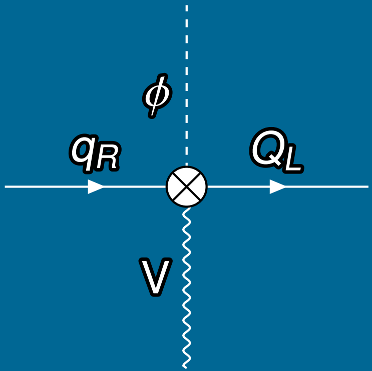

Scale Evolution of the SMEFT dipole operators in the presence of an ALP

RADCOR-LoopFest

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Outline

- **RG Evolution, ALP Lagrangian**
- **ALP-SMEFT Interference**
- **Effects on the UV Scale Evolution of the Dipole Coefficients**
- **Summary and Outlook**

Effective Field Theories

basic idea: separate the different scales and
determine the relevant degrees of freedom!

$\hookrightarrow$ Operator-Product Expansion:

$$\mathcal{L}_{\mathrm{EFT}} = \sum_d \frac{1}{\Lambda^{d-4}} \sum_{i=1}^{n_d} C_i^{(d)}(\mu) Q_i^{(d)}(\mu)$$

RG Evolution Equation:

$$\frac{d\, C_i^{(d)}(\mu)}{d \log \mu} = \gamma_{ji}^{(d)}(\mu) C_j^{(d)}(\mu)$$

Effective Lagrangian for the Axion-Like Particle

Assume an ALP a that is

- classically shift symmetric ($a \to a + c$)
- a gauge singlet
- a pseudoscalar
- massive with mass m_a

most general Lagrangian:

[H. Georgi, D. B. Kaplan, L. Randall: Phys.Lett.B 169 (1986) 73-78]

$$
\begin{aligned}
\mathcal{L}_{\mathrm{ALP}}^{D\le 5} = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{\partial^\mu a}{f} \sum_F \bar{\Psi}_F \, \boldsymbol{c}_F \, \gamma_\mu \, \Psi_F \\
+ \; c_{GG} \, \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \, \tilde{G}^{\mu\nu,a} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^A \, \tilde{W}^{\mu\nu,A} + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \, \tilde{B}^{\mu\nu}
\end{aligned}
$$

Alternative Form of the Effective Lagrangian

Assume an ALP a that is

- classically shift symmetric ($a \to a + c$)
- a gauge singlet
- a pseudoscalar
- massive with mass m_a

alternative form of the Lagrangian:

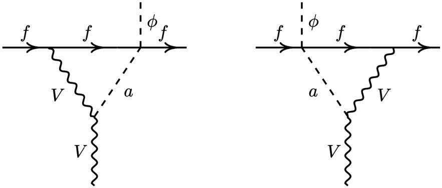
[M. Bauer, et al.: arXiv:2012.12272]

$$
\begin{aligned}
\mathcal{L}_{\mathrm{ALP}}^{D\leq 5} = {} & \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 \\
& - \frac{a}{f}\left(\bar{Q}_L\,\phi\,\hat{\boldsymbol{Y}}_{\boldsymbol{d}}\,d_R + \bar{Q}_L\tilde{\phi}\,\hat{\boldsymbol{Y}}_{\boldsymbol{u}}\,u_R + \bar{L}\phi\,\hat{\boldsymbol{Y}}_{\boldsymbol{e}}\,e_R + \mathrm{h.c.}\right) \\
& + C_{GG}\frac{a}{f}\,G_{\mu\nu}^a\tilde{G}^{\mu\nu,a} + C_{BB}\frac{a}{f}\,B_{\mu\nu}\tilde{B}^{\mu\nu} + C_{WW}\frac{a}{f}\,W_{\mu\nu}^A\tilde{W}^{\mu\nu,A}
\end{aligned}
$$

$\Rightarrow$ effective Higgs-Fermion-Fermion-ALP vertex!

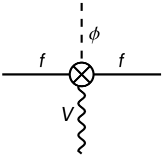
$\hat{\boldsymbol{Y}}_{\boldsymbol{d}} = i(\boldsymbol{Y}_{\boldsymbol{d}}\,\boldsymbol{c}_d - \boldsymbol{c}_Q\,\boldsymbol{Y}_{\boldsymbol{d}})\,,\qquad C_{GG} = \frac{\alpha_s}{4\pi}\left[c_{GG} + \frac{1}{2}\mathrm{Tr}(\boldsymbol{c}_d + \boldsymbol{c}_u - 2\boldsymbol{c}_Q)\right]$ etc.

virtual **ALP** exchange induces **UV-divergent** one-loop graphs, first studied in the case of $(g-2)_\mu$ [Marciano, Masiero, Paradisi, Passera (2016); Bauer, Neubert, Thamm (2017)]



$\sim 1/\epsilon$

→ requires local dimension-6 operators as **counterterms**!



ALP-SMEFT Interference

consistent treatment: embedding of the ALP model in SMEFT via

[Buchmüller, Wyler (1986)]

$$\underbrace{\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{ALP}} + \mathcal{L}_{\text{SMEFT}}}$$

ALP contributes source terms to the $D = 6$ SMEFT Wilson coefficients

$$\frac{d}{d\ln\mu}\,C_i^{\text{SMEFT}} - \gamma_{ji}^{\text{SMEFT}}\,C_j^{\text{SMEFT}} = \frac{S_i}{(4\pi f)^2} \quad \text{for}\ \mu < 4\pi f$$

[AG, Neubert, Renner: 2105.01078]

$\hookrightarrow$ SMEFT Wilson coefficients are generated at the scale $\Lambda = 4\pi f$ **independent of the ALP mass**!

ALP-SMEFT Interference

systematic study: consider a redundant basis of $D = 6$ operators made out of SM fields

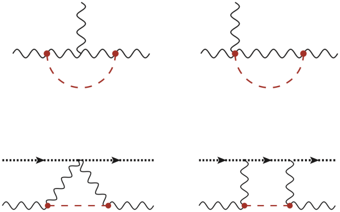
$\hookrightarrow$ compute the one-loop divergent Green's functions with virtual ALP exchange

[AG, Neubert, Renner: 2105.01078]

for instance:

Operator class	Warsaw basis	Way of generation	
Purely bosonic			
X^3	yes	direct	—
X^2D^2	no	direct	
X^2H^2	yes	direct	—
XH^2D^2	no	—	
H^6	yes	—	EOM
H^4D^2	yes	—	EOM
H^2D^4	no	—	

[Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)]



ALP-SMEFT Interference

Example: Three gluon amplitudes

need the *redundant* operator: $\hat{Q}_{G,2} = (D^\rho\, G_{\rho\mu})^a\, (D_\omega\, G^{\omega\mu})^a$

$$\mathcal{A}(ggg) = -\frac{1}{\Lambda^2}\,\frac{C_{GG}^2}{\epsilon}\left[4\, g_s \,\langle Q_G \rangle + \frac{4}{3}\,\langle \hat{Q}_{G,2}\rangle - 2\, m_a^2\, \langle G^a_{\mu\nu} G^{\mu\nu,a}\rangle\right] + \text{finite}$$

Weinberg operator
(present in the Warsaw basis)

not present in the Warsaw basis
$\hookrightarrow$ needs to be transformed!

ALP-SMEFT Interference

Example: Three gluon amplitudes

transformation into the Warsaw basis:

$\hat{Q}_{G,2} = (D^\rho\, G_{\rho\mu})^a\, (D_\omega\, G^{\omega\mu})^a$

needs the SM equation of motion

$D_\rho\, G^{\rho\mu,a} = -\, g_s\, (\bar{Q}_L\, \gamma^\mu\, t^a\, Q_L + \bar{u}_R\, \gamma^\mu\, t^a\, u_R + \bar{d}_R\, \gamma^\mu\, t^a\, d_R)$

Thus,

$$\begin{aligned}\hat{Q}_{G,2} &= g_s^2\, (\bar{Q}_L\, \gamma^\mu\, t^a\, Q_L + \bar{u}_R\, \gamma^\mu\, t^a\, u_R + \bar{d}_R\, \gamma^\mu\, t^a\, d_R)^2 \\ &= g_s^2 \Big[\frac{1}{4} \left([Q_{qq}^{(1)}]_{prrp} + [Q_{qq}^{(3)}]_{prrp} \right) - \frac{1}{2N_c} [Q_{qq}^{(1)}]_{pprr} + \frac{1}{2} [Q_{uu}]_{prrp} - \frac{1}{2N_c} [Q_{uu}]_{pprr} \\ &\quad + \frac{1}{2} [Q_{dd}]_{prrp} - \frac{1}{2N_c} [Q_{dd}]_{pprr} + 2[Q_{qu}^{(8)}]_{pprr} + 2[Q_{qd}^{(8)}]_{pprr} + 2[Q_{ud}^{(8)}]_{pprr} \Big]\end{aligned}$$

ALP-SMEFT Interference: From Diagrams to Source Terms

$$\mathcal{A}(ggg) = -\frac{1}{\Lambda^2}\,\frac{C_{GG}^2}{\epsilon}\,\Big[4\,g_s\,\langle Q_G\rangle + \frac{4}{3}\,\langle \hat{Q}_{G,2}\rangle - 2\,m_a^2\,\langle G^a_{\mu\nu}G^{\mu\nu,a}\rangle\Big] + \text{finite}$$

To cancel the $1/\epsilon$ terms, the *bare* Wilson coefficients must contain

$$C_{G,0} \ni \frac{4g_s}{(4\pi f)^2}\,C_{GG}^2\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{M^2} + \dots\right)$$

M: characteristic mass scale of the UV theory

$\ln\mu^2$: generic for one-loop diagrams in dimensional regularization

Thus, after removing the pole: $\frac{d}{d\ln\mu}C_G(\mu) \ni \frac{8g_s}{(4\pi f)^2}C_{GG}^2$

$$\frac{d}{d\ln\mu}\,C_i^{\text{SMEFT}} - \gamma_{ji}^{\text{SMEFT}}C_j^{\text{SMEFT}} = \frac{S_i}{(4\pi f)^2} \quad \text{for}\ \mu < 4\pi f$$

$\longrightarrow$ $S_G = 8g_s\,C_{GG}^2$

ALP-SMEFT Interference

	Operator Q	Source Term D
Q_G	$g_3\, f^{abc}\, G_\mu^{\nu,a} G_\nu^{\rho,b} G_\rho^{\mu,c}$	$8\left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\tilde{G}}$	$g_3\, f^{abc}\, \tilde{G}_\mu^{\nu,a} G_\nu^{\rho,b} G_\rho^{\mu,c}$	0
Q_W	$g_2\, \epsilon^{IJK}\, W_\mu^{\nu,I} W_\nu^{\rho,J} W_\rho^{\mu,K}$	$8\left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\tilde{W}}$	$g_2\, \epsilon^{IJK}\, \tilde{W}_\mu^{\nu,I} W_\nu^{\rho,J} W_\rho^{\mu,K}$	0
$Q_{\phi G}$	$g_3^2\, \phi^\dagger\phi\, G_{\mu\nu}^a G^{\mu\nu,a}$	0
$Q_{\phi\tilde{G}}$	$g_3^2\, \phi^\dagger\phi\, \tilde{G}_{\mu\nu}^a G^{\mu\nu,a}$	0
$Q_{\phi W}$	$g_2^2\, \phi^\dagger\phi\, W_{\mu\nu}^I W^{\mu\nu,I}$	$-2\left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\phi\tilde{W}}$	$g_2^2\, \phi^\dagger\phi\, \tilde{W}_{\mu\nu}^I W^{\mu\nu,I}$	0
$Q_{\phi B}$	$g_1^2\, \phi^\dagger\phi\, B_{\mu\nu} B^{\mu\nu}$	$-2\left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\phi\tilde{B}}$	$g_1^2\, \phi^\dagger\phi\, \tilde{B}_{\mu\nu} B^{\mu\nu}$	0
$Q_{\phi WB}$	$g_1 g_2\, \phi^\dagger\sigma^I\phi\, W_{\mu\nu}^I B^{\mu\nu}$	$-4\left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)\left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\phi\tilde{W}B}$	$g_1 g_2\, \phi^\dagger\sigma^I\phi\, \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	0
$Q_{\phi\Box}$	$(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$	$g_1^2\, \frac{8}{3}\, \mathcal{Y}_\phi^2 \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 + 2\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\phi D}$	$(\phi^\dagger D_\mu\phi)^*\, (\phi^\dagger D^\mu\phi)$	$g_1^2\, \frac{32}{3}\, \mathcal{Y}_\phi^2 \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
Q_ϕ	$(\phi^\dagger\phi)^3$	$g_2^2\, \frac{8}{3}\, \lambda \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$

	Operator Q	Source Term D
$Q_{\underset{ij}{eW}}$	$g_2\, (\bar{L}_L^i\, \sigma^{\mu\nu} e_R^j)\, \tau^I \phi\, W_{\mu\nu}^I$	$-2\, i\, [\hat{\boldsymbol{Y}}_e]^{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)$
$Q_{\underset{ij}{eB}}$	$g_1\, (\bar{L}_L^i\, \sigma^{\mu\nu} e_R^j)\, \phi\, B_{\mu\nu}$	$-2\, i\, (\mathcal{Y}_L + \mathcal{Y}_e)[\hat{\boldsymbol{Y}}_e]^{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\underset{ij}{uG}}$	$g_3\, (\bar{Q}_L^i\, \sigma^{\mu\nu}\, t^a u_R^j)\, \tilde{\phi}\, G_{\mu\nu}^a$	$-4\, i\, [\hat{\boldsymbol{Y}}_u]^{ij} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)$
$Q_{\underset{ij}{uW}}$	$g_2\, (\bar{Q}_L^i\, \sigma^{\mu\nu} u_R^j)\, \tau^I \tilde{\phi}\, W_{\mu\nu}^I$	$-2\, i\, [\hat{\boldsymbol{Y}}_u]^{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)$
$Q_{\underset{ij}{uB}}$	$g_1\, (\bar{Q}_L^i\, \sigma^{\mu\nu} u_R^j)\, \tilde{\phi}\, B_{\mu\nu}$	$-2\, i\, (\mathcal{Y}_Q + \mathcal{Y}_u)\, [\hat{\boldsymbol{Y}}_u]^{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\underset{ij}{dG}}$	$g_3\, (\bar{Q}_L^i\, \sigma^{\mu\nu}\, t^a d_R^j)\, \phi\, G_{\mu\nu}^a$	$-4\, i\, [\hat{\boldsymbol{Y}}_d]^{ij} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)$
$Q_{\underset{ij}{dW}}$	$g_2\, (\bar{Q}_L^i\, \sigma^{\mu\nu} d_R^j)\, \tau^I \phi\, W_{\mu\nu}^I$	$-2\, i\, [\hat{\boldsymbol{Y}}_d]^{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)$
$Q_{\underset{ij}{dB}}$	$g_1\, (\bar{Q}_L^i\, \sigma^{\mu\nu} d_R^j)\, \phi\, B_{\mu\nu}$	$-2\, i\, (\mathcal{Y}_Q + \mathcal{Y}_d)\, [\hat{\boldsymbol{Y}}_d]^{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\underset{ij}{e\phi}}$	$(\phi^\dagger\phi)\, (\bar{L}_L^i e_R^j\, \phi)$	$-2\, [\hat{\boldsymbol{Y}}_e\, \boldsymbol{Y}_e^\dagger\, \hat{\boldsymbol{Y}}_e]^{ij} - \frac{1}{2}\, [\hat{\boldsymbol{Y}}_e\, \hat{\boldsymbol{Y}}_e^\dagger\, \boldsymbol{Y}_e]^{ij} - \frac{1}{2}\, [\boldsymbol{Y}_e\, \hat{\boldsymbol{Y}}_e^\dagger\, \hat{\boldsymbol{Y}}_e]^{ij} + \frac{4}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 [\boldsymbol{Y}_e]^{ij}$
$Q_{\underset{ij}{u\phi}}$	$(\phi^\dagger\phi)\, (\bar{Q}_L^i u_R^j\, \tilde{\phi})$	$-2\, [\hat{\boldsymbol{Y}}_u\, \boldsymbol{Y}_u^\dagger\, \hat{\boldsymbol{Y}}_u]^{ij} - \frac{1}{2}\, [\hat{\boldsymbol{Y}}_u\, \hat{\boldsymbol{Y}}_u^\dagger\, \boldsymbol{Y}_u]^{ij} - \frac{1}{2}\, [\boldsymbol{Y}_u\, \hat{\boldsymbol{Y}}_u^\dagger\, \hat{\boldsymbol{Y}}_u]^{ij} + \frac{4}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 [\boldsymbol{Y}_u]^{ij}$
$Q_{\underset{ij}{d\phi}}$	$(\phi^\dagger\phi)\, (\bar{Q}_L^i d_R^j\, \phi)$	$-2\, [\hat{\boldsymbol{Y}}_d\, \boldsymbol{Y}_d^\dagger\, \hat{\boldsymbol{Y}}_d]^{ij} - \frac{1}{2}\, [\hat{\boldsymbol{Y}}_d\, \hat{\boldsymbol{Y}}_d^\dagger\, \boldsymbol{Y}_d]^{ij} - \frac{1}{2}\, [\boldsymbol{Y}_d\, \hat{\boldsymbol{Y}}_d^\dagger\, \hat{\boldsymbol{Y}}_d]^{ij} + \frac{4}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 [\boldsymbol{Y}_d]^{ij}$
$Q_{\underset{ij}{\phi L}}^{(1)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{L}_L^i \gamma^\mu L_L^j)$	$\frac{1}{4}\, [\hat{\boldsymbol{Y}}_e\, \hat{\boldsymbol{Y}}_e^\dagger]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_\phi\, \mathcal{Y}_L \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}$
$Q_{\underset{ij}{\phi L}}^{(3)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu^I \phi)\, (\bar{L}_L^i \sigma^I\, \gamma^\mu L_L^j)$	$\frac{1}{4}\, [\hat{\boldsymbol{Y}}_e\, \hat{\boldsymbol{Y}}_e^\dagger]^{ij} + \frac{4}{3}\, g_2^2 \left(\frac{\tilde{c}_{WW}}{4\pi}\right)^2 \delta_{ij}$
$Q_{\underset{ij}{\phi e}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{e}_R^i \gamma^\mu e_R^j)$	$-\frac{1}{2}\, [\hat{\boldsymbol{Y}}_e^\dagger\, \hat{\boldsymbol{Y}}_e]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_e\, \mathcal{Y}_\phi \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}$
$Q_{\underset{ij}{\phi Q}}^{(1)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{Q}_L^i \gamma^\mu Q_L^j)$	$\frac{1}{4}\, [\hat{\boldsymbol{Y}}_d\, \hat{\boldsymbol{Y}}_d^\dagger]^{ij} - \frac{1}{4}\, [\hat{\boldsymbol{Y}}_u\, \hat{\boldsymbol{Y}}_u^\dagger]^{ij} + \frac{16}{3}\, \mathcal{Y}_\phi\, \mathcal{Y}_Q\, g_1^2\, \delta_{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ij}{\phi Q}}^{(3)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu^I \phi)\, (\bar{Q}_L^i \sigma^I\, \gamma^\mu Q_L^j)$	$\frac{1}{4}\, [\hat{\boldsymbol{Y}}_d\, \hat{\boldsymbol{Y}}_d^\dagger]^{ij} + \frac{1}{4}\, [\hat{\boldsymbol{Y}}_u\, \hat{\boldsymbol{Y}}_u^\dagger]^{ij} + \frac{4}{3}\, g_2^2\, \delta_{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\underset{ij}{\phi u}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{u}_R^i \gamma^\mu u_R^j)$	$\frac{1}{2}\, [\hat{\boldsymbol{Y}}_u^\dagger\, \hat{\boldsymbol{Y}}_u]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_\phi\, \mathcal{Y}_u\, \delta_{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ij}{\phi d}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{d}_R^i \gamma^\mu d_R^j)$	$-\frac{1}{2}\, [\hat{\boldsymbol{Y}}_d^\dagger\, \hat{\boldsymbol{Y}}_d]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_\phi\, \mathcal{Y}_u\, \delta_{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ij}{\phi u}} + \text{h.c.}$	$i\, (\tilde{\phi}^\dagger D_\mu \phi)\, (\bar{u}_R^i \gamma^\mu d_R^j)$	$-[\hat{\boldsymbol{Y}}_u^\dagger\, \hat{\boldsymbol{Y}}_d]^{ij}$

	Operator Q	Source Term D
$Q_{\underset{ijkl}{LL}}$	$(\bar{L}_L^i\, \gamma_\mu L_L^j)\, (\bar{L}_L^k \gamma^\mu L_L^l)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_L^2 \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}\delta_{kl} + \frac{2}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 (2\, \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl})$
$Q_{\underset{ijkl}{QQ}}^{(1)}$	$(\bar{Q}_L^i\, \gamma_\mu Q_L^j)\, (\bar{Q}_L^k \gamma^\mu Q_L^l)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_Q^2 \left(\frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}\delta_{kl} + \frac{2}{3}\, g_3^2 \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2 \left(\delta_{il}\delta_{jk} - \frac{2}{N_c}\delta_{ij}\delta_{kl}\right)$
$Q_{\underset{ijkl}{QQ}}^{(3)}$	$(\bar{Q}_L^i\, \gamma_\mu\, \sigma^I\, Q_L^j)\, (\bar{Q}_L^k \gamma^\mu\, \sigma^I\, Q_L^l)$	$\frac{2}{3}\, g_3^2 \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2 \delta_{il}\delta_{jk} + \frac{2}{3}\, g_2^2\, \delta_{ij}\delta_{kl} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{LQ}}^{(1)}$	$(\bar{L}_L^i\, \gamma_\mu L_L^j)\, (\bar{Q}_L^k \gamma^\mu Q_L^l)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_Q\, \mathcal{Y}_L \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{LQ}}^{(3)}$	$(\bar{L}_L^i\, \gamma_\mu\, \sigma^I\, L_L^j)\, (\bar{Q}_L^k \gamma^\mu\, \sigma^I\, Q_L^l)$	$\frac{4}{3}\, g_2^2\, \delta_{ij}\delta_{kl} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{ee}}$	$(\bar{e}_R^i\, \gamma_\mu e_R^j)\, (\bar{e}_R^k \gamma^\mu e_R^l)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_e^2\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{uu}}$	$(\bar{u}_R^i\, \gamma_\mu u_R^j)\, (\bar{u}_R^k \gamma^\mu u_R^l)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_u^2\, \delta_{ij}\delta_{kl} \left(\frac{\tilde{c}_{BB}}{4\pi}\right)^2 + \frac{4}{3}\, g_3^2 \left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\, \delta_{ij}\delta_{kl}\right) \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{dd}}$	$(\bar{d}_R^i\, \gamma_\mu d_R^j)\, (\bar{d}_R^k \gamma^\mu d_R^l)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_d^2\, \delta_{ij}\delta_{kl} \left(\frac{\tilde{c}_{BB}}{4\pi}\right)^2 + \frac{4}{3}\, g_3^2 \left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\, \delta_{ij}\delta_{kl}\right) \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{eu}}$	$(\bar{e}_R^i\, \gamma_\mu e_R^j)\, (\bar{u}_R^k \gamma^\mu u_R^l)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_e\, \mathcal{Y}_u\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{ed}}$	$(\bar{e}_R^i\, \gamma_\mu e_R^j)\, (\bar{d}_R^k \gamma^\mu d_R^l)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_e\, \mathcal{Y}_d\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{ud}}^{(1)}$	$(\bar{u}_R^i\, \gamma_\mu u_R^j)\, (\bar{d}_R^k \gamma^\mu d_R^l)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_u\, \mathcal{Y}_d\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{ud}}^{(8)}$	$(\bar{u}_R^i\, \gamma_\mu\, t^a u_R^j)\, (\bar{d}_R^k \gamma^\mu\, t^a\, d_R^l)$	$\frac{16}{3}\, g_3^2\, \delta_{ij}\delta_{kl} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Le}}$	$(\bar{L}_L^i\, \gamma_\mu L_L^j)\, (\bar{e}_R^k \gamma^\mu e_R^l)$	$[\hat{\boldsymbol{Y}}_e]^{il}\, [\hat{\boldsymbol{Y}}_e^\dagger]^{kj} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_e\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Lu}}$	$(\bar{L}_L^i\, \gamma_\mu L_L^j)\, (\bar{u}_R^k \gamma^\mu u_R^l)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_u\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Ld}}$	$(\bar{L}_L^i\, \gamma_\mu L_L^j)\, (\bar{d}_R^k \gamma^\mu d_R^l)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_d\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Qe}}$	$(\bar{Q}_L^i\, \gamma_\mu Q_L^j)\, (\bar{e}_R^k \gamma^\mu e_R^l)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_Q\, \mathcal{Y}_e\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Qu}}^{(1)}$	$(\bar{Q}_L^i\, \gamma_\mu Q_L^j)\, (\bar{u}_R^k \gamma^\mu u_R^l)$	$\frac{1}{N_c}\, [\hat{\boldsymbol{Y}}_u]^{il}\, [\hat{\boldsymbol{Y}}_u^\dagger]^{kj} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_u\, \mathcal{Y}_Q\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Qu}}^{(8)}$	$(\bar{Q}_L^i\, \gamma_\mu\, t^a\, Q_L^j)\, (\bar{u}_R^k \gamma^\mu\, t^a\, u_R^l)$	$2\, [\hat{\boldsymbol{Y}}_u]^{il}\, [\hat{\boldsymbol{Y}}_u^\dagger]^{kj} + \frac{16}{3}\, g_3^2\, \delta_{ij}\delta_{kl} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Qd}}^{(1)}$	$(\bar{Q}_L^i\, \gamma_\mu Q_L^j)\, (\bar{d}_R^k \gamma^\mu d_R^l)$	$\frac{1}{N_c}\, [\hat{\boldsymbol{Y}}_d]^{il}\, [\hat{\boldsymbol{Y}}_d^\dagger]^{kj} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_d\, \mathcal{Y}_Q\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Qu}}^{(8)}$	$(\bar{Q}_L^i\, \gamma_\mu\, t^a\, Q_L^j)\, (\bar{d}_R^k \gamma^\mu\, t^a\, d_R^l)$	$2\, [\hat{\boldsymbol{Y}}_d]^{il}\, [\hat{\boldsymbol{Y}}_d^\dagger]^{kj} + \frac{16}{3}\, g_3^2\, \delta_{ij}\delta_{kl} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$

	Operator Q	Source Term D
$Q_{\underset{ijkl}{LedQ}}$	$(\bar{L}_L^i\, e_R^j)\, (\bar{d}_R^k\, Q_L^l)$	$-2\, [\hat{\boldsymbol{Y}}_e]^{ij}\, [\hat{\boldsymbol{Y}}_d]^{kl}$
$Q_{\underset{ijkl}{QuQd}}^{(1)}$	$(\bar{Q}_L^{i,m}\, u_R^j)\, \epsilon_{mn} (\bar{Q}_L^{k,n}\, d_R^l)$	$-2\, [\hat{\boldsymbol{Y}}_u]^{ij}\, [\hat{\boldsymbol{Y}}_d]^{kl}$
$Q_{\underset{ijkl}{QuQd}}^{(8)}$	$(\bar{Q}_L^{i,m}\, t^a\, u_R^j)\, \epsilon_{mn}\, (\bar{Q}_L^{k,n}\, t^a\, d_R^l)$	0
$Q_{\underset{ijkl}{LeQu}}^{(1)}$	$(\bar{L}_L^{i,m}\, e_R^j)\, \epsilon_{mn}\, (\bar{Q}_L^{k,n}\, u_R^l)$	$2\, [\hat{\boldsymbol{Y}}_e]^{ij}\, [\hat{\boldsymbol{Y}}_u]^{kl}$
$Q_{\underset{ijkl}{LeQu}}^{(3)}$	$(\bar{L}_L^{i,m} \sigma_{\mu\nu}\, e_R^j)\, \epsilon_{mn}\, (\bar{Q}_L^{k,n} \sigma^{\mu\nu}\, u_R^l)$	0

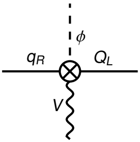
Nearly the whole Warsaw basis is sourced by the ALP at one-loop order!

UV Running of the Dipole Coefficients

Dipole Operators above the Weak Scale

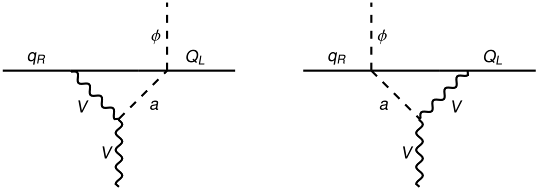
$$
\begin{aligned}
\mathcal{L}_{\text{SMEFT}} \supset & \, C^{ij}_{uB} \bar{Q}^i \tilde{\phi} \, \sigma_{\mu\nu} B^{\mu\nu} u^j_R + C^{ij}_{dB} \bar{Q}^i \phi \, \sigma_{\mu\nu} B^{\mu\nu} d^j_R + C^{ij}_{eB} \bar{L}^i \phi \, \sigma_{\mu\nu} B^{\mu\nu} e^j_R \\
& + C^{ij}_{uW} \bar{Q}^i \tau_A \tilde{\phi} \, \sigma_{\mu\nu} W^{\mu\nu}_A u^j_R + C^{ij}_{dW} \bar{Q}^i \tau_A \phi \, \sigma_{\mu\nu} W^{\mu\nu}_A d^j_R \\
& \qquad + C^{ij}_{eW} \bar{L}^i \tau_A \phi \, \sigma_{\mu\nu} W^{\mu\nu}_A e^j_R \\
& + C^{ij}_{uG} \bar{Q}^i \tilde{\phi} \, \sigma_{\mu\nu} G^{\mu\nu}_a t_a u^j_R + C^{ij}_{dG} \bar{Q}^i \phi \, \sigma_{\mu\nu} G^{\mu\nu}_a t_a d^j_R
\end{aligned}
$$

Wilson coefficients C^{ij}_{fV}: 3×3 matrices in generation space



quark-sector dipole operator

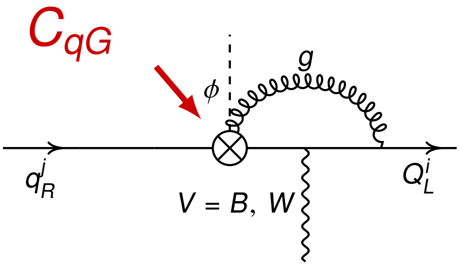
quark-sector:



$$\boldsymbol{S}_{qB} = 2\, g_1\, C_{BB}\, (\mathcal{Y}_Q + \mathcal{Y}_q)(\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q)$$
$$\boldsymbol{S}_{qW} = g_2\, C_{WW}\, (\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q)$$
$$\boldsymbol{S}_{qG} = 4\, g_s\, C_{GG}\, (\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q)$$

$q = u,\, d$

mixing between the dipole Wilson coefficients:
[E. Jenkins et al: arXiv: 1310.4838, 1312.2014]



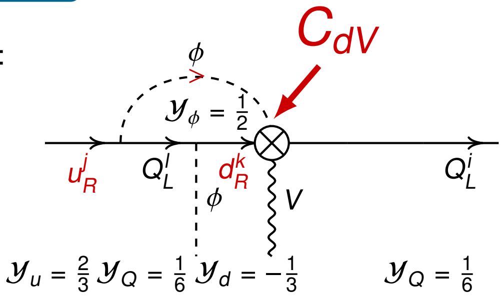
$\hookrightarrow$ QCD-effects mix $\boldsymbol{C_{qG}}$, $\boldsymbol{C_{qW}}$ and $\boldsymbol{C_{qB}}$

Mixing of C_{uV} and C_{dV}

mixing between the dipole Wilson coefficients:
[E. Jenkins et al: arXiv: 1310.4838, 1312.2014]

$$\alpha_t \sim \alpha_s$$

for instance:



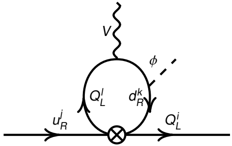
$\hookrightarrow$ the Higgs mixes $\boldsymbol{C_{uV}}$ and $\boldsymbol{C_{dV}}$

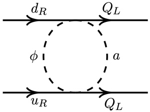
Mixing with other SMEFT coefficients

[E. Jenkins et al: arXiv: 1310.4838, 1312.2014]

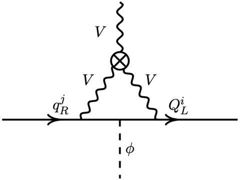
generated **for instance** via:

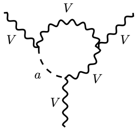
four-fermion operator:



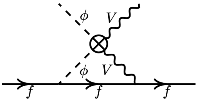


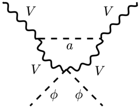
Weinberg operator:





$Q_{HV(V')}$-type operator:





Application: Top Chromo-Magnetic Moment

Chromo-magnetic and chromo-electric dipole moments:

$$\mathcal{L}_{\rm eff} = \hat{\mu}_q \frac{g_3}{2m_q} \bar{q}\, \sigma_{\mu\nu} G_a^{\mu\nu}\, t_a\, q + i\, \hat{d}_q \frac{g_3}{2m_q} \bar{q}\, \sigma_{\mu\nu}\gamma_5 G_a^{\mu\nu}\, t_a q\,,$$

top quark:

$$\hat{\mu}_t = -\frac{y_t v^2}{\Lambda^2} \mathfrak{Re}\, C_{uG}^{33}\,, \qquad \hat{d}_t = -\frac{y_t v^2}{\Lambda^2} \mathfrak{Im}\, C_{uG}^{33}$$

$$\frac{d}{d\ln\mu} \mathfrak{Re}\, C_{uG}^{33} = \left(\frac{15\alpha_t}{8\pi} - \frac{17\alpha_s}{12\pi}\right) \mathfrak{Re}\, C_{uG}^{33} + \frac{9\alpha_s}{4\pi}\, y_t\, C_G + \frac{g_s\, y_t}{4\pi^2}\, C_{HG} + \frac{S_{uG}^{33}}{(4\pi f)^2}$$

$$\frac{d}{d\ln\mu} C_G = \frac{15\,\alpha_s}{4\pi} C_G + \frac{S_G}{(4\pi f)^2}$$

$$\frac{d}{d\ln\mu} C_{HG} = \left(\frac{3\alpha_t}{2\pi} - \frac{7\alpha_s}{2\pi}\right) C_{HG} + \frac{g_s\, y_t}{4\pi^2} \mathfrak{Re}\, C_{uG}^{33}$$

$\Lambda = 4\pi f$: scale of global symmetry breaking

Application: Top Chromo-Magnetic Moment

To lowest logarithmic order: [AG, Neubert, Renner: 2105.01078]

$$\hat{\mu}_t \approx -\frac{8\,m_t^2}{(4\pi f)^2}\left[c_{tt}\,C_{GG}\ln\frac{4\pi f}{m_t} - \frac{9\alpha_s}{4\pi}\,C_{GG}^2\ln^2\frac{4\pi f}{m_t}\right]$$

$$\approx -(5.87 c_{tt}\,C_{GG} - 1.98 C_{GG}^2)\times\left[\frac{1\,\text{TeV}}{f}\right]^2$$

c_{tt}: ALP-top coupling below EWSB

for $m_t(m_t) = 163.4\,\text{GeV}$, $\alpha_s(m_t) = 0.1084$ and $f = 1\,\text{TeV}$

Combined with experimental bounds from CMS (2019) this gives:

$$-0.68 < (c_{tt}\,C_{GG} - 0.34\,C_{GG}^2)\times\left[\frac{1\,\text{TeV}}{f}\right]^2 < 2.38 \quad (95\%\,\text{CL})$$

Comparable to the strongest bounds following from collider and flavor physics for $m_a > 1\text{GeV}$!

Summary and Outlook

In this talk, we have . . .

- ✓ seen the ALP Lagrangian and an alternative form for the coupling to the SM
- ✓ analyzed the effects of an ALP on the $D = 6$ SMEFT operators
- ✓ solved the RG equation of C_{uG}^{33} to lowest logarithmic order
  $\hookrightarrow$ model independent framework for studying virtual ALP contributions to precision measurements

Open Tasks:

- **!** A more comprehensive analysis of virtual ALP effects based on a global fit to precision data is left for future work

Take Home Message

The ALP generates SMEFT operators above the weak scale by means of inhomogeneous source terms.

Thank You!