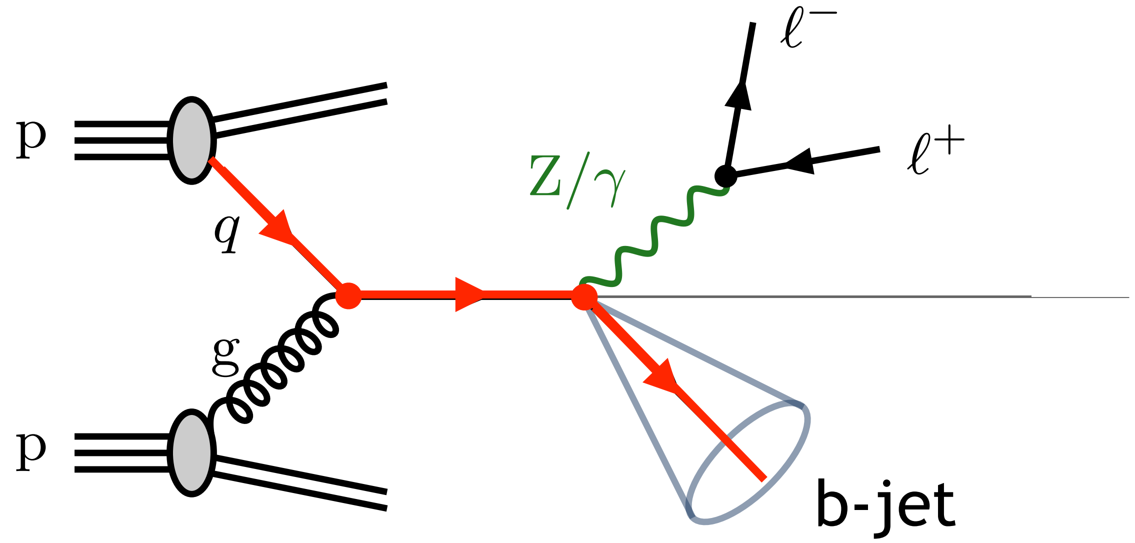


Z boson production with a b-jet at $\mathcal{O}(\alpha_s^3)$

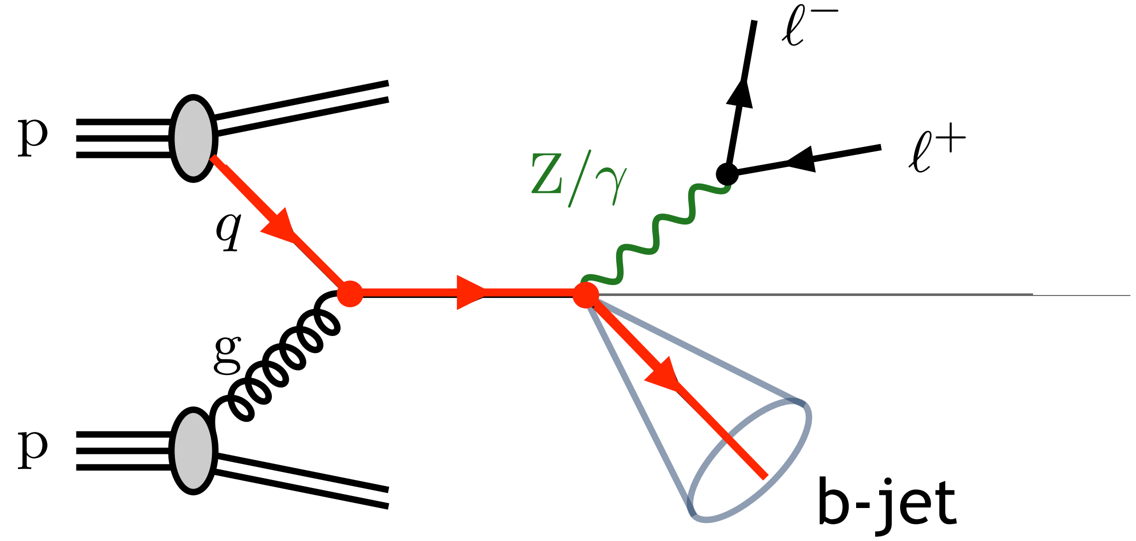
Rhorry Gauld

Radcor & LoopFest 2021: Parallel III.A
May 18th

NWO Netherlands Organisation for Scientific Research

Nikhef

Z boson production with a b-jet at $\mathcal{O}(\alpha_s^3)$



***RG**, A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, I. Majer* https://arxiv.org/abs/2005.03016
PRL 125 (2020) 22, 222002

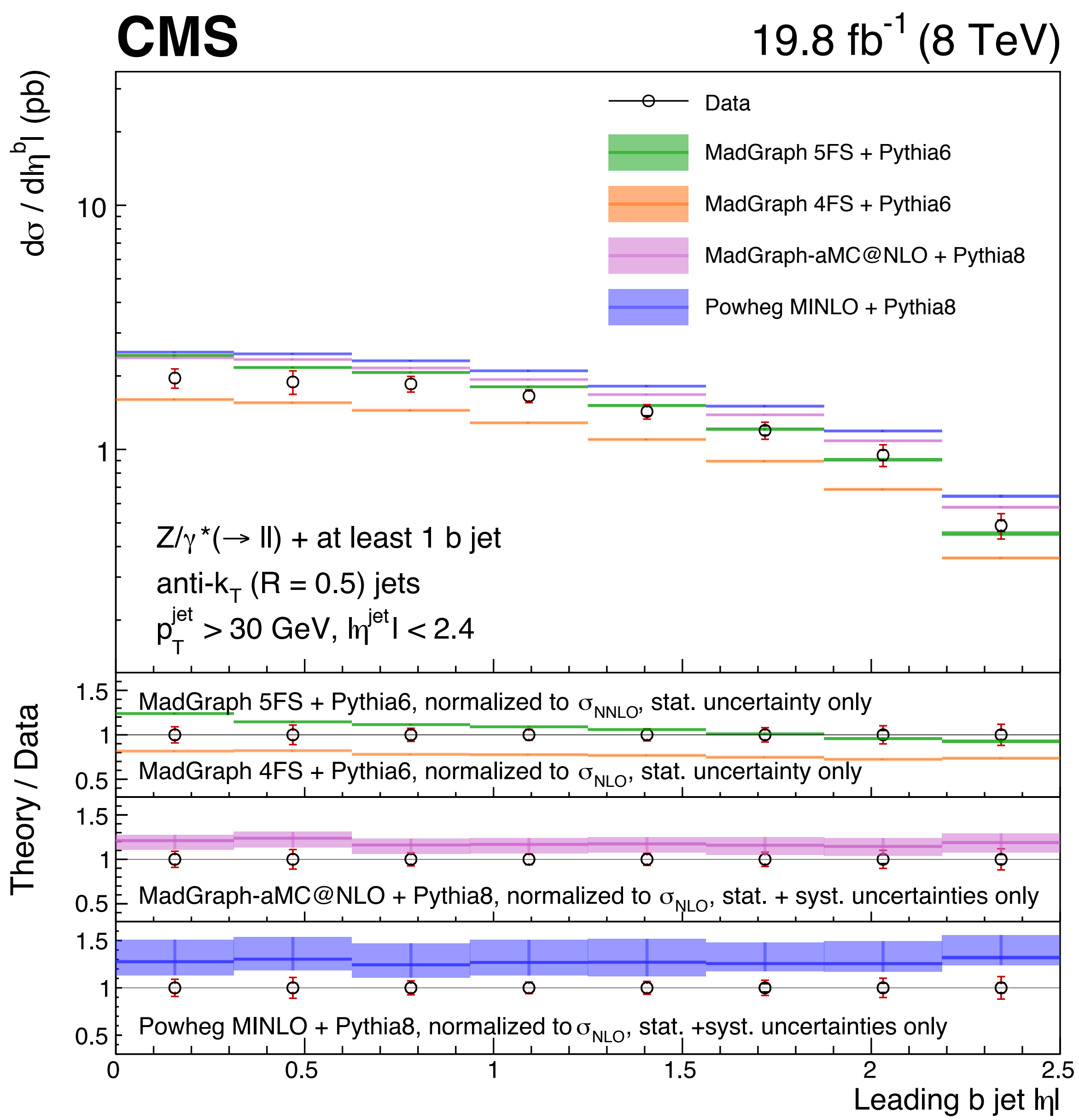
Physics relevance/potential:

(i) Study of processes involving flavoured jets (test theory framework)

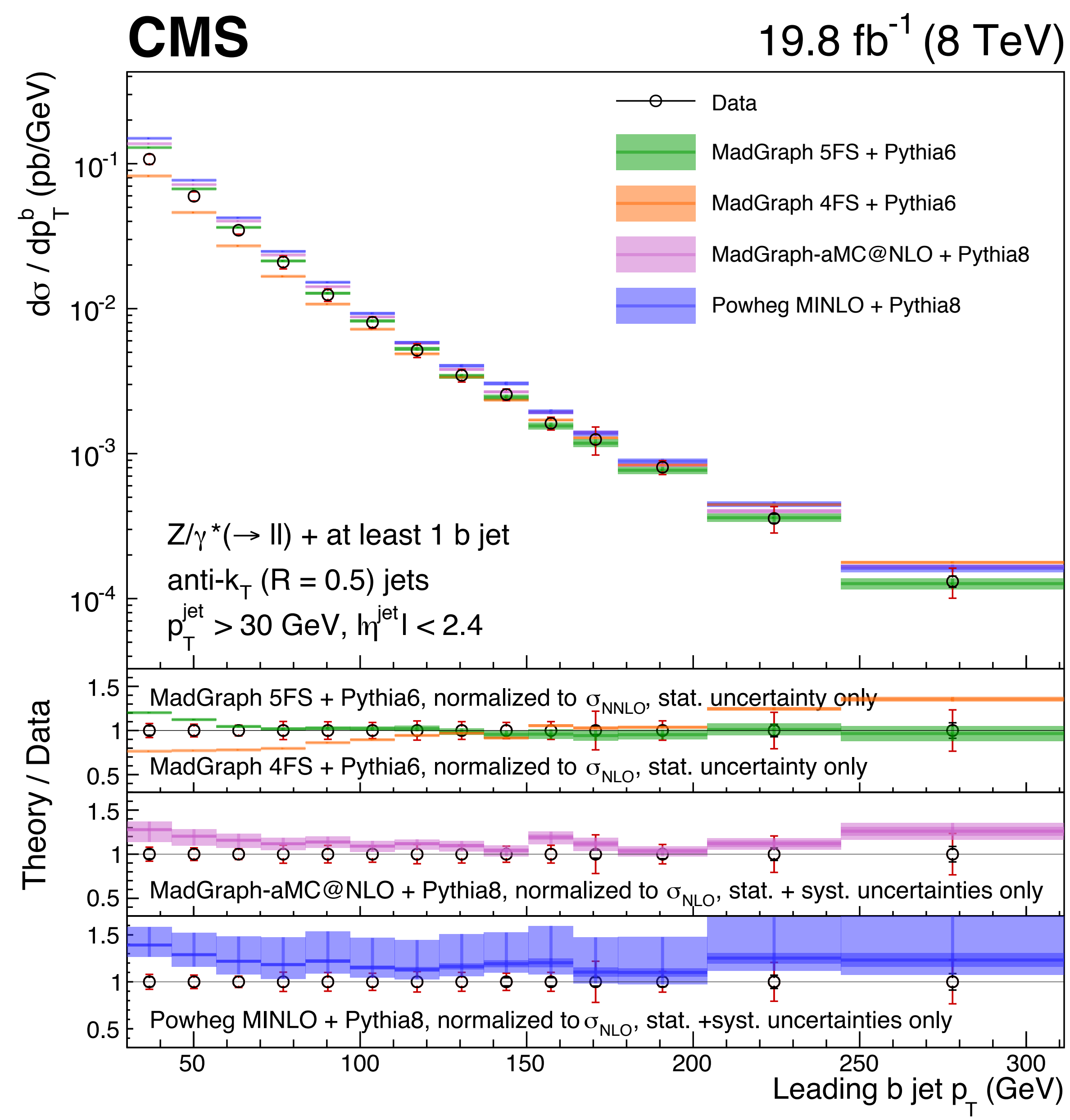
(ii) Backgrounds for Higgs measurements, direct searches (Dark Matter)

(iii) Probing the flavour content of the proton

Theory motivation



η_b



$p_{T,b}$

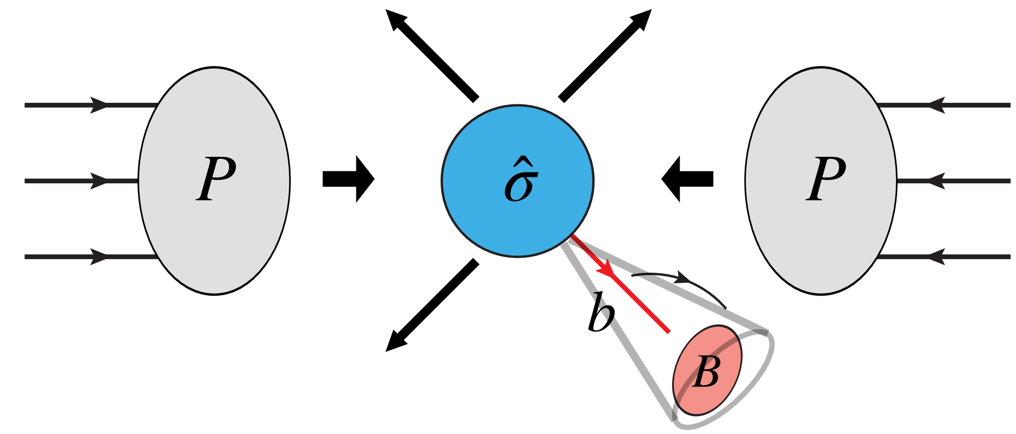
CMS Run I (8 TeV) measurement of $pp \to Z + b - \text{jet(s)}$ https://arxiv.org/abs/1611.06507

Eur. Phys. J. C 77, 11 (2017) 751

Overview of this talk

Anatomy of heavy-flavour processes

- General structure of computation
- The b-quark PDF

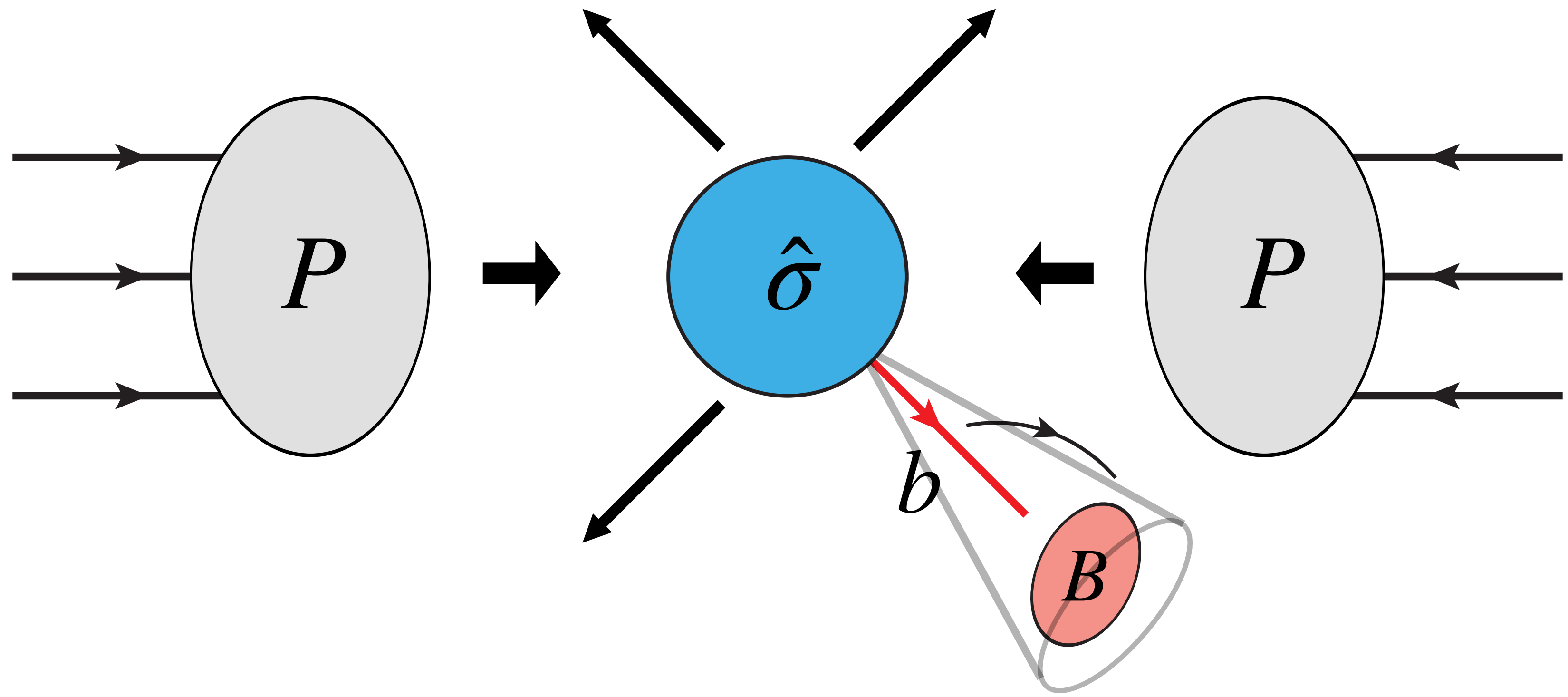


Flavoured jets & perturbative computations

- A flavoured jet algorithm
- NNLO QCD predictions (with m_b effects)

Comparison to CMS data, and conclusions

Anatomy of heavy-flavour processes



Factorisation theorem

$$d\sigma_{pp\to X} = \sum_{i,j} \int dx_1 dx_2\, f_i(x_1)\, f_j(x_2)\, d\hat{\sigma}_{ij\to\hat{X}}(\hat{s},\ldots)\, T(\hat{X}\to X)$$

Partonic cross-section

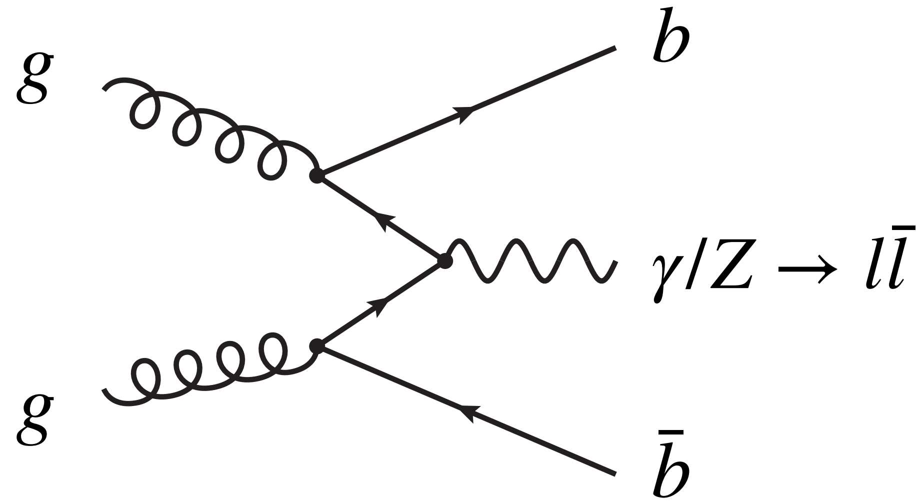
$$d\hat{\sigma}_{ij\to\hat{X}} = d\hat{\sigma}^{\mathrm{LO}}_{ij\to\hat{X}} + \alpha_s\, d\hat{\sigma}^{\mathrm{NLO}}_{ij\to\hat{X}} + \alpha_s^2\, d\hat{\sigma}^{\mathrm{NNLO}}_{ij\to\hat{X}} + \ldots$$

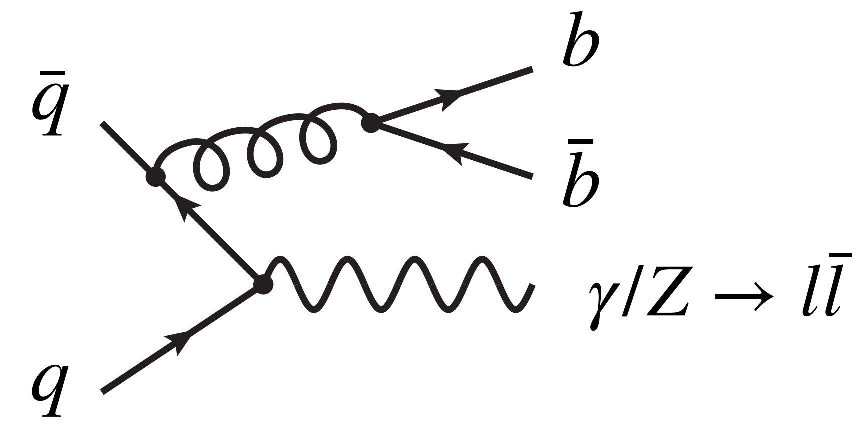
At fixed-order, define parton-level jets [corrected with $T(\hat{X}\to X)$]

[from NLO+PS for example]

$pp \to Z + b - \text{jet} + \ldots$

Computed in a scheme: e.g. "4fs" aka ZM-VFNS $(n_f^{\max} = 4)$

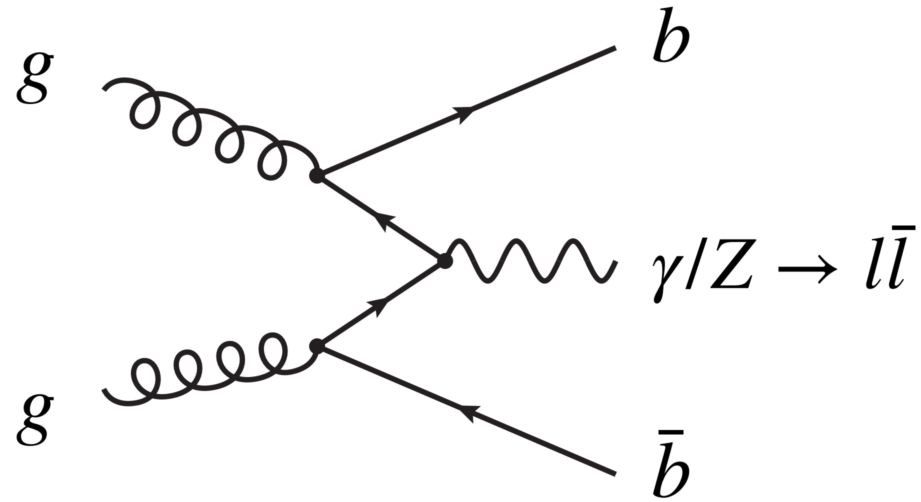


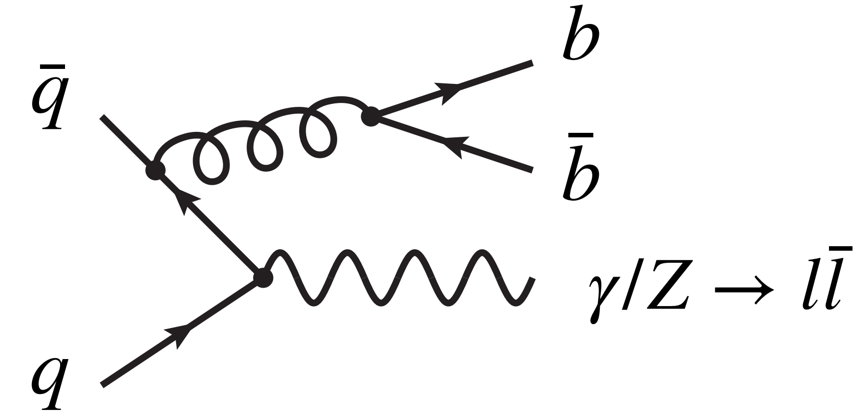


LO computation in 4fs $\mathcal{O}(\alpha_s^2)$, $\mathrm{d}\hat{\sigma}_{ij}^{\mathrm{LO}} \sim \int \mathrm{d}\phi_{\hat{X}} \, |M_{ij\to\hat{X}}|^2$

$pp \to Z + b - \text{jet} + \ldots$

Computed in a scheme: e.g. "4fs" aka ZM-VFNS ($n_f^{\max} = 4$)



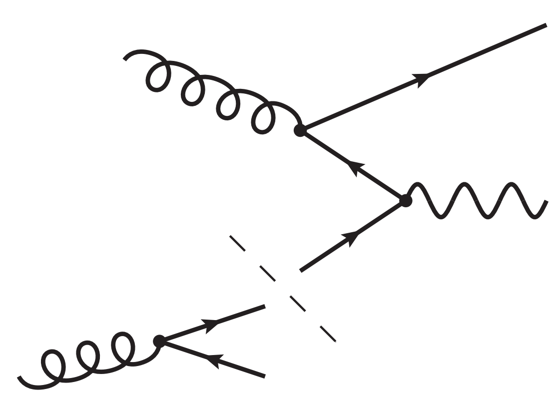


LO computation in 4fs $\mathcal{O}(\alpha_s^2)$, $\mathrm{d}\hat{\sigma}_{ij}^{\mathrm{LO}} \sim \int \mathrm{d}\phi_{\hat{X}} \, |M_{ij \to \hat{X}}|^2$

$$\mathrm{d}\sigma^{4fs} = \mathrm{d}\sigma^{m_b=0} \quad + \quad \mathrm{d}\sigma^{\ln[m_b]} \quad + \quad \mathrm{d}\sigma^{m_b}$$

Massless component

$\mathcal{O}(\alpha_s^2 n_f)$ in 5fs

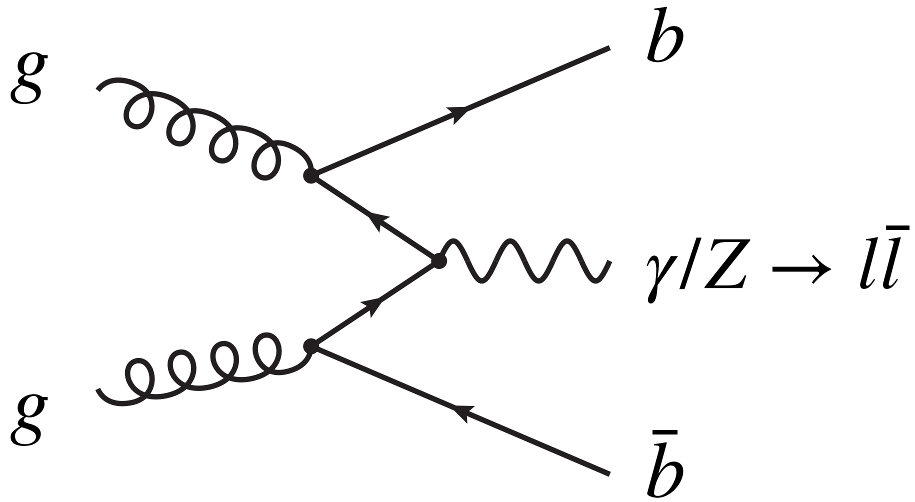


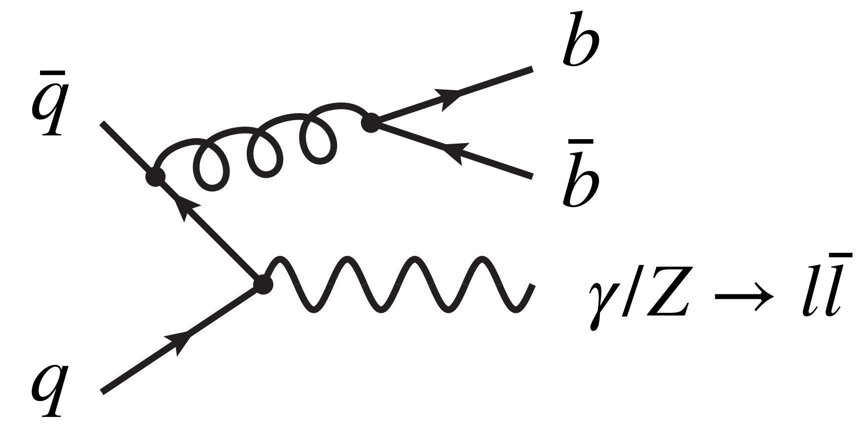
$\mathcal{O}(m_b^2)$ effects

exact kinematics

$pp \to Z + b - \text{jet} + \ldots$

Computed in a scheme: e.g. "4fs" aka ZM-VFNS ($n_f^{\max} = 4$)





LO computation in 4fs $\mathcal{O}(\alpha_s^2)$, $\mathrm{d}\hat{\sigma}_{ij}^{\mathrm{LO}} \sim \int \mathrm{d}\phi_{\hat{X}} \, |M_{ij \to \hat{X}}|^2$

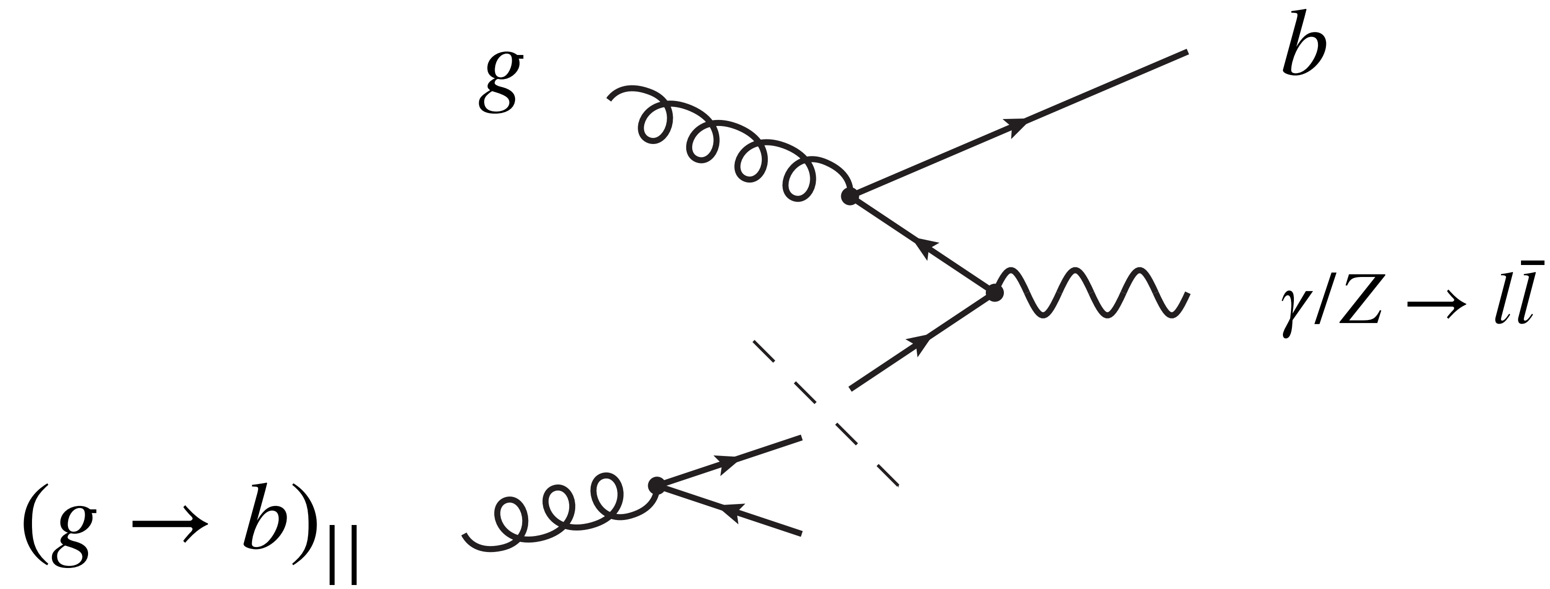
$$\mathrm{d}\sigma^{4fs} = \mathrm{d}\sigma^{m_b=0} \quad + \quad \mathrm{d}\sigma^{\ln[m_b]} \quad + \quad \mathrm{d}\sigma^{m_b}$$

[pb] 1.4	-0.45	1.9	-0.05
100%	-32%	135%	-3%

The b-quark PDF

Clearly, we wish to understand the logarithmic part (it is the largest)

$$\mathrm{d}\sigma^{\ln[m_b]} \sim$$

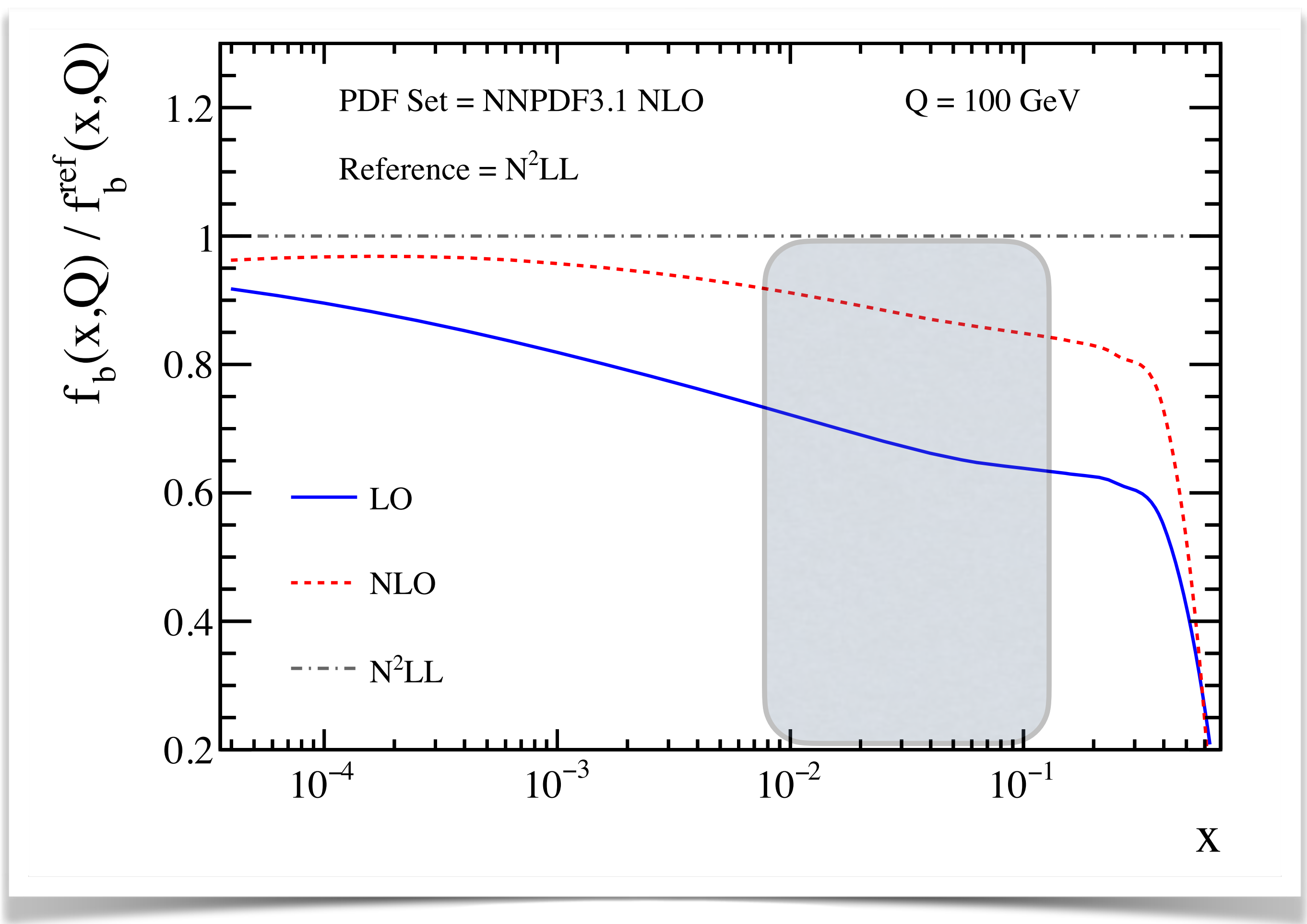


$$\mathrm{d}\sigma^{\ln[m_b]} \sim f_g(x_1, \mu_F^2) \otimes f_b^{(1)}(x_2, \mu_F^2) \otimes \mathrm{d}\hat{\sigma}^{m_b=0}_{bg\to Zb}$$

The b-quark PDF

Clearly, we wish to understand the logarithmic part (it is the largest)

Actually, we know it well (use the renormalisation group, "PDF evolution")



I am showing fixed-order pdf versus a resummed one (PDF evolution)

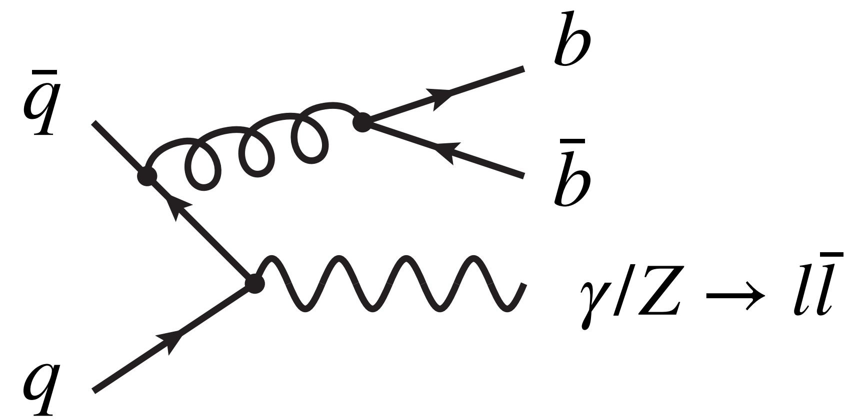
$\alpha_s^m \ln^n[\mu_F^2/m_b^2], \quad m \geq n$

Note! $\alpha_s \ln[m_Z^2/m_b^2] \approx 0.7$

“Choice” of jet algorithm

What happens if we apply anti-k_T alg. as in an experimental set-up

Collinear safety



“Choice” of jet algorithm

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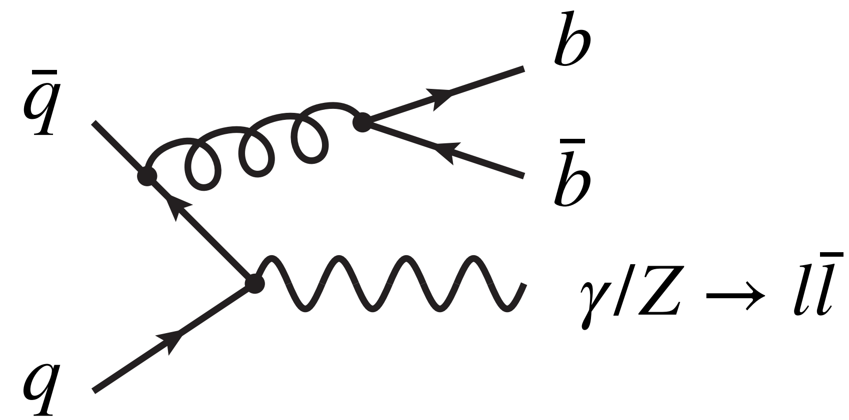
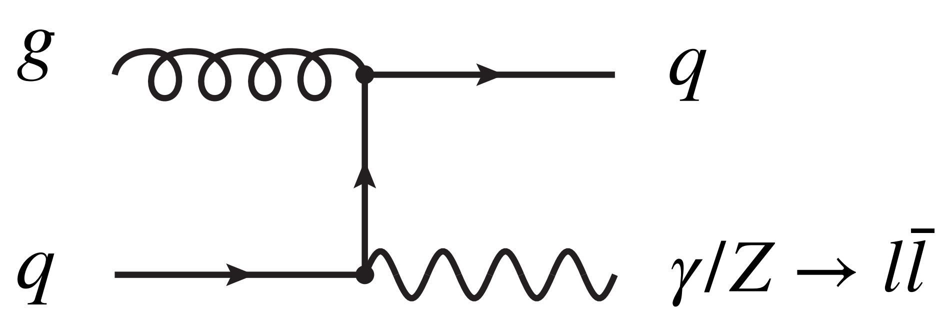


Soft safety



“Choice” of jet algorithm

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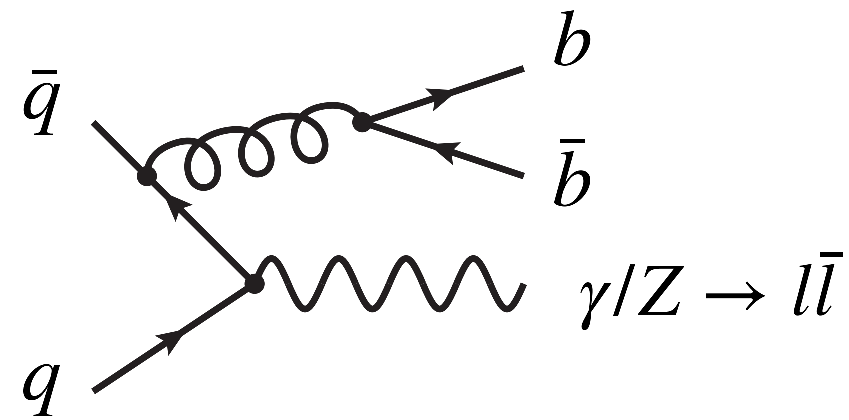
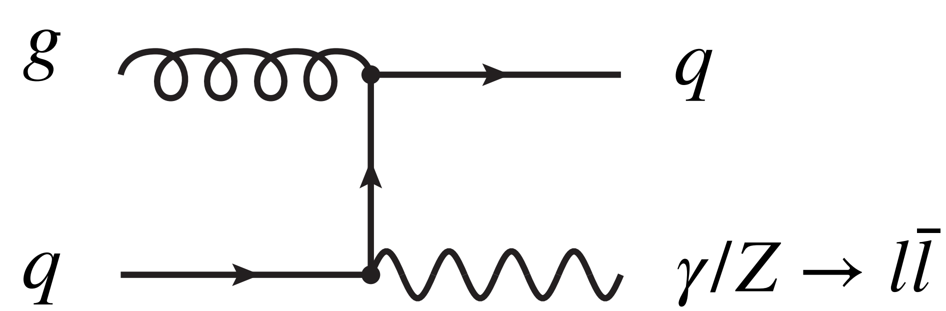


Soft safety



Massless(5fs): infinite

Massive(4fs): finite, but contains large corrections like

$$C \propto \alpha_s \ln[Q^2/m_b^2]$$

$$S \propto \alpha_s^2 \ln^2[Q^2/m_b^2]$$

The flavour-k_T algorithm

We want to use resummed b-pdf, and avoid large "C, S" corrections

Theoretical Physics | Published: 19 May 2006

Infrared-safe definition of jet flavour

A. Banfi, G.P. Salam & G. Zanderighi

The European Physical Journal C - Particles and Fields **47**, 113–124(2006) | Cite this article

(1) Quantum flavour assignment:

$$b = +1, \; \bar{b} = -1$$

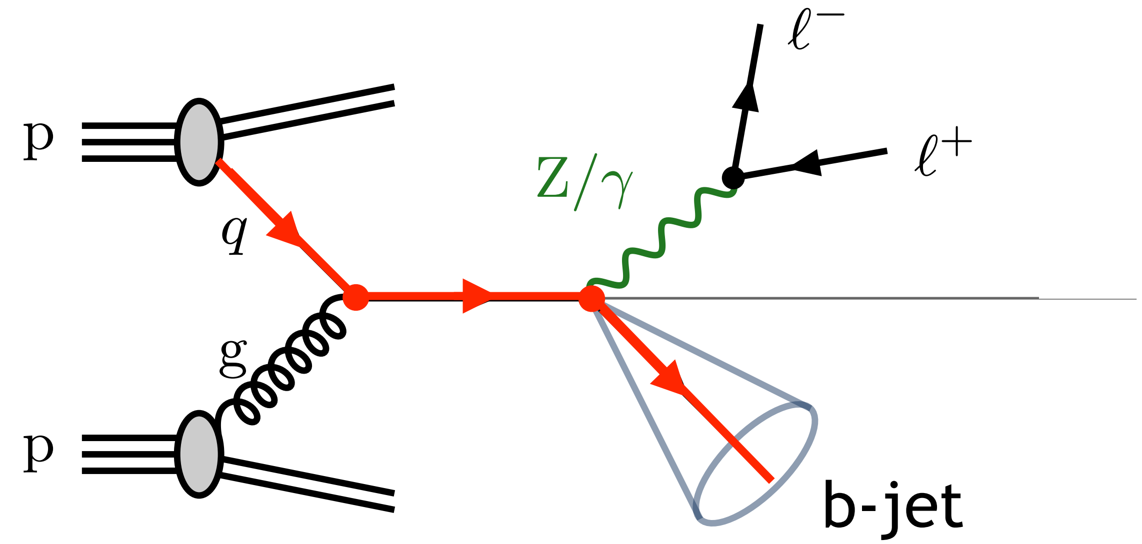
(2) Flavour specific clustering

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha} & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}, k_{tj})^\alpha & \text{softer of } i, j \text{ is unflavoured} \end{cases}$$

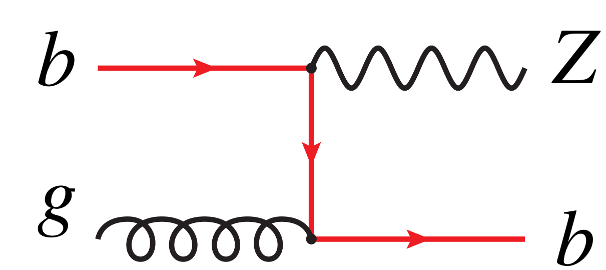
Note! anti-k_T clustering not well suited (preference of soft-hard clustering)

The massless calculation

$$d\hat{\sigma}_{ij\to\hat{X}} = d\hat{\sigma}^{\mathrm{LO}}_{ij\to\hat{X}} + \alpha_s\, d\hat{\sigma}^{\mathrm{NLO}}_{ij\to\hat{X}} + \alpha_s^2\, d\hat{\sigma}^{\mathrm{NNLO}}_{ij\to\hat{X}} + \ldots$$



At LO: 4 distinct channels, $d\hat{\sigma}^{\mathrm{LO}}_{ij} \sim \int d\phi_{\hat{X}}\, |M_{ij\to\hat{X}}|^2$



At NNLO: 1000 distinct channels (amplitudes also become complicated)

Use a "flavoured dressed" version of the computation of $Z + 1j$ @ NNLO

Gehrmann-De Ridder et al., https://arxiv.org/abs/1507.02850

PRL 117, 022001 (2016)

The massless calculation

$$\mathrm{d}\hat{\sigma}_{ij\to\hat{X}} = \mathrm{d}\hat{\sigma}^{\mathrm{LO}}_{ij\to\hat{X}} + \alpha_s\,\mathrm{d}\hat{\sigma}^{\mathrm{NLO}}_{ij\to\hat{X}} + \alpha_s^2\,\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{ij\to\hat{X}} + \ldots$$

Theory collaboration between CERN, Durham, Lisbon, Nikhef, Zurich

A (parton level) Monte Carlo generator, antenna subtraction formalism

(Gehrmann et al. 2005-2013)

My collaborators for this work, i.e. $pp \to Z + \mathrm{b-jet}$, *PRL* 125 (2020) 22, 222002

RG

A. Gehrmann-De Ridder

E.W.N. Glover

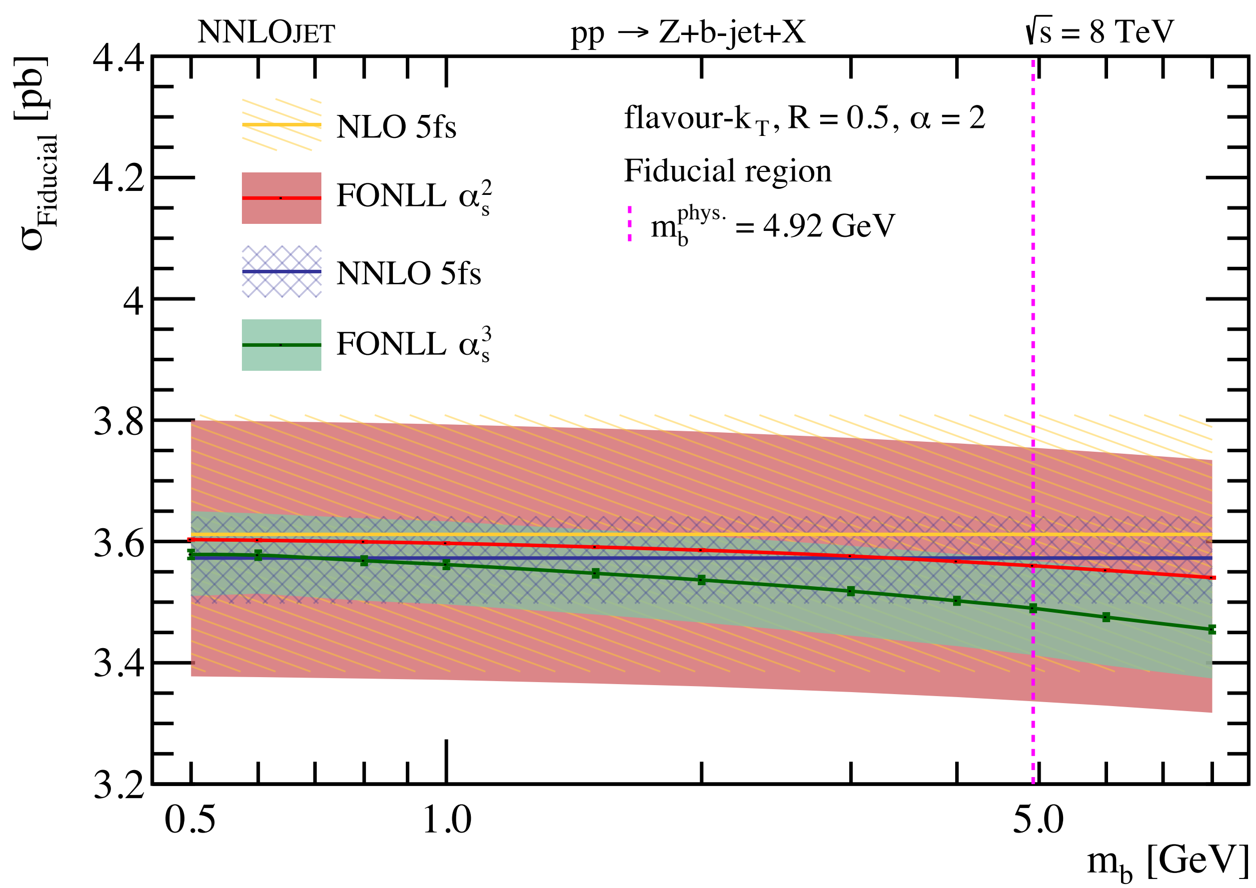
A. Huss

I. Maier

Including mass corrections

Construct a massive variable flavour number scheme (M-VFNS)

$$d\sigma^{\mathrm{M-VFNS}} = d\sigma^{5fs} + \left(d\sigma^{4fs} - d\sigma^{4fs}_{m_b \to 0} \right)$$



$$d\sigma^{4fs} = d\sigma^{m_b=0} + d\sigma^{\ln[m_b]} + \left(d\sigma^{m_b} \right) \approx -3\,\%$$

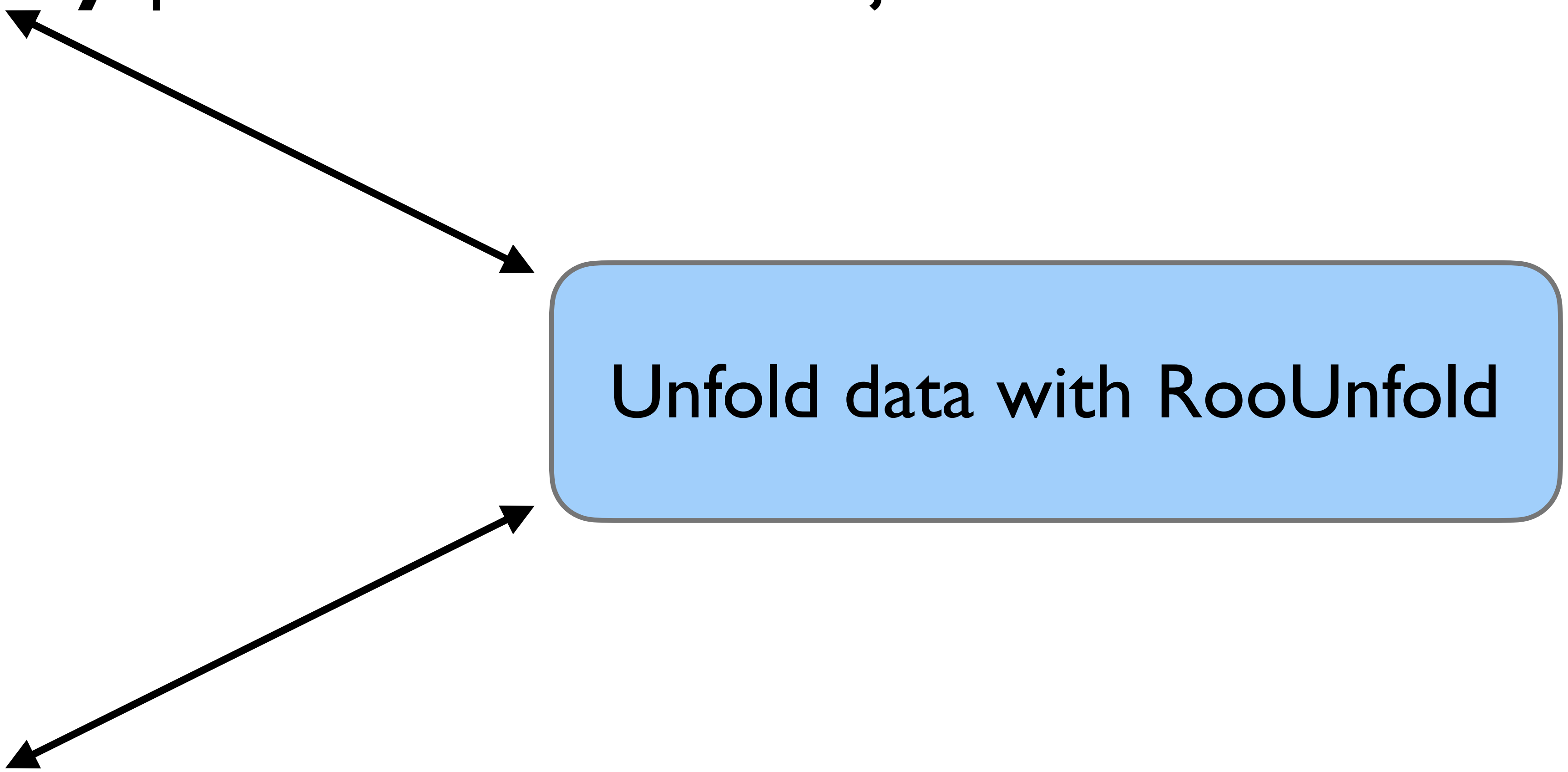
aMC@NLO — dedicated computation

*M-VFNS = FONLL

Comparison to CMS data

$$d\sigma_{pp\to X} = \sum_{i,j} \int dx_1 dx_2\, f_i(x_1)\, f_j(x_2)\, d\hat{\sigma}_{ij\to\hat{X}}(\hat{s},\ldots)\, T(\hat{X}\to X)$$

Theory: parton-level flavour-k_T jets



This includes:

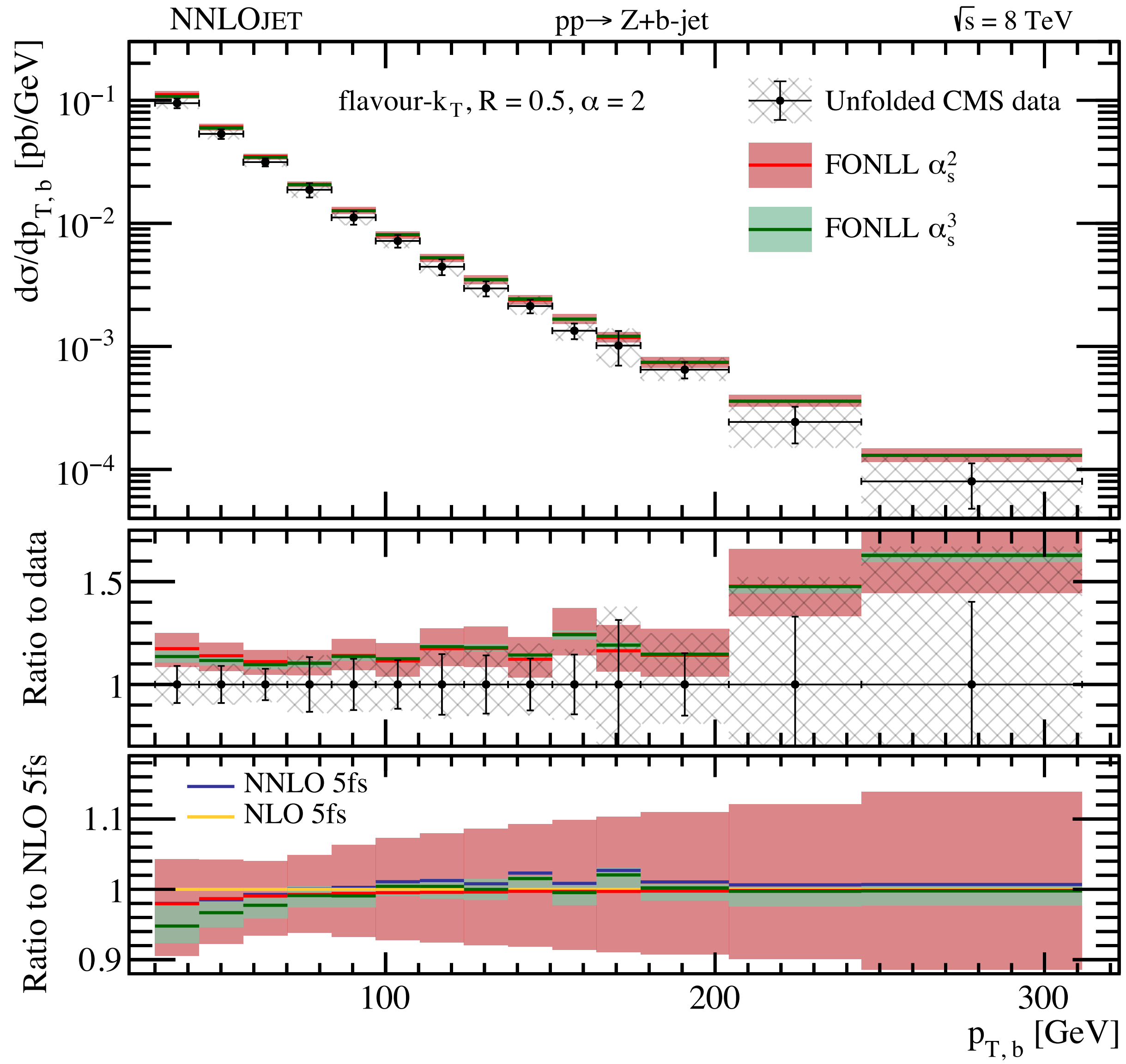
$T(\hat{X}\to X)$

Data: hadron-level anti-k_T jets

Model: NLO+PS, aMC@NLO+Pythia8

Main effect: to remove ‘double-tagged’ jets. i.e. a pseudo-collinear $g\to b\bar{b}$

Comparison to CMS data



$p_{T,b}$

m_b $\mathcal{O}(\alpha_s^3)$

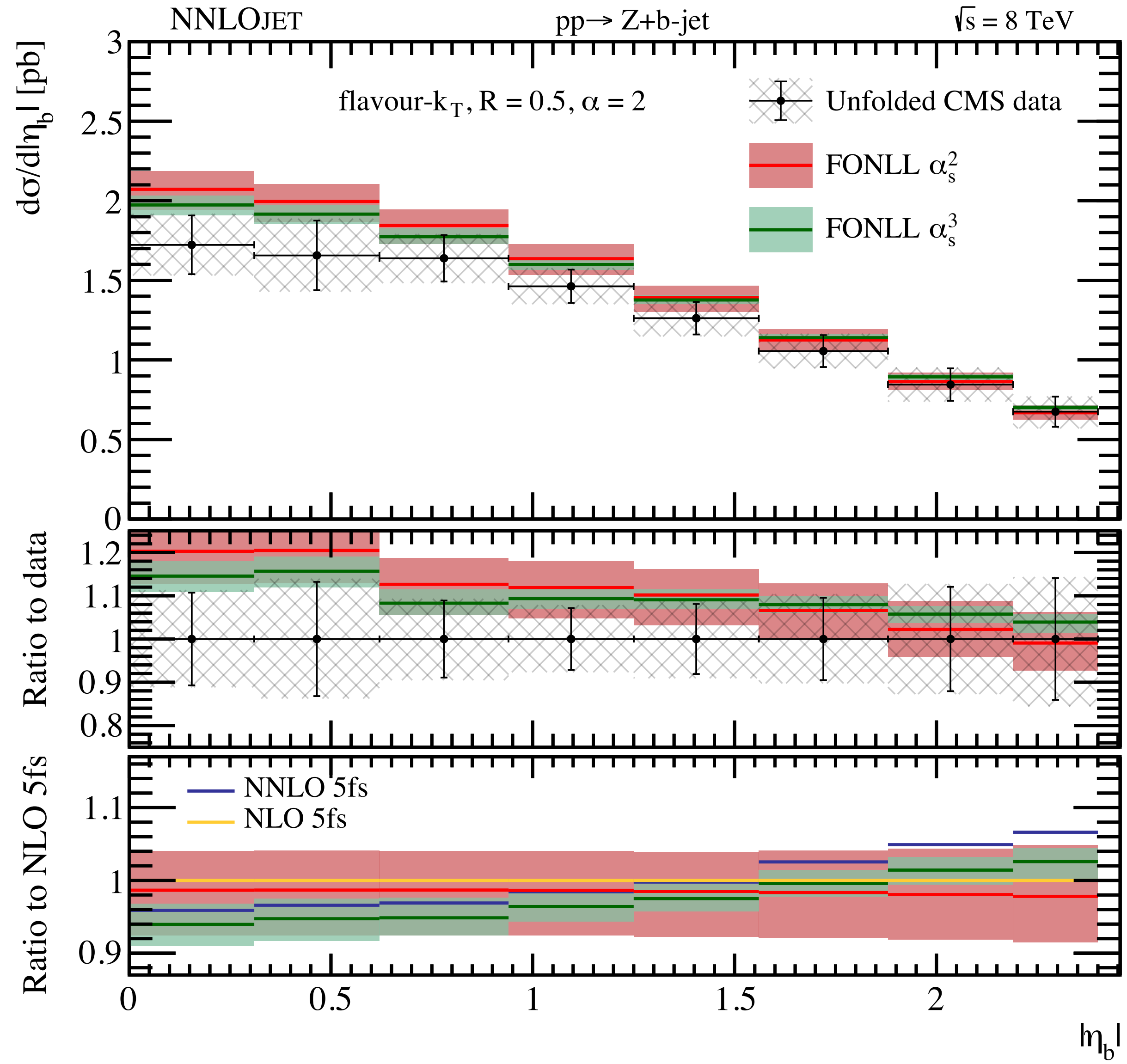
χ^2/N_{dat} 1.9 → 1.7 → 1.5

NLO (5fs)

$$\chi^2 = \sum_{\text{bins}} \frac{(\sigma_{\text{theory}} - \sigma_{\text{data}})^2}{\delta\sigma_{\text{data}}^2}$$

Note: used smaller uncertainty in χ^2

Comparison to CMS data



η_b

m_b $\mathcal{O}(\alpha_s^3)$

χ^2/N_{dat} 2.0 → 1.6 → 1.0

NLO (5fs)

$$\chi^2 = \sum_{\text{bins}} \frac{(\sigma_{\text{theory}} - \sigma_{\text{data}})^2}{\delta\sigma_{\text{data}}^2}$$

Note: used smaller uncertainty in χ^2

Conclusions

Calculational setup:

- Resummation of dominant ISR logarithmic corrections
- Removal of troublesome FSR logarithmic corrections (jet algorithm)
- Inclusion of the finite b-quark mass corrections
- Percent level uncertainties (2-3)% + accurate!!!

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The jet algorithm:

- Tackle inconsistency between data and theory with unfolding
- Motivates further study into this issue (exp. and th. side)

Conclusions

Calculational setup:

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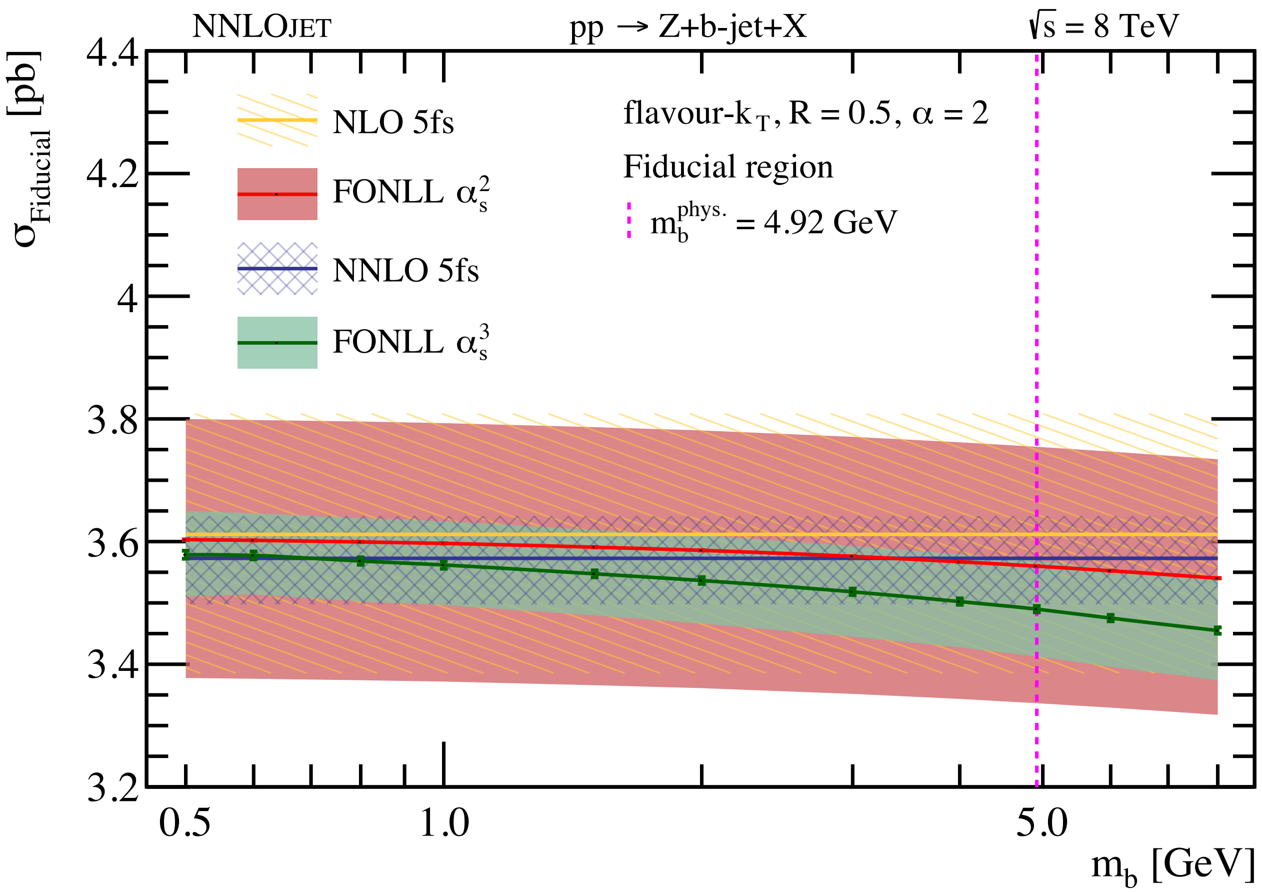
- Inclusion of the finite b-quark mass corrections
- Percent level uncertainties (2-3)% + accurate!!!

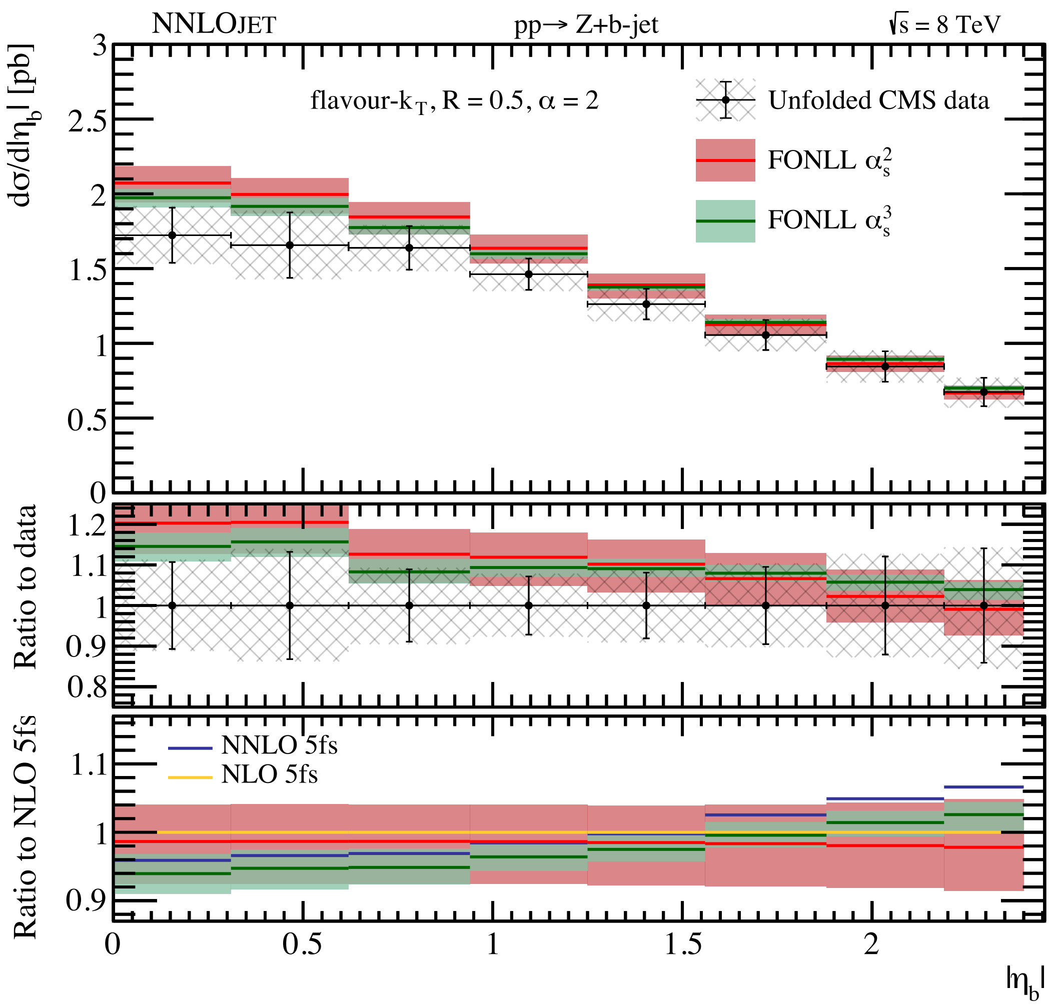
The jet algorithm:

- Tackle inconsistency between data and theory with unfolding
- Motivates further study into this issue (exp. and th. side)

Future applications:

- Precise signal <> control region corrections (searches, etc.)
- Scaling of MC (reduce theory uncertainty of backgrounds)
- Ideally, direct comparison to data (PDF analyses etc.)
- Interface to Parton Shower





Much obliged! Questions?

Whiteboard

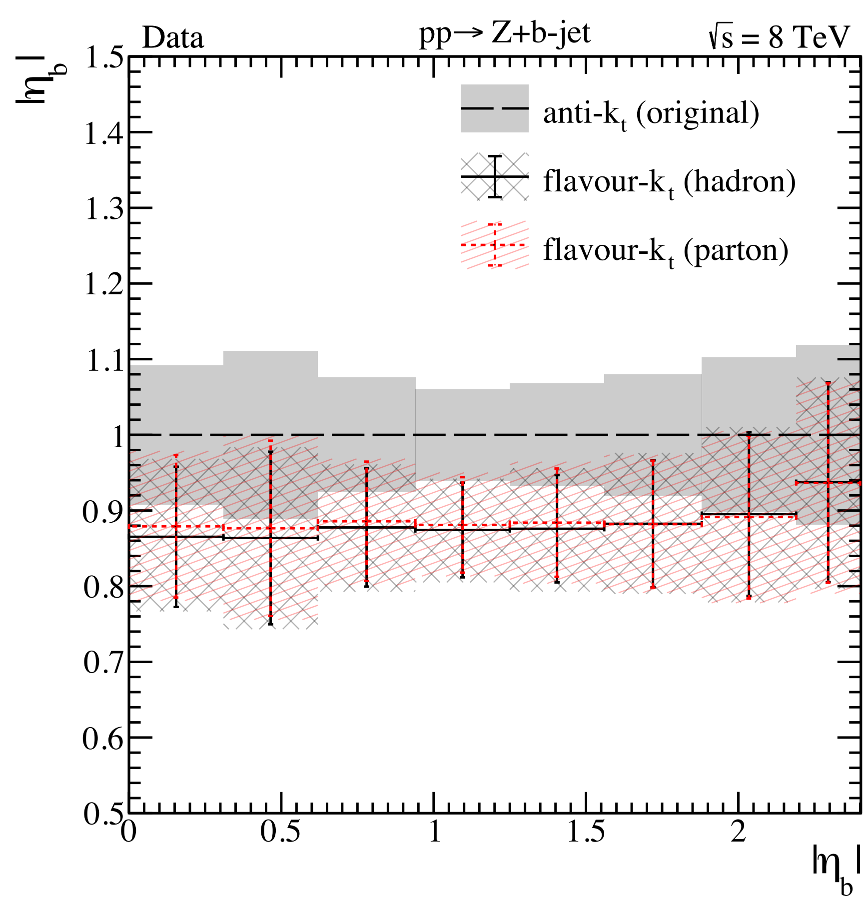
Whiteboard

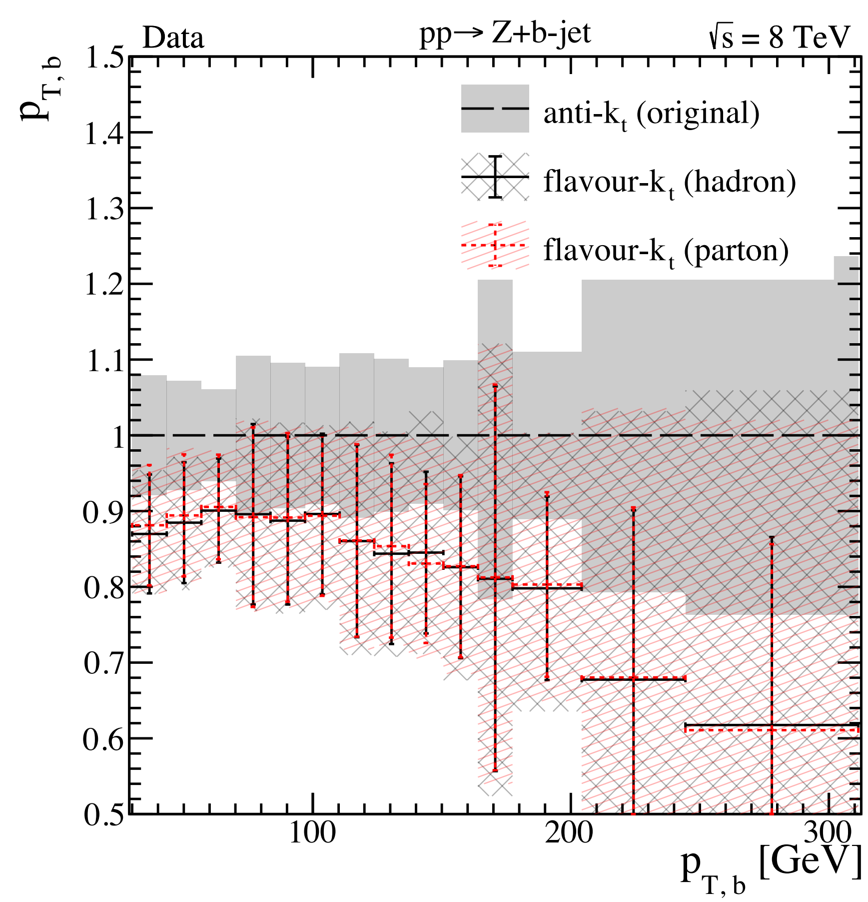
Size of unfolding correction

How to account for theory-experiment mismatch?

Use an NLO + Parton Shower prediction (which can evaluate both)
1) Prediction at parton-level, flavour-k_T algorithm **(Theory)**
2) Prediction at hadron-level, anti-k_T algorithm **(Experiment)**

Calculate an "Unfolding" correction from 2) Experiment → 1) Theory





We use RooUnfold (following the procedure used in the exp. analyses)

NNLO flavoured jet cross-sections

$$\mathrm{d}\hat{\sigma}_{ij,\mathrm{NLO}} = \int_{n+1} \left[\mathrm{d}\hat{\sigma}^{R}_{ij,\mathrm{NLO}} - \mathrm{d}\hat{\sigma}^{S}_{ij,\mathrm{NLO}}\right] + \int_{n} \left[\mathrm{d}\hat{\sigma}^{V}_{ij,\mathrm{NLO}} - \mathrm{d}\hat{\sigma}^{T}_{ij,\mathrm{NLO}}\right], \qquad (2.1)$$

Generic structure of higher-order terms —see Gauld et al. arXiv: 1907.05836

$$\mathrm{d}\hat{\sigma}^{R}_{ij,\mathrm{NLO}} = \mathcal{N}^{R}_{\mathrm{NLO}}\, \mathrm{d}\Phi_{n+1}\left(\{p_3, \ldots, p_{n+3}\}; p_1, p_2\right) \frac{1}{S_{n+1}} \times \left[M^{0}_{n+3}\left(\{p_{n+3}\}, \{f_{n+3}\}\right) J^{(n+1)}_{n}(\{p_{n+1}\}, \{f_{n+1}\})\right]. \qquad (2.2)$$

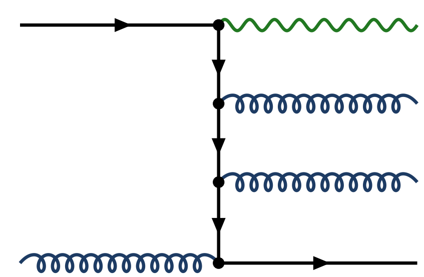
Jet function acts on flavour of reduced MEs. In general (i, j, k) => (I, K) flavour momentum

$$\mathrm{d}\hat{\sigma}^{S}_{ij,\mathrm{NLO}} = \mathcal{N}^{R}_{\mathrm{NLO}} \sum_{k} \mathrm{d}\Phi_{n+1}\left(\{p_3, \ldots, p_{n+3}\}; p_1, p_2\right) \frac{1}{S_{n+1}} \times \left[X^{0}_{3}(\cdot, k, \cdot)\, M^{0}_{n+2}\left(\{\tilde{p}_{n+2}\}, \{\tilde{f}_{n+2}\}\right) J^{(n)}_{n}\left(\{\tilde{p}_{n}\}, \{\tilde{f}_{n}\}\right)\right], \qquad (2.3)$$

The ~ functions denoted mapped (in soft/collinear limits) momenta/flavour sets

NNLO corrections, Z+(b)-jet

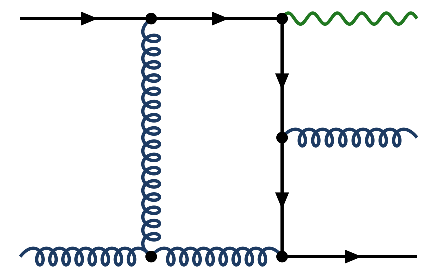
$$\sigma_{\mathrm{NNLO}} = \int_{\phi_{n+2}} \mathrm{d}\sigma^{RR}_{\mathrm{NNLO}}$$



Single unresolved

Double unresolved

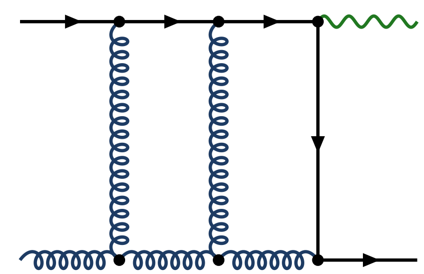
$$+ \int_{\phi_{n+1}} \mathrm{d}\sigma^{RV}_{\mathrm{NNLO}}$$



Single unresolved

$1/\epsilon,\ 1/\epsilon^2$

$$+ \int_{\phi_{n+0}} \mathrm{d}\sigma^{VV}_{\mathrm{NNLO}}$$



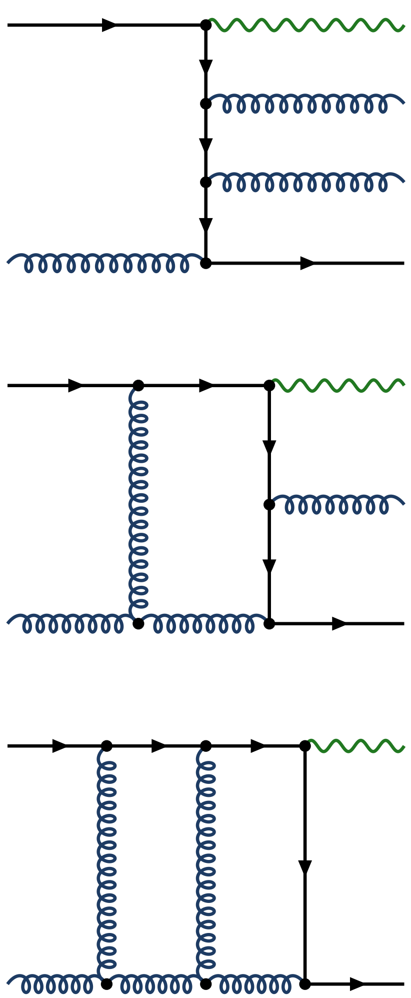
$1/\epsilon,\ 1/\epsilon^2,\ 1/\epsilon^3,\ 1/\epsilon^4$

$$\sum = \mathrm{Finite}$$

Non-trivial cancellation of IR divergences

NNLO corrections, Z+(b)-jet

$$\sigma_{\rm NNLO} = \int_{\phi_{n+2}} d\sigma^{RR}_{\rm NNLO} + \int_{\phi_{n+1}} d\sigma^{RV}_{\rm NNLO} + \int_{\phi_{n+0}} d\sigma^{VV}_{\rm NNLO}$$



Hagiwara, Zeppenfeld `89
Berends, Giele, Kuijf `89
Falck, Graudenz, Kramer `89

Glover, Miller `97
Bern, et al. `97
Campbell, Glover, Miller `97
Ben, Dixon, Kosower `98

Moch, Uwer, Weinzierl `02
Garland et al. `02
Gehrmann, Tancredi `12

$$\sum = \text{Finite}$$

Non-trivial cancellation of IR divergences

NNLO corrections, Z+(b)-jet

$$\sigma_{\rm NNLO} = \int_{\phi_{n+2}} {\rm d}\sigma^{RR}_{\rm NNLO} + \int_{\phi_{n+1}} {\rm d}\sigma^{RV}_{\rm NNLO} + \int_{\phi_{n+0}} {\rm d}\sigma^{VV}_{\rm NNLO}$$

$$\sum = {\rm Finite}$$

Antenna subtraction
Gehrmann(-De Ridder), Glover `05

CoLorFul subtraction
Del Duca, Somogyi, Trocsanyi `05

qT subtraction
Catani, Grazzini `05

Sector-Improved residue subtraction
Czakon `10
Boughezal, Melnikov, Petriello `11

N-jettiness subtraction
Gaunt, Stahlhofen, Tackmann, Walsh `15
Boughezal, Melnikov, Petriello `15

Projection-to-Born
Cacciari et al. `15

Organisation of calculation to allow numerical integration

NNLO corrections, Z+(b)-jet

$$\sigma_{\mathrm{NNLO}} = \int_{\phi_{n+2}} \left(\mathrm{d}\sigma^{RR}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{S}_{\mathrm{NNLO}}\right)$$

$$+ \int_{\phi_{n+1}} \left(\mathrm{d}\sigma^{RV}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{T}_{\mathrm{NNLO}}\right)$$

$$+ \int_{\phi_{n+0}} \left(\mathrm{d}\sigma^{VV}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{U}_{\mathrm{NNLO}}\right)$$

mimic unresolved

explicit pole cancellation

$$\sum = \mathrm{Finite} - 0$$

Each line individually finite, can be integrated in 4-d

Framework - antenna subtraction

- Exploits factorisation properties in IR limits
- Formalism operates on colour-ordered amplitudes

$$|\mathcal{M}^0_{m+1}(..,i,j,k,..)|^2 \xrightarrow{\text{j unresolved}} X^0_3(i,j,k)\,|\mathcal{M}^0_m(..,I,K,..)|^2$$

Partial amplitude

Antenna function

Reduced amplitude $\{p_i, p_j, p_k\} \to \{p_I, p_K\}$

The antenna function captures multiple IR limits, e.g.

limit	$X^0_3(i,j,k)$	mapping
$p_j \to 0$	$\frac{2s_{ik}}{s_{ij}s_{kj}}$	$p_i \to p_I,\, p_k \to p_K$
$p_j \parallel p_k$	$\frac{1}{s_{kj}}P_{kj}(z)$	$p_i \to p_I,\, (p_k + p_j) \to p_K$
$p_j \parallel p_i$	$\frac{1}{s_{ij}}P_{ij}(z)$	$(p_i + p_j) \to p_I,\, p_k \to p_K$

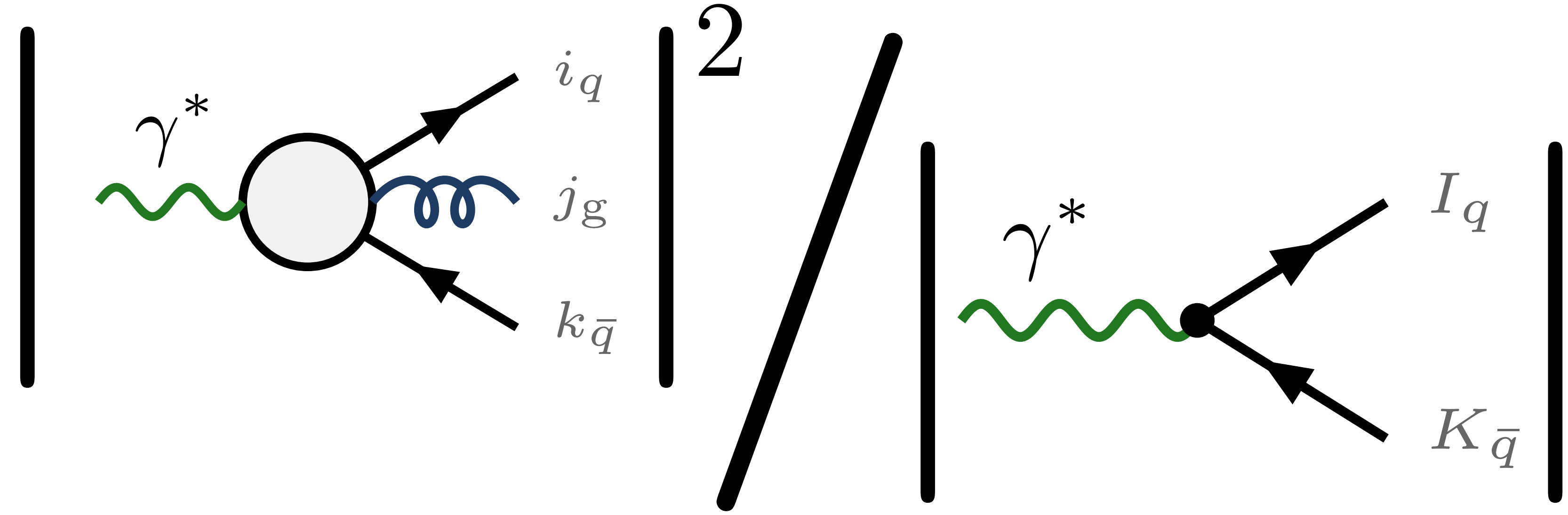
Framework - antenna subtraction

- Exploits factorisation properties in IR limits
- Formalism operates on colour-ordered amplitudes

j unresolved

Example: tree-level quark, anti-quark antenna

$$X_3^0(i_q, j_g, k_{\bar{q}}) \sim$$



$p_j \parallel p_i$	$\frac{1}{s_{ij}} P_{ij}(z)$	$(p_i + p_j) \to p_I\,,\; p_k \to p_K$

Framework - antenna subtraction

Real subtraction then numerically integrated

$$d\sigma^S = \frac{1}{S_{m+1}} \, d\phi_{m+1}(..., p_i, p_j, p_k, ...) \, X^0_{ijk}(i,j,k) \, |\mathcal{M}^0_m(.., I, K, ..)|^2 J^{(m)}_m(.., p_I, p_K, ..)$$

m+1 parton phase-space — Antenna function — Reduced amplitude — Jet function

Note that (phase-space factorisation):

$$d\phi_{m+1}(..., p_i, p_j, p_k, ...) = d\phi_m(..., p_I, p_K, ...) \cdot d\phi_{X_{ijk}}(p_i, p_j, p_k; p_I + p_K)$$

can be used to re-write subtraction term according to:

$$d\phi_m(..., p_I, p_K, ...) \, |\mathcal{M}^0_m(.., I, K, ..)|^2 J^{(m)}_m(.., p_I, p_K, ..) \int d\phi_{X_{ijk}} X^0_{ijk}(i,j,k)$$

analytically integrated in d-dimensions

which allows to construct integrated subtraction term $d\sigma^T$.

Flavour-k_T Jet algorithm

Original work: Banfi, Salam, Zanderighi et al. hep-ph/0601139

These details from — Gauld et al. arXiv: 1907.05836

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha} & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}, k_{tj})^\alpha & \text{softer of } i, j \text{ is unflavoured,} \end{cases} \tag{2.4}$$

and

$$d_{i\overleftrightarrow{B}} = \begin{cases} \max(k_{ti}, k_{t\overleftrightarrow{B}}(y_i))^\alpha \min(k_{ti}, k_{t\overleftrightarrow{B}}(y_i))^{2-\alpha} & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}, k_{t\overleftrightarrow{B}}(y_i))^\alpha & \text{softer of } i, j \text{ is unflavoured.} \end{cases} \tag{2.5}$$

Introduction of a beam momentum, controls clusterings

$$k_{tB}(y) = \sum_i k_{ti} \left(\Theta(y_i - y) + \Theta(y - y_i)\, \mathrm{e}^{y_i - y}\right), \tag{2.6}$$

$$k_{t\bar{B}}(y) = \sum_i k_{ti} \left(\Theta(y - y_i) + \Theta(y_i - y)\, \mathrm{e}^{y - y_i}\right), \tag{2.7}$$

The mass of the b-quark

$$m_b^{\text{pole}} \sim 5 \text{ GeV}$$

The NNLO QCD computation assumes $m_b = 0$, what does this mean

The b-quark is **active** in the running of α_s and PDF. Meaning?

$$\frac{d\alpha_s}{d\ln\mu} = -\beta_0 \frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3)$$

$$\alpha_s(\mu_R) = \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{2\pi}\alpha_s(\mu_0)\ln[\mu_r^2/\mu_0^2]}$$

$$\beta_0 = \left(\frac{11}{6}c_a - \frac{2}{3}t_r n_f\right)$$

$$\frac{x}{1+Bx} \approx x\left(1 - Bx + B^2x^2\right) + \mathcal{O}(x^4)$$

Use of 'RG' improved perturbation theory to (re)sum $\ln[\mu_r/m_b]$ terms

The zero mass limit

$$\mathrm{d}\sigma^{GMVFNS} = \mathrm{d}\sigma^{m=0} + \left(\mathrm{d}\sigma^{m} - \mathrm{d}\sigma^{m\to 0}\right)$$

$$\mathrm{d}\sigma^{m} = \mathrm{d}\sigma^{m=0,n_f} + \mathrm{d}\sigma^{L[m]} + \mathrm{d}\sigma^{\mathcal{O}(m^2)}$$

$\mathrm{d}\sigma^{L[m]}$ is built from:

1) convolutions of a massless partonic cross section and OME
2) explicit virtual corrections, implicit via $\alpha_s^{n_f}$

Example: gg-channel at $\mathcal{O}(\alpha_s^2)$.

$$\mathrm{d}\sigma_{gg,L} = \int \mathrm{d}x_1\,\mathrm{d}x_2\, g(x_1,\mu_F^2)\left[\hat{A}_{g\to b}(z,\mu_F^2/m_b^2)\otimes g(x_2/z,\mu_F^2)\right]\hat{\sigma}^{[5fs]}_{bg\to Zb}(\alpha_s(\mu_r),\mu_r,\mu_F,\hat{s})$$

$$f_b(x,\mu_F^2) = \int_x^1 g(x/z,\mu_F^2)\hat{A}_{gb} + \mathcal{O}(\alpha_s^2)\,, \quad \hat{A}_{gb} = \frac{\alpha_s(\mu_F^2)}{2\pi}P_{g\to q}(z)\,\ln[\mu_F^2/m_b^2]$$