The muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

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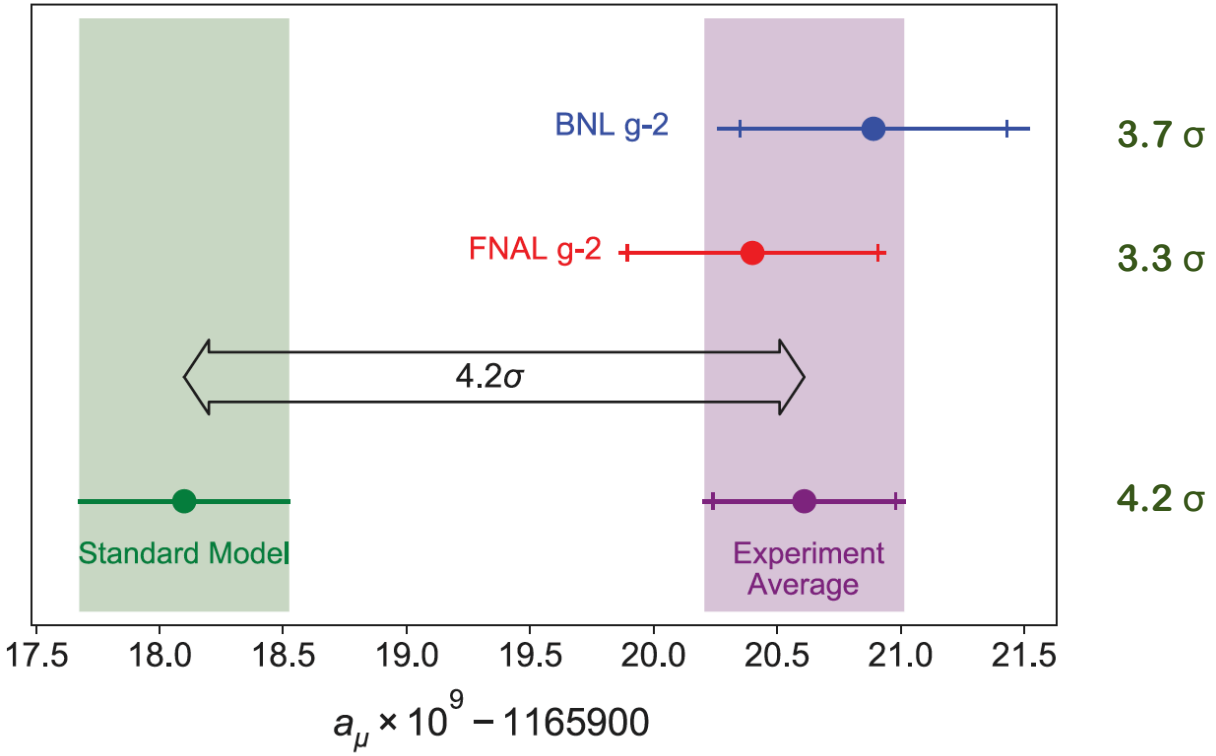
INFN Padova

Radcor & LoopFest 2021
Florida State University (virtual conference)
May 17th 2021

Muon g-2: FNAL confirms BNL



$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

- FNAL aims at 16 x 10^{-11}. First 3 runs completed, 4th in progress.
- Muon g-2 proposal at J-PARC: Phase-1 with ~ BNL precision.

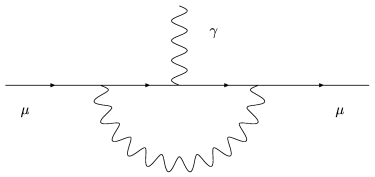
- **Muon g-2: the Standard Model prediction**
- **Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection**
- **The MUonE project**

Muon g-2: the Standard Model prediction

WP20 = White Paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

Muon g-2: the QED contribution

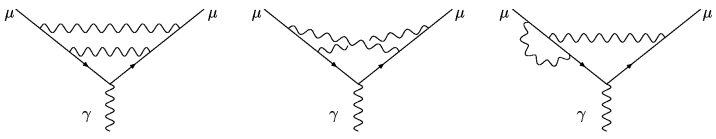
$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948



$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

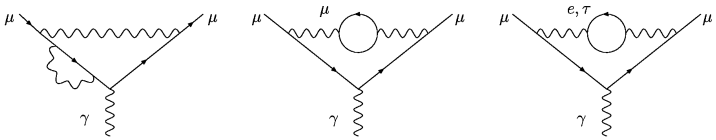
Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$



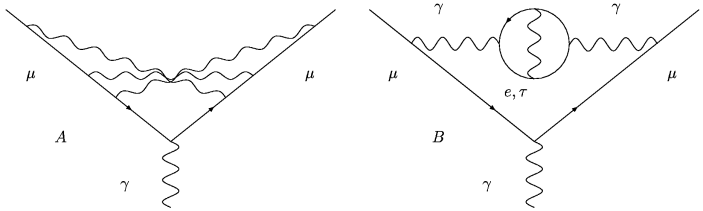
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek '99; MP '04;
Friot, Greynat & de Rafael '05, Ananthanarayan, Friot, Ghosh 2020

$+ \; 130.8780\,(60)\,(\alpha/\pi)^4$

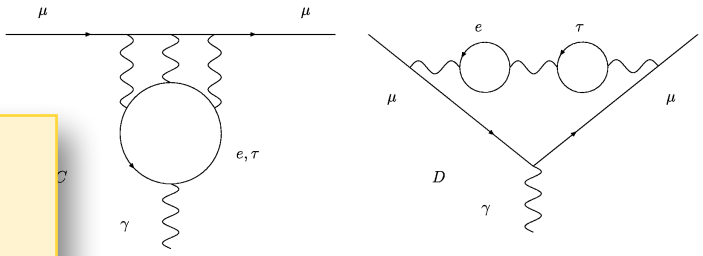


Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term) COMPLETED[2]!

$+ \; 750.86\,(88)\,(\alpha/\pi)^5$ COMPLETED!



Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$[no lept loops] at variance, but negligible $\delta a_\mu \sim 6\times10^{-14}$



Adding up, we get:

$a_\mu^{QED} = 116584718.931\,(19)(100)(23) \times 10^{-11}$

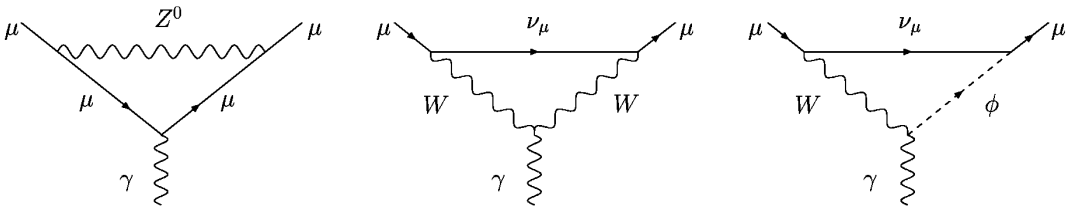
mainly from 4-loop coeff. unc. 6-loop from α(Cs)

α = 1/137.035999046(27) [0.2ppb] Parker et al 2018 WP20 value

Shift down in $a_\mu^{QED} \sim 1.4 \times 10^{-12}$ with new LKB Paris α(Rb) value (Morel et al 2020)

The electroweak contribution

- **One-loop term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

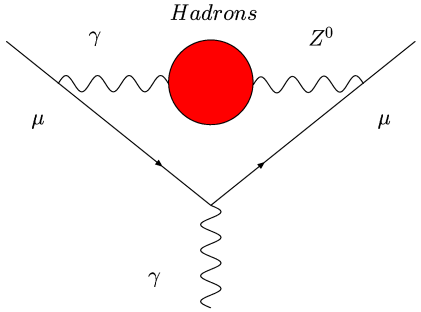
- **One-loop plus higher-order terms:**

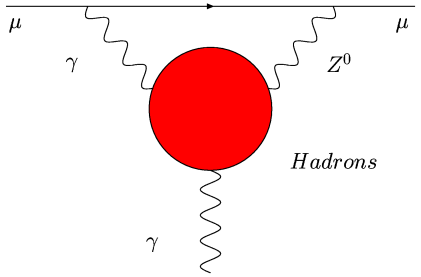
$a_\mu^{\rm EW} = 153.6\ (1.0) \times 10^{-11}$

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.

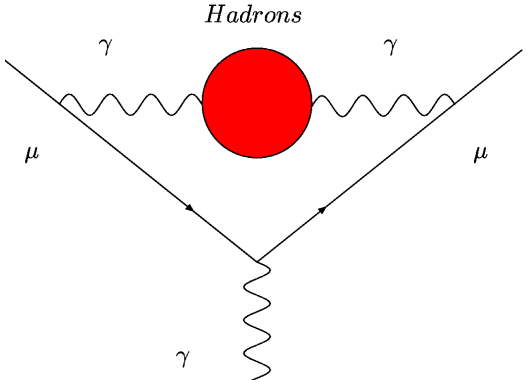
Hadronic loop uncertainties (and 3-loop nonleading logs).

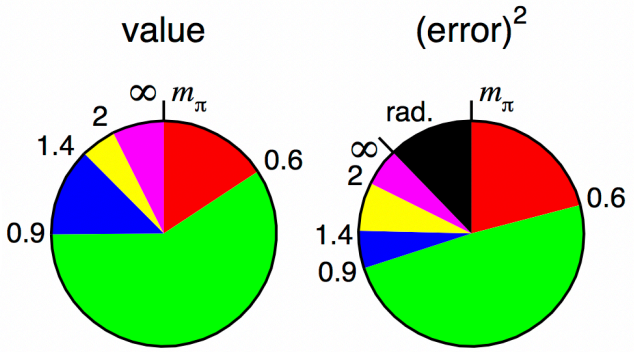
WP20 value





The hadronic LO contribution





Keshavarzi, Nomura, Teubner 2018

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\,\sigma_{\rm had}^{(0)}(s) \qquad K(s) = \int_0^1 dx \frac{x^2\,(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

$a_\mu^{\rm HLO}$ = 6895 (33) x 10^{-11} F. Jegerlehner, arXiv:1711.06089

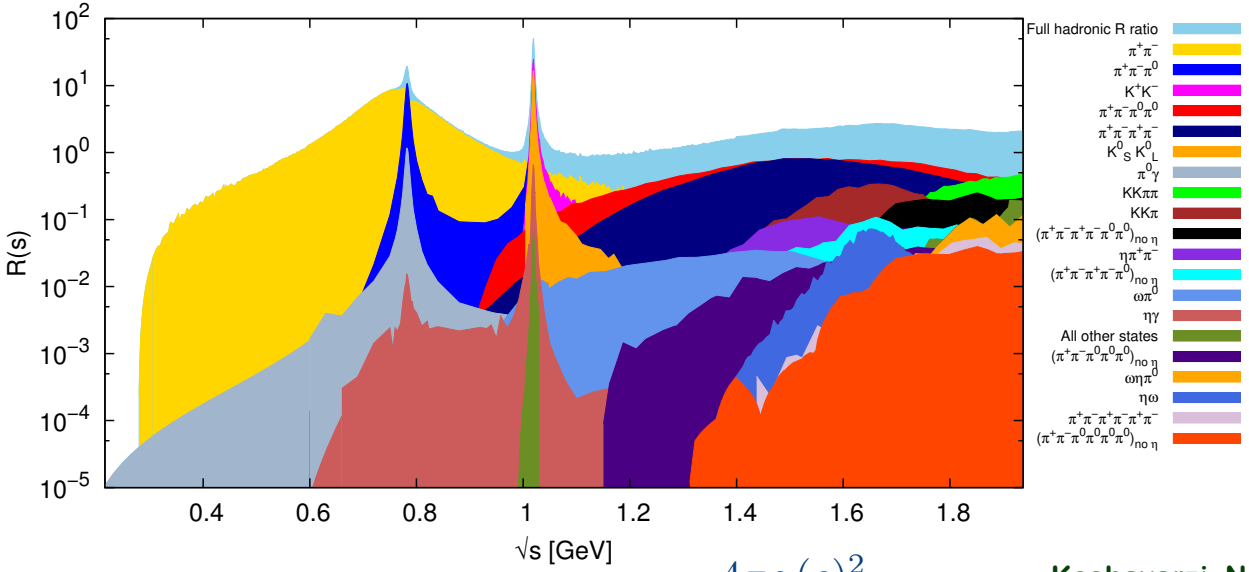
= 6939 (40) x 10^{-11} Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

= 6928 (24) x 10^{-11} Keshavarzi, Nomura, Teubner, arXiv:1911.00367

= 6931 (40) x 10^{-11} (0.6%) WP20 value

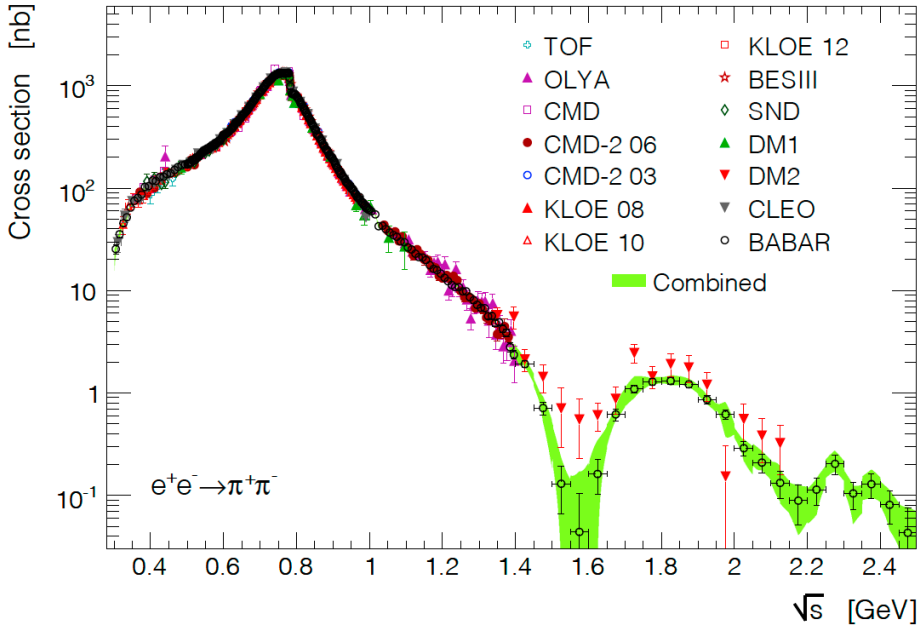
- **WP20 value obtained merging conservatively DHMZ + KNT + constraints from CHHKS**
  Colangelo, Hoferichter, Hoid, Kubis, Stoffer 2018-19
- **Radiative Corrections to σ(s) are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585

The low-energy hadronic cross section



$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

Keshavarzi, Nomura Teubner
PRD 2018



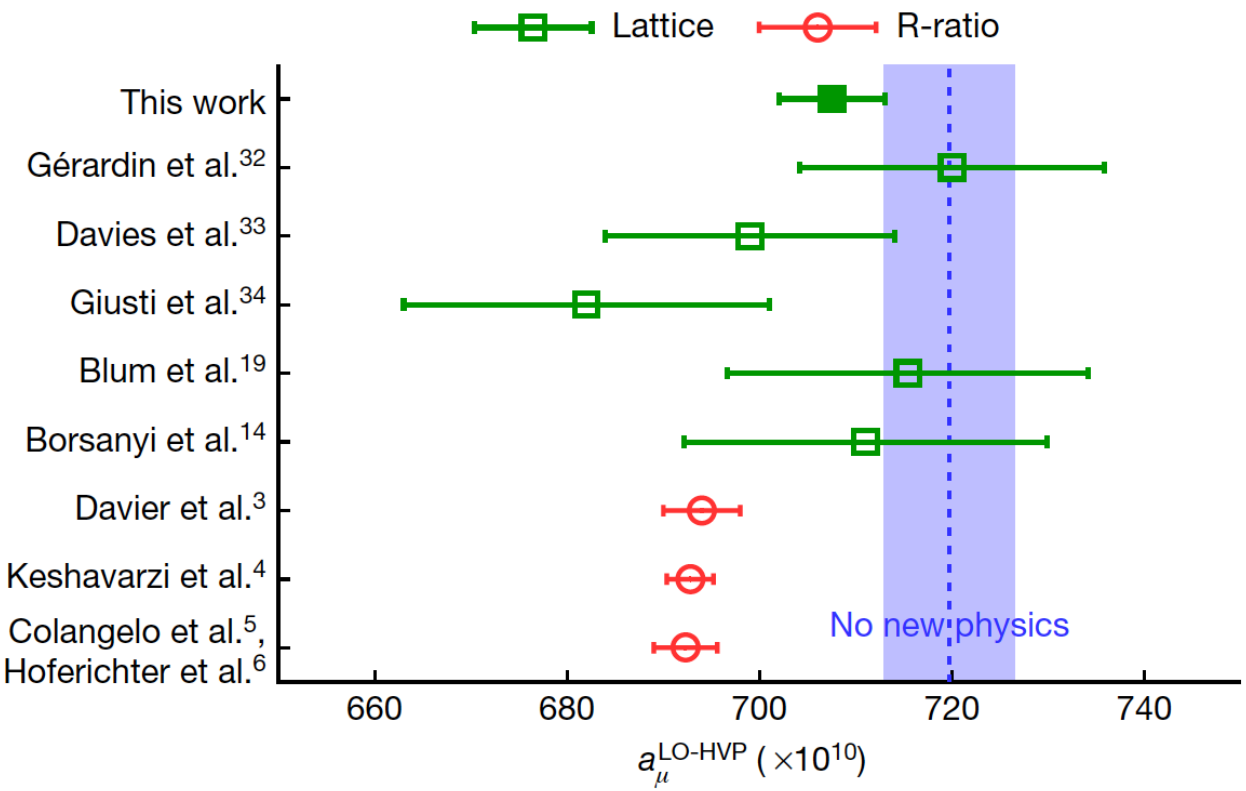
Davier, Hoecker, Malaescu, Zhang
EPJC 2020

The hadronic LO contribution from lattice QCD

- **Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:**

$$a_\mu^{HLO} = 7075(23)_{stat}(50)_{syst}\,[55]_{tot} \times 10^{-11}$$

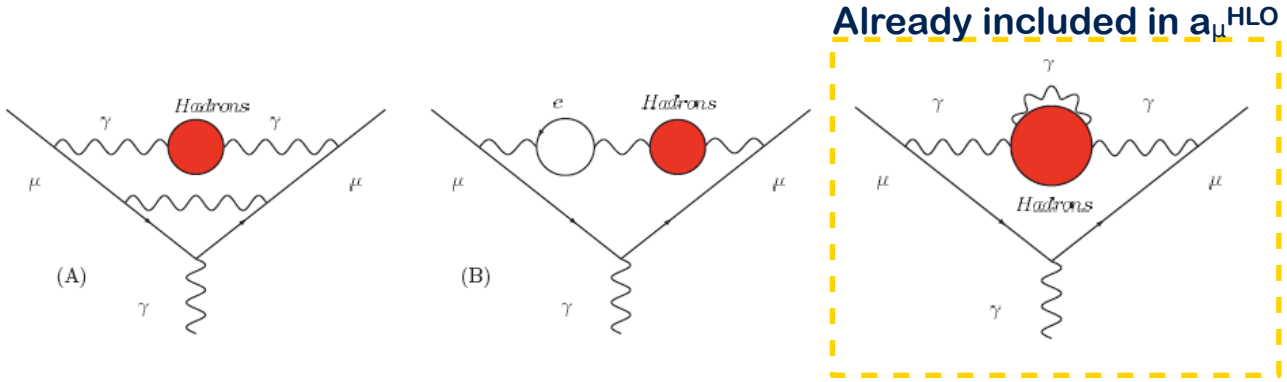
- **2–2.5σ tension with the dispersive evaluations.** BMW collaboration 2021



Borsanyi et al (BMWc), Nature 2021

The hadronic HO VP contribution

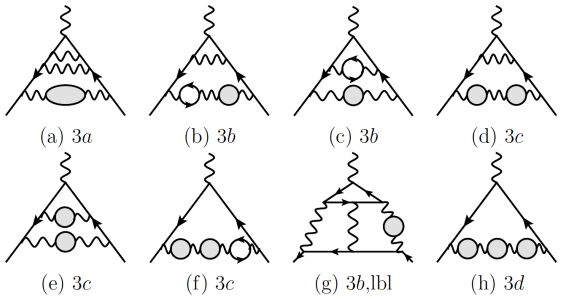
- **O(α³) contributions of diagrams containing HVP insertions:**



$$a_\mu^{HNLO}(vp) = -98.3\ (7) \times 10^{-11}$$

Krause '96; Keshavarzi, Nomura, Teubner 2019; WP20.

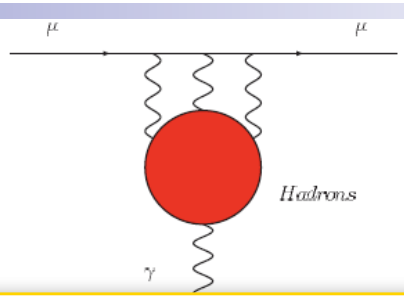
- **O(α⁴) contributions of diagrams containing HVP insertions:**



$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

- **Hadronic light-by-light at O(α^3)**



  - **This term had a troubled life! But nowadays:**

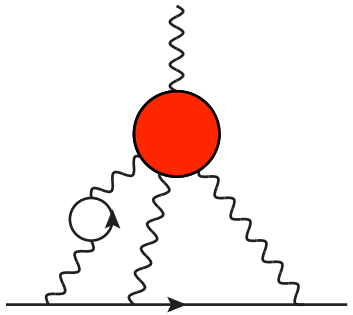
$a_\mu^{HNLO}(lbl) =$	80 (40) x 10^{-11}	Knecht & Nyffeler '02
=	136 (25) x 10^{-11}	Melnikov & Vainshtein '03
=	105 (26) x 10^{-11}	Prades, de Rafael, Vainshtein '09
=	100 (29) x 10^{-11}	Jegerlehner, arXiv:1705.00263
=	92 (19) x 10^{-11}	WP20 (phenomenology)

  - **Significant improvements due to data-driven dispersive approach.**
    Colangelo, Hoferichter, Procura, Stoffer, 2014–17; Pauk, Vanderhaeghen 2014.
  - **Lattice: RBC: 82(35)x10^{-11}** 1911.08123 **Mainz: 110(15)x10^{-11}** 2104.02632

- **Hadronic light-by-light at O(α^4)**

$a_\mu^{HNNLO}(lbl) = 2\ (1) \times 10^{-11}$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; WP20



Muon g-2: the discrepancy

Comparing the SM prediction with the measured muon g-2 value:

$a_\mu^{EXP} = 116592061\ (41) \times 10^{-11}$ — BNL+FNAL

$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$ — WP20

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 251\ (59) \times 10^{-11}$ — 4.2 σ

Using the BMW 2021 HLO lattice result instead of the WP20 one, EXP and SM differ only by 1.6σ

Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

Marciano, MP, Sirlin 2008 & 2010

Keshavarzi, Marciano, MP, Sirlin 2020

- Can Δa_μ be due to missing contributions in the hadronic σ(s)?
- An upward shift of σ(s) also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

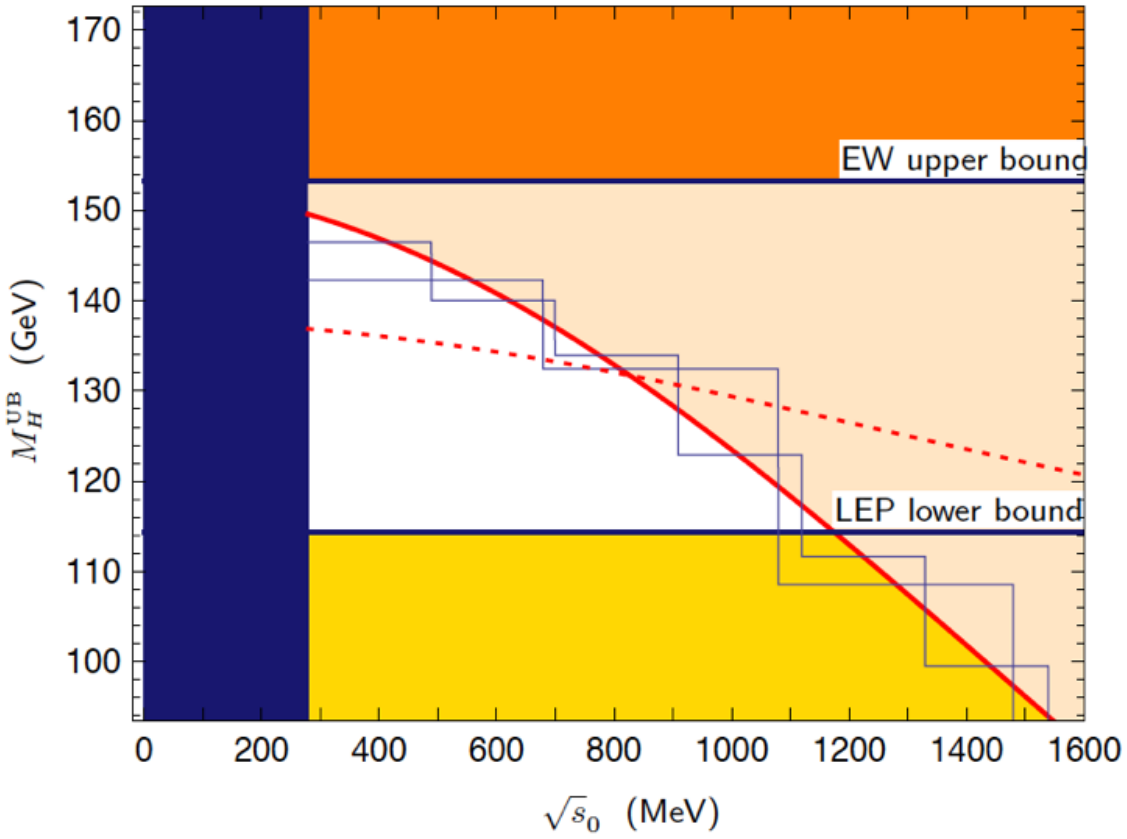
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

ε>0, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

Muon g-2: connection with the SM Higgs mass (2008)

How much does the M_H upper bound from the EW fit change when we shift up σ(s) by Δσ(s) [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to fix Δa_μ ?



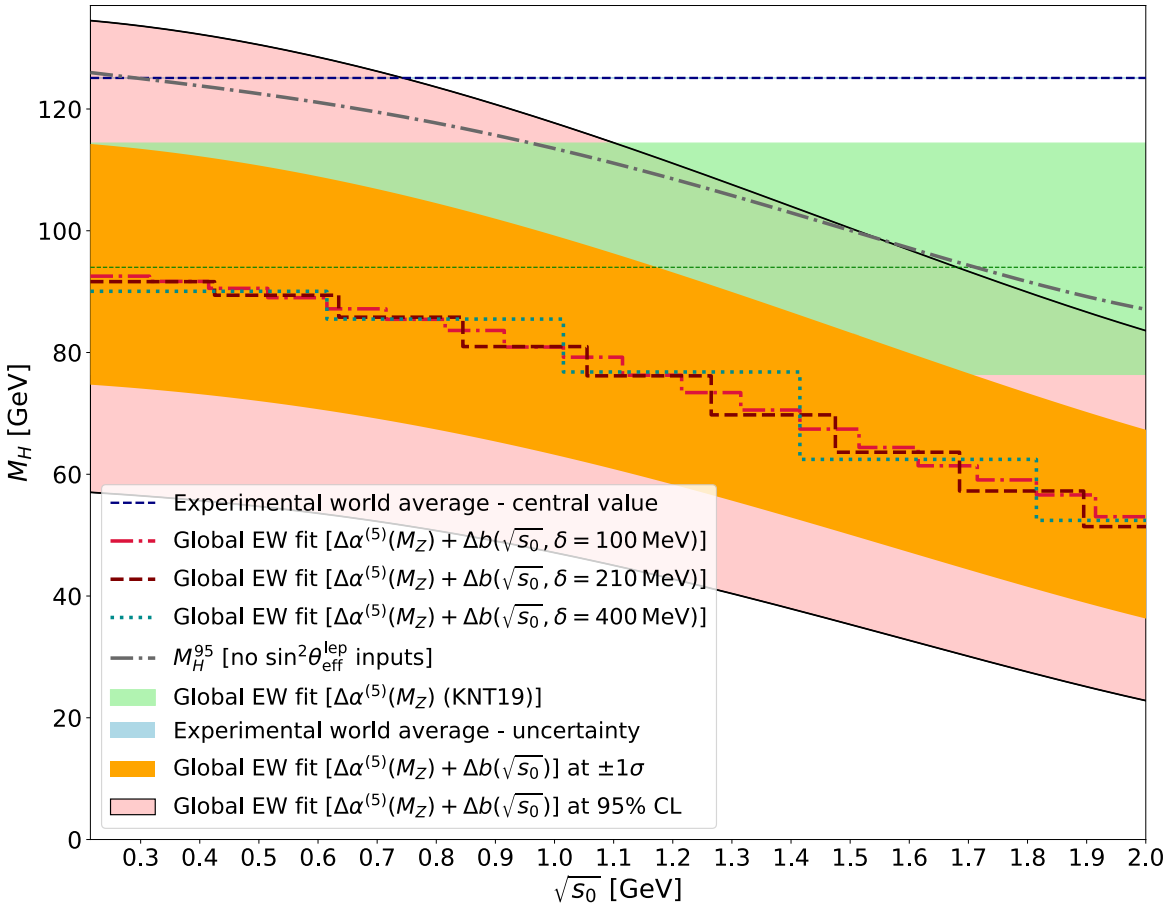
Marciano, MP, Sirlin, PRD 2008

Major update: Higgs discovered, improved EW observables (M_W, $\sin^2\theta$, M_{top}, …), updates to σ(s), theory improvements, global fit, …

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	[5]	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	[5]	173.0(4)	···
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	···
M_Z (GeV)	91.1876(21)	[5]	91.1883(20)	···
Γ_Z (GeV)	2.4952(23)	[5]	2.4940(4)	···
Γ_W (GeV)	2.085(42)	[5]	2.0903(4)	···
$\sigma^0_{\rm had}$ (nb)	41.541(37)	[108]	41.490(4)	···
R^0_l	20.767(25)	[108]	20.732(4)	···
R^0_c	0.1721(30)	[108]	0.17222(8)	···
R^0_b	0.21629(66)	[108]	0.21581(8)	···
$\bar{m}_c$ (GeV)	1.27(2)	[5]	1.27(2)	···
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	···
$A^{0,l}_{\rm FB}$	0.0171(10)	[108]	0.01622(7)	···
$A^{0,c}_{\rm FB}$	0.0707(35)	[108]	0.0737(2)	···
$A^{0,b}_{\rm FB}$	0.0992(16)	[108]	0.1031(2)	···
A_ℓ	0.1499(18)	[75,108]	0.1471(3)	···
A_c	0.670(27)	[108]	0.6679(2)	···
A_b	0.923(20)	[108]	0.93462(7)	···
$\sin^2\theta^{\rm lep}_{\rm eff}(Q_{\rm FB})$	0.2324(12)	[108]	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\rm lep}_{\rm eff}$(Had Coll)	0.23140(23)	[100]	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (using Gfitter)

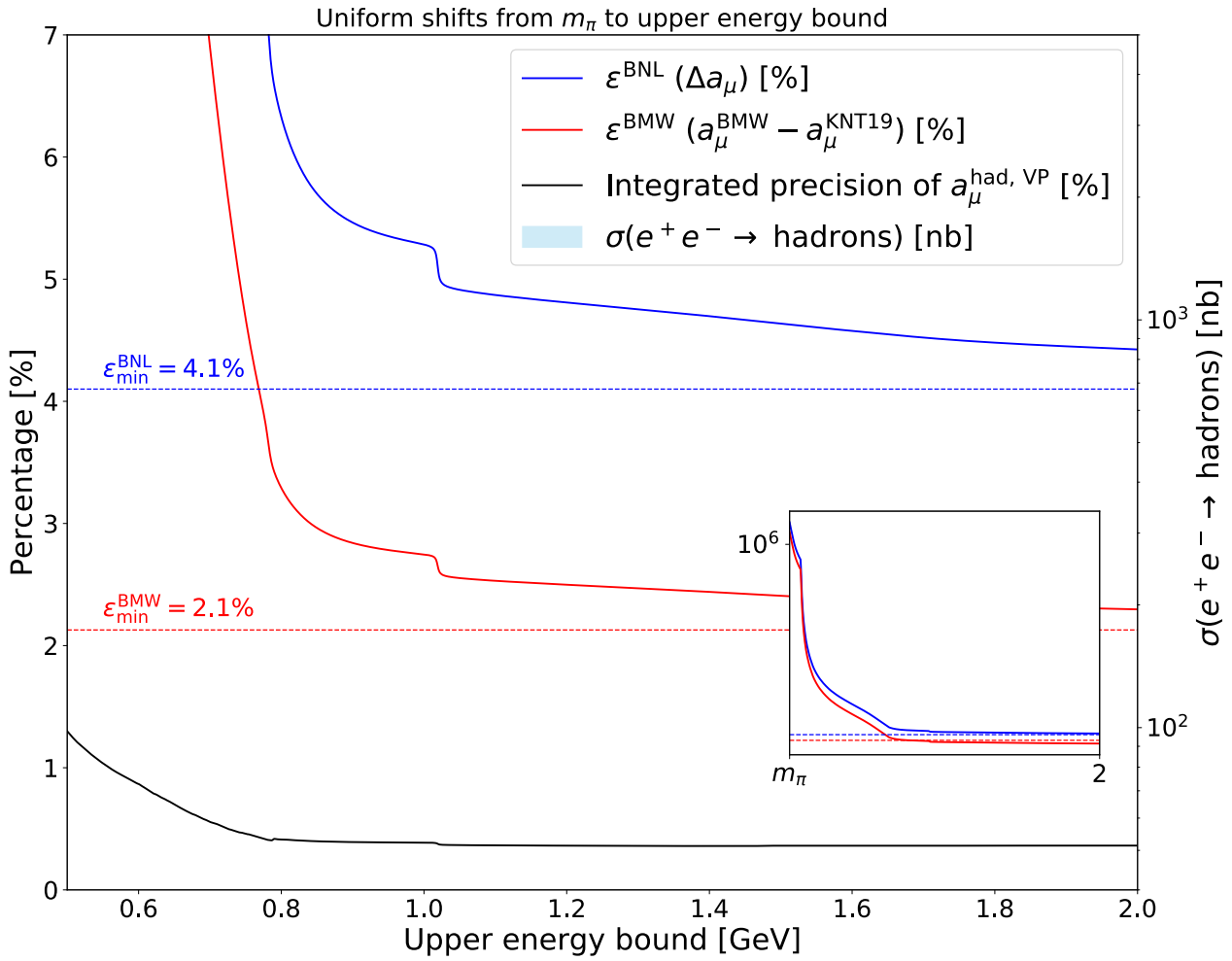
Muon g-2: connection with the SM Higgs mass (2020)



Shifts Δσ(s) to fix Δa_μ are possible,
but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts Δσ(s) ?



Shifts below ~1 GeV conflict with the quoted exp. precision of σ(s)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)

- Crivellin, Hoferichter, Manzari and Montull, "Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits," arXiv:2003.04886.

- Eduardo de Rafael, "On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,'' arXiv:2006.13880.

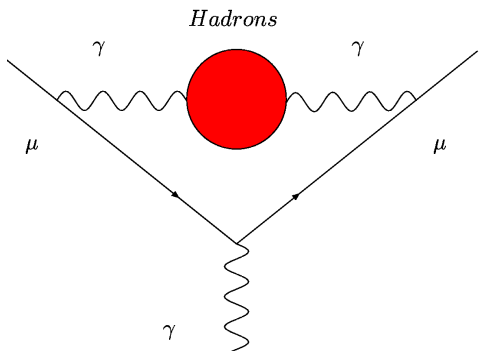
- Malaescu and Schott, "Impact of correlations between a_μ and α_{QED} on the EW fit," arXiv:2008.08107.

- Colangelo, Hoferichter and Stoffer, "Constraints on the two-pion contribution to hadronic vacuum polarization," arXiv:2010.07943.

The MUonE project

Spacelike proposal for a_μ^{HLO}

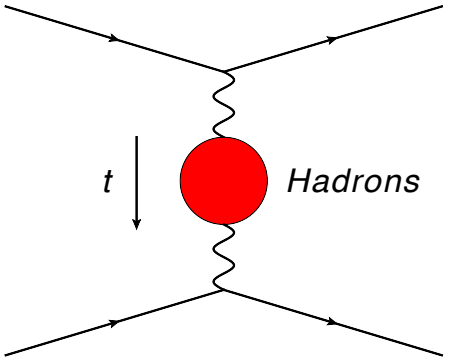
- The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\,\sigma_{\rm had}^{(0)}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,\left(s/m_\mu^2\right)}$$

- Alternatively, simply exchanging the x and s integrations:



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

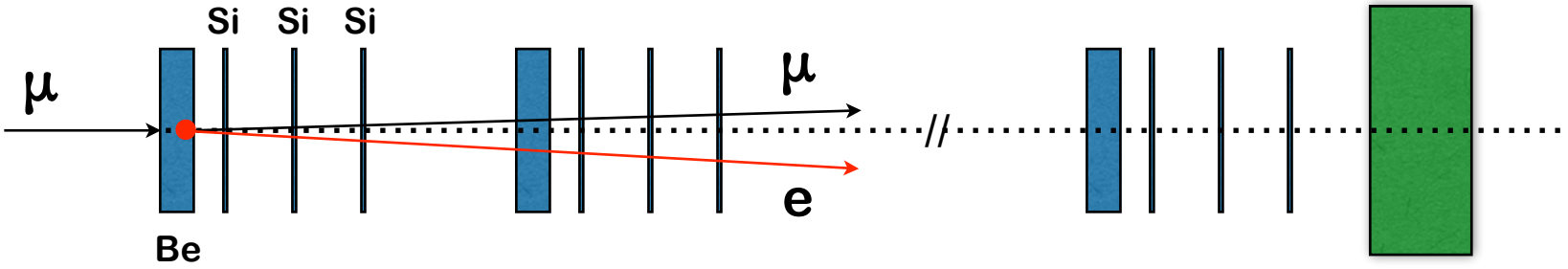
Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.

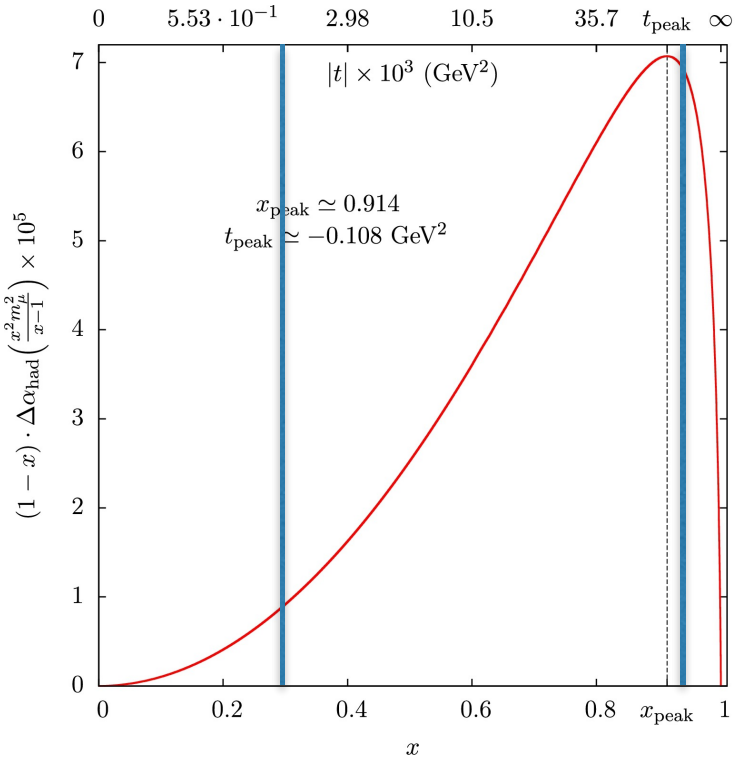


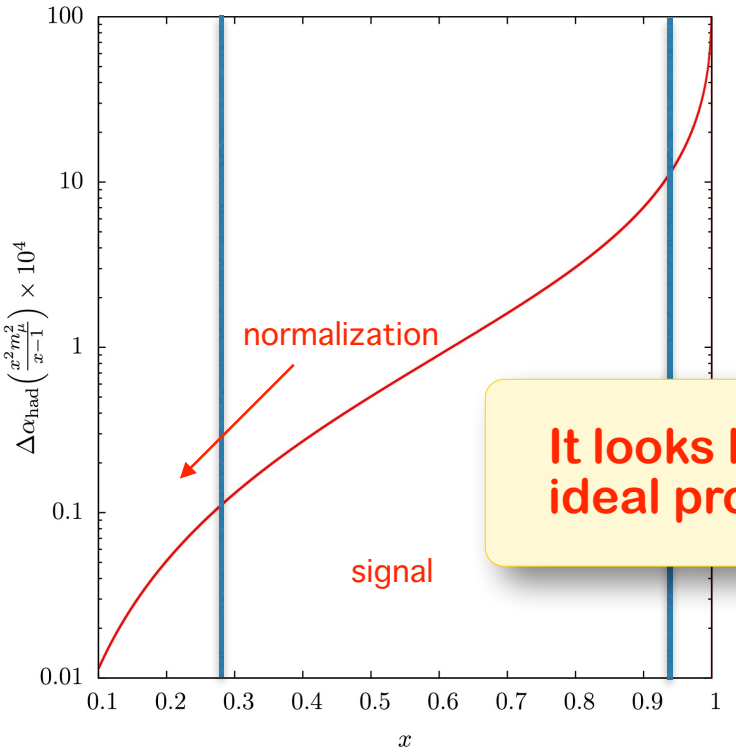
Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni

EPJC 2017 - arXiv:1609.08987

MUonE: Muon-electron scattering @ CERN (II)

- For a 150 GeV muon beam ($\sqrt{s}$~400 MeV), MUonE's scan region extends up to x=0.932, ie beyond the peak! (the peak is at x=0.914)
- The high-energy region inaccessible to MUonE contributes only 13% of a_μ^{HLO} integral. It can be determined with timelike data and/or lattice QCD results. Already obtained via lattice QCD! Giusti&Simula and Marinkovic´&Cardoso 2019





It looks like an ideal process!

MUonE — Status

- **Statistics**: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total thickness 60cm), 2-3 years of data taking (2×10^7 s/yr) → $\mathscr{L}_{\rm int} \sim 1.5 \times 10^7\,{\rm nb}^{-1}$.

- With this $\mathscr{L}_{\rm int}$ we estimate that measuring the shape of dσ/dt we can reach a statistical sensitivity of ~0.3% on $a_\mu^{\rm HLO}$, ie $\sim 20 \times 10^{-11}$.

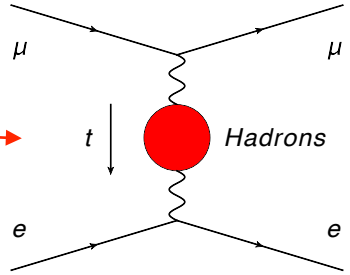
- **Systematic** effects must be known at ≲ 10ppm!

- Test beams performed at CERN in 2017 & 2018 arXiv:1905.11677, 2102.11111

- LoI submitted to CERN SPSC in 2019: Test run approved for 2021.

- Full-statistics run hopefully in 2022–24.

- To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at ≲ 10ppm!



- Fully differential fixed-order MC @ NLO ready Pavia and PSI 2018-19
- NNLO QED: Master Integrals for 2-loop box diagrams computed. Full 2-loop amplitude close to completion. Padova 2017 - present
- Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation Pavia and PSI 2020
- NNLO hadronic effects computed Padova and KIT 2019
- Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes PSI 2019-present
- …

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

MUonE — Theory workshops

Muon-electron scattering: Theory kickoff workshop

4-5 September 2017

Padova

Europe/Rome timezone

- **Overview**
- Venue
- Timetable
- Logistic
- Map

Support

MUonE theory workshops: Padova 2017, Mainz 2018, Zurich 2019

Next MUonE theory workshop: MITP Mainz 2020-21 postponed to 2022

Conclusions

- Fermilab's Muon g-2 experiment confirms BNL's result: the discrepancy between experiment and SM increases to 4.2σ.
- The BMWc lattice QCD result weakens the exp-SM discrepancy. It must be confirmed or refuted by other lattice calculations.
- Is Δa_μ due to missed contributions in the hadronic cross section? Shifts above 1 GeV to fix Δa_μ conflict with the electroweak fit.
- Leading hadronic contribution to a_μ: dispersive vs lattice. MUonE will provide a new independent & alternative determination.