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Two-Loop renormalization of QCD operators and Higgs EFT amplitudes

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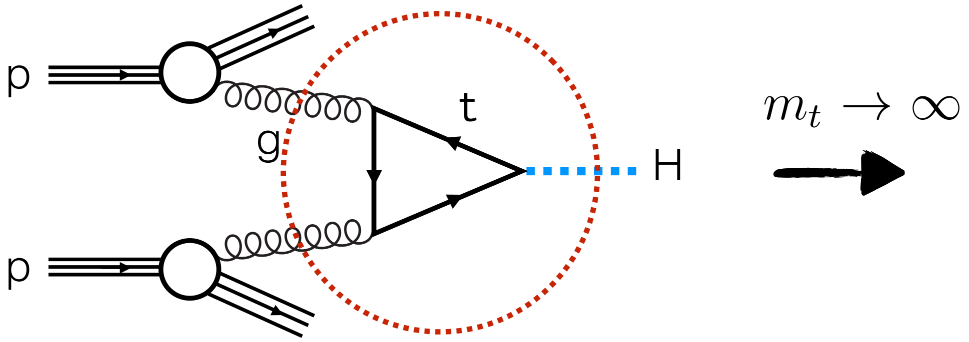
Based on:
- arXiv:1910.09384 (JHEP) Qingjun Jin, GY
- arXiv:2011.02494 (JHEP) Qingjun Jin, Ke Ren, GY
- In progress, Qingjun Jin, Ke Ren, GY, Rui Yu

Motivation

Gauge invariant operators are important in QFT.

- Anomalous dimensions (~spectrum of hadrons, RG, OPE, …)
- Correlation functions

Local operators also appear as vertices in EFT Lagrangian. For example: Higgs EFT obtained by integrating Top quark loop:



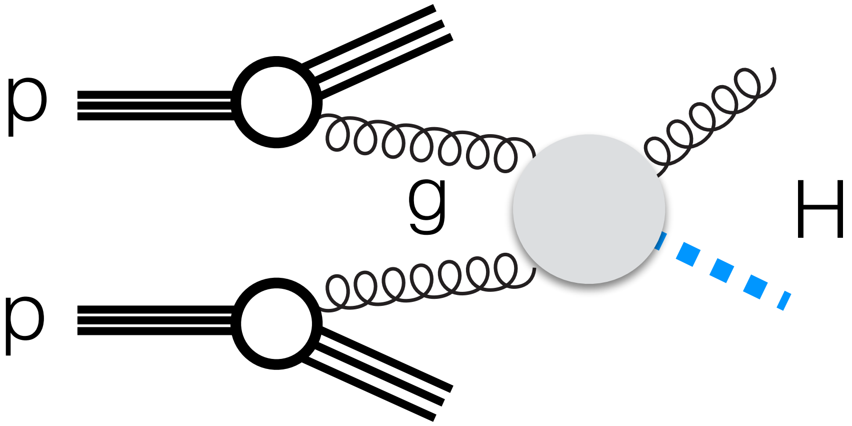
$$\mathcal{L}_{\text{eff}} = \hat{C}_0 H \mathcal{O}_{4;0} + \sum_{k=1}^{\infty} \frac{1}{m_{\text{t}}^{2k}} \sum_i \hat{C}_i H \mathcal{O}_{4+2k;i}$$

Wilczek, 1977; Shifman et.al., 1979; Dawson, 1991; Djouadi et.al. 1991, ….

Motivation

Higgs plus jet production

Boughezal, Caola, Melnikov, Petriello, Schulze 2013; Chen, Gehrmann, Glover, Jaquier 2014; Boughezal, Focke, Giele, Liu, Petriello 2015; Harlander, Liebler, Mantler 2016; Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger 2016; Lindert, Kudashkin, Melnikov, Wever 2018; Jones, Kerner, Luisoni 2018; Neumann 2018; …



$p_T \sim 2m_t$ → High-dimension operators become important.

$$\mathcal{L}_{\text{eff}} = C_0 O_0 + \frac{1}{m_{\text{t}}^2} \sum_{i=1}^{4} C_i O_i + \mathcal{O}\left(\frac{1}{m_{\text{t}}^4}\right)$$

Dimension-5 operator

$$O_0 = H\text{tr}(F_{\mu\nu}F^{\mu\nu})$$

Gehrmann, Jaquier, Glover, Koukoutsakis 2011

Dimension-7 operators

$$O_1 = H\text{tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu)\,,$$
$$O_2 = H\text{tr}(D_\rho F_{\mu\nu} D^\rho F^{\mu\nu})\,,$$
$$O_3 = H\text{tr}(D^\rho F_{\rho\mu} D_\sigma F^{\sigma\mu})\,,$$
$$O_4 = H\text{tr}(F_{\mu\rho} D^\rho D_\sigma F^{\sigma\mu})\,.$$

Dawson, Lewis, Zeng 2014
Jin, GY 2019

Related S-matrix computations

Setup of the problem

Operators:

$$\mathcal{O} \sim \; c(a_1, ..., a_n)\big(D_{\mu_{11}}...D_{\mu_{1m_1}}F_{\nu_1\rho_1}\big)^{a_1} \cdots \big(D_{\mu_{n1}}...D_{\mu_{nm_n}}F_{\nu_n\rho_n}\big)^{a_n} X(\eta, \epsilon)$$

$$D_\mu \star = \partial_\mu + ig[A_\mu, \star]\,, \qquad [D_\mu, D_\nu] \star = ig[F_{\mu\nu}, \star] \qquad\qquad F_{\mu\nu} = F^a_{\mu\nu}T^a, \qquad [T^a, T^b] = if^{abc}T^c$$

Classical dimension $\quad \dim(\mathcal{O}) = \Delta_0(\mathcal{O}) = (\#\text{ of } D\text{'s}) + 2 \times (\#\text{ of } F\text{'s})$

Length of operator $\quad \mathrm{len}(\mathcal{O}) = (\#\text{ of } F\text{'s})$

Lorentz indices $\quad F^{\mu_1\mu_2}D_{\mu_1}D_{\mu_5}F^{\mu_3\mu_4}D_{\mu_2}D^{\mu_5}F_{\mu_3\mu_4} \Rightarrow F_{12}D_{15}F_{34}D_{25}F_{34}$

Setup of the problem

Operators:

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Problems to address in this talk:

- Independent operator basis (classical)
- Renormalization of operators (quantum UV)
- Higgs amplitudes (finite remainder)

Content

Operator basis

Basis of operators (classical)

Operators are in general not independent:

$$\mathcal{O} \sim \; c(a_1, ..., a_n) \big(D_{\mu_{11}} ... D_{\mu_{1m_1}} F_{\nu_1 \rho_1}\big)^{a_1} \cdots \big(D_{\mu_{n1}} ... D_{\mu_{nm_n}} F_{\nu_n \rho_n}\big)^{a_n} X(\eta, \epsilon)$$

Equation of motion: $D_\mu F^{\mu\nu} = 0$

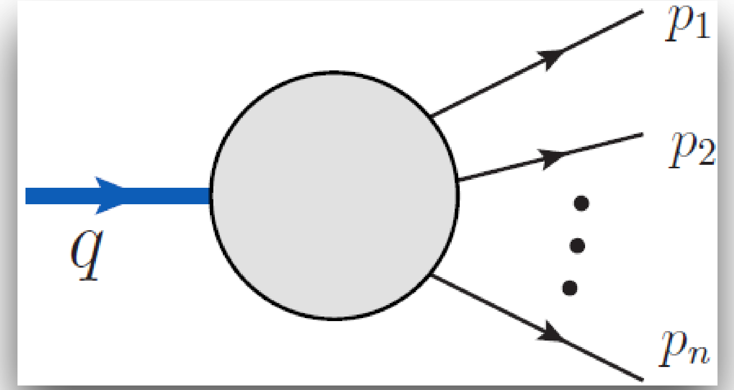
Bianchi identities: $D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} + D_\rho F_{\mu\nu} = 0$

One needs to remove such relations to find a set of independent basis operators.

Observable: form factors

Hybrids of on-shell states and off-shell operators:

$$F = \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{O}(x)|p_1\, p_2 \cdots p_n\rangle$$
$$= \delta^4\Big(\sum_{i=1}^{n} p_i - q\Big)\langle 0|\mathcal{O}(q)|p_1 p_2 \cdots p_n\rangle$$



$$q = \sum_i p_i, \qquad q^2 \neq 0$$

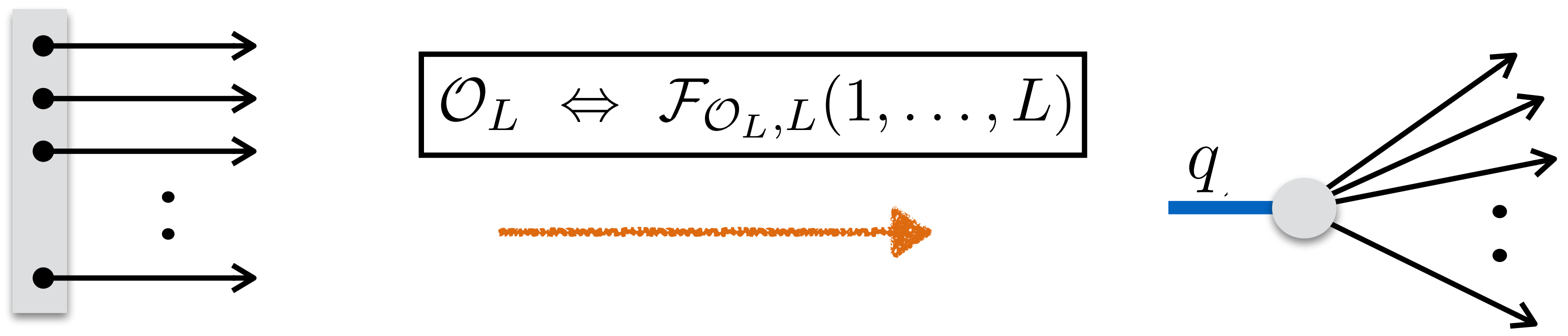
$\langle 0|p_1 p_2 \cdots p_n\rangle$

Amplitudes

$\langle 0\,|\mathcal{O}(x_1)\mathcal{O}(x_2)\ldots\mathcal{O}(x_n)|\,0\rangle$

Correlation functions

Minimal tree form factors



$$\boxed{\mathcal{O}_L \;\Leftrightarrow\; \mathcal{F}_{\mathcal{O}_L,L}(1,\ldots,L)}$$

Dictionary:

Used in N=4 SYM: Zwiebel 2011, Wilhelm 2014

operator	D_μ	$F_{\mu\nu}$
kinematics	p_μ	$p_\mu \varepsilon_\nu - p_\nu \varepsilon_\mu$

D-dim

operator	$D_{\dot{\alpha}\alpha}$	$f_{\alpha\beta}$	$\bar{f}_{\dot{\alpha}\dot{\beta}}$
spinor	$\tilde{\lambda}_{\dot{\alpha}}\lambda_\alpha$	$\lambda_\alpha\lambda_\beta$	$-\tilde{\lambda}_{\dot{\alpha}}\tilde{\lambda}_{\dot{\beta}}$

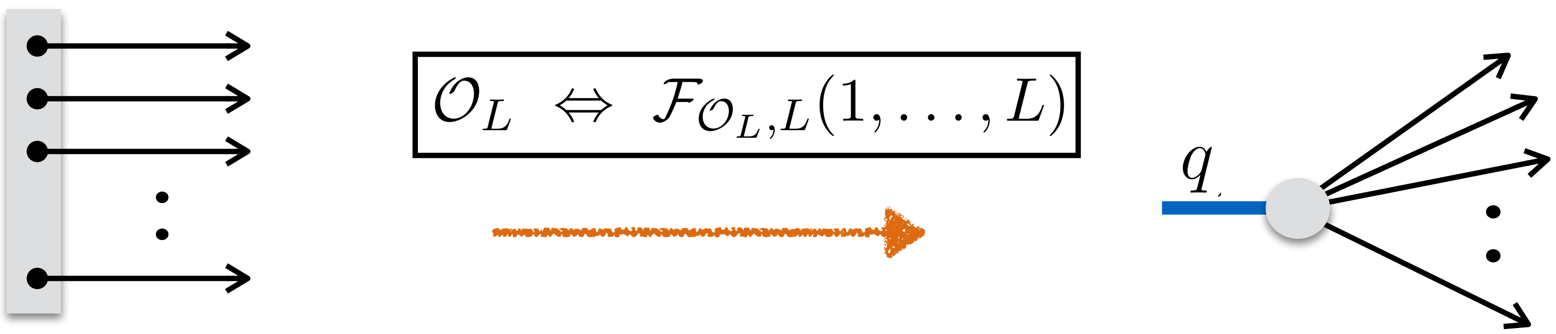
4-dim

$$F_{\mu\nu} \to F_{\alpha\dot{\alpha}\beta\dot{\beta}} = \epsilon_{\alpha\beta}\bar{f}_{\dot{\alpha}\dot{\beta}} + \epsilon_{\dot{\alpha}\dot{\beta}}f_{\alpha\beta}$$

One can translate any local operator into “on-shell” kinematics:

$$\mathrm{tr}(\bar{F}_{\dot{\alpha}}{}^{\dot{\beta}}\bar{F}_{\dot{\beta}}{}^{\dot{\gamma}}\bar{F}_{\dot{\gamma}}{}^{\dot{\alpha}}) \to \tilde{\lambda}_1^{\dot{\alpha}}\tilde{\lambda}_{1\dot{\beta}}\tilde{\lambda}_2^{\dot{\beta}}\tilde{\lambda}_{2\dot{\gamma}}\tilde{\lambda}_3^{\dot{\gamma}}\tilde{\lambda}_{3\dot{\alpha}} = [1\,2][2\,3][3\,1]$$

Minimal tree form factors



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operator	D_μ	$F_{\mu\nu}$
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4-dim

$F_{\mu\nu} \to F_{\alpha\dot\alpha\beta\dot\beta} = \epsilon_{\alpha\beta}\bar{f}_{\dot\alpha\dot\beta} + \epsilon_{\dot\alpha\dot\beta}f_{\alpha\beta}$

Important for capturing
Evanescent operators

In preparation, Qingjun Jin, Ke Ren, GY, Rui Yu

A good set of operators

For the convenience of the loop computation, it is also important to provide a set of "**good**" operators.

Color sectors

$$f^{abc} = \mathrm{Tr}(T^a T^b T^c) - \mathrm{Tr}(T^a T^c T^b)\,, \qquad d^{abc} = \mathrm{Tr}(T^a T^b T^c) + \mathrm{Tr}(T^a T^c T^b)$$

Helicity sectors

$$\begin{aligned} &\alpha\text{-sector}: \quad \mathcal{F}_{\mathcal{O}}^{(0),\mathrm{min}} \neq 0 \text{ only for } (-,-,+),(+,+,-),\\ &\beta\text{-sector}: \quad \mathcal{F}_{\mathcal{O}}^{(0),\mathrm{min}} \neq 0 \text{ only for } (-,-,-),(+,+,+). \end{aligned}$$

Examples

dim 6

$$\mathcal{O}''_{6;1} = \frac{1}{3}\mathrm{Tr}(F_{12}F_{13}F_{23})\,.$$

dim 8

$$\mathcal{O}''_{8;1} = \mathrm{Tr}(D_1F_{23}D_4F_{23}F_{14});\ \mathcal{O}''_{8;2} = \mathrm{Tr}(D_1F_{23}D_1F_{24}F_{34})\,.$$

dim 10

$$\mathcal{O}''_{10;1} = \mathrm{Tr}(D_{12}F_{34}D_{15}F_{34}F_{25}),\ \mathcal{O}''_{10;2} = \mathrm{Tr}(D_{12}F_{34}D_5F_{34}D_1F_{25}),\ \mathcal{O}''_{10;3} = \mathrm{Tr}(D_2F_{34}D_{15}F_{34}D_1F_{25});$$

$$\mathcal{O}''_{10;4} = \mathrm{Tr}(D_{12}F_{34}D_1F_{35}D_2F_{45}),\ \mathcal{O}''_{10;5} = \mathrm{Tr}(D_{12}F_{34}D_{12}F_{35}F_{45})\,.$$

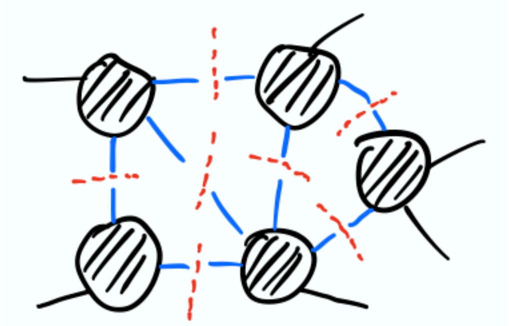
Good basis operator	$\mathcal{F}^{(0)}(-,-,+)$	$\mathcal{F}^{(0)}(-,-,-)$	color factor
$\mathcal{O}_{6;\beta;f;1} = \mathcal{O}''_{6;1}$	0	A_2	f^{abc}
$\mathcal{O}_{8;\alpha;f;1} = \mathcal{O}''_{8;1} - \frac{1}{2}\partial^2\mathcal{O}_{6;\beta;f;1}$	A_1	0	f^{abc}
$\mathcal{O}_{8;\beta;f;1} = \frac{1}{2}\partial^2\mathcal{O}_{6;\beta;f;1}$	0	$\frac{1}{2}s_{123}\ A_2$	f^{abc}
$\mathcal{O}_{10;\alpha;f;1} = \frac{1}{2}\partial^2\mathcal{O}_{8;\alpha;f;1}$	$\frac{1}{2}s_{123}A_1$	0	f^{abc}
$\mathcal{O}_{10;\alpha;f;2} = \mathcal{O}''_{10;1} - \mathcal{O}''_{10;5}$	$\frac{1}{2}s_{123}A_1\ u$	0	f^{abc}
$\mathcal{O}_{10;\alpha;d;1} = \mathcal{O}''_{10;2} - \mathcal{O}''_{10;3}$	$\frac{1}{2}s_{123}A_1\ (w-v)$	0	d^{abc}
$\mathcal{O}_{10;\beta;f;1} = \frac{1}{4}\partial^4\mathcal{O}_{6;\beta;f;1}$	0	$\frac{1}{4}s_{123}^2A_2$	f^{abc}
$\mathcal{O}_{10;\beta;f;2} = \mathcal{O}''_{10;5}$	0	$\frac{1}{4}s_{123}^2A_2\ (u^2+v^2+w^2)$	f^{abc}

$$A_1 = \langle 12\rangle^3[13][23]\,, \qquad A_2 = \langle 12\rangle\langle 13\rangle\langle 23\rangle$$

$$u = \frac{s_{12}}{s_{123}}, \qquad v = \frac{s_{23}}{s_{123}}, \qquad w = \frac{s_{13}}{s_{123}}$$

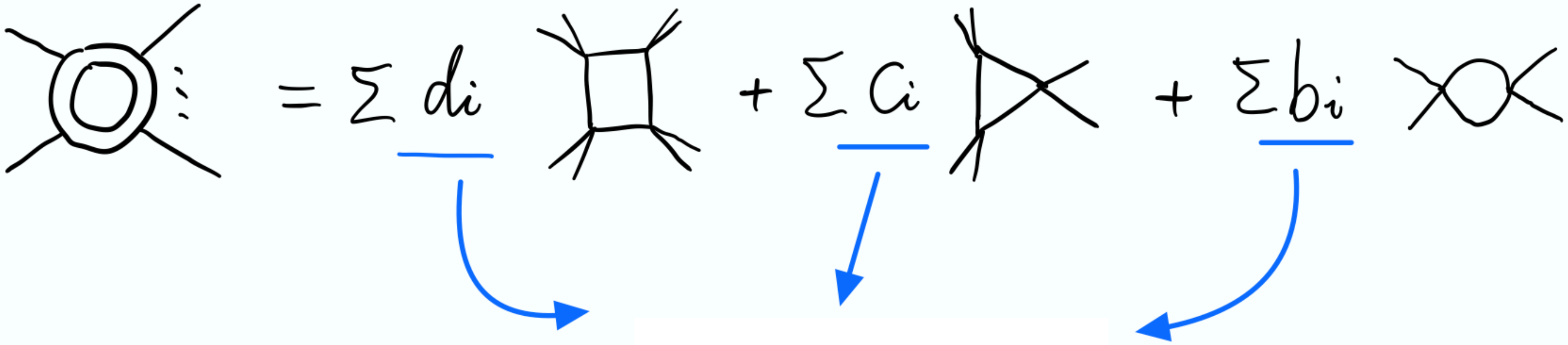
Loop computation

On-shell unitarity



Unitarity cuts

Consider one-loop amplitudes:

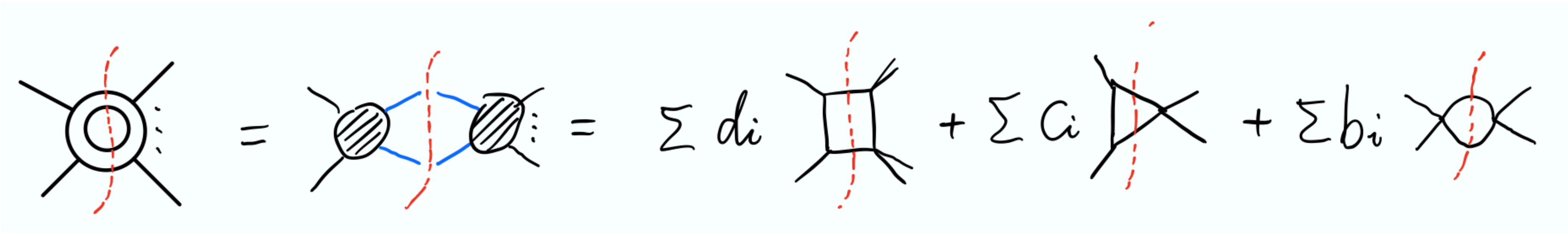


What we really want

Unitarity cuts

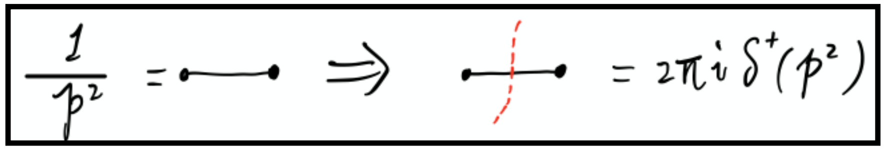
One can perform unitarity cuts:

[Bern, Dixon, Dunbar, Kosower 1994]
[Britto, Cachazo, Feng 2004]



and from tree products, one derives the coefficients more directly.

Cutkosky cutting rule:



Unitarity cuts

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[Bern, Dixon, Dunbar, Kosower 1994]

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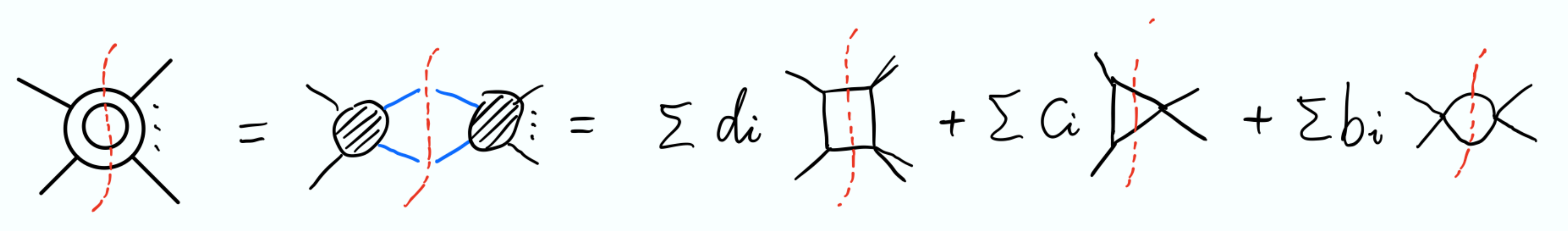


and from tree products, one derives the coefficients more directly.

Challenges at higher loops:

- Need D-dimensional cuts (rational term issue)
- Not trivial to reconstruct the full integrand and then reduce it, e.g. via IBP

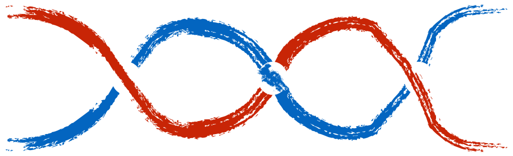
Unitarity-IBP strategy

Loop amplitudes = Sum (**coefficient** x IBP masters)

what we want

$$\mathcal{F}^{(l)}\Big|_{\rm cut} = \prod (\text{tree blocks}) = \text{cut integrand} \;\; = \sum_i c_i\, M_i|_{\rm cut}$$

On-shell unitarity



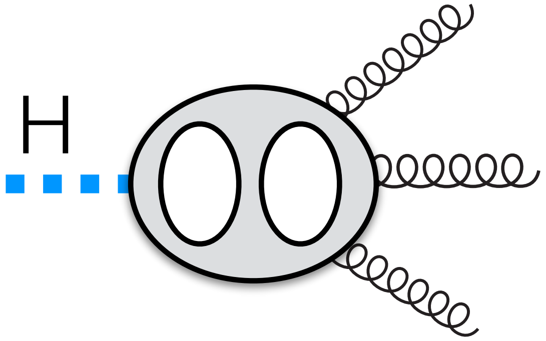
(cut) IBP reduction

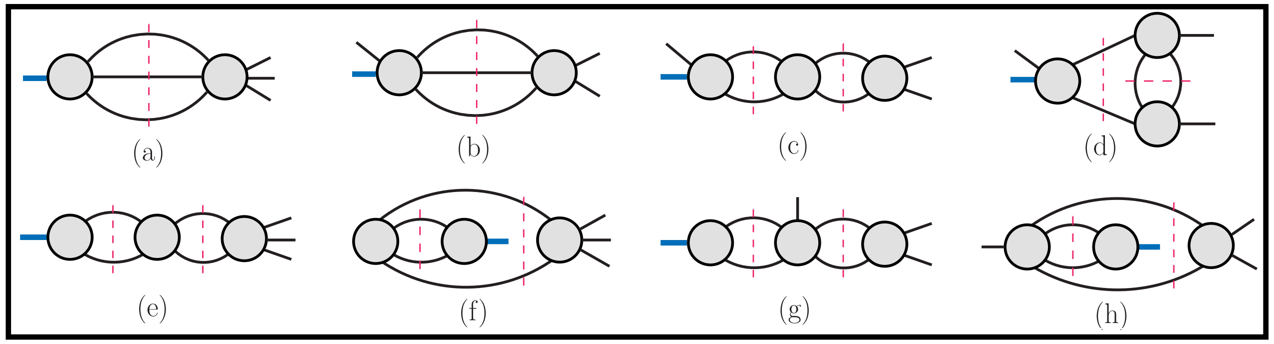
Jin, GY 2018 Boels, Jin, Luo 2018

Numerical unitarity: Abreu, Cordero, Ita, Jaquier, Page, Zeng 2017

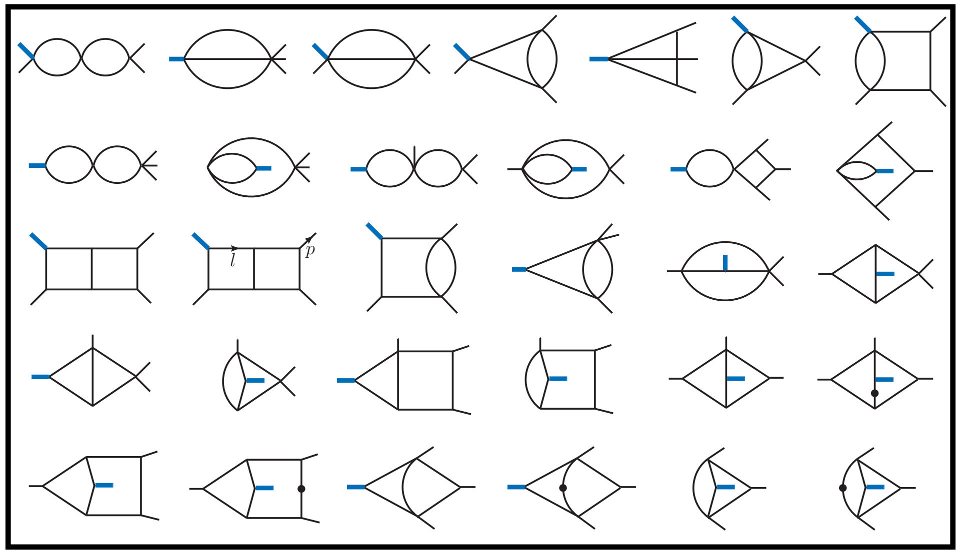
Higgs plus three gluons

All cuts that are needed:





Master integrals are known in terms of 2d Harmonic polylogarithms.



[Gehrmann, Remiddi 2001]

Results and analysis

UV renormalization

Finite remainder

Loop structure of form factors

General structure of (bare) amplitudes/form factors:

$$\text{Loop correction} = \underbrace{\text{IR} + \text{UV}}_{\text{Mixed in dim-reg}} + \text{finite remainder} + \mathcal{O}(\epsilon)$$

Loop structure of form factors

General structure of (bare) amplitudes/form factors:

Loop correction = IR + UV + finite remainder + $\mathcal{O}(\epsilon)$

IR structure is “universal”: [Catani 1998]

$$\mathcal{F}^{(1)}_{\mathcal{O},R} = I^{(1)}(\epsilon)\mathcal{F}^{(0)}_{\mathcal{O}} + \mathcal{F}^{(1)}_{\mathcal{O},\text{fin}} + \mathcal{O}(\epsilon)\,,$$
$$\mathcal{F}^{(2)}_{\mathcal{O},R} = I^{(2)}(\epsilon)\mathcal{F}^{(0)}_{\mathcal{O}} + I^{(1)}(\epsilon)\mathcal{F}^{(1)}_{\mathcal{O},R} + \mathcal{F}^{(2)}_{\mathcal{O},\text{fin}} + \mathcal{O}(\epsilon)$$

$$I^{(1)}(\epsilon) = -\frac{e^{\gamma_E \epsilon}}{\Gamma(1-\epsilon)}\Big(\frac{N_c}{\epsilon^2} + \frac{\beta_0}{2\epsilon}\Big)\sum_{i=1}^{E}(-s_{i,i+1})^{-\epsilon}\,,$$
$$I^{(2)}(\epsilon) = -\frac{1}{2}\big(I^{(1)}(\epsilon)\big)^2 - \frac{\beta_0}{\epsilon}I^{(1)}(\epsilon) + \frac{e^{-\gamma_E\epsilon}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\Big(\frac{\beta_0}{\epsilon} + \frac{67}{9} - \frac{\pi^2}{3}\Big)I^{(1)}(2\epsilon)$$
$$+ E\frac{e^{\gamma_E\epsilon}}{\epsilon\Gamma(1-\epsilon)}\Big(\frac{\zeta_3}{2} + \frac{5}{12} + \frac{11\pi^2}{144}\Big)\,.$$

UV renormalization: operator mixing

By subtracting the universal IR, one can obtain the UV renormalization matrix.

- Operators (of same classical dimension) can mix with each other at quantum level via renormalization:

$$\mathcal{O}_{R,i} = Z_i^{\ j} \mathcal{O}_{B,j}$$

- From the renormalization matrix, one can obtain the dilatation operator:

$$\mathcal{D} = -\frac{d \log Z}{d \log \mu}$$

- The anomalous dimensions are are given by the eigenvalues of dilatation operator:

$$\mathcal{D} \cdot \mathcal{O}_{\text{eigen}} = \gamma \cdot \mathcal{O}_{\text{eigen}}$$

Example

$$\mathcal{F}^{(2)}_{\mathcal{O}_{8;\alpha;f;1}}(1^-,2^-,3^+)\Big|_{\frac{1}{\epsilon}\text{ UV-div.}} = \mathcal{F}^{(0)}_{\mathcal{O}_{8;\alpha;f;1}}(1^-,2^-,3^+)\times\frac{N_c^2}{\epsilon}\Big(-\frac{1}{3vw}+\frac{269}{72}\Big),$$

$$\mathcal{F}^{(2),\alpha}_{\mathcal{O}_{8;\beta;f;1}}(1^-,2^-,3^+)\Big|_{\frac{1}{\epsilon}\text{ UV-div.}} = \mathcal{F}^{(0)}_{\mathcal{O}_{8;\alpha;f;1}}(1^-,2^-,3^+)\times\frac{N_c^2}{\epsilon}\Big(-\frac{1}{vw}\Big).$$

→ $$(Z^{(2)})_{\mathcal{O}_{8;\alpha;f;1}}^{\ \mathcal{O}_{8;0}} = -\frac{N_c^2}{3\epsilon}, \quad (Z^{(2)})_{\mathcal{O}_{8;\alpha;f;1}}^{\ \mathcal{O}_{8;\alpha;f;1}} = \frac{269N_c^2}{72\epsilon}, \quad (Z^{(2)})_{\mathcal{O}_{8;\beta;f;1}}^{\ \mathcal{O}_{8;0}} = -\frac{N_c^2}{\epsilon}.$$

……

$$\mathcal{F}^{(2)}_{\mathcal{O}_{8;\alpha;f;1}}(1^-,2^-,3^-)\Big|_{\frac{1}{\epsilon}\text{ UV-div.}} = \mathcal{F}^{(0)}_{\mathcal{O}_{8;\beta;f;1}}(1^-,2^-,3^-)\times\frac{N_c^2}{\epsilon}\Big(-\frac{1}{3uvw}+\frac{5}{2}\Big),$$

$$\mathcal{F}^{(2)}_{\mathcal{O}_{8;\beta;f;1}}(1^-,2^-,3^-)\Big|_{\frac{1}{\epsilon}\text{ UV-div.}} = \mathcal{F}^{(0)}_{\mathcal{O}_{8;\beta;f;1}}(1^-,2^-,3^-)\times\frac{N_c^2}{\epsilon}\Big(-\frac{1}{uvw}+\frac{25}{12}\Big).$$

→ $$(Z^{(2)})_{\mathcal{O}_{8;\alpha;f;1}}^{\ \mathcal{O}_{8;0}} = -\frac{N_c^2}{3\epsilon}, \qquad (Z^{(2)})_{\mathcal{O}_{8;\alpha;f;1}}^{\ \mathcal{O}_{8;\beta;f;1}} = \frac{5N_c^2}{2\epsilon},$$

$$(Z^{(2)})_{\mathcal{O}_{8;\beta;f;1}}^{\ \mathcal{O}_{8;0}} = -\frac{N_c^2}{\epsilon}, \qquad (Z^{(2)})_{\mathcal{O}_{8;\beta;f;1}}^{\ \mathcal{O}_{8;\beta;f;1}} = \frac{25N_c^2}{12\epsilon}.$$

$$\{\mathcal{O}_{8;0}, \mathcal{O}_{8;\alpha;f;1}, \mathcal{O}_{8;\beta;f;1}\}$$

$$Z^{(2)}_{\mathcal{O}_8}\Big|_{\frac{1}{\epsilon}\text{-part.}} = \frac{N_c^2}{\epsilon}\begin{pmatrix} -\frac{34}{3} & 0 & 0 \\ -\frac{1}{3} & \frac{269}{72} & \frac{5}{2} \\ -1 & 0 & \frac{25}{12} \end{pmatrix}$$

$$\mathbb{D}_{\mathcal{O}_8} = \begin{pmatrix} -\frac{22}{3}\hat{\lambda} - \frac{136}{3}\hat{\lambda}^2 & 0 & 0 \\ -\frac{\hat{\lambda}^2}{\hat{g}} & \frac{7}{3}\hat{\lambda} + \frac{269}{18}\hat{\lambda}^2 & 10\hat{\lambda}^2 \\ -3\frac{\hat{\lambda}^2}{\hat{g}} & 0 & \hat{\lambda} + \frac{25}{3}\hat{\lambda}^2 \end{pmatrix}$$

→ $$\hat{\gamma}^{(1)}_{\mathcal{O}_8} = \Big\{-\frac{22}{3}; 1; \frac{7}{3}\Big\}, \qquad \hat{\gamma}^{(2)}_{\mathcal{O}_8} = \Big\{-\frac{136}{3}; \frac{25}{3}; \frac{269}{18}\Big\}.$$

Mixing matrix and spectrum

Results were known previously at one-loop up to dimension-8.

See e.g.: Gracey 2002; Dawson, Lewis, Zeng 2014

We obtain new one- and two-loop results up to dimension16.

$$\mathbb{D}_{\mathcal{O}_8} = \begin{pmatrix} -\frac{22}{3}\hat{\lambda} - \frac{136}{3}\hat{\lambda}^2 & 0 & 0 \\ -\frac{\hat{\lambda}^2}{\hat{g}} & \frac{7}{3}\hat{\lambda} + \frac{269}{18}\hat{\lambda}^2 & 10\hat{\lambda}^2 \\ -3\frac{\hat{\lambda}^2}{\hat{g}} & 0 & \hat{\lambda} + \frac{25}{3}\hat{\lambda}^2 \end{pmatrix} \qquad \hat{\gamma}^{(1)}_{\mathcal{O}_8} = \left\{-\frac{22}{3}; 1; \frac{7}{3}\right\}, \qquad \hat{\gamma}^{(2)}_{\mathcal{O}_8} = \left\{-\frac{136}{3}; \frac{25}{3}; \frac{269}{18}\right\}$$

$$\mathbb{D}_{\mathcal{O}_{10,f}} = \begin{pmatrix} -\frac{22\hat{\lambda}}{3} - \frac{136}{3}\hat{\lambda}^2 & 0 & 0 & 0 & 0 \\ -\frac{\hat{\lambda}^2}{\hat{g}} & \frac{7\hat{\lambda}}{3} + \frac{269}{18}\hat{\lambda}^2 & 0 & 10\hat{\lambda}^2 & 0 \\ -\frac{209}{300}\frac{\hat{\lambda}^2}{\hat{g}} & -\frac{6\hat{\lambda}}{5} - \frac{5579\hat{\lambda}^2}{4500} & \frac{71\hat{\lambda}}{15} + \frac{2848}{125}\hat{\lambda}^2 & \frac{1493}{300}\hat{\lambda}^2 & \frac{5}{9}\hat{\lambda}^2 \\ -3\frac{\hat{\lambda}^2}{\hat{g}} & 0 & 0 & \hat{\lambda} + \frac{25}{3}\hat{\lambda}^2 & 0 \\ -\frac{19}{12}\frac{\hat{\lambda}^2}{\hat{g}} & \frac{139}{600}\hat{\lambda}^2 & \frac{499}{200}\hat{\lambda}^2 & -2\hat{\lambda} - \frac{143}{72}\hat{\lambda}^2 & \frac{17\hat{\lambda}}{3} + \frac{2195}{72}\hat{\lambda}^2 \end{pmatrix}$$

$$\hat{\gamma}^{(1)}_{\mathcal{O}_{10,f}} = \left\{-\frac{22}{3}; 1; \frac{7}{3}; \frac{71}{15}, \frac{17}{3}\right\}, \qquad \hat{\gamma}^{(2)}_{\mathcal{O}_{10,f}} = \left\{-\frac{136}{3}; \frac{25}{3}; \frac{269}{18}; \frac{2848}{125}, \frac{2195}{72}\right\}$$

Mixing matrix and spectrum

Anomalous dimensions for length-3 operators up to dimension 16:

dim	4	6	8	10	12	14	16
$\gamma^{(1)}_{f,\alpha}$	$-\frac{22}{3}$	/	$\frac{7}{3}$	$\frac{71}{15}$	$\frac{241}{30}, \frac{101}{15}$	$\frac{61}{6}, \frac{172}{21}$	$\frac{331}{35}, \frac{1212\pm\sqrt{3865}}{105}$
$\gamma^{(2)}_{f,\alpha}$	$-\frac{136}{3}$	/	$\frac{269}{18}$	$\frac{2848}{125}$	$\frac{49901119}{1404000}, \frac{8585281}{234000}$	$\frac{4392073141}{87847200}, \frac{685262197}{15373260}$	$\frac{231568398949}{4253886000},$ $\frac{355106171452034\pm95588158951\sqrt{3865}}{6576507756000}$
$\gamma^{(1)}_{f,\beta}$	$-\frac{22}{3}$	1	/	$\frac{17}{3}$	9	$\frac{43}{5}$	$\frac{67}{6}$
$\gamma^{(2)}_{f,\beta}$	$-\frac{136}{3}$	$\frac{25}{3}$	/	$\frac{2195}{72}$	$\frac{79313}{1800}$	$\frac{443801}{9000}$	$\frac{63879443}{1058400}$
$\gamma^{(1)}_{d,\alpha}$	/	/	/	$\frac{13}{3}$	$\frac{41}{6}$	$\frac{551\pm3\sqrt{609}}{60}$	$\frac{321\pm\sqrt{1561}}{30}$
$\gamma^{(2)}_{d,\alpha}$	/	/	/	$\frac{575}{36}$	$\frac{46517}{1440}$	$\frac{5809305897\pm19635401\sqrt{609}}{131544000}$	$\frac{229162584707\pm225658792\sqrt{1561}}{4130406000}$
$\gamma^{(1)}_{d,\beta}$	/	/	/	/	9	/	$\frac{67}{6}$
$\gamma^{(2)}_{d,\beta}$	/	/	/	/	$\frac{150391}{3600}$	/	$\frac{174229}{3150}$

Finite remainder

Finite remainder

The finite part of the form factor:

$$\mathcal{F}^{(2)}_{\mathcal{O},R} = I^{(2)}(\epsilon)\mathcal{F}^{(0)}_{\mathcal{O}} + I^{(1)}(\epsilon)\mathcal{F}^{(1)}_{\mathcal{O},R} \boxed{+\, \mathcal{F}^{(2)}_{\mathcal{O},\text{fin}}} + \mathcal{O}(\epsilon)$$

They provide two-loop H plus 3-gluon amplitudes for the top mass correction in the Higgs effective theory.

$$\mathcal{R}^{(2),\pm}_{\mathcal{O}} = \underline{\sum_{n=0}^{4} \mathcal{R}^{(2),\pm}_{\mathcal{O}}\Big|_{\text{deg-}n}} + \mathcal{R}^{(2),\pm}_{\mathcal{O}}\Big|_{\log^2(-q^2)} + \mathcal{R}^{(2),\pm}_{\mathcal{O}}\Big|_{\log(-q^2)}$$

$$\boxed{u = \frac{s_{12}}{s_{123}}, \qquad v = \frac{s_{23}}{s_{123}}, \qquad w = \frac{s_{13}}{s_{123}}}$$

Degree 4 part

The transcendentality degree-4 part is universal:

$$\boxed{\begin{aligned}&-\frac{3}{2}\mathrm{Li}_4(u)+\frac{3}{4}\mathrm{Li}_4\left(-\frac{uv}{w}\right)-\frac{3}{4}\log(w)\left[\mathrm{Li}_3\left(-\frac{u}{v}\right)+\mathrm{Li}_3\left(-\frac{v}{u}\right)\right]\\&+\frac{\log^2(u)}{32}\left[\log^2(u)+\log^2(v)+\log^2(w)-4\log(v)\log(w)\right]\\&+\frac{\zeta_2}{8}\left[5\log^2(u)-2\log(v)\log(w)\right]-\frac{1}{4}\zeta_4+\mathrm{perms}(u,v,w)\,,\end{aligned}}$$

It also appears as a universal function for length-3 operators in N=4 SYM

[Brandhuber, Kostacinska, Penante, Travaglini, Wen, Young 2014, 2016]

[Loebbert, Nandan, Sieg, Wilhelm, GY 2015, 2016]

“Maximal transcendentality principle” [Kotikov, Lipatov, Onishchenko, Velizhanin 2004]

Lower degree parts

Degree-3 part and degree-2 part are consist of universal building blocks $\{T_3, T_2\}$, plus simple log functions:

$$\boxed{\begin{aligned} T_3(u,v,w) :=& \Big[-\mathrm{Li}_3\left(-\frac{u}{w}\right) + \log(u)\mathrm{Li}_2\left(\frac{v}{1-u}\right) - \frac{1}{2}\log(u)\log(1-u)\log\left(\frac{w^2}{1-u}\right) \\ & + \frac{1}{2}\mathrm{Li}_3\left(-\frac{uv}{w}\right) + \frac{1}{2}\log(u)\log(v)\log(w) + \frac{1}{12}\log^3(w) + (u \leftrightarrow v)\Big] \\ & + \mathrm{Li}_3(1-v) - \mathrm{Li}_3(u) + \frac{1}{2}\log^2(v)\log\left(\frac{1-v}{u}\right) - \zeta_2\log\left(\frac{uv}{w}\right) . \end{aligned}}$$

$$\boxed{T_2(u,v) := \mathrm{Li}_2(1-u) + \mathrm{Li}_2(1-v) + \log(u)\log(v) - \zeta_2 .}$$

The coefficients of $\{T_3, T_2\}$ and log functions are non-trivial rational functions, which they contain spurious poles.

Example

For dimension-8 operator, the apparent leading pole is $1/u^6$:

$$\mathcal{R}^{(2),+}_{\mathcal{O}_{8;\alpha;f;1}}\Big|_{\deg 2} = T_2(v,w)\Big(\frac{v^2w^2}{2u^4} - \frac{5vw(v^2+w^2)}{3u^4} + \frac{11v^2w^2(v+w)}{6u^5} + \frac{5v^3w^3}{u^6}\Big) + \mathcal{O}(\frac{1}{u^3})$$

$$T_2(v,w) = \Big(-\frac{\log v}{1-v} - \frac{\log(1-v)}{v}\Big)u + \mathcal{O}(u^2)$$

$$\mathcal{R}^{(2),+}_{\mathcal{O}_{8;\alpha;f;1}}\Big|_{\deg 2} = \frac{5v^2(1-v)^2}{u^5}\Big(-v\log v - (1-v)\log(1-v)\Big) + \mathcal{O}(u^{-4})$$

which is to be cancelled by the degree one part.

Remark: to cancel spurious poles, one needs to combine functions of different transcendentality weights.

Example

For the $1/u^2$ pole, there are non-trivial cancellation involving deg-0 to deg-3 parts:

$\underline{1/u^2\text{-pole}}$

$$\text{deg-0}: \ \frac{1}{72}(93v^2 + 81v - 52),$$

$$\text{deg-1}: \ \frac{1}{12}(-31v^2 - 37v + 11) + \frac{(63v^3 - 311v^2 + 187v - 22)}{36v}\log(1-v) - \frac{(63v^3 - 302v^2 + 246v - 29)}{36(v-1)}\log(v),$$

$$\begin{aligned}\text{deg-2}: \ & \frac{1}{3}(1-2v)\mathrm{Li}_2(v) - \frac{1}{6}v^2\log^2(1-v) + \frac{1}{3}(v-1)^2\log(1-v)\log(v) - \frac{1}{6}(v-1)^2\log^2(v) - \frac{1}{18}\pi^2(v-1)^2 \\ & - \frac{(63v^3 - 347v^2 + 223v - 34)}{36v}\log(1-v) + \frac{63v^3 - 338v^2 + 282v - 41}{36(v-1)}\log(v) + \frac{1}{72}(93v^2 + 141v - 14),\end{aligned}$$

$$\begin{aligned}\text{deg-3}: \ & \frac{1}{3}(2v-1)\mathrm{Li}_2(v) + \frac{1}{6}v^2\log^2(1-v) - \frac{1}{3}(v-1)^2\log(1-v)\log(v) + \frac{1}{6}(v-1)^2\log^2(v) + \frac{1}{18}\pi^2(v-1)^2 \\ & - \frac{(3v^2 - 3v + 1)}{3v}\log(1-v) + \frac{3v^2 - 3v + 1}{3(v-1)}\log(v).\end{aligned}$$

Summary and Outlook

Summary

- We preform explicit two-loop computation for a large class of high dimensional QCD operators and related Higgs+3-gluon amplitudes.
- On-shell methods (minimal form factor and unitarity-IBP strategy) are used.

On-shell Amplitudes

Form Factors

Off-shell Operators

Outlook

- Consider more generic operators: higher length (twist) operators, operators with Fermion or massive fields, non-local operators, etc.
- Explore hidden structure of renormalization matrices and finite remainders.
- **Goal**: provide a two-loop framework for general EFT renormalization and EFT amplitudes.

Outlook

- Consider more generic operators: higher length (twist) operators, operators with Fermion or massive fields, non-local operators, etc.
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Thank you!

Extra slides

Strategy of basis construction

Construct first "primitive operators" at a given length:

Length-2 $\mathcal{O}_{\mathrm{P0}} = \mathrm{Tr}\big(F_{12}F_{12}\big)$

Length-3 $\mathcal{O}_{\mathrm{P1}} = \mathrm{Tr}\big(F_{12}F_{23}F_{31}\big)$, $\mathcal{O}_{\mathrm{P2}} = \mathrm{Tr}\big(D_1F_{23}D_4F_{23}F_{14}\big)$

High dimension operators can be obtained by inserting pairs of D_μ .

Can be generalized to high lengths. This strategy allows constructing basis operators of arbitrary high dimensions at a given length.

In preparation, Qingjun Jin, Ke Ren, GY, Rui Yu

Examples

dim 16

$$\mathcal{O}''_{16;1} = \mathrm{Tr}(D_{12345}F_{67}D_{12348}F_{67}F_{58}),\ \mathcal{O}''_{16;2} = \mathrm{Tr}(D_{12345}F_{67}D_{1238}F_{67}D_4F_{58}),\ \mathcal{O}''_{16;3} = \mathrm{Tr}(D_{1235}F_{67}D_{12348}F_{67}D_4F_{58}),$$

$$\mathcal{O}''_{16;4} = \mathrm{Tr}(D_{12345}F_{67}D_{128}F_{67}D_{34}F_{58}),\ \mathcal{O}''_{16;5} = \mathrm{Tr}(D_{1235}F_{67}D_{1248}F_{67}D_{34}F_{58}),\ \mathcal{O}''_{16;6} = \mathrm{Tr}(D_{125}F_{67}D_{12348}F_{67}D_{34}F_{58})$$

$$\mathcal{O}''_{16;7} = \mathrm{Tr}(D_{12345}F_{67}D_{18}F_{67}D_{234}F_{58}),\ \mathcal{O}''_{16;8} = \mathrm{Tr}(D_{1235}F_{67}D_{148}F_{67}D_{234}F_{58}),\ \mathcal{O}''_{16;9} = \mathrm{Tr}(D_{125}F_{67}D_{1348}F_{67}D_{234}F_{58}),$$

$$\mathcal{O}''_{16;10} = \mathrm{Tr}(D_{15}F_{67}D_{12348}F_{67}D_{234}F_{58}),\ \mathcal{O}''_{16;11} = \mathrm{Tr}(D_{12345}F_{67}D_8F_{67}D_{1234}F_{58}),\ \mathcal{O}''_{16;12} = \mathrm{Tr}(D_{1235}F_{67}D_{48}F_{67}D_{1234}F_{58}),$$

$$\mathcal{O}''_{16;13} = \mathrm{Tr}(D_{125}F_{67}D_{348}F_{67}D_{1234}F_{58}),\ \mathcal{O}''_{16;14} = \mathrm{Tr}(D_{15}F_{67}D_{2348}F_{67}D_{1234}F_{58}),\ \mathcal{O}''_{16;15} = \mathrm{Tr}(D_5F_{67}D_{12348}F_{67}D_{1234}F_{58});$$

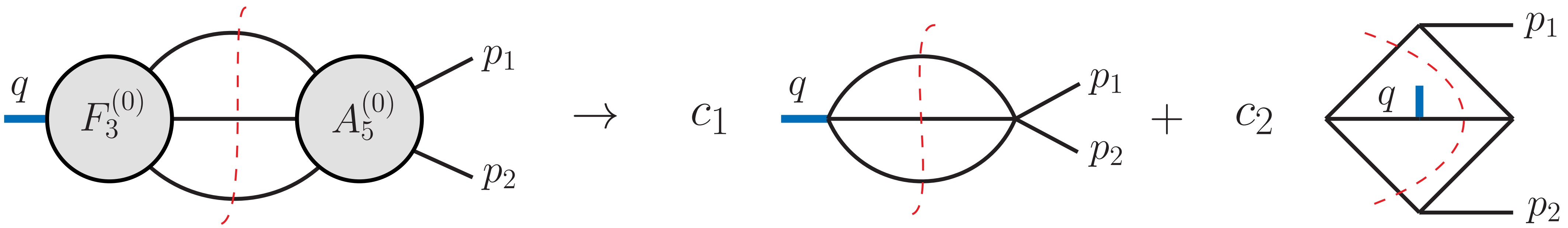
$$\mathcal{O}''_{16;16} = \mathrm{Tr}(D_{1234}F_{67}D_{125}F_{68}D_{345}F_{78}),\ \mathcal{O}''_{16;17} = \mathrm{Tr}(D_{123}F_{67}D_{12345}F_{68}D_{45}F_{78}),\ \mathcal{O}''_{16;18} = \mathrm{Tr}(D_{1234}F_{67}D_{1235}F_{68}D_{45}F_{78}),$$

$$\mathcal{O}''_{16;19} = \mathrm{Tr}(D_{12345}F_{67}D_{123}F_{68}D_{45}F_{78}),\ \mathcal{O}''_{16;20} = \mathrm{Tr}(D_{1234}F_{67}D_{12345}F_{68}D_5F_{78}),\ \mathcal{O}''_{16;21} = \mathrm{Tr}(D_{12345}F_{67}D_{1234}F_{68}D_5F_{78}),$$

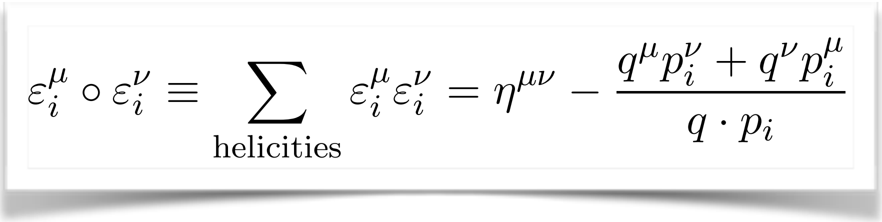
$$\mathcal{O}''_{16;22} = \mathrm{Tr}(D_{12345}F_{67}D_{12345}F_{68}F_{78})\,.$$

Good basis operator	$\mathcal{F}^{(0)}(-,-,+)$	$\mathcal{F}^{(0)}(-,-,-)$	color factor
$\mathcal{O}_{16;\alpha;f;1} = \frac{1}{16}\partial^8\mathcal{O}_{8;\alpha;f;1}$	$\frac{1}{16}s_{123}^4A_1$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;2} = \frac{1}{8}\partial^6\mathcal{O}_{10;\alpha;f;2}$	$\frac{1}{16}s_{123}^4A_1\ u$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;3} = \frac{1}{4}\partial^4\mathcal{O}_{12;\alpha;f;3}$	$\frac{1}{16}s_{123}^4A_1\ u^2$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;4} = \frac{1}{4}\partial^4\mathcal{O}_{12;\alpha;f;4}$	$\frac{1}{16}s_{123}^4A_1\ (u^2+v^2+w^2)$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;5} = \frac{1}{2}\partial^2\mathcal{O}_{14;\alpha;f;5}$	$\frac{1}{16}s_{123}^4A_1\ u^3$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;6} = \frac{1}{2}\partial^2\mathcal{O}_{14;\alpha;f;6}$	$\frac{1}{16}s_{123}^4A_1\ (u^3+v^3+w^3)$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;7} = \mathcal{O}''_{16;1} - \mathcal{O}''_{16;22}$	$\frac{1}{16}s_{123}^4A_1\ u^4$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;8} = \mathcal{O}''_{16;1} + \mathcal{O}''_{16;7} + \mathcal{O}''_{16;10}$ $-\mathcal{O}''_{16;17} - \mathcal{O}''_{16;19} - \mathcal{O}''_{16;22}$	$\frac{1}{16}s_{123}^4A_1\ u(u^3+v^3+w^3)$	0	f^{abc}
$\mathcal{O}_{16;\alpha;f;9} = \mathcal{O}''_{16;1} + \mathcal{O}''_{16;11} + \mathcal{O}''_{16;15}$ $-\frac{1}{2}\partial^2\mathcal{O}_{14;\beta;f;4}$	$\frac{1}{16}s_{123}^4A_1\ (u^4+v^4+w^4)$	0	f^{abc}
$\mathcal{O}_{16;\alpha;d;1} = \frac{1}{8}\partial^6\mathcal{O}_{10;\alpha;d;1}$	$\frac{1}{16}s_{123}^4A_1\ (w-v)$	0	d^{abc}
$\mathcal{O}_{16;\alpha;d;2} = \frac{1}{4}\partial^4\mathcal{O}_{12;\alpha;d;2}$	$\frac{1}{16}s_{123}^4A_1\ u(w-v)$	0	d^{abc}
$\mathcal{O}_{16;\alpha;d;3} = \frac{1}{2}\partial^2\mathcal{O}_{14;\alpha;d;3}$	$\frac{1}{16}s_{123}^4A_1\ u^2(w-v)$	0	d^{abc}
$\mathcal{O}_{16;\alpha;d;4} = \frac{1}{2}\partial^2\mathcal{O}_{14;\alpha;d;4}$	$\frac{1}{16}s_{123}^3A_1\ (w^3-v^3)$	0	d^{abc}
$\mathcal{O}_{16;\alpha;d;5} = \mathcal{O}''_{16;7} - \mathcal{O}''_{16;10} + \mathcal{O}''_{16;20} - \mathcal{O}''_{16;21}$ $-\frac{1}{4}\partial^4\mathcal{O}_{12;\beta;d;1}$	$\frac{1}{16}s_{123}^4A_1\ u(w^3-v^3)$	0	d^{abc}
$\mathcal{O}_{16;\alpha;d;6} = \mathcal{O}''_{16;11} - \mathcal{O}''_{16;15} - \mathcal{O}''_{16;20} + \mathcal{O}''_{16;21}$	$\frac{1}{16}s_{123}^3A_1\ (w^4-v^4)$	0	d^{abc}
$\mathcal{O}_{16;\beta;f;1} = \frac{1}{32}\partial^{10}\mathcal{O}_{6;\beta;f;1}$	0	$\frac{1}{32}s_{123}^5A_2$	f^{abc}
$\mathcal{O}_{16;\beta;f;2} = \frac{1}{8}\partial^6\mathcal{O}_{10;\beta;f;2}$	0	$\frac{1}{32}s_{123}^5A_2\ (u^2+v^2+w^2)$	f^{abc}
$\mathcal{O}_{16;\beta;f;3} = \frac{1}{4}\partial^4\mathcal{O}_{12;\beta;f;3}$	0	$\frac{1}{32}s_{123}^5A_2\ (u^3+v^3+w^3)$	f^{abc}
$\mathcal{O}_{16;\beta;f;4} = \frac{1}{2}\partial^2\mathcal{O}_{14;\beta;f;4}$	0	$\frac{1}{32}s_{123}^5A_2\ (u^4+v^4+w^4)$	f^{abc}
$\mathcal{O}_{16;\beta;f;5} = \mathcal{O}''_{16;22}$	0	$\frac{1}{32}s_{123}^5A_2\ (u^5+v^5+w^5)$	f^{abc}
$\mathcal{O}_{16;\beta;d;1} = \frac{1}{4}\partial^4\mathcal{O}_{12;\beta;d;1}$	0	$\frac{1}{32}s_{123}^5A_2(u-v)(u-w)(v-w)$	d^{abc}
$\mathcal{O}_{16;\beta;d;2} = \mathcal{O}''_{16;17} - \mathcal{O}''_{16;19} - \mathcal{O}''_{16;20} + \mathcal{O}''_{16;21}$	0	$\frac{1}{32}s_{123}^5A_2(u-v)(u-w)(v-w)$ $\times(u^2+v^2+w^2)$	d^{abc}

Example of cut

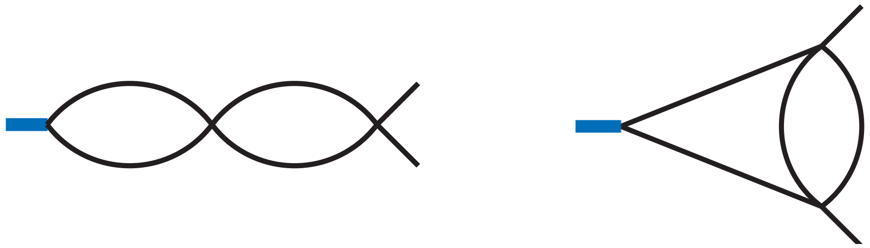


Tree product → sum internal helicities → project to gauge invariant basis → IBP for the cut integrand



$$\varepsilon_i^\mu \circ \varepsilon_i^\nu \equiv \sum_{\text{helicities}} \varepsilon_i^\mu \varepsilon_i^\nu = \eta^{\mu\nu} - \frac{q^\mu p_i^\nu + q^\nu p_i^\mu}{q \cdot p_i}$$

There are masters that do not contain this triple cut, such as:



→ need different cuts

UV renormalization: operator mixing

Form factor renormalization:

$$O^L_{R,i} = \mathcal{O}^L_{B,i} + \sum_{\ell=1}^{\infty} \left[\left(\frac{\alpha_s}{4\pi}\right)^{\ell} \sum_j \left(Z^{(\ell)}_{L\to L}\right)_i{}^j \mathcal{O}^L_{B,j} \; + \sum_k \left(\frac{\alpha_s}{4\pi}\right)^{\ell-\frac{k}{2}} \sum_j \left(Z^{(\ell)}_{L\to(L-k)}\right)_i{}^j \mathcal{O}^{L-k}_{B,j} \; + \sum_k \left(\frac{\alpha_s}{4\pi}\right)^{\ell+\frac{k}{2}} \sum_j \left(Z^{(\ell)}_{L\to(L+k)}\right)_i{}^j \mathcal{O}^{L+k}_{B,j} \right]$$

$$\begin{aligned}
\mathcal{F}^{(1)}_{\mathcal{O}_i,R} &= \mathcal{F}^{(1)}_{\mathcal{O}_i,B} + \sum_j (Z^{(1)}_{3\to3})_i{}^j \mathcal{F}^{(0)}_{\mathcal{O}_j} \,, \\
\mathcal{F}^{(2)}_{\mathcal{O}_i,R} &= \mathcal{F}^{(2)}_{\mathcal{O}_i,B} + \sum_j (Z^{(1)}_{3\to3})_i{}^j \mathcal{F}^{(1)}_{\mathcal{O}_j,B} - \frac{\beta_0}{\epsilon}\mathcal{F}^{(1)}_{\mathcal{O}_i,B} + \sum_j (Z^{(2)}_{3\to3})_i{}^j \mathcal{F}^{(0)}_{\mathcal{O}_j} + (Z^{(2)}_{3\to2})_i{}^0 \mathcal{F}^{(0)}_{\mathcal{O}_0} \,.
\end{aligned}$$

$$\mathbb{D} = -\frac{d\log Z}{d\log\mu} \qquad \longrightarrow$$

$$\begin{aligned}
\mathbb{D}^{(1)} &= 2\epsilon\Big(Z^{(1)}_{2\to2} + Z^{(1)}_{3\to3}\Big), \\
\mathbb{D}^{(\frac{3}{2})} &= 3\epsilon\Big(Z^{(1)}_{2\to3} + Z^{(2)}_{3\to2}\Big) = 3\epsilon Z^{(2)}_{3\to2}, \\
\mathbb{D}^{(2)} &= 4\epsilon\Big(Z^{(2)}_{2\to2}\big|_{\frac{1}{\epsilon}-\mathrm{part}} + Z^{(2)}_{3\to3}\big|_{\frac{1}{\epsilon}-\mathrm{part}}\Big).
\end{aligned}$$

$$Z^{(2)}_{L\to L}\big|_{\frac{1}{\epsilon^2}-\mathrm{part}} - \frac{1}{2}(Z^{(1)}_{L\to L})^2 + \frac{\beta_0}{2\epsilon} Z^{(1)}_{L\to L} = 0$$

Mixing matrix and spectrum

Dim-16 at 1-loop:

$$Z^{(1)}_{\mathcal{O}_{16,f}} = \frac{N_c}{\epsilon}\left(\begin{array}{c|ccccccccc|ccccc}
-\frac{11}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & \frac{7}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{3}{5} & \frac{71}{30} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{5}{4} & \frac{221}{60} & -\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & \frac{1}{10} & -\frac{19}{30} & \frac{37}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{17}{84} & -\frac{17}{28} & -\frac{47}{70} & -\frac{17}{28} & \frac{337}{84} & \frac{5}{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{3}{20} & \frac{9}{20} & -1 & -\frac{31}{20} & -\frac{1}{4} & \frac{31}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{13}{30} & -\frac{13}{15} & \frac{13}{10} & -\frac{13}{10} & -\frac{5}{2} & \frac{13}{15} & \frac{961}{210} & \frac{8}{15} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{71}{105} & -\frac{212}{105} & \frac{141}{35} & -\frac{71}{35} & -\frac{141}{35} & \frac{79}{105} & -\frac{38}{35} & \frac{223}{35} & \frac{5}{14} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{17}{70} & \frac{19}{105} & -\frac{19}{70} & -\frac{121}{70} & -\frac{11}{42} & \frac{16}{105} & -\frac{6}{5} & \frac{127}{210} & \frac{559}{105} & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \frac{17}{6} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \frac{9}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -2 & \frac{1}{3} & \frac{43}{10} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{5}{2} & \frac{5}{2} & -\frac{11}{4} & \frac{67}{12}
\end{array}\right)$$

$$Z^{(1)}_{\mathcal{O}_{16,d}} = \frac{N_c}{\epsilon}\left(\begin{array}{cccccc|cc}
\frac{13}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & \frac{41}{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & -2 & \frac{301}{60} & -\frac{2}{3} & 0 & 0 & 0 & 0 \\
-1 & 1 & -\frac{3}{10} & \frac{25}{6} & 0 & 0 & 0 & 0 \\
-\frac{2}{5} & \frac{1}{5} & 0 & -\frac{1}{5} & \frac{307}{60} & \frac{7}{20} & 0 & 0 \\
\frac{1}{3} & -1 & \frac{1}{2} & -\frac{7}{3} & \frac{13}{12} & \frac{67}{12} & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{12} & \frac{67}{12}
\end{array}\right)$$

Mixing matrix and spectrum

Dim-16 at 2-loop:

$$Z^{(2)}_{\mathcal{O}_{16,f}}\Big|_{\frac{1}{\epsilon}-\text{part.}} = \frac{N_c^2}{\epsilon}\left(\begin{array}{c|cccccccccccccc}
-\frac{34}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-\frac{1}{3} & \frac{269}{72} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{2} & 0 & 0 & 0 & 0 \\
-\frac{209}{900} & -\frac{5579}{18000} & \frac{712}{125} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1493}{1200} & \frac{5}{36} & 0 & 0 & 0 \\
-\frac{31}{180} & \frac{53}{3600} & -\frac{36227}{28800} & \frac{3575983}{432000} & \frac{9793}{21600} & 0 & 0 & 0 & 0 & 0 & \frac{13}{16} & \frac{16877}{14400} & -\frac{7319}{14400} & 0 & 0 \\
-\frac{181}{900} & -\frac{60979}{36000} & \frac{78487}{72000} & -\frac{2177}{2000} & \frac{704167}{72000} & 0 & 0 & 0 & 0 & 0 & \frac{1229}{1200} & \frac{115501}{43200} & -\frac{9803}{43200} & 0 & 0 \\
-\frac{523}{3920} & -\frac{2201287}{29635200} & \frac{605939}{1975680} & -\frac{64128769}{24696000} & \frac{3303367}{9878400} & \frac{332422343}{29635200} & \frac{6699071}{14817600} & 0 & 0 & 0 & \frac{37547}{78400} & \frac{75071}{39200} & -\frac{497}{576} & \frac{103}{1440} & 0 \\
-\frac{809}{5600} & -\frac{12166789}{21168000} & \frac{11202299}{7056000} & -\frac{73487}{36750} & -\frac{9182209}{7056000} & \frac{37249}{156800} & \frac{26302879}{2116800} & 0 & 0 & 0 & \frac{1613}{3360} & \frac{17401}{6720} & \frac{19}{225} & \frac{1187}{2880} & 0 \\
-\frac{269}{2520} & \frac{125599}{10584000} & \frac{50369}{1323000} & -\frac{98317}{1176000} & \frac{73489}{392000} & -\frac{8625329}{3528000} & -\frac{97913}{756000} & \frac{90760559}{7408800} & \frac{25354501}{21168000} & \frac{40519}{56448} & \frac{184259}{1058400} & \frac{65297}{23520} & -\frac{420373}{211680} & \frac{248791}{235200} & -\frac{2747}{9408} \\
-\frac{19717}{176400} & \frac{3374557}{7408800} & -\frac{102465523}{74088000} & \frac{5260289}{1764000} & -\frac{6201763}{4939200} & -\frac{115070197}{24696000} & \frac{10687837}{9261000} & \frac{6498287}{9261000} & \frac{1025255701}{74088000} & -\frac{25511}{493920} & \frac{347437}{1764000} & \frac{863371}{302400} & -\frac{230747}{105840} & \frac{938797}{705600} & -\frac{78243}{196000} \\
-\frac{19717}{176400} & -\frac{2733089}{9261000} & \frac{88146899}{74088000} & -\frac{5678651}{3528000} & -\frac{1966229}{12348000} & \frac{17842339}{18522000} & -\frac{6878309}{4630500} & -\frac{58976629}{37044000} & \frac{8569667}{9261000} & \frac{179275483}{12348000} & \frac{28489}{661500} & \frac{54403}{14700} & -\frac{228689}{88200} & \frac{687461}{264600} & -\frac{485507}{5292000} \\
\hline
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{25}{12} & 0 & 0 & 0 & 0 \\
-\frac{19}{36} & \frac{139}{2400} & \frac{499}{800} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{143}{288} & \frac{2195}{288} & 0 & 0 & 0 \\
-\frac{1}{3} & \frac{4}{15} & \frac{121}{400} & \frac{637}{800} & -\frac{211}{800} & 0 & 0 & 0 & 0 & 0 & \frac{119}{120} & -\frac{15643}{7200} & \frac{79313}{7200} & 0 & 0 \\
-\frac{209}{900} & \frac{6299}{21168} & \frac{6767}{35280} & \frac{71063}{88200} & -\frac{34723}{176400} & \frac{25841}{58800} & -\frac{36091}{264600} & 0 & 0 & 0 & \frac{22723}{21600} & -\frac{35}{48} & -\frac{2861}{5400} & \frac{443801}{36000} & 0 \\
-\frac{31}{180} & \frac{13843}{105840} & \frac{8317}{15120} & -\frac{797}{35280} & \frac{5477}{35280} & \frac{2417}{3528} & \frac{611}{105840} & \frac{13975}{14112} & -\frac{5377}{10584} & -\frac{3581}{10080} & \frac{114221}{151200} & \frac{6017}{15120} & \frac{121}{216} & -\frac{3661627}{1411200} & \frac{63879443}{4233600}
\end{array}\right)$$

$$Z^{(2)}_{\mathcal{O}_{16,d}}\Big|_{\frac{1}{\epsilon}-\text{part.}} = \frac{N_c^2}{\epsilon}\begin{pmatrix}
\frac{575}{144} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{23347}{14400} & \frac{46517}{5760} & 0 & 0 & 0 & 0 & \frac{487}{1800} & 0 \\
\frac{3883}{4032} & -\frac{171823}{37800} & \frac{36597791}{3024000} & -\frac{29581}{16800} & 0 & 0 & -\frac{1789}{4800} & 0 \\
-\frac{9271}{11200} & -\frac{35239}{50400} & \frac{74209}{168000} & \frac{188599}{18900} & 0 & 0 & \frac{2101}{4800} & 0 \\
\frac{3287}{84000} & -\frac{2048479}{1176000} & \frac{422283}{392000} & -\frac{2501309}{1764000} & \frac{49211483}{3528000} & \frac{293221}{392000} & \frac{2764807}{2116800} & -\frac{61}{20160} \\
\frac{947587}{1058400} & -\frac{1555357}{705600} & \frac{16831}{29400} & -\frac{239641}{75600} & -\frac{381527}{2116800} & \frac{5839021}{423360} & -\frac{5807}{201600} & \frac{118933}{1411200} \\
\frac{3349}{7200} & -\frac{2591}{2400} & 0 & 0 & 0 & 0 & \frac{150391}{14400} & 0 \\
-\frac{45083}{44100} & \frac{16564}{11025} & \frac{5447}{117600} & \frac{380791}{176400} & \frac{1063}{29400} & -\frac{545189}{352800} & \frac{1176541}{1058400} & \frac{174229}{12600}
\end{pmatrix}$$

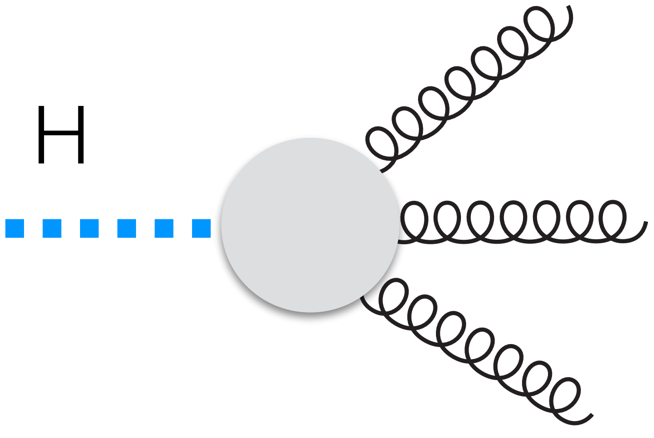
Orders of spurious poles

The coefficients of {T_3, T_2} and log functions are non-trivial rational functions, which they contain spurious poles.

operator	external	u	v, w	$u+v, u+w$
$\mathcal{O}_{\Delta_0,\alpha,f}$	$(-,-,+)$	$\frac{\Delta_0}{2}+2$	$\frac{\Delta_0}{2}+1$	2
	$(-,-,-)$	$\frac{\Delta_0}{2}+1$	$\frac{\Delta_0}{2}+1$	0
$\mathcal{O}_{\Delta_0,\beta,f}$	$(-,-,+)$	$\frac{\Delta_0}{2}+1$	$\frac{\Delta_0}{2}$	2
	$(-,-,-)$	$\frac{\Delta_0}{2}$	$\frac{\Delta_0}{2}$	0
$\mathcal{O}_{\Delta_0,\alpha,d}$	$(-,-,+)$	$\frac{\Delta_0}{2}+1$	$\frac{\Delta_0}{2}$	1
	$(-,-,-)$	$\frac{\Delta_0}{2}-1$	$\frac{\Delta_0}{2}-1$	0
$\mathcal{O}_{\Delta_0,\beta,d}$	$(-,-,+)$	$\frac{\Delta_0}{2}$	$\frac{\Delta_0}{2}-1$	1
	$(-,-,-)$	$\frac{\Delta_0}{2}-2$	$\frac{\Delta_0}{2}-2$	0

Operator in full QCD

Two-loop Higgs amplitudes with dim-7 operators



$$\mathcal{L}_{\text{eff}} = C_0 O_0 + \frac{1}{m_{\text{t}}^2} \sum_{i=1}^{4} C_i O_i + \mathcal{O}\left(\frac{1}{m_{\text{t}}^4}\right)$$

$$O_0 \ = \ H \text{tr}(F^2)$$

$$\boxed{\begin{aligned} O_1 &= H \text{tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu)\,, \\ O_2 &= H \text{tr}(D_\rho F_{\mu\nu} D^\rho F^{\mu\nu})\,, \end{aligned}}$$

for pure YM

$$O_3 = H \text{tr}(D^\rho F_{\rho\mu} D_\sigma F^{\sigma\mu})\,,$$

$$O_4 = H \text{tr}(F_{\mu\rho} D^\rho D_\sigma F^{\sigma\mu})\,.$$

$$D_\rho F^{\rho\mu} = -g \sum_{i=1}^{n_f} \bar{\psi}_i \gamma^\mu \psi_i \quad \longrightarrow$$

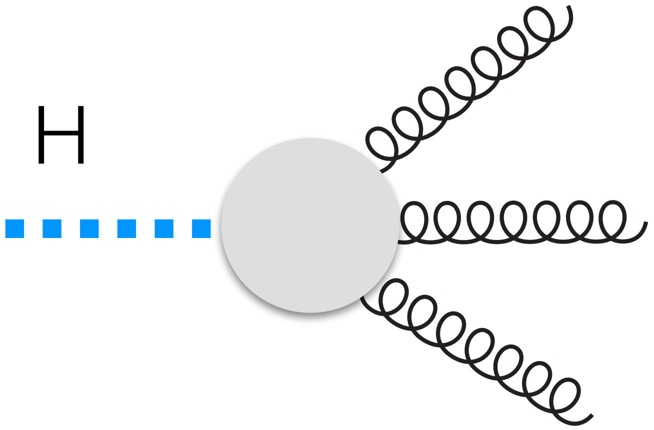
$$\mathcal{O}_3 \ \to \ \mathcal{O}_3' = g^2 \sum_{i,j=1}^{n_f} (\bar{\psi}_i \gamma^\mu \psi_i)(\bar{\psi}_j \gamma_\mu \psi_j)$$

$$\mathcal{O}_4 \ \to \ \mathcal{O}_4' = g F_{\mu\nu} D^\mu \sum_{i,j=1}^{n_f} (\bar{\psi}_i \gamma^\nu T^A \psi_i)$$

Operator in full QCD

Qingjun Jin, GY, 2019

Two-loop Higgs amplitudes with dim-7 operators



$$\mathcal{L}_{\text{eff}} = C_0 O_0 + \frac{1}{m_{\text{t}}^2} \sum_{i=1}^{4} C_i O_i + \mathcal{O}\left(\frac{1}{m_{\text{t}}^4}\right)$$

$$O_0 = H\text{tr}(F^2)$$

$$\boxed{\begin{aligned} O_1 &= H\text{tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu)\,, \\ O_2 &= H\text{tr}(D_\rho F_{\mu\nu} D^\rho F^{\mu\nu})\,, \\ O_3 &= H\text{tr}(D^\rho F_{\rho\mu} D_\sigma F^{\sigma\mu})\,, \\ O_4 &= H\text{tr}(F_{\mu\rho} D^\rho D_\sigma F^{\sigma\mu})\,. \end{aligned}}$$

for full QCD

$$D_\rho F^{\rho\mu} = -g \sum_{i=1}^{n_f} \bar{\psi}_i \gamma^\mu \psi_i \quad \longrightarrow \quad \mathcal{O}_3 \to \mathcal{O}_3' = g^2 \sum_{i,j=1}^{n_f} (\bar{\psi}_i \gamma^\mu \psi_i)(\bar{\psi}_j \gamma_\mu \psi_j)$$

$$\mathcal{O}_4 \to \mathcal{O}_4' = g F_{\mu\nu} D^\mu \sum_{i,j=1}^{n_f} (\bar{\psi}_i \gamma^\nu T^A \psi_i)$$