Mixed QCD×QED corrections to Drell-Yan and Higgs production beyond soft-virtual at N^{3}LO

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RADCOR-LoopFest2021

MOTIVATION

- Precision physics at the LHC
- Need for precise theoretical predictions
- QCD radiative corrections are crucial
- The measurements and predictions from QCD have reached the level that demand the inclusion of electroweak effects
- DY, Higgs productions in hadron colliders - known to N^3LO in QCD
- $\alpha_s^2 \sim \alpha$ - necessary to include the corrections from QED
- EW corrections compete with QCD under relevant kinematic conditions for the LHC
- Theory point of view - perturbative structure of mixed gauge group , universality of IR singularities ...

PREVIOUS WORKS

NLO EW corrections to DY → [S. Dittmaier and M. Krmer]
[U. Baur and D. Wackeroth]
[C. M. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini]
[U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackeroth]
[U. Baur, S. Keller and W. K. Sakumoto]

Two-loop EW corrections to Higgs boson production → [U. Aglietti, R. Bonciani, G. Degrassi and A. Vicini]

Mixed QCD×EW corrections to DY → [S. Dittmaier, A. Huss and C. Schwinn]
[R. Bonciani, F. Buccioni, N. Rana, I. Triscari and A. Vicini]

QED corrections to the Altarelli-Parisi splitting functions → [D. de Florian, G. F. R. Sborlini and G. Rodrigo]

Two-Loop Master Integrals for the mixed EW-QCD virtual corrections → [R. Bonciani, S. Di Vita, P. Mastrolia and U. Schubert]
[Heller, von Manteuffel, Schabinger]

Double-real emission → [R. Bonciani, F. Buccioni, R. Mondini and A. Vicini]

NNLO QCD⊕QED corrections → [D. de Florian, M. Der and I. Fabre]
[A. H. Ajjath, A. Chakraborty, P. K. Dhani, P. Mukherjee, N. Rana, V. Ravindran et al.]

Threshold N^3LO QCD⊕QED corrections to DY and Higgs production → [A.H. Ajjath, P. Mukherjee and V. Ravindran]

And many other works…

See talks by N.Rana, G.Pellicciolі, L. Buonocore, Andreas von Manteuffel ..

THE GOAL OF OUR WORK

$$\sigma(S,q^2) = \sigma_0(\mu_R^2)\sum_{cd}\int dx_1 dx_2 f_c(x_1,\mu_F^2) f_d(x_2,\mu_F^2) \times \Delta_{cd}(s,q^2,\mu_F^2,\mu_R^2)$$

$$a_s = \frac{g_s^2(\mu_R^2)}{16\pi^2}$$

$$a_e = \frac{e^2(\mu_R^2)}{16\pi^2}$$

Partonic cross-section/ Partonic coefficient function (CF) (Perturbatively calculable)

$$\Delta_{cd}(z,q^2) = \sum_{i,j=0}^{\infty} a_s^i a_e^j \Delta_{cd}^{(i,j)}(z,q^2)$$

THE GOAL OF OUR WORK

General structure of the CF near z→ 1
Threshold expansion:

$$z = \frac{q^2}{s}$$

Partonic scaling variable

$$\Delta_{cd}^{(i,j)}(z, q^2) = \sum_{k=0}^{2(i+j)-1} c_{ijk}^{\mathcal{D}}\, \mathcal{D}_k + c_{ij}^{\delta}\, \delta(1-z) + \sum_{k=0}^{2(i+j)-1} c_{ijk}^{L}\, \ln^k(1-z) + \mathcal{O}(1-z)$$

Inv mass square

$$\mathcal{D}_k = \left(\frac{\ln^k(1-z)}{(1-z)} \right)_+$$

Plus distribution

Most singular terms when z→ 1
Soft-virtual (SV) corrections
Well understood in

Next-to-dominant singular terms when z→ 1.
Collinear logarithms
Not much studied

SV+Next-to-soft virtual(NSV)

THE GOAL OF OUR WORK

To compute $\Delta_{cd}^{(i,j)}$ for $i + j \leq 3$ beyond SV/ Next-to-soft virtual (NSV) for DY and Higgs production in bottom quark annihilation

Infrared (IR) structure of various ingredients that contribute to Δ_{cd} in the mixed $\mathrm{SU(N)} \times U(1)$ gauge theory

NSV – HISTORY

The problem of NSV/NLP(next-to-leading power) logarithms has been of interest for a long time, and several different approaches have been proposed.

- The earliest evidence that IR effects can be studied at NLP
  [Low, Burnett, Kroll]

- Factorisation approach to study NLP logarithms
  [Del Duca, Laenen, Magnea et al]

- Important results using physical evolution equations method
  [Moch, Vogt et al]

- Resummation of NLP logarithms at leading order
  [Kramer, Laenen, Spira], [Ravindran, Grunberg], [Bonocore, Laenen, Magnea]

- SCET techniques
  [Beneke et al], [Stewart, Fleming et al]

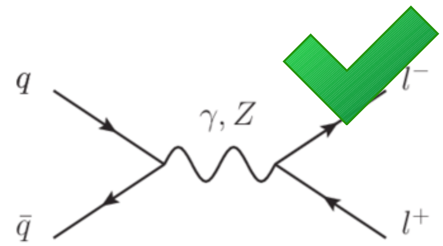
- Factorisation and RG invariance approach to study NSV resummation effects
  [Ajjath, Pooja, Ravindran]

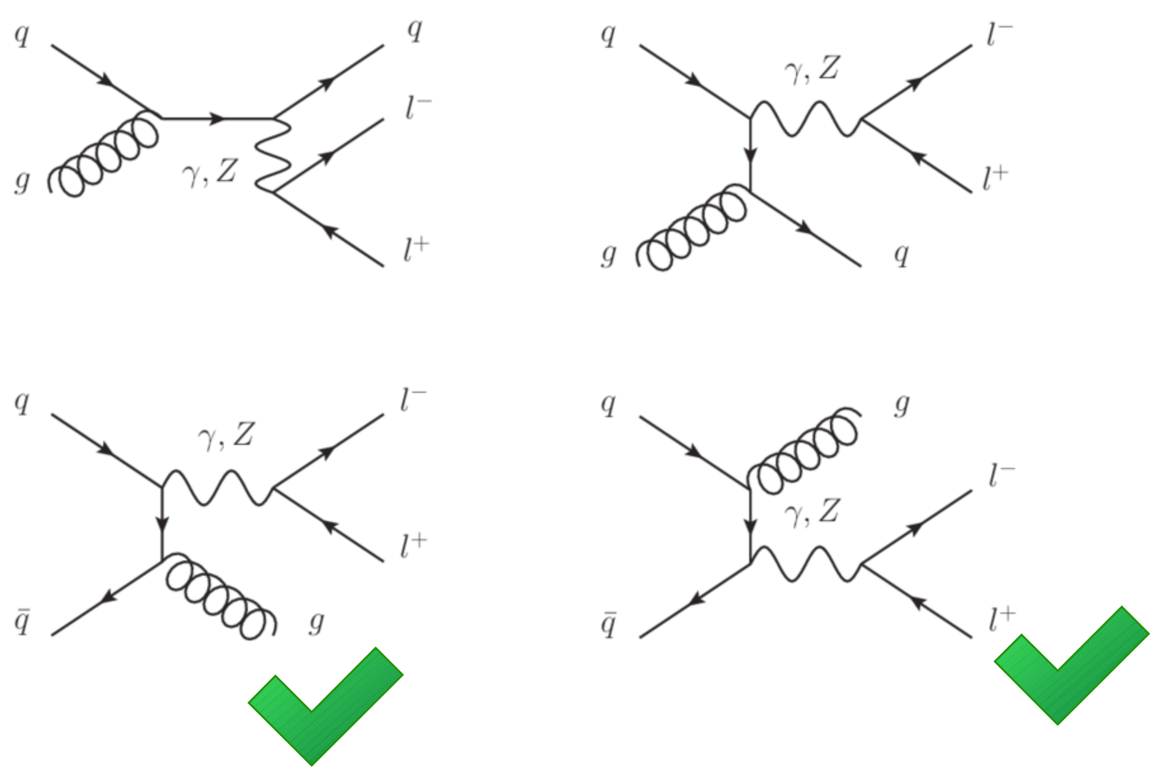
See talks by Melissa,
X.Wang, Pooja, Ajjath..

And many other works...

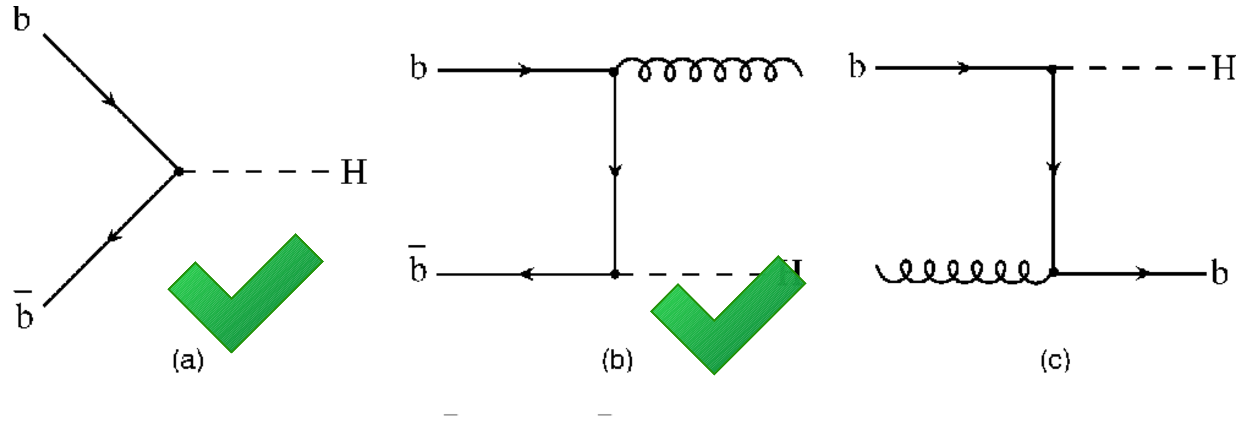
Only diagonal channels :

$\Delta_{q\bar{q}}$ - Drell-Yan





$\Delta_{b\bar{b}}$ - Higgs production in bottom quark annihilation



- We use:
  collinear factorisation
  Renormalisation group (RG) invariance
  Abelianisation rules

For the diagonal channels, the SV+NSV partonic coefficient function can be factorised

$$\Delta_I^{SV+NSV}(z,,q^2,\mu_F^2,\mu_R^2,\epsilon) = \left(Z_{UV,I}(\hat{a}_c,\mu_R^2,\mu^2,\epsilon)\right)^2 |\hat{F}_I(\hat{a}_c,\mu^2,q^2,\epsilon)|^2$$
$$\times\, \delta(1-z) \otimes \mathcal{S}_I\left(\hat{a}_c,\mu^2,q^2,z,\epsilon\right)$$
$$\otimes\, \Gamma_{II}^{-1}\left(\hat{a}_c,\mu^2,\mu_F^2,z,\epsilon\right) \otimes \Gamma_{\bar{I}\bar{I}}^{-1}(\hat{a}_c,\mu^2,\mu_F^2,z,\epsilon)$$

$Z_{UV,I}$ – Renormalisation constant

$\hat{F}_I$ – Form Factor

Γ_{II} – Altarelli-Parisi splitting kernel

$\mathcal{S}_I$ – Soft +next-to-soft contributions

$$\hat{a}_c = \{\hat{a}_s, \hat{a}_e\}$$

FORM FACTOR (FF)

FF – pure virtual corrections

[Sen,sterman,Magnea]

IR singularities, resulting from QCD and QED interactions factorise

[Moch,Vogt,Vermaseren; Ravindran]

$$\hat{F}_I(Q^2,\mu^2,\epsilon) = Z_{IR}(Q^2,\mu^2,\mu_R^2,\epsilon)\hat{F}_I^{fin}(Q^2,\mu^2,\mu_R^2,\epsilon)$$

universal IR counter term contains poles

Finite part

Differentiating both sides with respect to Q^2, we obtain **K+G equation** for the FFs

$$Q^2\frac{d}{dQ^2}\ln\hat{F}_I(Q^2,\mu^2,\epsilon) = \frac{1}{2}\left[K_I\left(\{\hat{a}_c\},\frac{\mu_R^2}{\mu^2},\epsilon\right) + G_I\left(\{\hat{a}_c\},\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)\right]$$

Poles

Finite

Cusp anomalous dimension

RG invariance -

$$\mu_R^2\frac{d}{d\mu_R^2}\ln\hat{F}_I(Q^2,\mu^2,\epsilon) = 0$$

$$\mu_R^2\frac{d}{d\mu_R^2}K_I\left(\{\hat{a}_c\},\frac{\mu_R^2}{\mu^2},\varepsilon\right) = -\mu_R^2\frac{d}{d\mu_R^2}G_I\left(\{\hat{a}_c\},\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\varepsilon\right) = -A_I(\{a_c(\mu_R^2)\})$$

solution in d=4+ϵ

$$\ln\hat{F}_I = \sum_{i,j}\hat{a}_s^i\hat{a}_e^j\left(\frac{Q^2}{\mu^2}\right)^{(i+j)\frac{\epsilon}{2}} S_\epsilon^{(i+j)}\hat{\mathcal{L}}_{F_I}^{(i,j)}(\epsilon)$$

$$\left\{A_I^{(i,j)},B_I^{(i,j)},f_I^{(i,j)},\gamma_I^{(i,j)}\right\}$$ + process dependent constants - $g_{I,ij}$

Cusp, collinear, soft and UV anomalous dimensions

Some Observations

- The radiative corrections resulting from QCD and QED interactions cannot be factored out independently

- Neither the IR singular function Z_{IR} nor the finite part of FF can be written as a product of pure QCD and pure QED contributions.

- Terms proportional to $a_s^{\,i}\, a_e^{\,j}$, where i,j > 0, which will not allow factorization of QCD and QED contributions for both Z_{IR} and finite part of FF

- For instance, take a look at the IR structure

Pure QCD: well known

Mixed QCD-QED : not much known, complicated

$$\hat{\mathcal{L}}_{F_I}^{(1,0)} = \frac{1}{\varepsilon^2}\Big(-2A_I^{(1,0)}\Big) + \frac{1}{\varepsilon}\Big(G_I^{(1,0)}(\varepsilon)\Big)\,,$$

$$\hat{\mathcal{L}}_{F_I}^{(0,1)} = \frac{1}{\varepsilon^2}\Big(-2A_I^{(0,1)}\Big) + \frac{1}{\varepsilon}\Big(G_I^{(0,1)}(\varepsilon)\Big)\,,$$

$$\hat{\mathcal{L}}_{F_I}^{(2,0)} = \frac{1}{\varepsilon^3}\Big(\beta_{00}A_I^{(1,0)}\Big) + \frac{1}{\varepsilon^2}\Big(-\frac{1}{2}A_I^{(2,0)} - \beta_{00}G_I^{(1,0)}(\varepsilon)\Big) + \frac{1}{2\varepsilon}\Big(G_I^{(2,0)}(\varepsilon)\Big)\,,$$

$$\hat{\mathcal{L}}_{F_I}^{(0,2)} = \frac{1}{\varepsilon^3}\Big(\beta'_{00}A_I^{(0,1)}\Big) + \frac{1}{\varepsilon^2}\Big(-\frac{1}{2}A_I^{(0,2)} - \beta'_{00}G_I^{(0,1)}(\varepsilon)\Big) + \frac{1}{2\varepsilon}\Big(G_I^{(0,2)}(\varepsilon)\Big)\,,$$

Mixed term

$$\hat{\mathcal{L}}_{F_I}^{(1,1)} = \frac{1}{\varepsilon^2}\Big(-\frac{1}{2}A_I^{(1,1)}\Big) + \frac{1}{2\varepsilon}\Big(G_I^{(1,1)}(\varepsilon)\Big)$$

Renormalisation Constant

To remove UV singularities in the bare FFs

[Moch,Vogt,Vermaseren]

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z_{UV,I} = \sum_{ij=0}^{\infty} a_s^i a_e^j \gamma_I^{(i,j)} \left[a_s(\mu_R^2), a_e(\mu_R^2)\right]$$

Renormalisation scale

UV anomalous dimension

For DY: $\gamma_q^{i,j} = 0$ (Conserved operator)

For Higgs: $\gamma_b^{i,j}$ (Yukawa coupling)

Altarelli-Parisi Splitting Kernel

Required to remove the initial state collinear singularities [Moch,Vogt,Vermaseren]

AP kernels which satisfy renormalisation group equations

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{II}(z, \mu_F^2) = \frac{1}{2} P_{II}(\mu_F^2) \otimes \Gamma_{II}(\mu_F^2)$$

Factorisation scale

AP Splitting function

$$P_{II}(z, \mu_F^2) = 2 \left[\frac{A_I(\{a_c(\mu_F^2)\})}{(1-z)_+} + B_I(\{a_c(\mu_F^2)\}) \, \delta(1-z) + C_I(\{a_c(\mu_F^2)\}) \, \ln(1-z) + D_I(\{a_c(\mu_F^2)\}) \right]$$

We consider only diagonal parts of splitting functions

known up to NNLO in QED, QCD$\times$ QED

[D. de Florian, G. F. R. Sborlini and G. Rodrigo]

What About The Soft + Next-to-soft Contributions ?

$$
\begin{aligned}
\Delta_I^{SV+NSV}(z,,q^2,\mu_F^2,\mu_R^2,\epsilon) = & \left(Z_{UV,I}(\hat{a}_c,\mu_R^2,\mu^2,\epsilon)\right)^2 |\hat{F}_I(\hat{a}_c,\mu^2,q^2,\epsilon)|^2 \\
& \times \delta(1-z) \otimes \mathcal{S}_I\left(\hat{a}_c,\mu^2,q^2,z,\epsilon\right) \\
& \otimes \Gamma_{II}^{-1}\left(\hat{a}_c,\mu^2,\mu_F^2,z,\epsilon\right) \otimes \Gamma_{\bar{I}\bar{I}}^{-1}\left(\hat{a}_c,\mu^2,\mu_F^2,z,\epsilon\right)
\end{aligned}
$$

$Z_{UV,I}$ – Renormalisation constant ✔

$\hat{F}_I$ – Form Factor ✔

Γ_{II} – Altarelli-Parisi splitting kernel ✔

$\mathcal{S}_I$ – Soft +next-to-soft contributions ?

↓ Collinear logarithmic contributions

Guiding Factors To Compute The Missing Soft + Next-to Soft Contributions

- Finiteness of the partonic coefficient function Δ_I
- Sudakov differential equation(K+G Eqn) of FFs
- RG equations of AP kernels and $Z_{UV,I}$

$\overline{K} + \overline{G}$ Eqn for $\mathcal{S}_{\mathcal{I}}$

[Ajjath, Pooja,Ravindran]

$$q^2 \frac{d}{dq^2} \mathcal{S}_I = \frac{1}{2} \left[\overline{K}_I \left(\hat{a}_c, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) + \overline{G}_I \left(\hat{a}_c, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) \right] \otimes \mathcal{S}_I$$

Poles

Finite

Solution takes convoluted exponential form

$$\mathcal{S}_I = \mathcal{C} \exp \left(2\Phi_I(\hat{a}_c, \mu^2, q^2, z, \epsilon) \right)$$

Soft+Next-to-soft distribution (will be discussed in detail)

$$\mathcal{C} \exp \left(2\Phi_I(z) \right) = \frac{\hat{\sigma}_{I\bar{I}}(z)}{Z_{UV,I}^2 \left| \hat{F}_I \right|^2}, \qquad I = q, b$$

No pure virtual , Only Real-Virtual (RV), Real-Real (RR) etc

All Oreder Structure – MASTER FORMULA

Factorisation + RG Eqns of building blocks
leads to an all order exponential form:

$$\Delta_I(q^2,\mu_R^2,\mu_F^2,z) \quad = \quad \mathcal{C}\exp\left(\Psi_I(q^2,\mu_R^2,\mu_F^2,z,\epsilon)\right)\Big|_{\epsilon=0}$$

$$\Psi_I = \left(\ln\left(Z_{UV,I}(\hat{a}_c,\mu^2,\mu_R^2,\epsilon)\right)^2 + \ln\left|\hat{F}_I(\hat{a}_c,\mu^2,Q^2,\epsilon)\right|^2\right)\delta(1-z)$$
$$+2\Phi_I(\hat{a}_c,\mu^2,q^2,z,\epsilon) - 2\mathcal{C}\ln\Gamma_{II}(\hat{a}_c,\mu^2,\mu_F^2,z,\epsilon)$$

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}(f\otimes f)(z) + \cdots$$

$\delta(1-z)$, $\left(\frac{\ln^k(1-z)}{(1-z)}\right)_+$ → Leading term : SV

$\ln^k(1-z)$ → Next-to Leading term : NSV

On The Structure Of Soft+Next-to-soft Distribution

[Ajjath, Pooja,Ravindran]

$$q^2 \frac{d}{dq^2} \Phi_I = \frac{1}{2} \left[\overline{K}_I \left(\{\hat{a}_c\}, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) + \overline{G}_I \left(\{\hat{a}_c\}, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) \right]$$

Singular

Finite – need to be determined

RG invariance implies

$$\mu_R^2 \frac{d}{d\mu_R^2} \overline{K}_I = -\mu_R^2 \frac{d}{d\mu_R^2} \overline{G}_I = A_I(\{a_c(\mu_R^2)\})\delta(1-z)$$

identical to the cusp anomalous dimension that appears in the FFs confirming the universality of IR structure of the underlying gauge theory(ies)

Solution verified up to two-loop , expanded around z=1

Parameterized in terms of $\ln(1-z)$

$$\Phi_I(\{\hat{a}_c\}, q^2, \mu^2, \epsilon, z) = \sum_{i,j} \hat{a}_s^i \hat{a}_e^j \left(\frac{q^2(1-z)^2}{\mu^2 z}\right)^{(i+j)\frac{\epsilon}{2}} S_\epsilon^{(i+j)} \left(\frac{(i+j)\epsilon}{1-z}\right) \left[\hat{\phi}_I^{A,(i,j)}(\epsilon) + (1-z)\, \hat{\phi}_I^{B,(i,j)}(z,\epsilon)\right]$$

Phase-space factor

From matrix elements

- Expanding the ansatz:

$$\frac{1}{(1-z)}\left[(1-z)^2\right]^{(i+j)\frac{\epsilon}{2}} = \frac{\delta(1-z)}{(i+j)\epsilon} + \sum_{k=0}^{\infty} \left[(i+j)\epsilon\right]^k \frac{\mathcal{D}_k}{k!}$$

→ Contribute to SV

$$z^{-(i+j)\frac{\epsilon}{2}} = \sum_{n=0}^{\infty} \frac{\left[\frac{-(i+j)\epsilon}{2} \ln(z)\right]^n}{n!}$$

→ Combining with SV, contribute to NSV

$$\left[(1-z)^2\right]^{(i+j)\frac{\epsilon}{2}} = \sum_{n=0}^{\infty} \frac{\left[(i+j)\epsilon\ \ln(1-z)\right]^n}{n!}$$

→ Contribute to NSV

Properties Of The SV Solution -Old

Can be extracted from FFs

* $\hat{\phi}_I^{A,(i,j)}(\epsilon)$ – function of $\left\{ A_I^{(i,j)}, f_I^{(i,j)}, \overline{\mathcal{G}}_{I,ij} \right\}$

Fixed using Abelianisation rules to 3rd order – due to missing N^{3}LO results

* Soft and collinear divergences proportional to $\delta(1-z)$ and $\mathcal{D}_0$ get cancelled against those resulting from the FFs entirely and the AP kernels partially

$$\left(\frac{1}{(1-z)}\right)_+$$

* Finite part correctly reproduces all the distributions in the SV part of CFs Δ_I

* $$\text{Universality} : \hat{\phi}_q^{A,(i,j)} = \hat{\phi}_b^{A,(i,j)}$$

$$\text{Casimir scaling} : \hat{\phi}_q^{A,(i,j)} = \hat{\phi}_b^{A,(i,j)} = C_F/C_A \; \hat{\phi}_g^{A,(i,j)}$$

Properties Of The NSV Solution -New

* $\hat{\phi}_I^{B,(i,j)}(z,\epsilon)$ - function of $\left\{C_I^{i,j}, D_I^{i,j}, \varphi_I(z)\right\}$ → Process dependent

* Singular part removes the remaining collinear divergences of the AP kernels

* NSV finite part + SV counterpart give rises to next to SV logarithmic terms to CFs Δ_I

* Universality breaks down beyond two-loop : $\hat{\phi}_q^{B,(i,j)} \neq \hat{\phi}_b^{B,(i,j)}$

  Casimir scaling fails : $\hat{\phi}_q^{B,(i,j)} \neq \hat{\phi}_b^{B,(i,j)} \neq C_F/C_A \; \hat{\phi}_g^{B,(i,j)}$

  NSV piece has z-dependence - consequence: shifting invariance

Recent findings from QCD
[Ajjath, P. Mukherjee, V. Ravindran]
arXiv:2006.06726 [hep-ph]
[Ajjath, P. Mukherjee, V. Ravindran, A.Sankar, S.Tiwari]
JHEP 04 (2021) 131

Abelianisation - Two-loops

certain transformation rules that relate color and charge factors for the relevant Feynman diagrams in QCD and QED

Two-loop : $q\bar{q}$ *initiated cases*

QCD	QCD×QED	QED
C_F^2	$2C_F e_q^2$	e_q^4
$C_F C_A$	0	0
$C_F n_f T_F$	0	$e_q^2 \left(N \sum_q e_q^2 + \sum_l e_l^2\right)$
$C_F T_F$	0	$N\ e_i^2 e_j^2$

NNLO QCD×QED results for DY were obtained using the Abelianisation: → [D. de Florian, M. Der and I. Fabre]

Explicit calculation verified the results obtained using Abelianisation → [A. H. Ajjath, A. Chakraborty, P. K. Dhani,P. Mukherjee, N. Rana, V. Ravindran et al.]

All the anomalous dimensions are found to obey the Abelianisation rules till two-loop

Abelianisation : Three-loops

- In QCD up to three loops, the cusp $A_I^{(i,0)}$, the soft $f_I^{(i,0)}$ and the constants $\overline{\mathcal{G}}_{I,ij}^{(k)}$ contain identical set of color factors, namely at one loop, we have $\{C_F\}$, at two loops $\{C_F C_A, C_F T_f n_f\}$ and at three loops $\{C_F C_A^2, C_F C_A T_f n_f, C_F^2 T_f n_f, C_F T_f^2 n_f^2\}$.

⬇

- $\hat{\phi}_I^{A,(i,j)}(\epsilon)$ in QCD demonstrate uniform color factor structure

⬇

- $\hat{\phi}_I^{A,(i,j)}(\epsilon)$ in QED and mixed QCD×QED obtained taking the Abelian limit assuming uniform color and charge factor structure holds true to third order

[Ajjath, P. Mukherjee, V. Ravindran]

Abelianisation : Three-loops

General set of Abelianisation rules obtained from the explicit calculation of FFs

Example diagram shown in next slide

QCD (a_s^3)	QCD×QED ($a_s^2 a_e$)	QCD×QED ($a_s a_e^2$)	QED (a_e^3)
C_F^3	$3C_F^2 e_I^2$	$3C_F e_I^4$	e_I^6
$C_A C_F^2$	$C_A C_F e_I^2$	0	0
$C_F^2 n_f T_F$	$a \ C_F n_f T_F e_I^2 + b \ C_F T_F\left(\sum_q e_q^2 + \sum_l e_l^2\right)$	$C_F e_I^2\left(N\sum_q e_q^2 + \sum_l e_l^2\right)$	$a \ e_I^4\left(N\sum_q e_q^2 + \sum_l e_l^2\right) + b \ e_I^2\left(N\sum_q e_q^4 + \sum_l e_l^4\right)$
$C_F C_A^2$	0	0	0
$C_F C_A n_f T_F$	0	0	0
$C_F n_f^2 T_F^2$	0	0	$e_I^2\left(N\sum_q e_q^2 + \sum_l e_l^2\right)^2$

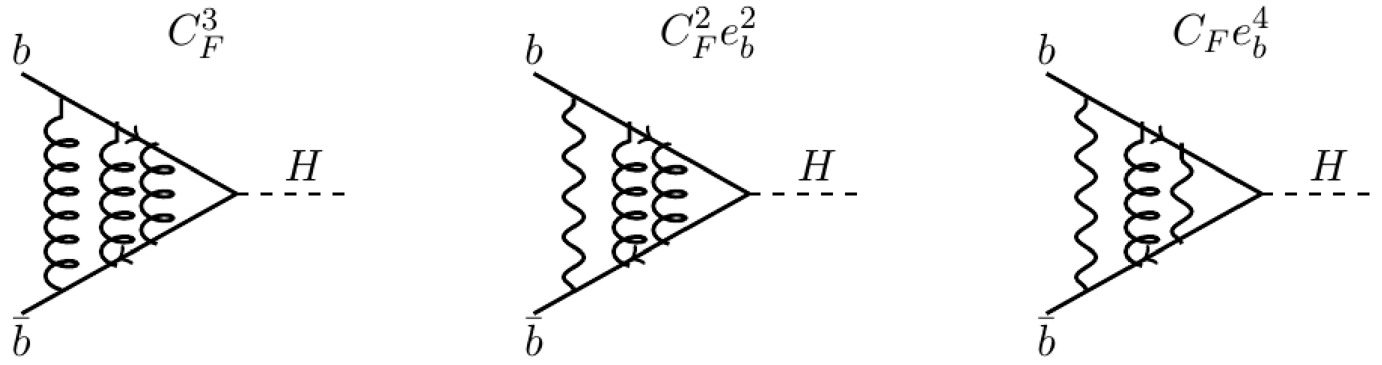
There is no one-one mapping: ambiguous color transformations – single fermion loop

The coefficients a, b against the corresponding color factors depend on the contribution from relevant topologies and are dependent on the FFs

Absence of Self-interaction vertices

Example : Three-loop

Transformation rule for C_F^3



This color factor arises from those diagrams where no fermion or gluon loops are present.
The numerical factor of three at a_s^2 a_e order accounts for the number of ways a gluon field can be replaced by a photon field in a pure QCD Feynman diagram.

Some Observations On Abelianisation

- Abelianisation procedure which succeeded in giving definite color transformation rules at the two loop level without explicit calculation fails at the three loop level

- Only $\left\{ A_I^{(i,j)}, f_I^{(i,j)}, \overline{\mathcal{G}}_{I,ij} \right\}$ follow unambiguous rules owing to their uniform color and charge factors

- At three loops, closed fermion loop configurations map to different charge-color factors in QED and QCD×QED hence taking abelian limit of the pure QCD FF results does not produce pure QED as well as QCD×QED results

- The coefficients {a,b} for QCD ×QED and pure QED color factors can only be fixed by explicit calculation, limits the use of abelianisation procedure beyond NNLO

Abelianisation – NSV Coefficients

- NSV anomalous dimensions $\left\{ \mathrm{C}_I^{i,j}, D_I^{i,j} \right\}$ and the coefficients $\varphi_I(z)$ do not exhibit any uniform color structure in QCD

$$+$$

- Universality of $\hat{\phi}_I^{B,(i,j)}(z,\epsilon)$ breakes beyond 2-loops in QCD sensitive to hard-vertex information

$$\Downarrow$$

- Not possible to assume that $\hat{\phi}_I^{B,(i,j)}(z,\epsilon)$ in QED and QCD×QED can be obtained by applying the rules at third order

Analytic Structure Of The Partonic CF - Third Order

Highest and next-to-highest logarithms predictable to all orders

Pure QED

Predictable from 2-loop results

Spoils the predictability unknown

$$\Delta_I^{(0,3)} = L_z^5\left\{8D_I^{(0,1)}\left(A_I^{(0,1)}\right)^2\right\} + L_z^4\left\{4A_I^{(0,1)}\varphi_{I,02}^{(2)} + 8A_I^{(0,1)}C_I^{(0,2)} + 4\left(A_I^{(0,1)}\right)^2\varphi_{I,01}^{(0)} + 16\left(A_I^{(0,1)}\right)^3 - 16D_I^{(0,1)}A_I^{(0,1)}f_I^{(0,1)} - \frac{40}{3}D_I^{(0,1)}\beta_{00}'A_I^{(0,1)}\right\}$$

$$+ L_z^3\left\{2\varphi_{I,03}^{(3)} - 4f_I^{(0,1)}\varphi_{I,02}^{(2)} - 8f_I^{(0,1)}C_I^{(0,2)} + 4A_I^{(0,1)}\varphi_{I,02}^{(1)} - 8A_I^{(0,1)}f_I^{(0,1)}\varphi_{I,01}^{(0)} - 32\left(A_I^{(0,1)}\right)^2 f_I^{(0,1)} - 8\beta_{00}'\varphi_{I,02}^{(2)} - 8\beta_{00}'C_I^{(0,2)} - \frac{32}{3}\beta_{00}'A_I^{(0,1)}\varphi_{I,01}^{(0)} - 16\beta_{00}'\left(A_I^{(0,1)}\right)^2 + 8D_I^{(0,1)}\left(f_I^{(0,1)}\right)^2 + 8D_I^{(0,1)}A_I^{(0,2)} + 16D_I^{(0,1)}A_I^{(0,1)}\overline{\mathcal{G}}_{I,01}^{(1)} + 16D_I^{(0,1)}A_I^{(0,1)}g_{I,01}^{(1)} - 40D_I^{(0,1)}\left(A_I^{(0,1)}\right)^2\zeta_2 + 16D_I^{(0,1)}\beta_{00}'f_I^{(0,1)} + \frac{16}{3}D_I^{(0,1)}(\beta_{00}')^2 + 8D_I^{(0,2)}A_I^{(0,1)}\right\} + \mathcal{O}(L_z^2)$$

$$L_z^k = \ln^k(1-z)$$

Mixed QCD $\times$ QED

Predictable from 2-loop results

Spoils the predictability unknown

$$
\begin{aligned}
\Delta_I^{(1,2)} =& L_z^5\Big\{4(A_I^{(0,1)})^2\varphi_{I,10}^{(1)} + 16D_I^{(0,1)}A_I^{(1,0)}A_I^{(0,1)} + 8D_I^{(1,0)}(A_I^{(0,1)})^2\Big\} \\
&+ L_z^4\Big\{2A_I^{(0,1)}\varphi_{I,11}^{(2)} + 8A_I^{(0,1)}C_I^{(1,1)} - 8A_I^{(0,1)}f_I^{(0,1)}\varphi_{I,10}^{(1)} + 4(A_I^{(0,1)})^2\varphi_{I,10}^{(0)} + 4A_I^{(1,0)}\varphi_{I,02}^{(2)} \\
&+ 8A_I^{(1,0)}C_I^{(0,2)} + 8A_I^{(1,0)}A_I^{(0,1)}\varphi_{I,01}^{(0)} + 48A_I^{(1,0)}(A_I^{(0,1)})^2 - \frac{8}{3}\beta'_{00}A_I^{(0,1)}\varphi_{I,10}^{(1)} \\
&- 16D_I^{(0,1)}A_I^{(0,1)}f_I^{(1,0)} - 16D_I^{(0,1)}A_I^{(1,0)}f_I^{(0,1)} - 8D_I^{(0,1)}\beta'_{00}A_I^{(1,0)} - 16D_I^{(1,0)}A_I^{(0,1)}f_I^{(0,1)} \\
&- \frac{16}{3}D_I^{(1,0)}\beta'_{00}A_I^{(0,1)}\Big\} \\
&+ L_z^3\Big\{\varphi_{I,12}^{(3)} - 2f_I^{(0,1)}\varphi_{I,11}^{(2)} - 8f_I^{(0,1)}C_I^{(1,1)} + 4(f_I^{(0,1)})^2\varphi_{I,10}^{(1)} \\
&- 4f_I^{(1,0)}\varphi_{I,02}^{(2)} - 8f_I^{(1,0)}C_I^{(0,2)} + 4A_I^{(0,2)}\varphi_{I,10}^{(1)} + 2A_I^{(0,1)}\varphi_{I,11}^{(1)} + 8A_I^{(0,1)}\overline{\mathcal{G}}_{I,01}^{(1)}\varphi_{I,10}^{(1)} + 8A_I^{(0,1)}g_{I,01}^{(1)}\varphi_{I,10}^{(1)} \\
&- 8A_I^{(0,1)}f_I^{(0,1)}\varphi_{I,10}^{(0)} - 8A_I^{(0,1)}f_I^{(1,0)}\varphi_{I,01}^{(0)} - 20(A_I^{(0,1)})^2\zeta_2\varphi_{I,10}^{(1)} - 32(A_I^{(0,1)})^2f_I^{(1,0)} + 4A_I^{(1,0)}\varphi_{I,02}^{(1)} \\
&- 8A_I^{(1,0)}f_I^{(0,1)}\varphi_{I,01}^{(0)} - 64A_I^{(1,0)}A_I^{(0,1)}f_I^{(0,1)} - 2\beta'_{00}\varphi_{I,11}^{(2)} - 4\beta'_{00}C_I^{(1,1)} + 4\beta'_{00}f_I^{(0,1)}\varphi_{I,10}^{(1)} \\
&- \frac{8}{3}\beta'_{00}A_I^{(0,1)}\varphi_{I,10}^{(0)} - 8\beta'_{00}A_I^{(1,0)}\varphi_{I,01}^{(0)} - 16\beta'_{00}A_I^{(1,0)}A_I^{(0,1)} + 16D_I^{(0,1)}f_I^{(1,0)}f_I^{(0,1)} + 8D_I^{(0,1)}A_I^{(1,1)} \\
&+ 16D_I^{(0,1)}A_I^{(0,1)}\overline{\mathcal{G}}_{I,10}^{(1)} + 16D_I^{(0,1)}A_I^{(0,1)}g_{I,10}^{(1)} + 16D_I^{(0,1)}A_I^{(1,0)}\overline{\mathcal{G}}_{I,01}^{(1)} + 16D_I^{(0,1)}A_I^{(1,0)}g_{I,01}^{(1)} \\
&- 80D_I^{(0,1)}A_I^{(1,0)}A_I^{(0,1)}\zeta_2 + 8D_I^{(0,1)}\beta'_{00}f_I^{(1,0)} + 8D_I^{(0,2)}A_I^{(1,0)} + 8D_I^{(1,1)}A_I^{(0,1)} + 8D_I^{(1,0)}(f_I^{(0,1)})^2 \\
&+ 8D_I^{(1,0)}A_I^{(0,2)} + 16D_I^{(1,0)}A_I^{(0,1)}\overline{\mathcal{G}}_{I,01}^{(1)} + 16D_I^{(1,0)}A_I^{(0,1)}g_{I,01}^{(1)} - 40D_I^{(1,0)}(A_I^{(0,1)})^2\zeta_2 + 8D_I^{(1,0)}\beta'_{00}f_I^{(0,1)}\Big\} + \mathcal{O}(L_z^2)
\end{aligned}
$$

Summary And Outlook

- The perturbative structure of the partonic coefficient function has been studied in the context of QED and mixed QCD×QED taking into account the NSV corrections.

- Factorisation and RG invariance properties of the the scattering cross-section has been used to achieve this.

- Partonic CF exhibits an exponential form followed from the factorisation and RG evolution Eqns. This exponential form allow the predictions of certain SV+NSV logarithms to all orders in pertuarbation theory.

- The Abelianisation procedure which worked for the case of NNLO QED and mixed QCD×QED failed to hold true at third order.

- The formalism for the Fixed-order NSV mixed QCD×QED computation has opened up opportunities to study the resummation effects of the NSV logarithms in this context.

Future Directions

- Phenomenological analysis of the impacts of QCD$\times$QED NSVcorrections
- Resummation of NSV logarithms in mixed QCD$\times$QED
- Extending the formalism to study the off-diagonal channel contributions

Thanks for the attention!

Backup Slides

Anomalous Dimensions In QCD-QED

$$X_I = \sum_{i,j=0}^{\infty} a_s^i a_e^j X_I^{(i,j)}$$

Extracted from AP splitting function up to 2-loops

$$X_I^{(i,j)} \in \left(A_I^{(i,j)}, B_I^{(i,j)}, f_I^{(i,j)}, \gamma_I^{(i,j)}, C_I^{(i,j)}, D_I^{(i,j)} \right)$$

Extracted directly from the FFs up to 3-loops

Universal: $A_q = A_b$

Cannot be disentangled from FFs alone

Single pole structure in FFs can be extracted

$$G_I^{(i,j)}(\varepsilon) = 2(B_I^{(i,j)} - \gamma_I^{(i,j)}) + f_I^{(i,j)} + \sum_{k=0} \varepsilon^k g_{I,ij}^k$$

Operator independent

$B_q = B_b, f_q = f_b$

$\Longrightarrow$ $\gamma_b^{(i,j)}$ $(\gamma_q^{(i,j)} = 0)$ can be computed using $G_b^{(i,j)} - G_q^{(i,j)}$

Only $2B_I + f_I$ can be extracted from the FFs up to 3-loops

We need to study the soft corrections to fix f_I

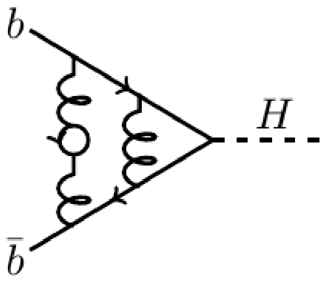
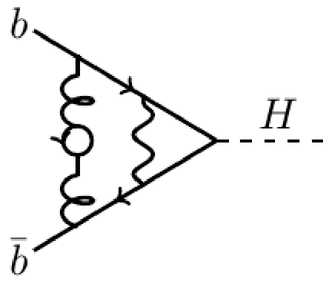
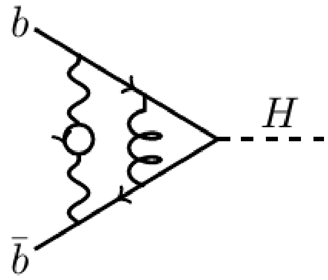
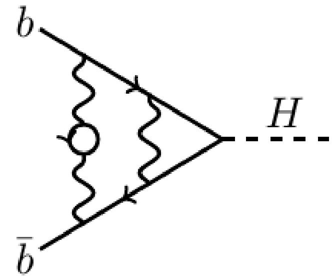
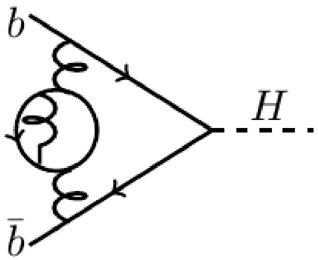
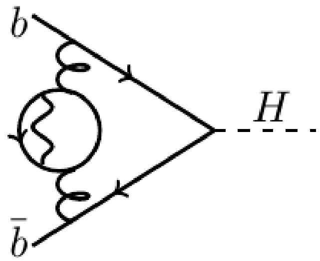
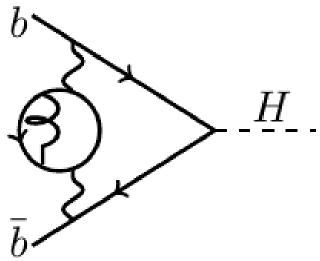
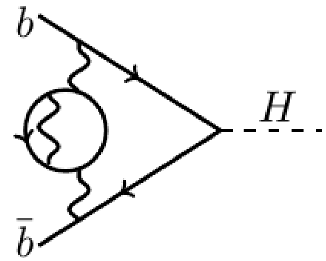
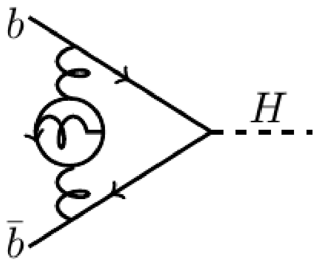
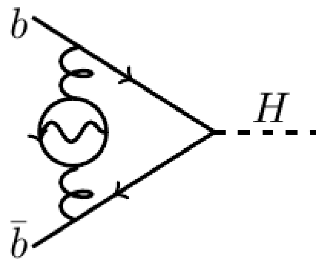
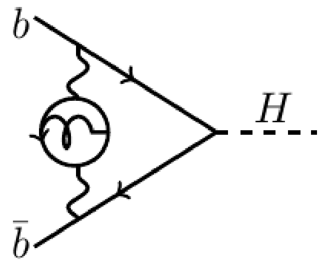
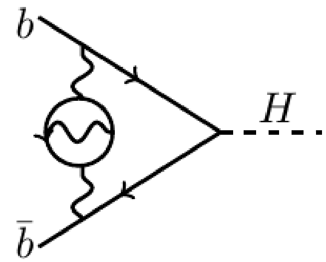
NSV Anomalous Dimensions

$$C_1 = 0\,, \qquad C_2 = -\sigma A_1^2\,, \qquad C_3 = -2\sigma A_1 A_2\,, \qquad C_4 = -\sigma(A_2^2 + 2A_1 A_3)\,,$$

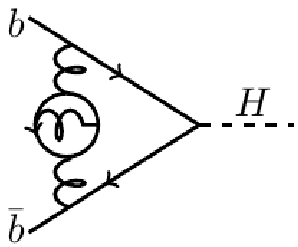
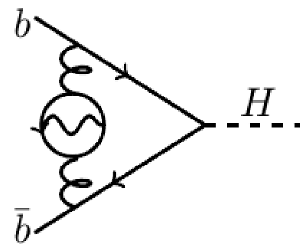
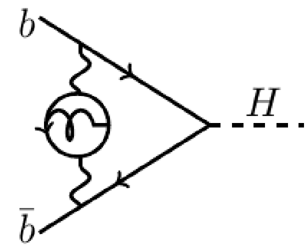
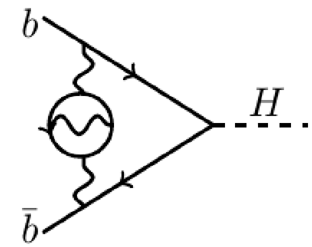
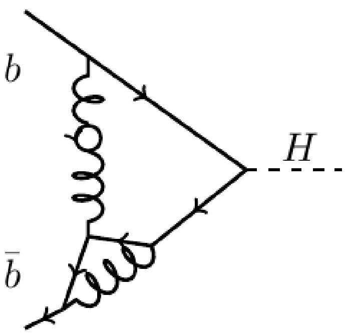
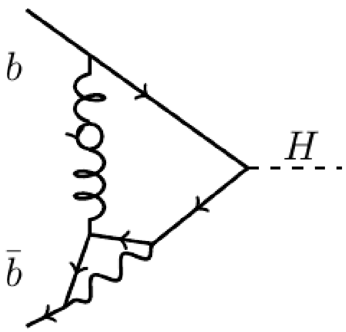
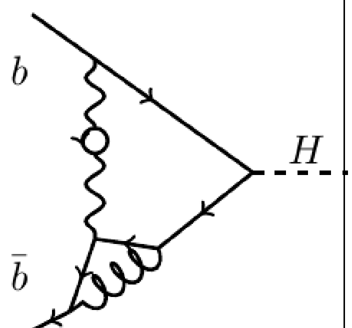
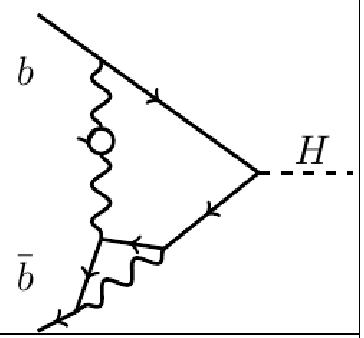
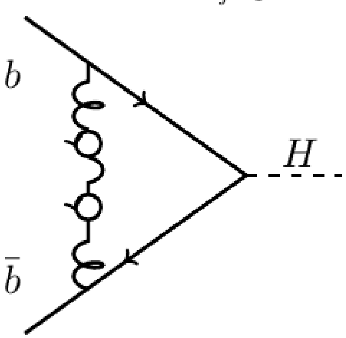
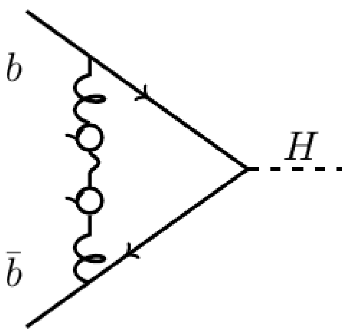
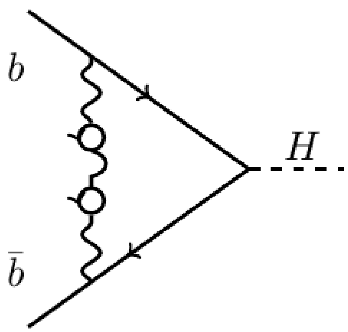
$$D_1 = 0, \quad D_2 = -A_1(\sigma B_1 + \beta_0)\,, \quad D_3 = -A_1(\sigma B_2 + \beta_1) - A_2(\sigma B_1 + \mathbf{2} \cdot \beta_0)\,,$$

[Dokshitzer,Marchesini,Salam (DMS)]

Color Transformation Rules- Three-loop

a_s^3	$a_s^2 a_e$	$a_s a_e^2$	a_e^3
$C_F^2 n_f T_F$ 	$C_F n_f T_F e_q^2$ 	$C_F e_b^2 (N \sum_q e_q^2 + \sum_l e_l^2)$ 	$e_b^4 (N \sum_q e_q^2 + \sum_l e_l^2)$ 
$C_F^2 n_f T_F$ 	$C_F T_F (\sum_q e_q^2 + \sum_l e_l^2)$ 	$C_F e_b^2 (N \sum_q e_q^2 + \sum_l e_l^2)$ 	$e_b^2 (N \sum_q e_q^4 + \sum_l e_l^4)$ 
$(C_F^2 - \frac{C_A C_F}{2}) n_f T_F$ 	$C_F T_F (\sum_q e_q^2 + \sum_l e_l^2)$ 	$C_F e_b^2 (N \sum_q e_q^2 + \sum_l e_l^2)$ 	$e_b^2 (N \sum_q e_q^4 + \sum_l e_l^4)$ 

Color Transformation Rules- Three-loop

a_s^3	$a_s^2 a_e$	$a_s a_e^2$	a_e^3
$(C_F^2 - \frac{C_A C_F}{2}) n_f T_F$ 	$C_F T_F (\sum_q e_q^2 + \sum_l e_l^2)$ 	$C_F e_b^2 (N \sum_q e_q^2 + \sum_l e_l^2)$ 	$e_b^2 (N \sum_q e_q^4 + \sum_l e_l^4)$ 
$(C_F^2 - \frac{C_A C_F}{2}) n_f T_F$ 	$C_F n_f T_F e_b^2$ 	$C_F e_b^2 (N \sum_q e_q^2 + \sum_l e_l^2)$ 	$e_b^4 (N \sum_q e_q^2 + \sum_l e_l^2)$ 
$C_F n_f^2 T_F^2$ 	0 	0 	$e_b^2 (N \sum_q e_q^2 + \sum_l e_l^2)^2$