On the universality of the KRK factorization scheme and improved Catani-Seymour scheme

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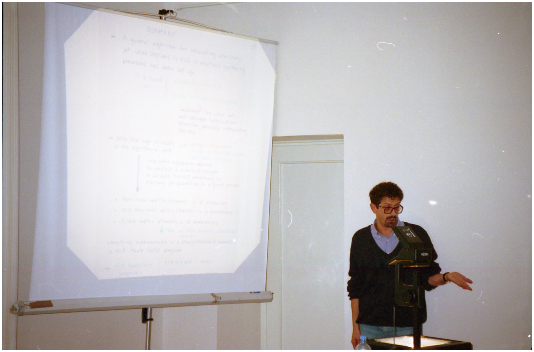
INTRODUCTION

One of the main highlights of the 1996 RADCOR in Kraków was work of Stefano Catani and Mike Seymour, Nucl.Phys. B485 (1997) 291-419, on “A General algorithm for calculating jet cross-sections in NLO QCD”, presently 1894 citations!



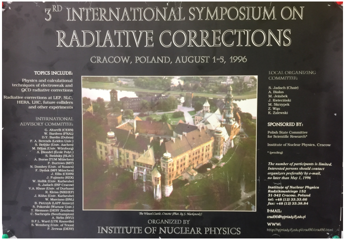


Photo: Stefano Catani at RADCOR 1996, Kraków, Wawel Royal Castle

INTRODUCTION

- In this talk I shall present paper published in Acta Phys.Polon. B51 (2020) 1363, arXiv:2004.04239 "On the universality of the MC factorization scheme" which showed that the Catani-Seymour (CS) scheme was not the optimal one.
- By means of the judicious choice of the so called "soft collinear counterterms" the final result of CS can be made dramatically simpler. For experts: The collinear terms **K** and **P** are eliminated!
- If this result was known in 2002 then `MC NLO` and `POWHEG` schemes of combining NLO corrections with parton shower would be obsolete. (Terms $\mathbf{K}+\mathbf{P}$ are the source of main complications in these schemes).
- Moreover, this work shows that the old dream of the "physical" factorization scheme of the PDFs, without sacrificing their universality (process independence) can be finally realized.

The above paper has presently 0 citations :(

KRK scheme inherits "universality" of PDFs from $\overline{MS}$

- KRK factorization scheme (FS) is a variant of the MS-bar system (including a new definition of the PDFs for initial hadrons). It is therefore trivially universal, that is process independent.
- The question of its universality is formulated differently: As the basic role of KRK FS is to simplify drastically NLO corrections, the question is whether the same **single** variant of the KRK FS is able to provide the same simplification of the NLO corrections for **all processes** with one or two initial hadrons and any number of final partons?
- The answer is positive and the proof is elaborated within the Catani-Seymour subtraction methodology.
- KRK FS is mandatory in KrkNLO matching NLO and the parton shower – a much simpler alternative of POWHEG and/or MC NLO
- The use of KRK FS simplifies NLO calculations not only for the CS scheme but for any other method and arbitrary process.

How KRK FS simplifies NLO Catani-Seymour master formula?

With PDFs in the (physical) KRK factorization scheme
and new modified soft-collinear counterterms,
the original Catani-Seymour NLO master formula

$$\begin{aligned}\sigma^{NLO}(p) &= \sigma^B(p) + \\ &+ \int_m \left[d\sigma^V(p) + d\sigma^B(p) \otimes \mathbf{I}\right]_{\varepsilon=0} + \int dz \int_m \left[d\sigma^B(zp) \otimes (\mathbf{P}+\mathbf{K})(z)\right]_{\varepsilon=0} \\ &+ \int\limits_{m+1} \left[d\sigma^R(p)_{\varepsilon=0} - \Big(\sum_{dipoles} d\sigma^B(p) \otimes dV_{dipole}\Big)_{\varepsilon=0}\right],\end{aligned} \tag{1}$$

turns into a much simpler one

$$\begin{aligned}\sigma^{NLO}(p) = \sigma^B(p) &+ \int_m \left[d\sigma^V(p) + d\sigma^B(p)\; I(\varepsilon)\right]_{\varepsilon=0} \\ &+ \int\limits_{m+1} \left[d\sigma^R(p)_{\varepsilon=0} - \Big(\sum_{dipoles} d\sigma^B(p) \otimes dV_{dipole}\Big)_{\varepsilon=0}\right]\end{aligned} \tag{2}$$

for ANY process with one or two initial hadrons
and any number m of final coloured partons.

Consequently, the KrkNLO matching scheme with parton shower
(much simpler alternative of POWHEG or MC@NLO)
applies not only to DY-like processes but to ANY process.

DY example of NLO for CS with PDFs in the KRK scheme

JHEP 1510 (2015) 052 [arXiv:1503.06849] (gluonstrahlung channel only):

Including measurement functions $J^F_{\rm LO}=J_{\rm LO}(x_F z, x_B)$, $J^B_{\rm LO}=J_{\rm LO}(x_F, x_B z)$, $J_{NLO}(x_F, x_B, z, k^T)$, the NLO x-section with CS dipole subtractions reads:

$$\sigma^{\overline{MS}}_{\rm NLO}[J] = \int dx_F\, dx_B\, dz\, dx\; \delta_{x=zx_F x_B} \Big\{$$

$$\delta_{1=z}(1+\Delta_{VS})\; d^2\sigma^{\rm LO}(sx,\hat\theta)\; J_{\rm LO} + \mathcal{G}(z)(J^F_{\rm LO}+J^B_{\rm LO})\; d^2\sigma^{\rm LO}(szx,\hat\theta)$$

$$+\Big(d^5\rho_1^{\rm NLO}\; J_{\rm NLO} - (d^3\rho_1^F J^F_{\rm LO} + d^3\rho_1^B J^B_{\rm LO}))\, d^2\sigma^{\rm LO}(\hat s,\hat\theta)\Big)\delta_{1-z=\alpha+\beta}\Big\} D^{\overline{MS}}{}_q(sx,x_F) D^{\overline{MS}}{}_{\bar q}(sx,x_B).$$

The dipole for real gluon emission in $d=4$ using Sudakov parametrization:

$$d^3\rho_1^F(s_1) = \frac{\alpha_s}{2\pi} H^{qq}(\alpha,\beta,\varepsilon)\Big|_{\varepsilon=0} = \frac{\alpha_s}{2\pi}\frac{d\beta_1\, d\alpha_1}{\beta_1}\frac{d\phi_1}{2\pi} P_{qq}(1-\alpha_1-\beta_1)$$ and ρ_1^B defined similarly.

In the KrkNLO matching, the absence of $\mathcal{G}(z)$ allows for single multiplicative MC weight:

$$W^{\rm MC}_{\rm NLO}(k)\Big|_{qq\ chan.} = (1+\Delta^{\rm MC}_{VS})\,\frac{d^5\rho_1^{\rm NLO}(k)}{(d^3\rho_1^F + d^3\rho_1^B)\; d^2\sigma^{\rm LO}(\hat s,\hat\theta)}.$$

NB. the finite virtual+soft corrections ($q\bar q$ channel) is:

$$\Delta^{\rm MC}_{VS} = \Delta^{virt.}_{q\bar q}(\varepsilon) + \frac{\alpha_s}{2\pi}\frac{\Gamma(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\frac{\hat s}{4\pi\mu^2}\right)^{\varepsilon}\int_0^1 dz\; z\widetilde{\mathcal{V}}^{q\leftarrow q}(z,\varepsilon) = \frac{C_F\alpha_s}{\pi}\left(\frac14+\frac23\pi^2\right)$$

Last but not least $\hat s=\mu^2$ was instrumental!

The KrkNLO method to be used in the KKMChh for DY process, see next talk by S. Yost

DY example of NLO for CS with PDFs in the KRK scheme

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$$\delta_{1=z}(1+\Delta^{\rm MC}_{VS})\; d^2\sigma^{\rm LO}(sx,\hat\theta)\; J_{\rm LO}$$

$$+\Big(d^5\rho^{\rm NLO}_1\; J_{\rm NLO} - \big(d^3\rho^F_1 J^F_{\rm LO} + d^3\rho^B_1 J^B_{\rm LO}\big)\big) d^2\sigma^{\rm LO}(\hat s,\hat\theta)\Big)\delta_{1-z=\alpha+\beta}\Big\} D^{\rm MC}{}_q(sx,x_F) D^{\rm MC}{}_{\bar q}(sx,x_B).$$

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The KrkNLO method to be used in the KKMChh for DY process, see next talk by S. Yost

Explicit transformation of LO PDFs from $\overline{MS}$ to KRK FS

At every $Q^2 = \mu^2$ the following "rotation" in the x and flavour space:

$$\begin{bmatrix} q(x,Q^2) \\ \bar{q}(x,Q^2) \\ G(x,Q^2) \end{bmatrix}_{\rm MC} = \begin{bmatrix} q \\ \bar{q} \\ G \end{bmatrix}_{\overline{MS}} + \frac{\alpha_s}{2\pi} \int dz dy \begin{bmatrix} \mathbb{K}^{\rm MC}_{qq}(z) & 0 & \mathbb{K}^{\rm MC}_{qG}(z) \\ 0 & \mathbb{K}^{\rm MC}_{\bar{q}\bar{q}}(z) & \mathbb{K}^{\rm MC}_{\bar{q}G}(z) \\ \mathbb{K}^{\rm MC}_{Gq}(z) & \mathbb{K}^{\rm MC}_{G\bar{q}}(z) & \mathbb{K}^{\rm MC}_{GG}(z) \end{bmatrix} \begin{bmatrix} q(y,Q^2) \\ \bar{q}(y,Q^2) \\ G(y,Q^2) \end{bmatrix}_{\overline{MS}} \delta(x-yz)$$

where

$$\mathbb{K}^{\rm MC}_{Gq}(z) = C_F \left\{ \frac{1+(1-z)^2}{z} \ln\frac{(1-z)^2}{z} + z \right\},$$

$$\mathbb{K}^{\rm MC}_{GG}(z) = C_A \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ + 2\left[\frac{1}{z} - 2 + z(1-z)\right] \ln\frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{341}{72} - \frac{59}{36}\frac{T_f}{C_A}\right) \right\},$$

$$\mathbb{K}^{\rm MC}_{qq}(z) = C_F \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ - (1+z)\ln\frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} + 1 - z - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{17}{4}\right) \right\},$$

$$\mathbb{K}^{\rm MC}_{qG}(z) = T_R \left\{ \left[z^2 + (1-z)^2\right] \ln\frac{(1-z)^2}{z} + 2z(1-z) \right\}.$$

All virtual parts $\sim \delta(1-z)$ are adjusted using momentum sum rules:

$$\sum_b \int dz \;\, z\, \mathbb{K}^{\rm MC}_{ba}(z) = 0$$

From *Eur. Phys. J. C76 (2016) 649* [arXiv:1606.00355].

Is $\mathbb{K}$-transformation on PDFs "universal"?

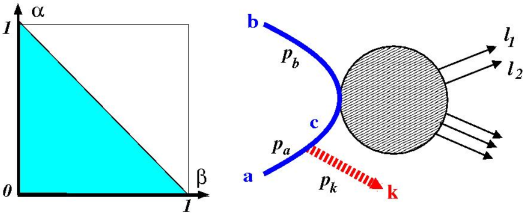
- $\mathbb{K}$ was adjusted semi-empirically in KrkNLO, Refs.(C,D) in the appendix, such that for $pp \to Z/\gamma$ and $pp \to \mathrm{Higgs}$ process the "collinear remnant" terms $\sim \delta(k_T)$ in the NLO calculations have disappeared.
  Since then the following question was pending:
- **Is it possible that the same $\mathbb{K}$ does the same for other processes?**
- To answer this question systematically I have re-derived $\mathbb{K}$ as integrals over subtraction terms of the NLO calculations,
  i.e. over "dipoles" of the Catani-Seymour subtraction scheme.
- As a byproduct I have found out that CS scheme can be significantly simplified!

For the initial state emitter and initial state spectator ($\mathcal{II}$ case) the CS dipoles are left unmodified.

However, I found a one-line formula for the integral over the $\mathcal{II}$ dipoles instead of equations stretching over several pages.

Do we get $\mathbb{K}_{qq}(z)$ from $\mathbb{JJ}$ CS dipole of Nucl.Phys. B485 (1997) ?

Start with kinematics of DY in Sudakov parametrization...



$$p_k = \alpha p_a + \beta p_b + p_k^T,\ \alpha = \frac{p_k p_b}{p_a p_b}, \beta = \frac{p_k p_a}{p_a p_b},$$

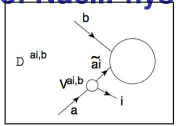
$$\alpha + \beta \le 1 \ \ |p_k^T|^2 = 2 p_a p_b \alpha\beta.$$

Some auxiliary variables:

$$s = 2p_a p_b,\ \hat{s} = Q^2 = (p_a + p_b - p_k)^2 = (1-\alpha-\beta)s = sz, \quad z = 1-\alpha-\beta.$$

Do we get $\mathbb{K}_{qq}(z)$ from $\mathbb{JJ}$ CS dipole of Nucl.Phys.B485 (1997)??

More details in the appendix

The initial-emitter initial-spectator $\mathcal{D}^{ai,b}$ dipole 

in CS ($d = 4 - 2\varepsilon$):

$$\frac{\alpha_S}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2}{2p_a p_b}\right)^{\epsilon}\tilde{\mathcal{V}}^{a,ai}(x;\epsilon) \equiv \int [dp_i(p_a,p_b,x)]\,\frac{1}{2p_a p_i}\,\frac{n_s(\widetilde{ai})}{n_s(a)}\,< \boldsymbol{V}^{ai,b}(x_{i,ab}) > \;.\qquad (5.152)$$

In our notation: $x = x_{i,ab} = 1 - \alpha - \beta, \quad \bar{v}_i = \beta$ and from **direct evaluation** one gets:

$$\tilde{\mathcal{V}}^{q,qG}(z,\varepsilon)|_{z\neq 1} = \frac{1}{\varepsilon}P_{qq}(z) + 2C_F(1+z^2)\frac{\ln(1-z)}{1-z} - C_F(1-z).$$

The same result in eq. (5.155-156) of CS paper looks mysteriously complicated:

$$\widetilde{\mathcal{V}}^{a,b}(x;\epsilon) = \mathcal{V}^{a,b}(x;\epsilon) + \delta^{ab}\boldsymbol{T}_a^2\left[\left(\frac{2}{1-x}\ln\frac{1}{1-x}\right)_+ + \frac{2}{1-x}\ln(2-x)\right] + \widetilde{K}^{ab}(x) + \mathcal{O}(\epsilon)\;, \qquad (5.155)$$

In fact $\sim \ln(2-x)$ term is in reality absent – it cancels out with another one in $\mathcal{V}^{a,b}(x,\varepsilon)$.
The term $\sim \frac{2}{1-x}\ln\frac{1}{1-x}$ cancels with another identical term inside $\mathcal{V}^{a,b}(x,\varepsilon)$.
$\tilde{K}$ corrects for the unlucky definition of $\mathcal{V}^{a,b}$ for DIS in CS paper, where $m_+ = \alpha/(\alpha+\beta)$ is applied only to soft part of DIS dipole, while in the DY it is applied to the entire dipole.

What about contributions to $\mathbb{K}$-matrix from dipoles with final emitter and initial spectator $\mathcal{FI}$ and with initial emitter and final spectator $\mathcal{IF}$?

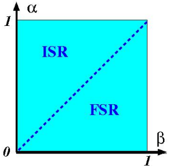
Final-final $\mathcal{FF}$ dipoles never contribute!

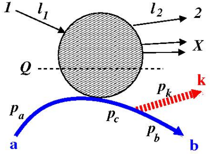
The main point is that it was possible to modify kinematic mapping in the final-initial $\mathcal{FI}$ dipole,
such that it does not contribute to $\mathbb{K}$-matrix!!!
Similarly as it is always true for the final-final dipole.

This transformation/mapping is present/known for ages in the BHLUMI Monte Carlo

New kinematic mapping in $\mathcal{FI}$ dipoles (initial spectator & final emitter)

This is the most important point!





$$p_k = \bar\alpha\, p_a + \bar\beta\, p_b + p_k^T,\quad \bar\alpha = \frac{p_k\cdot p_b}{p_a\cdot p_b},\ \bar\beta = \frac{p_k\cdot p_a}{p_a\cdot p_b},$$

$$\alpha = \frac{\bar\alpha}{1+\bar\beta} = \frac{p_k p_b}{p_a(p_k+p_b)},\ \beta = \frac{\bar\beta}{1+\bar\beta} = \frac{p_k p_a}{p_a(p_k+p_b)},$$

$$\bar\alpha = \frac{\alpha}{1-\beta},\ \bar\beta = \frac{\beta}{1-\beta},\ \max(\alpha,\beta)\le 1,$$

$$Q = p_b + p_k - p_a,\quad |Q^2| = 2p_a p_b\,\frac{1-\alpha}{1-\beta},$$

$$d\sigma^a_{bk} = d\Phi_{4+2\varepsilon}(p_k)\,\frac{1}{2p_b p_k}\,8\pi\mu^{-2\varepsilon}\alpha_s P^*_{b\leftarrow c}(\alpha,\beta)\,\frac{p_a\tilde p_b}{p_a(\tilde p_b - p_k)}\left\{\frac{1}{s}d\Phi(l_1' + \tilde p_a; \tilde p_b, l_2', \ldots)\ |\mathcal{M}(l_1', \tilde p_a; \tilde p_b, l_2', \ldots)|^2\right\}$$

$$= \frac{\alpha_s}{2\pi}\left(\frac{Q^2}{4\pi\mu^2}\right)^{\varepsilon}\frac{1}{\Gamma(1+\varepsilon)}\,\frac{d\Omega^{n-3}(p_k^T)}{\Omega^{n-3}}\ H_{bc}(\alpha,\beta,\varepsilon)\left\{d\sigma^{LO}(l_1', \tilde p_a; \tilde p_b, l_2', \ldots)\right\},$$

$$H_{bc}(\alpha,\beta,\varepsilon) = \left(\frac{\alpha\beta(1-\beta)}{(1-\alpha)}\right)^{\varepsilon}\ \frac{P^*_{b\leftarrow c}(\alpha,\beta,\varepsilon)}{\alpha},\qquad \tilde p_a = (1-\alpha)p_a,\quad \tilde p_b = Q - \tilde p_a.$$

$$P^*_{b\leftarrow c}(\alpha,\beta,\varepsilon)\big|_{\alpha\to 0} = P_{bc}(1-\beta,\varepsilon),$$ NEXT SLIDE

The essential difference with the original CS is an **additional active boost** B_x (tested in MC): $l_1' = B_x l_1,\ l_2' = B_x l_2,\ X' = B_x X$, in the plane perpendicular to Q, i.e. $B_x Q = Q$, with hyper-velocity η adjusted such that: $2l_1'\cdot\tilde p_a = (B_x(\eta)l_1)\cdot\tilde p_a = 2l_1\cdot p_a = s$.

The resulting LO part $\{d\sigma^{LO}(l_1', \tilde p_a; \tilde p_b, l_2', \ldots)\}$ does not depend on α and β anymore and to complete NLO calculations one needs to know only (as in $\mathcal{FF}$ case):

$$\mathcal{V}_{b\leftarrow c}(\varepsilon) = \int d\alpha d\beta\ H_{bc}(\alpha,\beta,\varepsilon).$$

$\mathbb{K}$-matrix from $\mathcal{IF}$, $\mathcal{FI}$, $\mathcal{FF}$ original and modified CS dipoles

Summary at this point and remaining problems:

(1) $\mathbb{K}$ matrix and $\mathcal{FF}$ dipoles (final emitter and final spectator) are unrelated. Hence $\mathcal{G}_{ab}(z)|_{z\neq 1} = 0$. Factor $\overline{\mathcal{V}}_{ab}(z,\varepsilon)$ decouples kinematically from PDFs. Only $\overline{\mathcal{V}}_{ab}(\varepsilon) = \int_0^1 dz\, \overline{\mathcal{V}}_{ab}(z,\varepsilon)$ matter (get combined with virt. corrs.)

(2) In CS paper, $\mathcal{V}_{ab}(z,\varepsilon)$ for $\mathcal{FI}$ dipoles (final emitter and initial spectator as in DIS) couples kinematically with PDFs and LO part through $\mathcal{G}_{ab}(z) \neq 0$.

(3) However, for the modified kinematic mapping in $\mathcal{FI}$ dipoles they kinematically decouple from PDFs, $\mathcal{G}_{ab}(z)|_{z\neq 1} = 0$, as for $\mathcal{FF}$. Previous slide.

(4) It remains to check whether $\mathbb{K}$-matrix from $\mathcal{IF}$ dipoles is the same as from $\mathcal{II}$.

(5) Not true for original $\mathcal{IF}$ dipoles of CS, however...

(6) Easy to modify *diagonal* $\mathcal{IF}$ dipoles such that $\mathbb{K}_{aa}(z)$ are the same. Next slide.

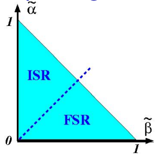
(7) For *nondiagonal* $\mathcal{IF}$ dipoles $a \neq b$ ($G \leftrightarrow q$) a workaround is found. Next slide.

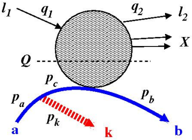
(8) Finally, it is possible to eliminate ALL collinear remnants $\mathcal{G}_{ab}(z)|_{z\neq 1}$ for ALL dipoles using common $\mathbb{K}$-rotation of PDFs from MS-bar to KRK FS.

(9) Last problem: collinear remnant terms $\sim \ln \frac{2p_i \cdot p_j}{\mu^2} P_{ab}(z)$ coupled with PDFs survive for more than two "legs"?? It looks that a recipe for zeroing them was found:)

Modified diagonal $\mathcal{IF}$ dipoles, (initial emitter & final spectator)

Exploiting freedom in $K^*_{c\leftarrow a}(\alpha,\beta)$ to get the same $\mathbb{K}_{ca}(z)$ as for $\mathcal{II}$.





$$d\sigma_b^{ak}(p_b,p_k,l_1,l_2,\dots) = \frac{\alpha_s}{2\pi}\left(\frac{Q^2}{4\pi\mu^2}\right)^{\varepsilon}\frac{1}{\Gamma(1+\varepsilon)}$$
$$\times H^{a\leftarrow c}(\alpha,\beta)\,d\alpha d\beta\,d\omega^{n-3}(p_k^T)\left\{d\sigma^{LO}(l_1,\tilde{p}_a;\tilde{p}_b,l_2,\dots)\right\}_{d=4+2\varepsilon},$$
$$H^{a\leftarrow c}(\alpha,\beta) = (\alpha\beta(1-\beta))^{\varepsilon}\,x_B^{-\varepsilon}\,\frac{P^*_{c\leftarrow a}(\alpha,\beta)}{\beta},\quad d\omega^{n-3} = \frac{d\Omega^{n-3}}{\Omega^{n-3}},$$

- $P^*_{a\leftarrow a}(\alpha,\beta)$ for $\mathcal{IF}$ and $\mathcal{FI}$ dipoles have to build together the correct soft limit.
- The CS choices for $\mathcal{IF}$, e.g. $P^*_{q\leftarrow q} = C_F\left[\frac{2}{\alpha+\beta} - (2-\alpha) + \varepsilon\alpha\right]$, are not good.
- The following general construction for diagonal $\mathcal{IF}$ and $\mathcal{FI}$ splittings was examined:

  $\mathcal{IF}$: $\quad P^*_{a\leftarrow a}(\alpha,\beta) = m_+(\alpha,\beta)\frac{1}{\alpha}[(1-z)P_{aa}(z)]\big|_{z=z(\alpha,\beta)},$

  $\mathcal{FI}$: $\quad P^*_{a\leftarrow a}(\alpha,\beta) = m_-(\alpha,\beta)\frac{1}{\alpha}[(1-z)P_{aa}(z)]\big|_{z=z(\alpha,\beta)},$

  with several choices of soft partition functions:

  $$m_+^{(a)}(\alpha,\beta) = \theta_{\beta<\alpha},\quad m_+^{(b)}(\alpha,\beta) = \frac{\alpha}{\alpha+\beta},\quad m_+^{(c)}(\alpha,\beta) = \frac{\alpha-\alpha\beta}{\alpha+\beta-\alpha\beta},\quad m_- = 1-m_+.$$

  and several choices of z-variable:

  $$z_A(\alpha,\beta) = 1-\max(\alpha,\beta),\quad z_B(\alpha,\beta) = 1-\alpha,\quad z_C(\alpha,\beta) = (1-\alpha)(1-\beta).$$
- The corresponding radiator functions for $\mathcal{IF}$ were calculated:

  $$\tilde{\mathcal{V}}^{c\leftarrow a}(z,\varepsilon) = \int d\alpha d\beta\; H^{a\leftarrow c}(\alpha,\beta)\;\delta(z - z_X(\alpha.\beta)),\; X = A,B,C.$$
- Good choices (compatible with $\mathcal{II}$) were found, for instance: *Aa*, *Ac*, *Ca* and *Cc*. The choice $z_B = 1-\alpha$ (Bjorken) used by CS is not good!

Problem and workaround for non-diagonal $\mathcal{IF}$ dipoles

- Non-diagonal dipoles, $a \neq b$, are not IR-divergent, hence $m_\pm$ not really needed:
  $$P^*_{c\leftarrow a}(\alpha,\beta) = P_{ca}\big(z(\alpha,\beta)\big) \qquad \text{in principle is OK.}$$
- However, we get slightly different $\tilde{\mathcal{V}}^{c\leftarrow a}(z,\varepsilon)$ than for $\mathcal{II}$ for ALL choices of $z = z(\alpha,\beta)$. The difference traced back to upper phase space limit: $\max(\alpha,\beta) \leq 1$ versus $\alpha+\beta \leq 1$.
- **The simplest workaround** is to split $\mathcal{IF}$ non-diag. dipoles into two parts:
  $$P^{*+}_{c\leftarrow a}(\alpha,\beta) = m^{(i)}_{+}(\alpha,\beta) P_{ca}(z)\big|_{z=z(\alpha,\beta)}, \quad c \neq a,$$
  $$P^{*-}_{c\leftarrow a}(\alpha,\beta) = m^{(i)}_{-}(\alpha,\beta) P_{ca}(z)\big|_{z=z(\alpha,\beta)},$$
  and treat $P^{*-}_{c\leftarrow a}$ as extra (non-singular) dipoles in the $\mathcal{FI}$ class (decoupled from PDFs).
- This above solution works for $m^{(a)}_{\pm}$ and $m^{(c)}_{\pm}$ and looks like **an affordable complication**.

Last problem: how to eliminate P term in the final CS formula?

The remaining collinear remnant P due to multiscales in NLO

P-matrix is a quite primitive object (CS eq.10.25):

$$\boldsymbol{P}^{a,a'}(p_1,\ldots,p_m,p_b;xp_a,x;\mu_F^2) = \frac{\alpha_S}{2\pi}P^{aa'}(x)\frac{1}{\boldsymbol{T}_{a'}^2}\left[\sum_i \boldsymbol{T}_i\cdot\boldsymbol{T}_{a'}\ \ln\frac{\mu_F^2}{2xp_a\cdot p_i}+\boldsymbol{T}_b\cdot\boldsymbol{T}_{a'}\ \ln\frac{\mu_F^2}{2xp_a\cdot p_b}\right].\ (10.25)$$

- It originates from normalization factors like $\left(\frac{xs_{ai}}{\mu_F^2}\right)^{\varepsilon}\times\frac{1}{\varepsilon}P_{aa'},\ \ s_{ai}=2p_a\cdot p_i$.
- For $hh\to Z\gamma, H, WW, ..$ and lepton-hadron DIS, only 2nd term is present. It is easily eliminated with $\mu_F^2=2xp_a\cdot p_b$ or $\mu_F^2=Q^2$, getting $\mathbf{P}=0$.
- The problematic 1-st term is from $\sum_i$ over $\mathcal{IF}$-dipoles with different s_{ai}.
- **Is there some choice of μ_F^2 in PDFs eliminating at once the entire 1-st term for all processes with more than two coloured "legs"?**

Zeroing collinear remnant P

$$\sigma_{ab}^{col.rem.} = \int dx_a dx_b\ f_b(\mu_F, x_b)\ f_a(\mu_F, x_a)\ \Big\{ d\sigma_{a,b}^{Born}(p_a, p_b) +$$

$$+ \sum_{a'} \int dx\ \Big\langle\ \frac{\alpha_S}{2\pi} P_{aa'}(x) \Big[\sum_i \frac{T_i \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2xs_{ai}} + \frac{T_b \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2xs_{ab}} \Big] d\sigma_{a',b}^{Born}(xp_a, p_b)\ \Big\rangle_{color} + \dots \Big\}$$

Using colour conservation $\langle\ T_{a'} + T_b + \sum_i T_i\ \rangle_{color} = 0$ and evolution equations for $f_a(\mu, x)$ we obtain the following identity:

$$\sigma_{ab}^{col.rem.} = \int dx_a dx_b\ f_b(\mu_F, x_b)\ f_a(\mu_1, x_a)\ \Big\{ d\sigma_{a,b}^{Born}(p_a, p_b) + \sum_{a'} \int dx\ \frac{\alpha_S}{2\pi} P_{aa'}(x)$$

$$\times \Big\langle\ \Big[\sum_i \frac{T_i \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2xs_{ai}} + \frac{T_b \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2xs_{ab}} + \ln \frac{\mu_1^2}{\mu_F^2} \Big] d\sigma_{a',b}^{Born}(xx_a p_1, x_b p_2)\ \Big\rangle_{color} + \dots \Big\}$$

μ_F^2 is local dummy parameter in [...] (colour conservation!), hence we substitute $\mu_F^2 = 2xs_{ab}$.
One has to solve for μ_1 the following equation at every phase space point:

$$\sum_{a'} \int_0^1 dz P_{aa'}(z) \sum_i \ln \frac{s_{ab}}{s_{ai}} \Big\langle \frac{T_i \cdot T_{a'}}{T_{a'}^2} d\sigma_{a',b}^{Born}(zp_a, p_b) \Big\rangle_{c.} + \sum_{a'} \int_0^1 dz P_{aa'}(z) d\sigma_{a',b}^{Born}(zp_a, p_b) \ln \frac{\mu_1^2}{2zs_{ab}} \equiv 0.$$

New scale μ_1 can be calculated numerically (1-dim. integral over z) at each point of the Born phase space, $h_1 + h_2 \to p_a + p_b \to 1 + 2 + \dots m$, or even analytically in some simple cases.

Collinear remnants of CS scheme in general picture

Total cross-section in CS for m partons schematically (hh scattering):

$$\sigma = \int_m d\sigma^{Born} + \left[\int_m d\sigma^{Virt.} + \int_{m+1} d\sigma^{A} + \int_{m+1} d\sigma^{Ct} \right] + \int_{m+1} \left[d\sigma^{Real}_{\varepsilon=0} - d\sigma^{A}_{\varepsilon=0} \right]$$

2-nd term [...] for $h(p_1)h'(p_2) \to a(p_a) + b(p_b) \to 1 + 2 + \ldots m$, eq.(10.30) in CS:

$$\sigma^{Virt.+A+Ct}_{ab} = \sum_{a'} \int dx_a dx_b dx\; f_a(x_a)\; f_b(x_b)\; \langle\; (\mathbf{K} + \mathbf{P})^{aa'}(x)\; d\sigma^{Born}_{a',b}(xp_a, p_b)\; \rangle_{color}$$
$$+ \sum_{b'} \int dx_a dx_b dx\; f_a(x_a)\; f_b(x_b)\; \langle\; (\mathbf{K} + \mathbf{P})^{bb'}(x)\; d\sigma^{Born}_{a,b'}(p_a, xp_b)\; \rangle_{color}, \text{ where}$$

$$\boldsymbol{K}^{a,a'}(x) = \frac{\alpha_S}{2\pi} \Big\{ \overline{K}^{aa'}(x) - K^{aa'}_{F.S.}(x) + \delta^{aa'} \sum_i \boldsymbol{T}_i \cdot \boldsymbol{T}_a \frac{\gamma_i}{\boldsymbol{T}_i^2} \left[\left(\frac{1}{1-x} \right)_+ + \delta(1-x) \right] \Big\} - \frac{\alpha_S}{2\pi} \boldsymbol{T}_b \cdot \boldsymbol{T}_{a'} \frac{1}{\boldsymbol{T}_{a'}^2} \widetilde{K}^{aa'}(x)$$

$$\widetilde{K}^{ab}(x) = P^{ab}_{\rm reg}(x)\; \ln(1-x) + \delta^{ab} \boldsymbol{T}_a^2 \left[\left(\frac{2}{1-x} \ln(1-x) \right)_+ - \frac{\pi^2}{3} \delta(1-x) \right]$$

$$K^{aa'}_{F.S.} \equiv 0$$

$$\overline{K}^{qg}(x) = P^{qg}(x) \ln\frac{1-x}{x} + C_F\, x\;, \qquad (8.32)$$
$$\overline{K}^{gq}(x) = P^{gq}(x) \ln\frac{1-x}{x} + T_R\, 2x(1-x)\;, \qquad (8.33)$$
$$\overline{K}^{qq}(x) = C_F \left[\left(\frac{2}{1-x} \ln\frac{1-x}{x} \right)_+ - (1+x) \ln\frac{1-x}{x} + (1-x) \right] - \delta(1-x) \left(5 - \pi^2\right) C_F\;, \qquad (8.34)$$
$$\overline{K}^{gg}(x) = 2C_A \left[\left(\frac{1}{1-x} \ln\frac{1-x}{x} \right)_+ + \left(\frac{1-x}{x} - 1 + x(1-x) \right) \ln\frac{1-x}{x} \right] - \delta(1-x) \left[\left(\frac{50}{9} - \pi^2 \right) C_A - \frac{16}{9} T_R N_f \right]. \qquad (8.35)$$

With our dipoles and PDFs in the KRK FS we are getting $\mathbf{K}^{a,a'} = 0$ **!!!**

This is for ANY process, with h+h beams or lepton+h beams (DIS)!

Also P gets eliminated!!! See previous slide...

Issues already explored but not covered in this talk:

due to its limited scope...

- Fine details of new modified dipoles, soft-coll. counterterms in $d = 4 + 2\varepsilon$ dimensions, including new kinematic mappings.
- Compatibility of CS scheme with LO parton shower MC. (Correct soft limit and and positivity).

Other important issues to be studied:

- More explicit examples of NLO calculations: $pp \to Z + jet,\ 2Jet, \ldots$.
- Extending KrkNLO to more processes.
- Does KRK FS extend to “NLO PDFs” $\otimes$ “NNLO Hard process”?
- Extending modified CS scheme to massive emitters as in hep-ph/0201036 of Catani, Dittmaier, Seymour and Trocsanyi.

Summary

- PDFs in the KRK scheme are formally and practically as universal (process independent) as in the $\overline{MS}$ scheme thanks to universality of the newly modified CS dipoles and/or related soft-collinear counterterms. NEW!
- Substantial simplification of the classic Catani-Seymour NLO calculation scheme is achieved. NEW!
- KrkNLO method with PDFs in the KRK factorization scheme (implementing NLO corrections with a single multiplicative MC weight) is NOT limited to processes with two coloured legs (DY, DIS)! NEW!

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KrkNLO method and PDFs in KRK factorization scheme

(A) 1-st idea of the KrkNLO for DY process and KRK FS:
Acta Phys.Polon. B42 (2011) 2433 , [arXiv:1111.5368] Ustron 2011 Proc.

(B) KrkNLO scheme for DY and DIS, PDFs in the KRK factorization scheme:
Phys.Rev. D87 (2013) 3, 034029 , [arXiv:1103.5015].

(C) Implementation for DY process of top of SHERPA and HERWIG in
JHEP 1510 (2015) 052 [arXiv:1503.06849],
comparisons with NLO and NNLO (fixed order), MC@NLO and POWHEG.

(D) PDFs in Monte Carlo factorization scheme, DY and Higgs production
Eur. Phys. J. C76 (2016) 649 [arXiv:1606.00355].

(E) MC simulations of Higgs-boson production at the LHC with the KrkNLO method
Eur.Phys.J. C77 (2017) 164 , [arXiv:1607.06799],

KrkNLO team: W. Płaczek, M. Sapeta, A. Siódmok, M. Skrzypek and S.J.

More details: $\mathbb{K}_{ba}(z)$ from CS initial-initial $\mathcal{JJ}$ dipoles

Let us recalculate $\mathcal{JJ}$ dipoles from the scratch, because in CS paper they are obscured by the unlucky choice of the $\mathcal{JF}$ dipoles (DIS/ISR) as a baseline objects.

Our compact elegant definition of all nine $\mathcal{JJ}$ dipoles, $K, I = q, \bar{q}, G$:

$$\tilde{\mathcal{V}}^{K\leftarrow I}(z,\varepsilon) = \int d\alpha d\beta\; \delta_{1-z=\alpha+\beta}\; H(\alpha,\beta,\varepsilon) = \int d\alpha d\beta\; \delta_{1-z=\alpha+\beta}\; (\alpha\beta)^{\varepsilon}\; z^{-\varepsilon} \frac{P^{*}_{K\leftarrow I}(\alpha,\beta)}{\beta}$$

$$= \delta_{z=1}\; \delta_{KI} \sum_{J=G,q,\bar{q}} \int_0^1 dz\; z\tilde{\mathcal{V}}^{J\leftarrow I}(z,\varepsilon) + \delta_{z=1} \frac{1}{\varepsilon}\; P_{KI}(z) + \mathcal{G}_{K\leftarrow I}(z),$$

$$\boxed{\mathbb{K}_{KI}(z) = \mathcal{G}_{K\leftarrow I}(z) = \delta_{z=1}\; \mathcal{G}^{0}_{KI} + \tfrac{1}{z}\Big[zP'_{KI}(z) + \ln\tfrac{(1-z)^2}{z}\; zP_{KI}(z)\Big]_{+}},$$

where $\mathcal{G}^{0}_{KI}$ are from momentum sum rules. Agrees with CS for DY.
Denoting $\bar{P}_{KI}(z) \equiv (1-z)P_{KI}(z)$ we are using CS choice of the "soft partition function":

$$P^{*}_{K\leftarrow K} = \frac{\bar{P}_{KK}(1-\alpha-\beta,\varepsilon)}{(\alpha+\beta)\beta}, \quad P^{*}_{K\leftarrow I} = \frac{P_{KI}(1-\alpha-\beta,\varepsilon)}{\beta},\; K \neq I.$$

NB. The same result is obtained with sharp "soft partition function" of paper (B):

$$P^{*}_{K\leftarrow K} = \frac{\bar{P}_{KK}(1-\alpha-\beta,\varepsilon)}{\alpha\beta}\theta_{\alpha>\beta}.$$

All $P_{KI}(z)$ kernels are here standard DGLAP splitting kernels.