Towards NNLO corrections to muon-electron scattering

RADCOR-LoopFest 2021

Syed Mehedi Hasan
(INFN, Sezione di Pavia)

In collaboration with

Carlo M. Carloni Calame, Mauro Chiesa,

Guido Montagna, Oreste Nicrosini, Fulvio Piccinini

based on: *JHEP* 11 (2020) 028

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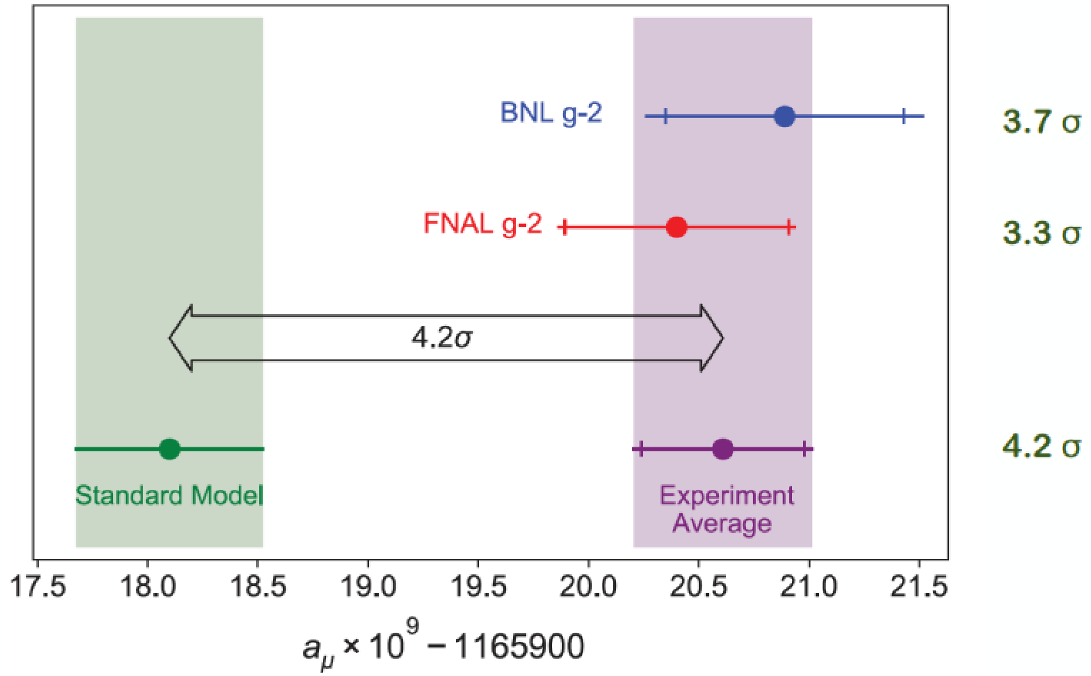
Email: syedmehe@pv.infn.it

Outline

- Motivation
- MUonE
- NNLO Photonic corrections
- NNLO Leptonic corrections
- MIs for Leptonic corrections on Vertices
- Outlook

Motivation



a_μ^{EXP} = (116592089 ± 63) x 10^{-11} [0.54ppm] BNL E821

a_μ^{EXP} = (116592040 ± 54) x 10^{-11} [0.46ppm] FNAL E989 Run 1

a_μ^{EXP} = (116592061 ± 41) x 10^{-11} [0.35ppm] WA

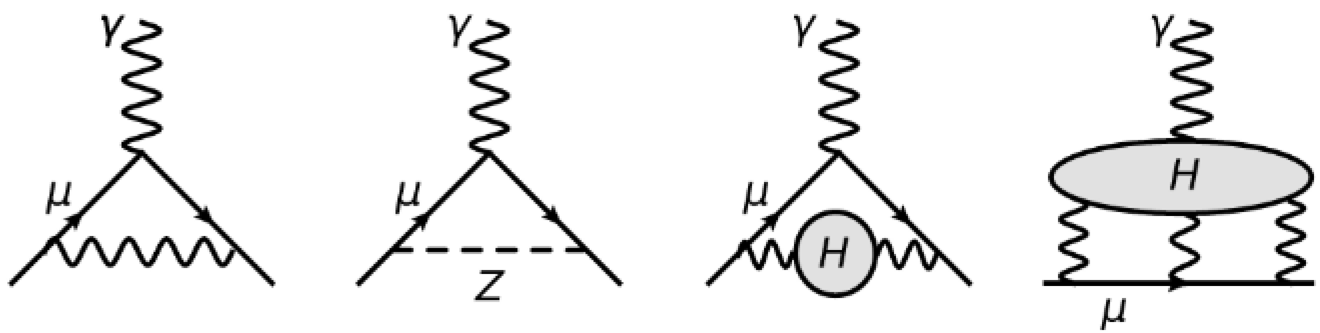
- **FNAL aims at 16 x 10^{-11}. First 3 runs completed, 4th in progress.**
- **Muon g-2 proposal at J-PARC: Phase-1 with ~ BNL precision.**

WP20 = White Paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

See M. Passera's talk

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP, LO}} + a_\mu^{\text{HVP, NLO}} + a_\mu^{\text{HVP, NNLO}} + a_\mu^{\text{HLbL}} + a_\mu^{\text{HLbL, NLO}}$$
$$= 116\,591\,810(43) \times 10^{-11}\,.$$

[WP20]



Types of corrections that goes inside SM prediction

$$a_\mu^{\text{QED}}(\alpha(\text{Cs})) = 116\,584\,718.931(104) \times 10^{-11}\,.$$

$$a_\mu^{\text{EW}} = 153.6(1.0) \times 10^{-11}$$

$$a_\mu^{\text{HVP, LO}} = 6931(40) \times 10^{-11}$$

$$a_\mu^{\text{HVP, NLO}} = -98.3(7) \times 10^{-11}$$

$$a_\mu^{\text{HVP, NNLO}} = 12.4(1) \times 10^{-11}$$

$$a_\mu^{\text{HLbL}}(\text{phenomenology + lattice QCD}) = 90(17) \times 10^{-11}$$

$$a_\mu^{\text{HLbL, NLO}} = 2(1) \times 10^{-11}$$

[WP20]

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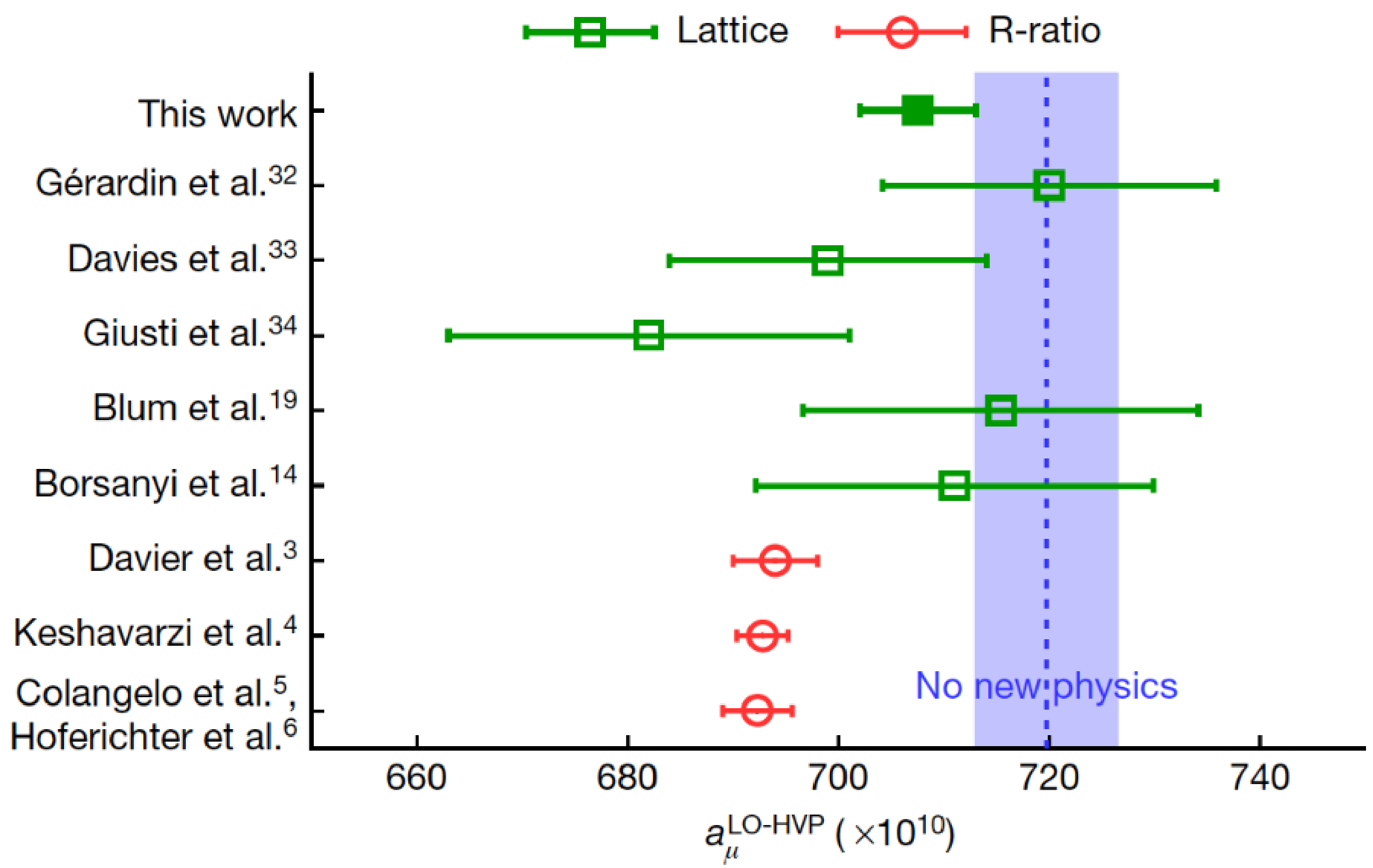
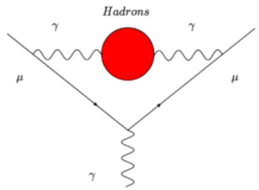


[Borsanyi et. al. (BMWC), Nature 2021]

- In the following, focus on $a_\mu^{\rm HLO}$, which contributes (with $a_\mu^{\rm HLbL}$) to the SM uncertainty



- Using dispersion relations and the Optical Theorem

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{e^+e^-\to {\rm had}}(s) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{K(s)R^{\rm had}(s)}{s^2} =$$

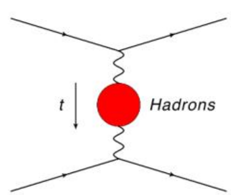
$$= \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{4m_\pi^2}^{E^2_{\rm cut}} ds \frac{K(s)R^{\rm had}_{\rm data}(s)}{s^2} + \int_{E^2_{\rm cut}}^{\infty} ds \frac{K(s)R^{\rm had}_{\rm pQCD}(s)}{s^2}\right]$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)\frac{s}{m_\mu^2}} \sim \frac{1}{s} \qquad R^{\rm had}(s) = \frac{\sigma^0_{e^+e^-\to {\rm had}}(s)}{\frac{4}{3}\frac{\pi\alpha^2}{s}}$$

- Alternatively (exchanging s and x integrations in $a_\mu^{\rm HLO}$)

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$



e.g. **Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193**

- Essentially the same formula used in lattice QCD calculation of $a_\mu^{\rm HLO}$

★ $\Delta\alpha_{\rm had}(t)$ (and $a_\mu^{\rm HLO}$) can be directly measured in a (single) experiment involving a space-like scattering process

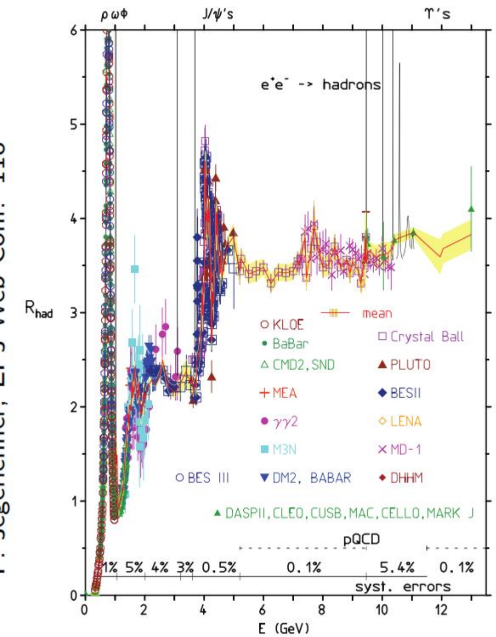
CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

Arbuzov *et al.* EPJC 34 (2004) 267

Abbiendi *et al.* (OPAL) EPJC 45 (2006) 1

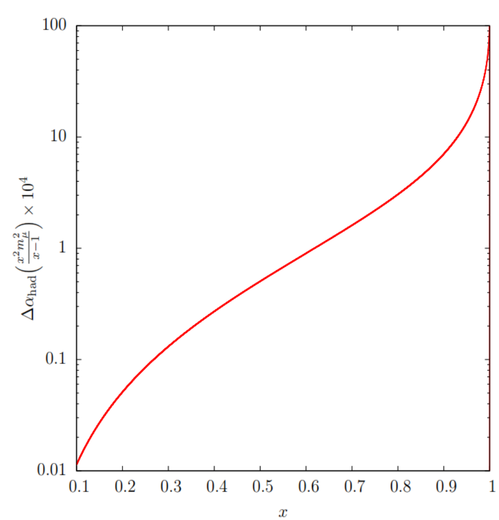
★ **Still a data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

Time-like



F. Jegerlehner, EPJ Web Conf. 118

Space-like



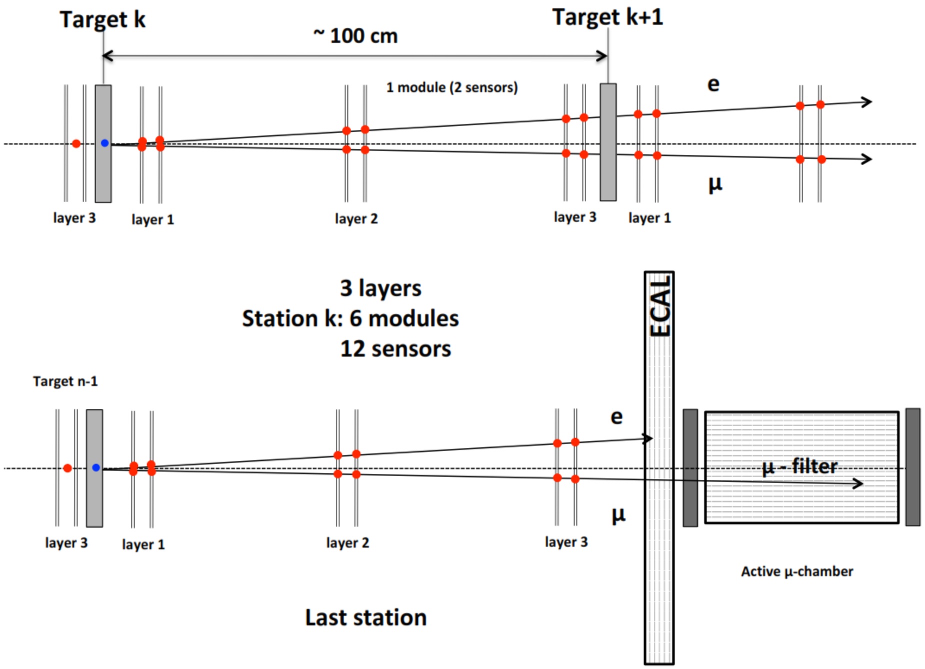
Smooth function

CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

From Carlo's talk, INFN Roma Tre, 2019

Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,
Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

$\Delta\alpha_{had}(t)$ can be measured from elastic $\mu e \to \mu e$ scattering.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,
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-> 150 GeV muon beam on a fixed electron target.
->Each module consists of a low-Z target (Berillium) and two silicon
tracking stations located at a distance of one meter.

-> Systematic effects must be known at ≤ 10 ppm
-> Test run approved for 2021.
> Hopefully full run from 2022-24.

- **Fully differential fixed-order MC @ NLO ready** Pavia and PSI 2018-19
- **NNLO QED: Master Integrals for 2-loop box diagrams computed. Full 2-loop amplitude close to completion.** Padova 2017 - present
- **Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation** Pavia and PSI 2020
- **NNLO hadronic effects computed** Padova and KIT 2019
- **Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes** PSI 2019-present
- **New Physics extracting $\Delta\alpha_{had}(t)$ at MUonE?** Padova and Heidelberg 2020
- …

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

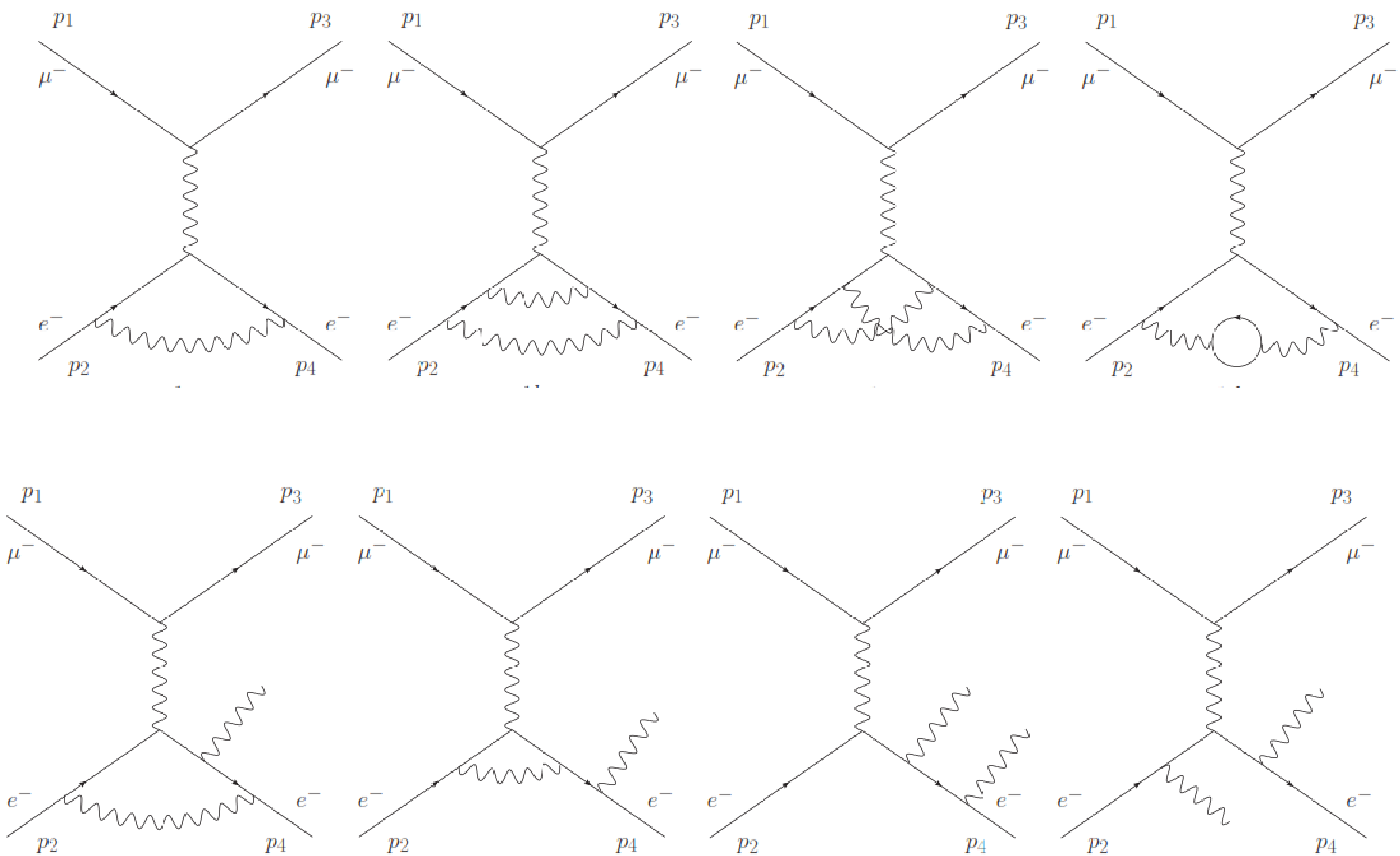
The ratio of the SM cross section in the signal and the normalization region must be known at ≤ 10 ppm

M. Passera's talk

NNLO Photonic Corrections

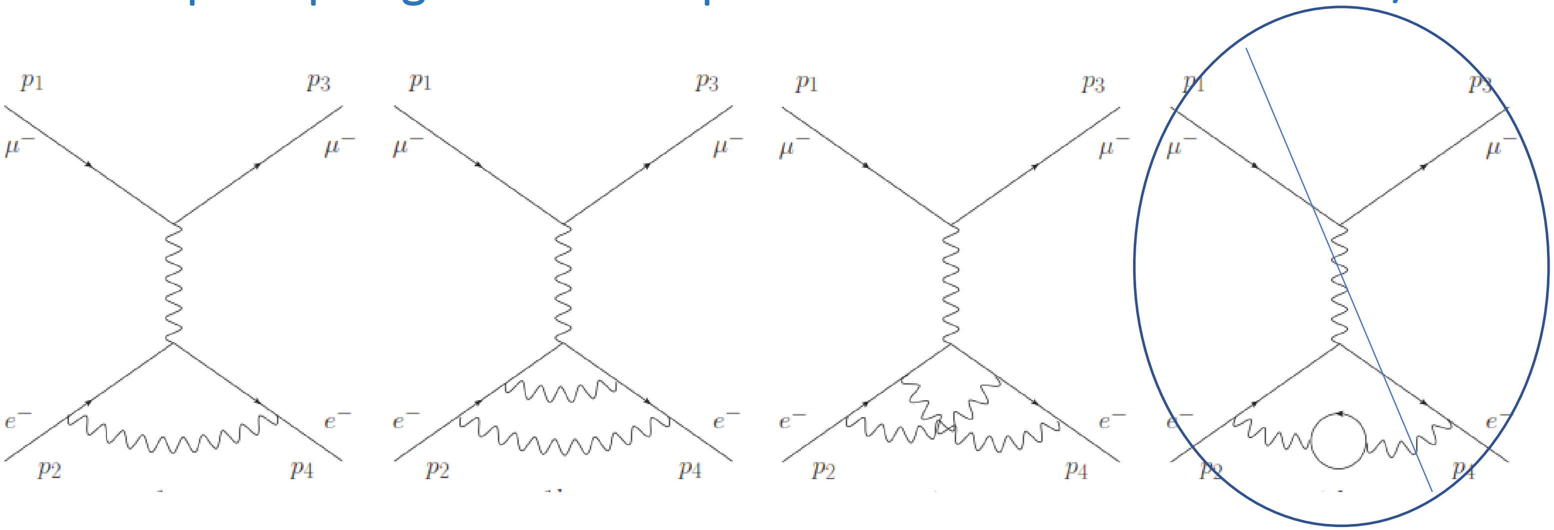
Published in --
JHEP 11 (2020) 028

Sample topologies for NNLO QED corrections on electron/muon line



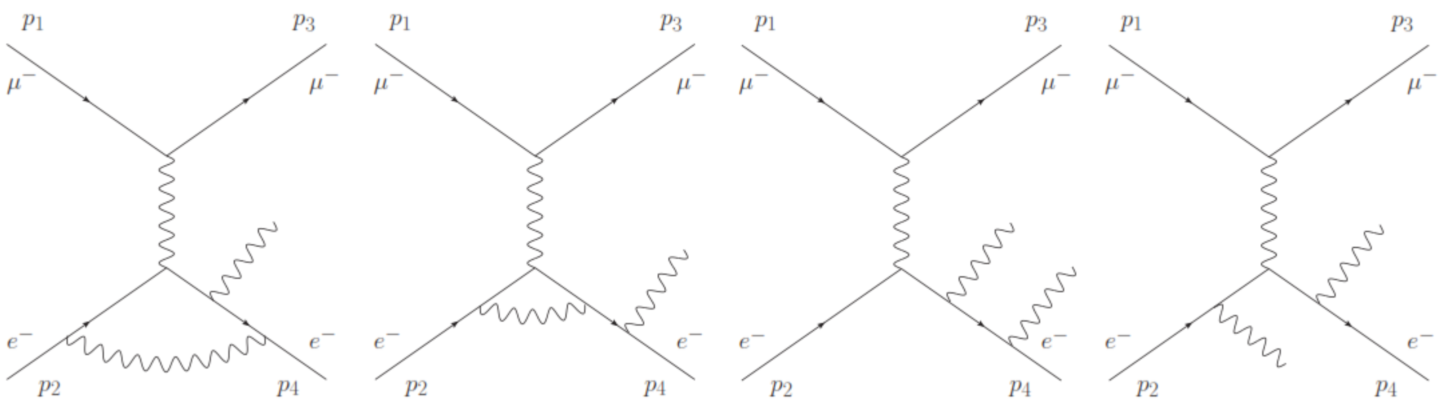
[Carlo Calame et. Al. ‘20]

Sample topologies for NNLO photonic corrections on electron/muon line



Nf=1 subset is removed

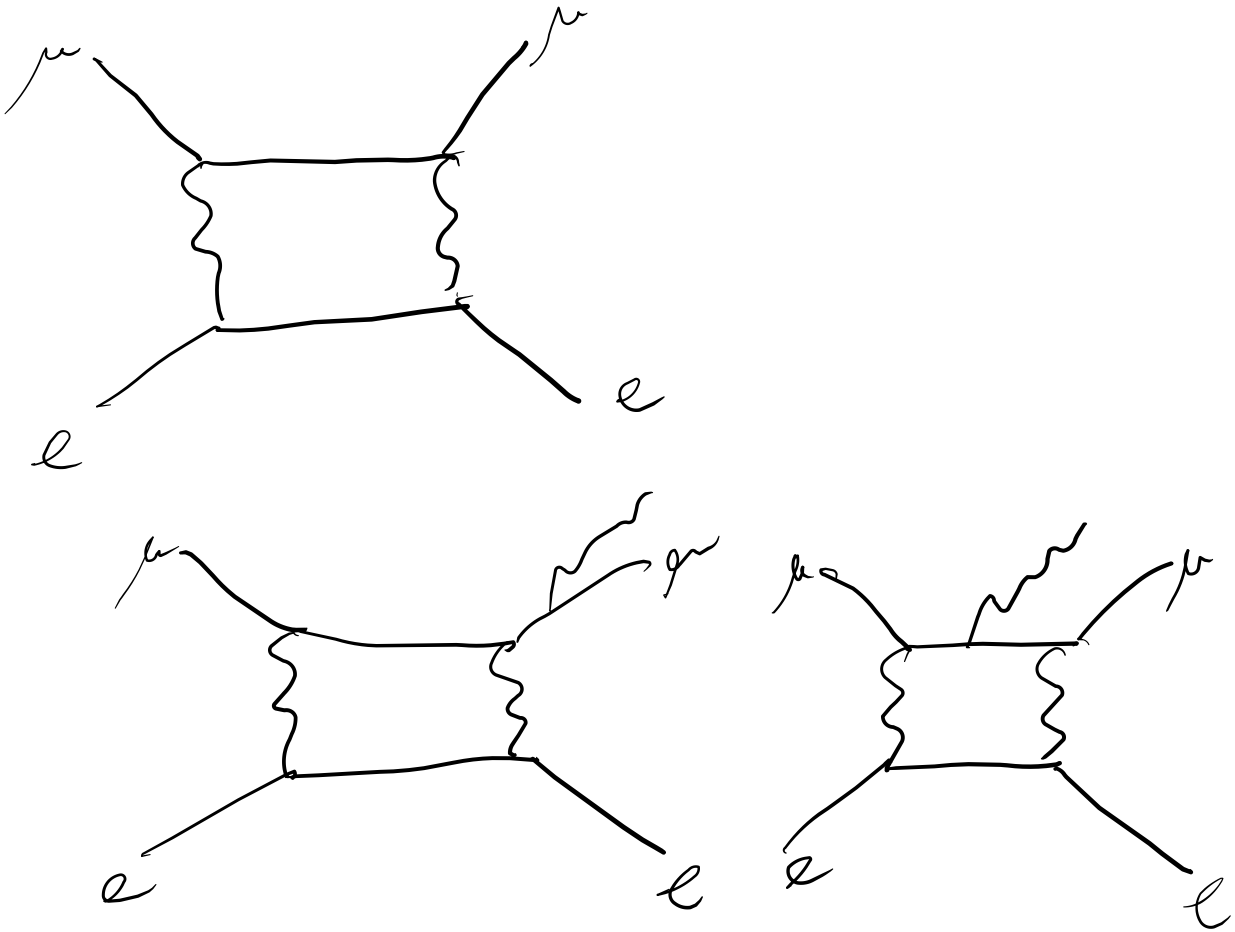
Exact calculation



Two loop Formfactors are taken from Mastrolia et. al. arXiv:hep-ph/0302162

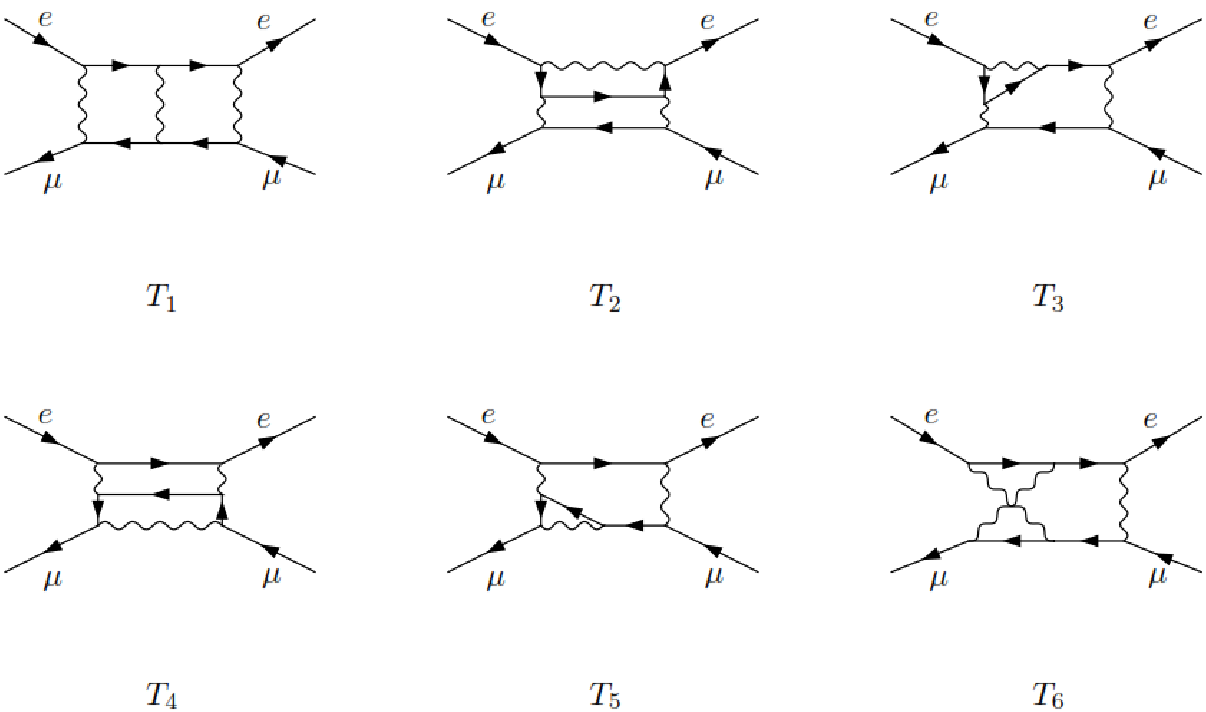
[Carlo Calame et. Al. ‘20]

Sample topologies for one loop boxes



All relevant one loop boxes and pentagons are calculated exactly

Sample topologies for NNLO photonic corrections to box like structure



Not known exactly with full mass effect. Recent progress in calculation of MIs keeping the muon mass and neglecting electron mass

[Mastrolia et. al. ’17]

Massification

Engel, Gnendiger, Signer, Ulrich, arXiv:1811.06461
Becher, Melnikov, arXiv:0704.3582
Mitov, Moch, arXiv:hep-ph/0612149
A. Penin, arXiv:hep-ph/0501120

Yennie-Frautschi-Suura (YFS) approximation used including full mass dependence

NNLO photonic corrections

$$\mathcal{M}^{\alpha^0} = \mathcal{T}$$

$$\widetilde{\mathcal{M}}^{\alpha^2} = \mathcal{M}_e^{\alpha^2} + \mathcal{M}_\mu^{\alpha^2} + \mathcal{M}_{e\mu,\,1\mathrm{L}\times 1\mathrm{L}}^{\alpha^2} + \frac{1}{2} Y_{e\mu}^2 \mathcal{T} + Y_{e\mu}\left(Y_e + Y_\mu\right)\mathcal{T} + (Y_e + Y_\mu)\mathcal{M}_{e\mu}^{\alpha^1,\mathrm{R}} + Y_{e\mu} M^{\alpha^1,\mathrm{R}}.$$

Only non-IR remnant of the two loop boxes are approximated

$$Y = \sum_{i,j=1,4}^{j\geq i} Y_{ij} = Y_e + Y_\mu + Y_{e\mu}$$

$$Y_{ij} = \begin{cases} \frac{1}{8}\frac{\alpha}{\pi} Q_i^2 \left[B_0\left(0, m_i^2, m_i^2\right) - 4 m_i^2 C_0\left(m_i^2, 0, m_i^2, \lambda^2, m_i^2, m_i^2\right)\right] & \text{for } i = j \\ \frac{\alpha}{\pi} Q_i Q_j \vartheta_i \vartheta_j \Big[p_i \cdot p_j\ C_0\left(m_i^2, (\vartheta_i p_i + \vartheta_i p_j)^2, m_j^2, \lambda^2, m_i^2, m_j^2\right) + & \\ \qquad + \frac{1}{4} B_0\left((\vartheta_i p_i + \vartheta_j p_j)^2, m_i^2, m_j^2\right)\Big] & \text{for } i \neq j \end{cases}$$

$$Y_e = Y_{24} + Y_{22} + Y_{44}$$

$$Y_\mu = Y_{13} + Y_{11} + Y_{33}$$

$$Y_{e\mu} = Y_{12} + Y_{14} + Y_{23} + Y_{34}$$

[Carlo Calame et. Al. '20]

NNLO photonic corrections

- Phenomenological results are obtained by using fully differential MC code, MESMER.
- Structure of the code is completely general. YFS can be replaced by exact calculation.
- We adopt the typical running condition of the MUonE experiment. Energy of the incoming muon beam is taken to be 150 GeV.
- The electron is assumed to be in rest inside a bulk target and thus $\sqrt{s} \simeq 0.405541$ GeV

[Carlo Calame et. al. '19]

1. $\vartheta_e, \vartheta_\mu < 100$ mrad and $E_e > 1$ GeV (i.e. $t_{ee} \lesssim -1.02 \cdot 10^{-3}$ GeV^2). The angular cuts model the typical acceptance conditions of the experiment and the electron energy threshold is imposed to guarantee the presence of two charged tracks in the detector (Setup 1);

2. the same criteria as above, with the additional acoplanarity cut $|\pi - |\phi_e - \phi_\mu|| \leq 3.5$ mrad. We remind the reader that this event selection is considered in order to mimic an experimental cut which allows to stay close to the elasticity curve given by the tree-level relation between the electron and muon scattering angles (Setup 2)

where $t_{ee} = (p_2 - p_4)^2$, $(\vartheta_e, \phi_e, E_e)$ and $(\vartheta_\mu, \phi_\mu, E_\mu)$ are the scattering and azimuthal angles and the energy, in the laboratory frame, of the outgoing electron and muon, respectively.

[Carlo Calame et. Al. '20]

NNLO photonic corrections

$\alpha = 1/137.03599907430637$ $\quad m_e = 0.510998928$ MeV $\quad m_\mu = 105.6583715$ MeV

σ (μb)	Setup 1		Setup 2	
	$\mu^- e^- \to \mu^- e^-$	$\mu^+ e^- \to \mu^+ e^-$	$\mu^- e^- \to \mu^- e^-$	$\mu^+ e^- \to \mu^+ e^-$
σ_{LO}	245.038910(1)			
σ^e_{NLO}	255.5500(7)		223.4387(6)	
$\sigma^\mu_{\mathrm{NLO}}$	244.9707(1)		244.4136(1)	
σ^f_{NLO}	255.1176(5)	255.8437(5)	222.8545(3)	222.7714(3)
σ^e_{NNLO}	255.5725(5)		224.4796(4)	
$\sigma^\mu_{\mathrm{NNLO}}$	244.9706(1)		244.4154(1)	
σ^f_{NNLO}	*255.205(1)*	*256.092(1)*	*224.041(1)*	*224.088(1)*

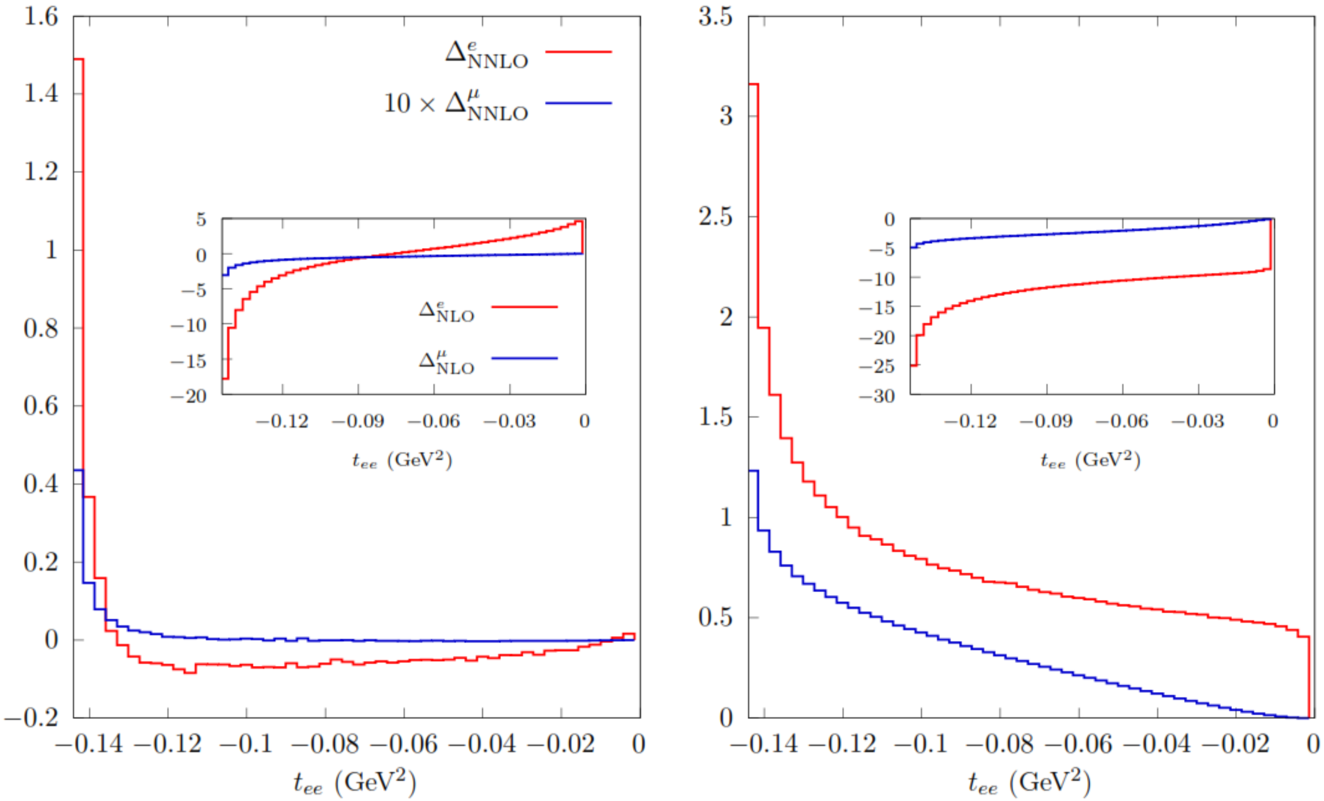
Cross sections (in μb) and relative corrections for the processes $\mu^- e^- \to \mu^- e^-$ and $\mu^+ e^- \to \mu^+ e^-$, in the two different setups described in the text. The symbols $\sigma^{e/\mu/f}_{\mathrm{(N)(N)LO}}$ stand for the cross sections with corrections along the electron line only, along the muon line only and the full approximate contributions, respectively, with the perturbative accuracy given by the subscripts. The digits in parenthesis correspond to 1σ MC error. Italicized numbers in the last row indicate that in this cross-section the full two-loop amplitude is approximated

[Carlo Calame et. Al. '20]

NNLO photonic corrections

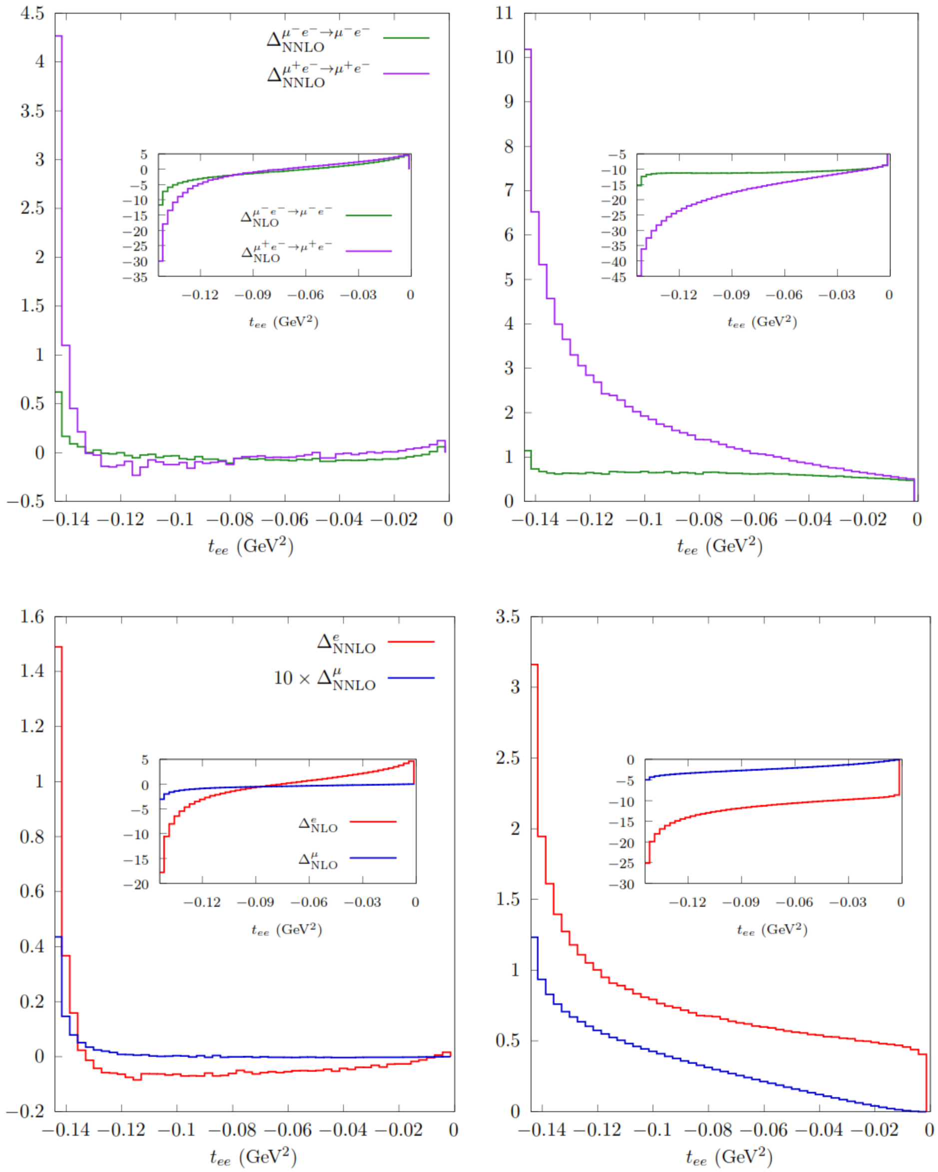
$$\Delta^i_{\rm NLO} = 100 \times \frac{d\sigma^i_{\rm NLO} - d\sigma_{\rm LO}}{d\sigma_{\rm LO}}$$

$$\Delta^i_{\rm NNLO} = 100 \times \frac{d\sigma^i_{\rm NNLO} - d\sigma^i_{\rm NLO}}{d\sigma_{\rm LO}}$$



[Carlo Calame et. Al. ‘20]

NNLO photonic corrections



[Carlo Calame et. Al. '20]

NNLO Leptonic Corrections

(Work in progress)

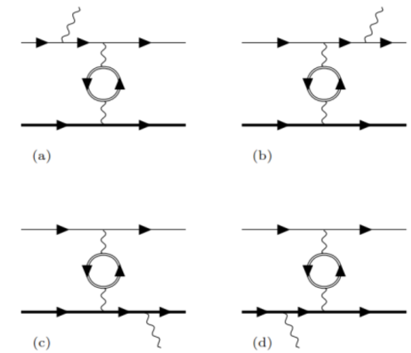
NNLO Leptonic Corrections

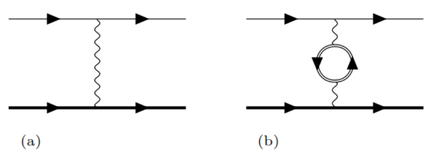
$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

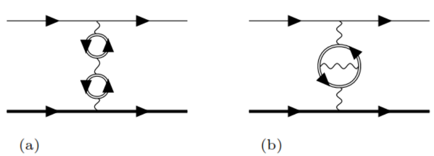
2 loop virtual

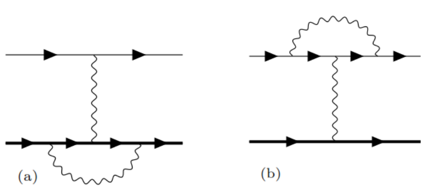
1 virtual 1 real

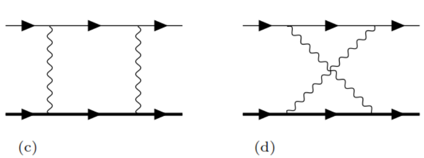
double real

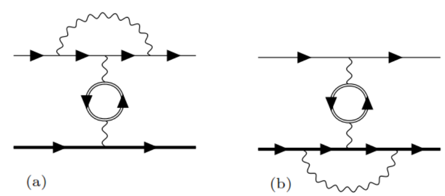


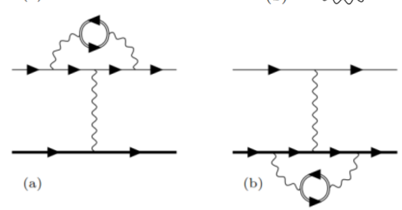


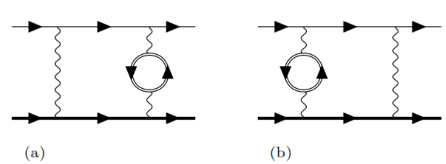


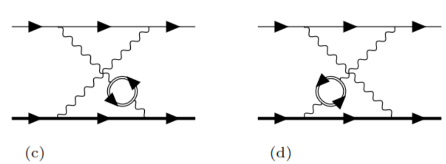


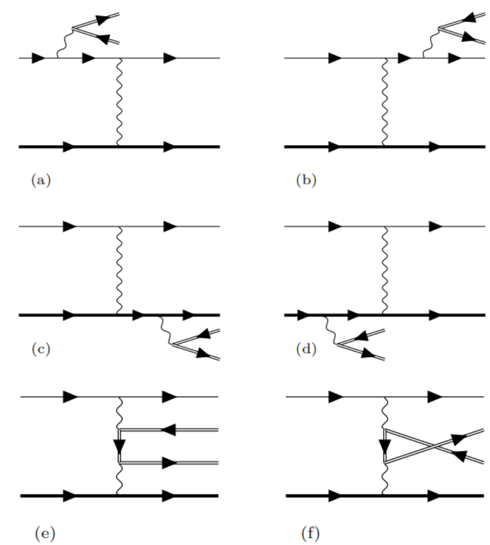








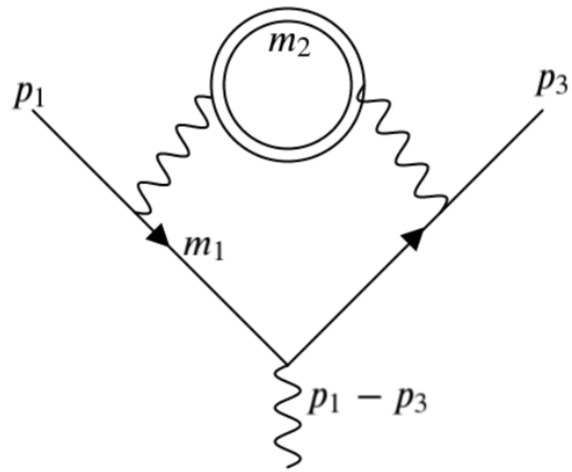




Ettore Budassi, Carlo M. Carloni Calame, Mauro Chiesa, Syed Mehedi Hasan, Guido Montagna, Oreste Nicrosini, Fulvio Picinini. (Work in progress)

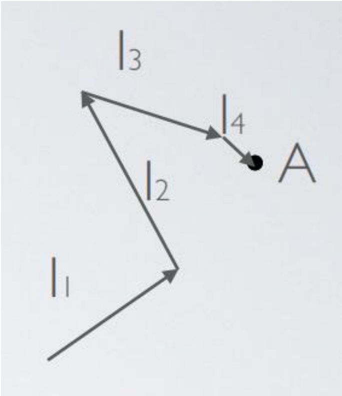
MIs for Leptonic corrections on Vertices

Vertex with two different mass:



-> Amplitude given by Feynman diagrams

$$A = \sum_i a_i I_i$$



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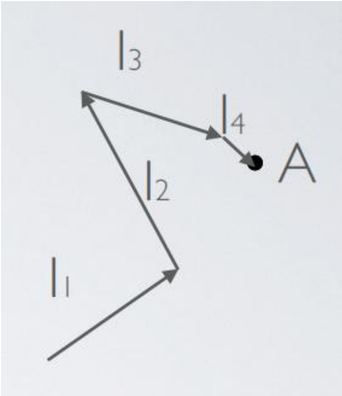
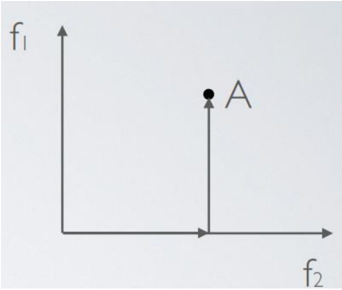


-> Project onto basis using Integration by Parts identities

(Tkachov; Chetyrkin, Tkachov)

$$A = \sum_i c_i f_i$$



Implemented in public codes

LiteRed	(Lee)
REDUZE	(Studerus, von Manteuffel)
Fire	(Smirnov)
Air	(Anastasiou, Lazopolus)
Kira	(Maierhoefer, Usovitsch, Uwer)

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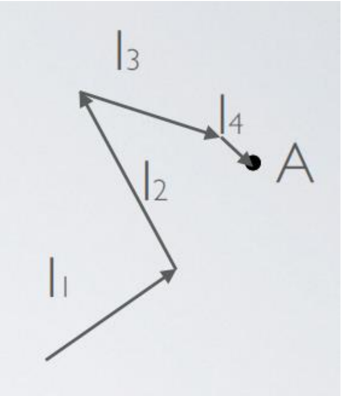

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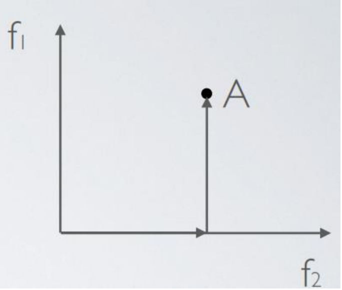

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Kira	(Maierhoefer, Usovitsch, Uwer)

-> Calculate basis elements via differential equations

Differential Equation

-> Kinematic derivative in space spanned by MIs

$$\partial_x \bar{f} = A_x \bar{f}$$

Differential Equation

-> Kinematic derivative in space spanned by MIs

$$\partial_x \bar{f} = A_x \bar{f}$$

-> Conjecture: There is a basis such that:

$$\partial_x \bar{g} = \epsilon \tilde{A}_x \bar{g}$$ (Henn)

->There are many strategies to get the epsilon factorized form

- Magnus Theorem (Ageri, Di Vita, Mastrolia, Mirabella, Schlenk, Tancredi, US)
- Unit leading Singularity (Henn)
- Reduction to fuchsian form and Eigenvalue normalization (Lee, Smirnov)
- Factorization of Picard-Fuchs operator (Adams, Chaubey, Weinzierl)

Solving Canonical Differential Equation

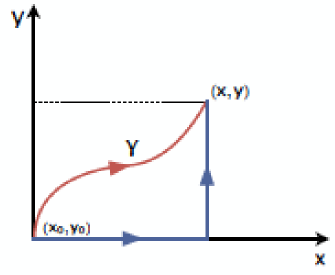
Canonical form

$$\partial_x \vec{g}(x,\epsilon) = \epsilon \tilde{A}_x(x)\vec{g}(x,\epsilon) \qquad d\vec{g}(x,\epsilon) = \epsilon \sum_i M_i dlog(\eta_i)\vec{g}(x,\epsilon)$$

- Kinematic dependence encoded in η
- η's form the alphabet

Solution given by

$$\vec{g}(x,\epsilon) = \left[1 + \sum_{i=1}^{\infty} \int_\gamma dA \dots dA\right] \vec{g}(x_0,\epsilon)$$



Algebraic ηs : Chen Iterated Integrals (Chen)

$$C(\vec{\eta_n}; x) = \int_\gamma dlog(\eta_1)\dots dlog(\eta_n)$$

Rational ηs : Generalized Polylogarithms (Goncharov)

$$G(\vec{0}_n; x) = \frac{1}{n!} Log(x)^n$$

$$G(\vec{w}_n; x) = \int_0^x \frac{dt}{t - w_1} G(\vec{w}_{n-1}; t)$$

Boundary Conditions

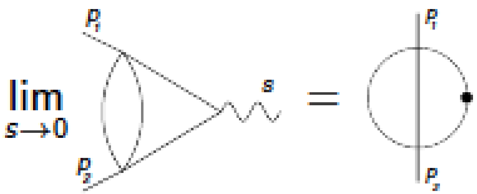
-> Solution given by

$$\vec{g}(x,\epsilon) = \left[1 + \sum_{i=1}^{\infty} \int_{\gamma} dA \ldots dA\right] \vec{g}(x_0,\epsilon)$$

-> Two general ways to fix the boundary

Known Limit

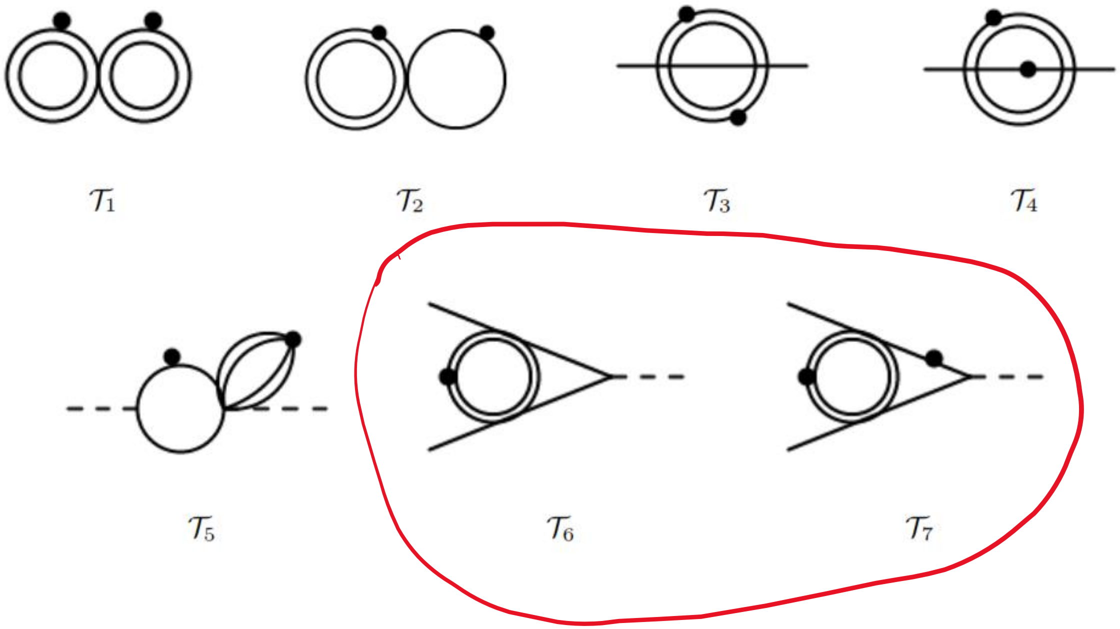
- Taking the limit x to x0
- Fix boundary constant by matching the solution to known function



Pseudo-thresholds

- Solution has unphysical divergences
- Demanding absence of unphysical divergences gives relations between boundary constant
- Leftover constants must be provided

MIs:



Boundary Constants are fixed using PSLQ

Numerically Checked against SecDec 3
(Borowka et. al arXiv:1502.06595)

Outlook

- MUonE is on track
- NNLO Photonic corrections to muon-electron scattering are presented
- NNLO Leptonic corrections are in progress
- New multi scale Master integrals are calculated

- Thank you
  for the attention