Field Theory of the "World Line"

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Goal:
- scan space of BV-field theories
- gain intuition on Background indep.

Plan:
1) Framework
2) Example(s)

Motivation:

BV SFT S.T.

$$\left(\underset{\text{fields}}{\mathcal{F}},\ \underset{\text{BV-op.}}{\Delta},\ \underset{\text{BV-action}}{S}\right) \rightsquigarrow \underset{\text{homol. v.f.}}{Q} \xrightarrow[\text{crit. point}]{\text{exp'n around}} \{Q^{(k)}\} \xrightarrow{\text{Feynman}} \{f^{(k)}\}$$

1) Construction of Q:

$$(\mathcal{F}, \Delta, S) \rightsquigarrow Q \rightsquigarrow \{Q^{(k)}\} \rightsquigarrow \{s^{(k)}\}$$

canonical construction of Q for given $\mathcal{A}$:

BRST algebra $\mathcal{A}$

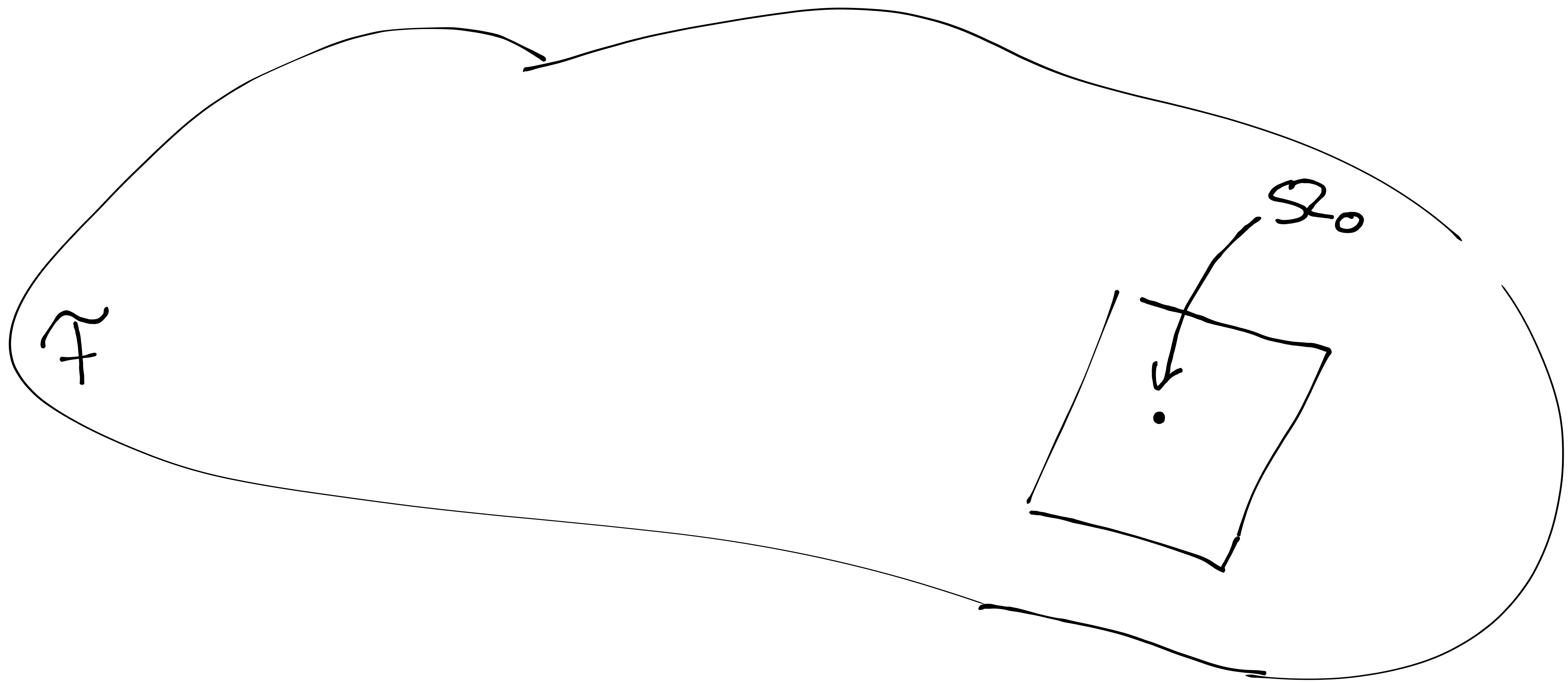
$\{P, X, b, c, \beta, \gamma \ldots\}$

coefficients

basis

$$\underset{(1)}{\Omega} = \underset{(1-k)}{\psi^A}\ \underset{(k)}{E_A} \quad ; \quad Q(\Omega) = \frac{1}{2}[\Omega, \Omega]$$

Chevalley-Eilenberg



critical points: $Q(\Omega) = \frac{1}{2}[\Omega, \Omega] = 0$

$\rightsquigarrow$ gauge system with $\delta\Omega = [\Omega, \Theta]$

$\therefore$ background independent within $D(\mathcal{F})$

What kind of gauge theory?

Choose a parametrisation:

$$\Omega = \psi^A E_A = \psi^a(x)\, e_a$$

generating function for background fields

$$Q(\Omega) = \frac{1}{2}[\Omega,\Omega] \overset{!}{=} 0 \implies$$ equations of motion

$\therefore$ no action

$\therefore$ no notion of space of states

2) Examples:

$\mathcal{A}$ from the spinning world line.

Rep:

$\mathcal{N} = 2 \rightsquigarrow$ BV-form'n of Yang-Mills

P. Dai, Y.-t. Huang and W. Siegel, '08

Proca field w/ M.Carosi, to appear

- SUSY Yang-Mills was obtained long before within the pure spinor formalism by Berkovits '01.

- $\mathcal{N} = 4$: $R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi - H_{\mu\lambda\sigma}\, H_\nu{}^{\lambda\sigma} = 0\,, \quad \nabla^\lambda H_{\lambda\mu\nu} - 2\, H_{\mu\nu\lambda}\nabla^\lambda\Phi = 0\,.$

- cf. pure spinor

$$\mathcal{A}^{\mathcal{N}=2} = \{ x^\mu, p_\mu, \theta^\mu, \bar\theta_\mu, b, c, \gamma, \bar\gamma, \beta, \bar\beta \}$$

$$[x^a, p_b] = i\delta^a_b\,, \qquad [c, b] = 1\,, \qquad [\theta^a, \bar\theta_b] = \delta^a_b\,, \quad [\gamma, \bar\beta] = 1\,, \quad [\bar\gamma, \beta] = 1$$

$$Q(\Omega) = \frac{1}{2}[\Omega, \Omega] \overset{!}{=} 0 \implies$$

Dilaton

$$\Omega = -c\,(p^2 - \partial^2\varphi + (\partial\varphi)^2 + 2\theta^\mu\bar\theta^\nu\,\partial_\mu\partial_\nu\varphi) + \gamma\bar q + \bar\gamma q + \bar\gamma\gamma\, b$$

$$+\,\Omega(\Psi_{(s)}),\ s \geq 2 \qquad q := \theta^\mu(p_\mu - i\partial_\mu\varphi)\,, \quad \bar q := \bar\theta^\mu(p_\mu + i\partial_\mu\varphi)\,.$$

$\therefore$ no Yang-Mills, but gravity (and dilaton) off shell!

$\therefore$ $\mathcal{N}=4$: $\quad A_\mu = 0\,,\ g_{\mu\nu} = \eta_{\mu\nu}\,,\ \partial_\mu\varphi = 0\,!$

- Filtration : $\mathcal{A}_{-1} \subset \mathcal{A}_0 \subset \mathcal{A}_1 \subset \cdots \subset \mathcal{A}$

(A_μ, φ) dimension $g_{\mu\nu}$

$$\boxed{[\mathcal{A}_r, \mathcal{A}_s] \subset \mathcal{A}_{r+s-1}}$$

- <u>Reduction of $\mathcal{A}$</u> :

If $\mathcal{H}$ module of $\mathcal{A}$ and $\mathcal{H}_0 \subset \mathcal{H}$. Then

$$\mathcal{A}'' = \{a \in \mathcal{A} : a\mathcal{H}_0 \subset \mathcal{H}_0\} \quad , \quad \mathcal{I} = \{a \in \mathcal{A}'' : a\mathcal{H}_0 = 0\}$$

$$\boxed{\mathcal{A}' = \mathcal{A}''/\mathcal{I}}$$

Here: $\mathcal{H}_0$ is eigenspace of $N = N_\theta + N_\gamma + N_\beta \overset{!}{=} 1$

"$\sim$" LEVEL TRUNCATION

generic deformation of Ω in $\mathcal{A}_0'$:

$$\Omega^{ext} = c(B^a(x)p_a + \mathbf{F}^{ab}(x)\theta_a\bar{\theta}_b + D(x) + \mathcal{E}(x)(\beta\bar{\gamma} + \gamma\bar{\beta})) + \gamma(\mathbf{A}^a(x)\bar{\theta}_a) + \bar{\gamma}(\bar{\mathbf{A}}^a(x)\theta_a)\,.$$

generic deformation of Ω in $\mathcal{A}'_0$:

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$= B(A,\varphi)$ $\quad = F(A,\varphi)$ $\quad = D(A,\varphi)$ $\quad = 0$ $\quad = A_a + i\partial_a\varphi$ $\quad = A_a - i\partial_a\varphi$

$$\boxed{Q(\Omega) \overset{!}{=} 0}$$

$$\partial_a F^{ab} = 2F^{ab}\partial_a\varphi\,. \qquad \text{for} \qquad F_{ab} = \partial_a A_b - \partial_b A_a\,.$$

- This is the complete classification in $\mathcal{A}'_0$
- Has a non-abelian generalisation by extending the filtration to $\mathcal{U}^{\sigma}$
- $N=4$: $\begin{cases} \varphi \text{ is determined by the metric} \in \mathcal{A}'_1 : R_{\mu\nu} - \Lambda g_{\mu\nu} + 2\nabla_\mu\nabla_\nu\varphi = 0\,, \\ A_\mu \equiv 0 \end{cases}$

- Interpretation of $\mathcal{A}'$:

  $\mathcal{A} \to \mathcal{A}'$ reduction means that we impose only that "particles" of spin and form degree ≤ 1 need to consistently defined on a classical solution (c.f. strings).

3) Action? given a non-degenerate, invariant trace on $\mathcal{A}$ of deg -3, then

$$S = \mathrm{tr}(\Omega, [\Omega, \Omega]) \text{ is gauge inv. and}$$

reproduces $[\Omega, \Omega] = 0$ as an equation of motion.

Unfortunately, we were able to find an inv. $\langle\ \rangle$ only for $a = \Omega_0$:

$$\Rightarrow \quad S^{(2)} = \langle \Omega [\Omega_0, \Omega] \rangle \text{ is inv.}$$

quadratic action.

but $\quad S = \langle \Omega [\Omega, \Omega] \rangle$ is not.

- Alternative:

$$|\phi\rangle = \mu(\Omega) \quad \text{wave function}$$

$$S = \langle \phi | \Omega_0 | \phi \rangle + \langle \phi | \left(|\phi\rangle \circ |\phi\rangle \right)$$

however, unlike, in string theory and pure spinor particle (Berkovits), Ω_0 is not a derivation of the natural product: •

Outlook:

- complete classification for $N = 4$.
- $N > 4 \longleftrightarrow$ higher spin backgrounds.
- Ramond-Ramond fields.
- Ext'n to Ambitwistor string.

Thank you!