

Based on 2102.05487 with Roberto Mondini

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# Bottom induced Higgs+jet at NNLO

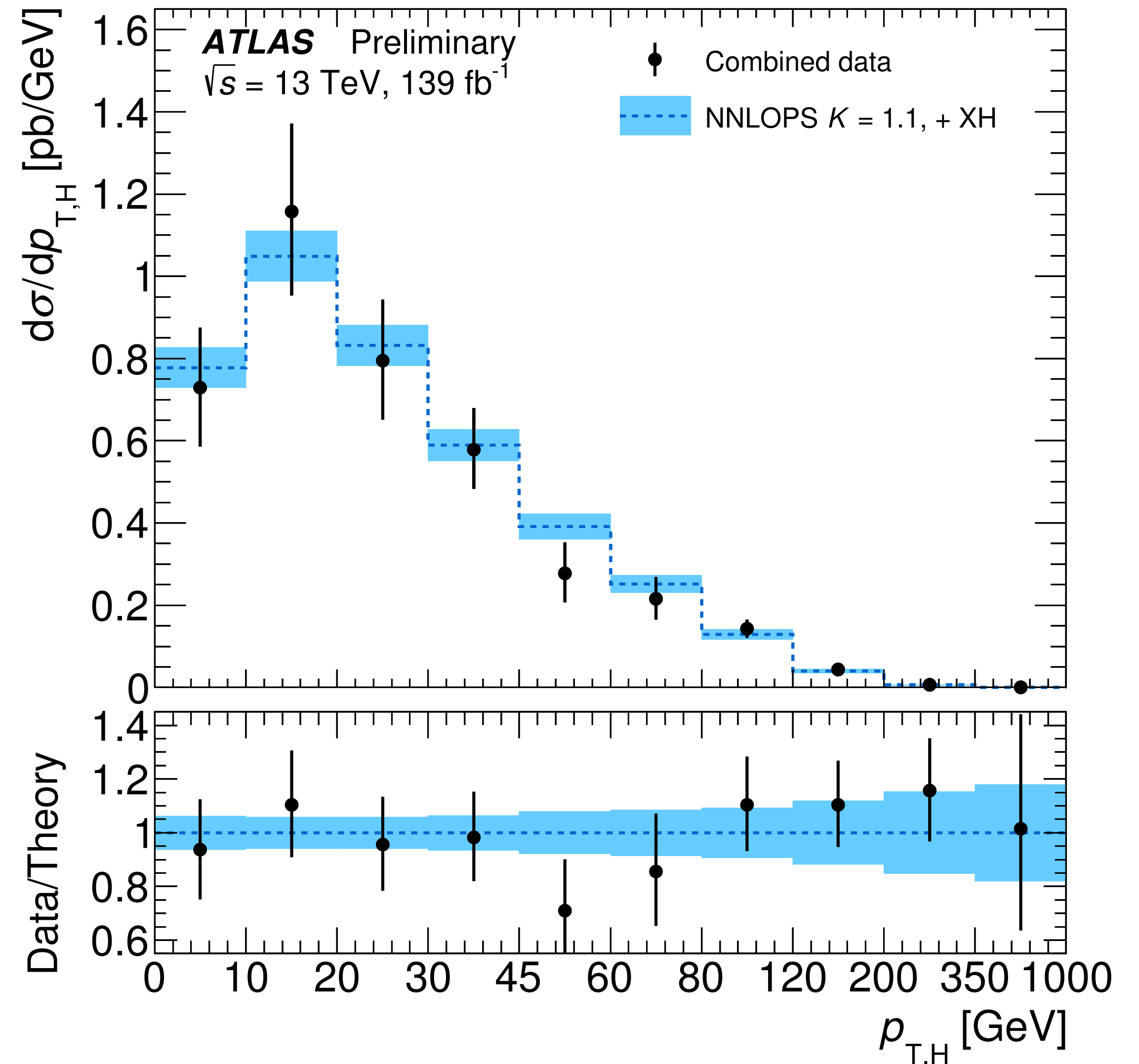
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# Introduction

Ongoing study of the Higgs boson differentially remains one of the pillars of the experimental program at the LHC.

An interesting component corresponds to the study of Higgs + 3rd generation quarks. Which dominate the production and decay of the Higgs boson. Need for precision has been demonstrated repeatedly.

The aim of this talk is to present an NNLO calculation of bottom induced contributions to  $H+j$ .



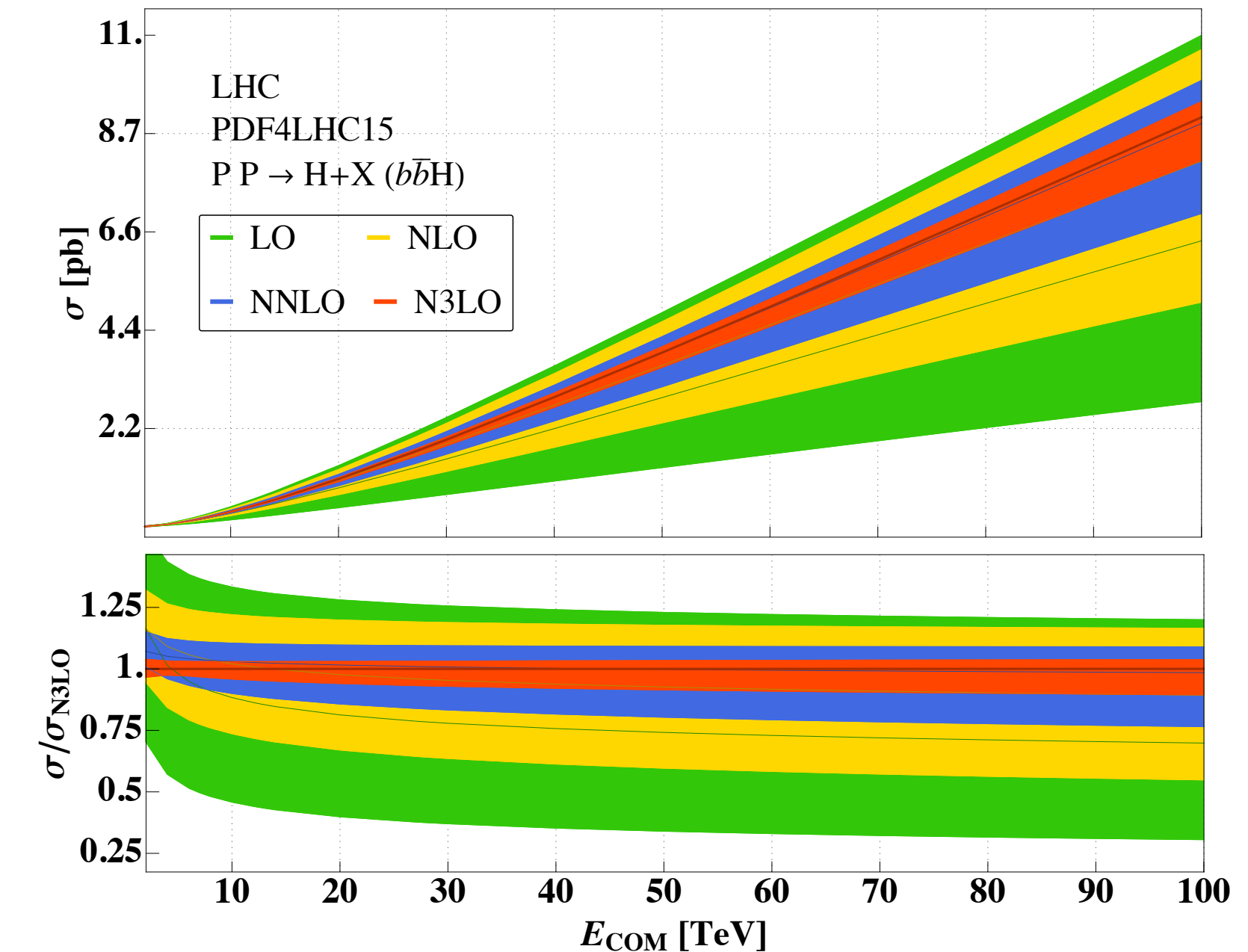
# Higgs in bottom fusion

One must first decide how to handle the bottom quark, known as the 4 or 5- flavor schemes.

The state of the art is at N3LO (Duhr, Dulat, Mistlberger 19) for the total cross section, matched in the 5FS matched to NLO in the 4FS (Duhr, Dulat, Hirschi Mistlberger 20)

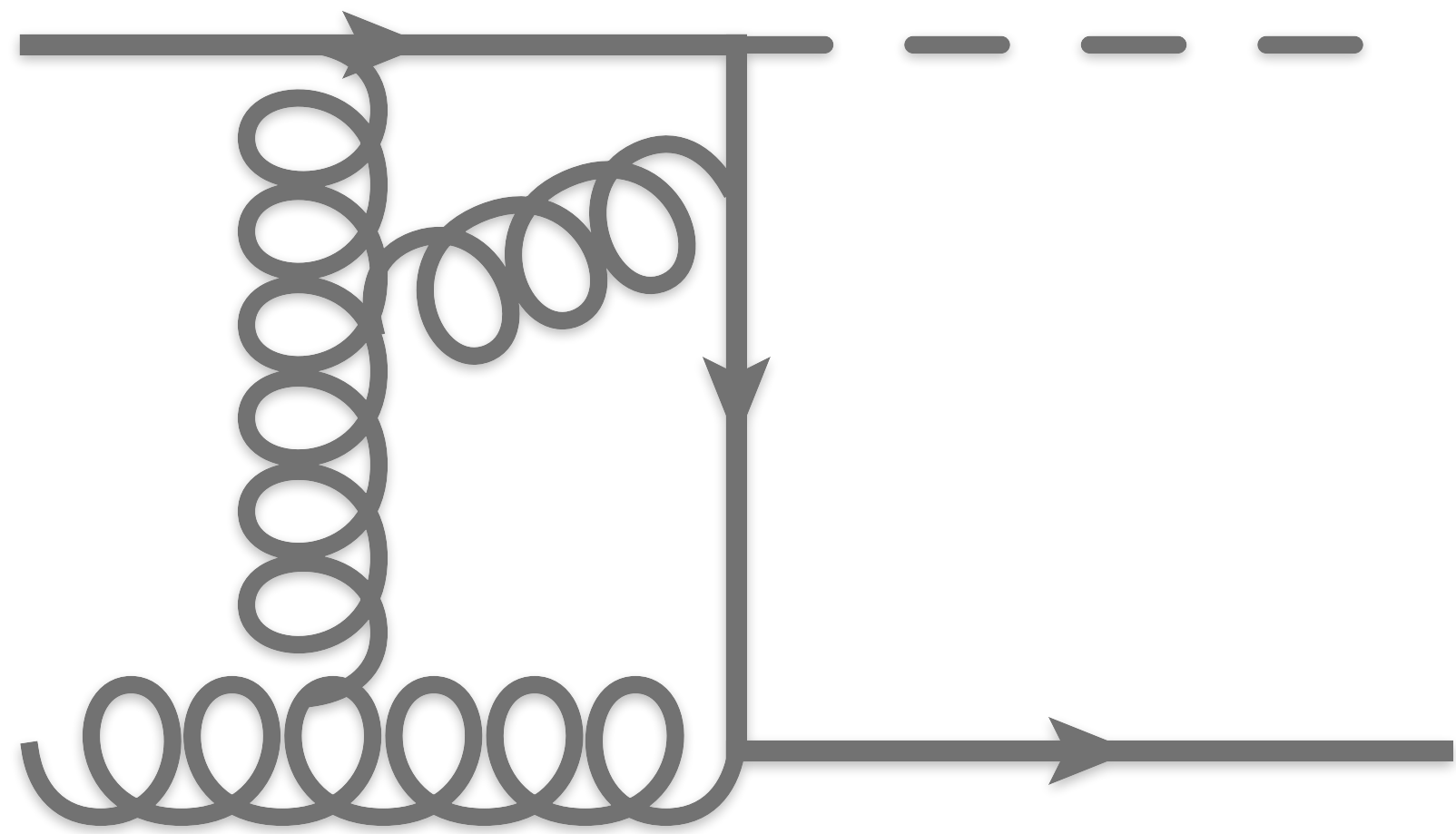
H+j through bottom fusion computed at NLO (Harlander Ozeren Wiesemann 10)

4FS 5FS Partonic channels (5FS)	– LO $b\bar{b}$	– NLO $b\bar{b}, bg$	LO NNLO $b\bar{b}, bg, bb, bq, b\bar{q}, gg, q\bar{q}$	NLO N <sup>3</sup> LO $b\bar{b}, bg, bb, bq, b\bar{q}, gg, q\bar{q}, qg$



Figs + table taken from (Duhr, Dulat, Mistlberger 19)

# 5FS and UV scheme



In the 5FS we take the bottom quark mass to zero. This allows us to resum large initial-state collinear logs into PDFs.

In order to keep non-zero coupling between the Higgs and bottom we keep  $y_b$  non-zero.

Specifically we use the mixed renormalization scheme in which the bottom Yukawa is evaluated in the  $\overline{MS}$  scheme. And the bottom quark mass is OS (and then taken to zero)

Cross sections in the 5FS scheme have a strong dependence on  $\mu_F$ , argued at NLO for bottom fusion that  $m_H/4$  is a sensible scale choice for this process **(Maltoni, Sullivan Willenbrock 03)**.



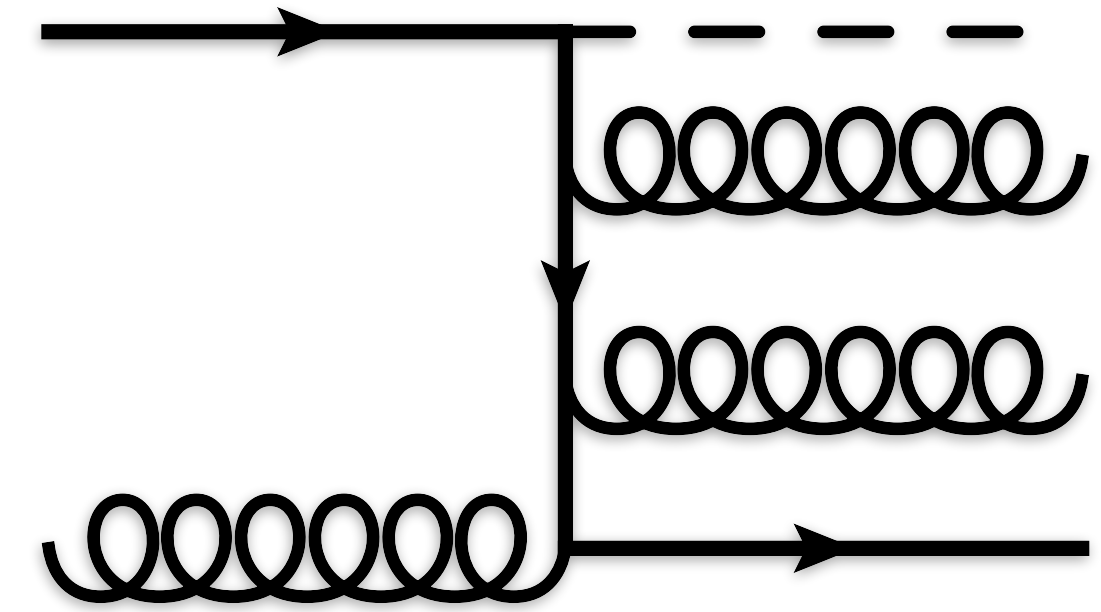
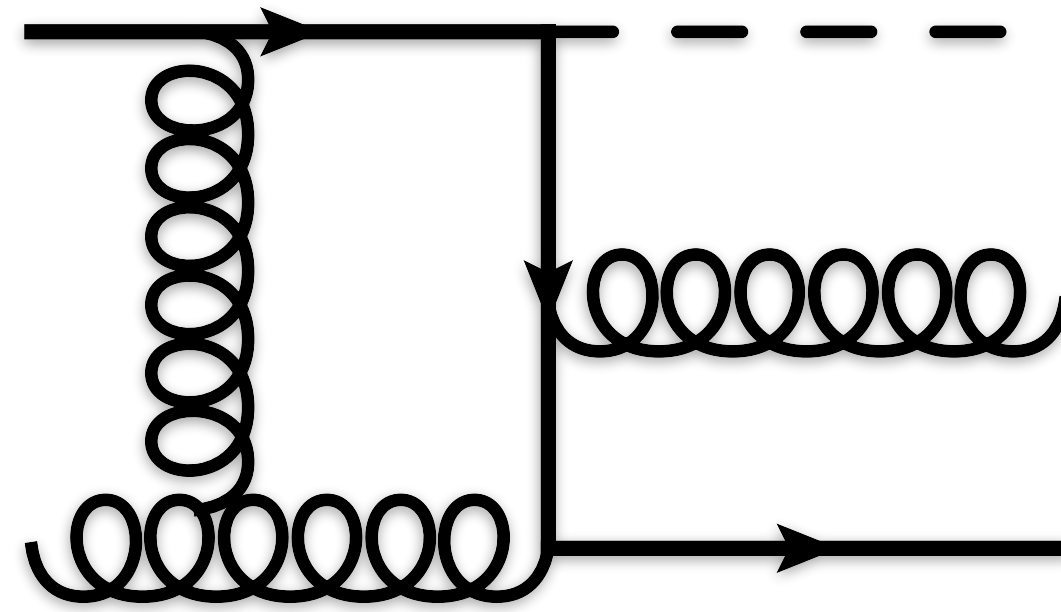
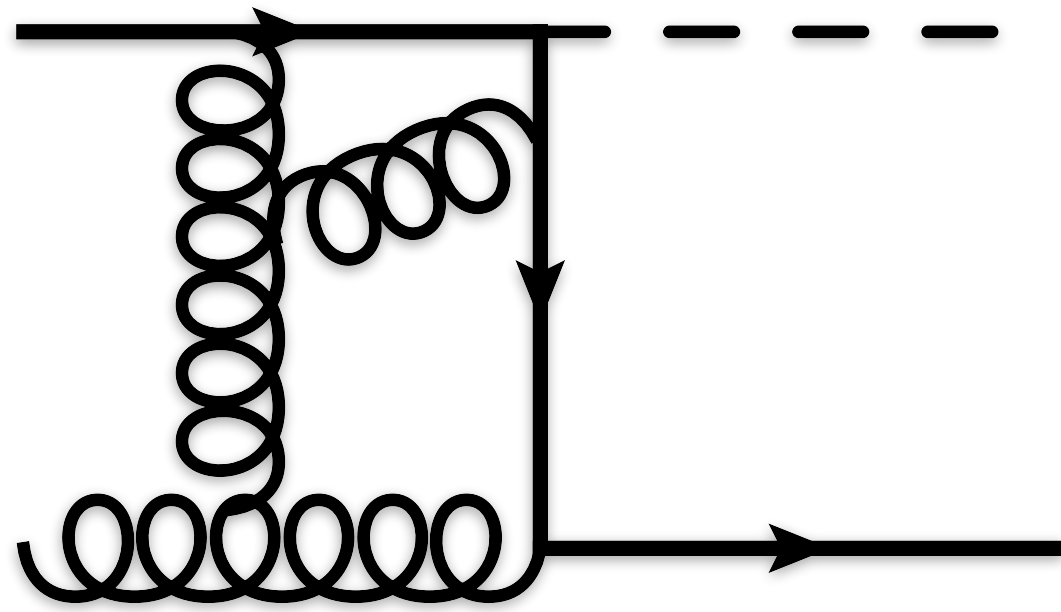
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# Calculation

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# Ingredients



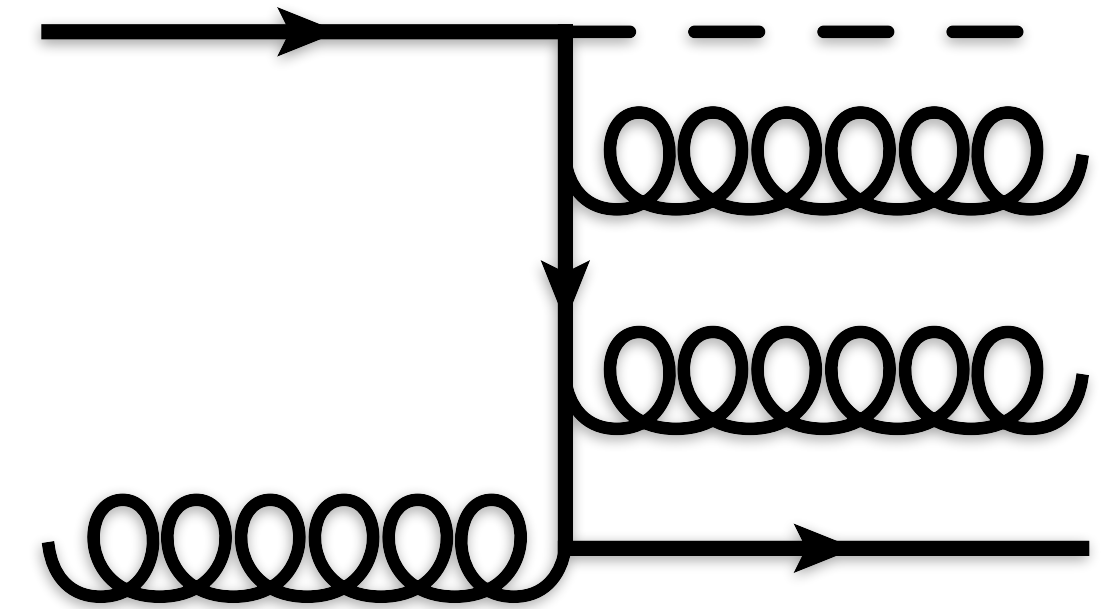
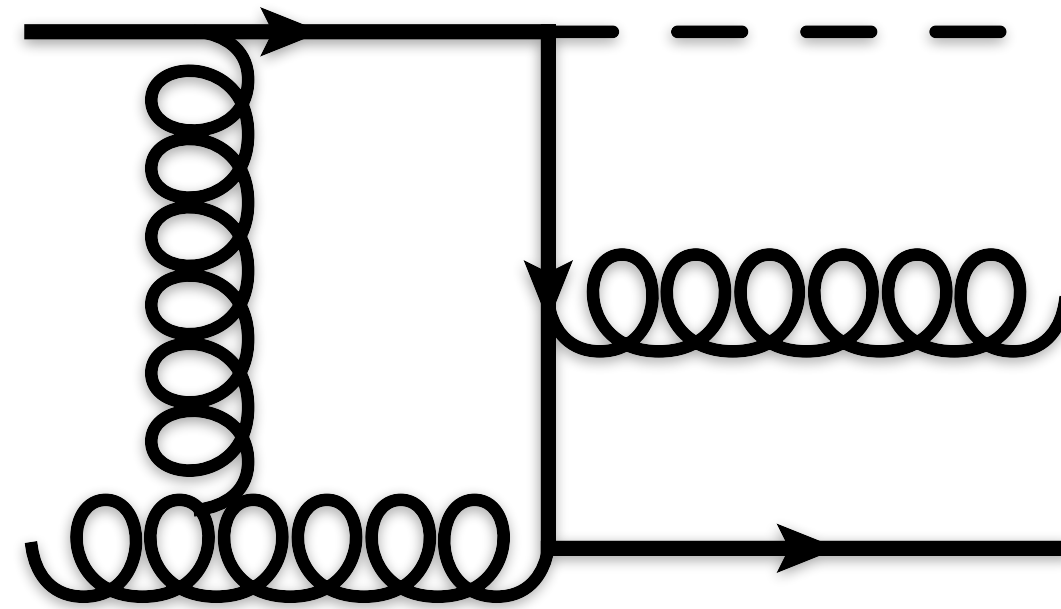
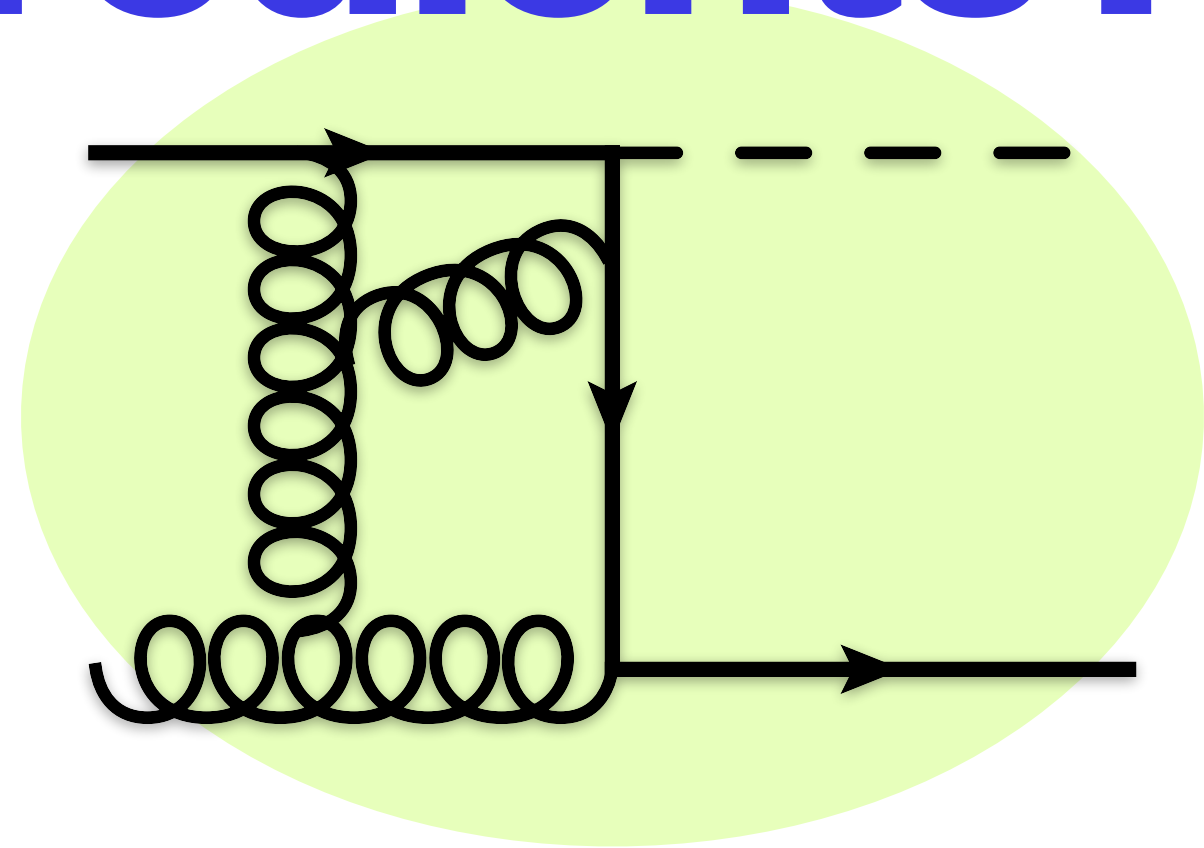
As is well-known we need three ingredients to complete our NNLO calculation:

- Double Virtual (two-loop amplitudes and one-loop squared)
- Real-Virtual ( $H+2j$ ) one-loop amplitudes
- Double Real ( $H+3j$ ) tree-level amplitudes

Plus a method to combine the disparate phase spaces.

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# Ingredients : Double Virtual

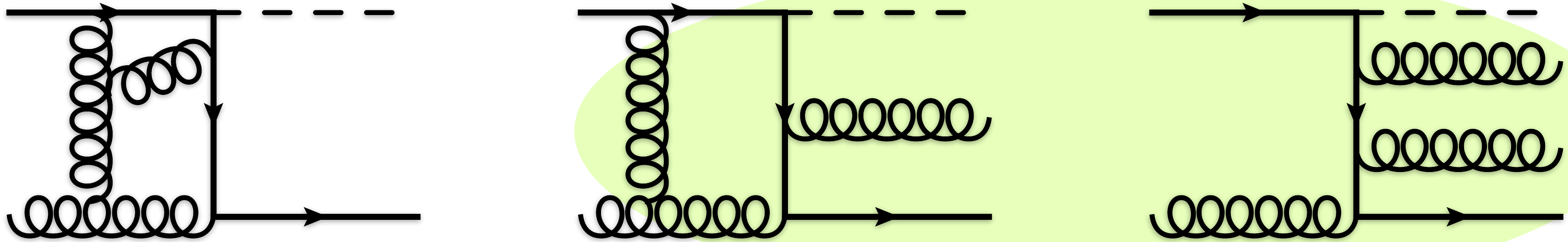


For the double virtual amplitude we take the results we computed for the  $H \rightarrow b\bar{b}$  calculation at N3LO (Mondini, Schiavi CW 19).

We then crossed them to the relevant LHC kinematics using the methodology outlined for  $V+j$  (Gehrmann and Remiddi 02)

Finally we confirmed the results using the soft (Li Zhu 13) and collinear (Badger, Glover 04) IR factorization as the gluon becomes unresolved.

# Ingredients: $H+2j$ @NLO



The amplitudes required for Higgs plus 2 jets were computed using analytic unitarity for the loop amplitudes. (Bern Dixon Kosower 94, Britto, Cachazo Feng 04, Forde 07, Mastrolia 09, Badger 08 ....)

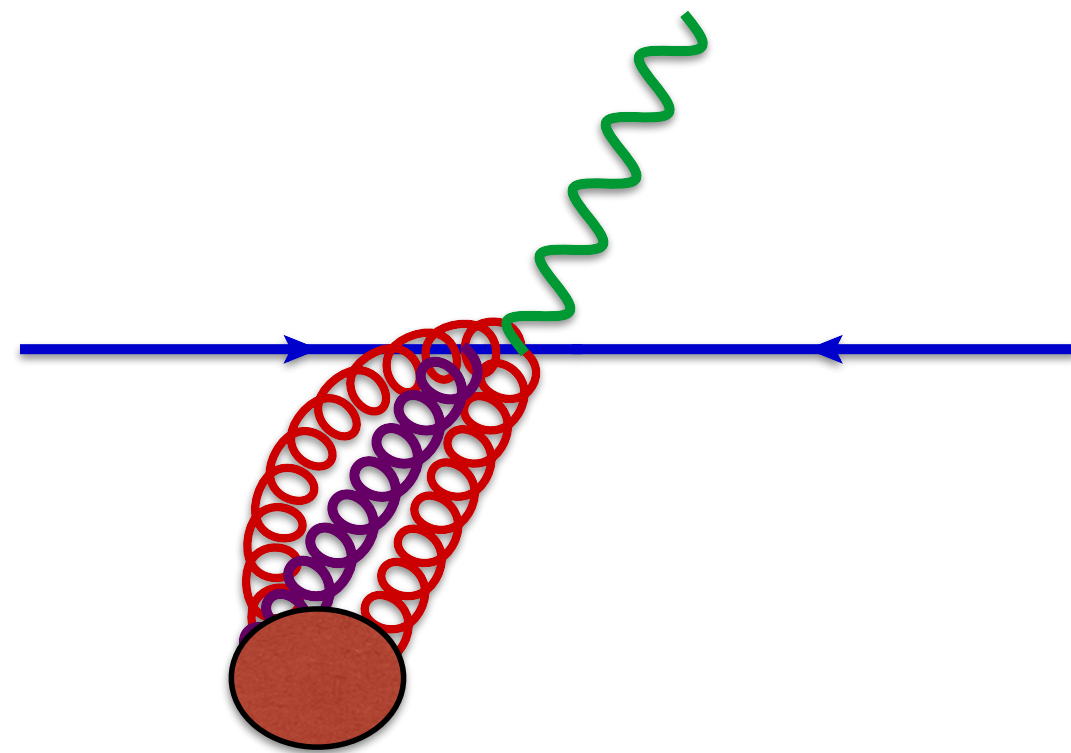
And BCFW (Britto Cachazo Feng Witten 05) recursion for the tree-level.

The final results are extremely compact analytic formula.

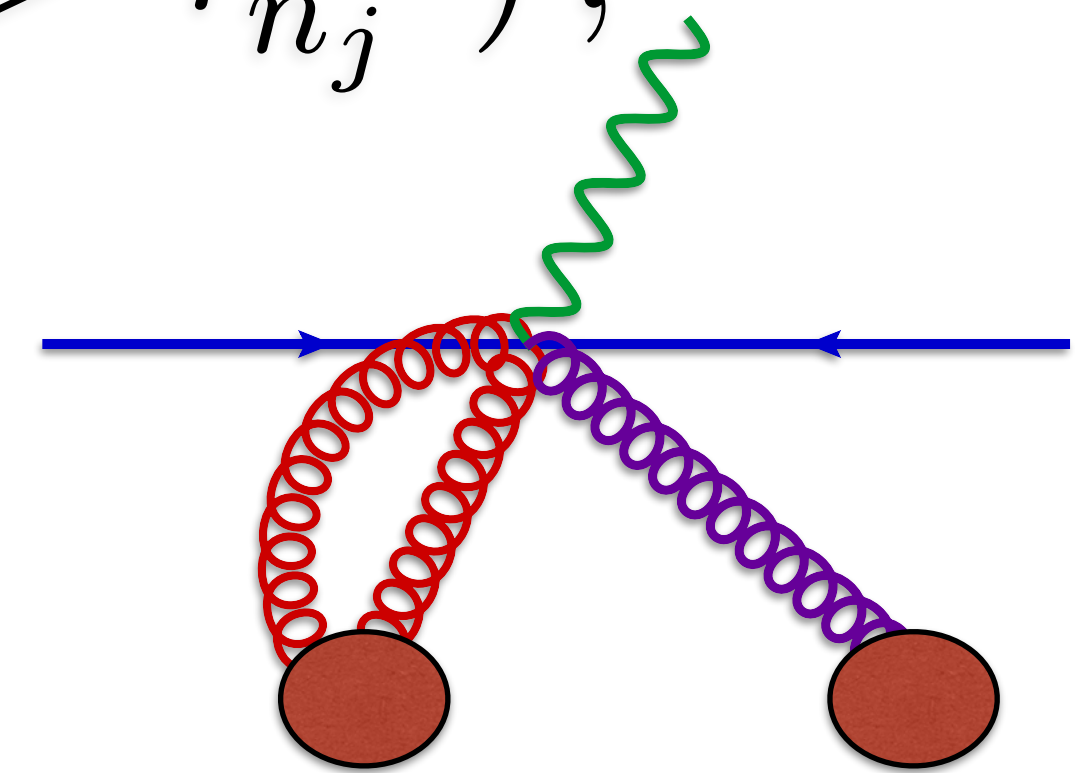
# Slicing @ NNLO

Idea behind a slicing approach is to split the phase space into two based on some suitable variable

$$\sigma^{\text{NNLO}} = \sigma(\tau_N \leq \tau_{n_j}^{\text{cut}}) + \sigma(\tau_N > \tau_{n_j}^{\text{cut}}),$$



Should contain all double unresolved limits, and be accessible via simplified result (i.e. factorization theorem)



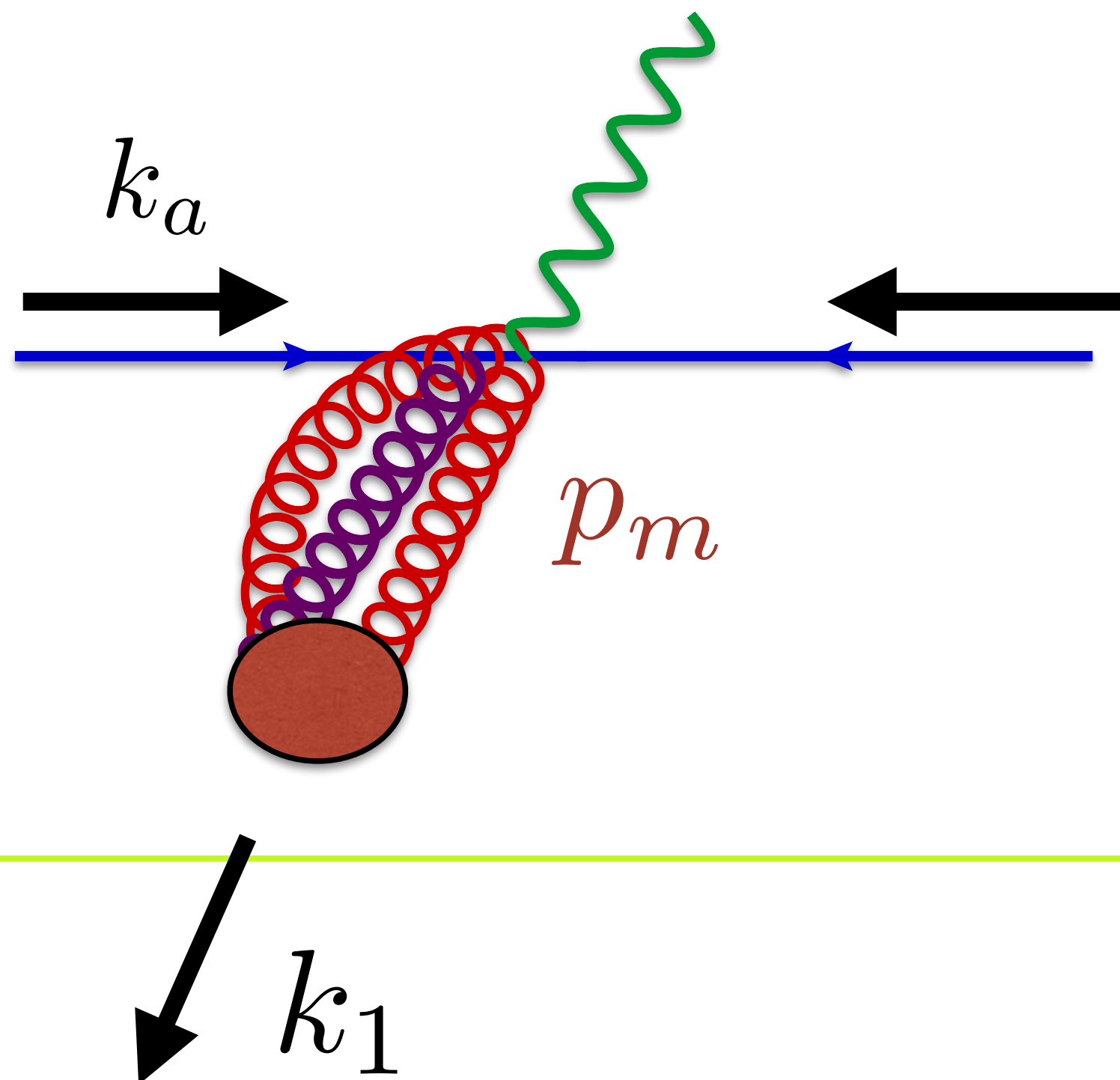
Should contain at most singly unresolved limits, (i.e. an NLO + extra parton) directly compute with suitable Monte Carlo codes

# N-jettiness slicing

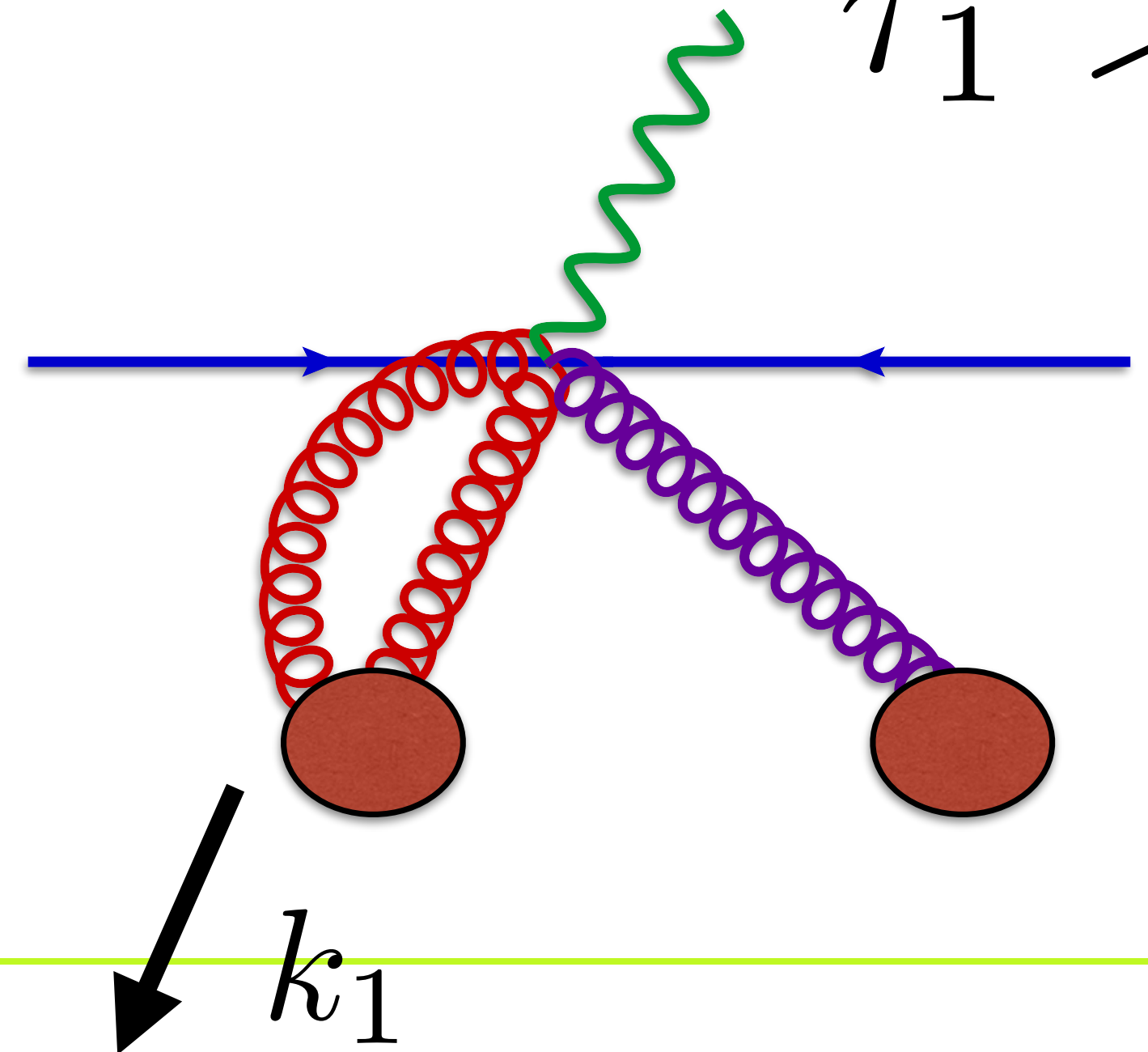
We use the N-jettiness event shape variable (Stewart, Tackmann Waalewijn 09) to split the regions to split the regions

$$\tau_1 = \sum_m \min_i \frac{2p_m \cdot k_i}{P_i},$$

$\tau_1 \rightarrow 0$



$\tau_1 > 0$





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# SCET factorization

To compute the below-cut piece we use the following factorization theorem, derived from SCET

$$\sigma(\tau \leq \tau_{n_j}^{\text{cut}}) = \int_0^{\tau_{n_j}^{\text{cut}}} d\tau \left( \mathcal{S} \otimes \prod_{i=1}^{n_j} \mathcal{J}_i \otimes \prod_{a=1,2} \mathcal{B}_a \otimes \mathcal{H} \right) + \mathcal{F}(\tau_{n_j}^{\text{cut}}),$$

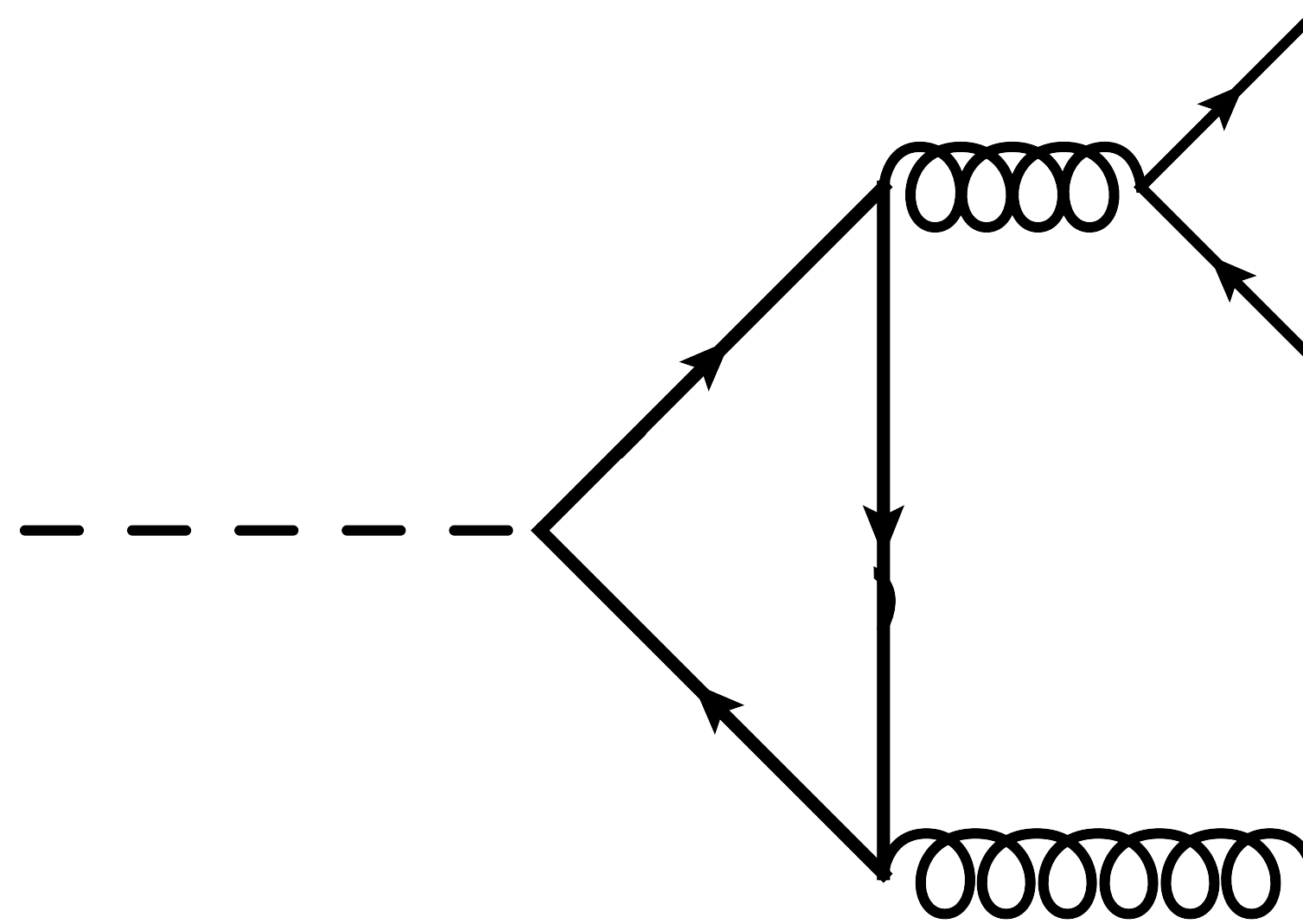
At  $\mathcal{O}(\alpha_s^2)$  the various pieces needed are :

- $\mathcal{S}$  - Soft function (for 3 partons) (**Boghezal Liu Petriello 15, Campbell Ellis Mondini CW 17**)
  - $\mathcal{J}_i, \mathcal{B}_a$  - Jet and beam functions (collinear behavior) (**Becher Bell 10, Gaunt Stahlhofen Tackmann 14**)
  - $\mathcal{H}$  - Hard function - process specific finite function.
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# Role of the Top Quark : I

From a point of view of strict expansion in  $\alpha_s$  the top quark first appears in H+j at  $\mathcal{O}(\alpha_s^3)$ , i.e. as a piece of our NNLO coefficient.

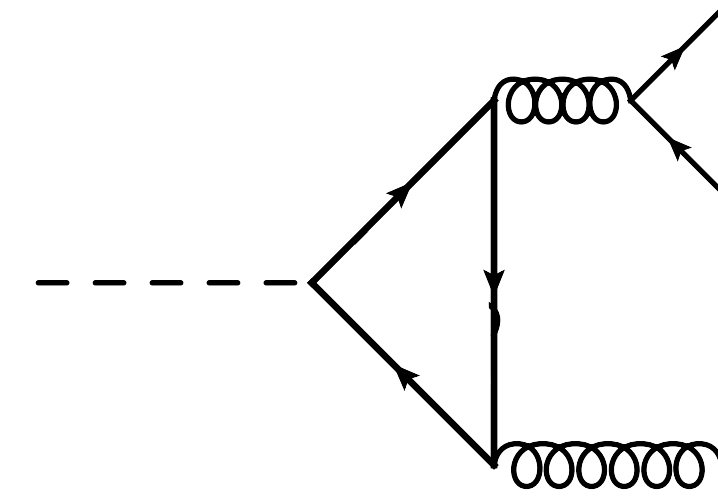


Of course since  $y_t \gg y_b$  from a pheno point of view this piece “is the LO”

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# Roll of the Top Quark: I



At this order we could, of course keep the full mass dependence of the top quark without any headache.

$H+j$  is now known with full top mass dependence at NLO (see e.g. [Jones Kerner Luisoni 18](#))

We could also consider the Effective Field Theory in which the top quark is integrated out, this of course is known to NNLO. ([Chen Gehrman Glover Jaquier 14](#), [Boughezal Caola Petriello Melnikov Schulze 15](#), [Boughezal Focke Giele Lui Petriello Schulze 15](#), [Chen Cruz-Martinez Gehrman Glover Jaquier 16](#), [Campbell Ellis Seth 19](#))

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# Our decision

The aim of this paper was to study the NNLO impact to the bottom induced corrections, and how amenable they were to a slicing based regulator.

With that in mind we remain agnostic to the top quark implementation and “punt” on the decision.

Of course to do a full pheno study of  $H+j$  we should include the top.



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# Roll of the Top Quark: I

A final point to bear in mind regarding EFT/FT choice for the top is related to the definition of the bottom coupling itself

Specifically if we were to work in the EFT we have an additional subtlety which is that the definition of the bottom-Yukawa is altered.

At our order of interest

$$y_b^{\text{EFT}} = y_b^{\text{SM}} \left( 1 + \left( \frac{\alpha_s}{\pi} \right)^2 \Delta_F^{(2)} + \mathcal{O}(\alpha_s^3) \right),$$

Our results here today will be in terms of  $y_b^{\text{SM}}$ , but the difference is very small.

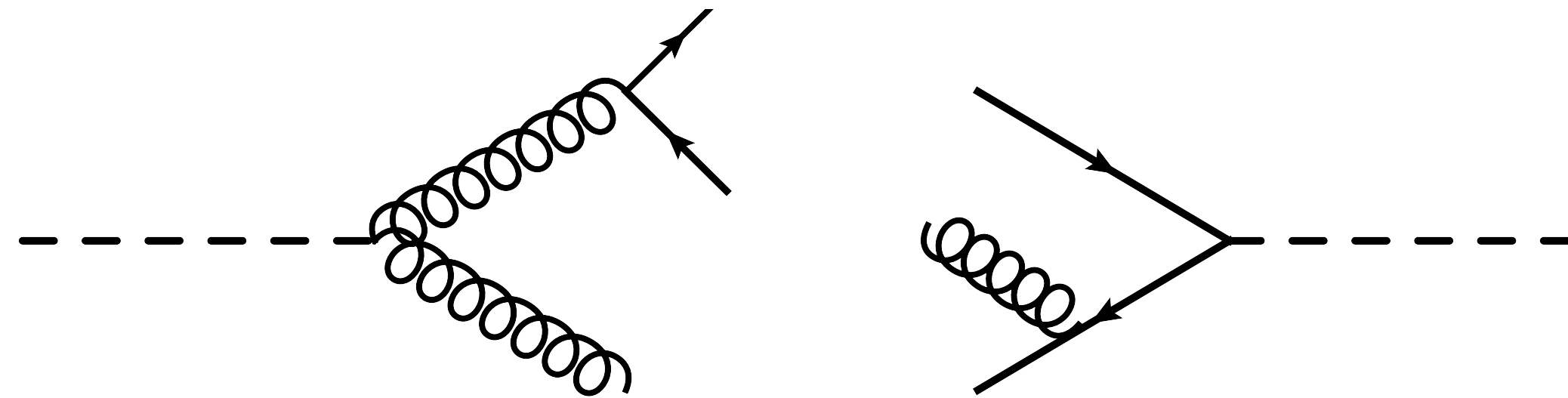
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$$\Delta_F^{(2)} = \left( \frac{5}{18} - \frac{1}{3} \log \frac{\mu^2}{m_t^2} \right).$$

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# Roll of the Top Quark: II

The top quark pops up again one last time in  $H+j$ , this time at  $\mathcal{O}(\alpha_s^2)$  (i.e. NLO) through an interference term.

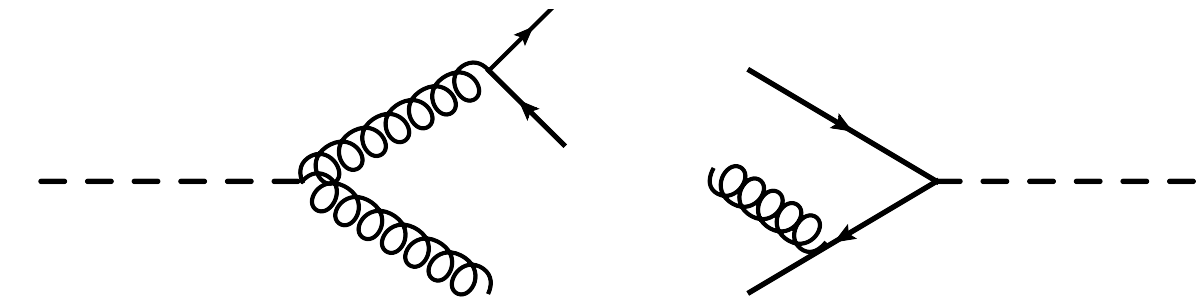


These pieces are interesting, they require a helicity flip to be non-zero and hence scale as.

$$2\text{Re}(A_{\text{EFT}}^\dagger A_{y_b}) \sim y_b \frac{m_b}{v}.$$



# Roll of the Top Quark: II



In the mixed renormalization scheme  $y_b$  is taken in the  $\overline{MS}$  scheme and  $m_b$  is taken in the OS scheme. So we would write

$$2\text{Re}(A_{EFT}^\dagger A_{y_b}) \sim y_b^{\overline{MS}} m_b^{OS}$$

Now in the 5FS we take  $m_b \rightarrow 0$  and these terms vanish.

However, there is an ambiguity. What if we used our freedom to re-write the on-shell mass in terms of the  $\overline{MS}$  mass before taking the limit  $m_b \rightarrow 0$ ?

We would now find the non-vanishing scaling

$$2\text{Re}(A_{EFT}^\dagger A_{y_b}) \sim (y_b^{\overline{MS}})^2$$

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# Roll of the Top Quark: II

At  $\mathcal{O}(\alpha_s^2)$ , requiring the presence of a final state jet yields a finite result, but the situation for us at  $\mathcal{O}(\alpha_s^3)$ , is considerably more intricate.

This is because of the presence of both a rich IR structure and UV renormalization with makes taking such a limit intricate.

In this talk we will take the first option and simply keep the original definition of the mixed renormalization scheme and take  $y_b$  as an independent parameter to  $m_b$  (with the latter being zero).

For two recent papers discussing this interference we refer to **ref [1]** (intf. Relating to H+c studies at the LHC) and **ref [2]** (intf in  $H \rightarrow b\bar{b}$  and  $H \rightarrow c\bar{c}$  decays)

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[1] Bizon, Melnikov Quarroz 21

[2] Mondini Schubert CW 20

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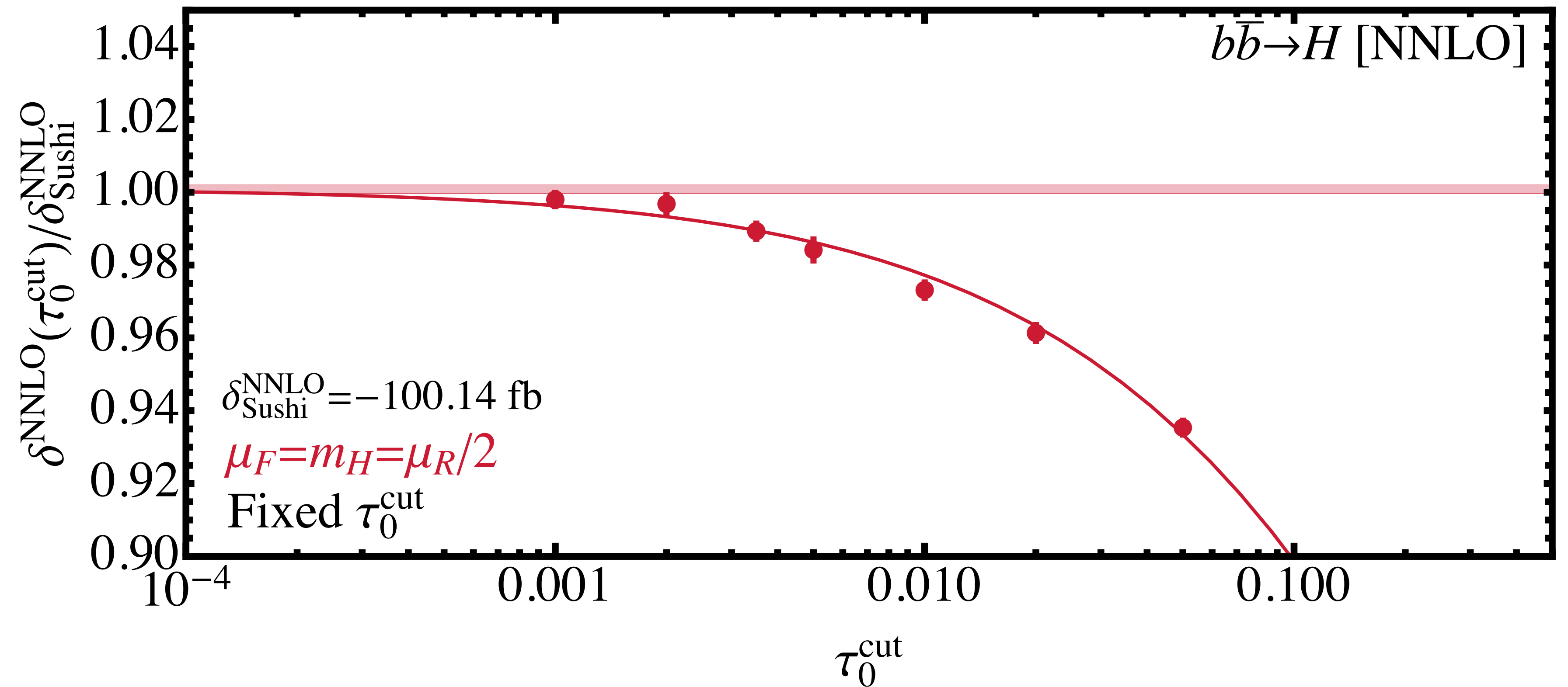
# Validation

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# $b\bar{b} \rightarrow H$

As a warm up we implement Higgs production in bottom fusion at NNLO using the N-jettiness slicing approach.

We can compare our results for the NNLO **coefficient** to the publicly available code **SusHi** (Harlander, Liebler Mantler 12) finding excellent agreement.



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# Dynamic/Boosted/fixed?

One of the practical decisions we have to make for the +1 jet calculation is how to define the cut parameter.

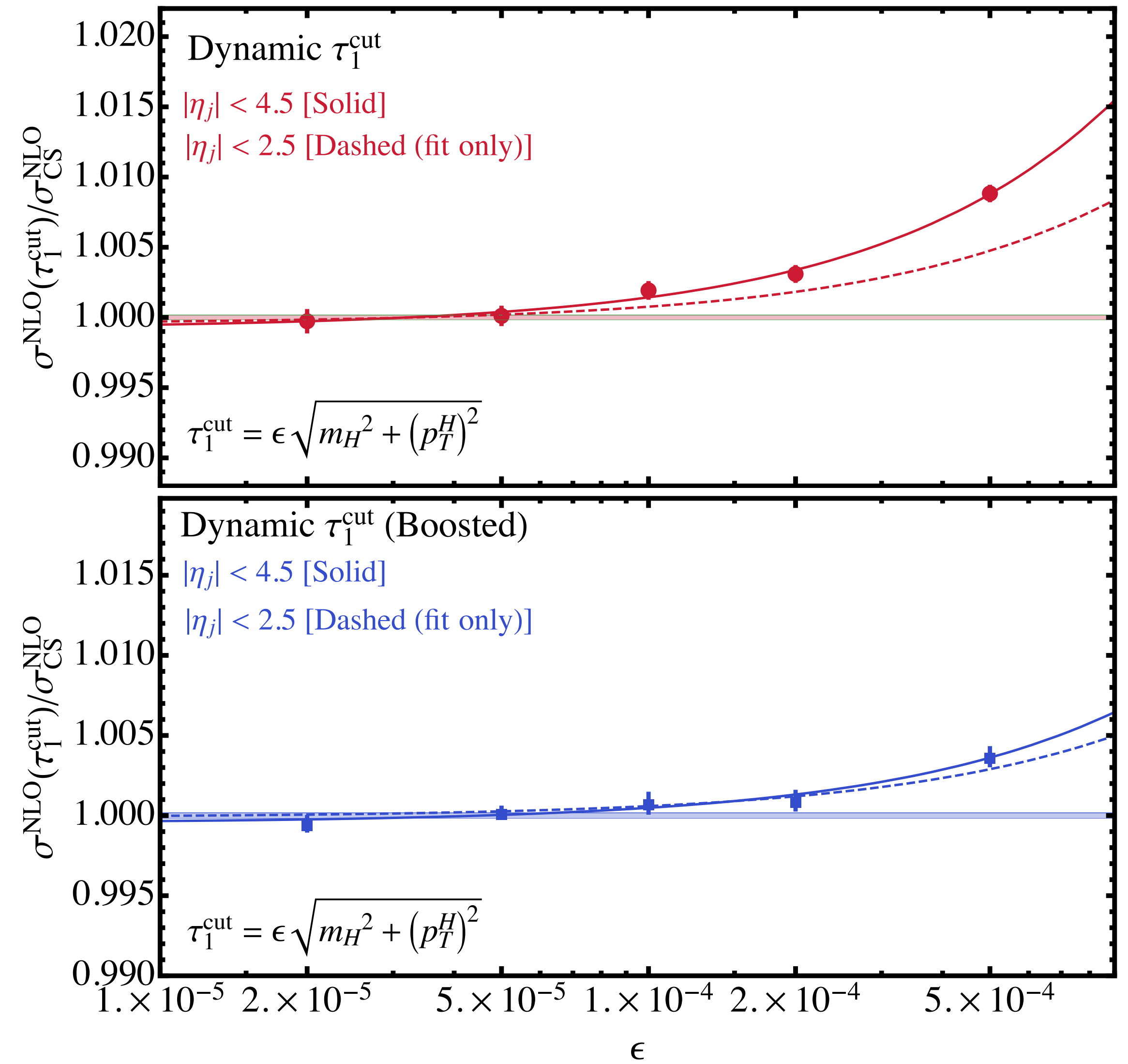
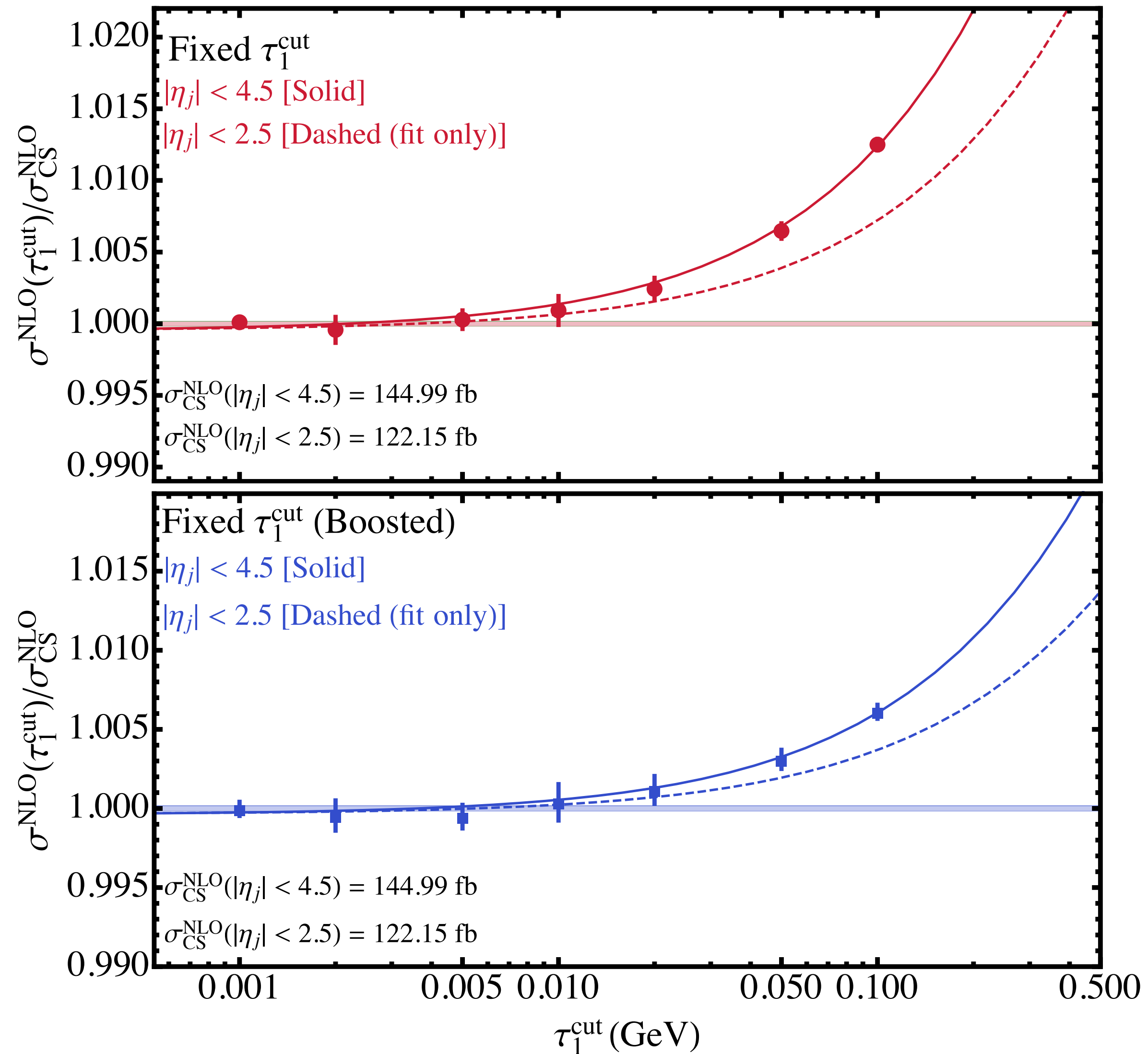
Its been seen that using a dynamic  $\tau_1^{\text{cut}}$  that its easier to get more stable results **(Campbell Ellis CW 16, Campbell Ellis Seth 20)** we take :

$$\tau_1^{\text{cut}} = \epsilon \sqrt{m_H^2 + (p_T^H)^2}.$$

Also its been argued that by evaluating the cut in the rest frame of the LO system then the power corrections will be smaller **(e.g. Moulth Rothen Stewart Tackmann 16)**

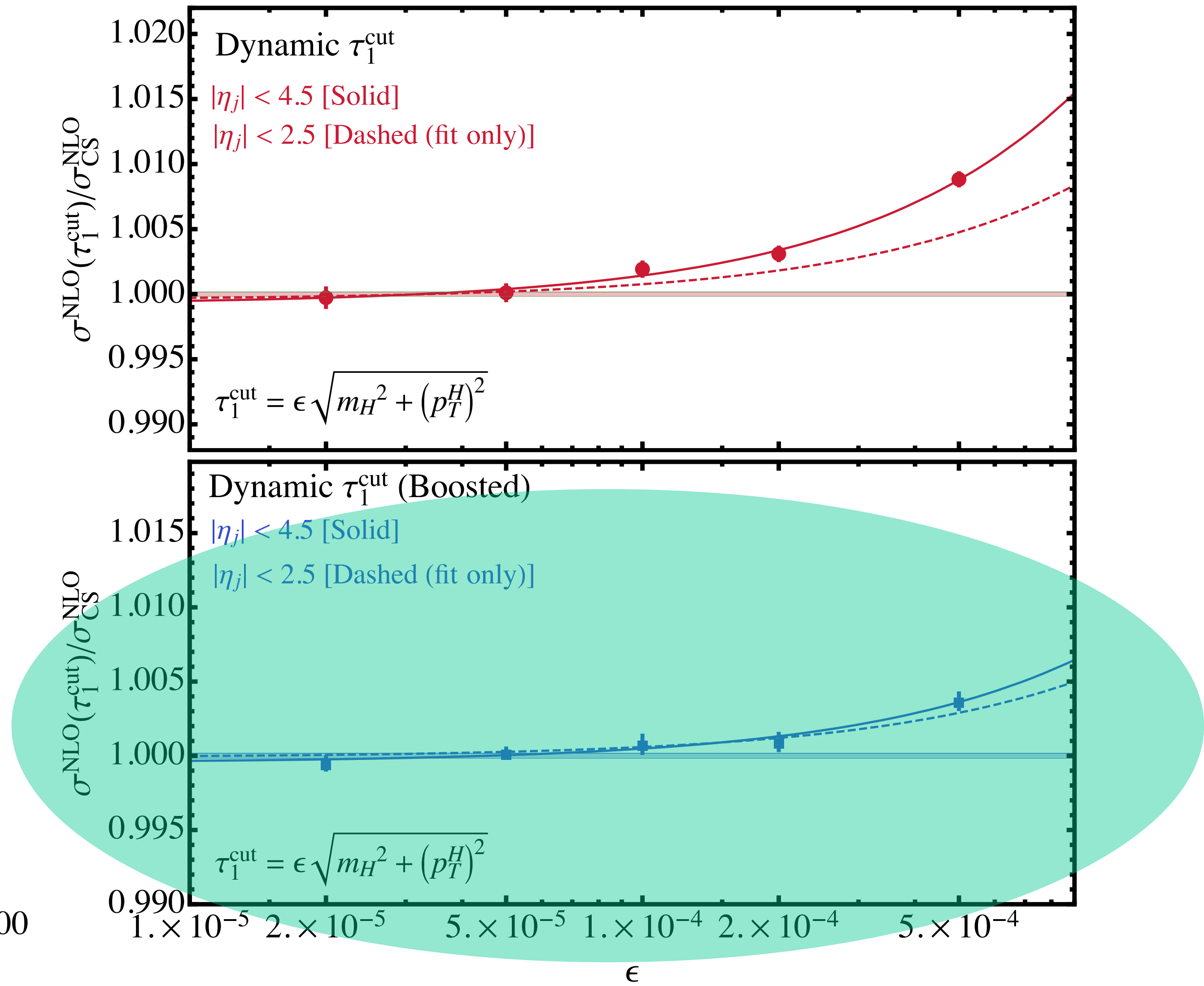
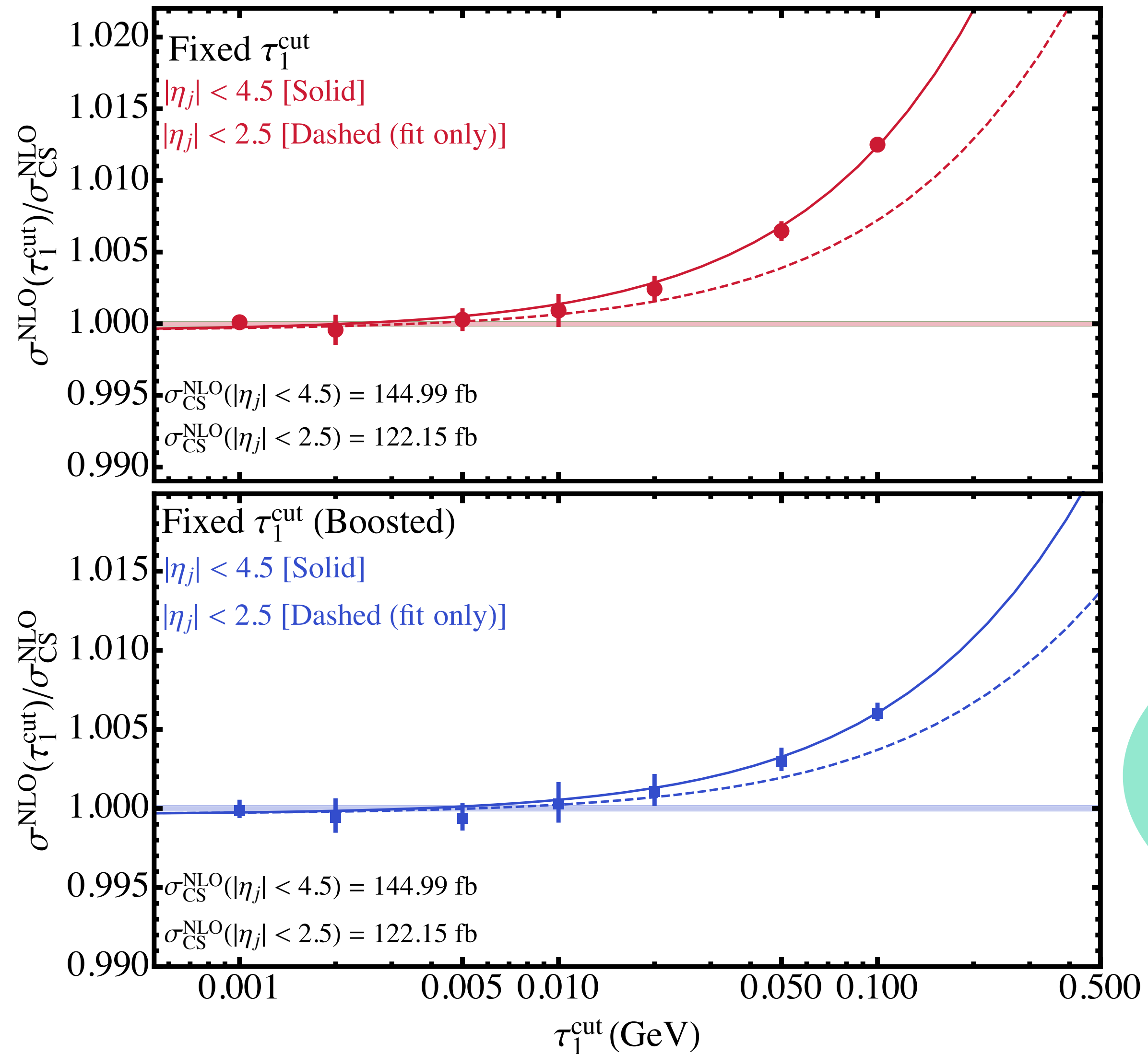
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# Dynamic/Boosted/fixed? @ NLO





# Dynamic/Boosted/fixed? @ NLO



# $\tau$ -dep for H+j

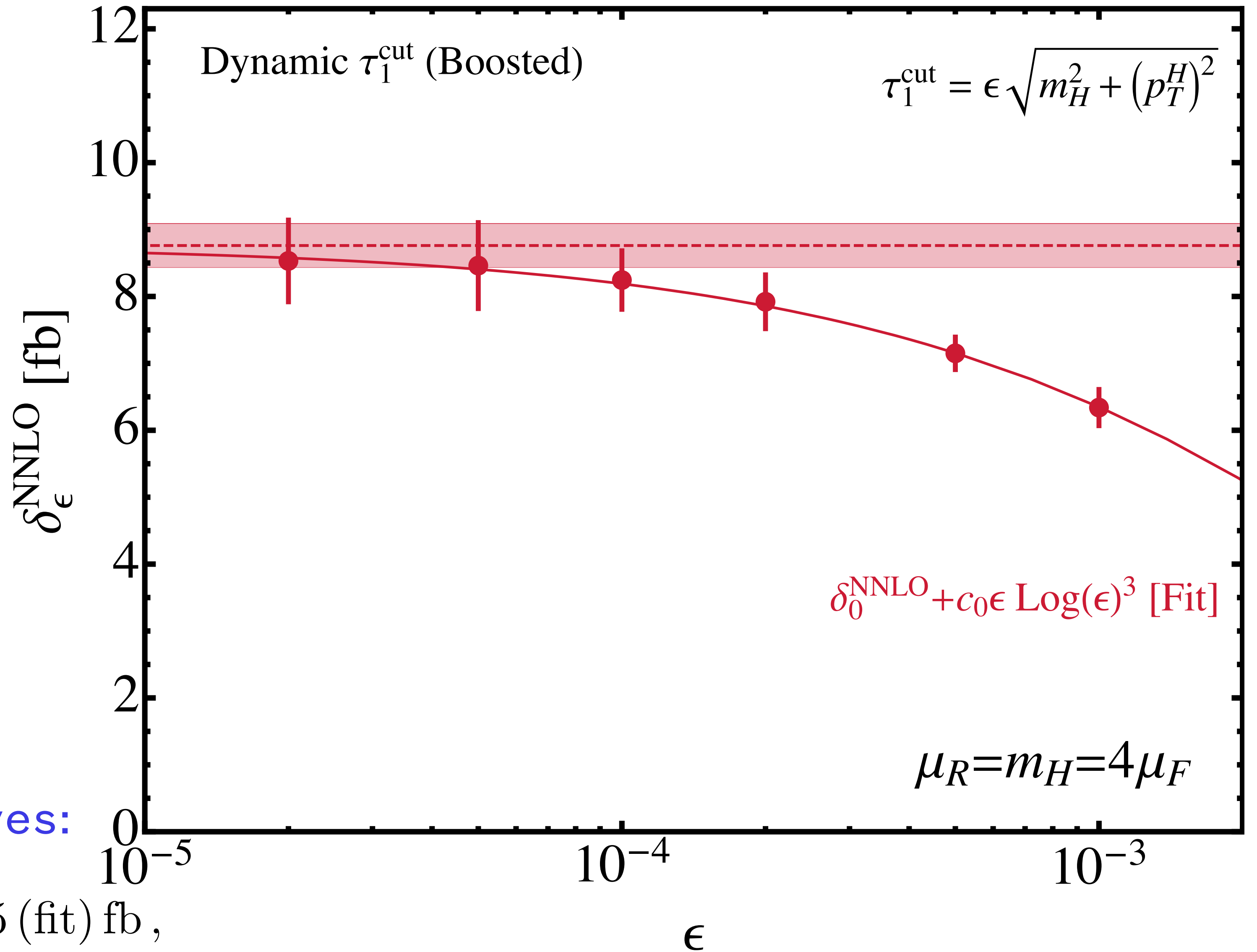
Our results have the same structure as those of H+j in EFT (**Campbell Ellis Seth 20**) in which the dynamic boosted  $\tau_1^{\text{cut}}$  performs the best

We study the  $\tau_1^{\text{cut}}$  dependence of the coefficient, fitting :

$$\delta_0^{\text{NNLO}} = 8.79 \pm 0.35 \text{ fb}$$

Which combined with the NLO gives:

$$\sigma_{H+j}^{\text{NNLO}}(\mu_R = 4\mu_F = m_H) = 153.78 \pm 0.35 \text{ (fit) fb},$$



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# Results

13 TeV LHC  $p_T^J > 30$  GeV  $|\eta_J| < 4.5$

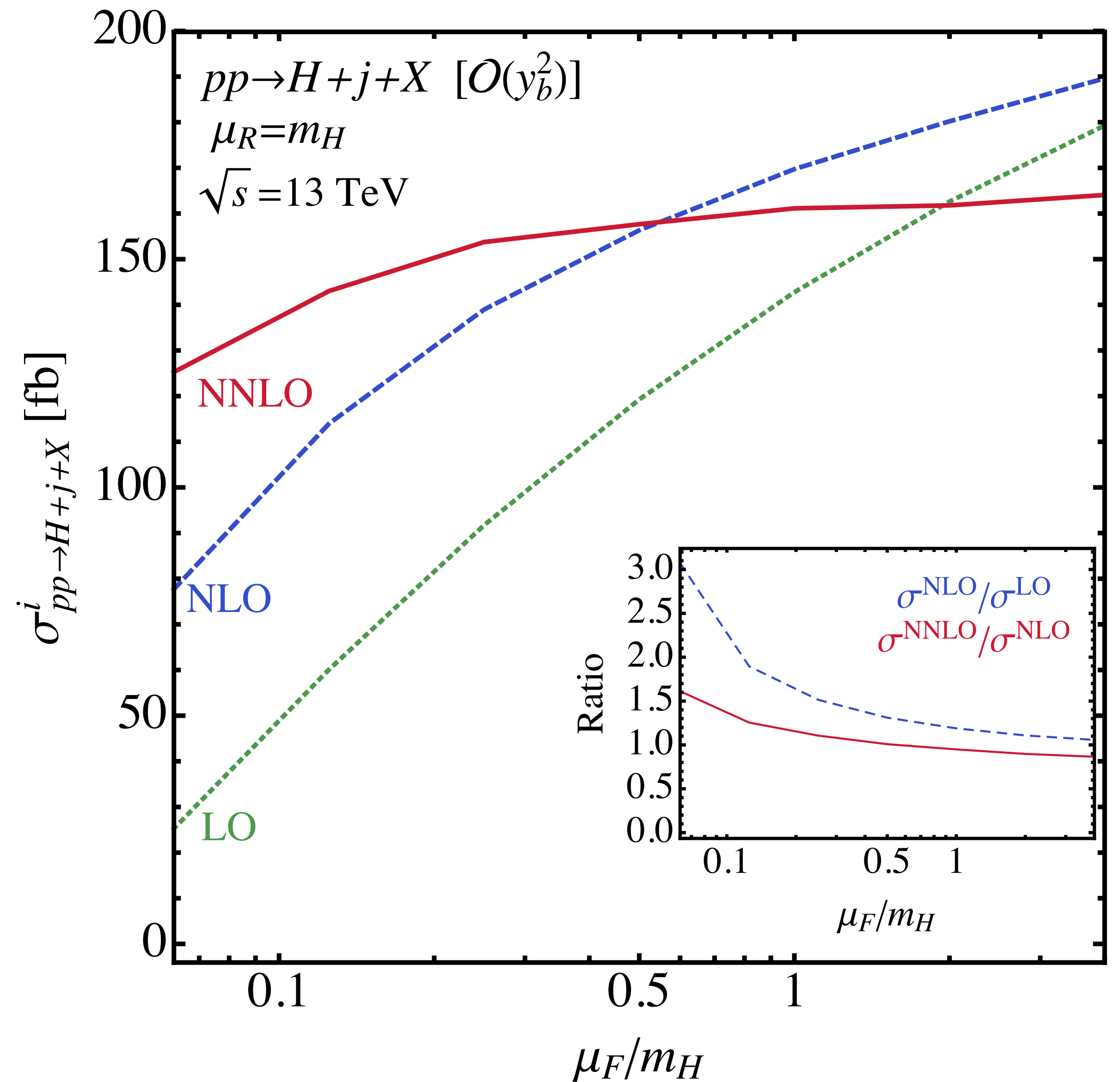
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# $\mu_F$ - dependence

We begin by looking at the factorization scale dependence.

At LO and NLO there is a striking dependence. A known issue of the 5FS calculations.

Our NNLO results significantly “flatten the curve” and are essentially independent of the scale for a large region around  $m_H/2$

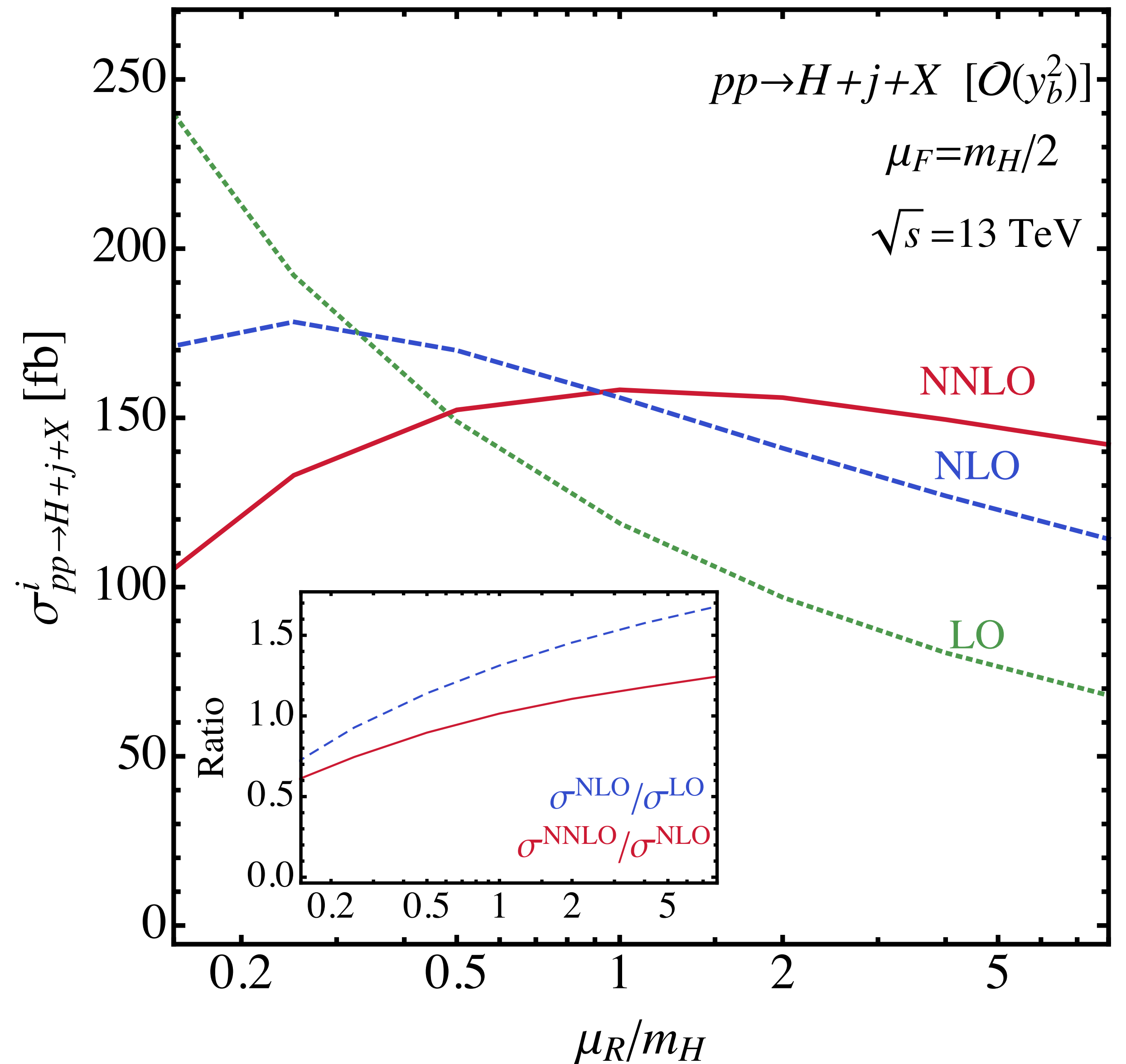


# $\mu_R$ - dependence

The renormalization scale dependence comes from the running of the strong and Yukawa couplings.

For scale choices in the region around the Higgs mass the cross section dependence is very mild at NNLO

These combined results suggest a central scale choice of  $(\mu_F, \mu_R) = (1/2, 1) \times m_H$  leads to perturbative stability.

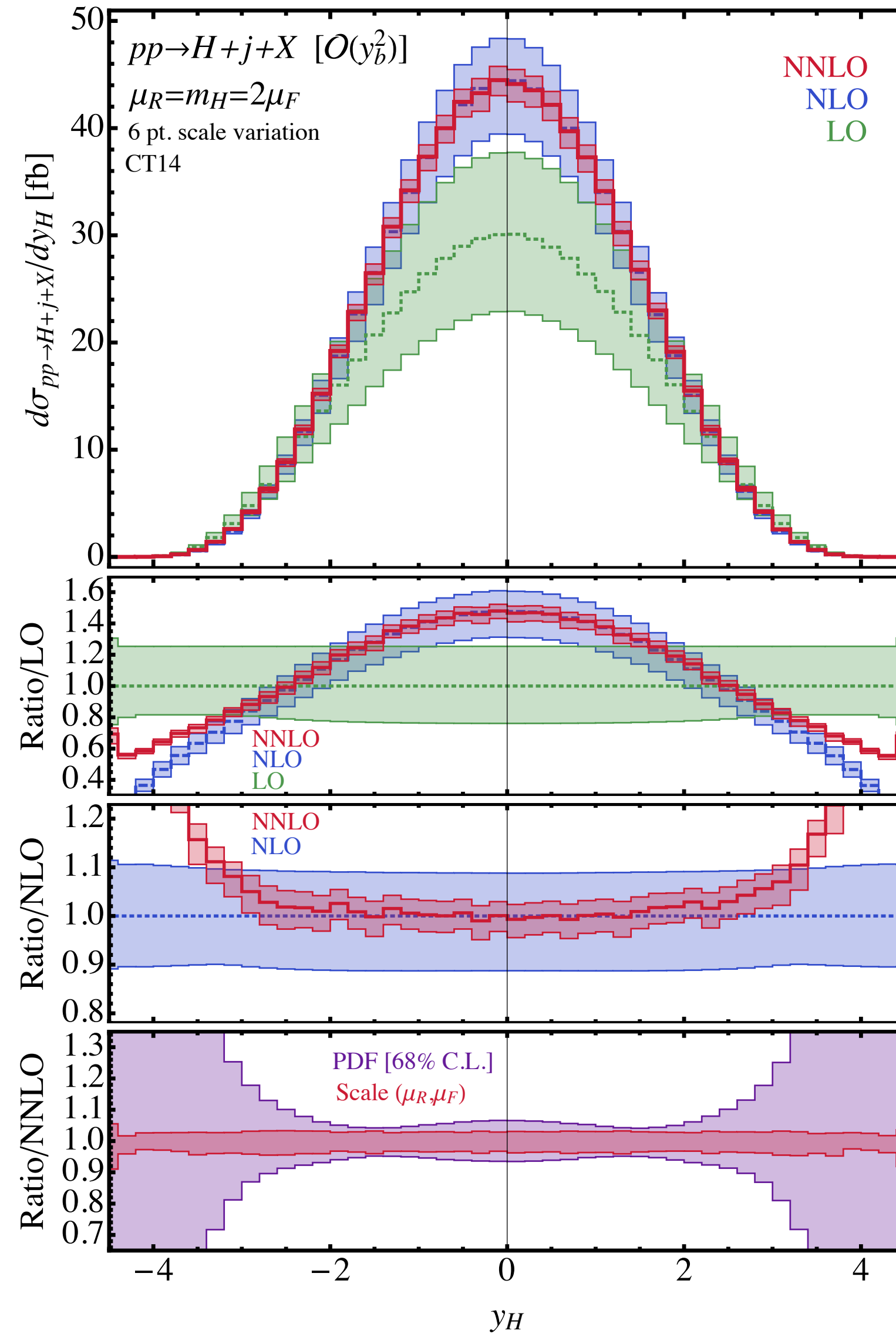


# Distributions

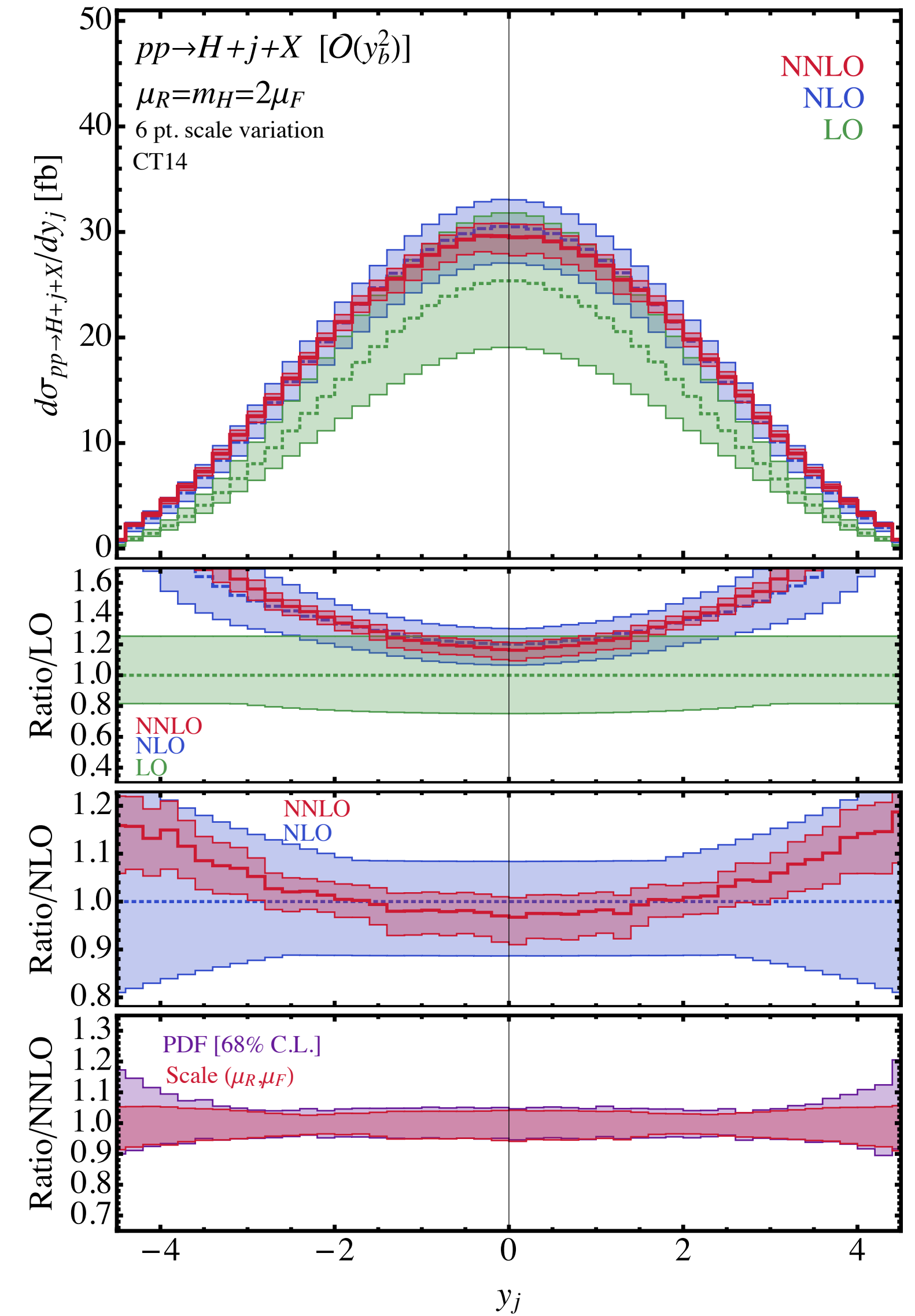
For this scale choice we plot the rapidity distributions for the Higgs boson and the leading jet

The Higgs has reasonably flat corrections, while the jet has more structure.

PDF errors comparable to 7-point scale var.



HIGGS



JET

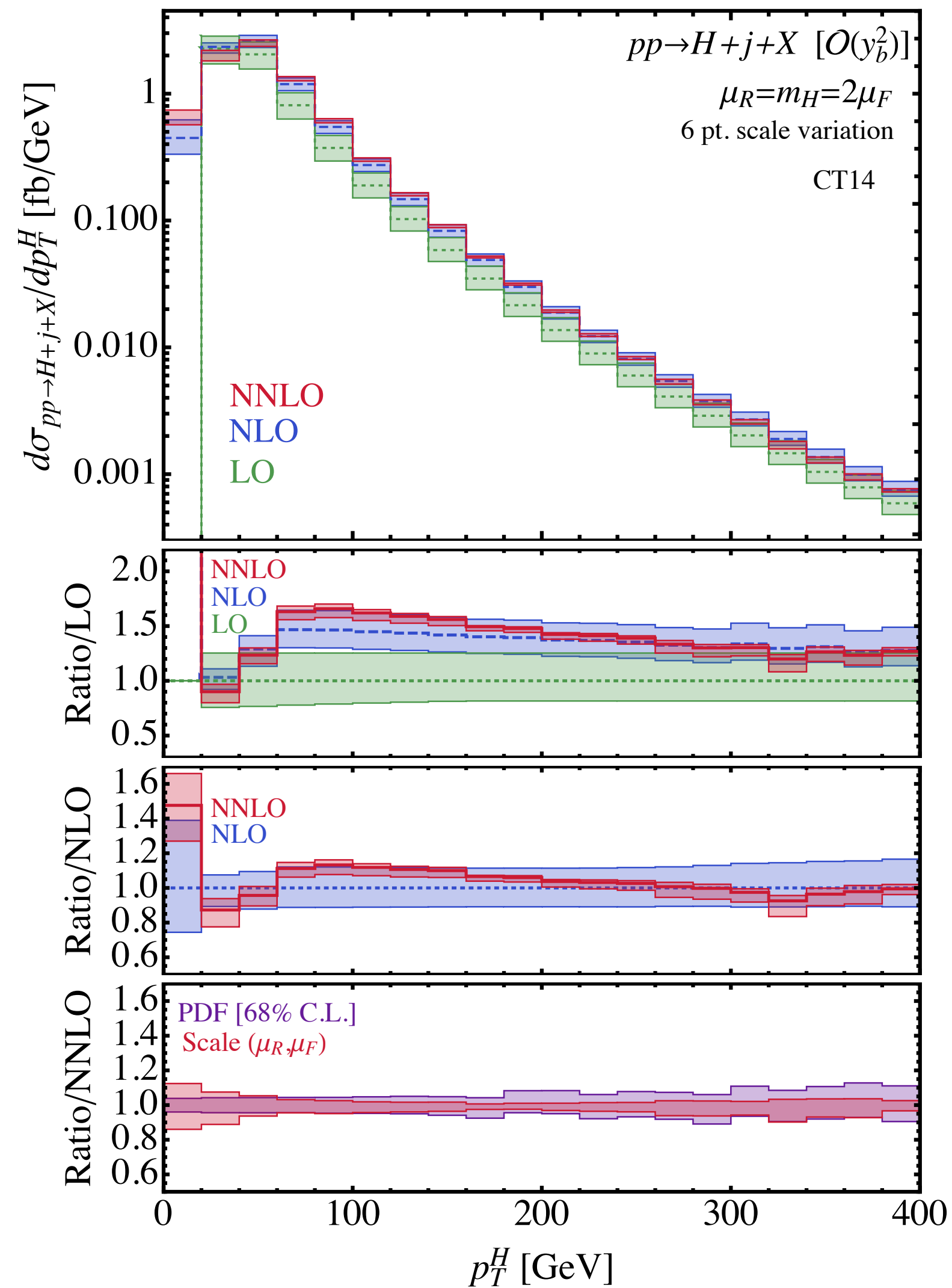


# Distributions

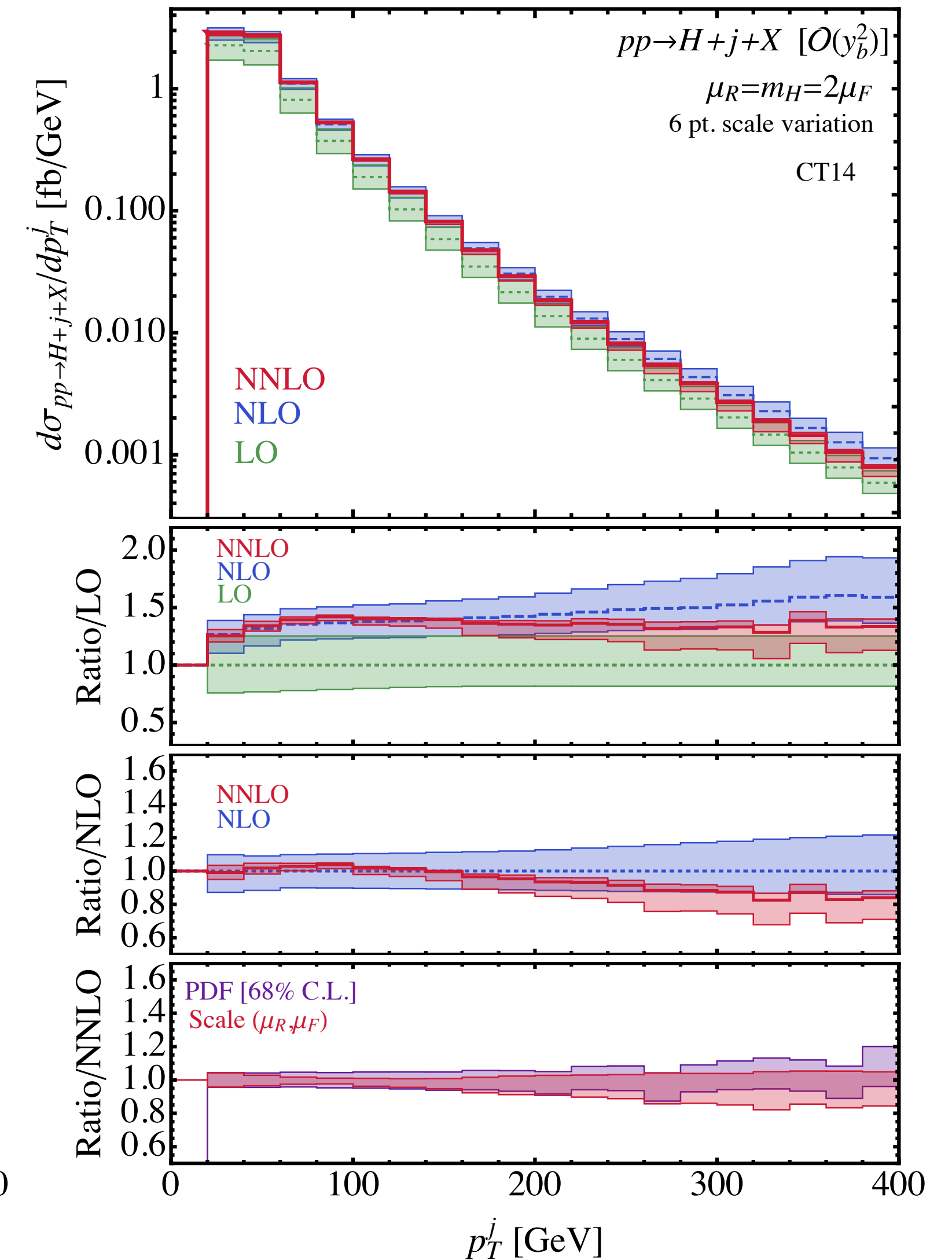
Next we look at transverse momentum

Both have corrections around 10% to NLO, tails soften compared to NLO.

Again PDF errors comparable to 7-point scale var.



HIGGS



JET

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# Future Directions

- Inclusion of the EFT NNLO calculation to make combined prediction for  $H+j$
  - Inclusion of interference effects
  - Implementation of b-tagging to target  $H+b$  final states.
  - Application to SUSY theories (specifically those with minimal coupling modifications or additional heavy scalars)
  - Application to Dark Matter/BSM searches for scalar mediators in models with Minimal Flavor Violation.
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# Conclusions

- Studying the Higgs coupling to fermions is an area of experimental and theoretical interest.
  - NNLO calculations are maturing , although final state jet processes at NNLO still require significant computational cost (and user skill/experience) making public releases somewhat tricky.
  - Presented results today for  $H+j$  through bottom quark fusion at NNLO. Complementary study to dominant  $H+j$  EFT production.
  - In the 5FS the factorization scale dependence is pretty sizable at lower orders.
  - NNLO corrections are of the order of 5-10% and result in a significant reduction in residual factorization and renormalization scale dependence.
-