New Results on Muons and Pions

Outline:

- Muon decay: NNNLO corrections to the rate!
  - useful expansion around equal muon and electron masses.

- Decay of a muonic (pionic) atom
  - daughter electron can remain bound
  - decay rate unexpectedly sensitive to components with E<0

RADCOR-LoopFest 2021

Andrzej Czarnecki University of Alberta

May 18, 2021

Part 1: Free muon decay

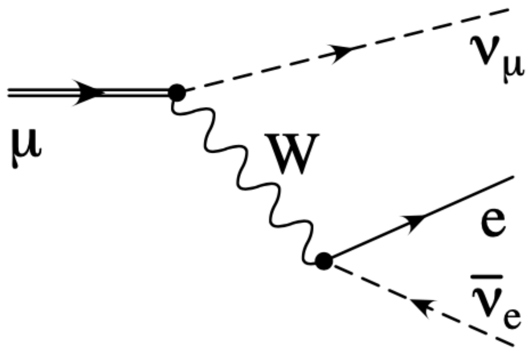
Three-loop corrections to the muon and heavy quark decay rates

Michał Czakon
Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany

Andrzej Czarnecki and Matthew Dowling
Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2E1

in press as a Letter in the Physical Review D
(accepted yesterday)

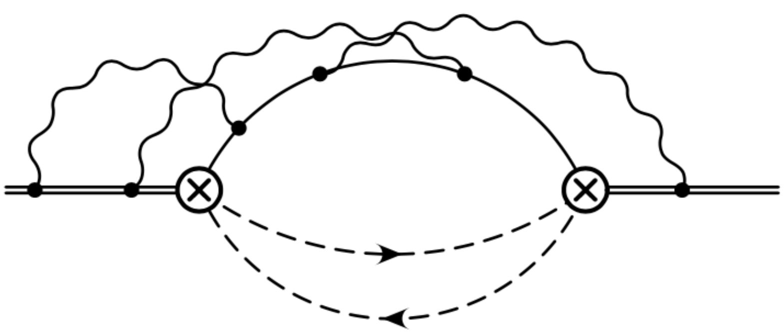
Free muon decay

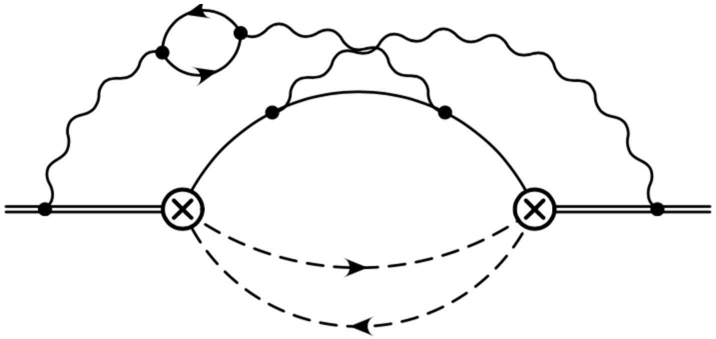


Pillar of precision tests: provides Fermi constant

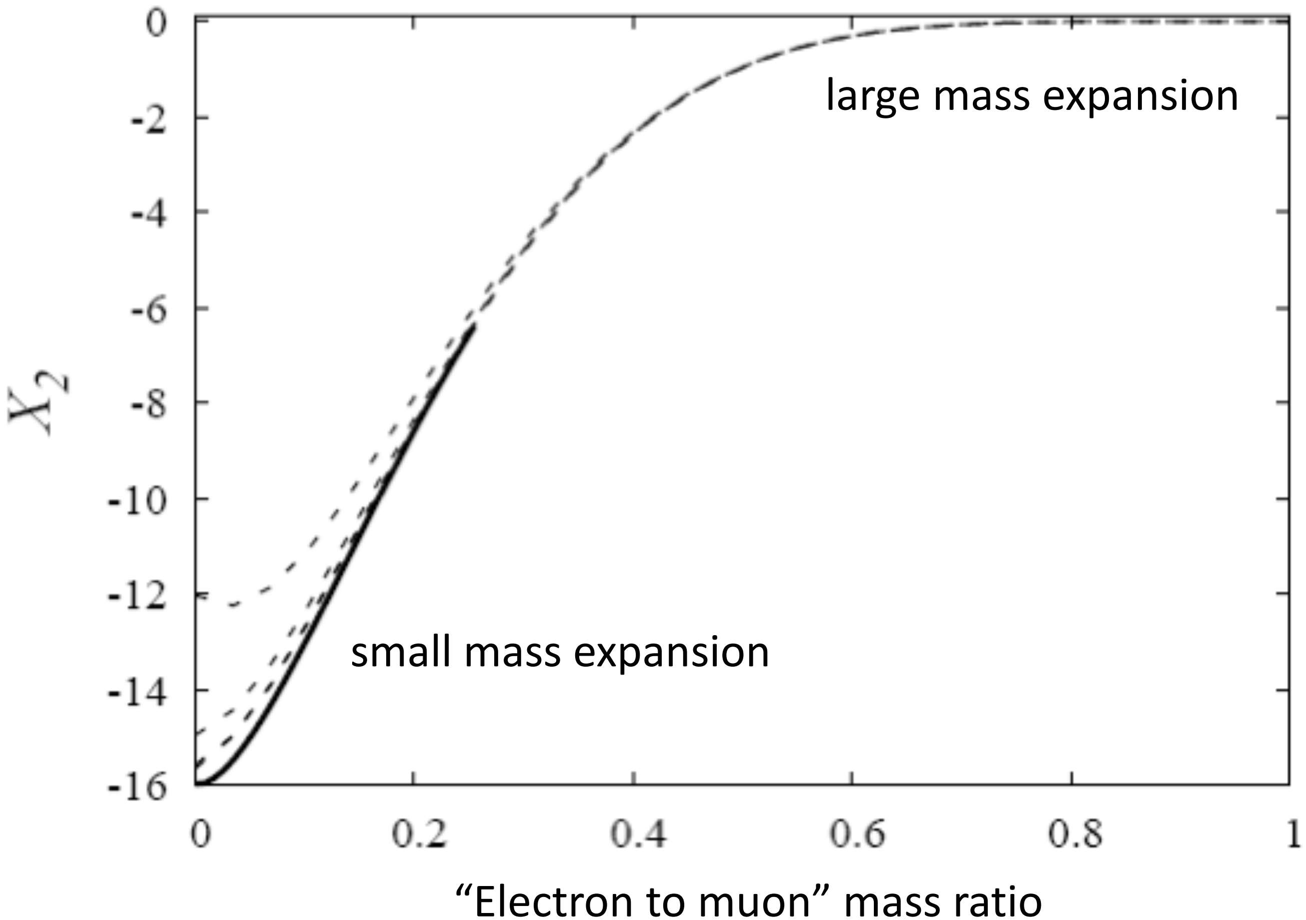
Radiative corrections:

O(α) 1956 Behrends, Finkelstein, Sirlin
$O(\alpha^2)$ 1999 van Rittbergen, Stuart ($m_e = 0$)
2008 Pak, Czarnecki ($m_e \neq 0$)
$O(\alpha^3)$ 2020-2021 Fael, Schönwald, Steinhauser;
Czakon, Dowling, Czarnecki (partial)
Expansion around $m_e = m_\mu$





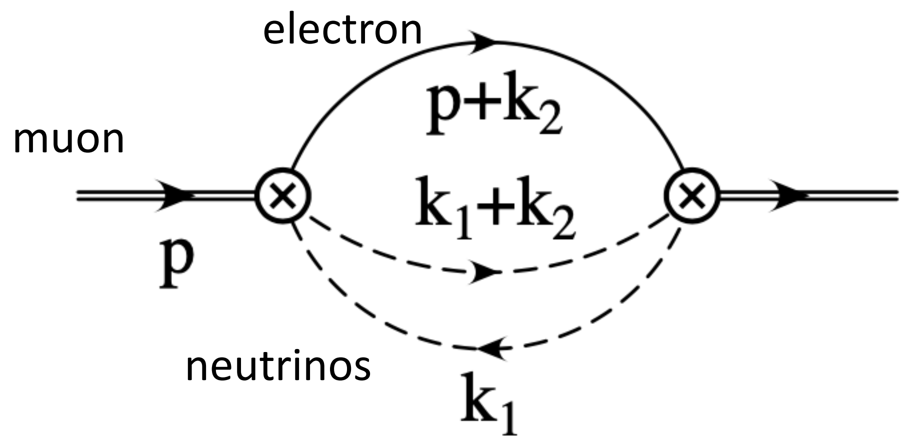
Good convergence of the large mass expansion



From Dowling, Piclum, AC

Note: the plot actually for QCD, and for two-loop corrections.
QED given by a subset of QCD results.

How is the expansion constructed?



1. Use optical theorem: $\Gamma(\mu \to e\bar{\nu}\nu) \sim \mathrm{Im} M$
2. Integrate over k_1
3. Consider hard $k_2 \sim m_\mu$ (no imaginary part);
   and soft $k_2 \sim m_\mu - m_e$ (technically easier to integrate)
4. Repeat for up to five-loop diagrams (with three photonic loops)

Typical soft integral: $$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2)^{n_1} [2p \cdot k]^{n_2} [2p \cdot k + \Delta]^{n_3}}$$

Resulting error budget in G_F : parts per million

$$G_F^2 \sim \frac{1}{\tau_\mu}\frac{1}{m_\mu^5}\left[1+\frac{\alpha}{\pi}\cdots+\left(\frac{\alpha}{\pi}\right)^2\cdots+\left(\frac{\alpha}{\pi}\right)^3\cdots\right]$$

$\frac{\Delta G_F}{G_F}$: 1 0.2 0.05

see Kay Schönwald's talk on Thursday

Part 2: Bound muon decay

PHYSICAL REVIEW D **102**, 073001 (2020)

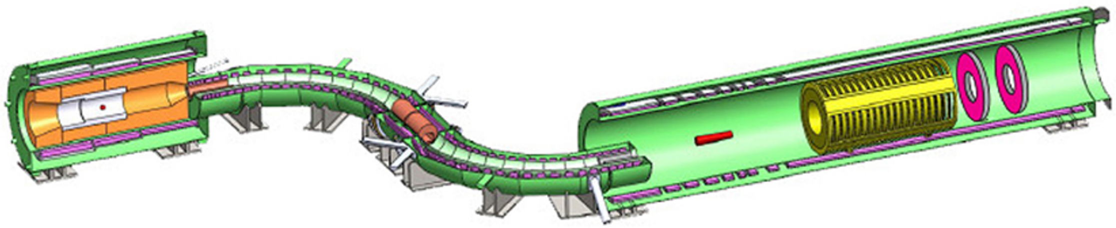
Decay of a bound muon into a bound electron

M. Jamil Aslam,[1,2] Andrzej Czarnecki,[1] Guangpeng Zhang,[1] and Anna Morozova[1]

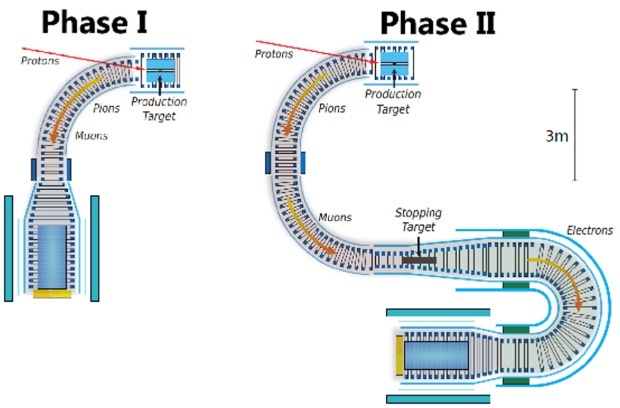
Bound muon decay: why do we care?

Muon-electron conversion searches

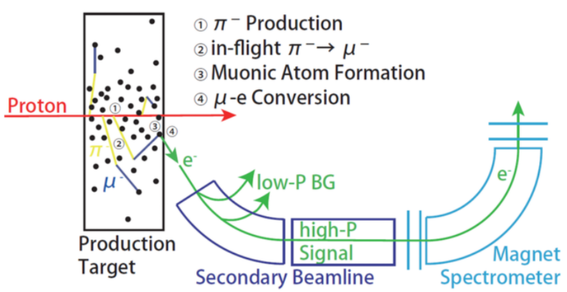
Mu2e
Fermilab



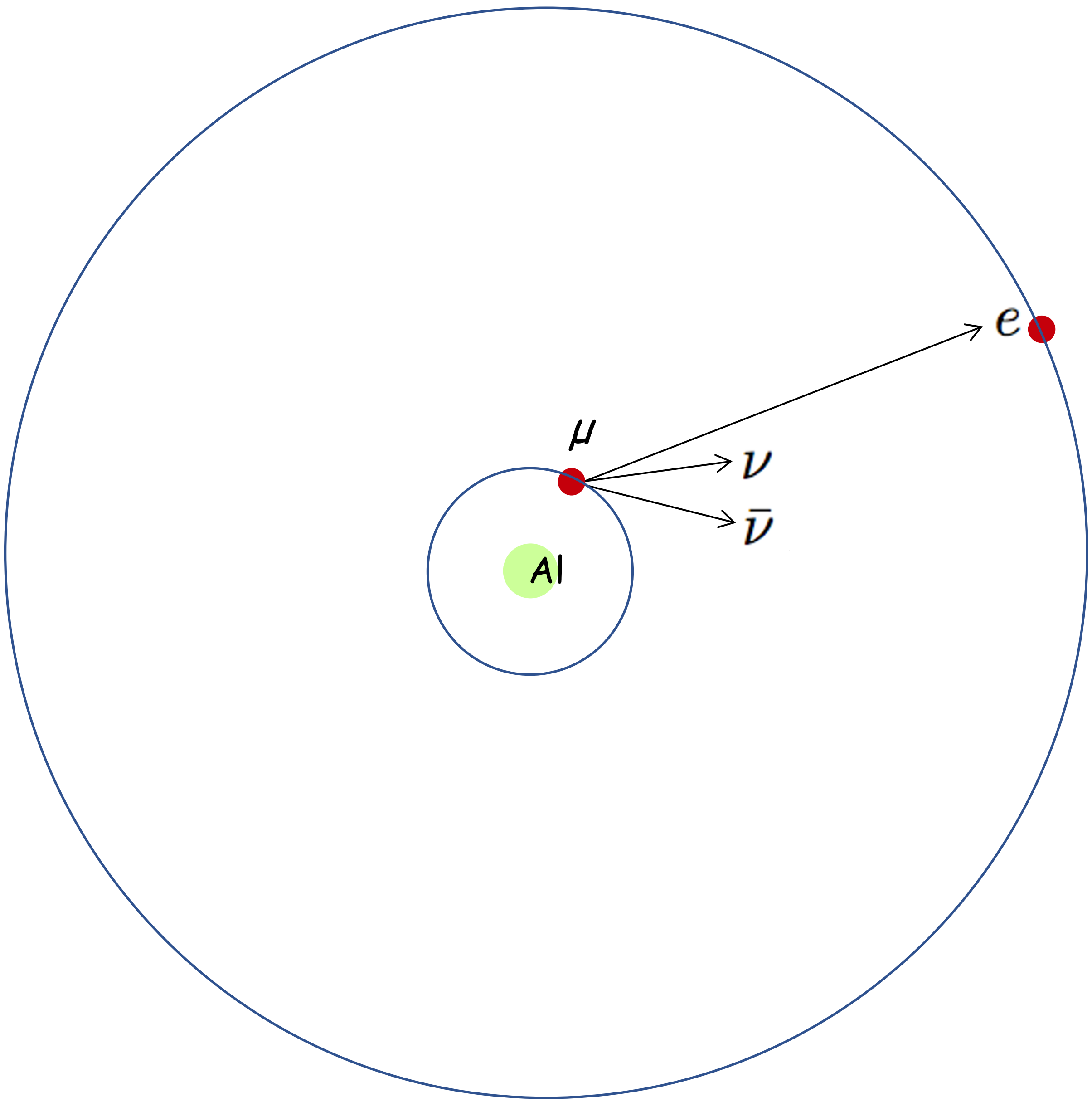
COMET
J-PARC



DeeMe
J-PARC



Bound muon decay into a bound electron



What is the probability of binding of the daughter electron?

Bound muon decay into a bound electron

PHYSICAL REVIEW D VOLUME 52, NUMBER 7 1 OCTOBER 1995

Atomic alchemy: Weak decays of muonic and pionic atoms into other atoms

C. Greub and D. Wyler
Institut für Theoretische Physik, Universität Zürich, Zürich, Switzerland

S. J. Brodsky and C. T. Munger
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

Bound particle wave function can be expanded in plane waves

$$\tilde{\Phi}(\vec{k}) = \sum_r \left[A_r(\vec{k}) \frac{u_r(\vec{k})}{\sqrt{2k^0}} + B_r^*(-\vec{k}) \frac{v_r(-\vec{k})}{\sqrt{2k^0}} \right]$$

negative energy states:
neglected in the "alchemy" paper:

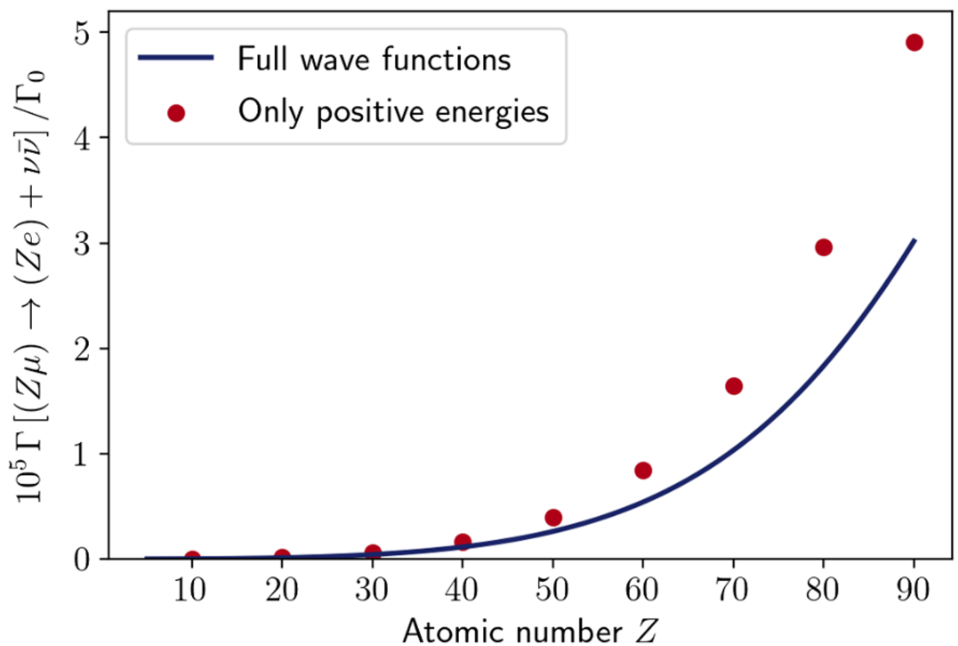
The integral $\int [d^3k/(2\pi)^3] \sum_r |B_r(\vec{k})|^2$ gives the probability to find a three particle Fock state ($e^+e^-e^-$) in the atom. Even for $Z = 80$ this fraction is tiny ($\approx 0.2\%$), so we only consider the one-Fock contribution characterized by $A_r(\vec{k})$.

Bound muon decay: our study

PHYSICAL REVIEW D **102,** 073001 (2020)

Decay of a bound muon into a bound electron

M. Jamil Aslam[1,2] Andrzej Czarnecki[1] Guangpeng Zhang[1] and Anna Morozova[1]



Why such large effect of negative energies?

Tentative explanation:

The decay happens where the muon and the electron wave functions overlap.
This is a tiny fraction of the electron's range, close to the nucleus.
In that region, E < 0 is relatively likely.

Check: position space calculation

$$\mathcal{M} = \frac{g}{\sqrt{2}} \int \mathrm{d}^3\boldsymbol{r}\, \exp\left(i\boldsymbol{q}\cdot\boldsymbol{r}\right) \bar{\Phi}_\mu(\boldsymbol{r}) \not{\epsilon}^{\lambda_A \star} L \Phi_e(\boldsymbol{r})$$

$$\frac{1}{\Gamma_0}\Gamma[(Z\mu^-) \to (Ze^-) + \nu\bar{\nu}] = 128 \int_0^{z_{\max}} (N_a^2 + N_b^2 + F_a^2 + F_b^2) k_A z^3 \mathrm{d}z.$$

Our position space and momentum space evaluations agree (if both A and B included).

Summary

Free muon decay: great new result by Fael, Schönwald, Steinhauser; we have checked the hardest pieces. Theoretical prediction ready for generations of experiments…. And useful for quark decays.

Decay of a bound muon into a bound electron:
- unexpected lesson in the Dirac equation
- negative energy components are important

Pion beta decay

PHYSICAL REVIEW D **101,** 091301(R) (2020)

Rapid Communications

Pion beta decay and Cabibbo-Kobayashi-Maskawa unitarity

Andrzej Czarnecki

Department of Physics, University of Alberta, Edmonton, Alberta T6G 2E1, Canada

William J. Marciano

Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA

Alberto Sirlin

Department of Physics, New York University, 726 Broadway, New York, New York 10003, USA

First row CKM unitarity

$$|V_{\rm ud}|^2 + |V_{\rm us}|^2 = 1$$ (V_{ub} negligible with current errors)

Agreement recently questioned; 1.7 – 5 sigma significance, dependent on nuclear corrections, kaon-pion form factors (lattice), etc.
[al. et Gorchtein et al. 2020]

V_{ud} best determined from superallowed nuclear beta decays.

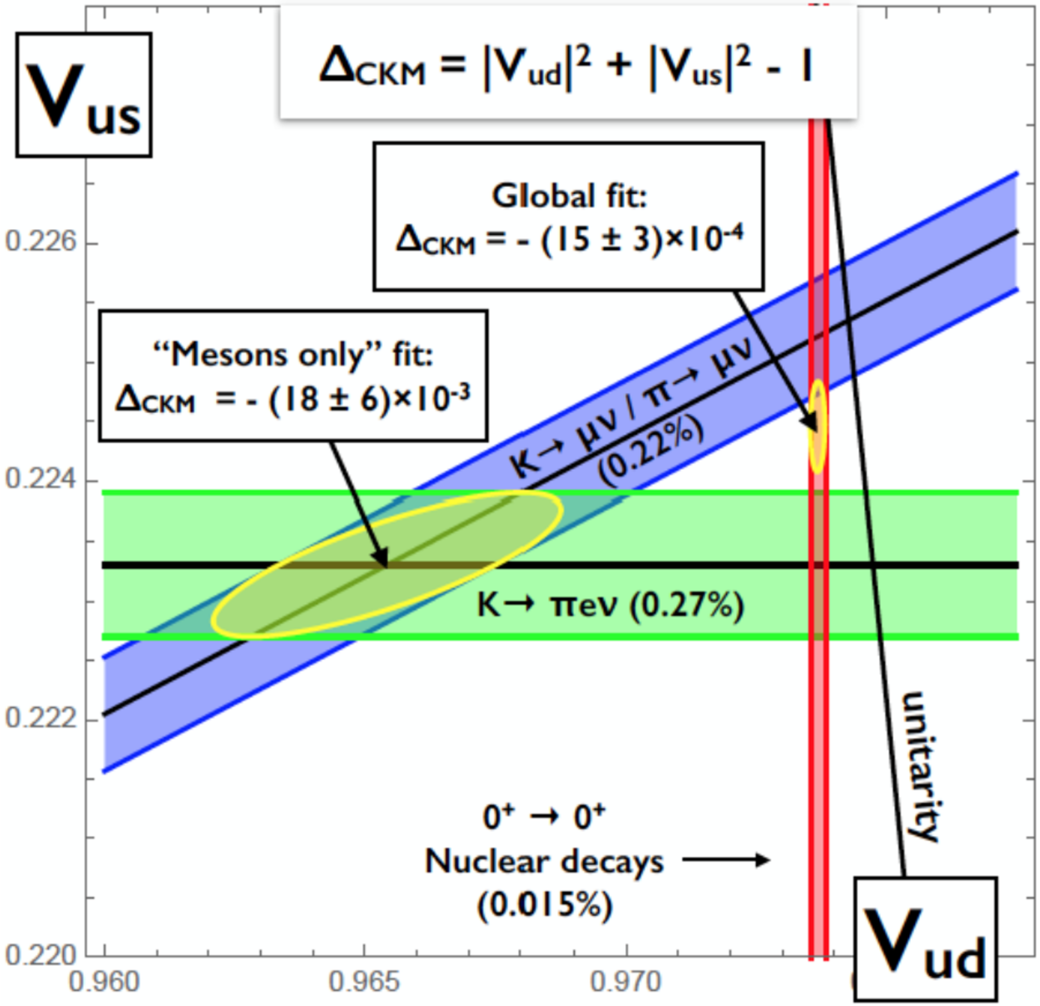
V_{us} from kaon decays: about 2 sigma discrepancy between K_{2l} and K_{3l}

Better constraint:

$$R_A = \frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} = 1.3367(28), \quad \frac{f_{K^+}}{f_{\pi^+}} = 1.1932(19) \qquad \frac{|V_{\rm us}|}{|V_{\rm ud}|} = 0.23131(45)$$

Tensions in the 1st-row CKM unitarity test

V. Cirigliano



R_V: a complementary constraint on the 1st row

We propose a complementary ratio, $$R_V = \frac{\Gamma(K_L \to \pi^\pm e^\mp \nu(\gamma))}{\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma))}$$

New role for the pion beta decay

$$R_V^{\exp} = \frac{\tau_\pi \times \mathrm{BR}(K_L \to \pi^\pm e^\mp \nu(\gamma))}{\tau_{K_L} \times \mathrm{BR}(\pi^+ \to \pi^0 e^+ \nu(\gamma))} = \frac{26.033(5)\ \mathrm{ns} \times 0.4056(9)}{51.16(21)\ \mathrm{ns} \times 1.038(6) \times 10^{-8}} = 1.9884(115)(93) \times 10^7$$

pion partial width

kaon

Results from the current value of R_V

Averaging over all K_{l3} modes:

$$\frac{f_+^K(0)V_{\rm us}}{f_+^\pi(0)V_{\rm ud}} = 0.22223(64)(40)$$

To agree with R_A, this ratio should be 0.9607(38), not 0.970(2); confirmed!

PACS Collaboration; Kakazu et al. PRD 101 (2020) 094504