NNLO Photon Fragmentation within Antenna Subtraction

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Outline

1. Photons in hadronic collisions: fragmentation and isolation
2. Antenna subtraction with identified final state particles
3. Integration of fragmentation antenna functions

Photon Production @ the LHC

γ (+jet) important observable:

1. Testing ground for precise QCD predictions

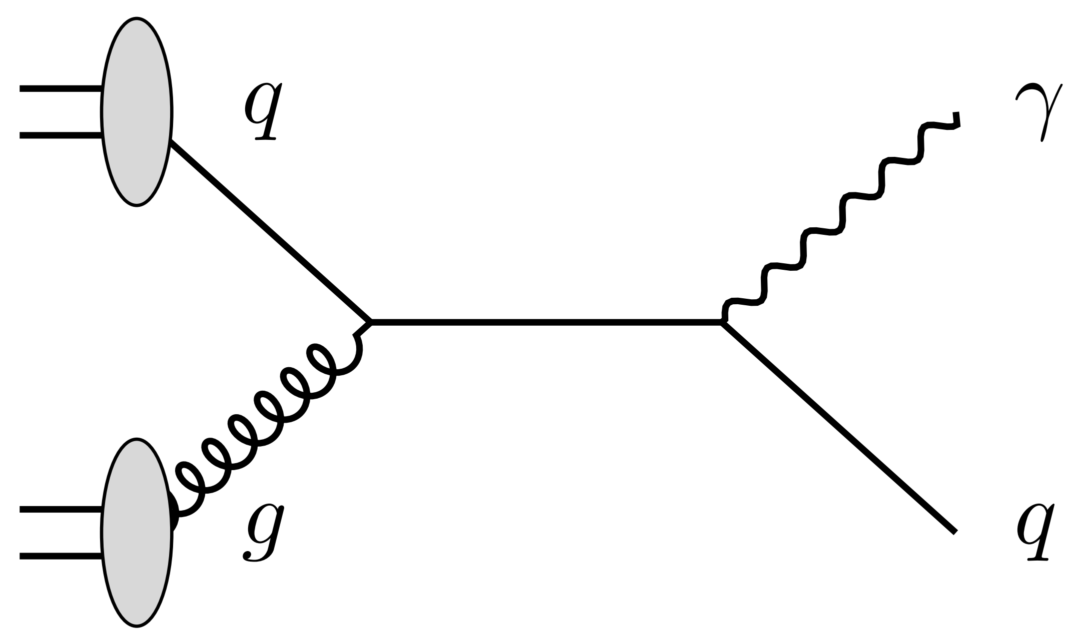
   → clean, well-reconstructable final state
   → precise data from experiment available
   Atlas, 2018, CMS, 2018

2. High sensitivity on gluon PDF

   →Compton scattering @ LO

3. Important background for new physics searches

   → new physics decaying into photons
   → γ + jet as data driven background estimate for dark matter searches

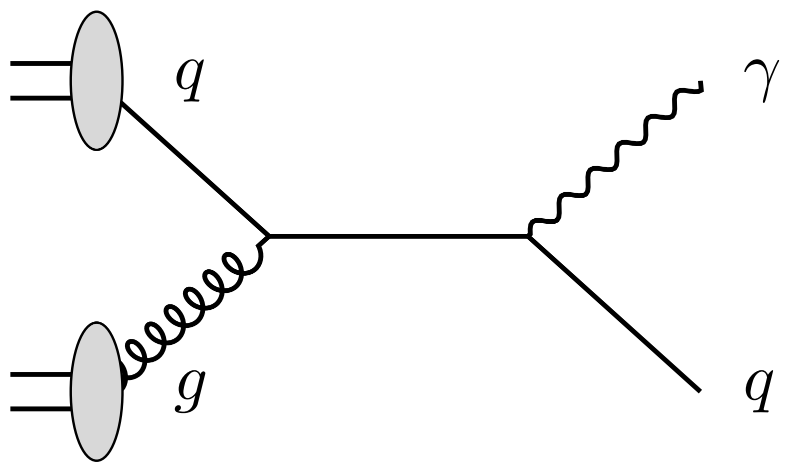


Photons @ the LHC

Three different kinds of photons in hadronic collisions:

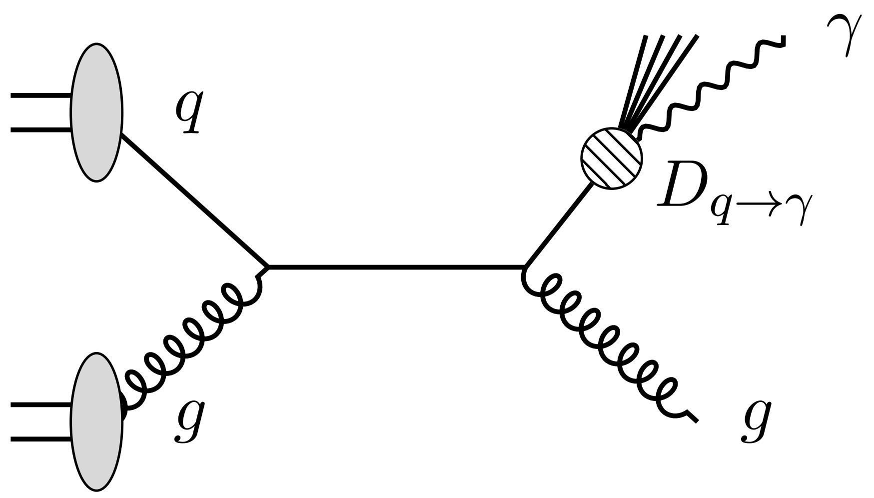
1. Direct photons

→ point-like coupling of quarks and photons

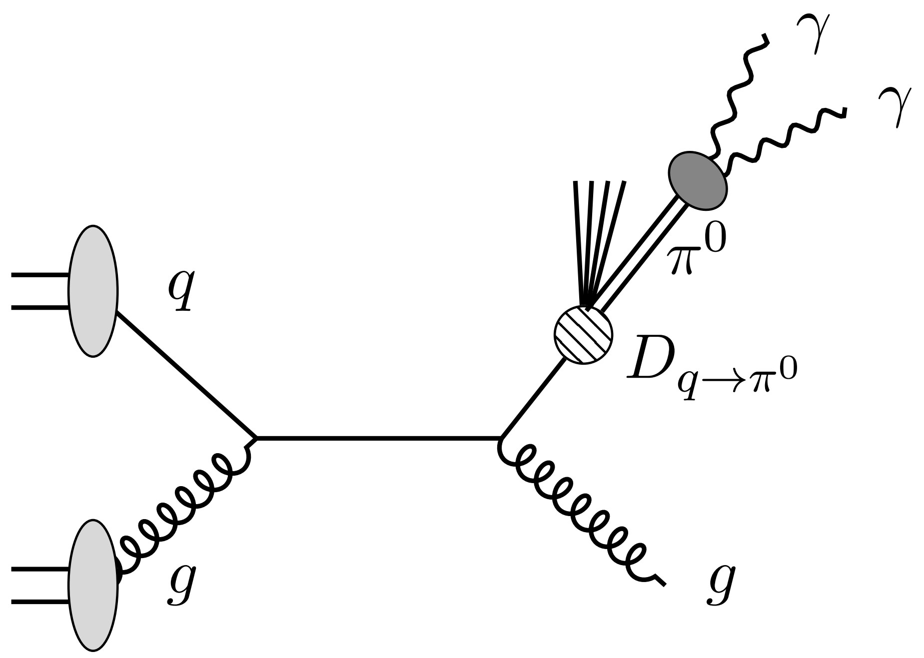


2. Partons fragmenting into photons

→ fragmentation functions (FF) $D_{k \to \gamma}(z)$

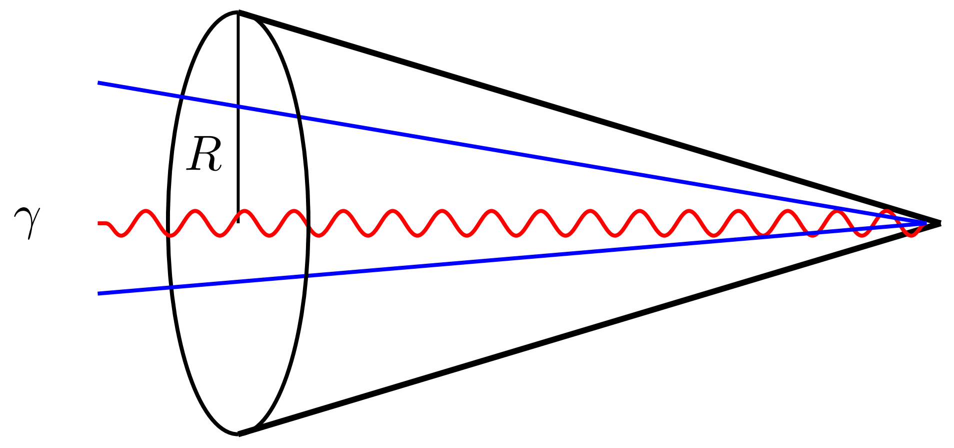


3. Photons from hadronic decays ($\pi^0 \to \gamma\gamma$)



Photon Isolation

Fixed cone isolation

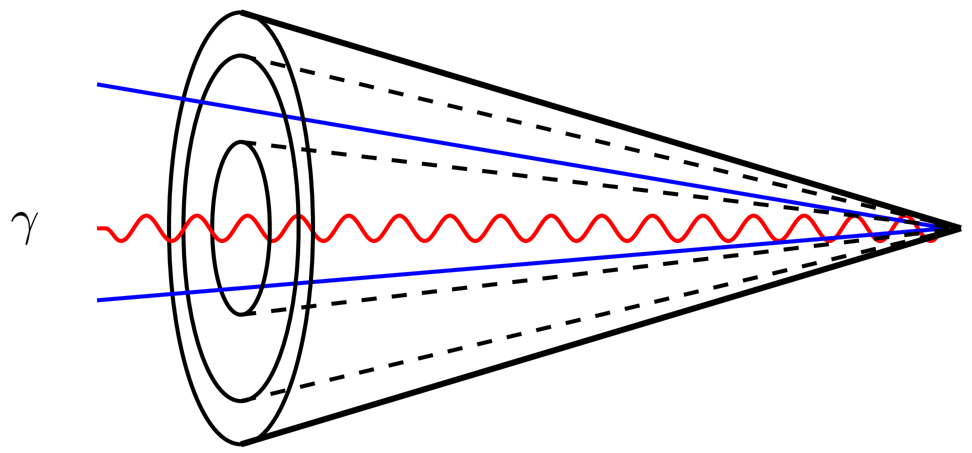


$$R^2 = \Delta\eta^2 + \Delta\phi^2$$

$$E_T^{\text{had}} < E_T^{\max}(E_T^{\gamma})$$

- Used in experimental analysis
- $q \parallel \gamma$ singularity
- σ contains fragmentation contribution

Smooth cone isolation S. Frixione 1998



arbitrary cones with $r_d < R$

$$E_T^{\text{had}}(r_d) < \epsilon E_T^{\gamma} \left(\frac{1 - \cos(r_d)}{1 - \cos(R)} \right)^n$$

- Idealised photon isolation
- No $q \parallel \gamma$ singularity
- σ has no fragmentation contribution

Theory Predictions

$$d\hat{\sigma}^{\gamma+X} = d\hat{\sigma}^{\gamma+X}_{\text{direct}} + \sum_k d\hat{\sigma}^{k+X} \otimes D_{k\to\gamma}$$

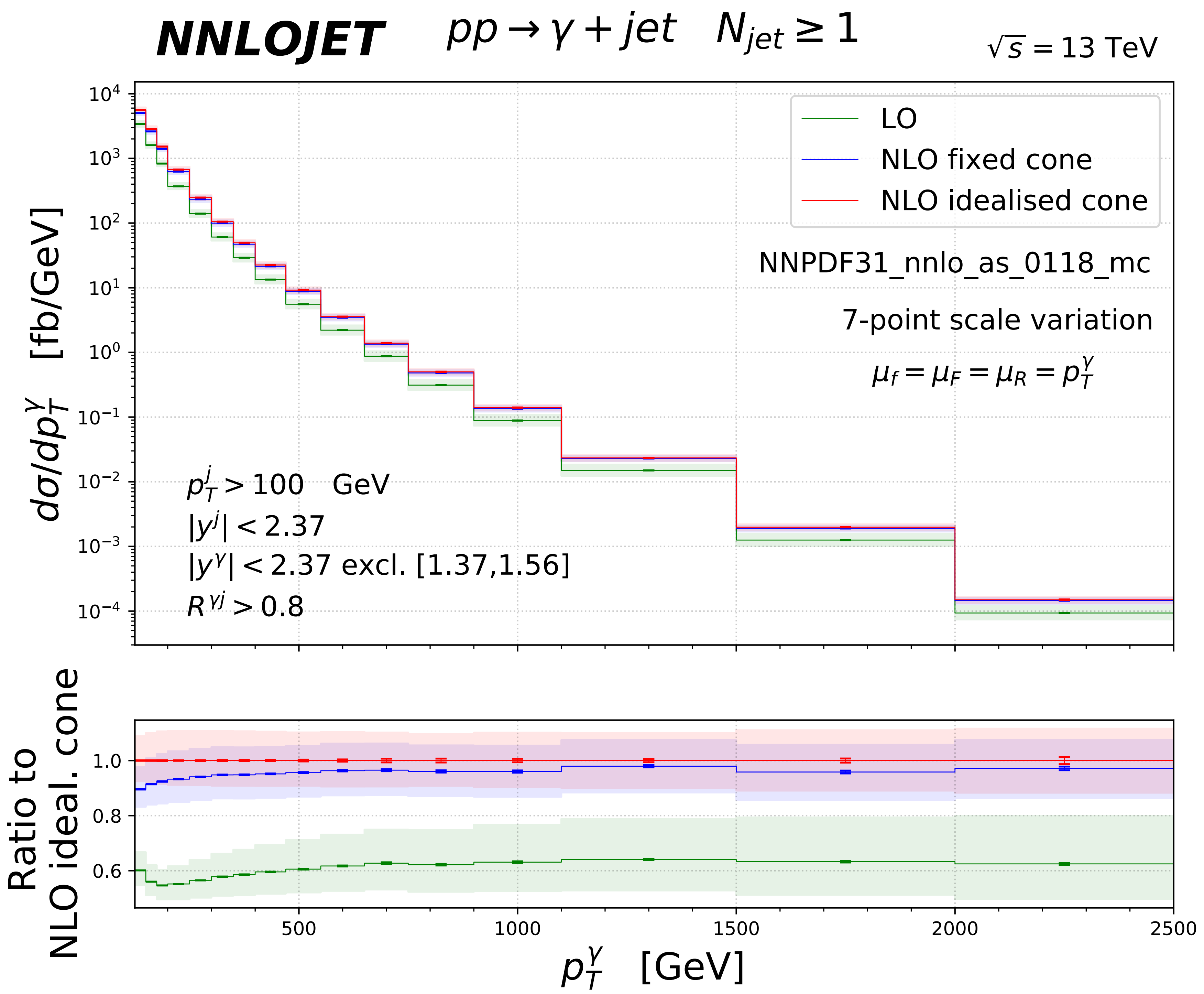
Only with fixed cone isolation

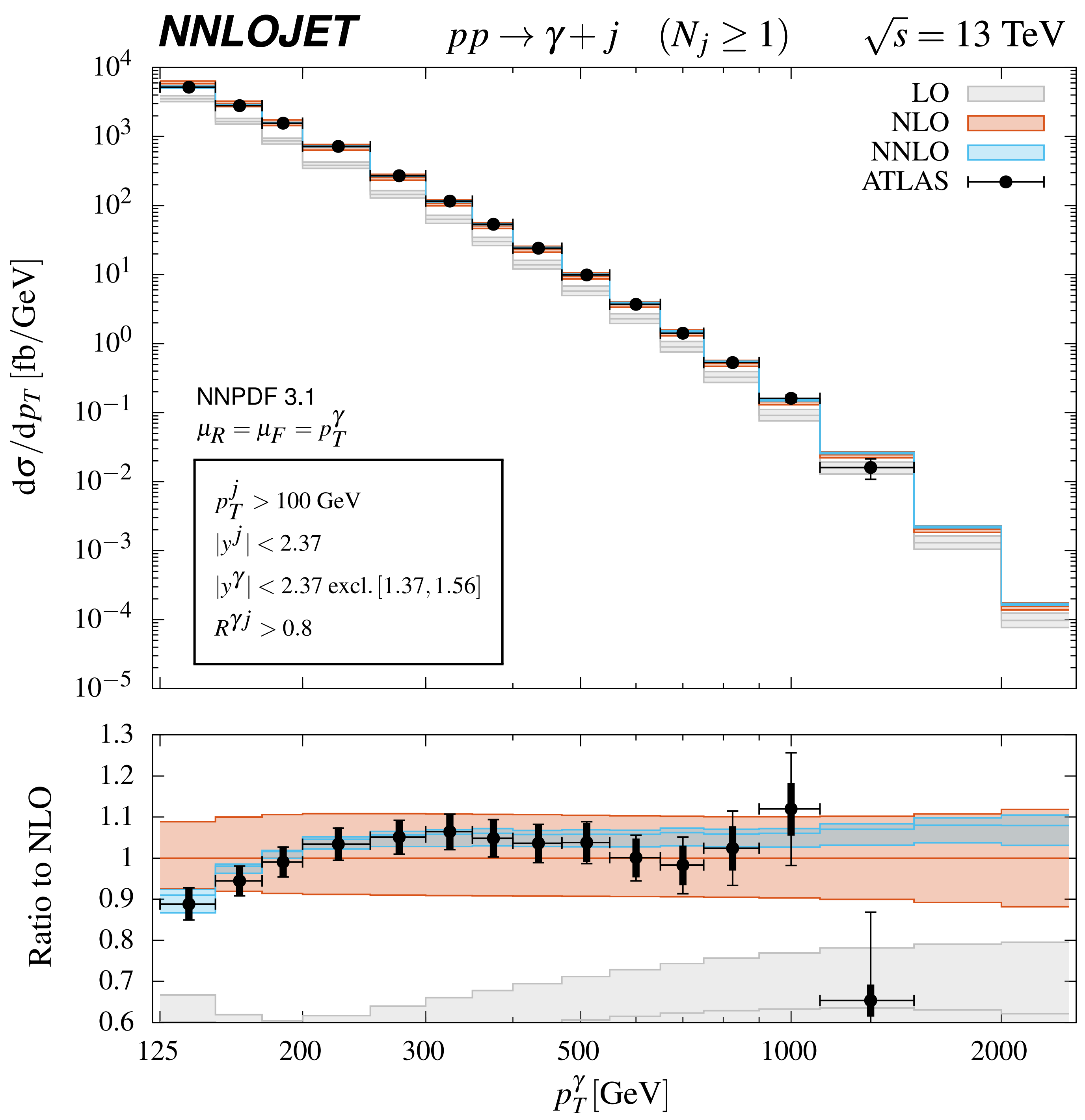
NNLO QCD with idealised isolation

J. M. Campbell et al., 2017
X. Chen et al., 2019

NLO QCD with fixed cone isolation

P. Aurenche et al., 1993
S. Catani et al., 2002

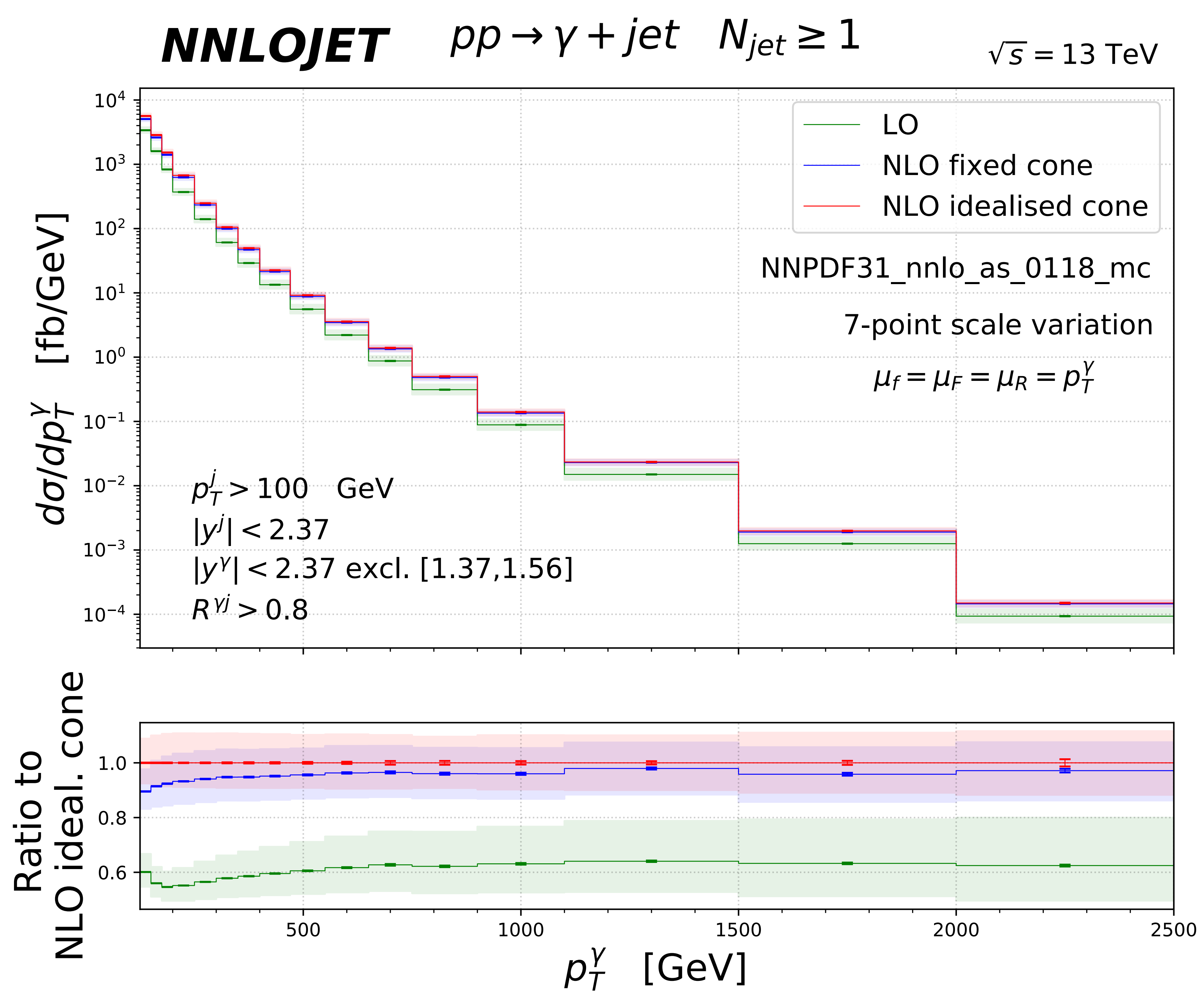




Theory Predictions

$$\mathrm{d}\hat{\sigma}^{\gamma+X} = \mathrm{d}\hat{\sigma}^{\gamma+X}_{\text{direct}} + \underline{\sum_k \mathrm{d}\hat{\sigma}^{k+X} \otimes D_{k\to\gamma}}$$

Only with fixed cone isolation



NNLO QCD with idealised isolation

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NLO QCD with fixed cone isolation

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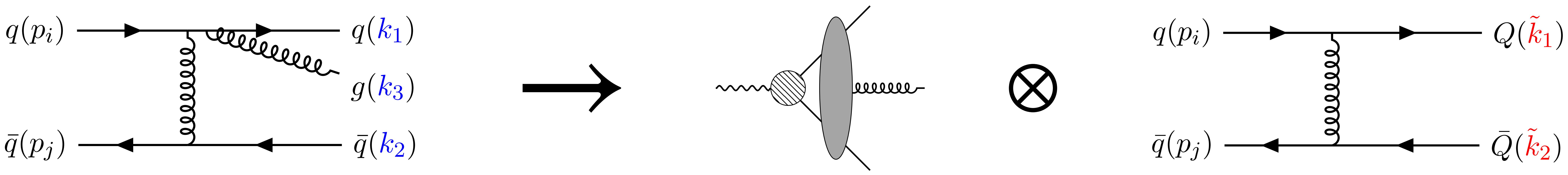
To overcome systematic uncertainty:

Use realistic fixed cone isolation
→Include fragmentation contribution @ NNLO
→subtract photonic limits

Antenna Subtraction



$$\mathrm{d}\sigma^S \propto A_3^0(q(k_1), g(k_3), \bar{q}(k_3))\,|\,M_2^0(\tilde{k}_1, \tilde{k}_2, p_i, p_j)\,|^2\, J(\tilde{k}_1, \tilde{k}_2)\,\mathrm{d}\Phi_3$$

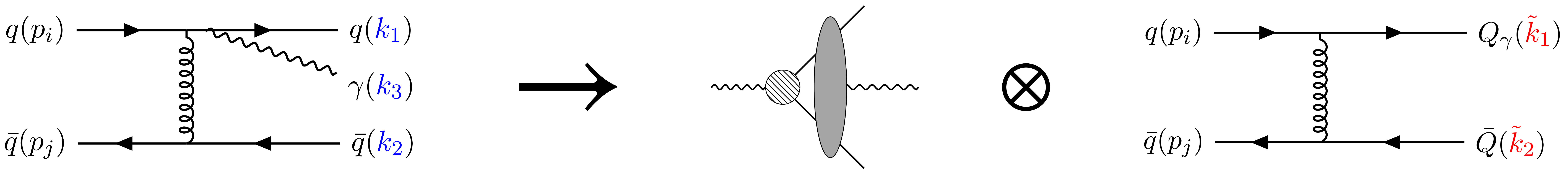
- Only dependence on $\{k_1, k_2, k_3\}$ in antenna function
- Jet function and reduced matrix element only depend on mapped momenta $\{\tilde{k}_1, \tilde{k}_2\}$

→ phase space can be factorised

$$\mathrm{d}\sigma^T \propto \underbrace{\left(\int \mathrm{d}\Phi_A A_3^0(q(k_1), g(k_2), \bar{q}(k_3))\right)}_{\mathscr{A}_3^0}\,|\,M_2^0(\tilde{k}_1, \tilde{k}_2; p_i, p_j)\,|^2\, J(\tilde{k}_1, \tilde{k}_2)\,\mathrm{d}\Phi_2$$

→ explicit ϵ-poles in $\mathscr{A}_3^0$

Fragmentation Antenna Functions



$$\mathrm{d}\sigma^S \propto A_3^0(q(k_1), \gamma^{\mathrm{id.}}(k_3), \bar{q}(k_3))\,|M_2^0(\tilde{k}_1, \tilde{k}_2, p_i, p_j)|^2\, J(\tilde{k}_1, \tilde{k}_2; z)\,\mathrm{d}\Phi_3$$

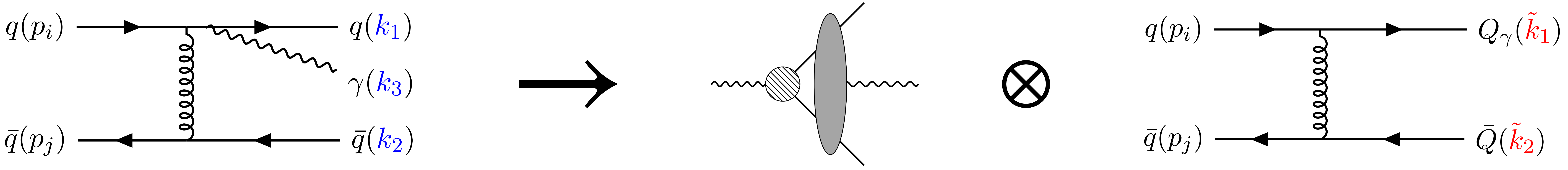
Jet function needs information about momentum fraction z of the photon within the quark-photon cluster Q_γ:

$$z = \frac{s_{23}}{s_{23} + s_{12}} \overset{q\|\gamma}{\rightarrow} \frac{E_\gamma}{E_\gamma + E_q}$$

Reconstruction of photon and quark momentum in photon isolation and jet algorithm:

$$\tilde{k}_1 \rightarrow \{z\,\tilde{k}_1, (1-z)\,\tilde{k}_1\}$$

Fragmentation Antenna Functions



$$\mathrm{d}\sigma^S \propto A_3^0(q(k_1), \gamma^{\mathrm{id.}}(k_3), \bar{q}(k_3))\,|M_2^0(\tilde{k}_1, \tilde{k}_2, p_i, p_j)|^2 J(\tilde{k}_1, \tilde{k}_2; z)\,\mathrm{d}\Phi_3$$

Jet function needs information about momentum fraction z of the photon within the quark-photon cluster Q_γ:

→ antenna phase space can not be fully integrated out

Integration over antenna phase space must remain differential in z:

$$\mathrm{d}\sigma_\gamma^T \propto -\int_0^1 \mathrm{d}z \underbrace{\left(\int \frac{\mathrm{d}\Phi_A}{\mathrm{d}z} A^0_{q\gamma\bar{q}}\right)}_{\mathscr{A}_3^0(z)} |M_2^0(\tilde{k}_1, \tilde{k}_2; p_i, p_j)|^2 J(\tilde{k}_1, \tilde{k}_2; z)\,\mathrm{d}\Phi_2$$

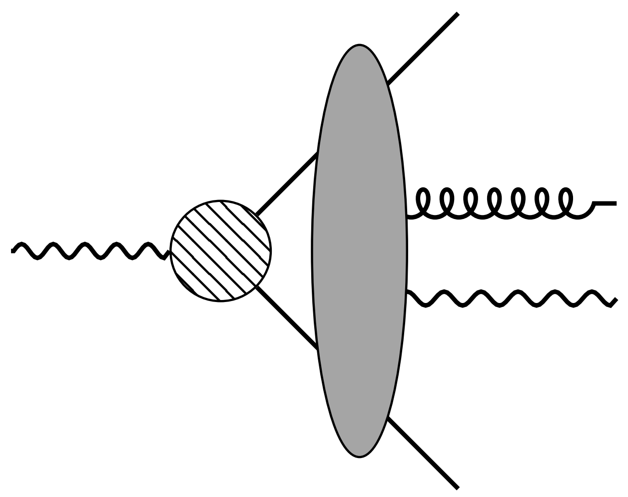
Towards NNLO

NLO:

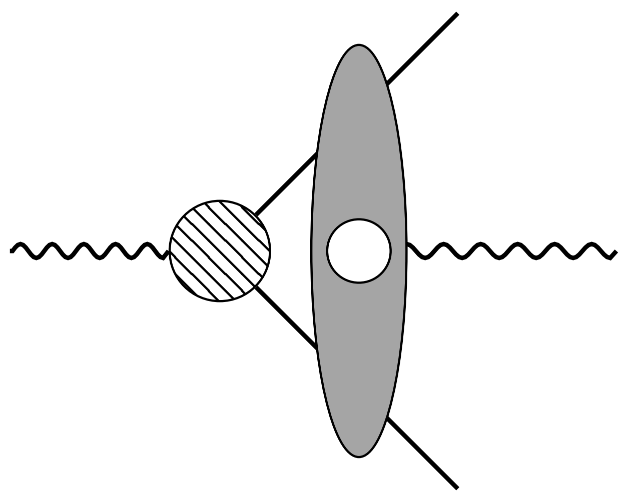
- Subtraction with identified final state particles @ NLO implemented in parton-level Monte Carlo event generator NNLOJET
- NLO results for isolated photon production and photon + jet production validated against JetPhox

NNLO:

- Subtraction of double unresolved limits (RR) and one-loop unresolved limits (RV)



X_4^0



X_3^1

- Integration of fragmentation X_4^0 and X_3^1 while retaining information on momentum fraction z in one kinematic configuration (initial-final)
- Inheritance of the momentum fraction z in consecutive unresolved limits

Integration of Fragmentation X_4^0

Necessary fragmentation antenna functions for photon production:

$\tilde{A}^0_{q,\gamma g q}$ subtracts $q \parallel g \parallel \gamma$ limit

$\tilde{E}^0_{q,q'\bar{q}'\gamma}$ subtracts the $q' \parallel \gamma \parallel \bar{q}'$ limit

Initial-final antenna phase space: $\mathrm{d}\Phi_A \propto \mathrm{d}\Phi_3(q(Q^2) + p \to k_1 + k_2 + \gamma(k_3))$

Additional δ-distribution in phase space integral fixes momentum fraction z:

$$\mathcal{X}_4^0(x, z) \propto \int \mathrm{d}^d k_1\, \mathrm{d}^d k_2\, \delta(k_1^2)\, \delta(k_2^2)\, \delta((p - q - k_1 - k_2)^2)\, \delta\left(z - \frac{s_{p3}}{s_{p1} + s_{p2} + s_{p3}}\right) X_4^0$$

For initial-final configuration: additional dependence on the initial-state momentum fraction x

Integration of Fragmentation X_4^0

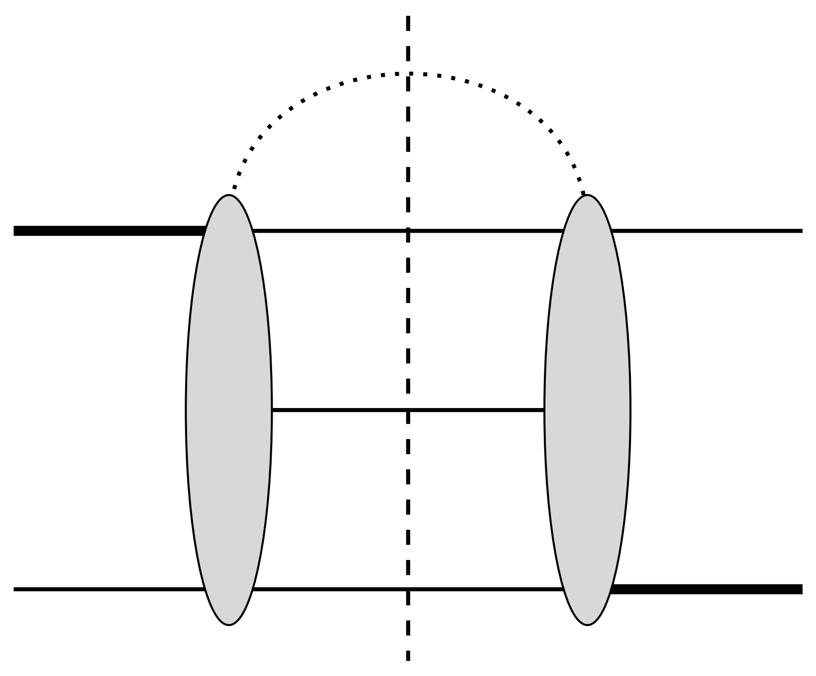
Strategy:

Unitarity → replace δ-distributions by propagators

$$2\pi i\delta(k_1^2) = \frac{1}{k_1^2 + i\epsilon} - \frac{1}{k_1^2 - i\epsilon}$$

Phase space integral = cut through loop integral:

- Reduction of integrals using IBP-relations to 9 master integrals (MI)
- MI are calculated by solving differential equations in x and z
- Integration constants fixed by integrating over z and comparing to inclusive result



Integration of Fragmentation X_3^1

Antenna phase space in initial-final configuration:

$$\mathrm{d}\Phi_A \propto \mathrm{d}\Phi_2(q(Q^2) + p \to k_1 + \gamma(k_2))\,; \qquad z = \frac{s_{p2}}{s_{p2} + s_{p1}}$$

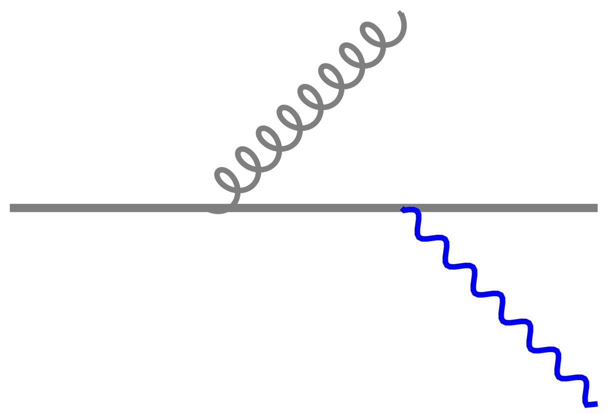
No actual integration has to be performed:

$$\mathcal{X}_3^1(x, z) = \frac{1}{C(\epsilon)} \int \frac{\mathrm{d}\Phi_2}{\mathrm{d}z} \frac{Q^2}{2\pi} X_3^1 = \frac{Q^2}{2} \frac{e^{\gamma_E \epsilon}}{\Gamma(1-\epsilon)} \left(Q^2\right)^{-\epsilon} \mathrm{J}^\gamma(x, z)\, X_3^1$$

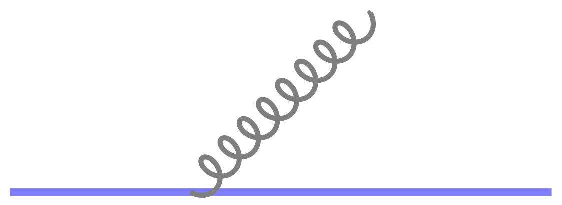
However, X_3^1 has to be cast into a form suitable for an expansion in distributions in $1 - x$ and z

Inheritance of z

In consecutive unresolved limits a proper inheritance of z is required:

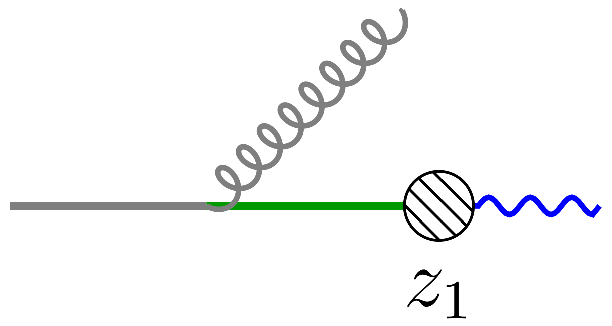


$\xrightarrow[z_1]{\gamma\|q}$

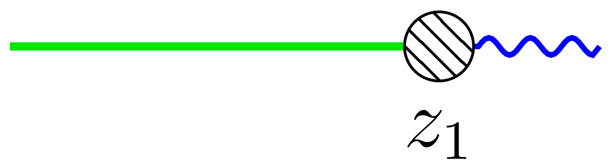


$\xrightarrow[z_2]{g\|\tilde{q}_\gamma}$

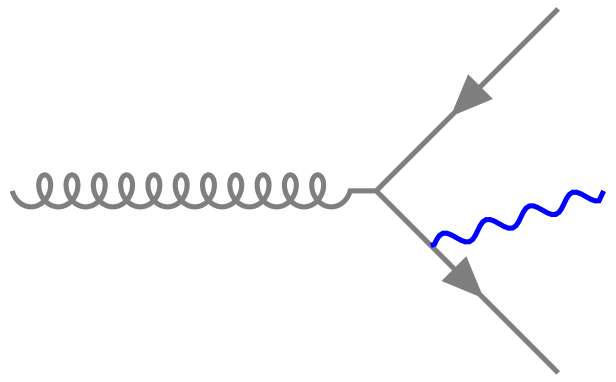
$$\underbrace{A_3^0(\bar{q},\gamma^{\text{id.}},q)}_{z_1}\ \underbrace{A_3^0(\bar{q},g,\tilde{q}_\gamma^{\text{id.}})}_{z_2}\dots J(z=z_1z_2)$$



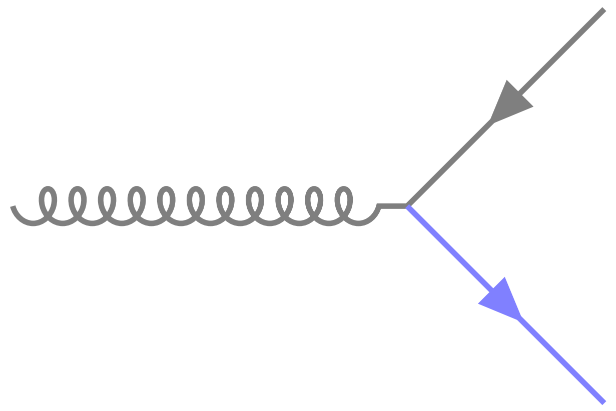
$\xrightarrow[z_2]{q\|g}$



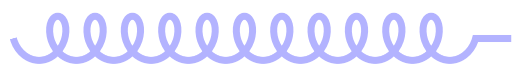
$$D_{q\to\gamma}(z_1)\ \underbrace{A_3^0(\bar{q},g,q^{\text{id.}})}_{z_2}\dots J(z=z_1z_2)$$



$\xrightarrow[z_1]{\gamma\|q}$



$\xrightarrow[z_2]{\tilde{q}_\gamma\|\bar{q}}$



$$\underbrace{A_3^0(\bar{q},\gamma^{\text{id.}},q)}_{z_1}\ \underbrace{E_3^0(q',\tilde{q}_\gamma^{\text{id.}},\bar{q})}_{z_2}\dots J(z=z_1z_2)$$

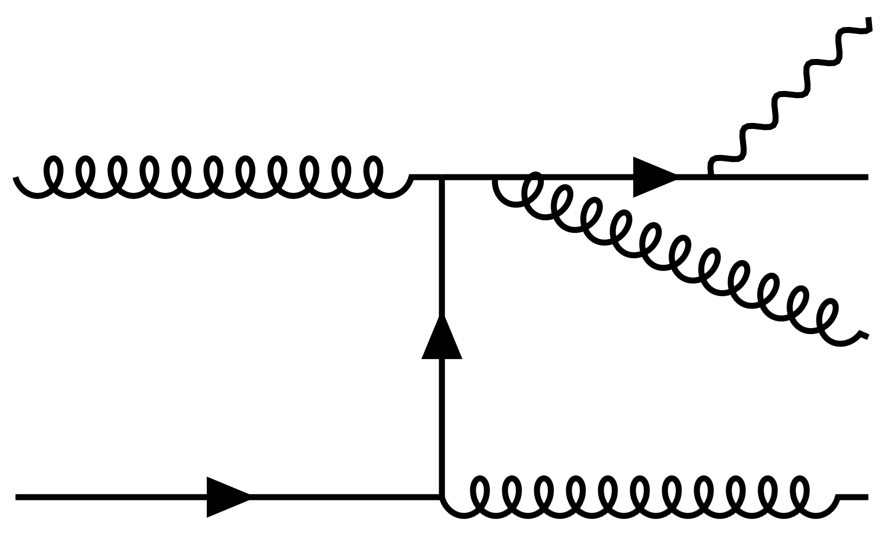
Subtraction Term

Subtraction term for subleading color matrix element $\tilde{\tilde{B}}_3^{\gamma,0}(\hat{q},\hat{g},g_1,g_2,q,\gamma)$

$$\mathrm{d}\sigma_{\mathrm{QCD}}^{S,a} = +A_{3,q}^0(\hat{q},g_1,q)\,\tilde{B}_2^{G,0}(\overline{\tilde{q}},\hat{g},g_2,\tilde{q}_{g_1},\gamma)\,J_1^{(3)}(\{\tilde{p}\}_3)$$

$$\mathrm{d}\sigma_{\gamma}^{S,a} = +A_{3,q}^0(\hat{q},\gamma^{\mathrm{id.}},q)\,\tilde{\tilde{B}}_3^0(\overline{\tilde{q}},\hat{g},g_1,g_2,\tilde{q}_\gamma)\,J_1^{(3)}(\{\tilde{p}\}_3;z)$$

$$\mathrm{d}\sigma_{\gamma}^{S,b} = +\tilde{A}_4^0(\hat{q},g_1,\gamma^{\mathrm{id.}},q)\,\tilde{B}_2^0(\overline{\tilde{q}},\hat{g},g_2,\tilde{q}_{\gamma g_1})\,J_1^{(2)}(\{\tilde{p}\}_2;z)$$
$$-A_{3,q}^0(\hat{q},g_1,q)\,A_{3,q}^0(\overline{\tilde{q}},\gamma^{\mathrm{id.}},\tilde{q}_{g_1})\,\tilde{B}_2^0(\overline{\overline{\tilde{q}}},\hat{g},g_2,(\gamma\tilde{q}_{g_1}))\,J_1^{(2)}(\{\tilde{\tilde{p}}\}_2;z)$$
$$-A_{3,q}^0(\hat{q},\gamma^{\mathrm{id.}},q)\,A_{3,q}^0(\overline{\tilde{q}},g_1,\tilde{q}_\gamma^{\mathrm{id.}})\,\tilde{B}_2^0(\overline{\overline{\tilde{q}}},\hat{g},g_2,(g_1\tilde{q}_\gamma))\,J_1^{(2)}(\{\tilde{\tilde{p}}\}_2;z=z_1z_2)$$



- $\mathrm{d}\sigma_{\mathrm{QCD}}^{S,a}$ and $\mathrm{d}\sigma_{\gamma}^{S,a}$ subtract the single unresolved limits of the matrix element
- $\mathrm{d}\sigma_{\gamma}^{S,b}$ subtract double unresolved limits of the matrix element

Full subtraction term: $\mathrm{d}\sigma^S = \mathrm{d}\sigma_{\mathrm{QCD}}^{S,a} + \mathrm{d}\sigma_{\gamma}^{S,a} + \mathrm{d}\sigma_{\mathrm{QCD}}^{S,b} + \mathrm{d}\sigma_{\gamma}^{S,b}$

Conclusion

- Including fragmentation contribution in predictions for $d\sigma^{\gamma+X}$ crucial to overcome irreducible systematic uncertainty (use of different isolation prescriptions in experiment and theory)

- Requires identification of final state photon in unresolved limits

- Within antenna subtraction: use of fragmentation antenna functions
  → new class of integrated antenna functions $(\mathcal{X}_3^0, \mathcal{X}_4^0, \mathcal{X}_3^1)$ depending on final state momentum fraction

- Applications of subtraction with identified final state particles beyond γ(+jet) production:
  →di-gamma, identified hadron production
  →complementary process: EW correction to γ(+jet), discriminate γ(+jet) final state from $\gamma(+\gamma)$ final state

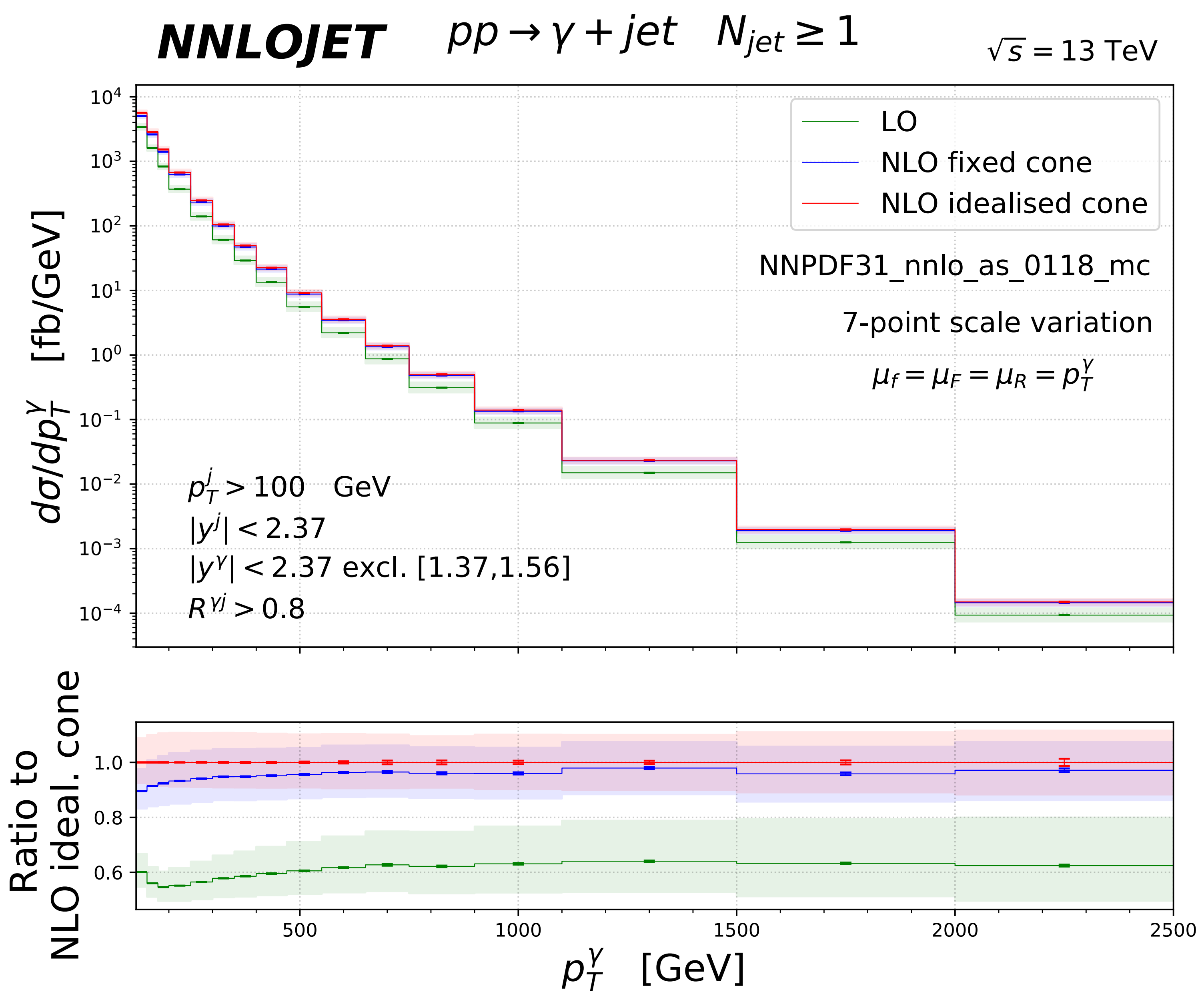
Thank you for your attention!

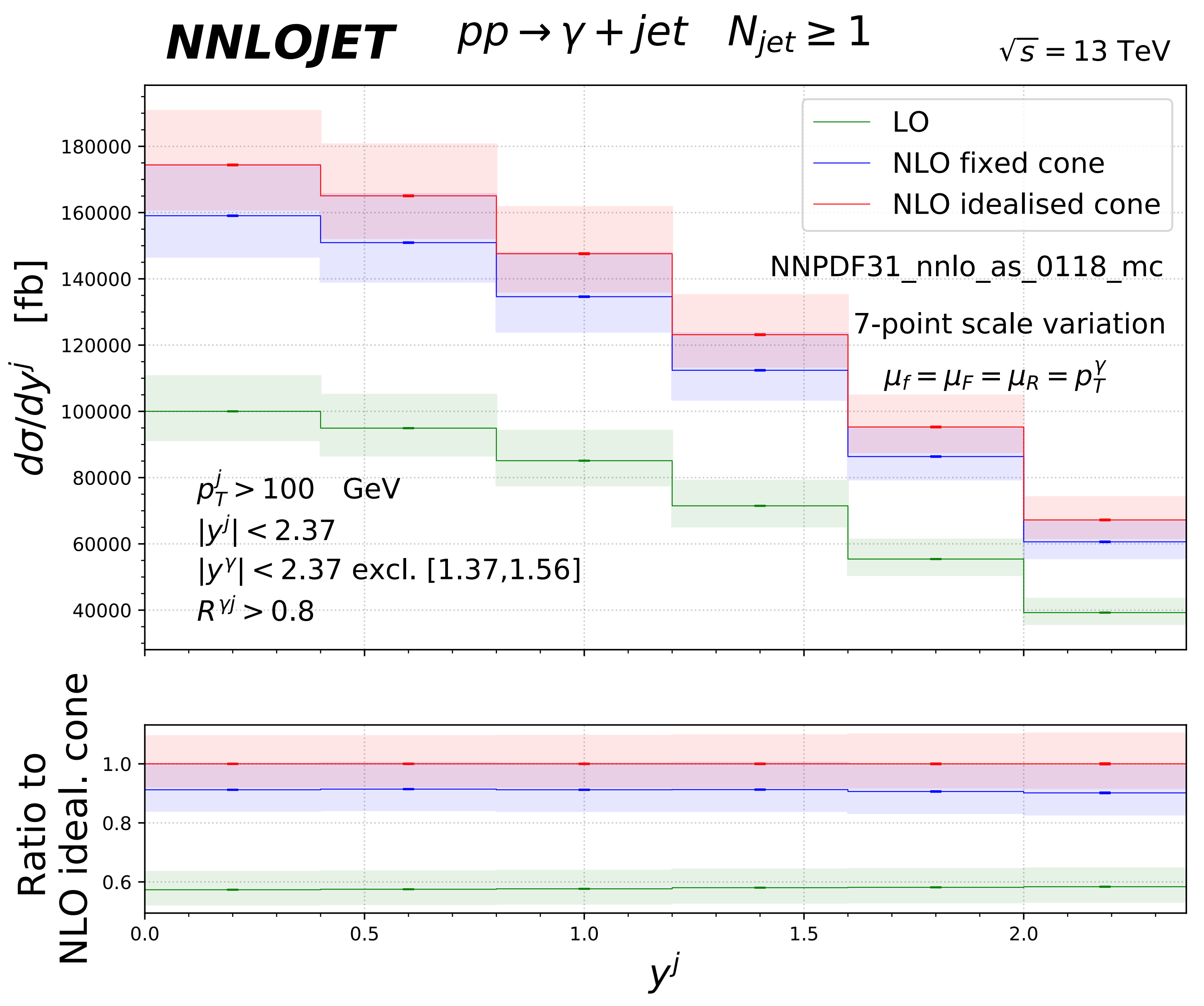
BACKUP

NLO Results

Comparison between fixed cone isolation and idealised isolation @ NLO

Set-up:
ATLAS 13 TeV photon+jet study Atlas, 2018 , Fragmentation functions: BFG2 set L. Bourhis et al., 1998
Fixed cone isolation: $R = 0.4$, $E_T^{\max} = 0.0042\, p_T^{\gamma} + 10\,\mathrm{GeV}$
Idealised isolation: dynamical cone $(R_d = 0.1, \epsilon = 0.1, n = 2)$ + fixed cone





Integration of Fragmentation X_3^1

X_3^1 can be expressed in terms of Box and Bubble MIs:

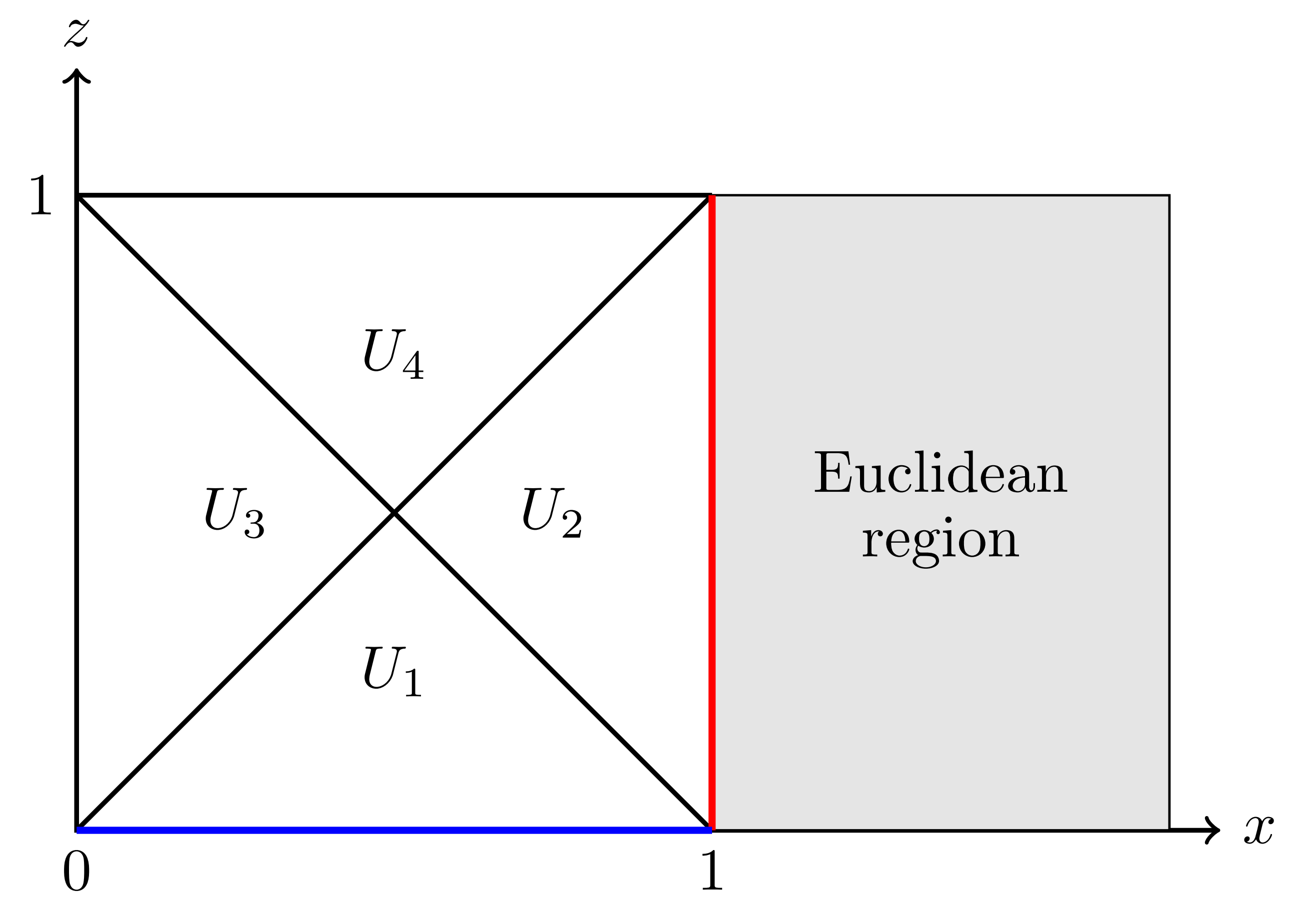
$$X_3^1(x, z) = \sum_{i=1}^{3} f_i(x, z)\,\mathrm{Box}_i(x, z) + \sum_{k=1}^{4} g_k(x, z)\,\mathrm{Bub}_k(x, z) + h(x, z)$$

$$\mathrm{Box}_i(x, z) \propto \sum_{j=1}^{3} \Big(r_{i,j}(x, z)\Big)^{-\epsilon} {}_2F_1(-\epsilon, -\epsilon; 1-\epsilon; a_{i,j}(x, z))$$

Box-integrals: real-valued and well defined
in Euclidean region only

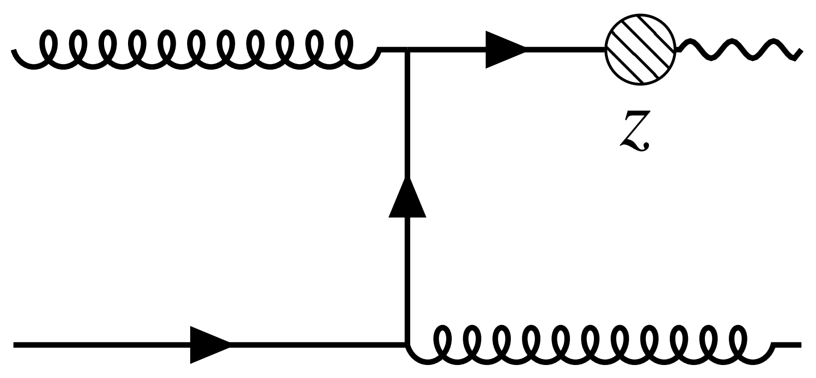
→ analytic continuation needed

Branch cuts ($a_{i,j}(x, z) = 1, \pm\infty$) within the physical region
→ distinguish different regions in the x-z-plane



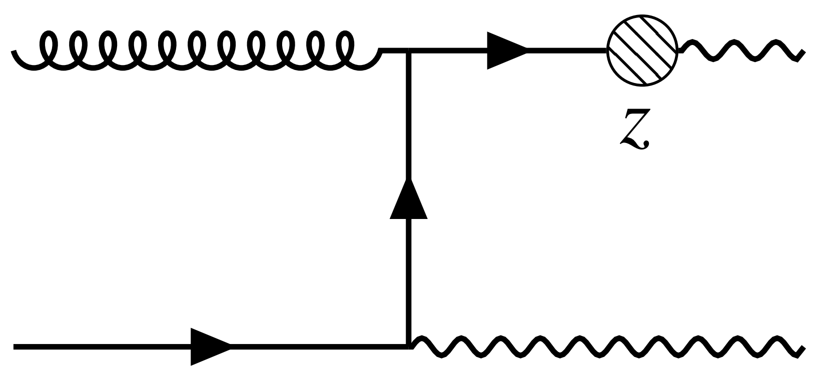
Fragmentation as EW correction

Fragmentation contribution to γ + jet:



To have a γ + jet final state:
Quark-photon cluster from fragmentation has to be identified as a photon
Lower limit on photon momentum fraction: $z > z_{\mathrm{min}}$

Fragmentation as EW contribution to γ + jet:



To have a γ + jet final state:
Quark-photon cluster from fragmentation has to be identified as a jet
Upper limit on photon momentum fraction: $z < z_{\mathrm{max}}$