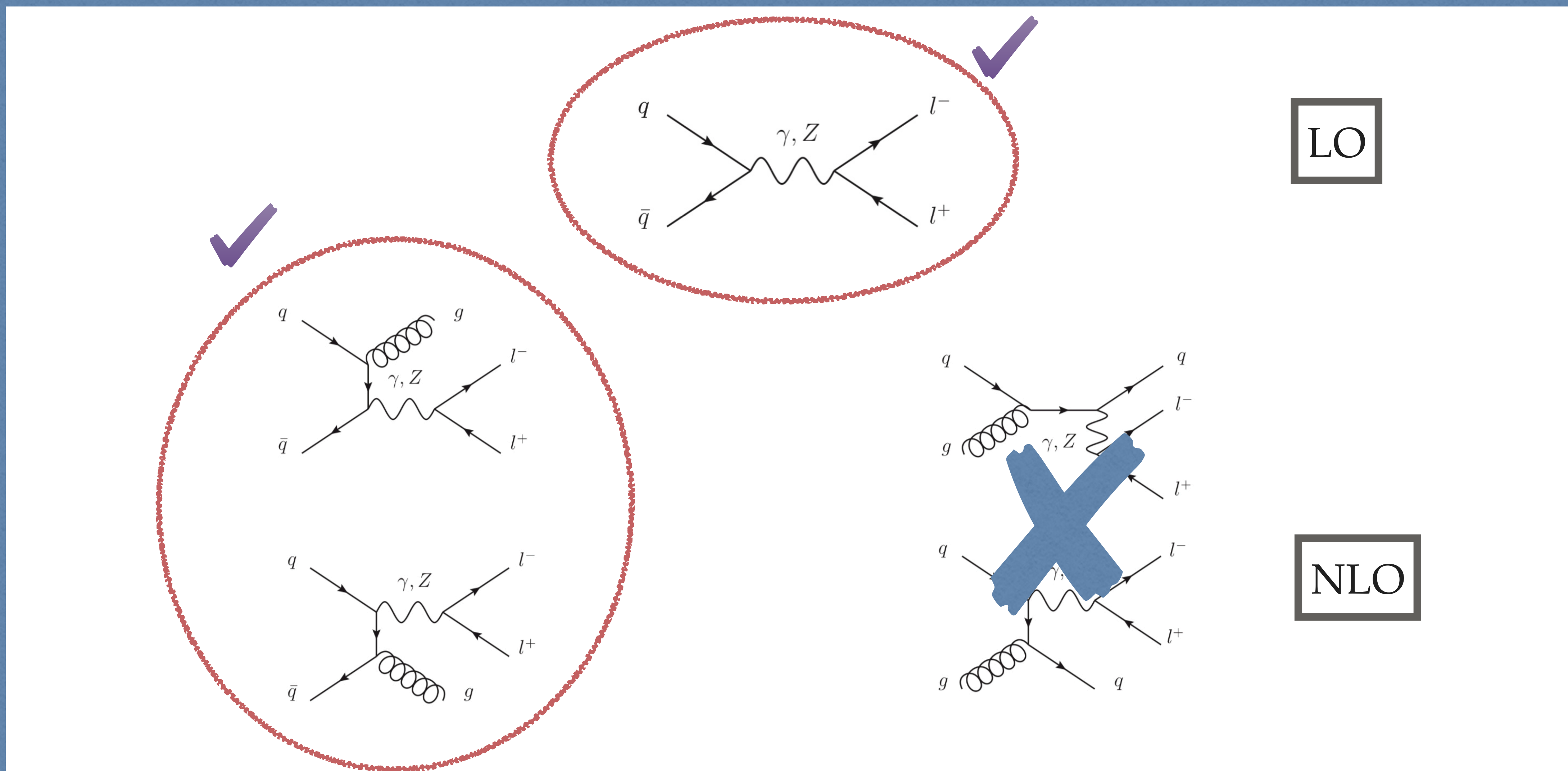
[3] *Lorentz Institute, University of Leiden, P.O. Box 9502, 2300 RA Leiden, The Netherlands.*

- Collinear factorization
- Renormalization group invariance
- Logarithmic structure of higher order perturbative results

Only for the diagonal channels

Taking forward....

For DY at NLO



Only for the diagonal channels

Formalism

- **Factorization :**

Born | Collinear singular | Altarelli-Parisi kernel

$$\Delta_{d,cc'}(\vec{z}, Q^2, \mu_F^2, \mu_R^2) = \sigma_B^{-1}(\mu_R^2) \sum_{ab,i,j} \left(\Gamma^T\right)^{-1}_{ca}(z_i, \mu_F^2, \epsilon) \otimes \frac{d\sigma_{ab}}{dy}(z_i, z_j, Q^2, \mu_R^2, \epsilon) \otimes \Gamma^{-1}_{bc'}(z_j, \mu_F^2, \epsilon)$$

IR finite coefficient function

SV+NSV contributions

$\mu_R \rightarrow$ Renormalization scale

$\mu_F \rightarrow$ Factorization scale

$$\Delta^{SV+NSV}_{d,c\bar{c}} = \sigma_B^{-1}(\mu_R^2) \sum_{i,j} \left(\Gamma^T\right)^{-1}_{cc}(z_i, \mu_F^2, \epsilon) \otimes \frac{d\sigma^{SV+NSV}_{c\bar{c}}}{dY}(z_i, z_j, Q^2, \mu_R^2, \epsilon) \otimes \Gamma^{-1}_{\bar{c}\,\bar{c}}(z_j, \mu_F^2, \epsilon)$$

Only diagonal contributions!

Formalism

- **Factorization :**

$$\Delta_{d,cc'}(\vec{z}, Q^2, \mu_F^2, \mu_R^2) = \sigma_B^{-1}(\mu_R^2) \sum_{ab,i,j} \left(\Gamma^T\right)^{-1}_{ca}(z_i,$$

IR finite coefficient function

SV+NSV contributions

Mass factorization

$$\Delta^2_{q\bar{q}} = \Gamma^{(0)}_{1,qq} \otimes \frac{\hat{\sigma}^{(2)}_{q\bar{q}}}{z_1 z_2} \otimes \Gamma^{(0)}_{2,\bar{q}\bar{q}} + \cdots$$

$$+ \Gamma^{(1)}_{1,gq} \otimes \frac{\hat{\sigma}^{(1)}_{qg}}{z_1 z_2} \otimes \Gamma^{(0)}_{2,\bar{q}\bar{q}}$$

$$+ \Gamma^{(1)}_{1,gq} \otimes \frac{\hat{\sigma}^{(1)}_{g\bar{q}}}{z_1 z_2} \otimes \Gamma^{(0)}_{2,\bar{q}\bar{q}}$$

beyond NSV

$$\Delta^{SV+NSV}_{d,c\bar{c}} = \sigma_B^{-1}(\mu_R^2) \sum_{i,j} \left(\Gamma^T\right)^{-1}_{cc}(z_i, \mu_F^2, \epsilon) \otimes \frac{d\sigma^{SV+NSV}_{c\bar{c}}}{dY}(z_i, z_j, Q^2, \mu_R^2, \epsilon) \otimes \Gamma^{-1}_{\bar{c}\,\bar{c}}(z_j, \mu_F^2, \epsilon)$$

Only diagonal contributions!

Formalism

- **Factoring out UV finite pure virtual corrections :**

Pure virtual corrections (renormalised)

Soft-collinear contributions from real emissions

$$\Delta_{d,c\bar{c}}^{\mathrm{SV+NSV}}\left(Q^2, z_1, z_2\right) = \left|F_c\left(Q^2\right)\right|^2 \delta(1-z_1)\delta(1-z_2) \otimes S_c^{\mathrm{SV+NSV}}\left(Q^2, \vec{z}\right)$$
$$\otimes \sum_{i,j} (\Gamma_{cc}^T)^{-1,\mathrm{SV+NSV}}\left(z_i, \mu_F^2\right) \otimes \Gamma_{\bar{c}\bar{c}}^{-1,\mathrm{SV+NSV}}\left(z_j, \mu_F^2\right)$$

Mass factorization counter terms

Formalism

Unrenormalized form factor

$$Q^2 \frac{d}{dQ^2} \ln \hat{F}^I \left(\hat{a}_s, Q^2, \mu^2, \varepsilon\right) = \frac{1}{2}\left[K^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^I\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

[Sen,Sterman,Magnea]

$$\Delta_{d,c\bar{c}}^{\rm SV+NSV}\left(Q^2, z_1, z_2\right) = \left|F_c\left(Q^2\right)\right|^2 \delta(1-z_1)\delta(1-z_2) \otimes S_c^{\rm SV+NSV}\left(Q^2, \vec{z}\right)$$

$$\otimes \sum_{i,j} (\Gamma_{cc}^T)^{-1,{\rm SV+NSV}}\left(z_i, \mu_F^2\right) \otimes \Gamma_{\bar{c}\bar{c}}^{-1,{\rm SV+NSV}}\left(z_j, \mu_F^2\right)$$

Mass factorization kernel

$$\mu_F^2 \frac{d}{d\mu_F^2} \mathcal{C} \ln \Gamma_{cc}(\mu_F^2, \bar{z}_l) = \frac{1}{2} P^c(a_s(\mu_F^2), \bar{z}_l) + \underbrace{\delta P^c}_{\text{beyond NSV}}$$

$$P^c(a_s, \bar{z}_l) = 2\left(\underbrace{\frac{A^c(a_s)}{(\bar{z}_l)_+} + B^c(a_s)\delta(\bar{z}_l)}_{\text{SV}} + \underbrace{C^c(a_s)\log(\bar{z}_l) + D^c(a_s)}_{\text{NSV}}\right)$$

[Moch, Vogt, Vermaseren]

Formalism

Unrenormalized form factor

$$Q^2 \frac{d}{dQ^2} \ln \hat{F}^I \left(\hat{a}_s, Q^2, \mu^2, \varepsilon\right) = \frac{1}{2}\left[K^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^I\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

[Sen,Sterman,Magnea]

$$\Delta_{d,c\bar{c}}^{\mathrm{SV+NSV}}\left(Q^2, z_1, z_2\right) = \left|F_c\left(Q^2\right)\right|^2 \delta(1-z_1)\delta(1-z_2) \otimes S_c^{\mathrm{SV+NSV}}\left(Q^2, \vec{z}\right)$$

$$\otimes \sum_{i,j} \left(\Gamma_{cc}^T\right)^{-1,\mathrm{SV+NSV}}\left(z_i, \mu_F^2\right) \otimes \Gamma_{\bar{c}\bar{c}}^{-1,\mathrm{SV+NSV}}\left(z_j, \mu_F^2\right)$$

?

Mass factorization kernel

$$\mu_F^2 \frac{d}{d\mu_F^2} \mathcal{C} \ln \Gamma_{cc}(\mu_F^2, \bar{z}_l) = \frac{1}{2} P^c(a_s(\mu_F^2), \bar{z}_l) + \delta P^c$$

beyond NSV

$$P^c(a_s, \bar{z}_l) = 2\left(\frac{A^c(a_s)}{(\bar{z}_l)_+} + B^c(a_s)\delta(\bar{z}_l) + C^c(a_s)\log(\bar{z}_l) + D^c(a_s)\right)$$

SV: $\frac{A^c(a_s)}{(\bar{z}_l)_+} + B^c(a_s)\delta(\bar{z}_l)$

NSV: $C^c(a_s)\log(\bar{z}_l) + D^c(a_s)$

[Moch, Vogt, Vermaseren]

Formalism

- Owing to :
  - IR factorization
  - RG evolution of splitting kernel
  - Finiteness of partonic cross section

$$Q^2 \frac{d}{dQ^2} \mathcal{S}_c(\vec{z}, Q^2, \epsilon) = \frac{1}{2} \left[\overline{K}^c_d(\mu_R^2, \vec{z}, \epsilon) + \overline{G}^c_d(Q^2, \mu_R^2, \vec{z}) \right] \otimes \mathcal{S}_c(\vec{z}, Q^2, \epsilon)$$

Infrared singular

IR finite

Soft-collinear contributions exhibits exponential behaviour

Formalism

- **Sudakov type diff. Eq. supplemented with RG invariance provides the structure of soft-collinear function :**

From matrix elements

$$\ln S_c^{\rm SV+NSV} = \sum_{i=1}^{\infty} \hat{a}_s^i S_\epsilon^i \left(\frac{q^2(1-z_1)(1-z_2)}{\mu^2 z_1 z_2}\right)^{i\frac{\epsilon}{2}} \left(\frac{(i\epsilon)^2}{(1-z_1)(1-z_2)}\right) \times \Big\{ \hat{\phi}_d^{c,(i)}(\epsilon) + (1-z_1)\sum_{k=0}^{i+j} \varphi_d^{c,(i,j,k)}\,\epsilon^j\,\ln^k(1-z_1) + (1-z_2)\sum_{k=0}^{i+j} \varphi_d^{c,(i,j,k)}\,\epsilon^j\,\ln^k(1-z_2)\Big\}$$

Phase space factor

$\hat{\phi}_d^{c,(i)}(\epsilon)$ → Pure SV

$(1-z_1)$ and $(1-z_2)$ terms → NSV

Upper limit $i+j$ of the sums → *Controlled by transcendality structure of Feynman integrals*

Formalism

Finite SV and NSV coefficients are obtained from the available inclusive corrections.

Total cross section at $N_1 = N_2 = N \to \infty$

$$\int_0^1 dx_1^0 \int_0^1 dx_2^0 \left(x_1^0 x_2^0\right)^{N-1} \frac{d\sigma^c}{dy} = \int_0^1 d\tau\ \tau^{N-1}\ \sigma^c\,,$$

$$\sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{\frac{i\epsilon}{2}} S_\epsilon^i \Big[t_1^i(\epsilon)\hat{\phi}_d^{c,(i)}(\epsilon) - t_2^i(\epsilon)\hat{\phi}^{c,(i)}(\epsilon)$$
$$+\sum_{k=0}^{\infty}\left(t_3^{(i,k)}(\epsilon)\varphi_{d,c}^{(i,k)}(\epsilon) - t_4^{(i,k)}(\epsilon)\varphi_c^{(i,k)}(\epsilon)\right)\Big] = 0\,.$$

$$t_1^i = \frac{i\epsilon(2-i\epsilon)}{4N^{i\epsilon}}\Gamma^2\left(1+i\frac{\epsilon}{2}\right), \quad t_2^i = \frac{i\epsilon(1-i\epsilon)}{2N^{i\epsilon}}\Gamma(1+i\epsilon),$$

$$t_3^{(i,k)} = \Gamma\left(1+i\frac{\epsilon}{2}\right)\frac{\partial^k}{\partial\alpha^k}\left(\frac{\Gamma(1+\alpha)}{N^{\alpha+i\epsilon/2}}\right)_{\alpha=i\frac{\epsilon}{2}},$$

$$t_4^{(i,k)} = \frac{\partial^k}{\partial\hat{\alpha}^k}\left(\frac{\Gamma(1+\hat{\alpha})}{N^{\hat{\alpha}}}\right)_{\hat{\alpha}=i\epsilon}.$$

From matrix elements

$$\left(\frac{(i\epsilon)^2}{(1-z_1)(1-z_2)}\right) \times \Big\{$$

$$\hat{\phi}_d^{c,(i)}(\epsilon) \quad \longrightarrow \textbf{Pure SV}$$

$$+ (1-z_1)\sum_{k=0}^{i+j}\varphi_d^{c,(i,j,k)}\,\epsilon^j\,\ln^k(1-z_1)$$
$$+ (1-z_2)\sum_{k=0}^{i+j}\varphi_d^{c,(i,j,k)}\,\epsilon^j\,\ln^k(1-z_2)\Big\}$$

NSV

Controlled by transcendality structure of Feynman integrals

Predictions

- Owing to the differential equations that F, S, Γ satisfies, the $\Delta_d^{\mathrm{SV+NSV}}$ exhibit an exponential structure, which helps to predict certain higher order terms.

GIVEN	PREDICTIONS					
FO Coefficients	$\Delta_{d,c}^{NSV,(2)}$	$\Delta_{d,c}^{NSV,(3)}$	$\Delta_{d,c}^{NSV,(4)}$	$\Delta_{d,c}^{NSV,(5)}$	$\Delta_{d,c}^{NSV,(6)}$	$\Delta_{d,c}^{NSV,(i)}$
χ_1	$L_{z_l}^3$	$L_{z_l}^5$	$L_{z_l}^7$	$L_{z_l}^9$	$L_{z_l}^{11}$	$L_{z_l}^{(2i-1)}$
χ_2		$L_{z_l}^4$	$L_{z_l}^6$	$L_{z_l}^8$	$L_{z_l}^{10}$	$L_{z_l}^{(2i-2)}$
χ_3			$L_{z_l}^5$	$L_{z_l}^7$	$L_{z_l}^9$	$L_{z_l}^{(2i-3)}$

Table 1: The all order predictions for NSV logarithms in $\Delta_{d,c}^{NSV,(i)}$ for a given set of fixed order coefficients. Here χ_i represents i-loop coefficients and $L_{z_l}^j = \ln^j(1-z_l)$ with $l = 1, 2$.

With known n-loop information, we can predict

$\ln^k(1 - z_l), k = \{2i - n, \ldots, 2i - 1\}$ at order a_s^i

3rd order NSV result

- **For $gg \to H$: known** [Dulat et al] [Tackmann et al]

- **For DY**

$$
\begin{aligned}
\Delta^{\mathrm{NSV},(3)}_{d,q} = {} & C_F^3 \Big\{ L_{z_1}^5 \big(-8\bar{\delta} \big) + L_{z_1}^4 \big(44\bar{\delta} - 40\overline{\mathcal{D}}_0 \big) + L_{z_1}^3 \Big[\bar{\delta} (132 + 32\zeta_2) + 160\overline{\mathcal{D}}_0 - 160\overline{\mathcal{D}}_1 \Big] + L_{z_1}^2 \Big[-\bar{\delta}\Big(\frac{1136}{3} + 320\zeta_3 + 96\zeta_2 \Big) \\
& + \overline{\mathcal{D}}_0 (416 + 96\zeta_2) + 416\overline{\mathcal{D}}_1 - 240\overline{\mathcal{D}}_2 \Big] + L_{z_1} \Big[\bar{\delta}\Big(848\zeta_3 - \frac{1675}{3} - \frac{88}{3}\zeta_2 + \frac{192}{5}\zeta_2^2 \Big) - \overline{\mathcal{D}}_0 (640 + 640\zeta_3 + 192\zeta_2) \\
& + \overline{\mathcal{D}}_1 (872 + 192\zeta_2) + 336\overline{\mathcal{D}}_2 - 160\overline{\mathcal{D}}_3 \Big] + \Big[\bar{\delta}\Big(\frac{557}{2} - 384\zeta_5 + 496\zeta_3 + \frac{700}{3}\zeta_2 + 128\zeta_2\zeta_3 - \frac{560}{3}\zeta_2^2 \Big) - \overline{\mathcal{D}}_0 \Big(697 - 816\zeta_3 \\
& - 64\zeta_2 - \frac{192}{5}\zeta_2^2 \Big) - \overline{\mathcal{D}}_1 (384 + 640\zeta_3 + 288\zeta_2) + \overline{\mathcal{D}}_2 (456 + 96\zeta_2) + 80\overline{\mathcal{D}}_3 - 40\overline{\mathcal{D}}_4 \Big] \Big\} + C_F^2 n_f \Big\{ L_{z_1}^4 \Big(-\frac{40}{9}\bar{\delta} \Big) \\
& + L_{z_1}^3 \Big(\frac{1040}{27}\bar{\delta} - \frac{160}{9}\overline{\mathcal{D}}_0 \Big) + L_{z_1}^2 \Big[\bar{\delta}\Big(32\zeta_2 - \frac{620}{9} \Big) + 112\overline{\mathcal{D}}_0 - \frac{160}{3}\overline{\mathcal{D}}_1 \Big] + L_{z_1} \Big[-\bar{\delta}\Big(\frac{9080}{81} + \frac{320}{3}\zeta_3 + \frac{32}{3}\zeta_2 \Big) \\
& - \overline{\mathcal{D}}_0 \Big(\frac{1040}{9} - 64\zeta_2 \Big) + \frac{640}{3}\overline{\mathcal{D}}_1 - \frac{160}{3}\overline{\mathcal{D}}_2 \Big] + \Big[\bar{\delta}\Big(\frac{1999}{27} + \frac{2032}{9}\zeta_3 - \frac{664}{9}\zeta_2 + \frac{256}{15}\zeta_2^2 \Big) - \overline{\mathcal{D}}_0 \Big(\frac{1448}{9} + \frac{320}{3}\zeta_3 + \frac{32}{9}\zeta_2 \Big) \\
& + \overline{\mathcal{D}}_1 \Big(64\zeta_2 - \frac{200}{3} \Big) + 96\overline{\mathcal{D}}_2 - \frac{160}{9}\overline{\mathcal{D}}_3 \Big] \Big\} + C_A C_F^2 \Big\{ L_{z_1}^4 \Big(\frac{220}{9}\bar{\delta} \Big) + L_{z_1}^3 \Big[\bar{\delta}\Big(32\zeta_2 - \frac{5756}{27} \Big) + \frac{880}{9}\overline{\mathcal{D}}_0 \Big] + L_{z_1}^2 \Big[\bar{\delta}\Big(\frac{3572}{9} \\
& - 168\zeta_3 - \frac{812}{3}\zeta_2 \Big) + \overline{\mathcal{D}}_0 (96\zeta_2 - 640) + \frac{880}{3}\overline{\mathcal{D}}_1 \Big] + L_{z_1} \Big[\bar{\delta}\Big(\frac{70763}{81} + 424\zeta_3 + \frac{20}{3}\zeta_2 + \frac{48}{5}\zeta_2^2 \Big) + \overline{\mathcal{D}}_0 \Big(\frac{6068}{9} - 336\zeta_3 \\
& - 512\zeta_2 \Big) + \overline{\mathcal{D}}_1 \Big(192\zeta_2 - \frac{3784}{3} \Big) + \frac{880}{3}\overline{\mathcal{D}}_2 \Big] + \Big[\bar{\delta}\Big(\frac{2260}{9}\zeta_2 - \frac{56101}{54} - 116\zeta_3 + 16\zeta_2\zeta_3 + 24\zeta_2^2 \Big) + \overline{\mathcal{D}}_0 \Big(\frac{11351}{9} + \frac{728}{3}\zeta_3 \\
& - \frac{1456}{9}\zeta_2 + \frac{48}{5}\zeta_2^2 \Big) + \overline{\mathcal{D}}_1 \Big(\frac{1088}{3} - 336\zeta_3 - 448\zeta_2 \Big) + \overline{\mathcal{D}}_2 (96\zeta_2 - 592) + \frac{880}{9}\overline{\mathcal{D}}_3 \Big] \Big\} + C_A C_F n_f \Big\{ L_{z_1}^3 \Big(\frac{176}{27}\bar{\delta} \Big) \\
& + L_{z_1}^2 \Big[\bar{\delta}\Big(\frac{16}{3}\zeta_2 - \frac{1678}{27} \Big) + \frac{176}{9}\overline{\mathcal{D}}_0 \Big] + L_{z_1} \Big[\bar{\delta}\Big(\frac{14648}{81} - \frac{212}{3}\zeta_2 \Big) + \overline{\mathcal{D}}_0 \Big(\frac{32}{3}\zeta_2 - \frac{3536}{27} \Big) + \frac{352}{9}\overline{\mathcal{D}}_1 \Big] + \Big[\bar{\delta}\Big(\frac{196}{3}\zeta_3 \\
& - \frac{118984}{729} + \frac{11816}{81}\zeta_2 - \frac{208}{15}\zeta_2^2 \Big) + \overline{\mathcal{D}}_0 \Big(\frac{16952}{81} - \frac{608}{9}\zeta_2 \Big) + \overline{\mathcal{D}}_1 \Big(\frac{32}{3}\zeta_2 - \frac{3896}{27} \Big) + \frac{176}{9}\overline{\mathcal{D}}_2 \Big] \Big\} + C_F n_f^2 \Big\{ L_{z_1}^3 \Big(-\frac{16}{27}\bar{\delta} \Big) \\
& + L_{z_1}^2 \Big(\frac{152}{27}\bar{\delta} - \frac{16}{9}\overline{\mathcal{D}}_0 \Big) + L_{z_1} \Big[\bar{\delta}\Big(\frac{32}{9}\zeta_2 - \frac{1264}{81} \Big) + \frac{304}{27}\overline{\mathcal{D}}_0 - \frac{32}{9}\overline{\mathcal{D}}_1 \Big] + \Big[\bar{\delta}\Big(\frac{10856}{729} + \frac{32}{27}\zeta_3 - \frac{304}{27}\zeta_2 \Big) + \overline{\mathcal{D}}_0 \Big(\frac{32}{9}\zeta_2 \\
& - \frac{1264}{81} \Big) + \frac{304}{27}\overline{\mathcal{D}}_1 - \frac{16}{9}\overline{\mathcal{D}}_2 \Big] \Big\} + C_A^2 C_F \Big\{ L_{z_1}^3 \Big(-\frac{484}{27}\bar{\delta} \Big) + L_{z_1}^2 \Big[\bar{\delta}\Big(\frac{4676}{27} - \frac{98}{3}\zeta_2 \Big) - \frac{484}{9}\overline{\mathcal{D}}_0 \Big] + L_{z_1} \Big[\bar{\delta}\Big(\frac{2560}{9}\zeta_2 \\
& - \frac{47386}{81} + 200\zeta_3 - \frac{176}{5}\zeta_2^2 \Big) + \overline{\mathcal{D}}_0 \Big(\frac{9496}{27} - \frac{176}{3}\zeta_2 \Big) - \frac{968}{9}\overline{\mathcal{D}}_1 \Big] + \Big[\bar{\delta}\Big(\frac{587684}{729} + 192\zeta_5 - \frac{21692}{27}\zeta_3 - \frac{40844}{81}\zeta_2 \\
& + \frac{176}{3}\zeta_2\zeta_3 + \frac{656}{15}\zeta_2^2 \Big) - \overline{\mathcal{D}}_0 \Big(\frac{49582}{81} - 176\zeta_3 - \frac{856}{3}\zeta_2 + \frac{176}{5}\zeta_2^2 \Big) + \overline{\mathcal{D}}_1 \Big(\frac{11476}{27} - \frac{176}{3}\zeta_2 \Big) - \frac{484}{9}\overline{\mathcal{D}}_2 \Big] \Big\} + (z_1 \leftrightarrow z_2) \,,
\end{aligned}
$$

- Confirmed the $D_i(z_l)\ln^j(1-z_m)$ terms with [Tackmann et al]
- $\delta(1-z_l)\ln^j(1-z_m)$ terms are new result

- **For $b\bar{b} \to H$**

$$
\begin{aligned}
\Delta^{\mathrm{NSV},(3)}_{d,b} = {} & \Delta^{NSV,(3)}_{d,q} + \Big[C_F^3 \Big\{ L_{z_1}^3 \big(-96\bar{\delta} \big) + L_{z_1}^2 \big(288\bar{\delta} - 288\overline{\mathcal{D}}_0 \big) + L_{z_1} \Big[\bar{\delta}(471 - 88\zeta_2) + 480\overline{\mathcal{D}}_0 - 576\overline{\mathcal{D}}_1 \Big] + \Big[-\bar{\delta}\Big(\frac{447}{2} + 384\zeta_3 \\
& 148\zeta_2 \Big) + \overline{\mathcal{D}}_0 (591 - 88\zeta_2) + 288\overline{\mathcal{D}}_1 - 288\overline{\mathcal{D}}_2 \Big] \Big\} + C_F^2 n_f \Big\{ L_{z_1}^2 \big(-16\bar{\delta} \big) + L_{z_1} \Big[\bar{\delta}\Big(\frac{1642}{9} - 32\zeta_2 \Big) - 32\overline{\mathcal{D}}_0 \Big] + \Big[-\bar{\delta}\Big(\frac{479}{3} \\
& - 48\zeta_2 \Big) + \overline{\mathcal{D}}_0 \Big(\frac{1642}{9} - 32\zeta_2 \Big) - 32\overline{\mathcal{D}}_1 \Big] \Big\} + C_A C_F^2 \Big\{ L_{z_1}^2 88\bar{\delta} + L_{z_1} \Big[\bar{\delta}\Big(144\zeta_3 + 256\zeta_2 - \frac{9925}{9} \Big) + 176\overline{\mathcal{D}}_0 \Big] \\
& + \Big[\bar{\delta}\Big(\frac{4615}{6} - 408\zeta_3 - 304\zeta_2 \Big) - \overline{\mathcal{D}}_0 \Big(\frac{10861}{9} - 144\zeta_3 - 256\zeta_2 \Big) + 176\overline{\mathcal{D}}_1 \Big] \Big\} + C_A^2 C_F \Big\{ L_{z_1} \; 8\bar{\delta} - \Big[16\bar{\delta} \Big] \Big\} + (z_1 \leftrightarrow z_2) \Big] .
\end{aligned}
$$

- New result

4th order predictions

- **4th order predictions for DY, $b\bar{b}H$, ggH respectively :**

$$\Delta^{\mathbf{NSV},(4)}_{d,q} = \ln^7(\bar{z}_1)\delta(\bar{z}_2)\Big(-\frac{16}{3}C_F^4\Big) + \ln^6(\bar{z}_1)\Big\{\delta(\bar{z}_2)\Big[\frac{128}{3}C_F^4 + C_F^3 n_f\Big(-\frac{56}{9}\Big) + \frac{308}{9}C_A C_F^3\Big] + \overline{\mathcal{D}}_0\, C_F^4\Big(-\frac{112}{3}\Big)\Big\} + \ln^5(\bar{z}_1)\Big\{\delta(\bar{z}_2)\Big[\frac{1864}{27}C_F^3 n_f + C_F^2 n_f^2\Big(-\frac{64}{27}\Big) + C_F^4(132+96\zeta_2) + \frac{704}{27}C_A C_F^2 n_f + C_A C_F^3\Big(48\zeta_2 - \frac{10576}{27}\Big) + C_A^2 C_F^2\Big(-\frac{1936}{27}\Big)\Big] + \overline{\mathcal{D}}_0\Big[240 C_F^4 + C_F^3 n_f\Big(-\frac{112}{3}\Big) + \frac{616}{3}C_A C_F^3\Big] - \overline{\mathcal{D}}_1\, 224 C_F^4\Big\} + (z_1 \leftrightarrow z_2)\,,$$

$$\Delta^{\mathbf{NSV},(4)}_{d,b} = \Delta^{NSV,(4)}_{d,q} - \Big[\ln^5(\bar{z}_1)\delta(\bar{z}_2)\; 96 C_F^4 + (z_1 \leftrightarrow z_2)\Big]\,,$$

$$\Delta^{\mathbf{NSV},(4)}_{d,g} = \ln^7(\bar{z}_1)\delta(\bar{z}_2)\Big(-\frac{16}{3}C_A^4\Big) + \ln^6(\bar{z}_1)\Big\{\delta(\bar{z}_2)\Big[\frac{692}{9}C_A^4 + C_A^3 n_f\Big(-\frac{56}{9}\Big)\Big] + \overline{\mathcal{D}}_0\, C_A^4\Big(-\frac{112}{3}\Big)\Big\} + \ln^5(\bar{z}_1)\Big\{\delta(\bar{z}_2)\Big[\frac{796}{9}C_A^3 n_f + C_A^2 n_f^2\Big(-\frac{64}{27}\Big) + C_A^4\Big(144\zeta_2 - \frac{12224}{27}\Big)\Big] + \overline{\mathcal{D}}_0\Big[\frac{1336}{3}C_A^4 + C_A^3 n_f\Big(-\frac{112}{3}\Big)\Big] - \overline{\mathcal{D}}_1\, 224 C_A^4\Big\} + (z_1 \leftrightarrow z_2)\,,$$

All order behaviour

- In order to study the **all order behaviour**, we formulated the Integral representation for Δ_d^c :

$$\ln \Delta_d^c = \frac{\delta(\bar{z}_1)}{2}\left(\int_{\mu_F^2}^{q^2\bar{z}_2} \frac{d\lambda^2}{\lambda^2} \mathscr{P}^c\left(a_s(\lambda^2), \bar{z}_2\right) + \mathscr{Q}_d^c\left(a_s(q_2^2), \bar{z}_2\right)\right)_+$$

$$+ \frac{1}{4}\left(\frac{1}{\bar{z}_1}\left\{\mathscr{P}^c(a_s(q_{12}^2), \bar{z}_2) + 2\ L^c(a_s(q_{12}^2), \bar{z}_2)\ + q^2 \frac{d}{dq^2}\left(\mathscr{Q}_d^c(a_s(q_2^2), \bar{z}_2) + 2\varphi_{d,c}^f(a_s(q_2^2), \bar{z}_2)\right)\right\}\right)_+$$

$$+ \frac{1}{2}\ \delta(\bar{z}_1)\delta(\bar{z}_2)\ \ln\left(g_{d,0}^c(a_s(\mu_F^2))\right) + \bar{z}_1 \leftrightarrow \bar{z}_2,$$

$$\mathscr{P}_{cc}\left(a_s, z_l\right) = 2\ A^c(a_s)\ \mathscr{D}_0(z_l) + 2\ L^c(a_s(q_{12}^2, \bar{z}_l))$$

$$\mathscr{Q}_d^c(a_s, \bar{z}_l) = \frac{2}{\bar{z}_l} D_d^{SV}(a_s) + 2\ \varphi_d^{c,f}(a_s, \bar{z}_l)$$

SV (first terms) — **NSV** (second terms)

Process independent

$\bar{z}_l = 1 - z_l$

$L^c = C^c(a_s)\ \ln(\bar{z}_l) + D^c(a_s)$

$q_l^2 = q^2\bar{z}_l \qquad q_{12}^2 = q^2\bar{z}_1\bar{z}_2$

All order behaviour

- In order to study the **all order behaviour**, we formulated the Integral representation for Δ^c_d :

$$\ln \Delta^c_d = \frac{\delta(\bar z_1)}{2}\left(\left(\int_{\mu_F^2}^{q^2\bar z_2} \frac{d\lambda^2}{\lambda^2}\mathscr{P}^c\left(a_s(\lambda^2),\bar z_2\right)+\mathscr{Q}^c_d\left(a_s(q_2^2),\bar z_2\right)\right)_+\right.$$

$$+\frac{1}{4}\left(\frac{1}{\bar z_1}\left\{\mathscr{P}^c(a_s(q_{12}^2),\bar z_2)+2\ L^c(a_s(q_{12}^2),\bar z_2)\ +q^2\frac{d}{dq^2}\left(\mathscr{Q}^c_d(a_s(q_2^2),\bar z_2)+2\varphi^f_{d,c}(a_s(q_2^2),\bar z_2)\right)\right\}\right)_+$$

$$+\frac{1}{2}\ \delta(\bar z_1)\delta(\bar z_2)\ \ln\left(g^c_{d,0}(a_s(\mu_F^2))\right)$$

$$\mathscr{P}_{cc}(a_s,z_l) = 2\ A^c(a_s)\ \mathscr{D}_0(z_l) + 2\ L^c(a_s(q_{12}^2,\bar z_l))$$

$$\mathscr{Q}^c_d(a_s,\bar z_l) = \frac{2}{\bar z_l}D^{SV}_d(a_s) + 2\ \varphi^{c,f}_d(a_s,\bar z_l)$$

SV **NSV**

Process independent

- $g^c_{d,0} \longrightarrow \delta(z_i)\ \delta(z_j)$ **contributions from** F^c **and** S^c
- $\mathscr{P}^c \longrightarrow$ **From splitting kernel after pole cancellation b/w** S^c **and** Γ_{cc}
- $\mathscr{Q}^c_d \longrightarrow$ **Finite contribution from SV and NSV** S^c

NSV in Mellin space

- Solving the integral representation in **Mellin space**, we get :

$$\Delta^c_{N_1,N_2} = \int_0^1 dz_1 z_1^{N_1-1} \int_0^1 dz_2 z_2^{N_2-1} \Delta^c_d(z_1, z_2)$$

$z_l \to 1 \longrightarrow N_l \to \infty$

SV NSV

$$\left(\frac{\ln(1-z_l)}{1-z_l}\right)_+ \to \frac{\ln^2 N_l}{2} - \frac{\ln N_l}{2N_l} + \frac{1}{2N_l} + \mathcal{O}\left(\frac{1}{N_l^2}\right)$$

$$\ln^k(1-z_l) \to \frac{\ln^k N_l}{N_l} + \mathcal{O}\left(\frac{1}{N_l^2}\right)$$

NSV in Mellin space

SV NSV

$$\Delta^{c}_{N_1,N_2} = 1 + a_s\Big[c_1^2 \ln^2 N_1N_2 + \cdots + c_1^0 + d_1^1 \frac{\ln N_1N_2}{N_1} + d_1^0 \frac{1}{N_1} + \mathcal{O}\Big(\frac{1}{N_1^2}\Big)\Big]$$

$$+a_s^2\Big[c_2^4 \ln^4 N_1N_2 + \cdots + c_2^0 + d_2^3 \frac{\ln^3 N_1N_2}{N_1} + \cdots + d_2^0 \frac{1}{N_1} + \mathcal{O}\Big(\frac{1}{N_1^2}\Big)\Big]$$

$$+\cdots$$

$$+a_s^n\Big[c_n^{2n} \ln^{2n} N_1N_2 + \cdots + c_n^0 + d_n^{2n-1} \frac{\ln^{2n-1} N_1N_2}{N_1} + \cdots + d_3^0 \frac{1}{N_1} + \mathcal{O}\Big(\frac{1}{N_1^2}\Big)\Big]$$

$$+ (N_1 \to N_2)$$

$a_s \ln N_1N_2 \sim \mathcal{O}(1)$ **when** a_s **is small : spoils the truncation of series**

NSV Resummation

- Solving the integral representation in **Mellin space**, we get :

$$\Delta^c_{d,\vec{N}}(q^2,\mu_R^2,\mu_F^2) = C_{d_0}(q^2,\mu_R^2,\mu_F^2)\exp\left(\Psi^{c,SV}_{d,\vec{N}}(q^2,\mu_F^2) + \Psi^{c,NSV}_{d,\vec{N}}(q^2,\mu_F^2)\right)$$

$\omega = a_s\beta_0 \ln(N_1 N_2)$

Known

[Ravindran et al]

$$\Psi^{c,sv}_{d,\vec{N}} = \ln g^c_{d_0} + g^c_{d,1}(\omega)\ln N_1 + \sum_{i=0}^{\infty} a_s^i \frac{1}{2} g^c_{d,i+2}(\omega) + N_1 \leftrightarrow N_2$$

New result !

$$\Psi^{c,\mathrm{NSV}}_{d,\vec{N}} = \frac{1}{N_1}\left(\sum_{i=0}^{\infty} a_s^i\, h^c_{d,i}(\omega, N_1)\right) + N_1 \leftrightarrow N_2$$

$$h^c_{d,i}(\omega, N_l) = \sum_{k=0}^{i} h^c_{d,ik}(\omega)\ \ln^k N_l$$

All order predictions for CFs

✦ **SV :**

Known structure

		Given	Resumed terms
	$g^c_{d,0,i}$ **up to**	$g^c_{d,i}$ **up to**	$a^i_s \ \ln^k N_l$
One loop info LL	0	1	$k = 2i$
Two loop info NLL	1	2	$k = \{2i-2,\cdots,2i\}$
Three loop info NNLL	2	3	$k = \{2i-4,\cdots,2i\}$
Four loop info NNNLL	3	4	$k = \{2i-6,\cdots,2i\}$

All order predictions for CFs

- **With known SV, the tower of logarithms resumed using lower order informations :**

One loop info

Given	Resumed terms
$h^c_{d,i}$ **up to**	$\frac{1}{N_l} a_s^i \ \ln^k N_l$
1	$k = 2i - 1$
2	$k = \{2i - 2,\ 2i - 1\}$
3	$k = \{2i - 3,\ 2i - 1\}$
n	$k = \{2i - n, \cdots, 2i - 1\}$

Resumed terms

$$a_s \frac{1}{N} \log N$$

$$a_s^2 \frac{1}{N} \log^3 N$$

$$a_s^3 \frac{1}{N} \log^5 N$$

⋮

$$a_s^i \frac{1}{N} \log^{2i-1} N$$

LL

All order predictions for CFs

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Two loop info

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n	$k = \{2i - n, \cdots, 2i - 1\}$

Resumed terms

NLL	LL
$a_s^2 \frac{1}{N} \log^2 N$	$a_s \frac{1}{N} \log N$
$a_s^3 \frac{1}{N} \log^4 N$	$a_s^2 \frac{1}{N} \log^3 N$
$a_s^4 \frac{1}{N} \log^6 N$	$a_s^3 \frac{1}{N} \log^5 N$
$\vdots$	$\vdots$
$a_s^i \frac{1}{N} \log^{2i-2} N$	$a_s^i \frac{1}{N} \log^{2i-1} N$

All order predictions for CFs

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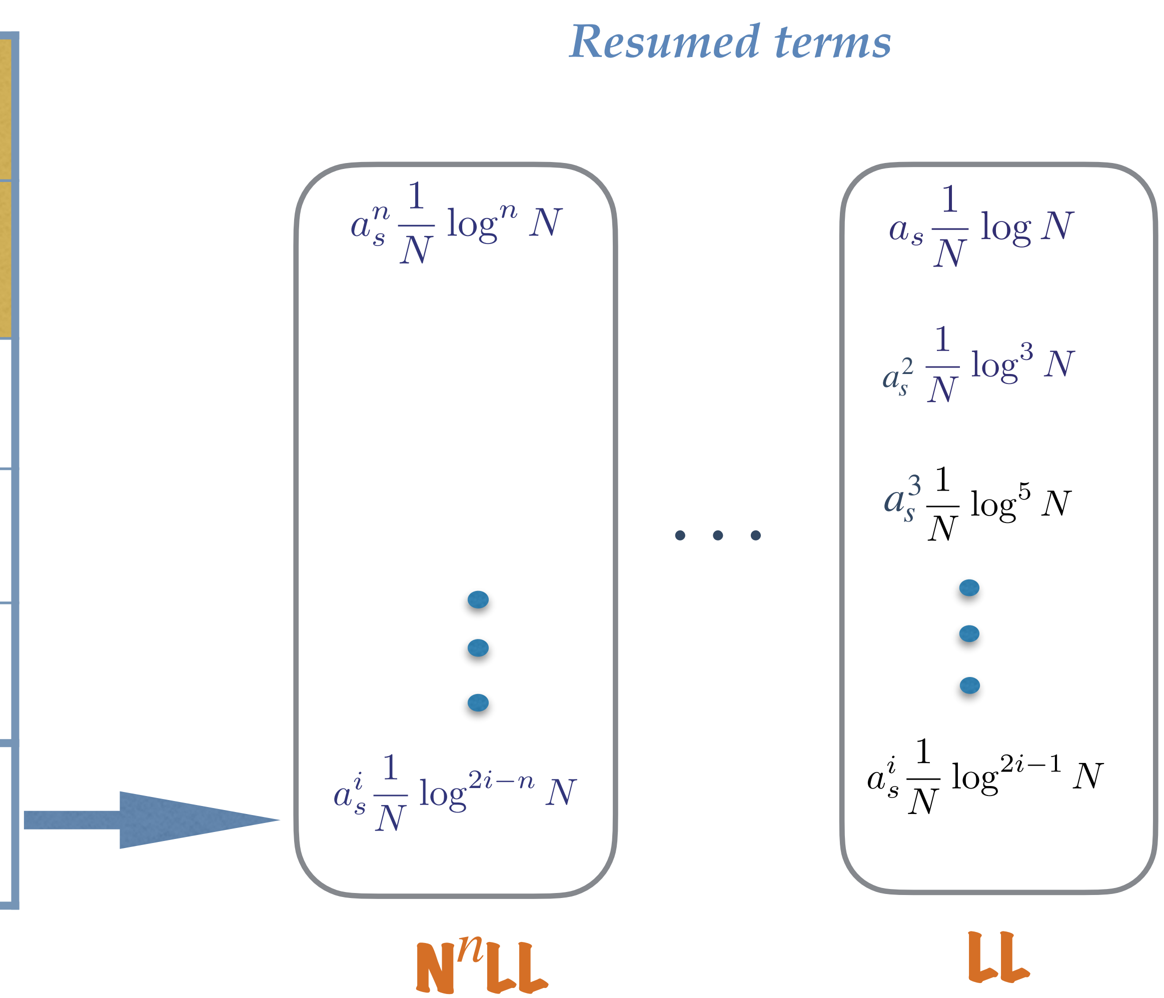
n- loop info



Summary and Outlook

- We propose a framework to compute the **effects of NSV logarithms in rapidity distributions**, using Collinear factorisation and RG invariance.

- The finite differential cross section **exhibits an exponential behaviour** which allows for all order predictions for certain SV+NSV logarithms.

- By formulating an integral representation, we propose an **SV+NSV resummation framework** in double Mellin space, which is first of the kind.

- Provide NSV contributions of rapidity distribution for DY (complete) and $b\bar{b} \to H$ at N^{3}LO and also predict certain NSV logarithms at N^{4}LO.

Summary and Outlook

- More to be done :
  - The numerical impact of the fixed order results and resummed result. Also, explore how resummed result affects different prescriptions.
  - By analysing the functional form of Soft-Collinear structure, one can look in to their impact on resummed result.
  - All the analysis has been limited to only diagonal channels. It will be interesting to explore off-diagonal channels as well and thereby develop a general framework for computing NSV effects on differential distributions.

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Thank You!