Heavy flavor corrections to DIS at 2- and 3-loop order

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DESY

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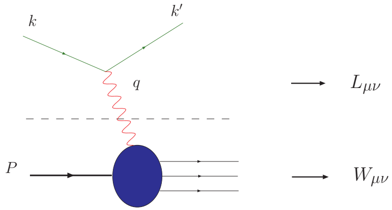
Outline

Theory of deep inelastic scattering



- Kinematic invariants:

$$Q^2 = -q^2, \qquad x = \frac{Q^2}{2P.q}$$

- The cross section factorizes into leptonic and hadronic tensor:

$$\frac{d^2\sigma}{dQ^2 dx} \sim L_{\mu\nu} W^{\mu\nu}$$

- The hadronic tensor can be expressed through structure functions:

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{4\pi} \int d^4\xi \exp(iq\xi) \langle P, | \left[J_\mu^{\text{em}}(\xi), J_\nu^{\text{em}}(\xi)\right] |P\rangle \\ &= \frac{1}{2x}\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) F_L(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2(x,Q^2) \\ &+ i\epsilon_{\mu\nu\rho\sigma} \frac{q^\rho S^\sigma}{q\cdot P} g_1(x,Q^2) + i\epsilon_{\mu\nu\rho\sigma} \frac{q^\rho \left(q\cdot PS^\sigma - q\cdot SP^\sigma\right)}{(q\cdot P)^2} g_2(x,Q^2) \end{aligned}$$

- F_L, F_2, g_1 and g_2 contain contributions from both, charm and bottom quarks.

Introduction

Why are Heavy Flavor Contributions important?

- They form a significant contribution to all structure functions particularly at small x and high Q^2.
- Concise 3-loop corrections are needed to determine $\alpha_s(M_Z)$, m_c and perhaps m_b.
- The accuracy of measurements at the LHC reaches a level of precision requiring 3-loop VFNS matching.

NNLO: [S. Alekhin, J. Blümlein, S. Moch and R. Placakyte (Phys. Rev. D96 (2017))]

$\alpha_s(M_Z^2) = 0.1147 \pm 0.0008$

$m_c(m_c) = 1.252 \pm 0.018(exp) \; {}^{+0.03}_{-0.02} \; (scale), \; {}^{+0.00}_{-0.07} \; (thy)\mathrm{GeV}$ ($\overline{\mathrm{MS}}$-scheme)

Yet approximate NNLO treatment for F_2 (non-negligible error) [H. Kawamura et al. (Nucl. Phys. B864 (2012))]
NS & PS corrections are exact [J. Ablinger et al. (Nucl. Phys. B886 & B890 (2014))]
EIC: many more high precision data ahead for varios detailed unpolarized and polarized precision measurements.

Factorization of the Structure Functions

At leading twist the structure functions factorize in terms of a Mellin convolution

$$F_{(2,L)}(x,Q^2) = \sum_j \underbrace{\mathbb{C}_{j,(2,L)}\left(x,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)}_{perturbative} \otimes \underbrace{f_j(x,\mu^2)}_{nonpert.}$$

into (pert.) Wilson coefficients and (nonpert.) parton distribution functions (PDFs).
$\otimes$ denotes the Mellin convolution

$$f(x)\otimes g(x) \equiv \int_0^1 dy \int_0^1 dz\, \delta(x-yz)f(y)g(z)\,.$$

The subsequent calculations are performed in Mellin space, where $\otimes$ reduces to a multiplication, due to the Mellin transformation

$$\hat{f}(N) = \int_0^1 dx\, x^{N-1} f(x)\,.$$

Wilson coefficients:

$$\mathbb{C}_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)=C_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2}\right)+H_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) .$$

At $Q^2 \gg m^2$ the heavy flavor part

$$H_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)=\sum_i C_{i,(2,L)}\left(N,\frac{Q^2}{\mu^2}\right)A_{ij}\left(\frac{m^2}{\mu^2},N\right)$$

[Buza, Matiounine, Smith, van Neerven (Nucl.Phys.B (1996))]

factorizes into the light flavor Wilson coefficients C and the massive operator matrix elements (OMEs) of local operators O_i between partonic states j

$$A_{ij}\left(\frac{m^2}{\mu^2},N\right)=\langle j \mid O_i \mid j\rangle .$$

$\rightarrow$ additional Feynman rules with local operator insertions for partonic matrix elements.

The unpolarized light flavor Wilson coefficients are known up to NNLO (for unpolarized scattering).

[Moch, Vermaseren, Vogt (Nucl.Phys.B (2005))]

For $F_2(x, Q^2)$: at $Q^2 \gtrsim 10m^2$ the asymptotic representation holds at the 1% level.

Status of OME calculations

Unpolarized

Leading Order: [Witten (1976); Babcock, Sivers, Wolfram (1978); Shifman, Vainshtein, Zakharov (1978); Leveille, Weiler (1979); Glück, Reya (1979); Glück, Hoffmann, Reya (1982)]

Next-to-Leading Order:

full m dependence (numeric) [Laenen, van Neerven, Riemersma, Smith (1993)]

$Q^2 \gg m^2$: via IBP [Buza, Matiounine, Smith, Migneron, van Neerven (1996)]

Compact results via ${}_pF_q$'s [Bierenbaum, Blümlein, Klein (2007)]

$O(\alpha_s^2 \varepsilon)$ (for general N) [Bierenbaum, Blümlein, Klein (2008, 2009)]

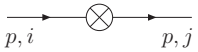
Next-to-Next-to-Leading Order: $Q^2 \gg m^2$

- Moments for F_2: $N = 2...10(14)$ [Bierenbaum, Blümlein, Klein (2009)] using `MATAD` [Steinhauser (2000)]
- Contributions to transversity: $N = 1...13$ [Blümlein, Klein, Tödtli 2009]
- Two masses $m_1 \neq m_2 \rightarrow$ Moments N = 2,4,6 [Blümlein, Wißbrock (2011)]

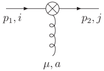
At 3-loop order for general values of N and extension to polarized scattering: Topic of this talk.

Calculation of the 3-loop operator matrix elements

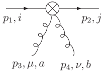
The OMEs are calculated using the QCD Feynman rules together with the following operator insertion Feynman rules:



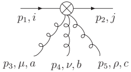
$$\delta^{ij}\not\Delta\gamma_\pm(\Delta\cdot p)^{N-1}\,,\quad N\geq 1$$



$$g t^a_{ji}\Delta^\mu\not\Delta\gamma_\pm\sum_{j=0}^{N-2}(\Delta\cdot p_1)^j(\Delta\cdot p_2)^{N-j-2}\,,\quad N\geq 2$$



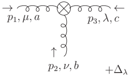
$$g^2\Delta^\mu\Delta^\nu\not\Delta\gamma_\pm\sum_{j=0}^{N-3}\sum_{l=j+1}^{N-2}(\Delta p_2)^j(\Delta p_1)^{N-l-2}$$
$$\left[(t^at^b)_{ji}(\Delta p_1+\Delta p_4)^{l-j-1}+(t^bt^a)_{ji}(\Delta p_1+\Delta p_3)^{l-j-1}\right]\,,$$
$$N\geq 3$$



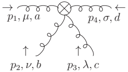
$$g^3\Delta_\mu\Delta_\nu\Delta_\rho\not\Delta\gamma_\pm\sum_{j=0}^{N-4}\sum_{l=j+1}^{N-3}\sum_{m=l+1}^{N-2}(\Delta.p_2)^j(\Delta.p_1)^{N-m-2}$$
$$\Big[\ (t^at^bt^c)_{ji}(\Delta.p_4+\Delta.p_5+\Delta.p_1)^{l-j-1}(\Delta.p_5+\Delta.p_1)^{m-l-1}$$
$$+(t^at^ct^b)_{ji}(\Delta.p_4+\Delta.p_5+\Delta.p_1)^{l-j-1}(\Delta.p_4+\Delta.p_1)^{m-l-1}$$
$$+(t^bt^at^c)_{ji}(\Delta.p_3+\Delta.p_5+\Delta.p_1)^{l-j-1}(\Delta.p_5+\Delta.p_1)^{m-l-1}$$
$$+(t^bt^ct^a)_{ji}(\Delta.p_3+\Delta.p_5+\Delta.p_1)^{l-j-1}(\Delta.p_3+\Delta.p_1)^{m-l-1}$$
$$+(t^ct^at^b)_{ji}(\Delta.p_3+\Delta.p_4+\Delta.p_1)^{l-j-1}(\Delta.p_4+\Delta.p_1)^{m-l-1}$$
$$+(t^ct^bt^a)_{ji}(\Delta.p_3+\Delta.p_4+\Delta.p_1)^{l-j-1}(\Delta.p_3+\Delta.p_1)^{m-l-1}\Big]\,,$$
$$N\geq 4$$

$$\gamma_+=1\,,\quad\gamma_-=\gamma_5\,.$$

$$\frac{1+(-1)^N}{2}\delta^{ab}(\Delta\cdot p)^{N-2}$$
$$\Big[g_{\mu\nu}(\Delta\cdot p)^2-(\Delta_\mu p_\nu+\Delta_\nu p_\mu)\Delta\cdot p+p^2\Delta_\mu\Delta_\nu\Big]\,,\quad N\geq 2$$



$$-ig\frac{1+(-1)^N}{2}f^{abc}\Bigg($$
$$\Big[(\Delta_\nu g_{\lambda\mu}-\Delta_\lambda g_{\mu\nu})\Delta\cdot p_1+\Delta_\mu(p_{1,\nu}\Delta_\lambda-p_{1,\lambda}\Delta_\nu)\Big](\Delta\cdot p_1)^{N-2}$$
$$+\Delta_\lambda\Big[\Delta\cdot p_1p_{2,\mu}\Delta_\nu+\Delta\cdot p_2p_{1,\nu}\Delta_\mu-\Delta\cdot p_1\Delta\cdot p_2g_{\mu\nu}-p_1\cdot p_2\Delta_\mu\Delta_\nu\Big]$$
$$\times\sum_{j=0}^{N-3}(-\Delta\cdot p_1)^j(\Delta\cdot p_2)^{N-3-j}$$
$$+\left\{\begin{matrix}p_1\to p_2\to p_3\to p_1\\ \mu\to\nu\to\lambda\to\mu\end{matrix}\right\}+\left\{\begin{matrix}p_1\to p_3\to p_2\to p_1\\ \mu\to\lambda\to\nu\to\mu\end{matrix}\right\}\Bigg)\,,\quad N\geq 2$$



$$g^2\frac{1+(-1)^N}{2}\Bigg(f^{abe}f^{cde}O_{\mu\nu\lambda\sigma}(p_1,p_2,p_3,p_4)$$
$$+f^{ace}f^{bde}O_{\mu\lambda\nu\sigma}(p_1,p_3,p_2,p_4)+f^{ade}f^{bce}O_{\mu\sigma\nu\lambda}(p_1,p_4,p_2,p_3)\Bigg)\,,$$
$$O_{\mu\nu\lambda\sigma}(p_1,p_2,p_3,p_4)=\Delta_\nu\Delta_\lambda\Big\{-g_{\mu\sigma}(\Delta\cdot p_3+\Delta\cdot p_4)^{N-2}$$
$$+[p_{4,\mu}\Delta_\sigma-\Delta\cdot p_4g_{\mu\sigma}]\sum_{i=0}^{N-3}(\Delta\cdot p_3+\Delta\cdot p_4)^i(\Delta\cdot p_4)^{N-3-i}$$
$$-[p_{1,\sigma}\Delta_\mu-\Delta\cdot p_1g_{\mu\sigma}]\sum_{i=0}^{N-3}(-\Delta\cdot p_1)^i(\Delta\cdot p_3+\Delta\cdot p_4)^{N-3-i}$$
$$+[\Delta\cdot p_1\Delta\cdot p_4g_{\mu\sigma}+p_1\cdot p_4\Delta_\mu\Delta_\sigma-\Delta\cdot p_4p_{1,\sigma}\Delta_\mu-\Delta\cdot p_1p_{4,\mu}\Delta_\sigma]$$
$$\times\sum_{i=0}^{N-4}\sum_{j=0}^{i}(-\Delta\cdot p_1)^{N-4-i}(\Delta\cdot p_3+\Delta\cdot p_4)^{i-j}(\Delta\cdot p_4)^j\Big\}$$
$$-\left\{\begin{matrix}p_1\leftrightarrow p_2\\ \mu\leftrightarrow\nu\end{matrix}\right\}-\left\{\begin{matrix}p_3\leftrightarrow p_4\\ \lambda\leftrightarrow\sigma\end{matrix}\right\}+\left\{\begin{matrix}p_1\leftrightarrow p_2,\ p_3\leftrightarrow p_4\\ \mu\leftrightarrow\nu,\ \lambda\leftrightarrow\sigma\end{matrix}\right\}\,,\quad N\geq 2$$

Method of calculation

- Resummation of operator insertion into propagator structure ($\Delta \cdot \Delta = 0$):

$$\sum_{N=0}^{\infty} t^N (\Delta \cdot k)^N = \frac{1}{1 - t\,\Delta \cdot k}$$

- Reduction to master integrals using IBP relations implemented in `Reduze2`[Manteuffel, Studerus (2012)] .
- Solution of master integrals obtained by different methods:
  - direct integration using Mellin-Barnes and ${}_pF_q$-techniques
  - differential equations in resummation variable t
- method of arbitrary high moments, i.e. reconstructing all-N solution from a large number of fixed moments

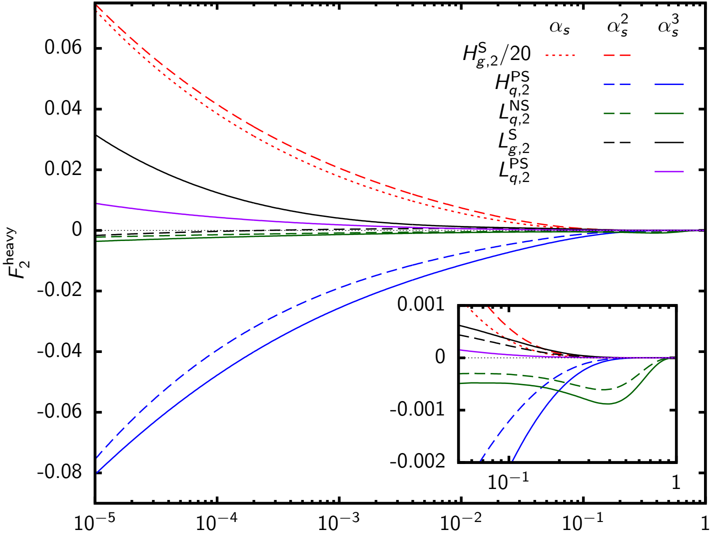
$\Rightarrow$ All these methods use the packages `Sigma`, `EvaluateMultiSums` [Schneider (2007-)] and `HarmonicSums` [Ablinger et al. (2010-)] which have been developed with these calculations.

The Wilson Coefficients at large Q^2

$$
\begin{aligned}
L_{q,(2,L)}^{\rm NS}(N_F+1) &= a_s^2\left[A_{qq,Q}^{(2),\rm NS}(N_F+1)\delta_2 + \hat{C}_{q,(2,L)}^{(2),\rm NS}(N_F)\right] + a_s^3\left[A_{qq,Q}^{(3),\rm NS}(N_F+1)\delta_2 + A_{qq,Q}^{(2),\rm NS}(N_F+1)C_{q,(2,L)}^{(1),\rm NS}(N_F+1) + \hat{C}_{q,(2,L)}^{(3),\rm NS}(N_F)\right] \\
L_{q,(2,L)}^{\rm PS}(N_F+1) &= a_s^3\left[A_{qq,Q}^{(3),\rm PS}(N_F+1)\delta_2 + N_F A_{gq,Q}^{(2),\rm NS}(N_F)\tilde{C}_{g,(2,L)}^{(1),\rm NS}(N_F+1) + N_F\hat{\tilde{C}}_{q,(2,L)}^{(3),\rm PS}(N_F)\right] \\
L_{g,(2,L)}^{\rm S}(N_F+1) &= a_s^2 A_{gg,Q}^{(1)}(N_F+1)N_F\tilde{C}_{g,(2,L)}^{(2)}(N_F+1) + a_s^3\left[A_{qg,Q}^{(3)}(N_F+1)\delta_2 + A_{gg,Q}^{(1)}(N_F+1)N_F\tilde{C}_{g,(2,L)}^{(2)}(N_F+1)\right. \\
&\quad \left. + A_{gg,Q}^{(2)}(N_F+1)N_F\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + A_{Qg}^{(1)}(N_F+1)N_F\tilde{C}_{q,(2,L)}^{(2),\rm PS}(N_F+1) + N_F\hat{\tilde{C}}_{g,(2,L)}^{(3)}(N_F)\right] \\
H_{q,(2,L)}^{\rm PS}(N_F+1) &= a_s^2\left[A_{Qq}^{(2),\rm PS}(N_F+1)\delta_2 + \tilde{C}_{q,(2,L)}^{(2),\rm PS}(N_F+1)\right] \\
&\quad + a_s^3\left[A_{Qq}^{(3),\rm PS}(N_F+1)\delta_2 + A_{gq,Q}^{(2)}(N_F+1)\tilde{C}_{g,(1,L)}^{(2)}(N_F+1) + A_{Qq}^{(2),\rm PS}(N_F+1)\tilde{C}_{q,(2,L)}^{(1),\rm NS}(N_F+1) + \tilde{C}_{q,(2,L)}^{(3),\rm PS}(N_F+1)\right] \\
H_{g,(2,L)}^{\rm S}(N_F+1) &= a_s\left[A_{Qg}^{(1)}(N_F+1)\delta_2 + \tilde{C}_{g,(2,L)}^{(1)}(N_F+1)\right] \\
&\quad + a_s^2\left[A_{Qg}^{(2)}(N_F+1)\delta_2 + A_{Qg}^{(1)}(N_F+1)\tilde{C}_{q,(2,L)}^{(1)}(N_F+1) + A_{gg,Q}^{(1)}(N_F+1)\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + \tilde{C}_{g,(2,L)}^{(2)}(N_F+1)\right] \\
&\quad + a_s^3\left[A_{Qg}^{(3)}(N_F+1)\delta_2 + A_{Qg}^{(2)}(N_F+1)\tilde{C}_{q,(2,L)}^{(1)}(N_F+1) + A_{gg,Q}^{(2)}(N_F+1)\tilde{C}_{g,(2,L)}^{(1)}(N_F+1)\right. \\
&\quad \left. + A_{Qg}^{(1)}(N_F+1)\tilde{C}_{q,(2,L)}^{(2),\rm S}(N_F+1) + A_{gg,Q}^{(1)}(N_F+1)\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + \tilde{C}_{g,(2,L)}^{(3)}(N_F+1)\right]
\end{aligned}
$$

- All first order factorizable contributions and $O(1000)$ fixed moments of $A_{Qg}^{(3)}$ are known.
- The case for two different masses obeys an analogous representation.

Heavy Flavor contribution to F_2



The NC PS contributions to $F_2(x, Q^2)$ and $F_L(x, Q^2)$

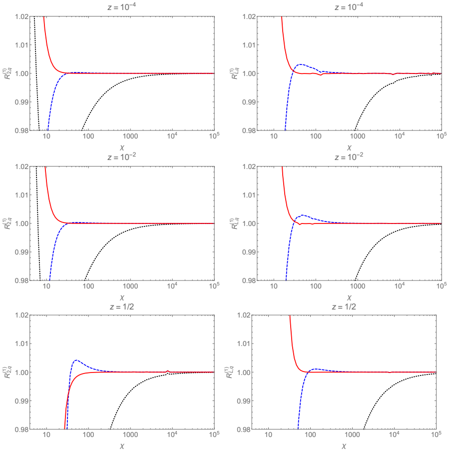


Figure 1: The ratios $R_{2,q}^{(1)}$ (left) and $R_{L,q}^{(1)}$ (right) as a function of $\chi = Q^2/m^2$ for different values of z gradually improved with κ suppressed terms. Dotted lines: asymptotic result; dashed lines: $O(m^2/Q^2)$ improved; solid lines : $O((m^2/Q^2)^2)$ improved.

Different convergence range for F_2 and F_L w.r.t Q^2 at $O(\alpha_s^2)$.

The variable flavor number scheme

- Matching conditions for parton distribution functions:

$$f_k(N_F+2) + f_{\overline{k}}(N_F+2) = A_{qq,Q}^{\rm NS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot \left[f_k(N_F) + f_{\overline{k}}(N_F)\right] + \frac{1}{N_F} A_{qq,Q}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot \Sigma(N_F)$$

$$+ \frac{1}{N_F} A_{qg,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot G(N_F)\,,$$

$$f_Q(N_F+2) + f_{\overline{Q}}(N_F+2) = A_{Qq}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2},\right) \cdot \Sigma(N_F) + A_{Qg}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot G(N_F)\,,$$

$$\Sigma(N_F+2) = \left[A_{qq,Q}^{\rm NS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) + A_{qq,Q}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) + A_{Qq}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right)\right] \cdot \Sigma(N_F)$$

$$+ \left[A_{qg,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) + A_{Qg}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right)\right] \cdot G(N_F)\,,$$

$$G(N_F+2) = A_{gq,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot \Sigma(N_F) + A_{gg,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot G(N_F)\,.$$

Polarized OMEs

- Polarized OMEs for heavy flavor production at 2-loop order have been calculated before.
  [Buza et al (1996), Klein (2009), Hasselhuhn (2013)]
- Calculation of OMEs relied on tensor-decomposition in order to arrive at Larin scheme, i.e.

$$\gamma_5 \to \frac{i}{24}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma.$$

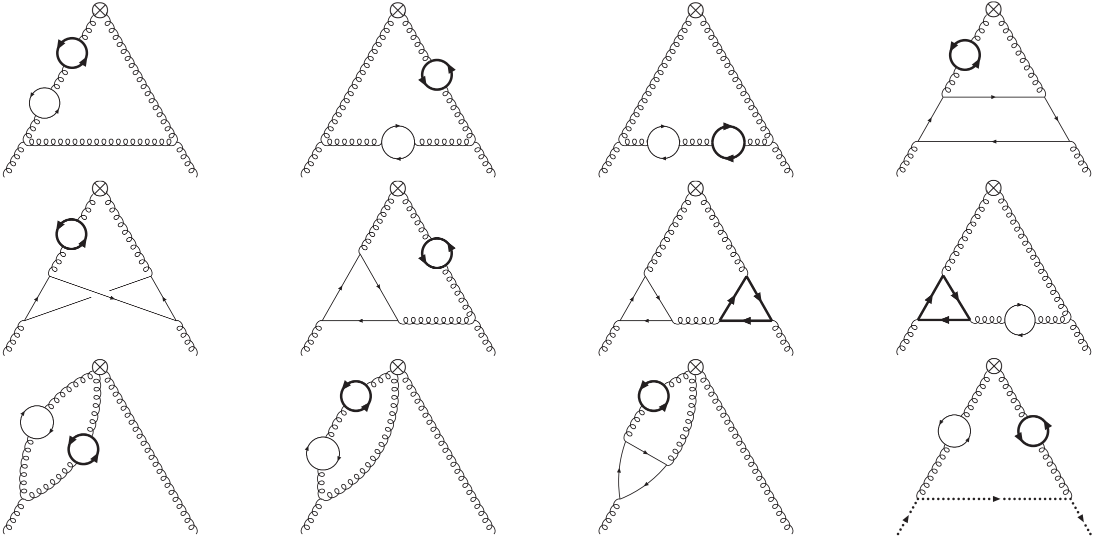
- We found out that a change of projector can accomplish the same:

$$\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left[\not p\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma G_q\right] \to \epsilon_{\mu\nu\rho\sigma}p^\rho\Delta^\sigma\mathrm{tr}\left[\not p\gamma^\mu\gamma^\nu G_q\right]$$

$\Rightarrow$ this allows to apply all technologies for the unpolarized OMEs directly to the polarized ones, i.e.

1. the terms $\sim T_F$ of the polarized 3-loop anomalous dimensions from a massive calculation
   [Behring et al. (Nucl. Phys. B948 (2020))]
2. $A_{qq,Q}^{(3),\mathrm{PS}}$, $A_{Qq}^{(3),\mathrm{PS}}$, $A_{gq,Q}^{(3)}$ (single and 2-mass contributions)
   [Ablinger et al. (Nucl. Phys. B952,953,955 (2020)), Behring et al. (Nucl. Phys. B964 (2021))]

2-mass contributions



2-mass contributions

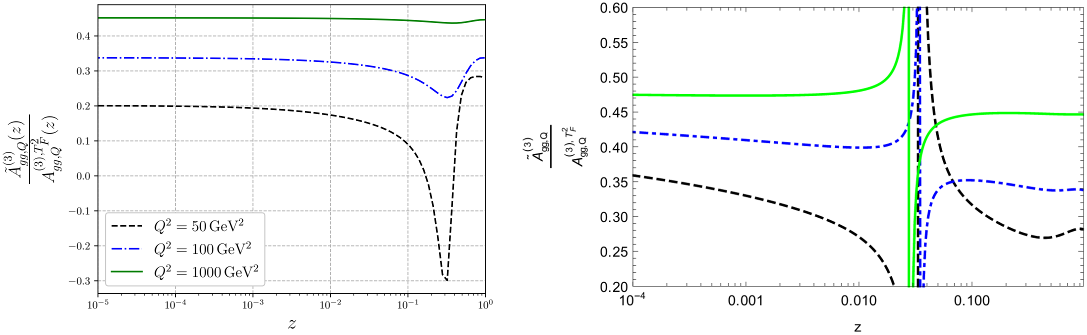
$A_{qq,Q}^{(3),\mathrm{NS}}, A_{gq,Q}^{(3)}$	Harmonic Sums [Vermaseren '98; Blümlein, Kurth '98] $\sum_{i=1}^{N} \frac{1}{i^3} \sum_{j=1}^{i} \frac{1}{i}$	HPLs [Remiddi, Vermaseren '99] $\int_0^x \frac{d\tau_1}{1+\tau_1} \int_0^{\tau_1} \frac{d\tau_2}{1-\tau_2}$
$A_{gg,Q}^{(3)}$	Generalized harmonic and binomial sums [Ablinger, Blülein, Schneider '13] [Ablinger, Blülein, Raab, Schneider '14] $\sum_{i=1}^{N} \frac{4^i (1-\eta)^{-i}}{i\binom{2i}{i}} \sum_{j=1}^{i} \frac{(1-\eta)^j}{j^2}$	Iterated integrals over root and η valued letters [Ablinger, Blümlein, Raab, Schneider '14] $\int_0^x d\tau_1 \frac{\sqrt{\tau_1(1-\tau_1)}}{1-\tau_1(1-\eta)} \int_0^{\tau_1} \frac{d\tau_2}{\tau_2}$
$A_{Qq}^{(3),\mathrm{PS}}$	–	Iterated integrals over root valued letters with restricted support $\theta(x-\eta_+) \int_0^{x(1-x)/\eta} d\tau \frac{\sqrt{1-4\tau}}{\tau}$

Results: $A_{gg,Q}^{(3)}$

$$\begin{aligned}
\tilde{a}_{gg,Q}^{(3)}(N) = \; & \frac{1}{2}\left(1+(-1)^N\right)\Biggl\{T_F^3\left\{\frac{32}{3}\left(L_1^3+L_2^3\right)+\frac{64}{3}L_1L_2\left(L_1+L_2\right)+32\zeta_2\left(L_1+L_2\right)+\frac{128}{9}\zeta_3\right\} \\
+ \; & C_F T_F^2\Biggl\{\cdots+32\left(\mathrm{H}_0^2(\eta)-\frac{1}{3}S_2\right)S_1+\frac{128}{3}S_{2,1}-\frac{64}{3}S_{1,1,1}\left(\frac{1}{1-\eta},1-\eta,1,N\right) \\
& -\frac{4P_{41}}{3(N-1)N^3(N+1)^2(N+2)(2N-3)(2N-1)}\left(\frac{\eta}{1-\eta}\right)^N\Biggl[\mathrm{H}_0^2(\eta) \\
& -2\mathrm{H}_0(\eta)S_1\left(\frac{\eta-1}{\eta},N\right)-2S_2\left(\frac{\eta-1}{\eta},N\right)+2S_{1,1}\left(\frac{\eta-1}{\eta},1,N\right)\Biggr]+\ldots\Biggr\} \\
+ \; & C_A T_F^2\Biggl\{\cdots+\Biggl[\frac{8P_{65}}{3645\eta(N-1)N^3(N+1)^3(N+2)(2N-3)(2N-1)} \\
& +\frac{8P_{37}\mathrm{H}_0(\eta)}{45\eta(N-1)N^2(N+1)^2(N+2)}+\frac{2P_{23}\mathrm{H}_0^2(\eta)}{9\eta(N-1)N(N+1)^2}+\frac{32}{27}\mathrm{H}_0^3(\eta)+\frac{128}{9}\mathrm{H}_{0,0,1}(\eta) \\
& +\frac{64}{9}\mathrm{H}_0^2(\eta)\mathrm{H}_1(\eta)-\frac{128}{9}\mathrm{H}_0(\eta)\mathrm{H}_{0,1}(\eta)\Biggr]S_1 \\
& +\frac{2^{-1-2N}P_{47}}{45\eta^2(N-1)N(N+1)^2(N+2)(2N-3)(2N-1)}\binom{2N}{N}\sum_{i=1}^{N}\frac{4^i\left(\frac{\eta}{\eta-1}\right)^i}{i\binom{2i}{i}}\Biggl\{\frac{1}{2}\mathrm{H}_0^2(\eta) \\
& S_{1,1}\left(\frac{\eta-1}{\eta},1,i\right)\Biggr\}+\ldots\Biggr\}
\end{aligned}$$

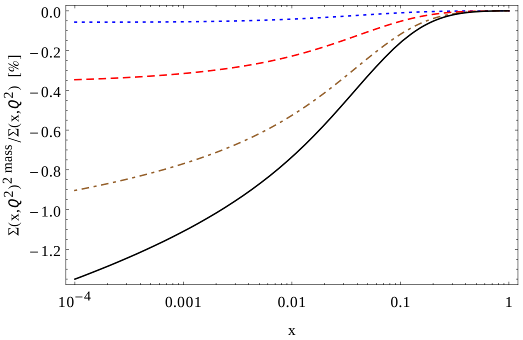
Results: $A_{gg,Q}^{(3)}$

The two mass contributions over the whole T_F^2- contributions to the OME $A_{gg,Q}^{(3)}$:

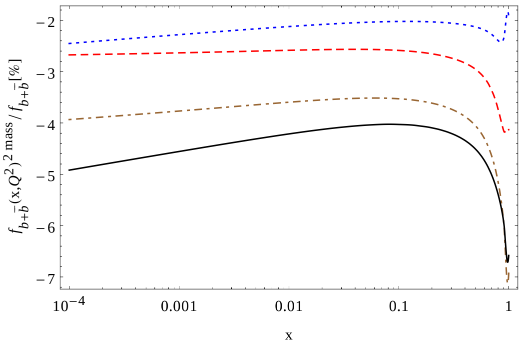


The variable flavor number scheme at NLO

$\Sigma(x, Q^2)^{\text{2-mass}} / \Sigma(x, Q^2)$



$f_{b+\bar{b}}(x, Q^2)^{\text{2-mass}} / f_{b+\bar{b}}(x, Q^2)$



- The ratio of the 2-mass contributions to the singlet parton distribution $\Sigma(x, Q^2)$ (left) and the heavy flavor parton distribution $f_{b+\bar{b}}(x, Q^2)$ (right) over their full form in percent for $m_c = 1.59$ GeV, $m_b = 4.78$ GeV in the on-shell scheme. Dash-dotted line: $Q^2 = 30$ GeV^2; Dotted line: $Q^2 = 30$ GeV^2; Dashed line: $Q^2 = 100$ GeV^2; Dash-dotted line: $Q^2 = 1000$ GeV^2; Full line: $Q^2 = 10000$ GeV^2.
- For the PDFs the NNLO variant of ABMP16 with $N_f = 3$ flavors was used.
  Alekhin et al., Phys. Rev. D 96 (2017) 1

Conclusions and Outlook

Conclusions

- The 2-loop contributions for the pure-singlet Wilson coefficients have been calculated for general kinematics.
- All operator matrix elements except of $A_{Qg}^{(3)}$ have been calculated for general values of N. $A_{Qg}^{(3)}$ is in progress.
- Calculation of 2-mass contributions to all operator matrix elements except of $A_{Qg}^{(3)}$ for general values of N. $A_{Qg}^{(3)}$ is in progress.
- Extension to the polarized case without the need of tensor-decomposition.
- The technologies have also been applied to QED ISR calculations for e^+e^- annihlation at higher order of massive leptons, incluing the electron.
- There were several new mathematica developments concerning massive Feynman integral calculations since RADCOR19.

Outlook

- Study of the VFNS in the polarized case.
- High accuracy reconstruction of x-space expression of $A_{Qg}^{(3)}$ from the large number of moments already constructed. Analytic solution requires more mathematical effort.
- 2-mass contributions to $A_{Qg}^{(3)}$ in an expansion in the mass ratio m_c/m_b.