Top quark mass corrections to single and double Higgs boson production in gluon fusion

Radcor-LoopFest 2021, Parallel III.B

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Higgs self coupling

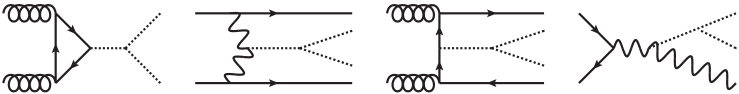
Standard Model Higgs potential:

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4,$$

where $\lambda = m_H^2/(2v^2) \approx 0.13$.

Want to measure λ, to determine if $V(H)$ is consistent with nature.

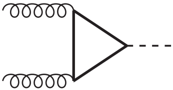
λ appears in various production channels:



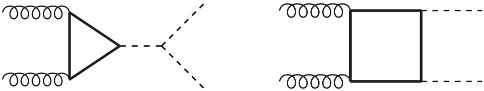
- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- H-strahlung

Gluon Fusion

Leading order (1 loop):



$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}^{\mu\nu} h$$



$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

$\mathcal{F}_{tri}$ contains the dependence on λ

$gg \to H$:

- LO, NLO: exact [Georgi et al. '78] [Spira et al. '95]
- NNLO: partial exact, approx (LME, THR) [Harlander, Prausa, Usovitch '19]
- N3LO: HEFT [Anastasiou et al. '16]

$gg \to HH$:

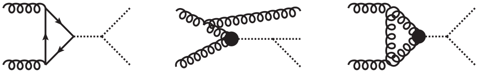
- LO: known exactly [Glover, van der Bij '88]
- Beyond LO... no exact (analytic) results to date

$gg \to HH$ Beyond LO

$m_t \to \infty$ limit used in many approximations:

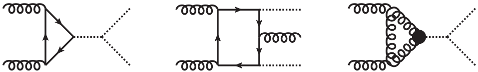
- NLO "Born-improved" HTL: [Dawson, Dittmaier, Spira '98]

  exact LO | B-i NLO real | B-i NLO virt



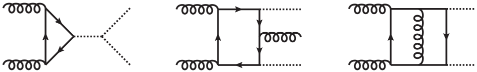
- NLO "FTapprox": [Maltoni, Vryonidou, Zaro '14]

  exact LO | exact NLO real | B-i NLO virt



- NLO Full: [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]

  exact LO | exact NLO real | exact NLO virt



B-i: $d\sigma_{NLO}(m_t) \approx \frac{d\sigma_{LO}(m_t)}{d\sigma_{LO}(\infty)} d\sigma_{NLO}(\infty)$

$gg \to HH$ Beyond LO

- NLO: large-m_t + threshold expansion [Gröber, Maier, Rauh ‘17]
- NLO: high-energy expansion [Davies, Mishima, Steinhauser, Wellmann ‘18,‘19]
- NLO: small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber ‘18]

- NNLO: large-m_t expansion of virtuals [Grigo, Hoff, Steinhauser ‘15]
- + numeric reals [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli ‘18]

- N3LO: Wilson coefficient C_{HH} [Spira ‘16][Gerlach, Herren, Steinhauser ‘18]
- N3LO: HEFT [Chen, Li, Shao, Wang ‘19]

The rest of the talk...

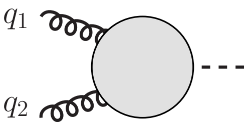
- Large-m_t expansion
  - expansion by subgraph
- NNLO $gg \to HH$
  - virtual corrections: more terms [Davies, Steinhauser '19]
    - combination with threshold expansion, Padé approximation [Davies, Gröber, Maier, Rauh, Steinhauser '19]
  - real corrections [Davies, Herren, Mishima, Steinhauser '19]
    $\to$ work in progress
- N3LO $gg \to H$
  - virtual corrections [Davies, Herren, Steinhauser '19]
- N3LO $H \to \gamma\gamma$ decay width [Davies, Herren '21]

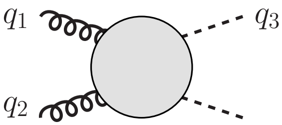
Large-m_t expansion

Expand integrals in the region $m_t \gg$ other scales.

Result: series in powers of $\{q_i \cdot q_i,\ q_i \cdot q_j,\ q_j \cdot q_j,\ \ldots\}/m_t^2$

- $gg \to H$: $q_1 \cdot q_2 / m_t^2$
- $gg \to HH$: $\{q_3 \cdot q_3,\ q_1 \cdot q_2,\ q_1 \cdot q_3,\ q_2 \cdot q_3\}/m_t^2$





Large-m_t expansion

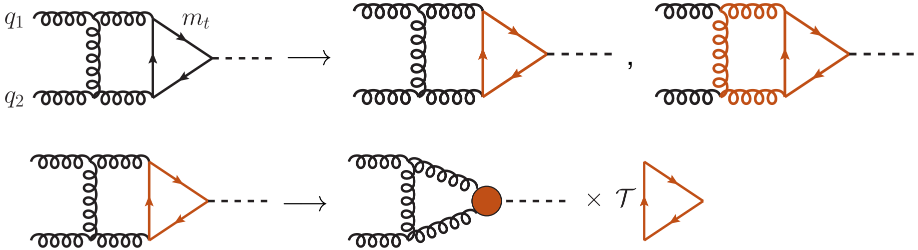
Expansion by sub-graph:

- sum over sub-graphs which contain m_t
- expand hard-scaling propagators in their small parameters

Diagrams factorize into:

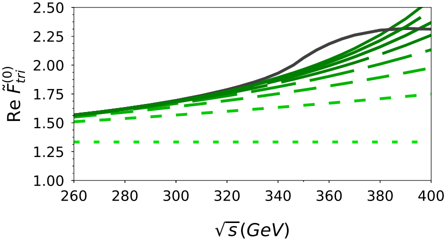
- massless integrals
- expanded hard sub-graphs $\to$ massive tadpole integrals

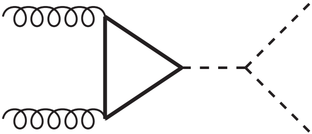
Example: 2-loop $gg \to H$ diagram:



Large-m_t expansion

Eg, LO $\mathcal{F}_{tri}$: expansion to $1/m_t^{14}$





Software:

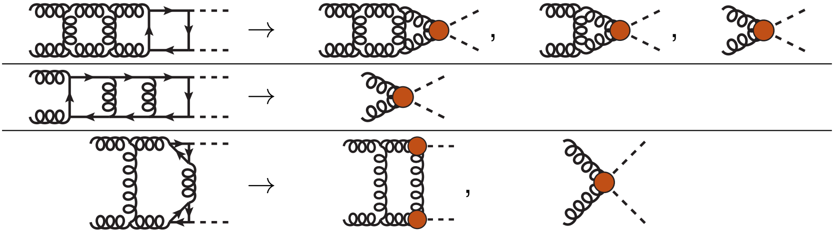
diagram generation	`qgraf`	[Nogueira '93]
large-m_t expansion	`q2e/exp`	[Harlander, Seidelsticker, Steinhauser '97]
	`FORM 4.2`	[Ruijl, Ueda, Vermaseren '17]
tadpoles (1-3 loop)	`MATAD`	[Steinhauser '00]
tadpoles (4 loop), massless integrals	`FIRE 6`	[Smirnov '19]

NNLO $gg \to HH$

Recall: $\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$.

We are interested in $\mathcal{F}_{tri}$, $\mathcal{F}_{box1}$, $\mathcal{F}_{box2}$ at 3L, in large m_t limit.

Large-m_t expansion proceeds as described before:

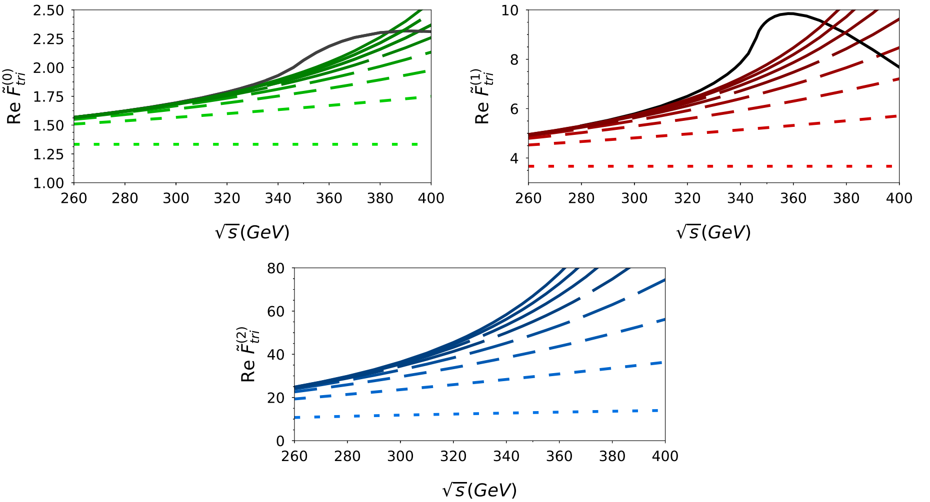


XS known to $1/m_t^4$; we want more terms, for FFs. [Grigo,Hoff,Steinhauser '15]

Computationally expensive: improve and optimize `FORM` routines.

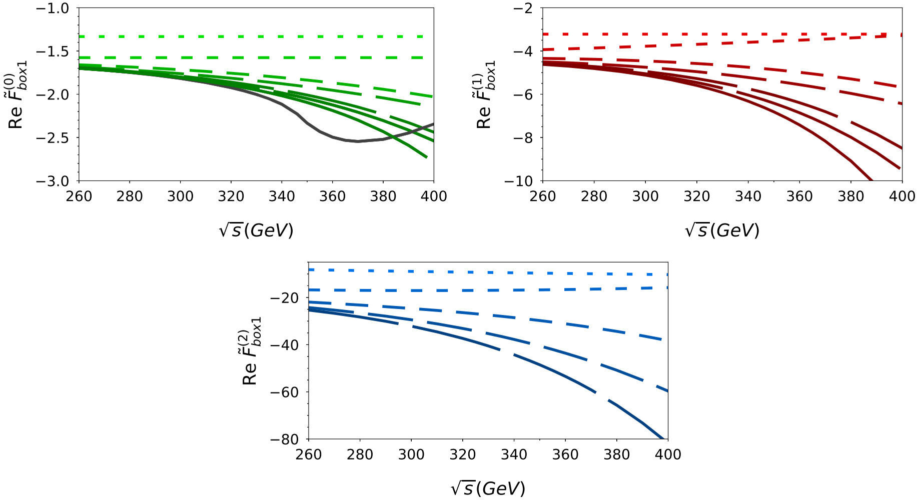
NNLO $gg \to HH$, expansion results

$\mathcal{F}_{tri}$: expansion to $1/m_t^{14}$



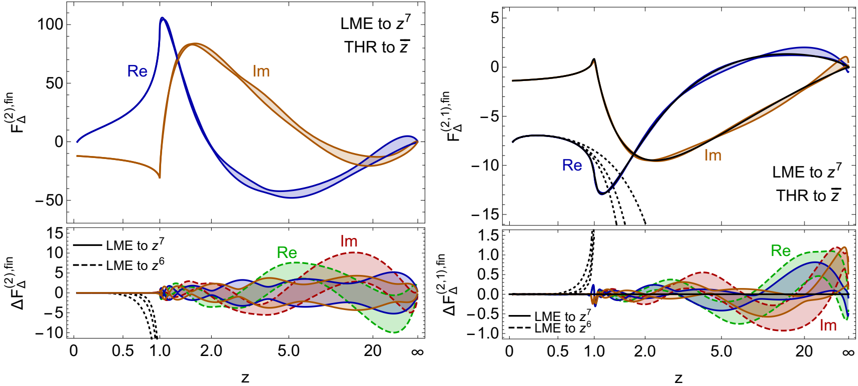
NNLO $gg \to HH$, expansion results

$\mathcal{F}_{box1}$: expansion to $1/m_t^8$



NNLO $gg \to H$ Padé approximation

Combine the large-m_t expansion (from $\mathcal{F}_{tri}$) and threshold expansion to produce approximation of NNLO m_t dependence for $gg \to H$.
($z = s/4m_t^2$) [Gröber,Maier,Rauh '18] [Davies,Gröber,Maier,Rauh,Steinhauser '19]



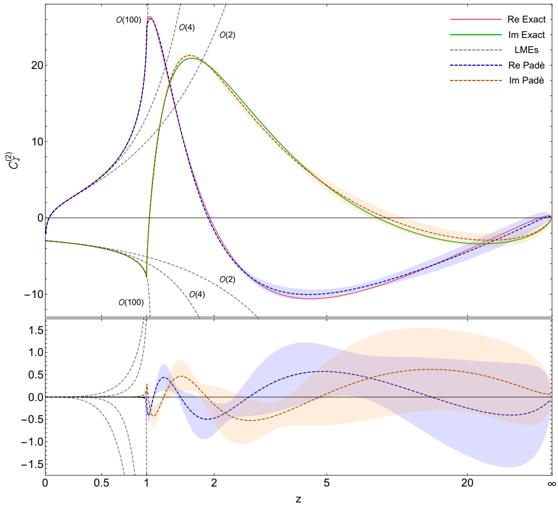
RHS: Light fermion part, known analytically. [Harlander, Prausa, Usovitsch '19]

Work in progress: same idea for $gg \to HH$.

NNLO $gg \to H$ Padé approximation

Also: NNLO $gg \to H$, semi-numerical full result [Czakon, Niggetiedt '20]

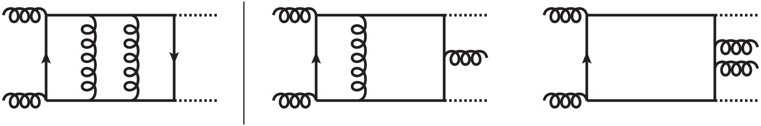
- large-m_t, threshold and high-energy exp. + interpolated numerical



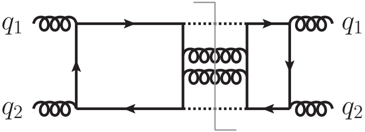
NNLO $gg \to HH$, Real Corrections

This has all been about virtual corrections.

What about real emission? $gg \to gHH$, $gg \to ggHH$?



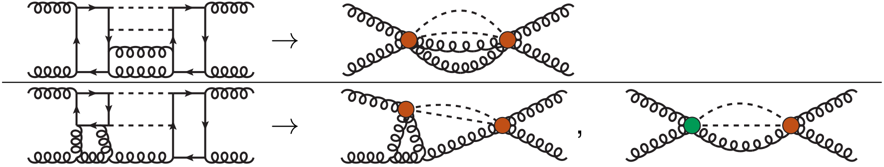
Proceed via reverse unitarity. 5-loop forward scattering diagrams:



Other channels also contribute: $q\bar{q}$, qq', gq.

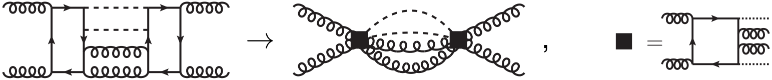
NNLO $gg \to HH$, Real Corrections

Direct expansion of such diagrams is very hard, computationally.



- we compute only the leading term $(1/m_t^0)$, as a cross-check.

Approach: construct “building blocks”, pre-exanded effective vertices:

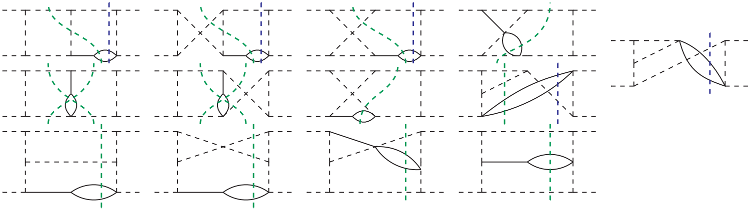


Here, instead of 3600, we generate 1 diagram.

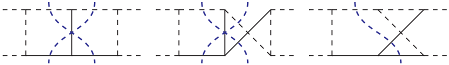
Need to compute and expand building blocks: *ggH*, *ggHH*, *gggH*, *gggHH*, *ggggH*, *ggggHH*.

NNLO $gg \to HH$, Real Corrections

After IBP, we are left with three and four particle cuts of 74 3-loop MIs belonging to 13 topologies,



and 16 2-loop MIs belonging to 3 topologies,



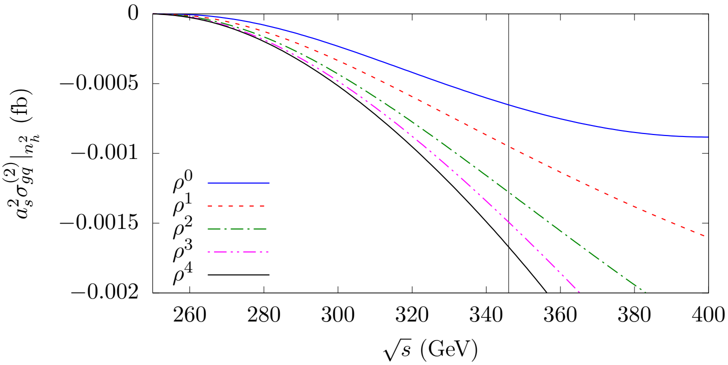
Compute via differential equations in $x = m_H^2/s$

- $\partial_x \vec{I} = M(\epsilon, x)\vec{I}$
- exact solutions and also via series ansatz: $\delta = \sqrt{1-4x} \to 0$

NNLO $gg \to HH$, Real Corrections

First results: dias with 3 top quark loops. [Davies, Herren, Mishima, Steinhauser '19]

Preliminary: gq channel: $(\rho = m_H^2/m_t^2)$



N3LO $gg \to H$

$$\text{amplitude} \sim \left(q_1 \cdot q_2 \, g^{\mu\nu} - q_1^\nu q_2^\mu\right) h(\rho)$$

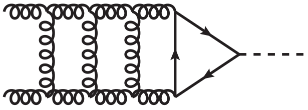
- known: N3LO HEFT [Anastasiou et al. '16] [Mistlberger '18]
- N3LO virtual amplitude to $1/m_t^4$. [Davies, Herren, Steinhauser '19]

Define $F = h(\rho)/h^{(0)}(\rho) = 1 + \mathcal{O}(\alpha_s)$. Then $\log F = \log F_{\text{poles}} + \log F_{\text{finite}}$.

$\log F_{\text{poles}}$ known, agree. [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]

$\log F_{\text{finite}}$ contains the ρ dependence:

Re $\log F_{\text{finite}}$	ρ^0	ρ^1	ρ^2
NLO	100%	0.63%	0.036%
NNLO	7.04%	0.32%	0.022%
N3LO	-0.79%	0.08%	0.008%



$H \to \gamma\gamma$ decay

Similarly to $gg \to H$,

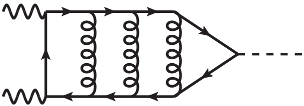
$$\text{amplitude} \sim \left(q_1 \cdot q_2\, g^{\mu\nu} - q_1^\nu q_2^\mu\right) A(s).$$

Decay width: $\Gamma_{H\to\gamma\gamma} = m_H^3/(64\pi) \cdot |A(s)|^2$.

- partial (fermionic) N3LO HEFT results available, [Sturm '14]
- we compute large-m_t corrections, to $1/m_t^6$. [Davies, Herren '21]

Numerical effect, m_t OS, $\mu = m_H$:

$$\Gamma_{H\to\gamma\gamma} \times 10^6\ \text{GeV}^{-1} =$$
$$+9.2581|_{\text{LO}} - 0.1502|_{\text{NLO,EW}}$$
$$+0.1569|_{\text{NLO,t}} + 0.0157|_{\text{NLO,bc}}$$
$$+0.0030|_{\text{NNLO,t}} + 0.0037|_{\text{NNLO,bc}}$$
$$-0.0031|_{\text{N3LO,t}}$$



Conclusion

We can compute large-mass expansions of difficult amplitudes

- multi loop
- multi scale

in a systematic way.

We can combine with information from other sources, for e.g.

- threshold exp
- high-energy exp $\leftarrow$ G. Mishima's talk, Wed. Parallel I.A
- small-p_T exp
- numerics

in order to produce approximations sufficient for pheno purposes.