Double Virtual Contributions for Massless $2 \to 3$ Scattering in NNLO QCD

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MAX-PLANCK-INSTITUT
FÜR PHYSIK

Era of precision physics at the LHC

- LHC will still be running for years
- Many observables will be probed at percent level precision
- Discovery via precision: search anomalous deviations from Standard Model

Theory must reach comparable precision target

NNLO QCD and NLO EW corrections generally required

($\oplus$ parton shower, resummation, etc.)

$$\sigma \;\sim\; \overbrace{\underbrace{\sigma_{\mathsf{LO}}\Big(1+\alpha_s\sigma^{(1)}}_{\text{NLO}}+\alpha_s^2\sigma^{(2)}}^{\text{NNLO}}+\mathcal{O}\big(\alpha_s^3\big)\Big)$$

naively $\lesssim 1\%$

NNLO QCD: status

Les Houches wishlist 2019 [2003.01700]

State of the art

- Many $2 \to 2$ NNLO QCD completed
- Current frontier is $2 \to 3$, first cross sections [see Rene's talk]

[Chawdry, Czakon, Mitov, Poncelet '19]
[Kallweit, VS, Wiesemann '20]
[Chawdry, Czakon, Mitov, Poncelet '21]

Bottlenecks

- IR divergences
- Two-loop amplitudes

process	known	desired
$pp \to 2\,\text{jets}$	NNLO_{QCD} $\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	
$pp \to 3\,\text{jets}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	NNLO_{QCD} — α_s running
⋮	⋮	⋮
$pp \to \gamma\gamma$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	
$pp \to \gamma\gamma + j$	NLO_{QCD} NLO_{EW}	H p_T spectrum $\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
$pp \to \gamma\gamma\gamma$	NNLO_{QCD}	anomalous gauge couplings
⋮	⋮	⋮
$pp \to V + j$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	hadronic decays
$pp \to V + 2j$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ NLO_{EW}	QCD precision physics NNLO_{QCD}
$pp \to V + b\bar{b}$	NLO_{QCD}	$H \to b\bar{b}$ decay $\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$
⋮	⋮	⋮
$pp \to H + 2j$	$\text{NLO}_{\text{HTL}} \otimes \text{LO}_{\text{QCD}}$ $\text{N}^3\text{LO}^{(\text{VBF}^*)}_{\text{QCD}}$ (incl.) $\text{NNLO}^{(\text{VBF}^*)}_{\text{QCD}}$ $\text{NLO}^{(\text{VBF})}_{\text{EW}}$	VBF studies $\text{NNLO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $\text{NNLO}^{(\text{VBF})}_{\text{QCD}} + \text{NLO}^{(\text{VBF})}_{\text{EW}}$

Five-point two-loop amplitudes: status

Work on integrals

Massless [Papadopoulos, Tommasini, Wever '15] [Abreu, Page, Zeng '18] [Gehrmann, Henn, Lo Presti '18] [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia '18] [Abreu, Dixon, Herrmann, Page, Zeng '18]

One external mass [Abreu, Ita, Moriello, Page, Tschernow, Zeng '20] [Syrrakos '20] [Canko, Papadopoulos, Syrrakos '20]

[Ben's talk] [Nikolaos's talk] [Costas's talk]

Work on two-loop amplitudes

[Gehrmann, Henn, Lo Presti '15] [Peraro '16] [Dunbar, Perkins '16] [Boels, Jin, Luo '18]
[Badger, Brønnum-Hansen, Hartanto, Peraro '18] [Abreu, Dormans, Febres Cordero, Ita, Page '18]
[Abreu, Febres Cordero, Ita, Page, VS '18] [Böhm, Georgoudis, Larsen, Schönemann, Zhang '18]
[Abreu, Dormans, Febres Cordero, Ita, Page, VS '19] [Chawdry, Lim, Mitov '18] [Wang, Li, Basat '19]
[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia '19]
[Hartanto, Badger, Brønnum-Hansen '19] [Guan, Liu, Ma '19] [Chawdry, Czakon, Mitov, Poncelet '19]
[de Laurentis, Maître '20] [Bendlea, Böhm, Heymann, Ma, Rahn, Ristau, Wittmann, Wu, Zhang '21]
[Badger, Hartanto, Zoia '21] [Bayu's talk]

Phenomenology-ready results

$pp \to \gamma\gamma\gamma$ (LC) [Abreu, Page, Pascual, VS '20] [Chawdry, Czakon, Mitov, Poncelet '20]

$q\bar{q} \to \gamma\gamma g$ (LC) [Chawdry, Czakon, Mitov, Poncelet '21] [Agarwal, Buccioni, von Manteuffel, Tancredi '21]

$pp \to jjj$ (LC) [Abreu, Febres Cordero, Ita, Page, VS '21]

$pp \to \gamma\gamma j$ [Agarwal, Buccioni, von Manteuffel, Tancredi '21]

[Herschel's talk] [Federico's talk]

[Chicherin, VS '20]

Anatomy of a loop amplitude

$$\mathcal{A} = \sum_{\mathbf{i}} r_{\mathbf{i}}(\mathbf{s}, \epsilon)\ \mathbf{f}^{\mathbf{i}}(\mathbf{s}, \epsilon)$$

Rational (algebraic) coefficients	Transcendental part
Process-dependent	Depends only on kinematics
Goal: analytic form	No consensus on how to organize: master integrals? functions? what functions?
Evaluation time subleading if known analytically	Dominates evaluation time

Good choice of transcendental part $\Longrightarrow$ compact rational coefficients, numerical stability

Transcendental part

Feynman integrals: differential equations method

If expressible through multiple polylogarithms (MPLs):

1. Find canonical differential equations (DEs) [Henn '13] for integrals

$$\mathrm{d}\vec{f} = \epsilon\, A\; \vec{f}, \qquad A = \sum_i \mathrm{d}\log W_i(\mathbf{s})\; A_i \tag{$\star$}$$

Letters of "alphabet"

Rational matrix

2. Initial values from physical consistency conditions, etc.
3. Integrate ($\star$) order-by-order in ϵ in terms of MPLs $\implies$ relations and numerical evaluation

Extremely successful strategy for problems with few scales!

Difficulties in multi-scale generalization

- Obtaining canonical DE case-by-case [see Ben's talk]
- Finding explicit MPL solutions from ($\star$) challenging due to algebraic letters
  (progress with SDE method [Papadopoulos '14] [Papadopoulos, Tommasini, Wever '15] [see Costas's, Dhimiter's, Nikolaos's talks])
- Explosion of number of MPLs required to write the solution
  $\Longrightarrow$ unacceptably slow evaluation: even the simplest five-point massless case impractical!
- Analytic continuation

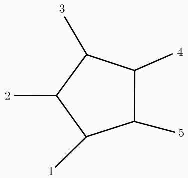
Our approach

- Give up MPLs in favor of iterated-integral representation
  $\Longrightarrow$ transparent analytic structure
- Dedicated numerical algorithms

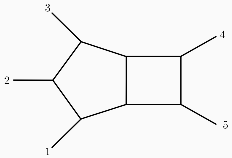
Interesting alternative: semi-numerical solution of DE by local generalized series expansions along a path [Moriello '19] [Hidding '20] [Abreu, Ita, Moriello, Page, Tschernow, Zeng '20]

[see Martijn's talk]

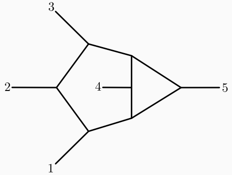
Integral topologies



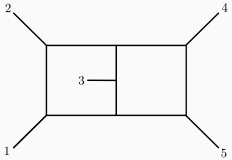
pentagon



planar pentagon-box



non-planar hexagon-box



non-planar double pentagon

Canonical differential equations

Bases of UT master integrals ($\iff$ DE in canonical form) known

[Gehrmann, Henn, Lo Presti '18] [Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser '18]

[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia '18] [Abreu, Dixon, Herrmann, Page, Zeng '18] [Abreu, Page, Zeng '18]

- Consider DEs for all $5!$ permutations $\sigma \in \mathcal{S}_5$ of each integral families τ
- Common base point s_0 in physical region

$$\mathrm{d}\vec{f}_{\tau,\sigma} = \epsilon\, A_{\tau,\sigma} \vec{f}_{\tau,\sigma}$$

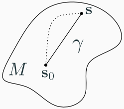
$$\Downarrow$$

No analytic continuation required

Iterated integrals

[Chen '77] (see also *"Iterated integrals in QFT"* [Brown '11])

Let $\omega_1, \ldots, \omega_n$ be differential 1-forms on M (phase space), and path $\gamma : [0,1] \to M$. Pull the forms back on the path $\omega_i(\mathbf{s}) \xrightarrow{\gamma^\star} w_i(t)\,\mathrm{d}t$. Iterated integrals iints are



$$I_\gamma[\omega_1, \ldots, \omega_n] = \int_0^1 w_n(t_n)\,\mathrm{d}t_n \ldots \int_0^{t_2} w_1(t_1)\,\mathrm{d}t_1 \qquad \text{(ii)}$$

We need only logarithmic forms $\omega_i = \mathrm{d}\log(W_i)$, use notation

$$[W_1, \ldots, W_n]_\gamma \coloneqq I_\gamma[\omega_1, \ldots, \omega_n]$$

Shuffle product

$$I_\gamma[\omega_1, \ldots, \omega_r]\ I_\gamma[\omega_{r+1}, \ldots, \omega_n] = \sum_{\boldsymbol{i} \in \{1,\ldots,r\} ⧢ \{r+1,n\}} I_\gamma[\omega_{i_1}, \ldots, \omega_{i_n}] \qquad (⧢)$$

- All functional relations manifest
- Similar to *symbol*, but complete information preserved

Classification strategy

[Chicherin, VS '20] (see also [Gehrmann, Henn, Lo Presti '18])

Goal

Construct complete set of algebraically independent transcendental functions

Proceed recursively order-by-order in ϵ (or transcendental weight w).
Denote weight w functions as $\boldsymbol{g}^{(w)}$.
Start from $w = 0$: integrals are rational numbers $\iff$ $\boldsymbol{g}^{(0)} := \{1\}$.

- Write solutions $\vec{f}^{(w)}$ of $4 \times 5!$ DEs in terms of iints (trivial).
- Consider all products of lower-weight functions $\boldsymbol{g}^{(w')}$ and construct a vector space

$$\mathbf{G} := \mathrm{span}\Big(\vec{f}^{(w)}\Big) \bigcup_{\vec{\boldsymbol{w}}} \boldsymbol{g}^{(\vec{\boldsymbol{w}})}, \qquad \boldsymbol{g}^{(\vec{\boldsymbol{w}})} = g_1^{(w_1)} \cdots g_n^{(w_n)}, \qquad \sum_k w_k = w$$

  Use shuffle algebra of iints to linearize identities in $\mathbf{G}$.

- Use linear algebra to choose basis in $\mathbf{G}$, preferring products of lower-weight functions.
  This basis is functions $\boldsymbol{g}^{(w)}$.

Classification strategy

Pentagon functions

Minimal and complete set of transcendental functions for massless five-point scattering

$$\boldsymbol{g}^{(w)} = \sum_{w'=0}^{w} \sum_{i_1,\ldots,i_{w'}=1}^{31} \boldsymbol{\kappa}^{(w-w')}_{i_1,\ldots,i_{w'}} \left[W_{i_1}, \ldots, W_{i_{w'}} \right]$$

weight $w - w'$ constants

- ✓ branch-cut free in whole physical phase space
- ✓ no linear or algebraic relations
- ✓ amplitudeology-friendly

# master integrals (MIs)	30360
# topologically independent	1917

$w \longrightarrow$	1	2	3	4
$\dim \vec{f}^{(w)} \leftrightarrow$ # linearly independent MIs	10	79	482	1462
$\dim \boldsymbol{g}^{(w)} \leftrightarrow$ # pentagon functions	10	24	111	472

Explicit representation

Weights 1 and 2

Explicit real-analytic $\log, \mathrm{Li}_2, \mathrm{Cl}_2$ functions, e.g.

$$g^{(1)}_{1,4} \equiv [W_4] = \log(s_{45})$$

$$g^{(2)}_{1,3} \equiv -[W_2, W_2] + [W_2, W_{14}] + [W_4, W_2] - [W_4, W_{14}] + ([W_2] - [W_4]) \log(2) + \frac{\pi^2}{12}$$

$$= -\mathrm{Li}_2\left(\frac{s_{45}}{s_{23}}\right) - \log\left(-\frac{s_{45}}{s_{23}}\right) \log\left(1 - \frac{s_{45}}{s_{23}}\right)$$

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Weight 3

One-fold integrals by construction:

$$g^{(3)}_i = \sum_{j=1}^{31} \int_0^1 \mathrm{d}\log W_j(t)\ h^{(2)}_{ij}(t)$$

weight 2 functions

Explicit representation

Weights 1 and 2

Explicit real-analytic $\log, \mathrm{Li}_2, \mathrm{Cl}_2$ functions, e.g.

$$g^{(1)}_{1,4} \equiv [W_4] = \log(s_{45})$$

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Weight 3

One-fold integrals by construction: $\quad g^{(3)}_i = \sum_{j=1}^{31} \int_0^1 \mathrm{d}\log W_j(t)\; h^{(2)}_{ij}(t)$

weight 2 functions

Weight 4

Change order of integration [Caron-Huot, Henn '14] $\Longrightarrow$ trivially integrate $\int_t^1 \mathrm{d}\log\left(W_n(u)\right)$ $\Longrightarrow$ one-fold integrals

$$g^{(4)}_i = \sum_{j,k=1}^{31} \int_0^1 \left(\mathrm{d}\log W_j(t)\; \log\left(\frac{W_k(1)}{W_k(t)}\right) h^{(2)}_{ijk}(t) \;+\; \mathrm{d}\log W_j(t)\, \kappa^{(3)}_{ij} \right)$$

weight 3 constants

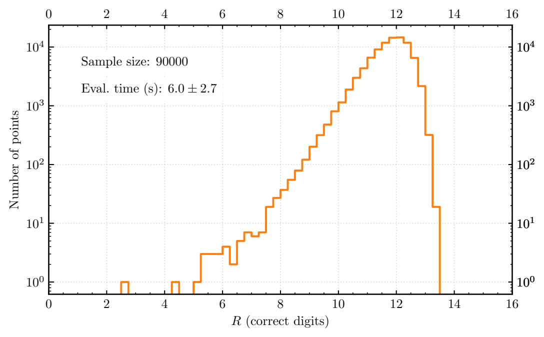
Integrands analytic on the integration domain $\Longrightarrow$ integration well-defined

Numerical evaluation

Numerical performance

Evaluation of all functions: any massless five-point amplitude in all "crossings"

Sample over physical phase-space (NNLO $pp \to \gamma\gamma\gamma$)



(vs. quad precision targets) [Chicherin, VS '20]

"Real-world" tested [Abreu, Page, Pascual, VS '20] [Kallweit, VS, Wiesemann '20] [Agarwal, Buccioni, von Manteuffel, Tancredi '21] [Abreu, Febres Cordero, Ita, Page, VS '21]

Available as a `C++` library `PentagonFunctions++`
`https://gitlab.com/pentagon-functions/PentagonFunctions-cpp`

Rational coefficients

Algebraic complexity

Integrand $\sum_i \frac{m_i(\mathbf{s},\epsilon;\,\ell)}{\prod_j \rho_{i,j}}$

Tensor & IBP reduction

$\sum_i c_i(\mathbf{s},\epsilon)\,\mathcal{I}_i$ ← pure MIs

UV & IR renormalization

$\sum_{\vec{\mathbf{i}}} r_{\vec{\mathbf{i}}}(\mathbf{s})\; \boldsymbol{g}^{\vec{\mathbf{i}}}(\mathrm{s}) \;+\mathcal{O}(\epsilon)$ ← pentagon functions

Key bottleneck
intermidiate expression swell

Central lesson

- Coefficients $r_{\vec{\mathbf{i}}}$ remarkably simple
- Bypass complexity with exact numerics (finite fields)
- Reconstruct analytic form from numerical samples

[von Manteuffel, Schabinger '14] [Peraro '16]

`FiniteFlow` [Peraro '19]
[Badger, Brønnum-Hansen, Hartanto, Peraro '18]
[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia '19]

Caravel [Abreu, Dormans, Febres Cordero, Ita, Kraus, Page, Pascual, Ruf, VS '20]

Better IBPs: important insights from alebraic geometry

[Gluza, Kadja, Kosower '11] [Ita '15] [Larsen, Zhang '15]
[Georgoudis, Larsen, Zhang '16] [Böhm, Georgoudis, Larsen, Schulze, Zhang '17]
[Agarwal, Jones, von Manteuffel '20]

Numerical unitarity

Generalization of one-loop unitarity methods [Bern, Dixon, Kosower, Dunbar '94, '95] [Britto, Feng, Cachazo '05] [Ossola, Papadopoulos, Pittau '07] [Ellis, Giele, Kunszt '08] [Giele, Kunszt, Melnikov '08]

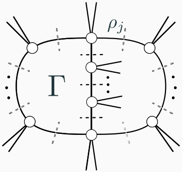
Obtain coefficients $c_{\Gamma,i}$ from linear systems of cut equations:

[Ita '15] [Abreu, Febres Cordero, Ita, Jaquier, Page, Zeng '17] [Abreu, Febres Cordero, Ita, Page, Zeng '17] [Abreu, Febres Cordero, Ita, Page, VS '18]

$$\lim_{\ell_l \to \ell_l^\Gamma} \Big(\mathcal{A}(\ell_l) \prod_{j \in P_\Gamma} \rho_j \Big) = \sum_i c_{\Gamma,i}\, m_{\Gamma,i}$$

on-shell limit for Γ

Unitarity $\to$ factorization into product of trees



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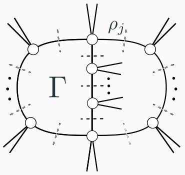
topologies with more propagators

$$\lim_{\ell_l \to \ell_l^\Gamma} \left(\mathcal{A}(\ell_l) \prod_{j \in P_\Gamma} \rho_j \right) = \sum_i c_{\Gamma,i}\, m_{\Gamma,i} \quad + \quad \sum_{\Gamma' > \Gamma} \sum_i c_{\Gamma',i} \frac{m_{\Gamma',i}}{\prod_{j \in P_{\Gamma'} \setminus P_\Gamma} \rho_j},$$

on-shell limit for Γ

$\|$ Unitarity $\to$ factorization into product of trees



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topologies with more propagators

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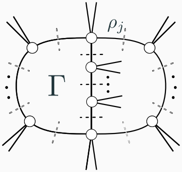
Unitarity $\rightarrow$ factorization into product of trees



Solve cut equations numerically

- ✓ analytic integrand or individual Feynman diagrams not needed
- ✓ suitable for floating point and finite fields

Analytic results from finite-field evaluations

Rational functions in amplitudes are not arbitrary, analytic structure constrained by physics

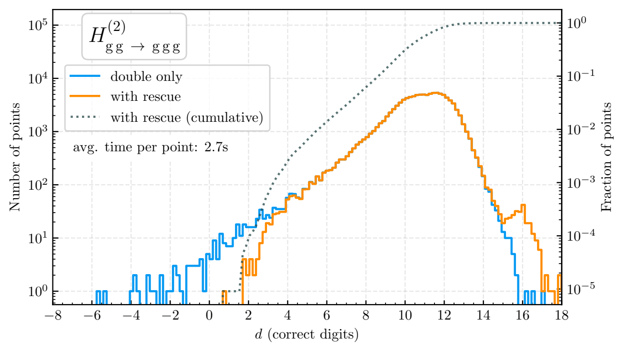
- Denominators are symbol letters [Abreu, Dormans, Febres Cordero, Ita, Page '18]
- Reconstruction can be seriously simplified by judicious ansatzing
  [Abreu, Dormans, Febres Cordero, Ita, Page, VS '19] [de Laurentis, Maître '20] [Badger, Hartanto, Zoia '21]
- Amenable for dramatic simplification by multivariate partial fractioning
  [Abreu, Dormans, Febres Cordero, Ita, Page, VS '19]
  $\Longrightarrow$ compact analytic amplitudes (typical size few Mb)
  $\Longrightarrow$ efficient numerical evaluation
  Public implementations now available: [Böhm, Wittmann, Wu, Xu, Zhang '20]
  [Heller, von Manteuffel '21] [Bendle, Böhm, Heymann, Ma, Rahn, Ristau, Wittmann, Wu, Zhang '21]
- Insights gained from amplitudes also invaluable for more traditional approaches
  [Böhm, Wittmann, Wu, Xu, Zhang '20] [Chawdry, Czakon, Mitov, Poncelet '20]
  [Agarwal, Buccioni, von Manteuffel, Tancredi '21]

Numerical benchmarks

$q\bar{q} \to \gamma\gamma\gamma$	0.8s
$\mathrm{gg} \to \mathrm{ggg}$	1.6s
$q\bar{q} \to \mathrm{ggg}$	1.0s
$q\bar{q} \to Q\bar{Q}\mathrm{g}$	0.7s
$q\mathrm{g} \to qq\bar{q}$	1.0s

(all LC)

- automated precision rescue system
- $\lesssim 90\%$ spent on pentagon functions



Numerical stability
(vs. quad precision targets)

[Abreu, Febres Cordero, Ita, Page, VS '21]

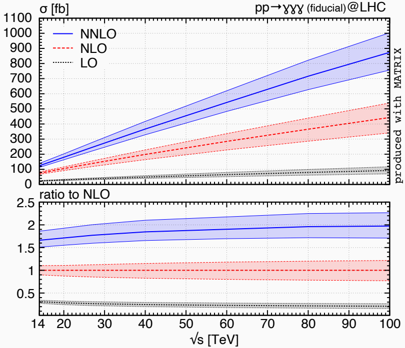
Available as C++ library:
`https://gitlab.com/five-point-amplitudes/FivePointAmplitudes-cpp`

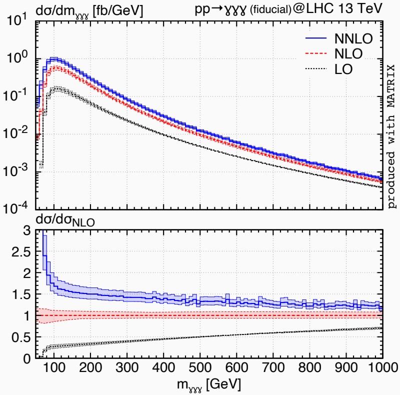
Employs `PentagonFunctions++` for evaluation of pentagon functions

see also $q\bar{q}(g) \to \gamma\gamma g(q)$ in `aajamp` [Agarwal, Buccioni, von Manteuffel, Tancredi '21]

Two-loop amplitudes in action

NNLO QCD $pp \to \gamma\gamma\gamma$ with q_T-subtraction in `MATRIX` [Kallweit, VS, Wiesemann '20]





- $2 \to 3$ NNLO on-the-fly
- Available in future release of `MATRIX` (or immediately on request)

Conclusions

First pheno-ready analytic two-loop five-point amplitudes

- Public codes for (LC) $pp \to \gamma\gamma\gamma$, $pp \to jjj$, $pp \to \gamma\gamma j$ available, seconds per phase-space point.

- Pentagon functions: basis of transcendental functions for five-point massless scattering
  - ✓ Facilitates compact amplitudes
  - ✓ Fast, stable, public numerical evaluation

- Intermediate expressions tamed (for now)

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Looking forward to more exciting NNLO phenomenology!

Acknowledgments

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European Research Council
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