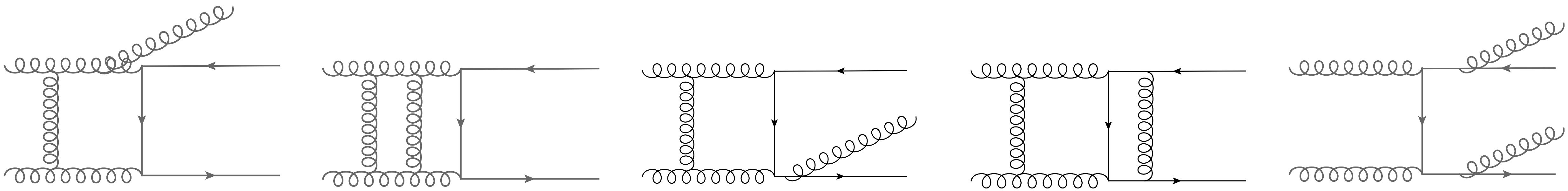


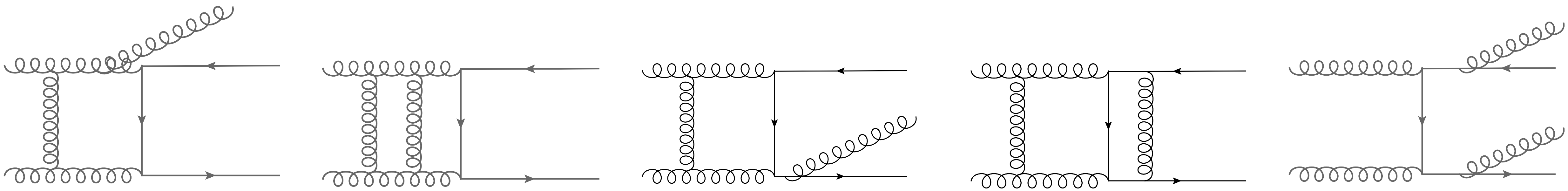
BOTTOM-QUARK HADROPRODUCTION IN NNLO QCD

RADCOR&LoopFest 2021, 18.05.21
Florida State University, Virtual Conference

UNIVERSITÀ
DEGLI STUDI
DI MILANO

Simone Devoto

In collaboration with:
S. Catani, M. Grazzini, S. Kallweit, J. Mazzitelli



CONTENTS

- Introduction;
- q_T subtraction formalism for heavy-quark production:
  - Top-pair production within q_T subtraction formalism;
  - Extension to bottom-pair production;
- Phenomenological results:
  - Inclusive cross section;
  - Differential distributions;
- Summary and outlook.

BOTTOM-QUARK PRODUCTION AT HADRON COLLIDERS

Bottom-quark production is extensively studied at hadron colliders:

- *CERN $Sp\bar{p}S$;*
- *Tevatron (CDF, D0);*
- *CERN LHC (ATLAS, CMS, LHCb).*

Heavy-quark production classic test of perturbative QCD!

Low bottom mass (~4-5 GeV) → Slow convergence → **NNLO corrections!**

Low bottom mass (~4-5 GeV) → Large theoretical uncertainties → **NNLO corrections!**

THEORETICAL STATUS: SHORT OVERVIEW

- **NLO QCD:**

  [Nason, Dawson, Ellis; '88], [W. Beenakker, H. Kuijf, W. L. van Neerven, and J. Smith; '89], [P. Nason, S. Dawson, and R. K. Ellis; '90], […]:

- b-**hadron production:**
  [B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger; 1109.2472], [G. Kramer and H. Spiesberger; 1809.04297];

- **Resummation:**

  [M. Cacciari and M. Greco; 9311260];

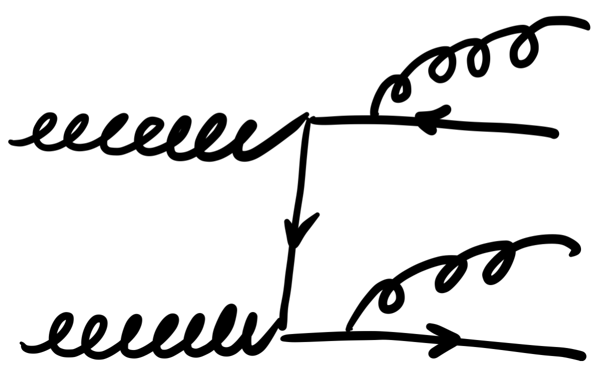
- **FONNL:**
  [M. Cacciari, M. Greco, and P. Nason; 9803400],[M. Cacciari, S. Frixione, N. Houdeau, M. L. Mangano, P. Nason, and G. Ridolfi; 1205.6344];

- **NNLO QCD - total cross section:**
  Results can be obtained with HATHOR *[M. Aliev, H. Lacker, U. Langenfeld, S. Moch, P. Uwer, and M. Wiedermann; 1007.1327]*, which implements the computation of *[Czakon et al.; 1204.5201, 1207.0236, 1210.6832, 1303.6254];*

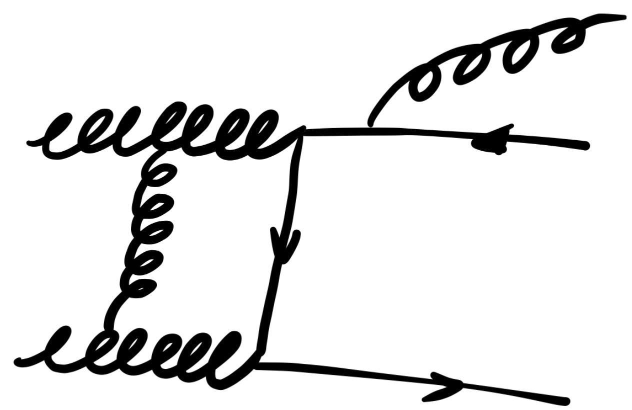
- **Differential NNLO QCD**
  [S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli; 2010.11906]

BOTTOM QUARK PRODUCTION AT NNLO – INGREDIENTS

DOUBLE REAL



REAL - VIRTUAL



TWO LOOP VIRTUAL

Fast and stable evaluation with OpenLoops 2
[Cascioli et al. (2012), Buccioni et al. (2018), Buccioni et al. (2019)]

Numerically available
[Czakon (2008); Barnreuther et al. (2013)]

IR divergent | **IR divergent** | **IR divergent**

IR divergences cancel once all contributions are combined (KLN theorem) but they do not allow a straightforward implementation of ***numerical techniques****.*

We need a method to handle and cancel IR singularities!

SUBTRACTION METHODS

- **NLO methods:**
  - *Catani-Seymour dipole subtraction [S. Catani, M. Seymour (1996)]*
  - *FKS subtraction [S. Frixione, Z. Kunszt, A. Signer (1996)]*
  - …
- **NNLO methods:**
  - *CoLoRFulNNLO [G. Somogyi, Z. Trocsanyi, V. Del Duca (2005)]*
  - *Antenna subtraction [T. Gehrmann, A. Gehrmann-De Ridder, N. Glover (2005)]*
  - q_T *subtraction formalism [S. Catani, M. Grazzini (2007)]*
  - *STRIPPER formalism [M. Czakon (2010); Boughezal et al (2011)]*
  - *N-jettiness subtraction [Boughezal, et al. (2015); Gaunt et al. (2015)]*
  - *Projection to Born [M. Cacciari, F. A. Dreyer, A. Karlberg, G. P. Salam (2015)]*
  - *Nested soft-collinear [F. Caola, K. Melnikov, R, Röntsch, (2017)]*
  - *Geometric subtraction [F. Herzog (2018)]*
  - *Local analytic sector [L. Magnea et al. (2018)]*
  - …

q_T SUBTRACTION FORMALISM

[S. Catani, M. Grazzini (2007)]

The q_T subtraction formalism is a method to handle and cancel IR divergences, originally developed for **colourless** final states.

$$d\sigma^F_{(N)NLO} = d\sigma^F_{(N)NLO}\Big|_{q_T=0} + d\sigma^F_{(N)NLO}\Big|_{q_T\neq 0}$$

q_T=*transverse momentum of the system F*

$$d\sigma^{F+jets}_{(N)LO}$$

$$d\sigma^F_{(N)NLO} = \mathscr{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right]$$

HARD COLLINEAR COEFFICIENT
Contains information on virtual corrections to the process.

Singularities for $q_t \neq 0$ can be computed with NLO subtraction techniques.

Extra singularities of NNLO type associated to the $q_t \to 0$ limit need additional subtraction. IR behaviour known from q_t resummation formalism allow us to construct a counterterm.
[J. C. Collins, D. E. Soper, G. Sterman (1985),G. Bozzi, S. Catani, D. de Florian, M. Grazzini: arXiv:0508068]

q_T SUBTRACTION FORMALISM FOR HEAVY QUARK PRODUCTION

With the inclusion of extra contributions, q_T subtraction formalism can be extended to **massive coloured** final states.

$$d\sigma^{Q\bar{Q}}_{NNLO} = \mathscr{H}^{Q\bar{Q}}_{NNLO} \otimes d\sigma^{Q\bar{Q}}_{LO} + \left[d\sigma^{Q\bar{Q}+jets}_{NLO} - d\sigma^{CT}_{NNLO} \right]$$

Hard Collinear Coefficient

Computable with NLO subtraction techniques.

IR behaviour known from studies in q_T resummation
[A. Ferroglia, M. Neubert, B. D. Pecjak, L. L. Yang (2009)];
[Hai Tao Li, Chong Sheng Li, Ding Yu Shao, Li Lin Yang, Hua Xing Zu (2013)];

Contains the integrations of the additional final-state soft singularities.
It was recently computed by some of us.
[S. Catani, SD, M.Grazzini, J.Mazzitelli, in preparation. See also R. Angeles-Martinez, M. Czakon, S. Sapeta (2018)]

THE MATRIX PROJECT

MUNICH

MUlti-ChaNnel Integrator at Swiss (CH) precision

OPENLOOPS

(Collier, CutTools…)

TWO-LOOP AMPLITUDES

(VVamp, GiNaC, tdhpl…)

q_T subtraction

↕

q_T resummation

↓

MATRIX

Munich Automates qT subtraction and Resummation to Integrate X-sections

MATRIX 1.0.0

[M.Grazzini, S. Kallweit, M. Wiesemann: arXiv 1711.06631]

Public code that allows the evaluation of fully differential cross sections at NNLO QCD for a wide class of processes with **colourless** final state.

- $pp \to H$
- $pp \to Z/\gamma^* (\to l^+l^-)$
- $pp \to W (\to l^+ \nu)$
- $pp \to \gamma\gamma$
- $pp \to Z\gamma \to l^+l^- \gamma$
- $pp \to W\gamma \to l\ \nu\ \gamma$
- $pp \to ZZ\ (\to 4l)$
- $pp \to WW\ (\to l\nu l'\nu')$
- $pp \to ZZ/WW \to ll\nu\nu$
- $pp \to WZ \to l\nu ll$

```
[devoto:/mnt/runs2/devoto/MATRIX_v1.0.0] ./matrix

   /---------------------------------------------------------------------\
   |                                                                     |
   |          __ __    ___    ____   ___    _    _                      |
   |         | _\/_ |  / _ \  |____| / _ \  ||  \\  //                   |
   |         || \/ ||  | |_| |   ||   ||_||  ||   \\ //                   |
   |         ||    ||  | ___ |   ||   ||\\   ||   // \\                   |
   |         ||    ||  ||   ||   ||   || \\  ||  //   \\                  |
   |                                                                     |
   |         Version: 1.0.0                              Nov 2017        |
   |         Reference: arXiv:1711.06631                                 |
   |                                                                     |
   | Munich -- the MUlti-chaNnel Integrator at swiss (CH) precision --   |
   | Automates qT-subtraction and Resummation to Integrate X-sections    |
   |                                                                     |
   | \          \ ____      \          \ ____     \ ____      \          |
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   |  /          /          |/          /____     |/          |/         |
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   |                                                                     |
   | M. Grazzini                             (grazzini@physik.uzh.ch)    |
   | S. Kallweit                            (stefan.kallweit@cern.ch)    |
   | M. Wiesemann                         (marius.wiesemann@cern.ch)     |
   |---------------------------------------------------------------------|
   | MATRIX is based on a number of different computations and tools     |
   | from various people and groups. Please acknowledge their efforts     |
   | by citing the list of references which is created with every run.   |
   \---------------------------------------------------------------------/

<<MATRIX-MAKE>> This is the MATRIX process compilation.
<<MATRIX-READ>> Type process_id to be compiled and created. Type "list" to show
                available processes. Try pressing TAB for auto-completion. Type
                "exit" or "quit" to stop.
|============>> list
-------------------------------------------------------------------------------------
process_id    ||  process                          ||  description
-------------------------------------------------------------------------------------
pph21         >>  p p --> H                        >>  on-shell Higgs production
ppz01         >>  p p --> Z                        >>  on-shell Z production
ppw01         >>  p p --> W^-                      >>  on-shell W- production with CKM
ppwx01        >>  p p --> W^+                      >>  on-shell W+ production with CKM
ppeex02       >>  p p --> e^- e^+                  >>  Z production with decay
ppnenex02     >>  p p --> v_e^- v_e^+              >>  Z production with decay
ppenex02      >>  p p --> e^- v_e^+                >>  W- production with decay and CKM
ppexne02      >>  p p --> e^+ v_e^-                >>  W+ production with decay and CKM
ppaa02        >>  p p --> gamma gamma              >>  gamma gamma production
ppeexa03      >>  p p --> e^- e^+ gamma            >>  Z gamma production with decay
ppnenexa03    >>  p p --> v_e^- v_e^+ gamma        >>  Z gamma production with decay
ppenexa03     >>  p p --> e^- v_e^+ gamma          >>  W- gamma production with decay
ppexnea03     >>  p p --> e^+ v_e^- gamma          >>  W+ gamma production with decay
ppzz02        >>  p p --> Z Z                      >>  on-shell ZZ production
ppwxw02       >>  p p --> W^+ W^-                  >>  on-shell WW production
ppemexmx04    >>  p p --> e^- mu^- e^+ mu^+        >>  ZZ production with decay
ppeeexex04    >>  p p --> e^- e^- e^+ e^+          >>  ZZ production with decay
ppeexnmnmx04  >>  p p --> e^- e^+ v_mu^- v_mu^+    >>  ZZ production with decay
ppemxnmnex04  >>  p p --> e^- mu^+ v_mu^- v_e^+    >>  WW production with decay
ppeexnenex04  >>  p p --> e^- e^+ v_e^- v_e^+      >>  ZZ/WW production with decay
ppemexnmx04   >>  p p --> e^- mu^- e^+ v_mu^+      >>  W-Z production with decay
ppeeexnex04   >>  p p --> e^- e^- e^+ v_e^+        >>  W-Z production with decay
ppeexmxnm04   >>  p p --> e^- e^+ mu^+ v_mu^-      >>  W+Z production with decay
ppeexexne04   >>  p p --> e^- e^+ e^+ v_e^-        >>  W+Z production with decay
|============>>
```
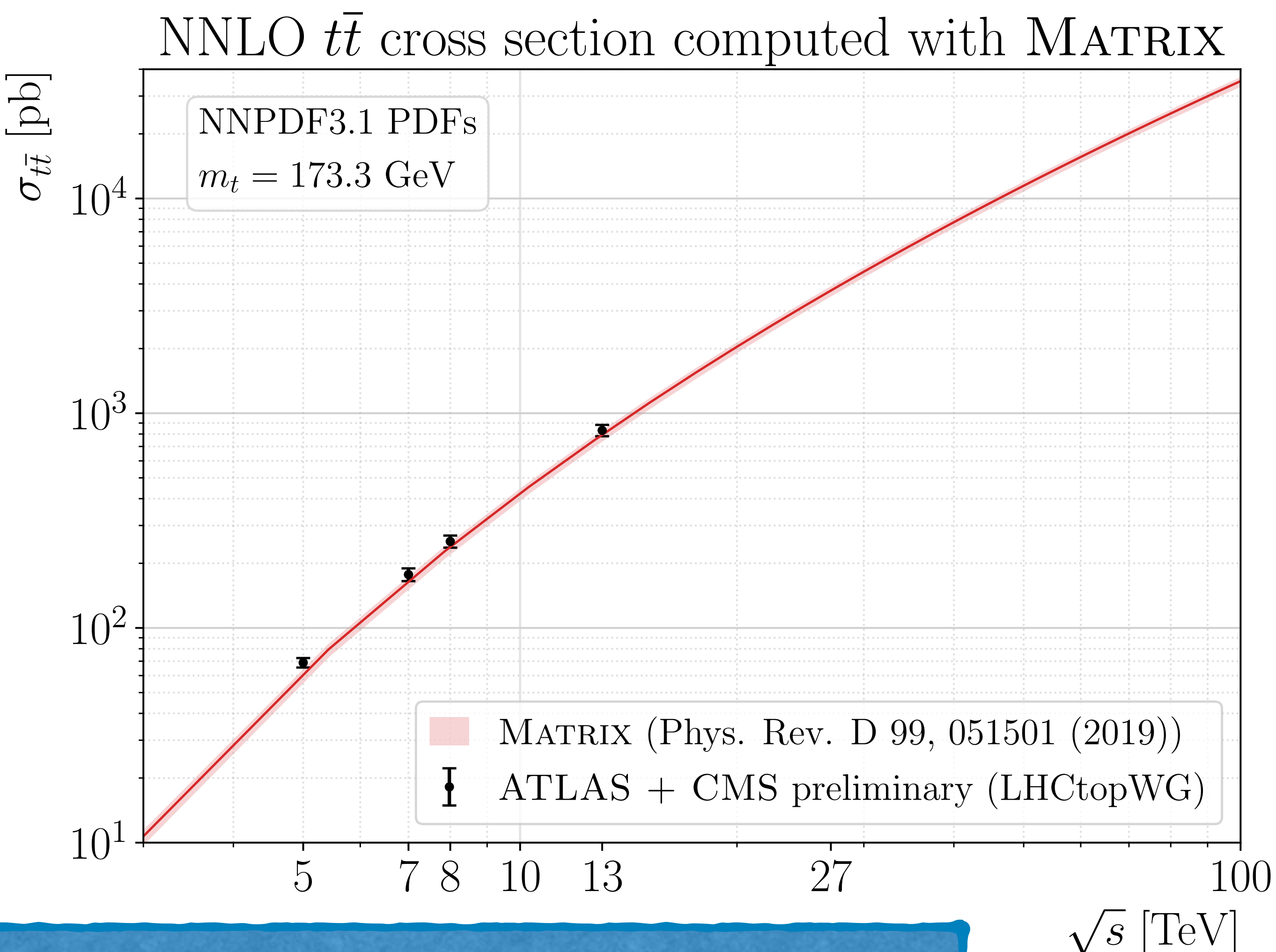

TOP QUARK PRODUCTION IN MATRIX

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli, H. Sargsyan (2019)]



Per-mille accuracy in ~1000 CPU days

Inclusive cross section can be compared with the results of TOP++

σ_{NNLO} [pb]	MATRIX	TOP++
8 TeV	$238.5(2)^{+3.9\%}_{-6.3\%}$	$238.6\,^{+4\%}_{-6.3\%}$
13 TeV	$794.0(8)^{+3.5\%}_{-5.7\%}$	$794.0^{+3.5\%}_{-5.7\%}$
100 TeV	$35215(74)^{+2.8\%}_{-4.7\%}$	$35216^{+2.9\%}_{-4.8\%}$

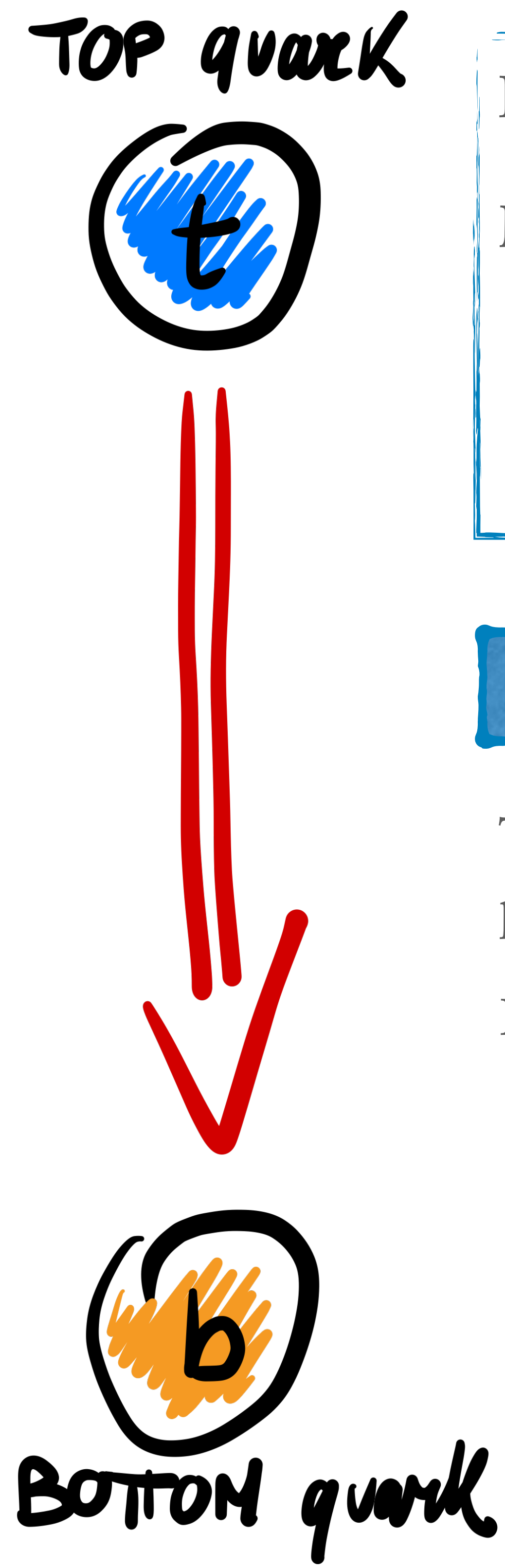
Statistical + systematic uncertainties

Scale uncertainties

- **Inclusive cross section** ~ *S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli, H. Sargsyan: Phys.Rev.D 99 (2019) 5, 051501*
- **Differential Distributions** ~ *S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP* 07 (2019) 100
- **Predictions in the $\overline{MS}$ scheme** ~ *S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP* 08 (2020) 08, 027

A TOP-BOTTOM APPROACH

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]



Implementation of **bottom-pair** production in the MATRIX framework:

Extend the implementation of **top-pair** production to:

- lower heavy quark **mass**: $m = 173.3$ GeV → $m = 4.92$ *GeV*
- different **number of light flavours**: $n_f = 5$ → $n_f = 4$

Massive bottom quark, top quark decoupled from the process

Two-loop grid *[Czakon (2008); Barnreuther et al.(2013)]* so far used only for top-pair production.

Is the phase space sufficiently sampled also for bottom-pair production?

- Effect of the two loop contribution: $\mathcal{O}(1\%)$
- Effect of reducing the number of points of the grid by a factor of 2: $\mathcal{O}(1‰)$

The results of *[Czakon (2008); Barnreuther et al.(2013)]* *can be safely used in our computation!*

A TOP-BOTTOM APPROACH

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]

The small mass of the bottom quark might challenge the cancellation of IR singularities

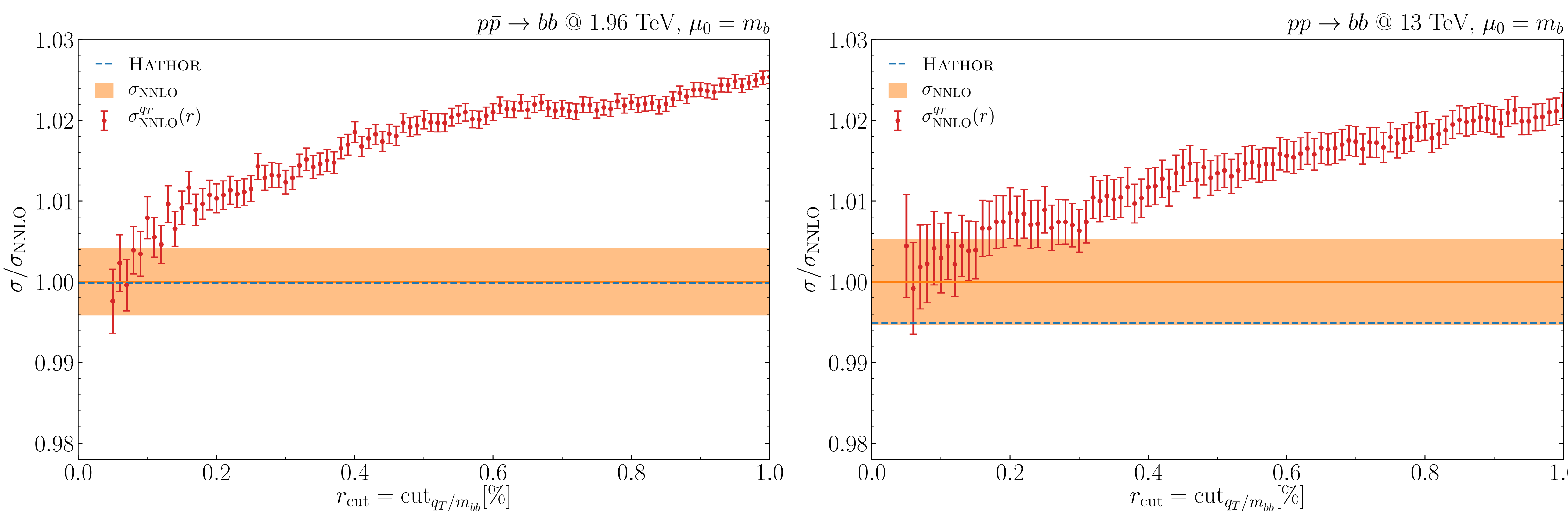
$$d\sigma^F_{(N)NLO} = \mathscr{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

$d\sigma^{F+jets}_{(N)LO}$ and $d\sigma^{CT}_{(N)LO}$ are separately divergent.

In practice, q_T subtraction is implemented as a **slicing method**:

- ➤ introducing a cutoff $r_{cut} = Q/M$;
- ➤ performing the limit $r_{cut} \to 0$

Stability of the subtraction procedure can be understood looking at the r_{cut} dependence:



INCLUSIVE CROSS SECTION

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]

Central scale: $\mu_0 = m_b = 4.92\,GeV$

7-points scale variations:

$$\frac{1}{2}\mu_0 < \mu_F, \mu_R < 2\mu_0 \qquad \frac{1}{2} < \frac{\mu_F}{\mu_R} < 2$$

Excellent agreement with HATHOR [*1007.1327*]!

σ_{NNLO} [μb]	MATRIX	HATHOR
$p\bar{p}$ @ 1.96 TeV	$75.4(3)^{+22\%}_{-21\%}$	$75.45^{+22\%}_{-21\%}$
pp @ 7 TeV	$288(2)^{+30\%}_{-24\%}$	$284.3^{+30\%}_{-24\%}$
pp @ 13 TeV	$508(3)^{+32\%}_{-25\%}$	$505.5^{+32\%}_{-25\%}$

MATRIX PREDICTIONS					
σ [μb]	LO	NLO	NNLO	K_{NLO}	K_{NNLO}
$p\bar{p}$ @ 1.96 TeV	$34.66^{+51\%}_{-32\%}$	$60.23^{+54\%}_{-28\%}$	$75.4(3)^{+22\%}_{-21\%}$	1.74	1.25
pp @ 7 TeV	$138.7^{+51\%}_{-46\%}$	$219.8^{+61\%}_{-39\%}$	$288(2)^{+30\%}_{-24\%}$	1.58	1.31
pp @ 13 TeV	$249.0^{+59\%}_{-51\%}$	$378.6^{+65\%}_{-45\%}$	$508(3)^{+32\%}_{-25\%}$	1.52	1.34

Poor perturbative behaviour:
- large **scale uncertainties**;
- large **K-factors**.

The inclusion of NNLO correction significantly improves both!

K-factor - ratio to the previous perturbative order: $K_{NLO} = \sigma_{NLO}/\sigma_{LO}$ — $K_{NNLO} = \sigma_{NNLO}/\sigma_{NLO}$

DIFFERENTIAL DISTRIBUTIONS

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]

- We computed single differential distributions at Tevatron (1.96 TeV) and LHC (7 and 13 TeV);
- We reduced theoretical uncertainties by considering ratios at different energies;
- We compared the pseudorapidity distribution with recent measurements from LHCb [LHCb-PAPER-2016-031].

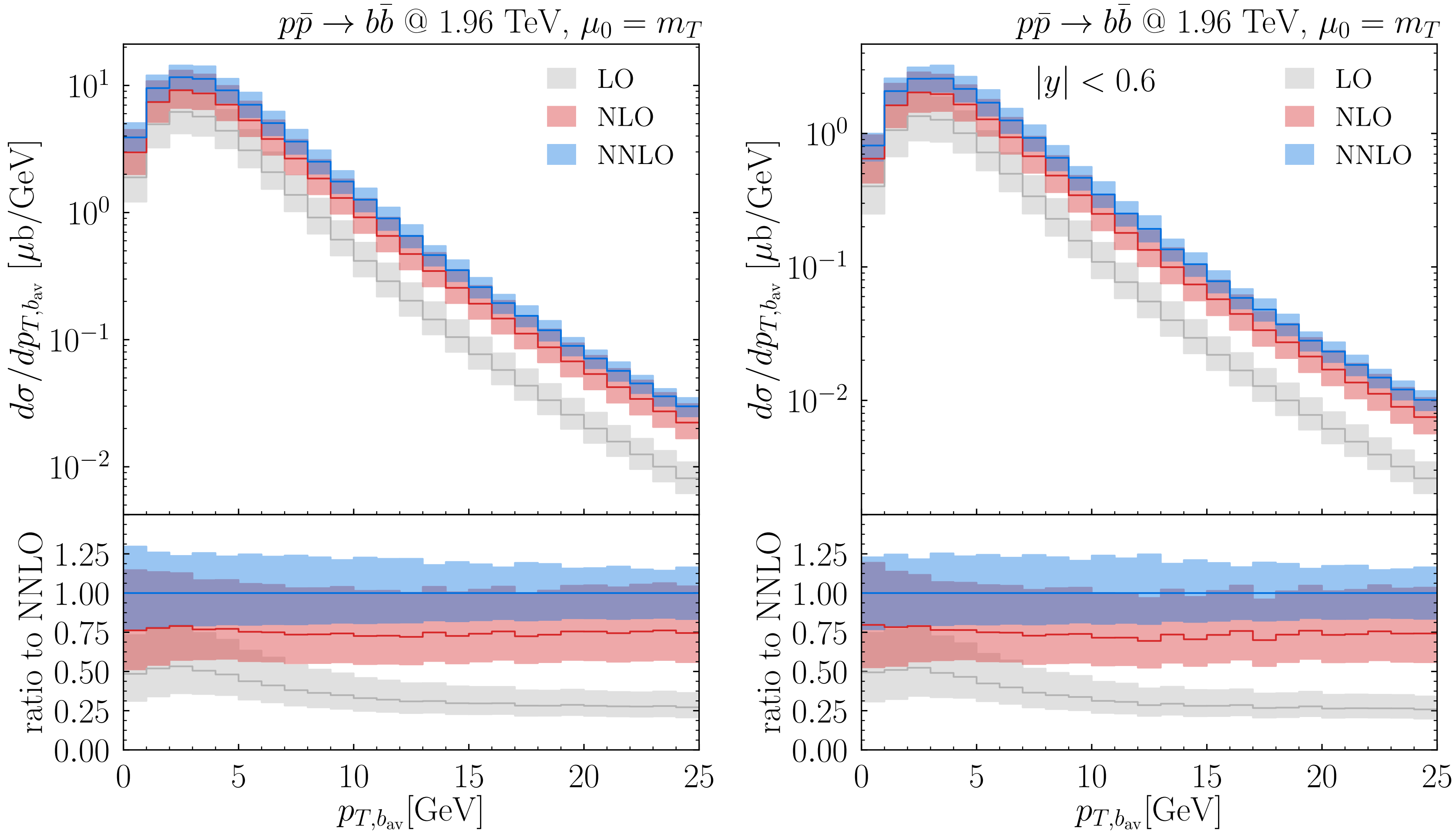
- We considered the **ratio** between predictions at LHC with $\sqrt{s} = 7$ and 13 TeV;
- Theoretical uncertainties are reduced by assuming **correlation** between scale variations at different energies (**reduced sensitivity**).

Renormalisation and factorisation scales, μ_R and μ_F, are chosen of the order of the characteristic hard scale.

Hard scale	
Total cross section	m_b
Rapidity distribution	m_b
Invariant mass distribution	$m_{b\bar{b}}$
Transverse momentum distribution	m_T

DIFFERENTIAL DISTRIBUTIONS: TEVATRON

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]



Total Cross Section with rapidity cut

$\sigma^{b/\bar{b}}_{LO}(|y_{b/\bar{b}}| < 0.6) = 7.840(3)^{+51\%}_{-34\%}\ \mu b$

$\sigma^{b/\bar{b}}_{NLO}(|y_{b/\bar{b}}| < 0.6) = 14.282(6)^{+53\%}_{-28\%}\ \mu b$

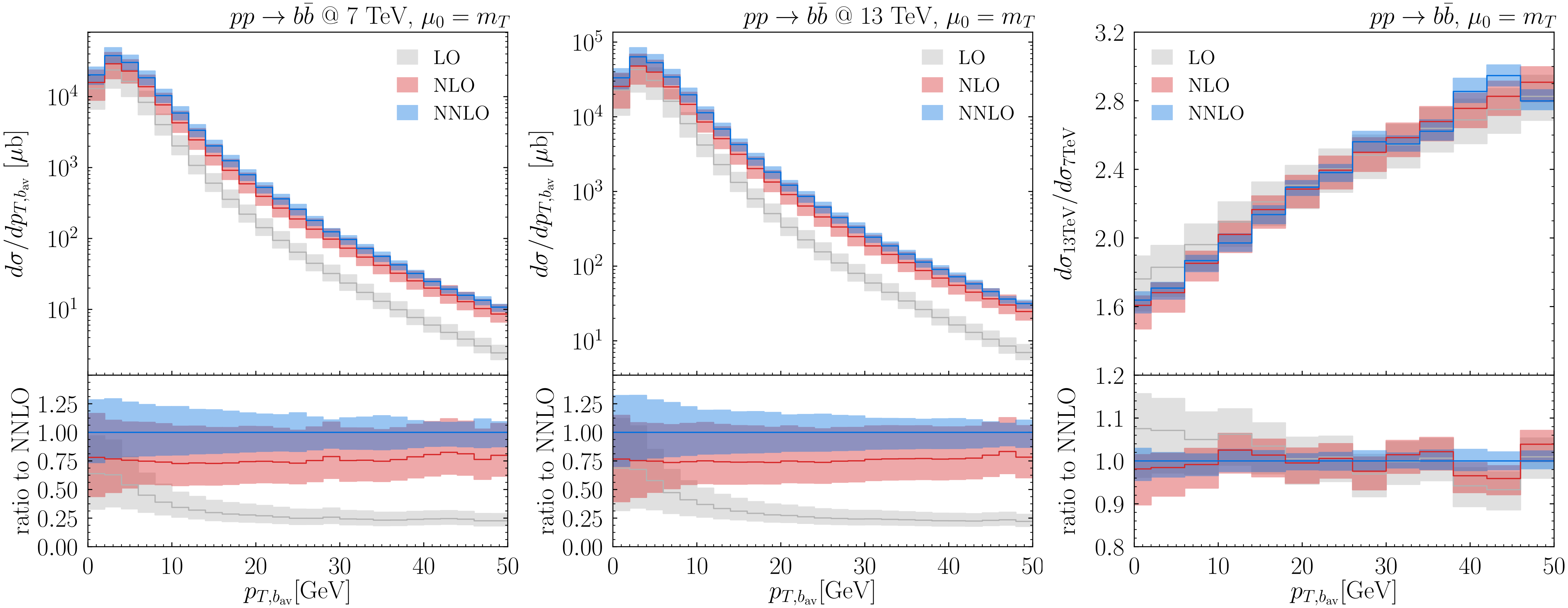
$\sigma^{b/\bar{b}}_{NNLO}(|y_{b/\bar{b}}| < 0.6) = 17.87(12)^{+22\%}_{-21\%}\ \mu b$

$\sigma^{H_b}(|y_{H_b}| < 0.6) = 17.6 \pm 0.4^{+2.5}_{-2.3}\ \mu b$ *(stat.) (syst.)*

- LO and NLO compatible only at low p_T, NLO and NNLO compatible in all the phase space;
- The inclusion of a rapidity cut does not change the shape of the distribution;
- Total Cross section with rapidity cut can be compared with **CDF measurements** for b-hadron production *[Phys. Rev. D 71 (2005) 032001]*.

DIFFERENTIAL DISTRIBUTIONS: LHC

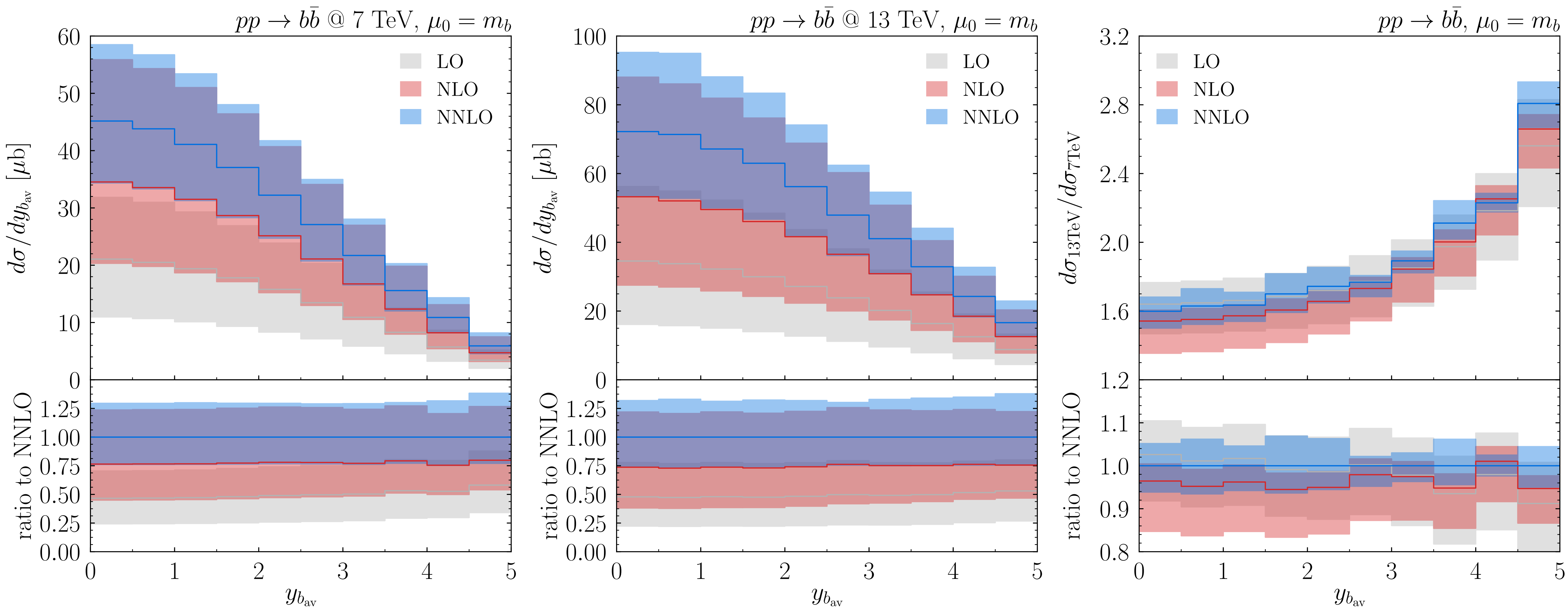
[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]



- LO and NLO compatible only at low p_T, NLO and NNLO compatible in all the phase space;
- Similar behaviour at different energies;
- Strong **reduction of uncertainty bands** in the ratio: from ~30% (at low p_T), ~10% (in the tail) to ~5%.

DIFFERENTIAL DISTRIBUTIONS: LHC

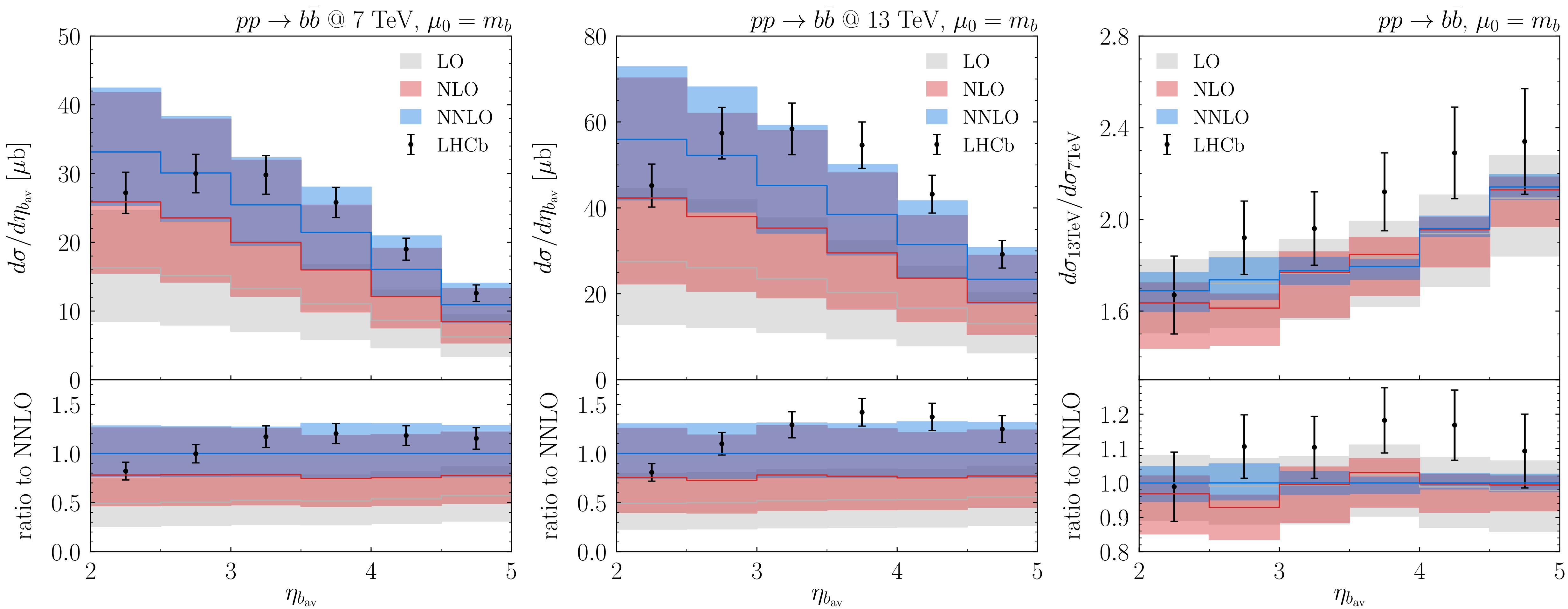
[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]



- Since the impact of QCD correction is **uniform in rapidity**, higher order corrections do not change the shape of the distribution;
- NNLO results completely **overlap** with the NLO predictions and reduce the scale uncertainties;
- The ratio strongly **reduces the scale uncertainties** to ~5%.
- The stability of the perturbative expansion in the ratio **confirms** the hypothesis of correlation of the scale variation.

DIFFERENTIAL DISTRIBUTIONS: LHC

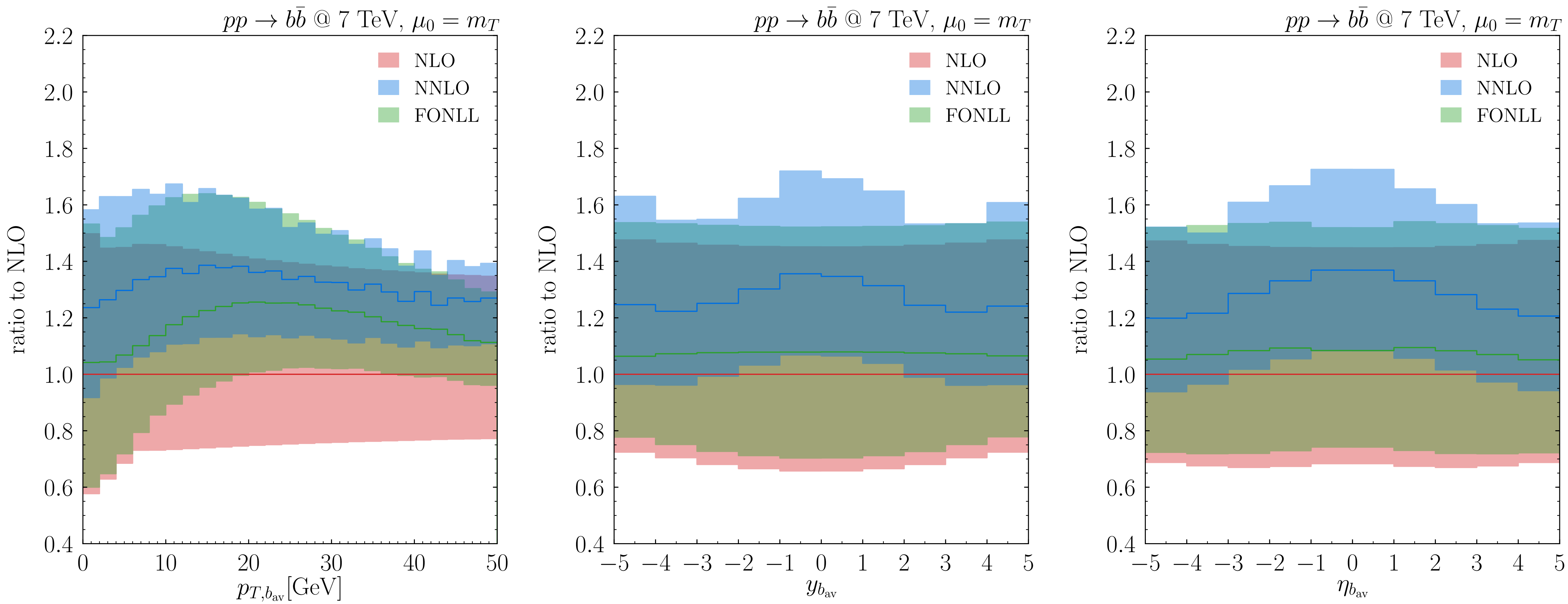
[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]



- We compare with the results presented by the **LHCb collaboration** in *[LHCb-PAPER-2016-031]*;
- Measured distributions **overlap** with NNLO predictions at both energies, but the shape is different.
- In the distributions the theoretical uncertainties are **larger** than the experimental errors, in their ratio the theoretical uncertainties are **smaller** than the experimental errors.
- In the ratio, data systematically **above** prediction (systematic and η-independent uncertainty?).

COMPARISON WITH FONLL

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]

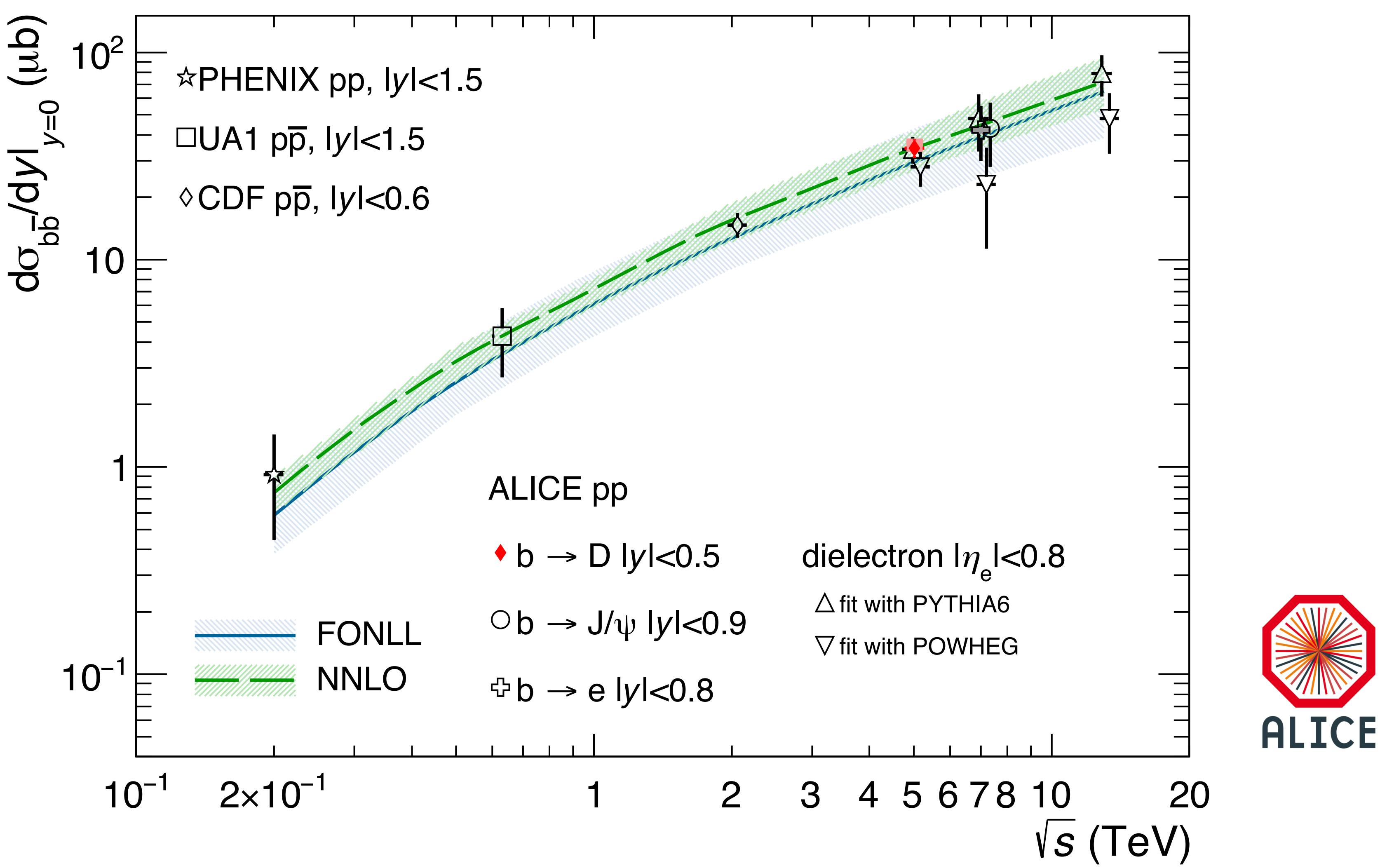


- At high p_T large logarithms in the expansion: they need to be **resummed** at all orders.
- **FONLL** predictions combine NLO calculations with resummed computations, our **NNLO** calculation **ignores** the large logarithms.
- Comparison between our NNLO and FONLL: **estimate** of the impact of large logs.
- FONLL and NNLO uncertainty bands **overlap**, NNLO has **smaller** uncertainty.

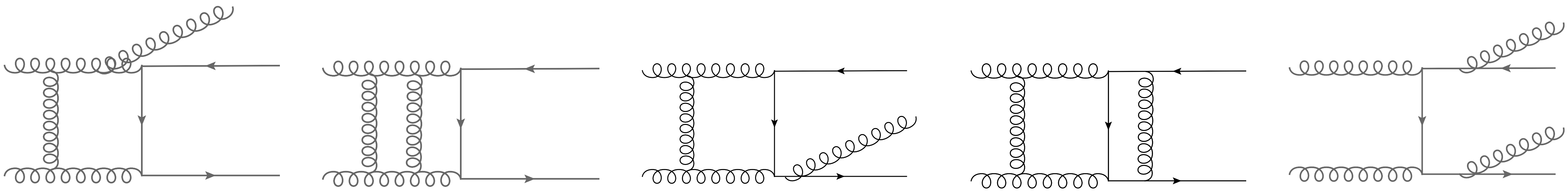
In the phase-space region considered, the NNLO prediction is reliable!

BOTTOM PAIR PRODUCTION IN MATRIX

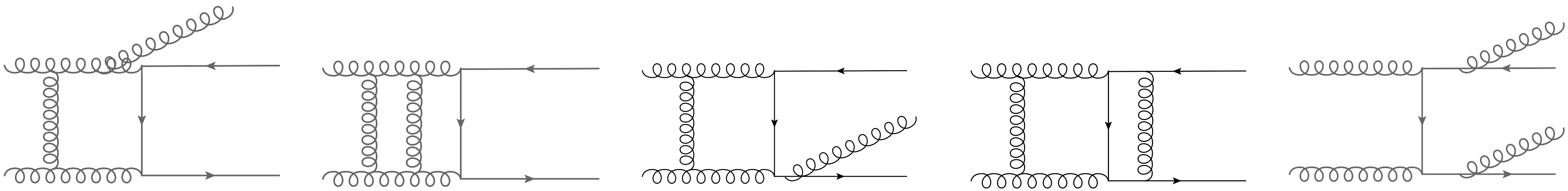
- The computation is **fully implemented** within a (at the moment) private release of **MATRIX**;
- Predictions provided upon request;
- Our computation has already been used by experimental collaborations for data-theory comparisons.



*Plot from arXiv:2102.13601 (ALICE collaboration). The NNLO prediction (green) has been computed with **MATRIX**.*

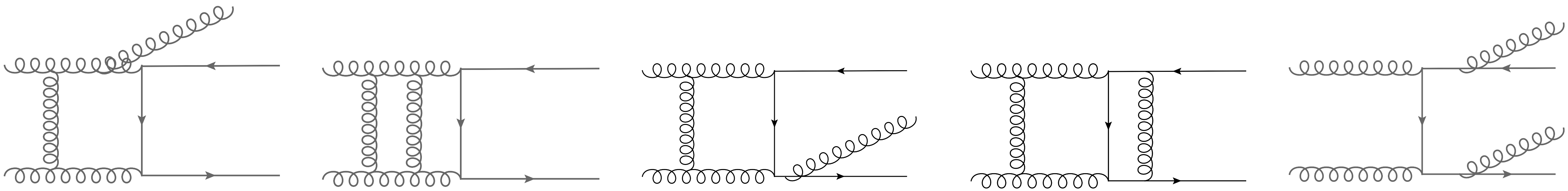


SUMMARY & OUTLOOK



SUMMARY & OUTLOOK

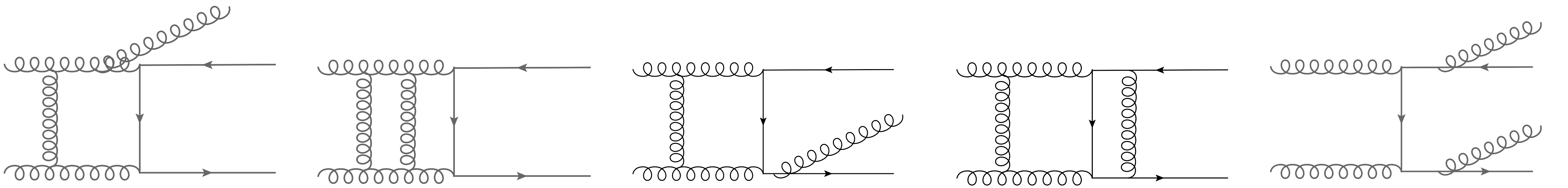
- We have presented the first differential computation for bottom-quark production at NNLO;
- We have shown results for inclusive and differential distributions, comparison with experimental data;
- The process is a extension of the computation for top-pair production and has been implemented into the MATRIX framework;
- Theoretical uncertainties are still large but significantly reduced by the inclusion of NNLO corrections.



SUMMARY & OUTLOOK

- New public MATRIX release with the inclusion of heavy quark production;
- Improvements of NNLO QCD:
  - b-jet production?
  - resummation effects?
  - fragmentation effects?

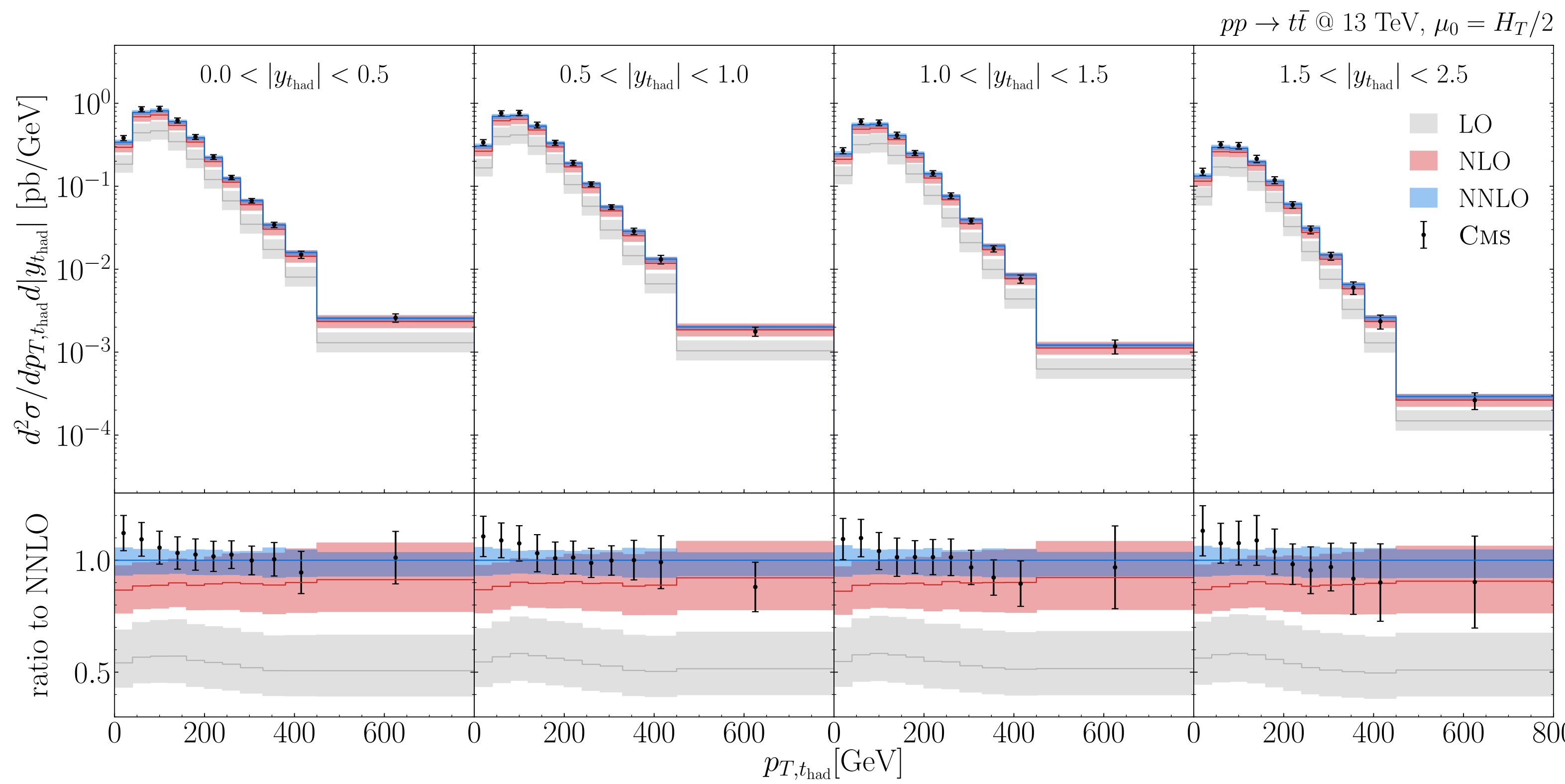
THANKS!



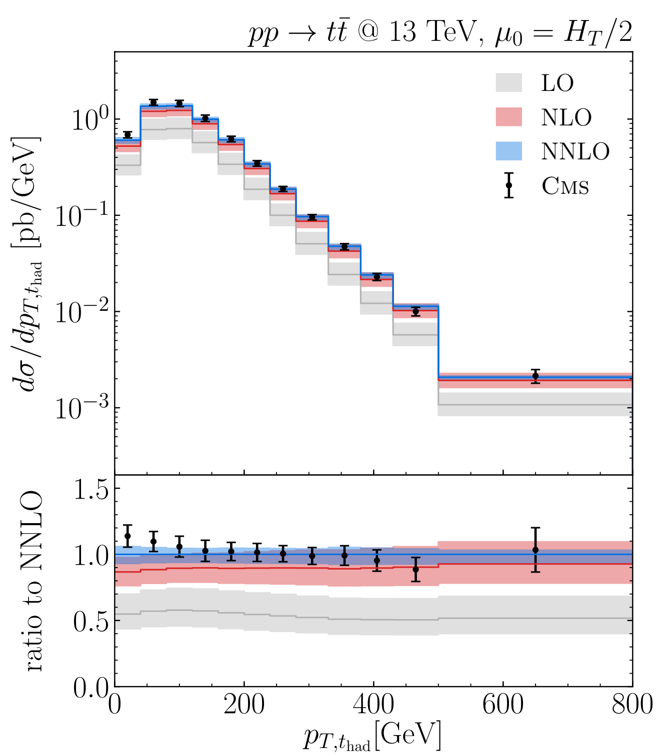
BACKUP SLIDES

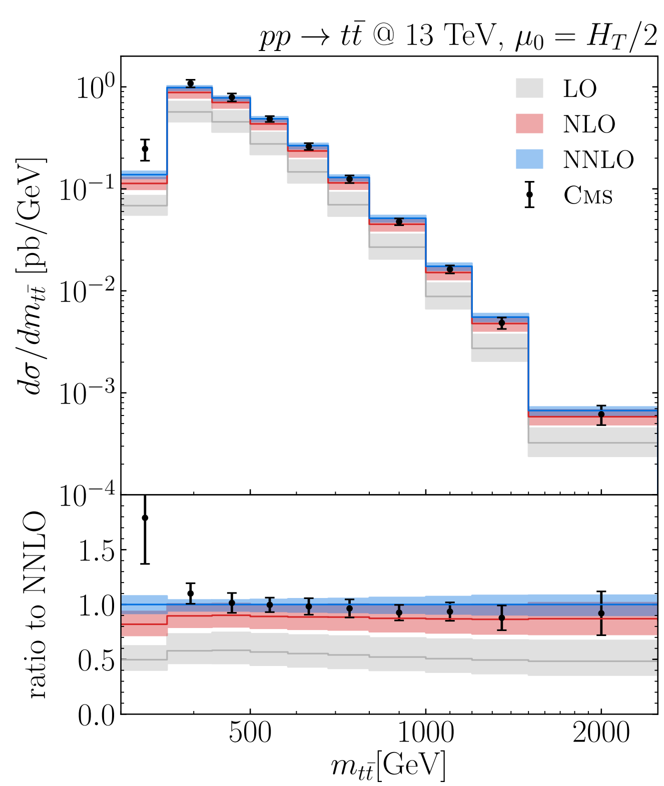
TOP QUARK PRODUCTION IN MATRIX

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli, H. Sargsyan (2019)]



Single and multi-differential distributions for top -pair production have also been implemented in the MATRIX framework!





*Shown here: plots from [**JHEP 07 (2019) 100**], where we compare our predictions with recent measurements from CMS in the leptons+jet channels [**CMS-TOP-17-002**].*

TOTAL CROSS SECTION AND UNCERTAINTIES

	$\sigma_{\rm NNLO}(\mu b)$	$\Delta\sigma_{\rm scale}$	$\Delta\sigma_{\rm mass}$	$\Delta\sigma_{\rm PDFs}$	$\Delta\sigma_{\alpha_S}$
$p\bar{p}$ @ 1.96 TeV	75.4(3)	$^{+22\%}_{-21\%}$	$^{+9.8\%}_{-8.7\%}$	$\pm 1.3\%$	$^{+0.9\%}_{-3.0\%}$
pp @ 7 TeV	288(2)	$^{+30\%}_{-24\%}$	$^{+7.9\%}_{-7.2\%}$	$\pm 2.8\%$	$^{+0.3\%}_{-2.9\%}$
pp @ 13 TeV	508(3)	$^{+32\%}_{-25\%}$	$^{+7.4\%}_{-6.8\%}$	$\pm 4.6\%$	$^{+0.0\%}_{-3.0\%}$

- **Bottom mass uncertainty**: variation between $m_b = 4.79$ GeV and $m_b = 5.05$ GeV ($m_b = 4.92 \pm 0.13$ GeV);
- **QCD coupling uncertainty**: evaluating the cross section with NNPDF31 NNLO PDFs obtained with $\alpha_S(m_Z) = 0.119$ and $\alpha_S(m_Z) = 0.117$

Despite the inclusion of NNLO corrections, the missing higher orders in the QCD perturbative expansion still represent the dominant source of theoretical uncertainty!

TOTAL CROSS SECTION AND SCALE CHOICES

σ [μb]	$p\bar{p}$ @ 1.96 TeV	pp @ 7 TeV	pp @ 13 TeV
$\mu_0 = m_b$			
LO	$34.66\ ^{+51\%}_{-32\%}$	$138.7\ ^{+51\%}_{-46\%}$	$249.0\ ^{+59\%}_{-51\%}$
NLO	$60.23\ ^{+54\%}_{-28\%}$	$219.8\ ^{+61\%}_{-39\%}$	$378.6\ ^{+65\%}_{-45\%}$
NNLO	$75.4(3)\ ^{+22\%}_{-21\%}$	$288(2)\ ^{+30\%}_{-24\%}$	$508(3)\ ^{+32\%}_{-25\%}$
$\mu_0 = 2m_b$			
LO	$30.94\ ^{+41\%}_{-25\%}$	$145.8\ ^{+41\%}_{-32\%}$	$281.9\ ^{+41\%}_{-37\%}$
NLO	$51.16\ ^{+33\%}_{-23\%}$	$203.3\ ^{+36\%}_{-26\%}$	$362.9\ ^{+34\%}_{-28\%}$
NNLO	$66.7(2)\ ^{+21\%}_{-18\%}$	$258(1)\ ^{+20\%}_{-18\%}$	$458(2)\ ^{+20\%}_{-18\%}$

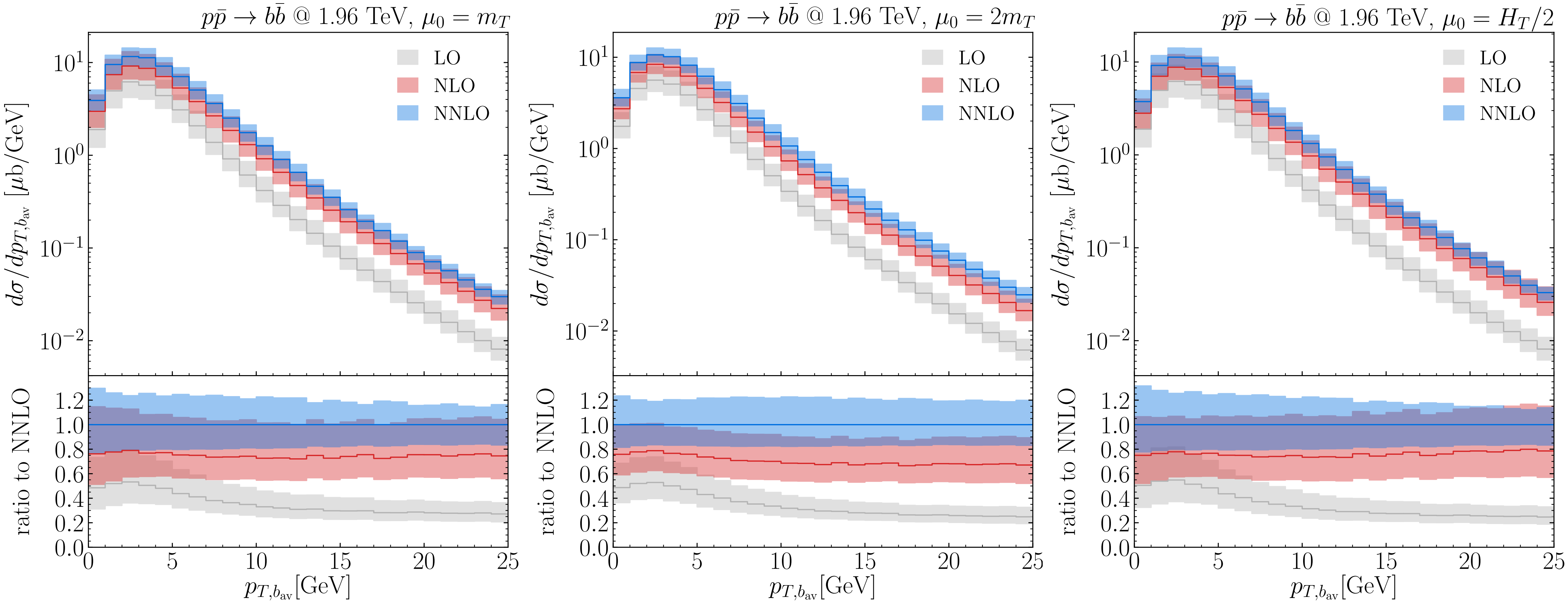
The use of the central scale $\mu_0 = m_b$ leads to **larger scale uncertainties** due to the lower values of μ_R involved.

Nevertheless, despite the low value of $m_b = 4.92$ GeV, we do **not** observe a worrisome perturbative behaviour.

DIFFERENTIAL DISTRIBUTIONS: TEVATRON

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]

Different central scale choices

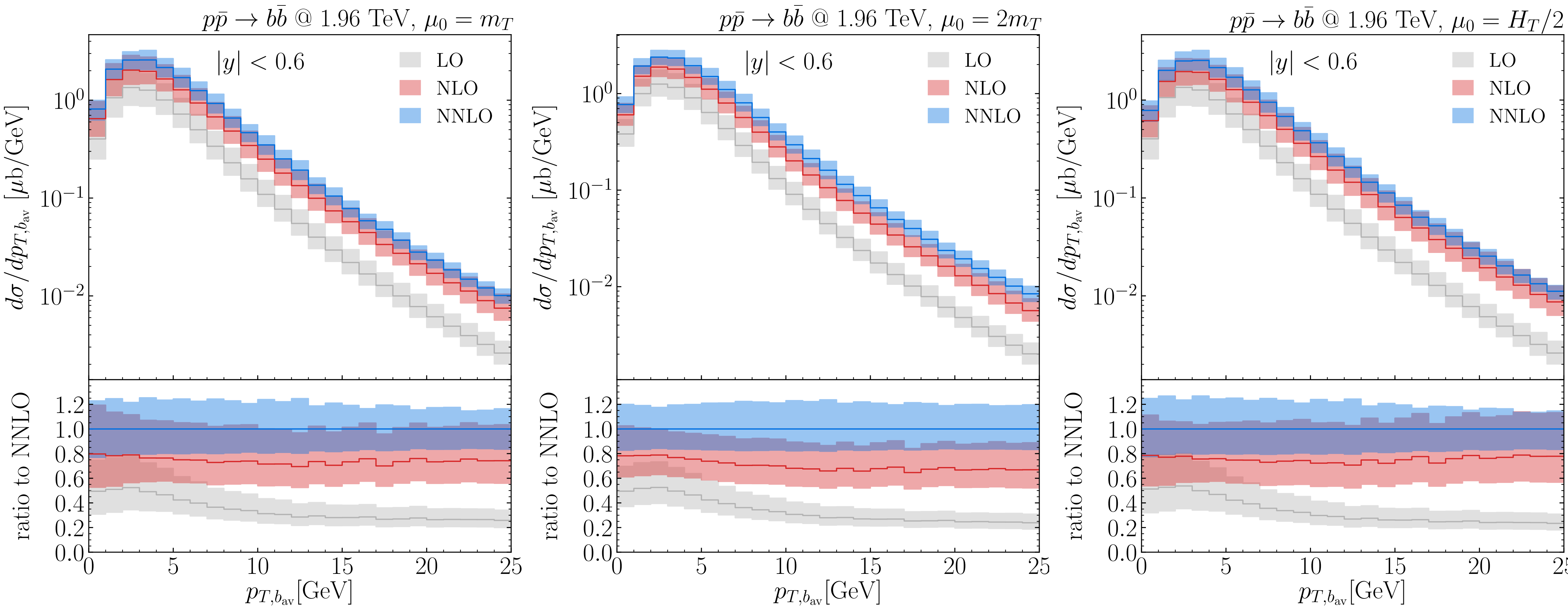


- We introduced the **dynamic scale** $\frac{1}{2}H_T = \frac{m_{T,b} + m_{T,\bar{b}}}{2}$;
- At LO, $H_T = m_T$: they indeed show a similar behaviour;
- The choice of a **smaller** scale $\mu_0 = m_T$ leads to a **better** overlap between NLO and NNLO uncertainty bands.

DIFFERENTIAL DISTRIBUTIONS: TEVATRON

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]

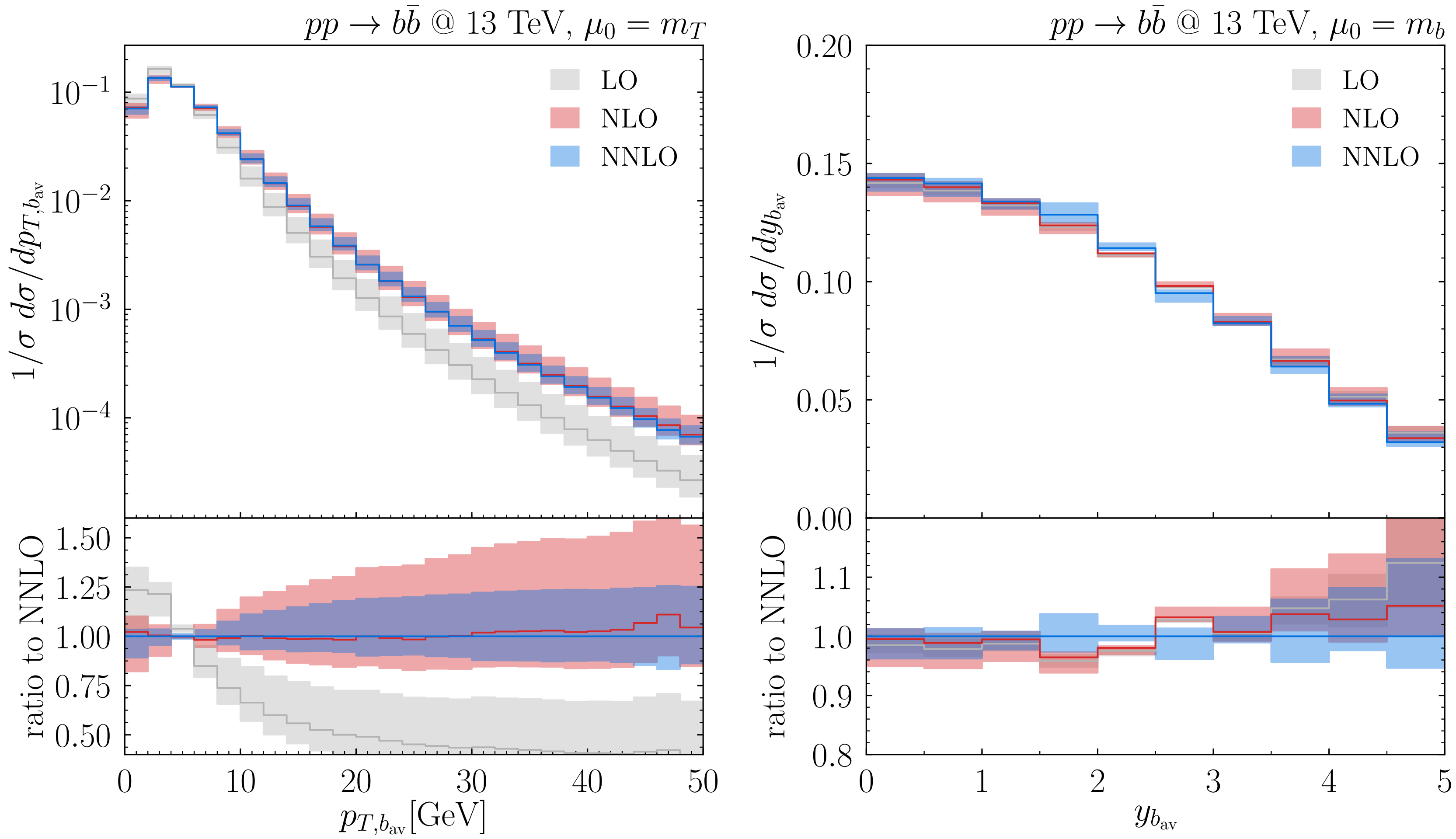
Different central scale choices



- Distributions with rapidity cut $|y| < 0.6$.

NORMALISED DISTRIBUTIONS: LHC

[S. Catani, SD, M. Grazzini ,S.Kallweit, J. Mazzitelli: JHEP 03 (2021) 029]

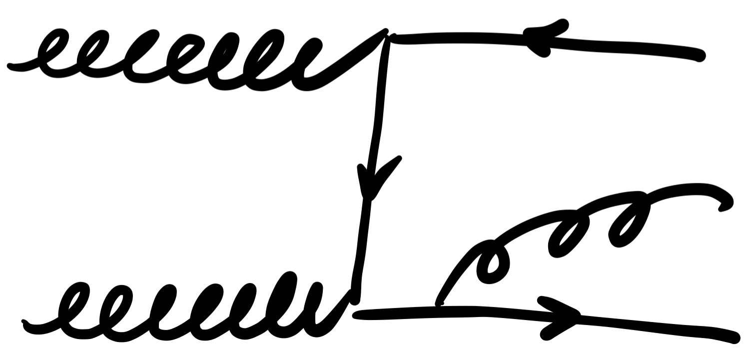


IR SINGULARITIES - NLO

$$\Delta\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1}\left[d\sigma^{R} - d\sigma^{CT}\right] + \left[\int_{m} d\sigma^{V} + \int_{1} d\sigma^{CT}\right]$$

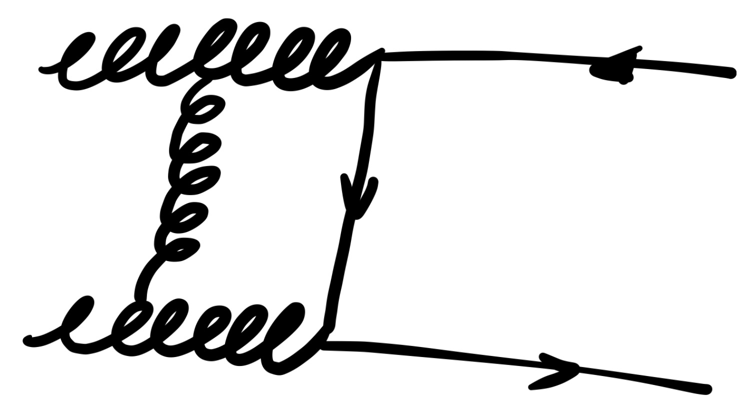
~~Divergent~~ CONVERGENT! ~~Divergent~~ CONVERGENT!

REAL



- No explicit ϵ poles;
- Singular in unresolved limit.

VIRTUAL

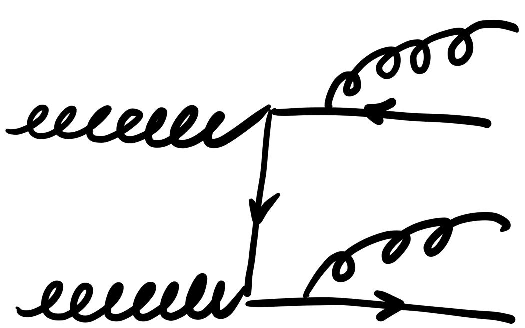


- Explicit poles up to order $1/\epsilon^2$;
- No additional PS singularity

IR SINGULARITIES - NNLO

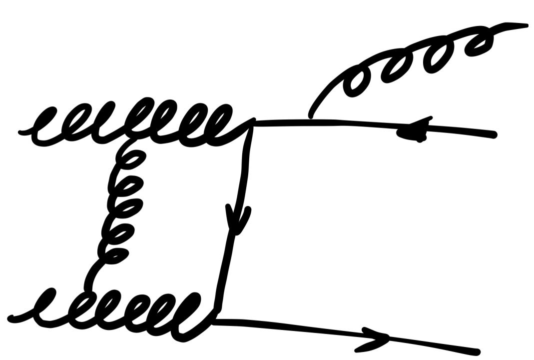
$$\Delta\sigma^{NNLO} = \underbrace{\int_{m+2} d\sigma^{RR}}_{\text{Divergent}} + \underbrace{\int_{m+1} d\sigma^{RV}}_{\text{Divergent}} + \underbrace{\int_{m} d\sigma^{VV}}_{\text{Divergent}}$$

More complicated structure due to overlapping singularities!



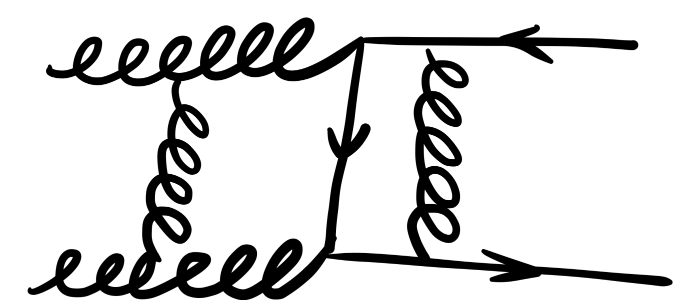
DOUBLE REAL

- No ϵ poles;
- Singular in (double) unresolved limit.



REAL - VIRTUAL

- Explicit $1/\epsilon^2$ poles;
- Singular in unresolved limit.



2LOOP VIRTUAL (AND 1LOOP SQUARED)

- Explicit $1/\epsilon^4$ poles;
- No additional PS singularity.

THE HARD-COLLINEAR COEFFICIENT

$$d\sigma^{F}_{(N)NLO} = \mathscr{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

Colourless final state

$$\mathscr{H} \propto \langle \tilde{M} | \tilde{M} \rangle$$

$$\left| \tilde{M} \right\rangle = (1 - I_C) \left| M \right\rangle$$

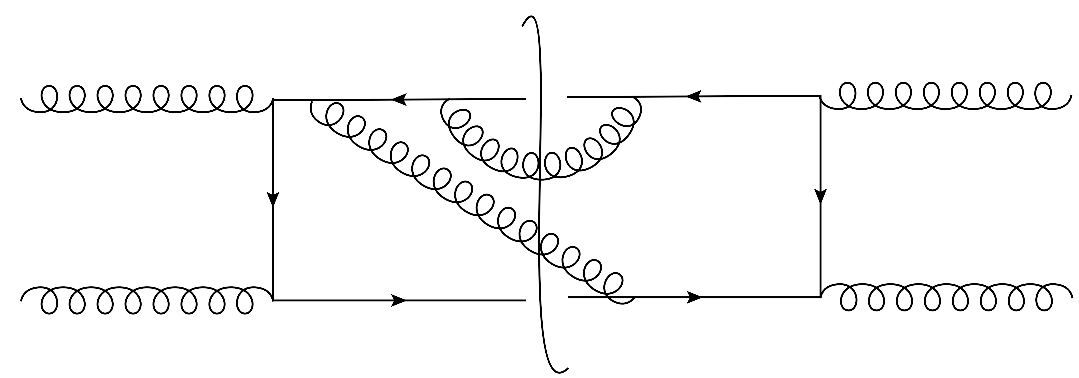
Subtraction operator
Removes the poles plus a finite part, taking into account what has been put in the counterterm.

All loop renormalised virtual amplitude

Colourful final state

$$\mathscr{H} \propto \langle \tilde{M} | \Delta | \tilde{M} \rangle$$

Additional soft radiative factor



- **COMPUTATION**: integration of the additional soft emission from the final state.

HARD COLLINEAR COEFFICIENT – NLO

[S. Catani, M. Grazzini, A. Torre: arXiv 1408:4564]

Computation of the soft contribution → *Integration of a suitably **subtracted** soft current.*

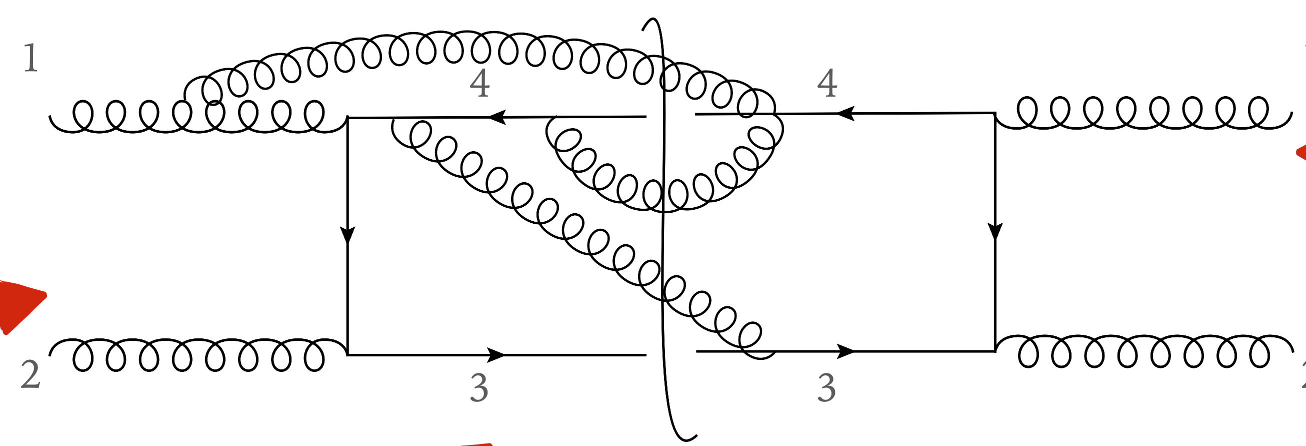
$$\langle \int \frac{d^n k}{2\pi^{n-1}} \delta_+(k) \left| J_{sub}(k) \right|^2 e^{i\vec{b}\cdot\vec{k}_T} \rangle_{\hat{b}}$$

Computation performed in Fourier space.

Average on the azimuthal degrees of freedom. $\hat{q}_T \to \hat{b}$.

- J_{sub} is the soft current after a proper subtraction of the (known) colourless contribution:

$$J_i^\mu(k) = \frac{p_i^\mu}{p_i \cdot k}$$



$$\left| J_{sub}(k) \right|^2 = \sum_{j=3,4} \frac{m_j^2}{(p_j \cdot k)^2} \mathbf{T}_j^2 + \frac{2\, p_3 \cdot p_4}{p_3 \cdot k\, p_4 \cdot k} \mathbf{T}_3 \cdot \mathbf{T}_4 + \sum_{\substack{i=1,2 \\ j=3,4}} \frac{2}{p_i \cdot k} \left(\frac{p_i \cdot p_j}{p_j \cdot k} - \frac{p_1 \cdot p_2}{(p_1 + p_2) \cdot k} \right) \mathbf{T}_i \cdot \mathbf{T}_j$$

HARD COLLINEAR COEFFICIENT – NLO

[S. Catani, M. Grazzini, A. Torre: arXiv 1408:4564]

Initial state contribution — *New soft contribution*

$$\tilde{I}^{(1)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon,\frac{M^2}{\mu_R^2}\right) = -\frac{1}{2}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}\left\{\left(\frac{1}{\epsilon^2}+i\pi\frac{1}{\epsilon}-\frac{\pi^2}{12}\right)(\mathbf{T}_1+\mathbf{T}_2)+\frac{2}{\epsilon}\gamma_c - \frac{4}{\epsilon}\mathbf{\Gamma}_t^{(1)}(y_{34}) + \mathbf{F}_t^{(1)}(y_{34})\right\}$$

Poles reproduce singularities in one-loop amplitudes

Required additional finite piece

- $$\mathbf{F}_t^{(1)}(y_{34}) = (\mathbf{T}_3^2+\mathbf{T}_4^2)\,\ln\left(\frac{m_T^2}{m^2}\right) + (\mathbf{T}_3+\mathbf{T}_4)^2\,\mathrm{Li}_2\left(-\frac{\mathbf{p}_\mathbf{T}^2}{m^2}\right) + \mathbf{T}_3\cdot\mathbf{T}_4\,\frac{1}{v}\,L_{34}$$

- $$\mathbf{\Gamma}_t^{(1)}(y_{34}) = -\frac{1}{4}\left\{(\mathbf{T}_3^2+\mathbf{T}_4^2)\,(1-i\pi) + \sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i\cdot\mathbf{T}_j\,\ln\frac{(2p_i\cdot p_j)^2}{M^2m^2} + 2\,\mathbf{T}_3\cdot\mathbf{T}_4\left[\frac{1}{2v}\ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v}+1\right)\right]\right\}\,.$$

- $$L_{34} = \ln\left(\frac{1+v}{1-v}\right)\ln\left(\frac{m_T^2}{m^2}\right) - 2\,\mathrm{Li}_2\left(\frac{2v}{1+v}\right) - \frac{1}{4}\ln^2\left(\frac{1+v}{1-v}\right) + 2\left[\mathrm{Li}_2\left(1-\sqrt{\frac{1-v}{1+v}}\,e^{y_{34}}\right) + \mathrm{Li}_2\left(1-\sqrt{\frac{1-v}{1+v}}\,e^{-y_{34}}\right) + \frac{1}{2}y_{34}^2\right]$$

$$v = \sqrt{1-\frac{m^4}{(p_3\cdot p_4)^2}}$$

$$y_{34} = y_3 - y_4$$

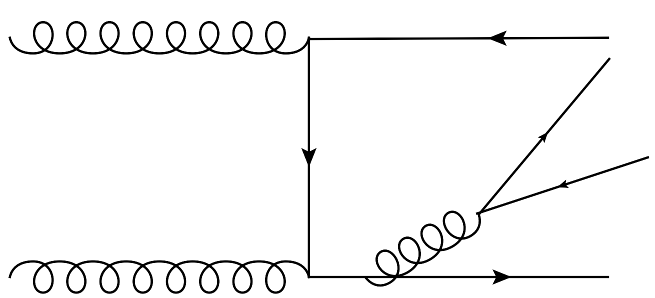
We need to do the same at the next order!

HARD COLLINEAR COEFFICIENT - NNLO

[S. Catani, SD, M.Grazzini, J.Mazzitelli, in preparation.]

NNLO: the soft current is more complicated. Contributions from:

- **Light quark pair production:**

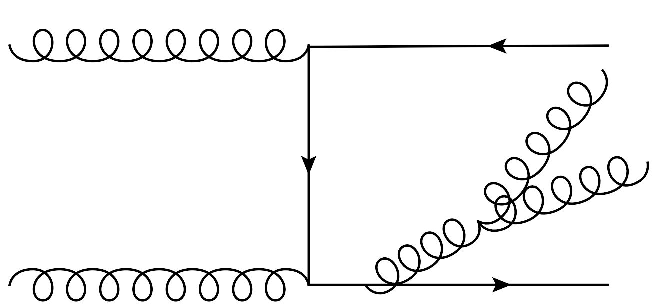


$$J_{sub}^{NNLO(q\bar{q})}(k_1,k_2)$$

[S.Catani, M.Grazzini (1999)]

$$\int \frac{d^nk_1}{2\pi^{n-1}}\frac{d^nk_2}{2\pi^{n-1}}\delta_+(k_1)\delta_+(k_2)\left|J_{sub}^{NNLO(q\bar{q})}(k_1,k_2)\right|^2 e^{i\vec{b}\cdot\left(\vec{k}_{T1}+\vec{k}_{T2}\right)}$$

- **Double gluon emission:**

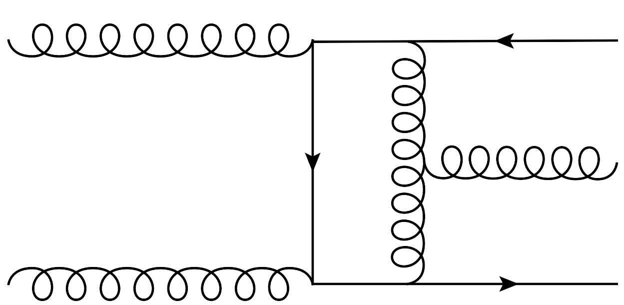


$$J_{sub}^{NNLO(gg)}(k_1,k_2)$$

[S.Catani, M.Grazzini (1999); M. Czakon (2011)]

$$\int \frac{d^nk_1}{2\pi^{n-1}}\frac{d^nk_2}{2\pi^{n-1}}\delta_+(k_1)\delta_+(k_2)\left|J_{sub}^{NNLO(gg)}(k_1,k_2)\right|^2 e^{i\vec{b}\cdot\left(\vec{k}_{T1}+\vec{k}_{T2}\right)}$$

- **One gluon emission at 1 loop:**



$$J_{sub}^{NNLO(1L)}(k)$$

[I. Bierenbaum, M. Czakon, A. Mitov (2011)]

$$\int \frac{d^nk}{2\pi^{n-1}}\delta_+(k)\left|J_{sub}^{NNLO(1L)}(k)\right|^2 e^{i\vec{b}\cdot\vec{k}_T}$$

AN EXAMPLE: DOUBLE GLUON EMISSION

Square of the soft current:

$$\left| J^{NNLO(gg)}(k_1,k_2) \right|^2 = \frac{1}{2}\left\{ \mathbf{J}^2(k_1), \mathbf{J}^2(k_2) \right\} - C_a \sum_{i,j=1}^{n} \mathbf{T}_i \cdot \mathbf{T}_j \, \mathcal{S}_{ij}(k_1,k_2)$$

$$\mathcal{S}_{ij}(k_1,k_2) = \mathcal{S}_{ij}^{m=0}(k_1,k_2) + \left(m_i^2 \mathcal{S}_{ij}^{m\neq 0}(k_1,k_2) + m_j^2 \mathcal{S}_{ji}^{m\neq 0}(k_1,k_2) \right)$$

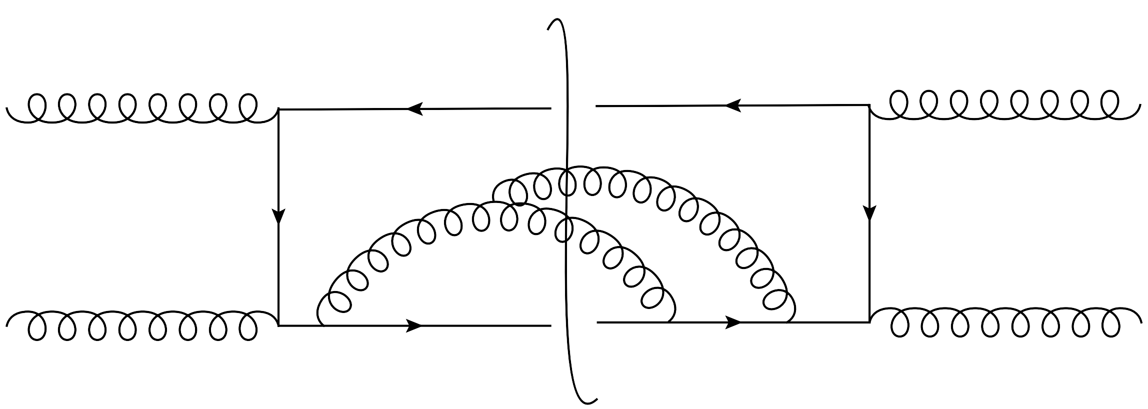
$$\begin{aligned}
\mathcal{S}_{ij}^{m=0}(q_1,q_2) = & \frac{(1-\epsilon)}{(q_1\cdot q_2)^2} \frac{p_i\cdot q_1\, p_j\cdot q_2 + p_i\cdot q_2\, p_j\cdot q_1}{p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)} \\
& - \frac{(p_i\cdot p_j)^2}{2\, p_i\cdot q_1\, p_j\cdot q_2\, p_i\cdot q_2\, p_j\cdot q_1} \left[2 - \frac{p_i\cdot q_1\, p_j\cdot q_2 + p_i\cdot q_2\, p_j\cdot q_1}{p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)} \right] \\
& + \frac{p_i\cdot p_j}{2\, q_1\cdot q_2} \left[\frac{2}{p_i\cdot q_1\, p_j\cdot q_2} + \frac{2}{p_j\cdot q_1\, p_i\cdot q_2} - \frac{1}{p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)} \right. \\
& \left. \times \left(4 + \frac{(p_i\cdot q_1\, p_j\cdot q_2 + p_i\cdot q_2\, p_j\cdot q_1)^2}{p_i\cdot q_1\, p_j\cdot q_2\, p_i\cdot q_2\, p_j\cdot q_1} \right) \right]
\end{aligned}$$

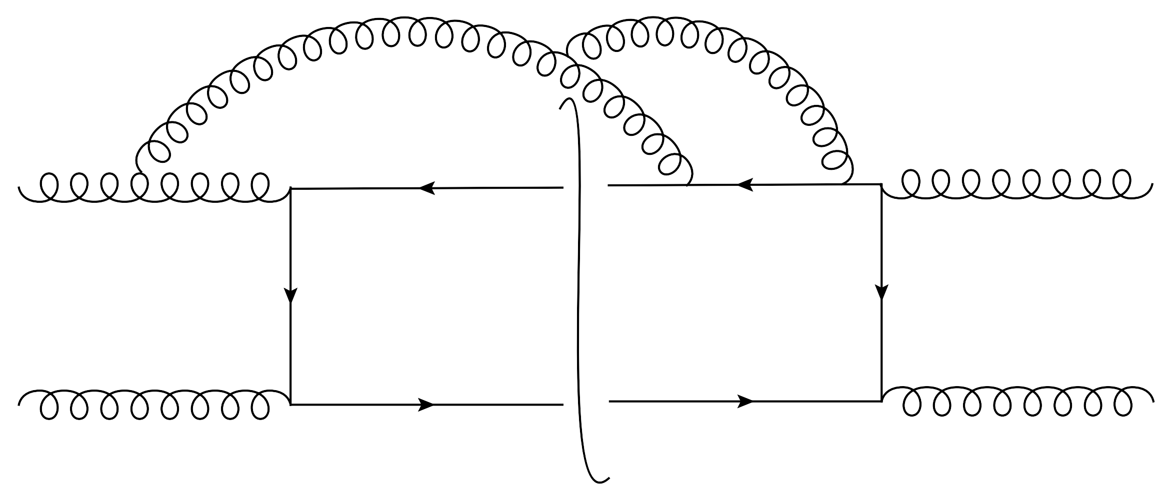
$$\begin{aligned}
\mathcal{S}_{ij}^{m\neq 0}(q_1,q_2) = & -\frac{1}{4\, q_1\cdot q_2\, p_i\cdot q_1\, p_i\cdot q_2} + \frac{p_i\cdot p_j\, p_j\cdot(q_1+q_2)}{2\, p_i\cdot q_1\, p_j\cdot q_2\, p_i\cdot q_2\, p_j\cdot q_1\, p_i\cdot(q_1+q_2)} \\
& - \frac{1}{2\, q_1\cdot q_2\, p_i\cdot(q_1+q_2)\, p_j\cdot(q_1+q_2)} \left(\frac{(p_j\cdot q_1)^2}{p_i\cdot q_1\, p_j\cdot q_2} + \frac{(p_j\cdot q_2)^2}{p_i\cdot q_2\, p_j\cdot q_1} \right)
\end{aligned}$$

FINAL RESULT - RECAP

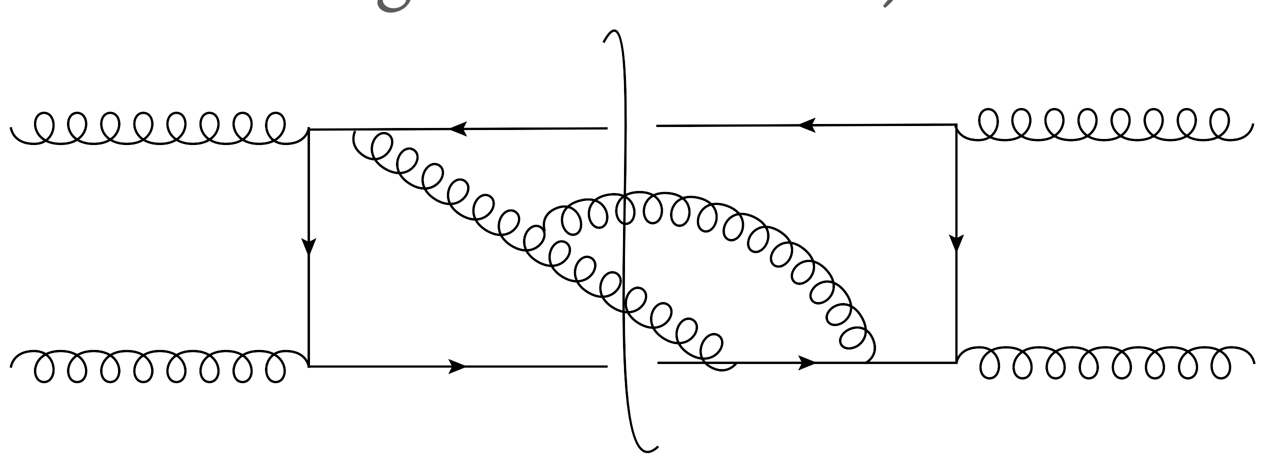
We computed all the needed integrals:

- **Analytic expression** for $\mathbf{T}_i\mathbf{T}_j\,,\mathbf{T}_j\mathbf{T}_j$ contributions:





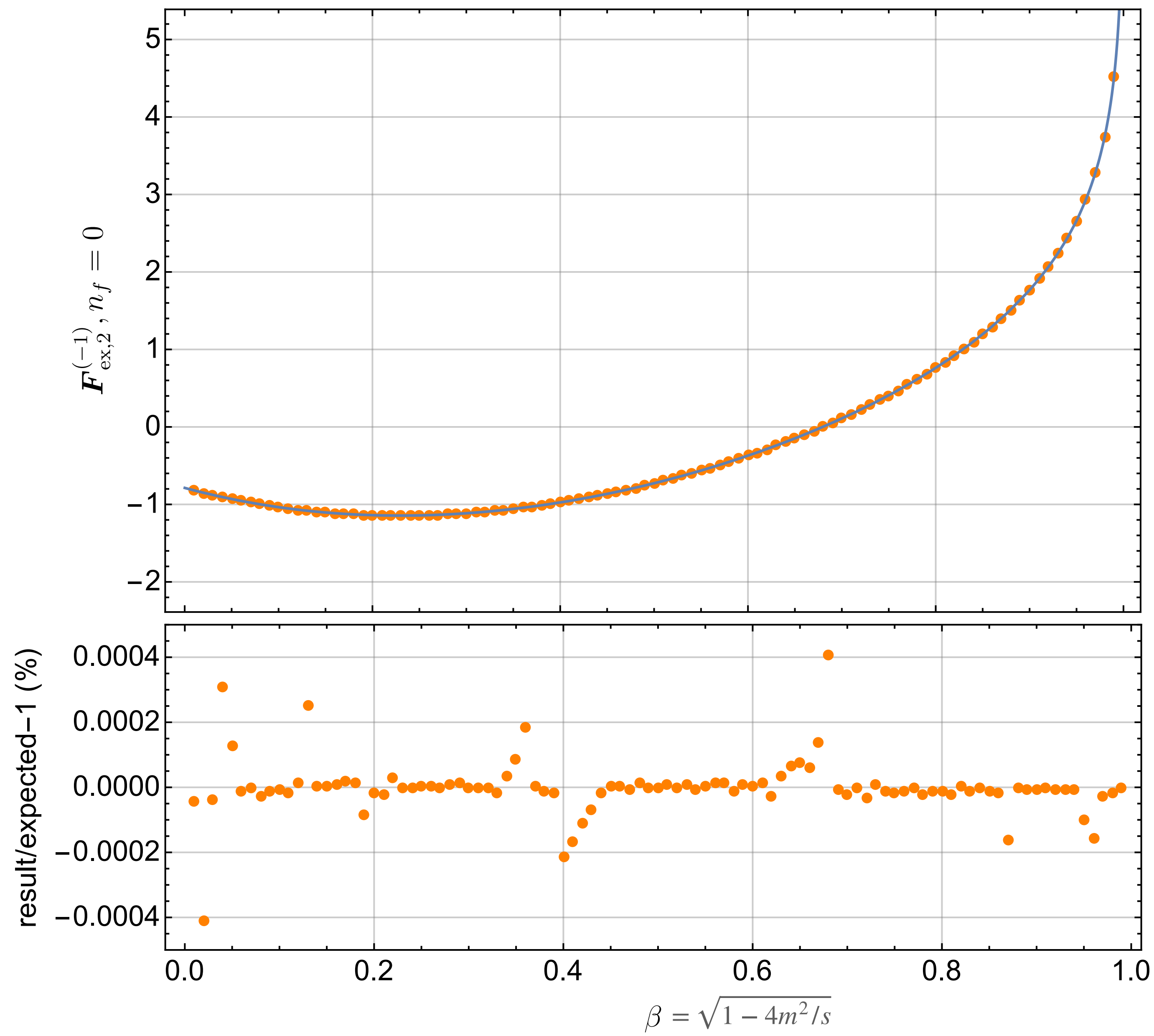
- **Numerical expression** for some pieces of the $\mathbf{T}_3\mathbf{T}_4$ contribution (regular part of the double gluon emission):



FINAL RESULT – POLES CANCELLATION

Pole structure can be predicted from 2-loop virtual contribution: it must cancel (part of) its IR singularities.

- **Triple poles**: cancel combining the 3 different contributions;
- **Double poles**: analytic cancellation;
- **Single** poles: analytic cancellation except $\mathbf{T}_3\mathbf{T}_4$.



The completion of this calculation allowed the implementation of **top pair production** and **bottom pair production** in the **MATRIX** framework!