The role of NLP threshold corrections in dQCD and SCET

Melissa van Beekveld

2101.07270 (with Eric Laenen, Jort Sinninghe Damste, Leonardo Vernazza)

Threshold expansion of cross sections

Consider hadronic observable of colour-singlet production $\sigma_{pp\to H+X}$

$$\mathrm{d}\sigma_{pp\to H+X} = \sigma_0^H \sum_{i,j} \int_\tau^1 \frac{\mathrm{d}z}{z} \mathscr{L}_{ij}\left(\frac{\tau}{z}\right) \Delta_{H,ij}(z) \qquad \tau = \frac{m_H^2}{S} = x_1 x_2 z$$

Threshold expansion of partonic coefficient function up to leading power (LP):

$$\Delta_{H,ij}(z) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(ij),\mathrm{LP}} \left(\frac{\ln^m(1-z)}{1-z}\right)_+ + c_n^\delta \delta(1-z) + \dots$$

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- Universal process-independent form
- Localized at threshold
- Linked to the soft and collinear divergences
- Resummation for all logarithmic accuracies well understood

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- Less well understood, general origin unclear
- No general resummation framework, LL for colour-singlet is available

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Understanding them is important because:

- Provide check of higher-order corrections
- Help to reduce scale uncertainties
- Stabilize automated fixed-order calculations
- Are sizable numerically

Resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

Partonic cross section at LP:

$$\mathrm{d}\sigma = \frac{1}{2s}\left[\int \mathrm{d}\Phi_{\mathrm{LP}}\,|\,\mathscr{M}\,|^2_{\mathrm{LP}} + \ldots\right]$$

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LP matrix element for DY and Higgs is governed by *soft emissions* only, which can be factorised from the hard scattering

$$\longrightarrow \quad |\mathscr{M}|^2_{\mathrm{LP}} = S_{\mathrm{eik,LP}} \times |\mathscr{M}_{\mathrm{LO}}|^2$$

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★Phase space for n soft-gluon radiations factorises (e.g. Forte and Ridolfi, 2002)

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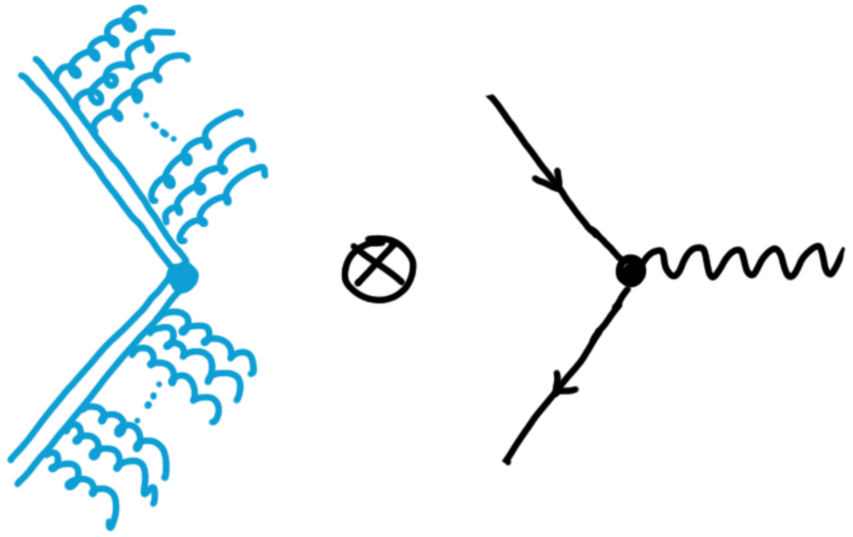
$\longrightarrow$ 

$$\sigma \propto \sigma_{\mathrm{hard}}(z, s) \times \sigma^{\mathrm{eik}}(z) \quad \text{with} \quad \sigma^{\mathrm{eik}} \propto \exp\left[\left[\int S_{\mathrm{LP}}(z)\right]\right]$$

Resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

$$\sigma^{\text{eik}} \propto \exp\left[\int S_{\text{LP}}(z)\right] \quad \text{with} \quad \sigma \propto \sigma_{\text{hard}}(z, s) \times \sigma^{\text{eik}}(z)$$

To obtain a closed form for the phase-space integrals: go to Mellin space

$$\int_0^1 \mathrm{d}z\, f(z)\, z^{N-1}$$

Threshold limit $z \to 1$ *'selected' for* $N \to \infty$

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Resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

$$\sigma^{\mathrm{res,LP}} = \sigma_{\mathrm{hard}} \exp\left[\int_0^1 \mathrm{d}z^{N-1} S_{\mathrm{LP}}(z)\right]$$

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Soft-collinear contributions (splitting functions)

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wide-angle contributions

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$$= \sigma_{\mathrm{hard}} \exp\left[\frac{2}{\alpha_s} g_a^{(1)}(\lambda) + g_a^{(2)}(\lambda) + \ldots\right]$$

NLP resummation for colour-singlet processes

[1905.13710]

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

Partonic cross section at NLP:

$$\mathrm{d}\sigma = \frac{1}{2s}\left[\int \mathrm{d}\Phi_{\mathrm{LP}}\,|\mathscr{M}|^2_{\mathrm{LP}} + \int \mathrm{d}\Phi_{\mathrm{LP}}\,|\mathscr{M}|^2_{\mathrm{NLP}} + \int \mathrm{d}\Phi_{\mathrm{NLP}}\,|\mathscr{M}|^2_{\mathrm{LP}} + \ldots\right]$$

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NLL only!

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LL

This contains only next-to-soft corrections at LL,
non-soft NLP enhancements are NLP NLL (and beyond)

[1410.6406, 1807.09246]

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Factorised ('external') next-to-soft-gluon emissions exponentiate [0811.2067,1010.1860]

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[1706.04018,1905.08741]

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$$\boxed{\sigma^{\mathrm{res,NLP}} = \sigma_{\mathrm{hard}} \exp\left[\int_0^1 \mathrm{d}z^{N-1} z\, S_{\mathrm{LP}}(z)\right]}$$

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$$P_{ii}^{\rm NLP} = \frac{\alpha_s}{2\pi} C_i \left[\left(\frac{1}{1-z}\right)_+ - 1 + \cdots\right] + \mathcal{O}(\alpha_s^2)$$

Key is that the LL LP and NLP contributions come from a pole in ϵ
that needs to be absorbed in parton distribution functions
$\rightarrow$the NLP expansion of the splitting function generates this information

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NLP resummation for colour-singlet processes

[1905.13710]

$$\sigma^{\text{res,NLP LL}} = \sigma_{\text{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1} \int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\text{NLP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1} \frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

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[2101.07270]

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[2101.07270]

NLP can be obtained from the LP with a derivative: $h_a^{(1)}(\lambda, N) = \frac{1}{2\alpha_s}\frac{\partial}{\partial N} g_a^{(1)}(\lambda)$

A subset of NLP contributions at arbitrary log order can be obtained this way

$$E^{\text{LP+NLP}} = \left(1 + \frac{1}{2}\frac{\partial}{\partial N}\right) \int_0^1 \mathrm{d}z^{N-1} \int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\text{LP}}(z, \alpha_s(q^2))$$

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Note that this only works at NLP LL for 'LP-induced' colour-singlet processes:

★ Beyond LL the phase space needs to be modified (leading to $Q^2(1-z)^2 \to Q^2(1-z)^2/z$)

★ What is the contribution from non-soft collinear emissions?

★ The qg-induced channels are not considered here

★ The kinematic shift for channels with more than two coloured legs is not factorisable

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Consider single Higgs and DY

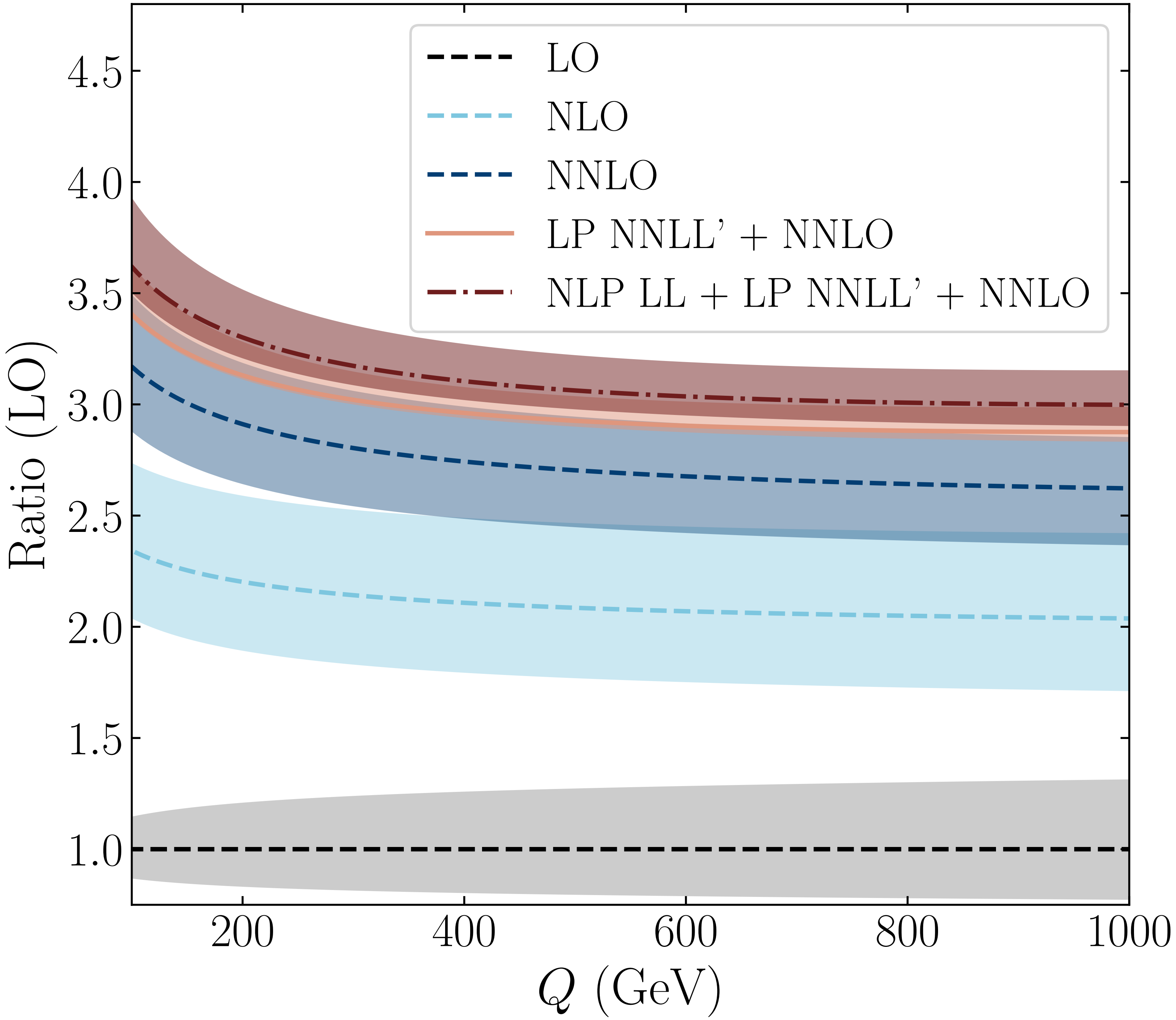
We take both processes at NNLL + NLP LL resummed and match to NNLO

Use PDF4LHC NNLO PDF set

Set $\mu_R = \mu_F$

Verified our set-up with the results from existing codes

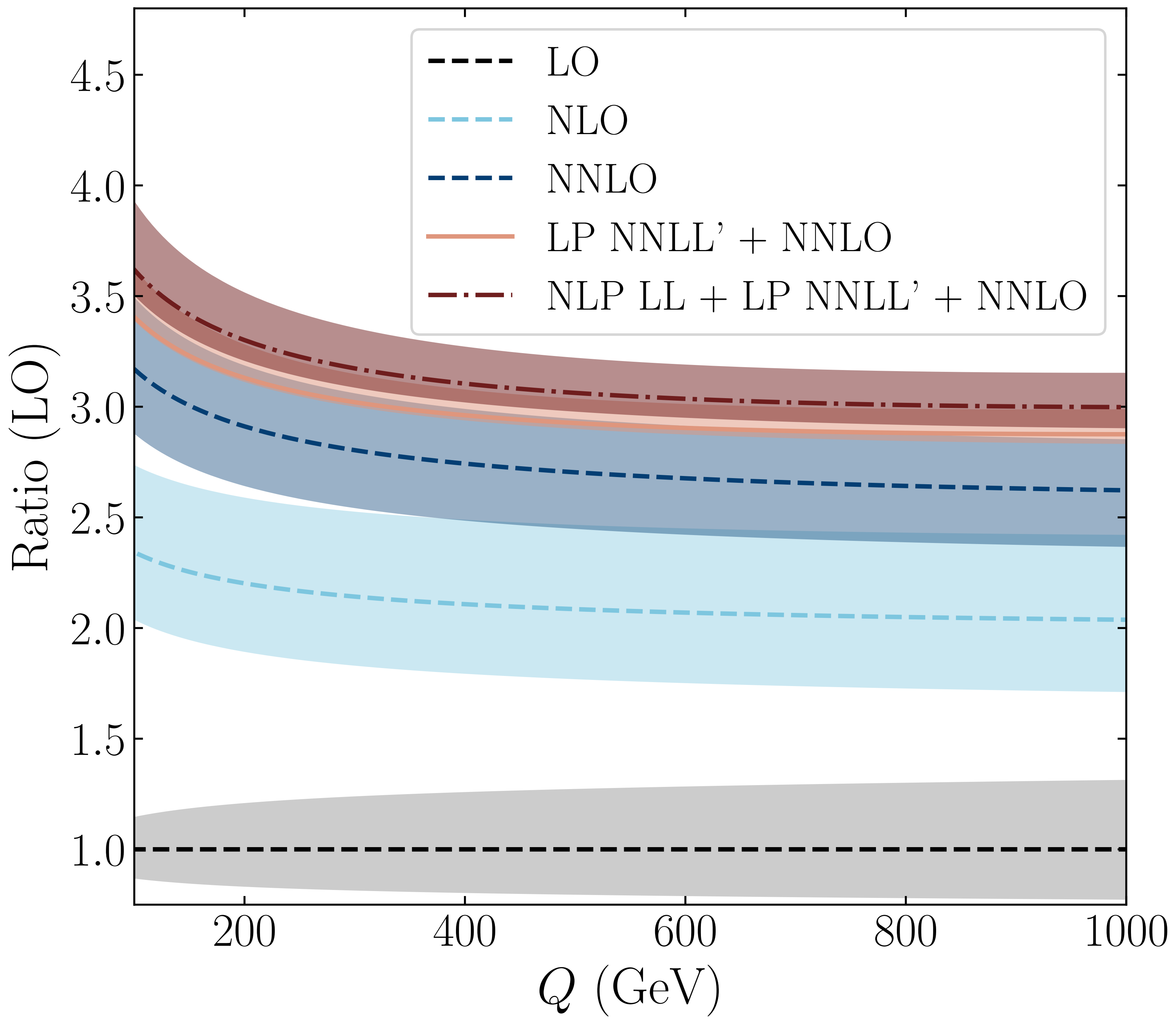
Consider single Higgs and DY

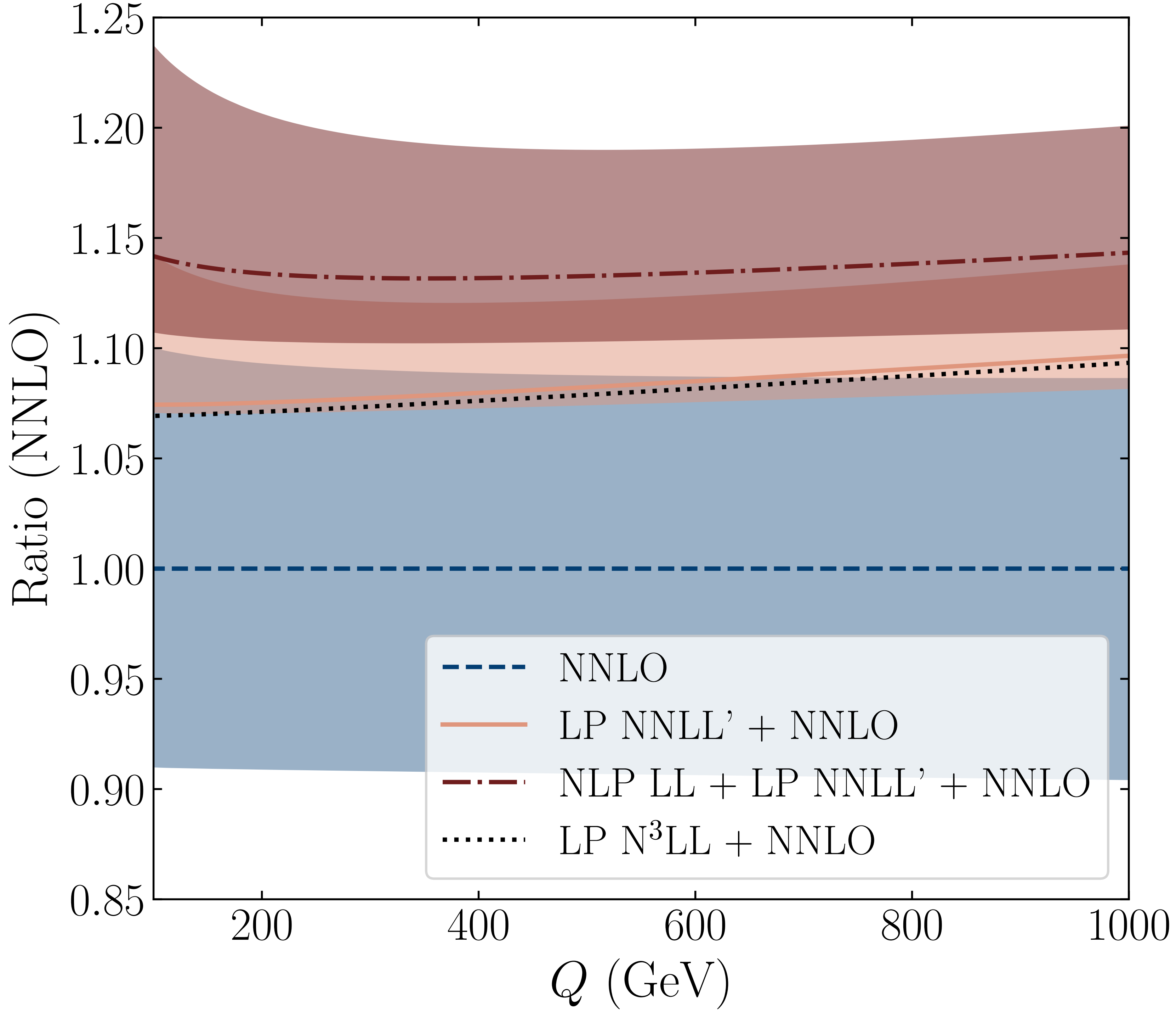


We vary $Q = m_h$

It seems that the NLP contribution is indeed subleading…

Consider single Higgs and DY





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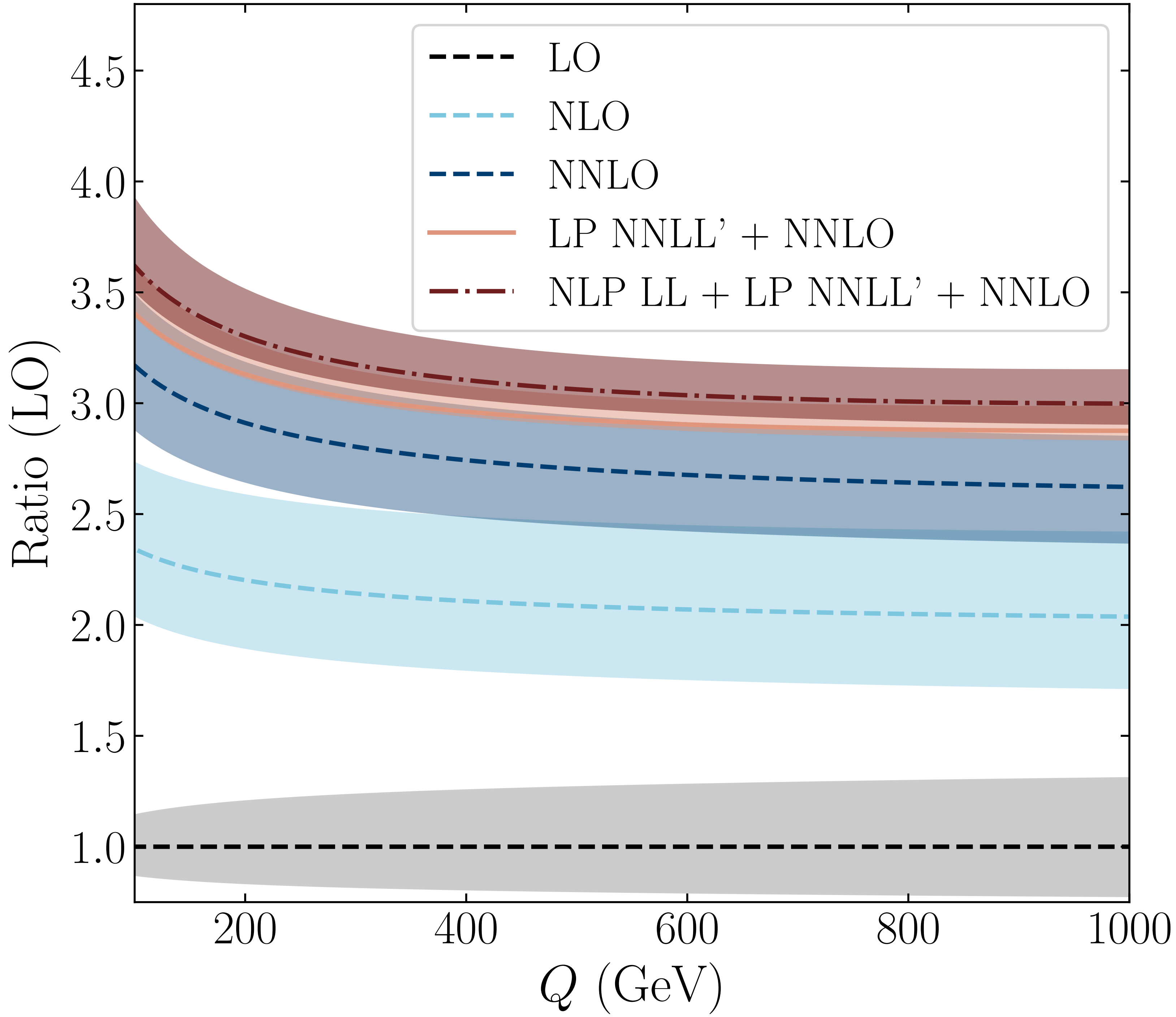
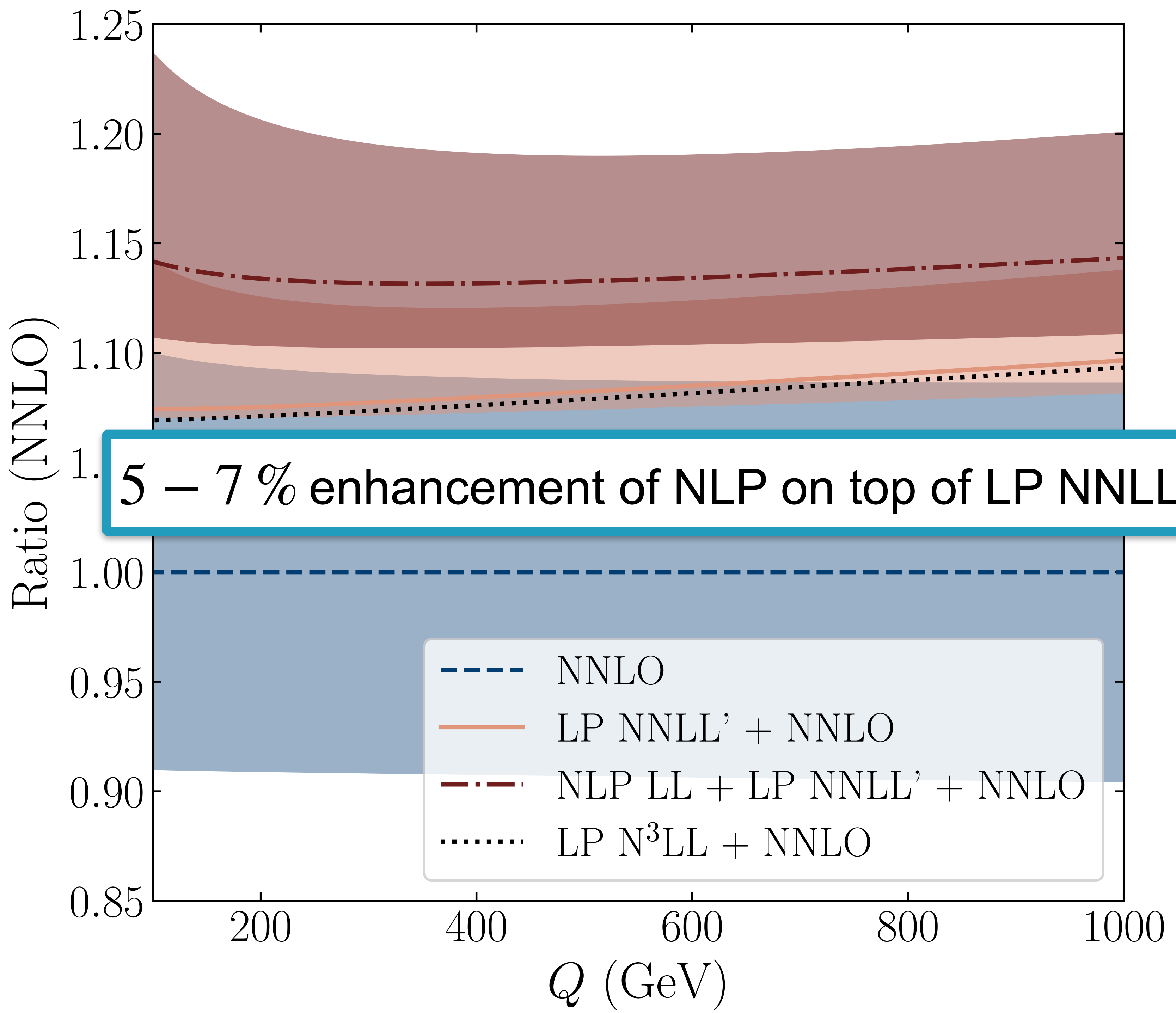
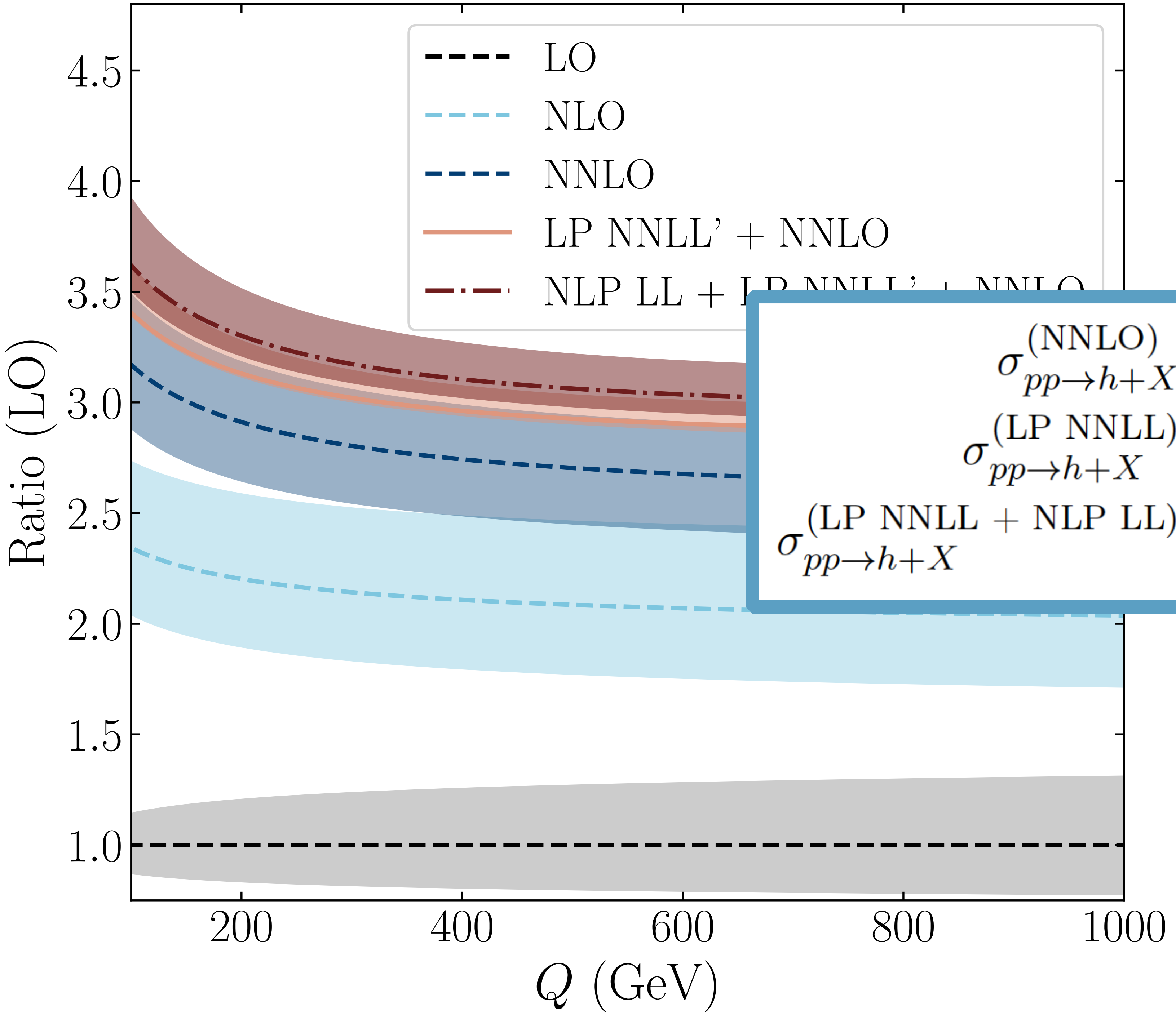
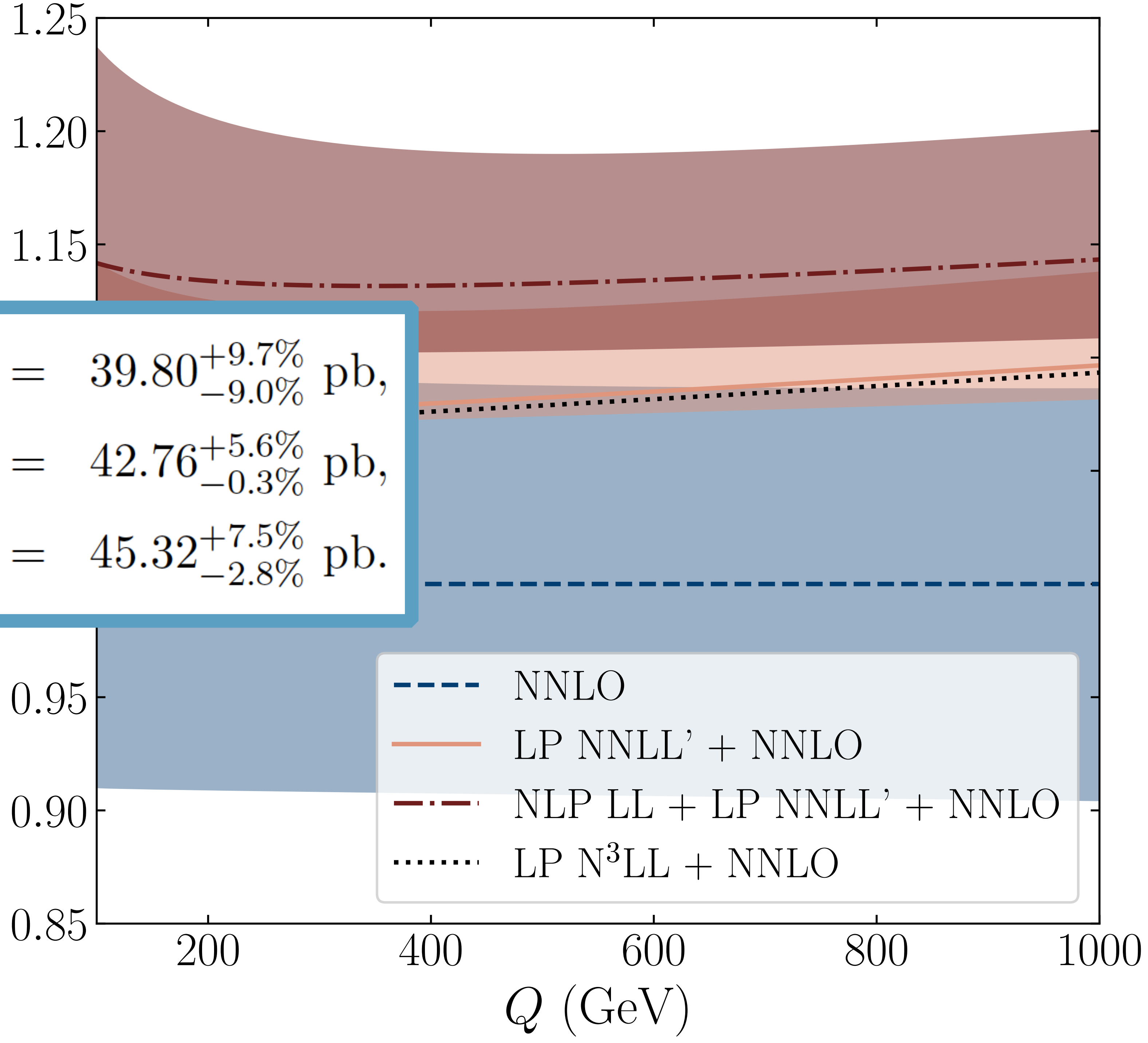




$5-7\,\%$ enhancement of NLP on top of LP NNLL

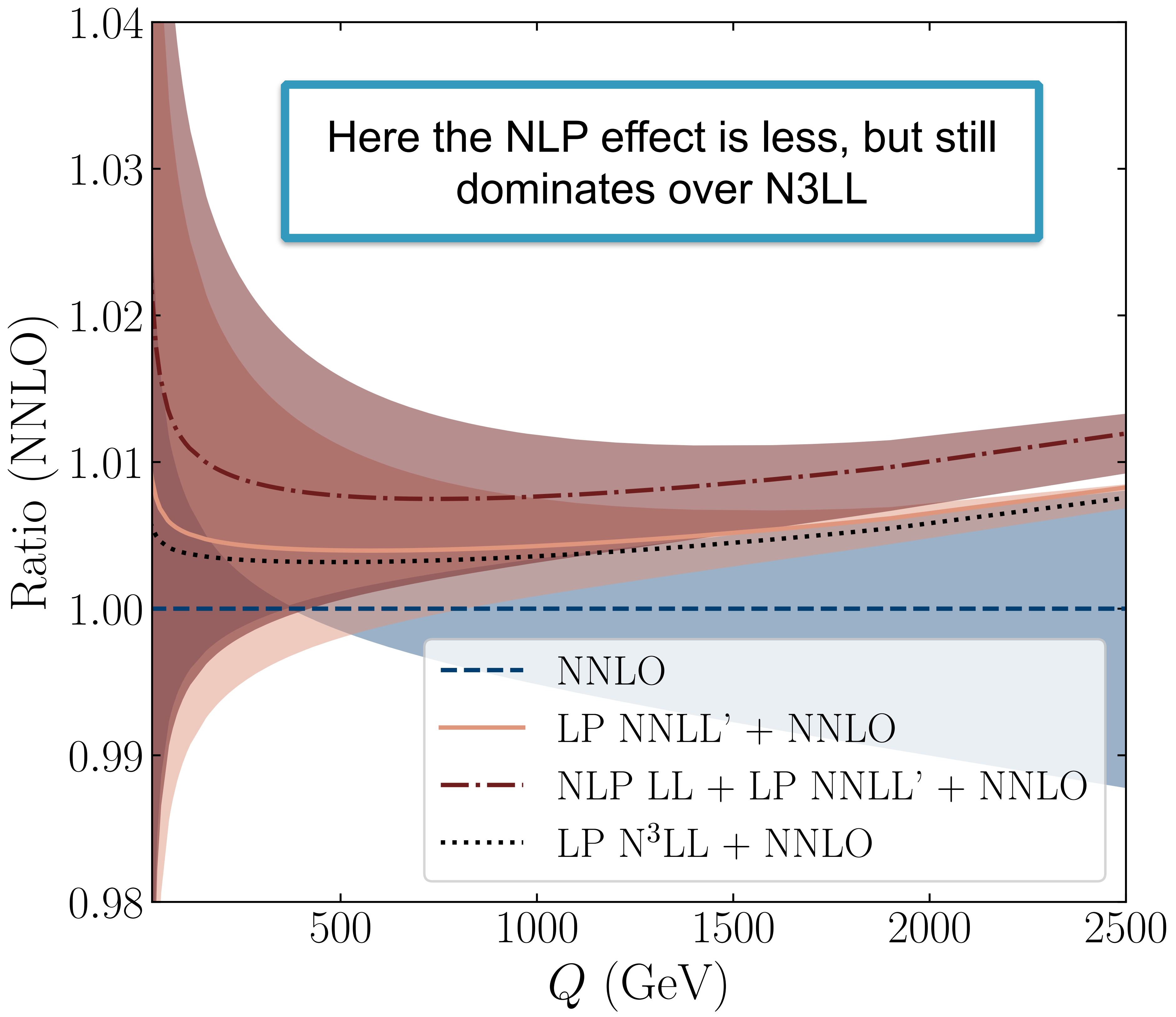
Consider single Higgs and DY





$$
\begin{aligned}
\sigma^{(\mathrm{NNLO})}_{pp\to h+X}(m_\mathrm{h}) &= 39.80^{+9.7\%}_{-9.0\%}\ \mathrm{pb},\\
\sigma^{(\mathrm{LP\ NNLL})}_{pp\to h+X}(m_\mathrm{h}) &= 42.76^{+5.6\%}_{-0.3\%}\ \mathrm{pb},\\
\sigma^{(\mathrm{LP\ NNLL\ +\ NLP\ LL})}_{pp\to h+X}(m_\mathrm{h}) &= 45.32^{+7.5\%}_{-2.8\%}\ \mathrm{pb}.
\end{aligned}
$$

Consider single Higgs and DY



What about qg channels?

We don’t know their resummation, but we can add the NLP LL $\mathcal{O}(\alpha_s^3)$ term

[1005.1606, 1407.1553, 2008.04943]

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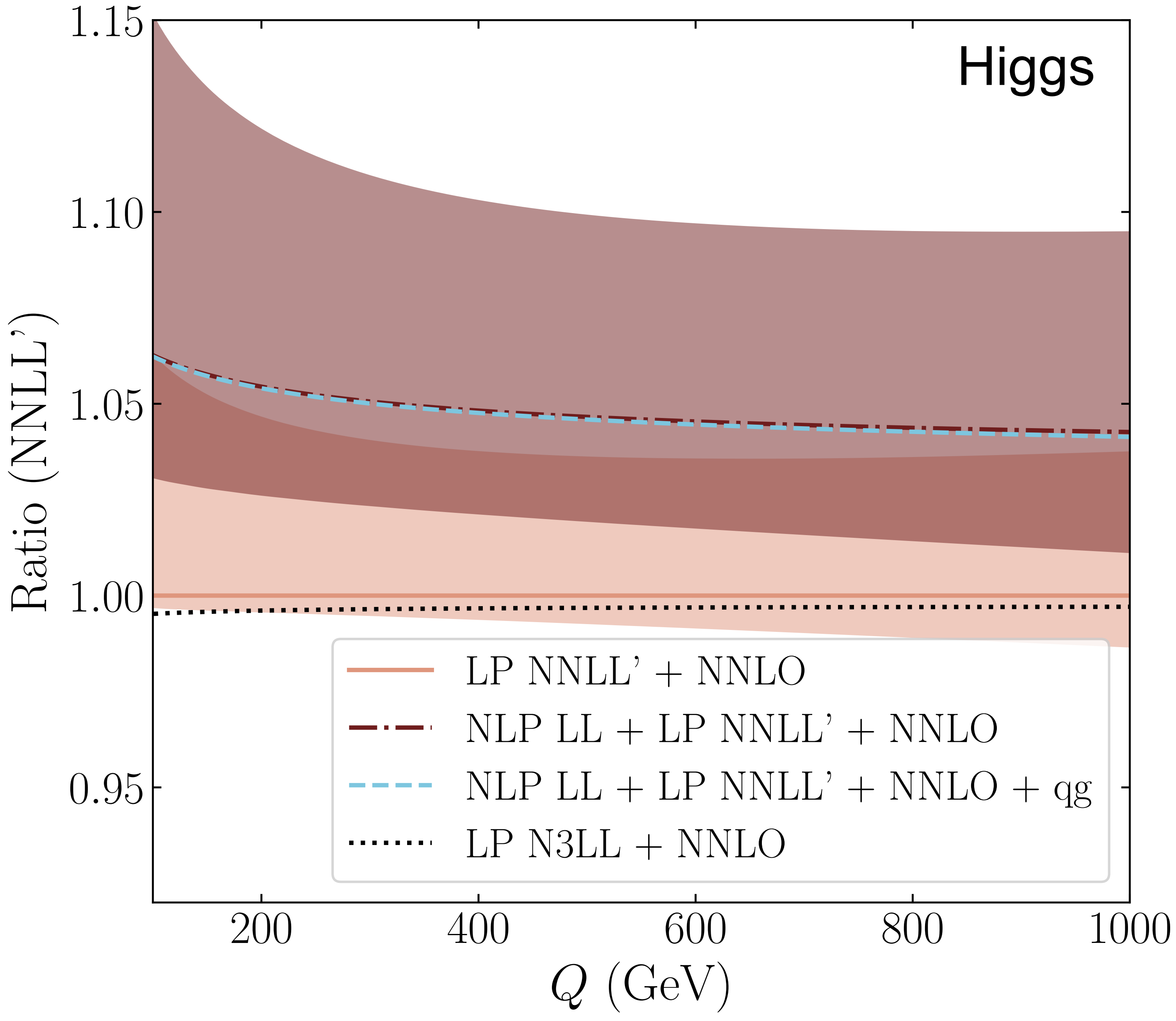
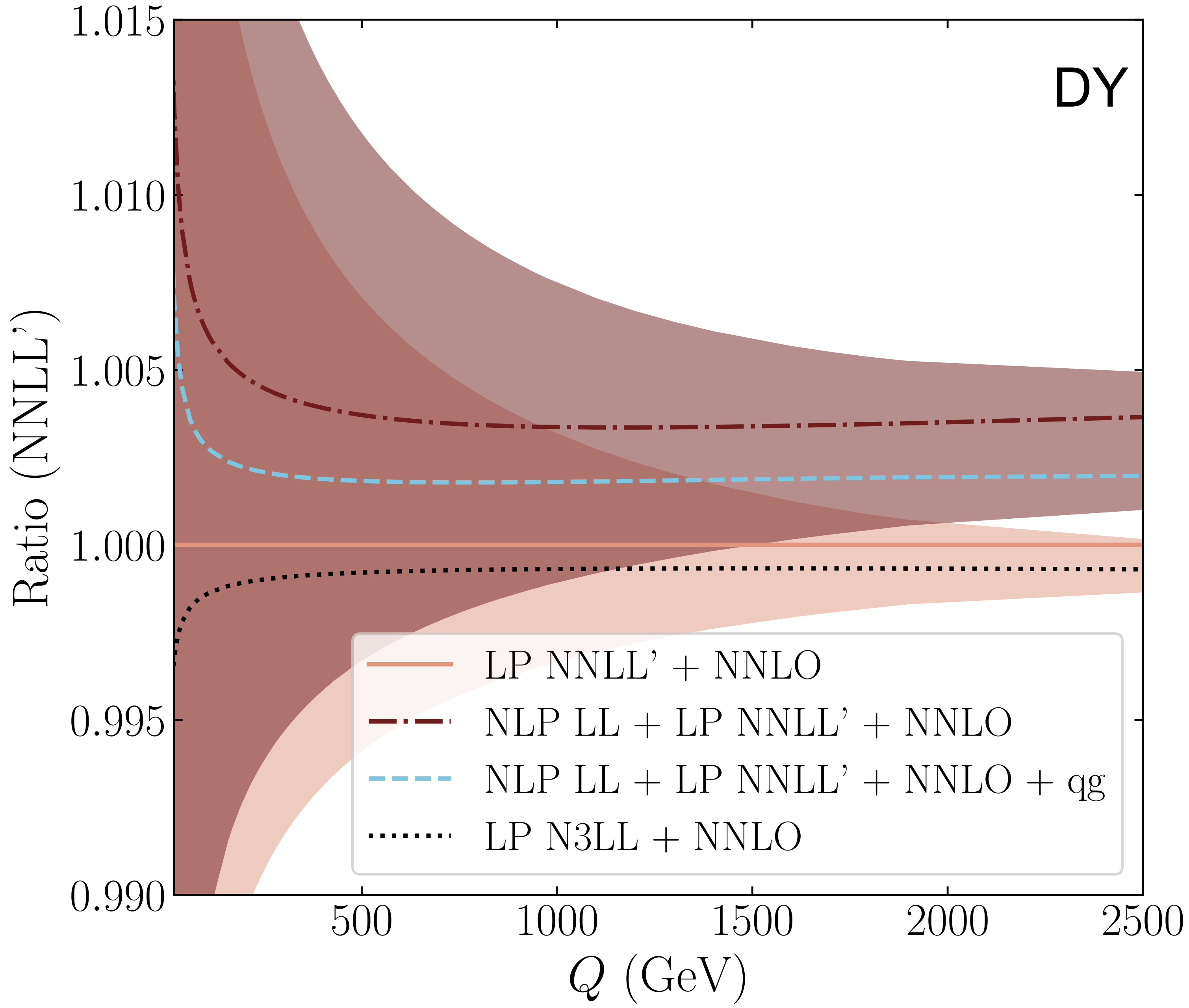




SCET vs dQCD at NLP

– Numerical differences can become sizable between two approaches at LP

– Shown that these differences originate from power-suppressed contributions

[0601048, 0809.4283, 1201.6364, 1301.4502, 1409.0864]

– Can we obtain analytical and numerical agreement at NLP LL?

SCET vs dQCD at NLP

Resummation at LP: [0710.0680, 0809.4283]

$$\Delta^{\mathrm{SCET,LP}} = H(Q,\mu)U(Q,\mu_s,\mu)\tilde{s}\left(\ln\frac{Q^2}{\mu_s^2}+\frac{\partial}{\partial\eta},\mu_s\right)\frac{z^{-\eta}}{(1-z)^{1-2\eta}}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

Resummation at NLP: $\Delta^{\mathrm{SCET,LP+NLP}} = \Delta^{\mathrm{SCET,LP}} + \Delta^{\mathrm{SCET,NLP}}$ [1809.10631,1910.12685]

SCET vs dQCD at NLP

Resummation at LP: [0710.0680, 0809.4283]

$$\Delta^{\mathrm{SCET,LP}} = H(Q,\mu)U(Q,\mu_s,\mu)\tilde{s}\left(\ln\frac{Q^2}{\mu_s^2}+\frac{\partial}{\partial\eta},\mu_s\right)\frac{z^{-\eta}}{(1-z)^{1-2\eta}}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

Resummation at NLP: $\Delta^{\mathrm{SCET,LP+NLP}} = \Delta^{\mathrm{SCET,LP}} + \Delta^{\mathrm{SCET,NLP}}$ [1809.10631,1910.12685]

$$\Delta^{\mathrm{SCET,NLP}} = -\beta(\alpha_s(\mu_s^2))\frac{\partial}{\partial\alpha_s(\mu_s^2)}U_{\mathrm{LL}}(Q,\mu_s)$$ [2101.07270]

As in the dQCD case, NLP contributions can be obtained directly from the LP ones with a derivative. In N-space, these forms are identical!

SCET vs dQCD at NLP

Important to include for analytical agreement at NLP

Resummation at LP:

$$\Delta^{\mathrm{SCET,LP}} = H(Q,\mu)U(Q,\mu_s,\mu)\tilde{s}\left(\ln\frac{Q^2}{\mu_s^2}+\frac{\partial}{\partial\eta},\mu_s\right)\frac{\boxed{z^{-\eta}}}{(1-z)^{1-2\eta}}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

Resummation at NLP: $\Delta^{\mathrm{SCET,LP+NLP}} = \Delta^{\mathrm{SCET,LP}} + \Delta^{\mathrm{SCET,NLP}}$

$$\Delta^{\mathrm{SCET,NLP}} = -\beta(\alpha_s(\mu_s^2))\frac{\partial}{\partial\alpha_s(\mu_s^2)}U_{\mathrm{LL}}(Q,\mu_s)$$

SCET vs dQCD at NLP

Important to include for analytical agreement at NLP

Resummation at LP:

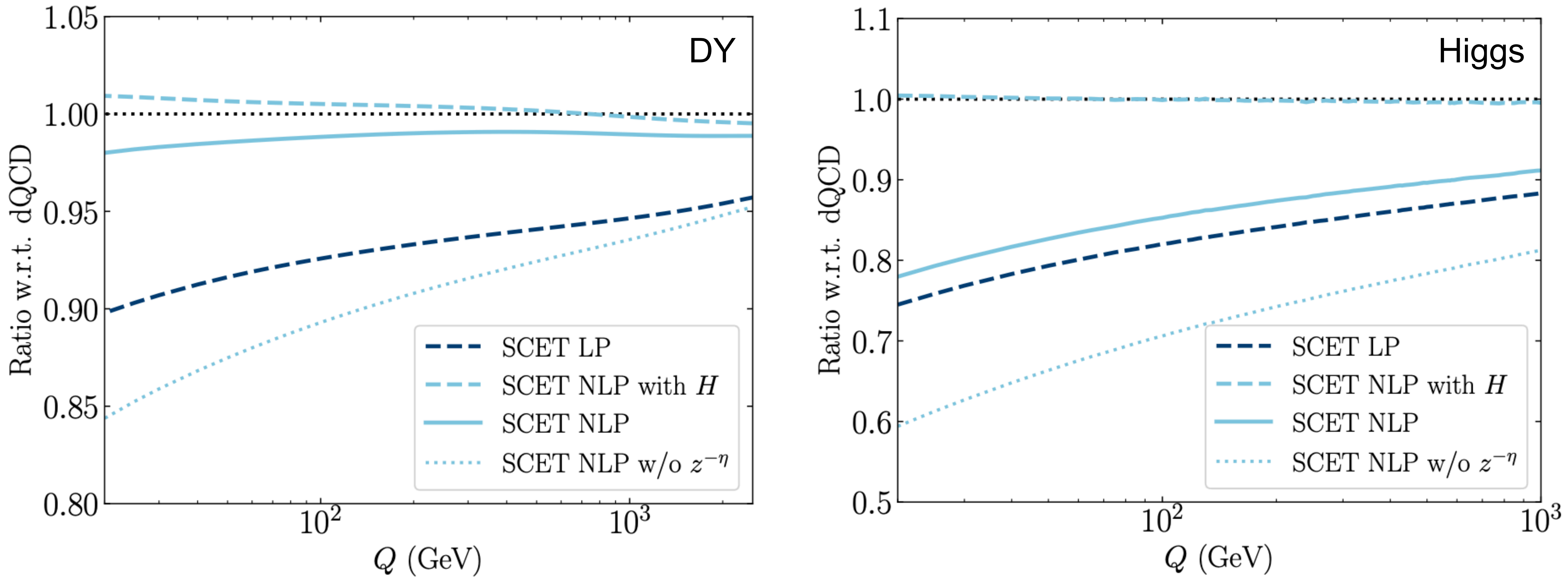
$$\Delta^{\mathrm{SCET,LP}} = H(Q,\mu)U(Q,\mu_s,\mu)\tilde{s}\left(\ln\frac{Q^2}{\mu_s^2}+\frac{\partial}{\partial\eta},\mu_s\right)\frac{\boxed{z^{-\eta}}}{(1-z)^{1-2\eta}}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

Resummation at NLP: $\Delta^{\mathrm{SCET,LP+NLP}} = \Delta^{\mathrm{SCET,LP}} + \Delta^{\mathrm{SCET,NLP}}$

$$\Delta^{\mathrm{SCET,NLP}} = -\boxed{H(Q,\mu)}\beta(\alpha_s(\mu_s^2))\frac{\partial}{\partial\alpha_s(\mu_s^2)}U_{\mathrm{LL}}(Q,\mu_s)$$

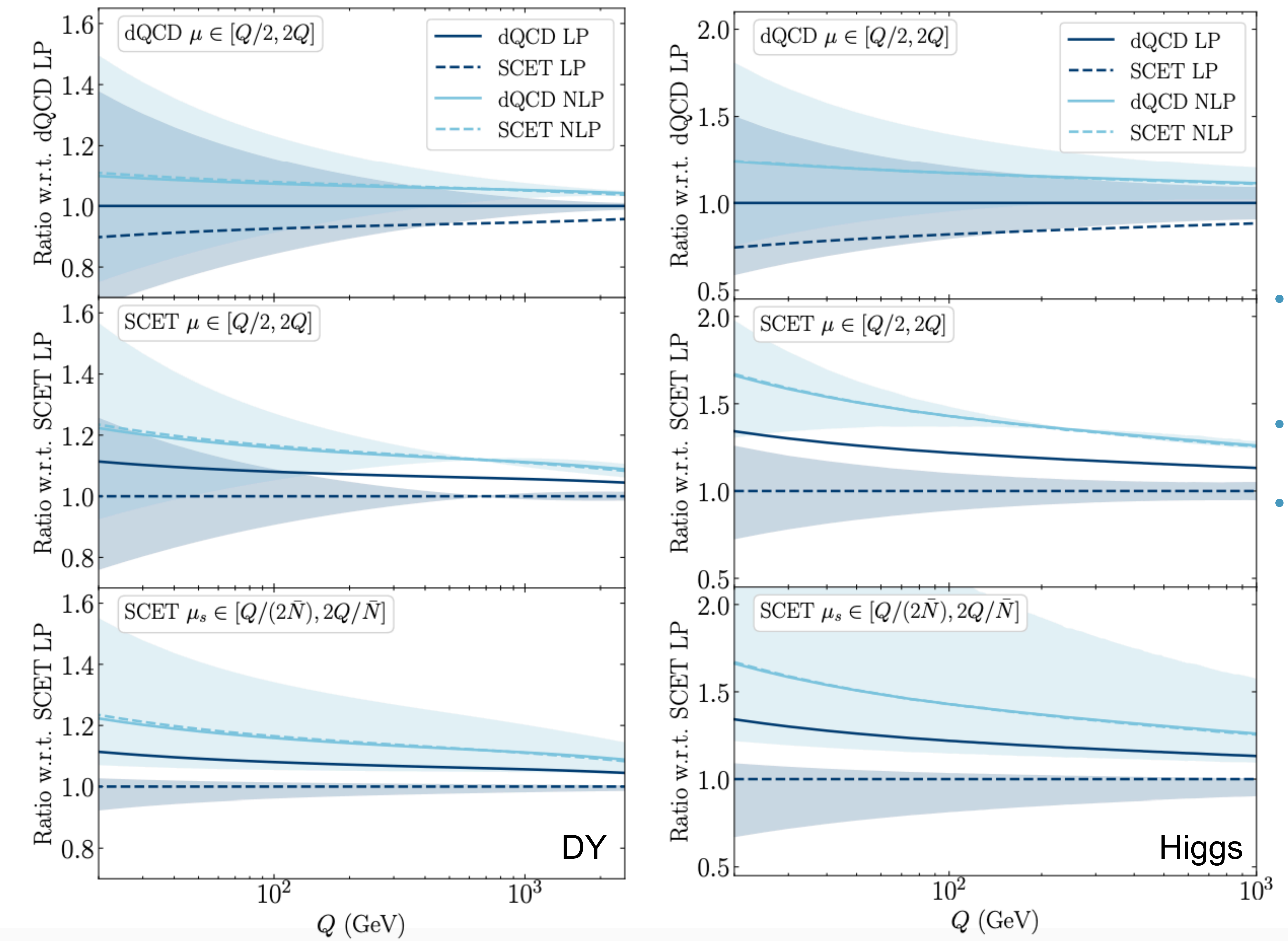
Important for numerical agreement

SCET vs dQCD at NLP



Remaining differences originate from:

- Truncations of higher-logarithmic terms
- Truncations of higher-power terms
- Running α_s effects



- dQCD uncertainty from varying μ is larger than that of SCET
- LP SCET does not contain central dQCD result
- Explicit soft scale variations in SCET induce a large uncertainty at NLP

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Conclusions

- NLP LL contributions for colour-singlet processes are directly linked to the LP LL ones, this allows their resummation

- Numerical contribution of LL NLP terms varies for different processes, but in general it is not ‘negligible’

- Including NLP effects in both dQCD and SCET results in analytical and numerical agreement between the two formalisms

Open questions:

- Relevant for NLP LL for colour-singlet: *Can we resum soft quarks?*
- Relevant for NLP LL in general: *How to deal with ‘wide-angle’ NLP emissions?*
- Relevant for NLP NLL: *What are ‘next-to-collinear/non-soft’ contributions?*

Back-up

Threshold expansion of cross sections

Start with hadronic observable of colour-singlet production $\sigma_{pp\to H+X}$

$$\sigma_{pp\to H+X} = \sigma_0^H \sum_{i,j} \int_\tau^1 \frac{\mathrm{d}z}{z} \mathscr{L}_{ij}\left(\frac{\tau}{z}\right) \Delta_{H,ij}(z) \qquad \tau = \frac{m_H^2}{S} = x_1 x_2 z$$

Threshold expansion of partonic coefficient function:

$$\Delta_{H,ij}(z) = \sum_{n=0}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \left[\sum_{k=-1}^{p} (1-z)^k \sum_{m=0}^{2n-1} c_{nm}^{(ij),k} \ln^m(1-z) + c_n^\delta \delta(1-z)\right]$$

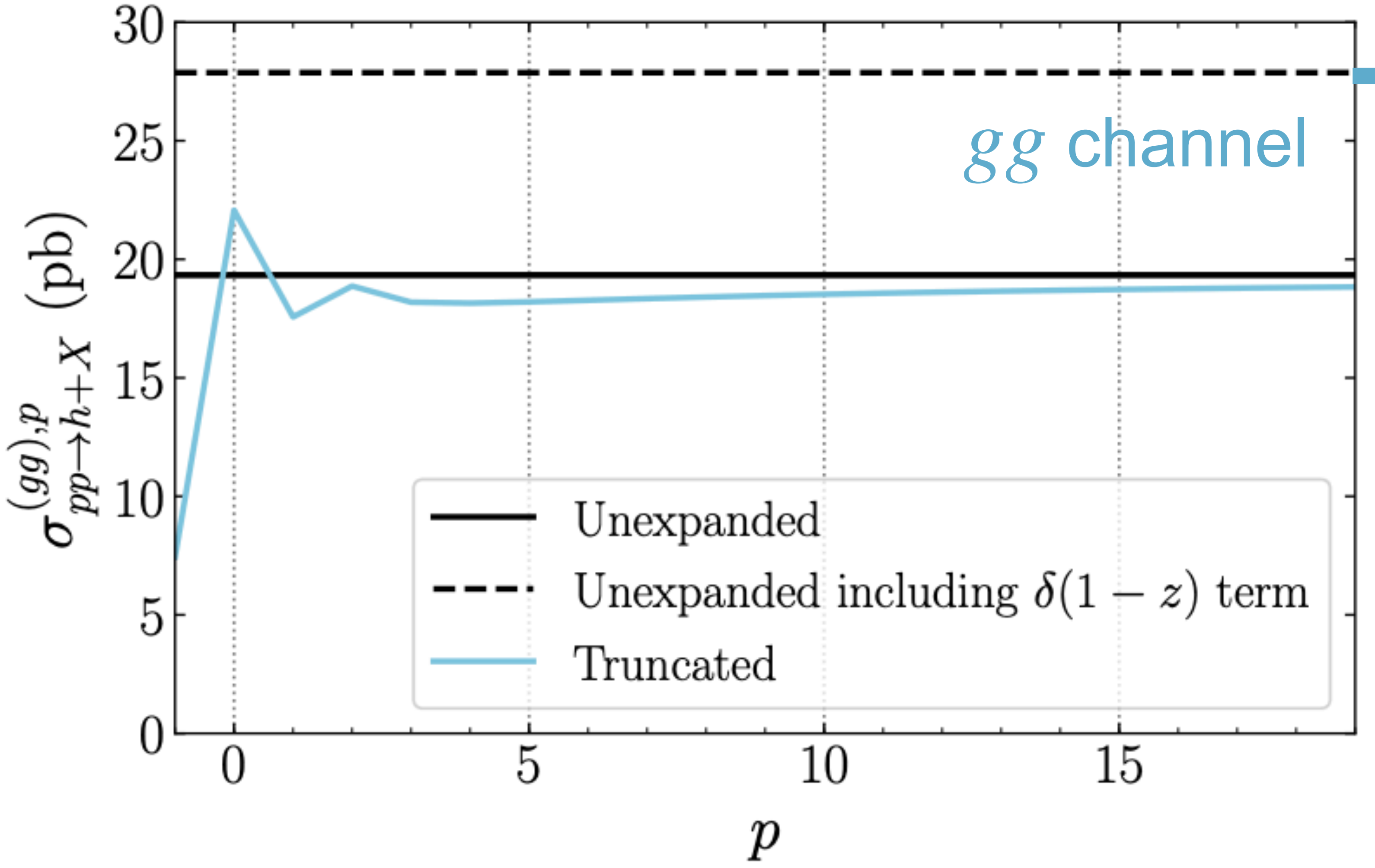
How does this behave for two benchmark colour-singlet production processes?

- Consider Higgs and DY
- Take up to NNLO ($n = 2$)
- $k = -1$ corresponds to LP, $m = 2n - 1$ corresponds to LL

Behaviour of threshold expansion: Higgs

[1411.3584, 1503.06056, 1602.00695, 2101.07270]

$$\sigma^{(ij),p}_{pp\to h+X} = \sigma_0^h \sum_{n=1}^{2} \left(\frac{\alpha_s}{\pi}\right)^n \int_\tau^1 \frac{\mathrm{d}z}{z} \mathscr{L}_{ij}\left(\frac{\tau}{z}\right) \left[\sum_{k=-1}^{p} (1-z)^k \sum_{m=0}^{2n-1} c_{nm}^{(ij),k} \ln^m(1-z) \right]$$



$+c_n^\delta \delta(1-z)$

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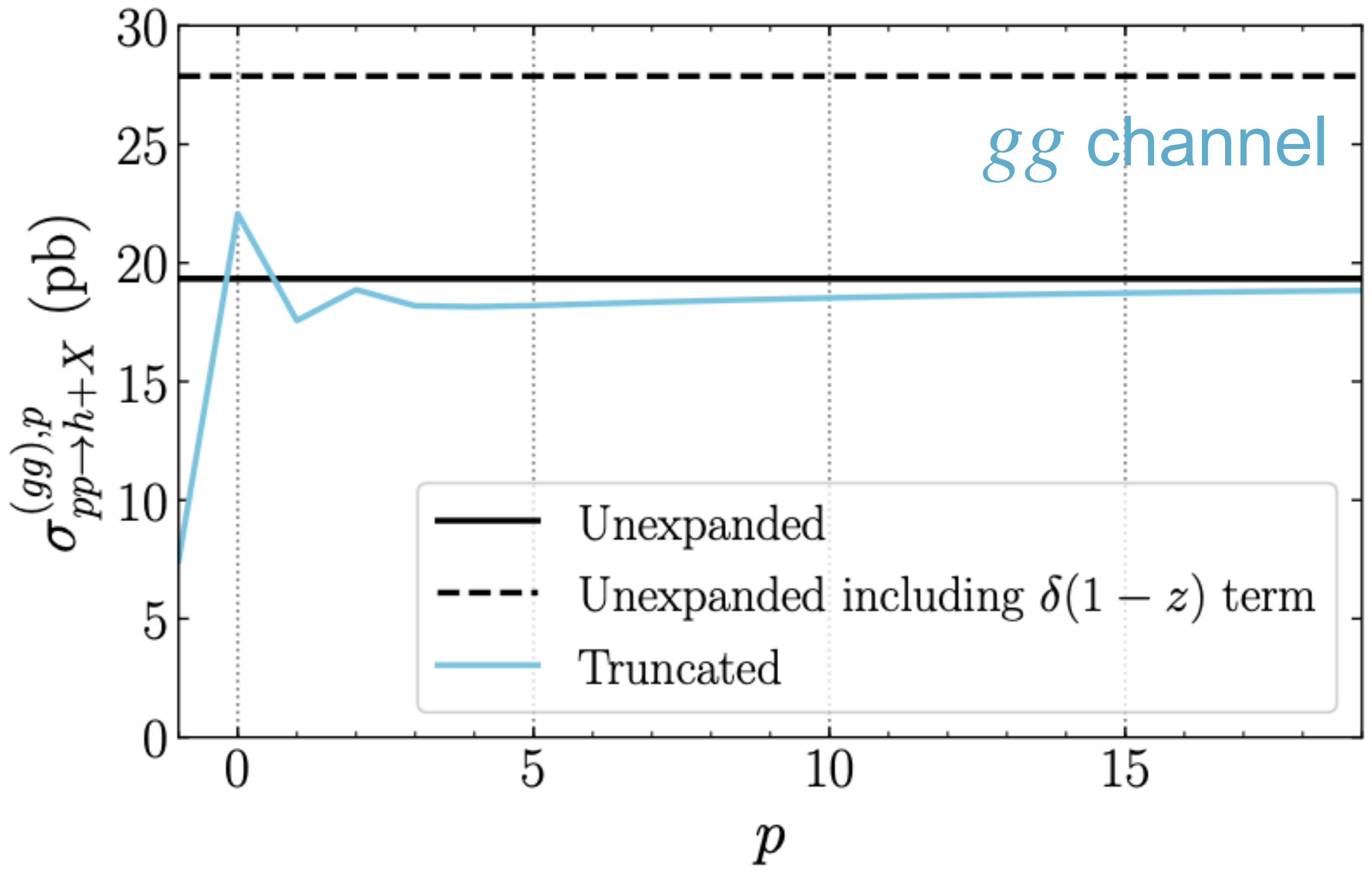
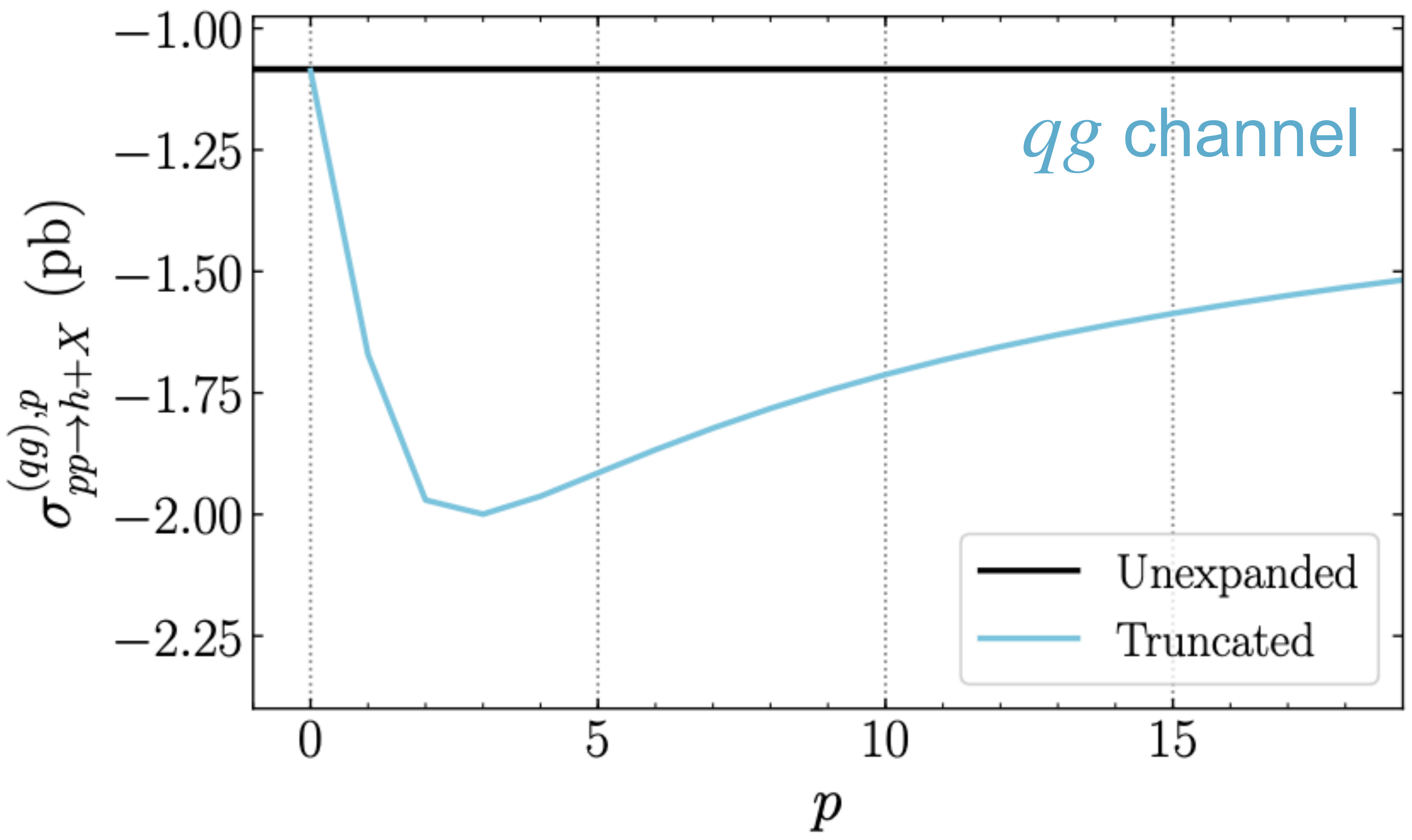
Behaviour of threshold expansion: Higgs

[1411.3584, 1503.06056, 1602.00695, 2101.07270]

$$\sigma^{(ij),p}_{pp\to h+X} = \sigma^h_0 \sum_{n=1}^{2} \left(\frac{\alpha_s}{\pi}\right)^n \int_\tau^1 \frac{\mathrm{d}z}{z} \mathscr{L}_{ij}\left(\frac{\tau}{z}\right) \left[\sum_{k=-1}^{p} (1-z)^k \sum_{m=0}^{2n-1} c^{(ij),k}_{nm} \ln^m(1-z)\right]$$





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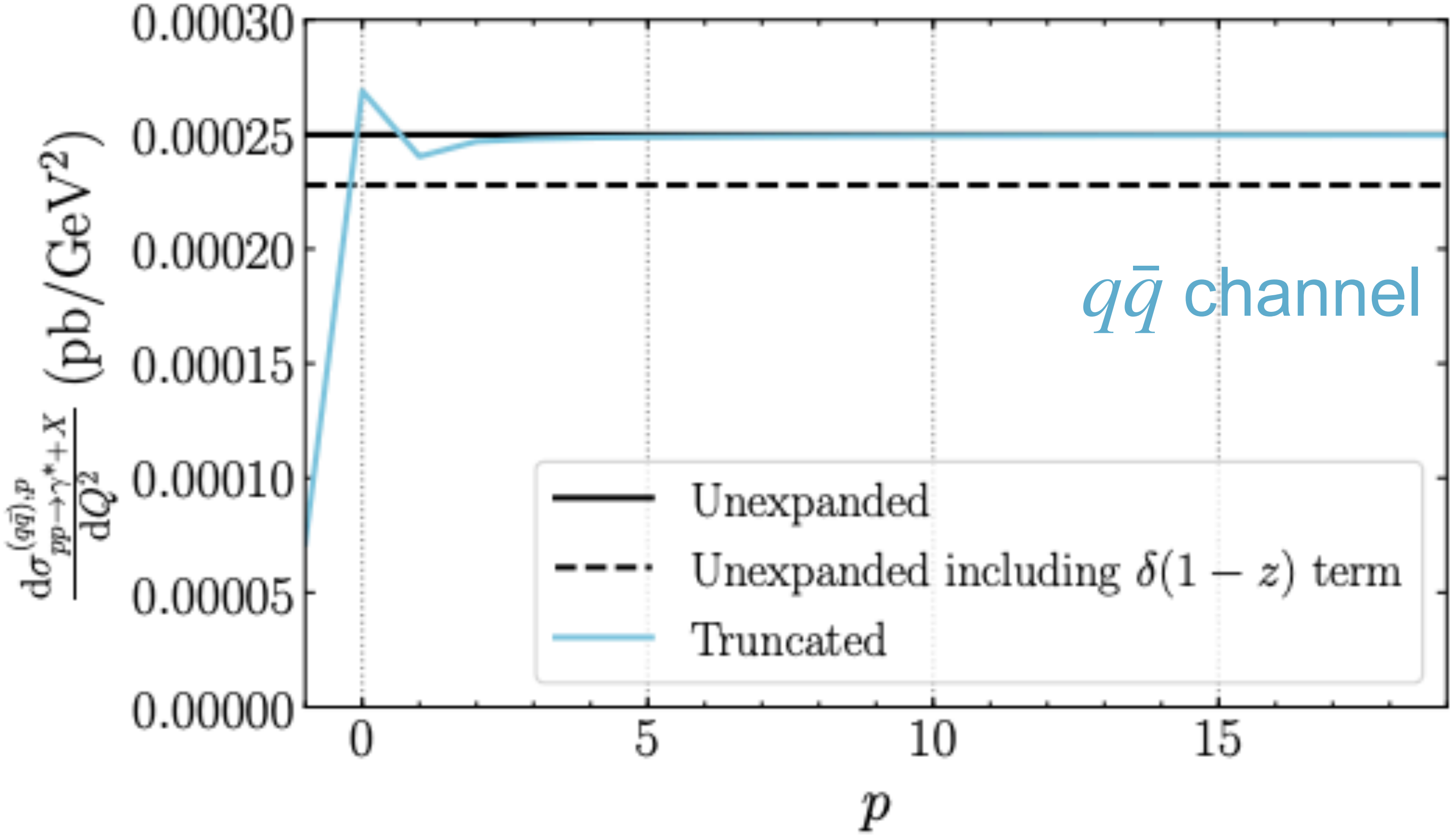
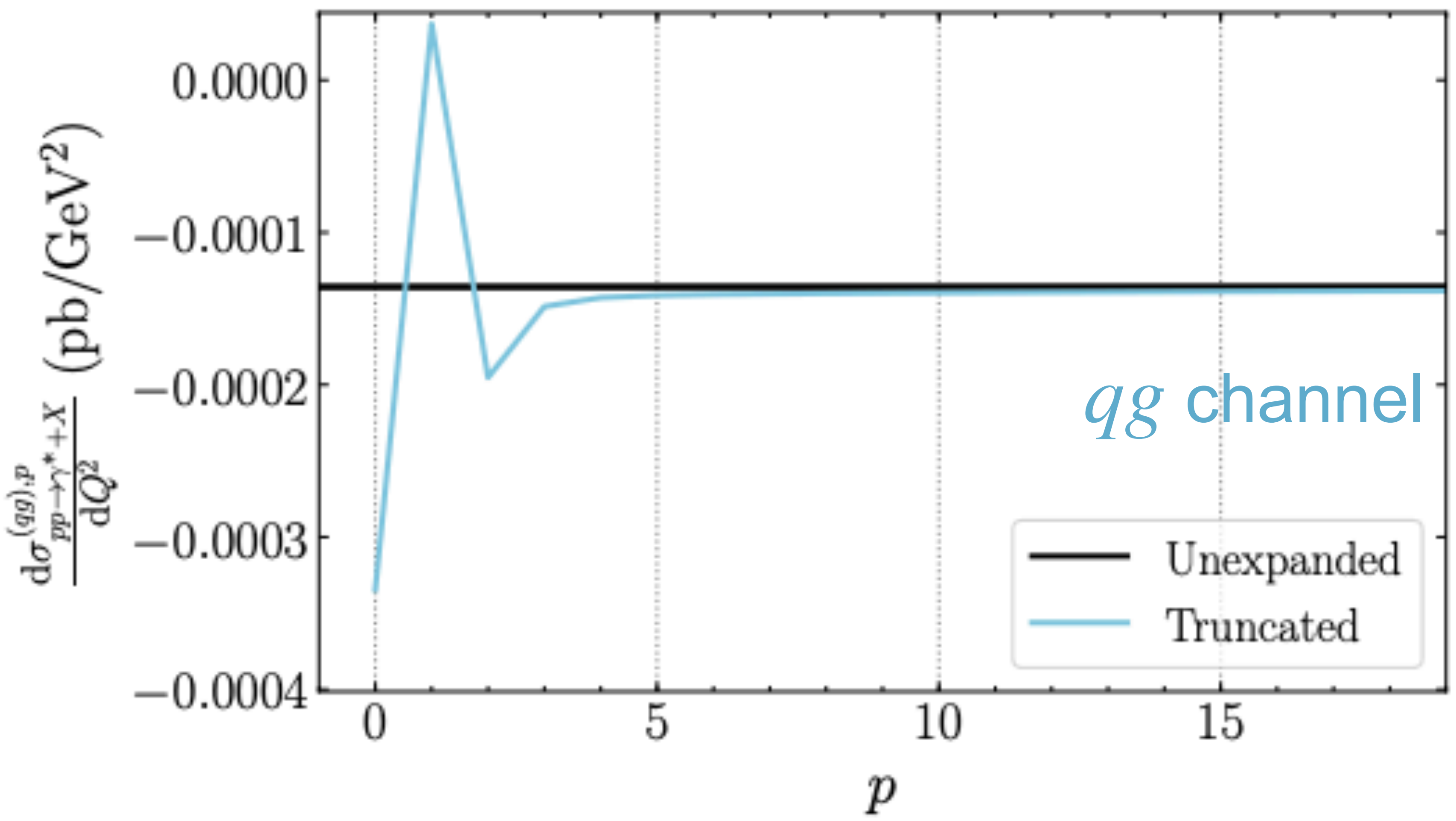
Behaviour of threshold expansion: DY

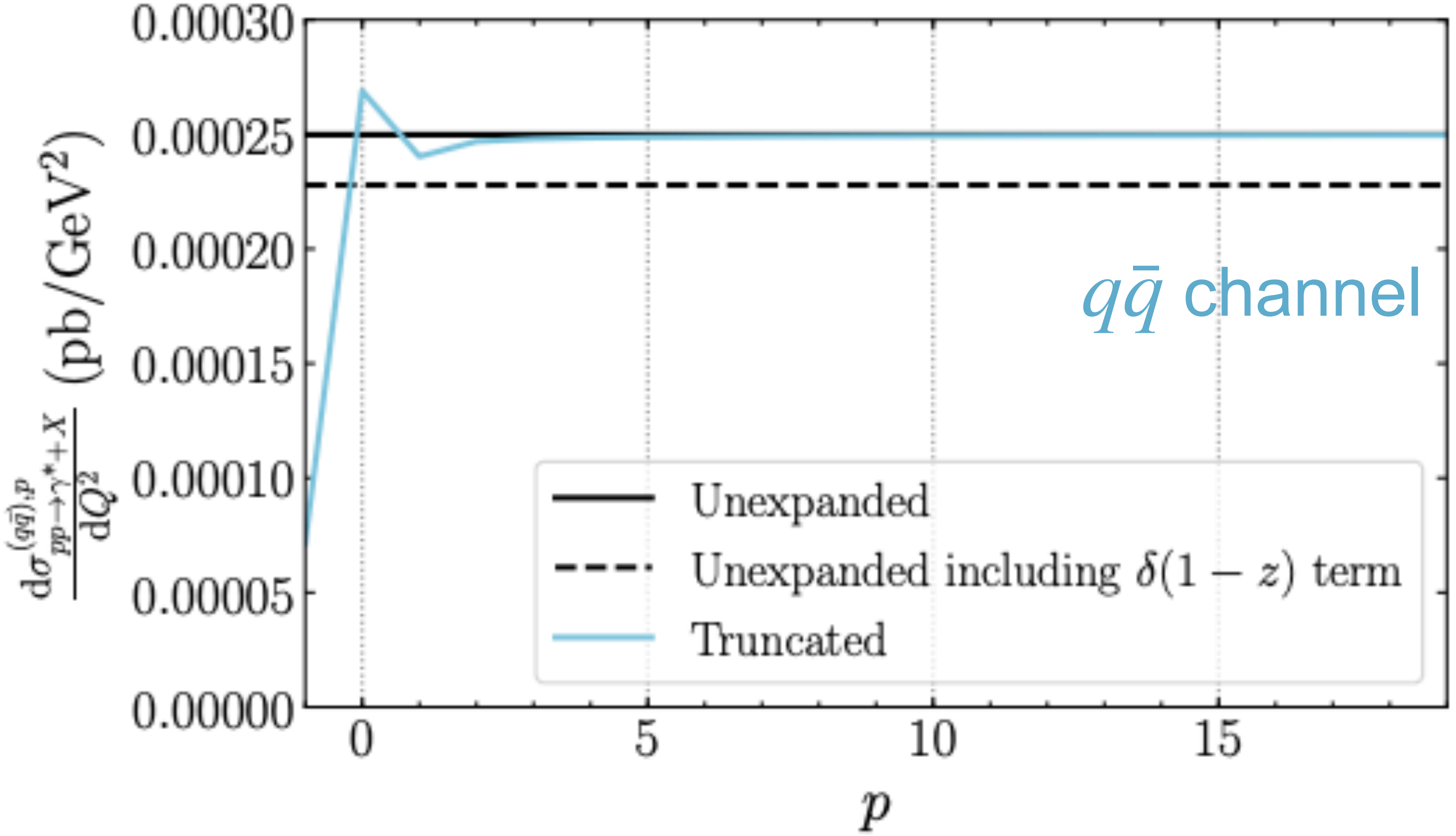
[2101.07270]

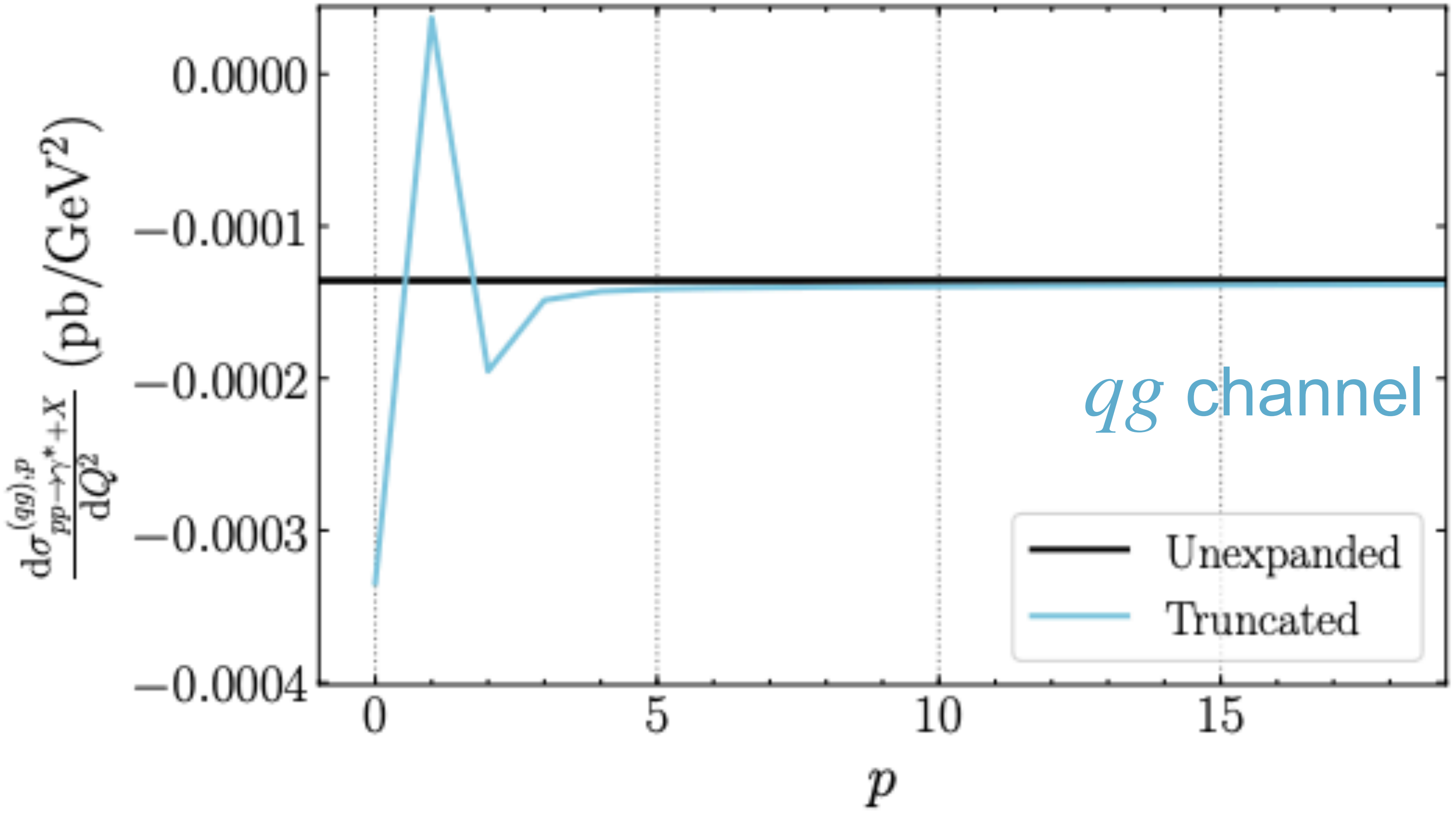




Behaviour of threshold expansion: DY

[2101.07270]





From this we conclude:

- The LP expansion is not enough to capture the entire result
- A better description would be given by including NLP

The NLP cross section

$$
\begin{aligned}
|\mathscr{A}_{\text{NLP}}|^2 &= \sum_{\text{colors}} |\mathscr{A}_{\text{scal}}|^2 + 2\text{Re}\left[(\mathscr{A}_{\text{spin}} + \mathscr{A}_{\text{orb}})^\dagger \mathscr{A}_{\text{scal}}\right] \\
&= K \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} |\mathscr{M}_{\text{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)|^2
\end{aligned}
$$

The NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,K_{\mathrm{NLP}}\,\sigma_{\mathrm{Born}}(Q)$$

[1706.04018]

The NLP cross section

Integration over phase space: $$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\, \boxed{K_{\mathrm{NLP}}}\, \sigma_{\mathrm{Born}}(Q)$$

$$K_{\mathrm{NLP}} = \frac{\alpha_s}{\pi}\left(\frac{\bar{\mu}^2}{s}\right)^{\epsilon}\left[-\frac{2}{\epsilon}\left(\left(\frac{1}{1-z}\right)_+ - 1\right) + 4\left(\frac{\ln(1-z)}{1-z}\right)_+ - 4\ln(1-z) + \frac{1}{\varepsilon^2}\delta(1-z) + \ldots\right]$$

[1706.04018]

The NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\, K_{\mathrm{NLP}}\, \sigma_{\mathrm{Born}}(Q)$$

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NLP log with the same coefficient as the LP log!

[1706.04018]

The NLP cross section

Integration over phase space: $$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,K_{\mathrm{NLP}}\,\sigma_{\mathrm{Born}}(Q)$$

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$$= zK_{\mathrm{LP}}$$

[1706.04018]

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Process with more colored legs

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

Shifts in Born amplitude

$$
\begin{aligned}
|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 \quad = \quad \frac{C_F}{C_A}\Bigg[& C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}_{\mathrm{LO}}^{q\bar{q}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \\
& +\frac{1}{2}C_A\frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)}\left|\mathscr{M}_{\mathrm{LO}}^{q\bar{q}}(p_1+\delta p_{1;R}, p_R-\delta p_{R;1})\right|^2 \\
& +\frac{1}{2}C_A\frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)}\left|\mathscr{M}_{\mathrm{LO}}^{q\bar{q}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2 \\
& -\frac{1}{2}C_A\frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}_{\mathrm{LO}}^{q\bar{q}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2\Bigg]
\end{aligned}
$$

Explicit forms - dQCD

$$g_a^{(1)}(\lambda, N) = \frac{A_a^{(1)}}{2\pi b_0^2}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right]$$

$$h_a^{(1)}(\lambda, N) = -\frac{A_a^{(1)}}{2\pi b_0}\frac{\ln(1-2\lambda)}{N}$$

$$\Rightarrow \quad \frac{1}{\alpha_s}g_a^{(1)}(\lambda, N) + h_a^{(1)}(\lambda, N) = \frac{1}{\alpha_s}\left(1 + \frac{1}{2}\frac{\partial}{\partial N}\right)g_a^{(1)}(\lambda)$$

Explicit forms - SCET

$$U_{aa}(Q,\mu_h,\mu,\mu_s) = \exp\left[S_a(\mu_h^2,\mu_s^2) - a_{\gamma_{V/S}}(\mu_h^2,\mu_s^2) + 2a_{\gamma_a}(\mu_s^2,\mu^2) - a_{\Gamma_{\text{cusp,a}}}(\mu_h^2,\mu_s^2)\ln\frac{Q^2}{\mu_h^2}\right]$$

$$\Delta^{\text{SCET,NLP}}(z,Q,\mu,\mu_s) = -\frac{2A_a^{(1)}}{\pi b_0}\ln\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_s^2)}\exp\left[S_{a,\text{LL}}(Q^2,\mu^2) - S_{a,\text{LL}}(\mu_s^2,\mu^2)\right]$$

$$S_{a,\text{LL}}(\mu^2,\nu^2) = \frac{A_a^{(1)}}{b_0^2\pi}\left[\frac{1}{\alpha_s(\mu^2)} - \frac{1}{\alpha_s(\nu^2)} - \frac{1}{\alpha_s(\mu^2)}\ln\frac{\alpha_s(\nu^2)}{\alpha_s(\mu^2)}\right]$$

$$\Delta^{\text{SCET,NLP}}(z,Q,\mu,\mu_s) = -\beta(\alpha_s(\mu^2))\frac{\partial}{\partial\alpha_s(\mu_s^2)}U_{aa,\text{LL}}(Q,\mu_h=Q,\mu,\mu_s)$$

$$= \frac{\beta(\alpha_s(\mu^2))}{\alpha_s^2 b_0^2}\frac{A_a^{(1)}}{\pi}\ln\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_s^2)}U_{aa,\text{LL}}(Q,\mu_h=Q,\mu,\mu_s)$$