Closed string deformations in Open String Field Theory

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+*work in progress*

with ***Jakub Vosmera (ETH)***

Open/Closed Strings and String Field Theory

- Interplay between open and closed strings is at the hearth of String Theory (Geometric Transitions, AdS/CFT, D-instantons, etc…)

- These interesting phenomena are beyond usual perturbation theory. But the whole non-perturbative physics should be in principle captured by the (open-closed) SFT path integral.

- Open-Closed String Field Theory is computationally hard (complete form? analytic techniques? classical solutions?)

- But there is a limit of O-C SFT which is fairly tractable: Witten-type Open String Field Theory deformed by the Shapiro-Thorn open-closed coupling (*limit of Zwiebach's interpolating O-C SFT's*)

- I will focus on this limit and see how far we can get.

- When problems arise I will try to understand them by recasting the deformed open string theory inside a fully regular open-closed string field theory.

Bosonic string

Bosonic Open String Field Theory with a deformation

- Deform Witten theory with the Shapiro-Thorn open-closed coupling

$$S^{(\mu)}[\Psi] = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{1}{3}\langle \Psi, \Psi^2 \rangle + \mu \langle \Psi, e \rangle$$

- The deformation is gauge invariant if $Qe = 0$ and $[e, \Psi] = 0$. This is realised by taking

$$e = \frac{1}{2\pi i} c(i)c(-i)V^{\text{matter}}_{(1,1)}(i, -i)\, I$$

- EOMS are deformed and $\Psi = 0$ is not a solution anymore (classical tadpole)

$$Q\Psi + \Psi^2 = -\mu e$$

- A classical solution Ψ_* (D-brane system) of the $\mu = 0$ theory will have to be deformed to solve the tadpole-sourced eom.

$$\Psi_* \to \Psi_*^{(\mu)}$$

- But it is not guaranteed that the deformed solution exists: the D-brane system could not survive the change in the closed string background!

Obstruction to vacuum shift and bulk-boundary OPE

- Assume the vacuum shift solution can be perturbatively expanded in μ

$$\Psi_\mu = \sum_{k=0}^{\infty} \mu^k \psi_k$$

- A deformation of the initial D-brane system will have $\psi_0 = 0$ and the first non trivial equation is

$$Q\psi_1 = -e \qquad\qquad e = \frac{1}{2\pi i} c(i)c(-i)V^{\text{matter}}_{(1,1)}(i,-i)\, I$$

- The "Ellwood state" needs to be BRST exact! But note that $c\bar{c}V(z,\bar{z})$ is not trivial in general. This is a requirement on the bulk-boundary OPE

$$V(z,\bar{z}) = M_{ij} V^i(z)\bar{V}^j(\bar{z}) \quad \rightarrow \quad M_{ij}\, {}^{(b)}\Omega^j{}_k\, V^i(z)V^k(z^*)$$

- We get problems (obstruction) if the chiral OPE $V^i(z)V^k(z^*)$ contains a weight-one field

$$V^i_1(z)V^k_1(z^*) = \cdots + \frac{1}{z-z^*} C^{ik}_p\, V^p_1(0) + \cdots$$

- If this happens e will contain a term in the gh=2 cohomology $c\partial c V_1$ and the equation cannot be solved.
- See related BCFT analysis by *Fredenhagen-Gaberdiel-Keller (2006)*

Vacuum shift solution

- Using the standard Siegel gauge propagator $h = \dfrac{b_0}{L_0}(1 - P_0)$ s.t. $[Q, h] = 1 - P_0$

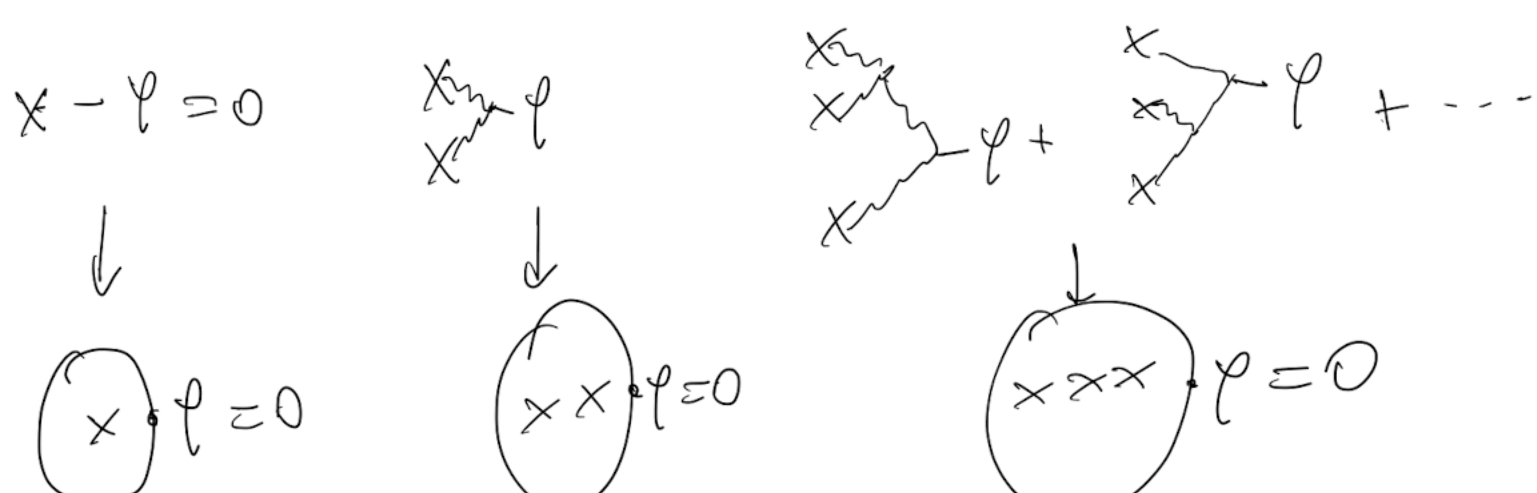
$$\psi_1 = -\,he, \quad \psi_2 = h(he)^2, \quad \psi_3 = -\,h[he, h(he)^2], \cdots$$

$$\Psi^{(\mu)} = \pi_1 \frac{\mathbf{1}}{\mathbf{1 + h(m_2 + \mu e)}} 1_{TH} = \sum_{k=1}^{\infty} \mu^k \psi_k$$

- This state solves the tadpole-sourced eom provided

$$P_0 e = 0, \quad P_0(he)^2 = 0, \quad P_0[he, h(he)^2] = 0, \cdots$$

- P_0 projects into the kernel of L_0 which contains the cohomology: ***Amplitudes involving one physical open string and arbitrary number of deforming closed strings should vanish***



Effective field theory description

- Integrate out (classically) the $L_0 \neq 0$ sector (at zero momentum: massive states). Homotopy transfer, effective products (tadpole+ m1+m2+m3+…).

$$S_{\rm eff}^{(\mu)}[\varphi] = S^{(\mu)}[\Psi(0)] + \sum_{k=0}^{\infty}\sum_{\alpha=0}^{\infty} \frac{\mu^\alpha}{k+1}\omega\left(\varphi, n_{k\alpha}\left(\varphi^{\otimes k}\right)\right)$$

Erbin, CM, Schnabl, Vosmera 2020
Koyama, Okawa, Suzuki

$$n_{01} = P_0 e\,,$$

$$n_{k\alpha}(\varphi^{\otimes k}) = \sum_{\substack{l_1,\ldots,l_\alpha \geq 0 \\ \sum_{i=1}^{\alpha+1} l_i = k}} (-1)^\alpha \tilde{m}_{k+\alpha}(\varphi^{\otimes l_1}, h_0 e, \varphi^{\otimes l_2}, h_0 e, \ldots, \varphi^{\otimes l_\alpha}, h_0 e, \varphi^{\otimes l_{\alpha+1}})$$

$$\begin{aligned}
\tilde{m}_2(\varphi_1,\varphi_2) &= P_0 m_2(\varphi_1,\varphi_2) \\
\tilde{m}_3(\varphi_1,\varphi_2,\varphi_3) &= -P_0\left[m_2\left(hm_2(\varphi_1,\varphi_2),\varphi_3\right) + m_2\left(\varphi_1, hm_2(\varphi_2,\varphi_3)\right)\right] \\
&\vdots\,,
\end{aligned}$$

- The previous (vanishing) obstructions are saying that the effective tadpole vanishes.

- Assuming the effective tadpole vanishes, $\Psi(0)$ is the vacuum shift solution. A cosmological constant is generated by the shift.

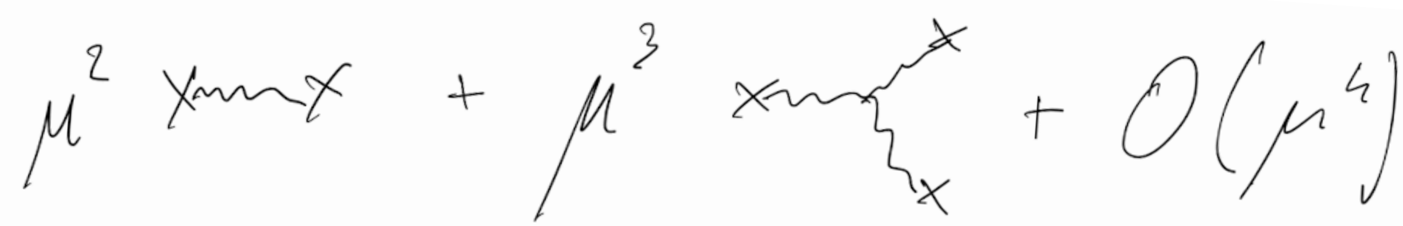
Shifted Theory

- After tadpole removal

$$S^{(\mu)}[\Psi_\mu + \psi] = S^{(\mu)}[\Psi_\mu] + \frac{1}{2}\langle\, \psi, Q_\mu \psi \,\rangle + \frac{1}{3}\langle\, \psi, \psi^2 \,\rangle$$

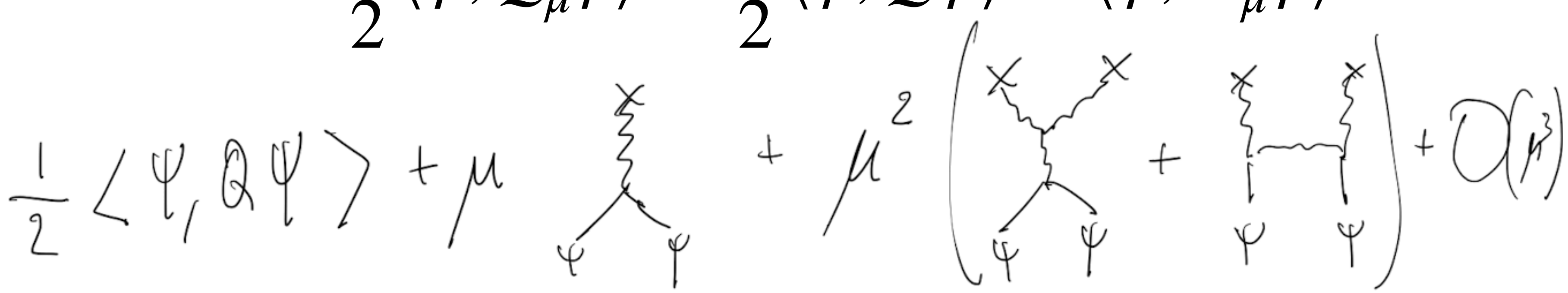
- Non-dynamical cosmological constant (closed string scattering off the disk)

$$S^{(\mu)}[\Psi_\mu] = \frac{\mu}{2}\langle\, e, \Psi_\mu \,\rangle - \frac{1}{6}\langle\, \Psi_\mu, \Psi_\mu^2 \,\rangle \; = \mu^2 \langle e, he \rangle + \mu^3 \langle he, (he)^2 \rangle + O(\mu^4)$$



- Deformed BRST operator (closed string scattering against 2 open strings)

$$\frac{1}{2}\langle \psi, Q_\mu \psi \rangle = \frac{1}{2}\langle \psi, Q\psi \rangle + \langle \psi, \Psi_\mu \psi \rangle$$



- This produces a shift in the mass formula for physical open strings. The D-brane adapts to the new background.

Example: Radius deformation

- Two examples of deformations for a D1 brane at the self-dual radius

$$e = \frac{1}{2\pi i} U_1^\dagger cj(i)cj(-i)|0\rangle \qquad P_0 e = 0 \qquad \text{Radius def.} \qquad j(z) \equiv i\sqrt{\tfrac{2}{\alpha'}}\partial Y(z)$$

$$e = \frac{1}{2\pi i} U_1^\dagger \, cJ_L^1(i) \, cJ_R^2(-i)|0\rangle \qquad P_0 e = c\partial c j(0)|0\rangle \neq 0 \qquad \begin{array}{l} J_L^2 = \sqrt{2}\sin(\frac{2}{\sqrt{\alpha'}}Y_L) \\ J_L^1 = \sqrt{2}\cos(\frac{2}{\sqrt{\alpha'}}Y_L) \end{array}$$

- The D1 brane (obviously) adapts to the radius deformation: mass deformations
- Initially there are three massless open string modes

$$\begin{aligned} \varphi_0 &= \phi_0 cj \\ \varphi_\pm &= \phi_\pm c e^{\pm \frac{2i}{\sqrt{\alpha'}} Y} \end{aligned}$$

- The new mass terms can be explicitly computed

$$\begin{aligned} \frac{1}{2}\phi_0 m_0^2 \phi_0 &= \mu \langle e | \frac{b_0}{L_0} | \varphi_0 * \varphi_0 \rangle = \mu\omega\left(m_2(\varphi_0, \varphi_0), he\right) \\ \phi_+ m_\pm^2 \phi_- &= \mu \langle e | \frac{b_0}{L_0} | \varphi_+ * \varphi_- \rangle + (+ \leftrightarrow -) = \mu\omega\left(m_2(\varphi_+, \varphi_-), he\right) + (+ \leftrightarrow -) \end{aligned}$$

Computation of mass corrections

- Reduce them to “standard” open-closed amplitudes on UHP. It can be done avoiding the complication of Giddings-like Schwarz-Christoffel map (adapt from *Berkovits-Schnabl 2003* strategy)

$$U_r = e^{\sum_{n>0} v_n^{(r)} L_{2n}}$$

$$(2\pi i)\langle e|\frac{b_0}{L_0}|\varphi_a * \varphi_b\rangle = \langle 0|cj(i)cj(-i)U_1\frac{b_0}{L_0}U_3^\dagger\varphi_a(\sqrt{3})\varphi_b(-\sqrt{3})|0\rangle =$$

$$\boxed{Qh + hQ + P_0 = 1}$$

$$U_r U_s^\dagger = U^\dagger_{2+\frac{2}{r}(s-2)}U_{2+\frac{2}{s}(r-2)}$$

$$\begin{aligned}
&= \langle 0|cj(i)cj(-i)hQU_1\,hU_3^\dagger\varphi_a(\sqrt{3})\varphi_b(-\sqrt{3})|0\rangle \\
&= \langle 0|cj(i)cj(-i)hU_1U_3^\dagger\varphi_a(\sqrt{3})\varphi_b(-\sqrt{3})|0\rangle \\
&= \langle 0|cj(i)cj(-i)hU_4^\dagger U_{\frac{4}{3}}\varphi_a(\sqrt{3})\varphi_b(-\sqrt{3})|0\rangle \\
&= \langle 0|cj(i)cj(-i)\frac{b_0}{L_0}\varphi_a(1)\varphi_b(-1)|0\rangle \\
&= \langle 0|cj(i)cj(-i)\frac{b_0}{L_0+\epsilon}\varphi_a(1)\varphi_b(-1)|0\rangle\Big|_{\epsilon=0} \\
&= \int_0^1\frac{dt}{t}t^\epsilon\,\langle cj(i)cj(-i)b_0\,(cj_a(t)cj_b(-t))\rangle\Big|_{\epsilon=0}
\end{aligned}$$

$$\begin{aligned}
U_r\phi(z)|0\rangle &= f_r\circ\phi(z)|0\rangle, \\
f_r(z) &= \tan\left(\frac{2}{r}\tan^{-1}z\right)
\end{aligned}$$

$$\boxed{\omega\left(m_2(\varphi_a,\varphi_b),he\right) = \frac{2\phi_a\phi_b}{\pi}\int_0^1 dt\,t^\epsilon(1+t^2)\,\langle j(i)j(-i)j_a(t)j_b(-t)\rangle\Big|_{\epsilon=0}}$$

$$\omega\left(m_2(\varphi_a,\varphi_b),he\right)=\frac{2\phi_a\phi_b}{\pi}\int_0^1 dt\,t^{\epsilon}(1+t^2)\left\langle j(i)j(-i)j_a(t)j_b(-t)\right\rangle\Big|_{\epsilon=0}$$

- The U(1) current remains massless (as it should). The epsilon reg. is crucial

$$\langle j(i)j(-i)j(t)j(-t)\rangle=\frac{1}{(t-i)^4}+\frac{1}{(t+i)^4}-\frac{1}{16t^2}.$$

$$\omega\left(e,hm_2(\varphi_0,\varphi_0)\right)=\frac{2\phi_0\phi_0}{\pi}\left(-\frac{i}{2}+\frac{i}{2}-\frac{\epsilon}{8(\epsilon^2-1)}\Big|_{\epsilon=0}\right)=0.$$

- The other two SU(2) currents become tachyonic (relate μ with δR by matching with the KK spectrum)

$$\left\langle j(i)j(-i)e^{\frac{2i}{\sqrt{\alpha'}}Y}(t)e^{-\frac{2i}{\sqrt{\alpha'}}Y}(-t)\right\rangle=\frac{1}{2t^2}\left(-\frac{1}{8}+\frac{4t^2}{(1+t^2)^2}\right)$$

$$\omega\left(e,hm_2(\varphi_+,\varphi_-)\right)=\frac{2\phi_+\phi_-}{\pi}\int_0^1 dt\,t^{\epsilon}\left(-\frac{1}{16t^2}-\frac{1}{16}+\frac{2}{1+t^2}\right)\Big|_{\epsilon=0}=\frac{1}{2}\phi_-\phi_+$$

$$\boxed{\mu=-2\frac{\delta R}{R}}$$

Superstring

NS Open superstring field theory in LHS with deformation

$$Q^2 = 0, \quad [\eta, Q] = 0, \quad \eta^2 = 0.$$

- Add an elementary open-closed coupling to (NS) WZW-like OSFT in LHS (*Michishita*)

$$S^{(\mu)}[\Phi] = -\int_0^1 dt \, \langle \, A_t, \eta A_Q - \mu e \, \rangle$$
$$= \frac{1}{2} \langle \, \eta\Phi, Q\Phi \, \rangle + \mathcal{I}(\Phi) + \mu \, \langle \, \Phi, e \, \rangle$$

$$A_Q = e^{-t\Phi} Q e^{t\Phi}$$
$$A_t = e^{-t\Phi} \partial_t e^{t\Phi} = \Phi$$
$$\mathcal{I}(\Phi) = \sum_{k=1}^{\infty} \frac{(-1)^k}{(k+2)!} \Big\langle \, \eta\Phi, \, \mathrm{ad}_\Phi^k \, Q\Phi \, \Big\rangle$$

- Gauge invariant deformation provided

$$\eta e = 0$$
$$Q e = 0.$$
$$[e, \Phi] = 0$$

- Solve perturbatively the tadpole-sourced equation of motion

$$\eta Q\Phi + \mathcal{J}(\Phi) = \mu e.$$

$$\mathcal{J}(\Phi) = -\frac{1}{2}[\eta\Phi, Q\Phi] + \frac{1}{6}\Big([\eta\Phi, [\Phi, Q\Phi]] - \frac{1}{2}[\Phi, [\eta\Phi, Q\Phi]] + \frac{1}{2}[\Phi, [\Phi, \eta Q\Phi]]\Big) + O(\Phi^4)$$

$$\Phi_\mu = \sum_{\alpha=1}^{\infty} \mu^\alpha \phi_\alpha.$$

$$\eta Q\phi_1 = e$$
$$\eta Q\phi_2 = \frac{1}{2}[\eta\phi_1, Q\phi_1]$$
$$\eta Q\phi_3 = \frac{1}{2}[\eta\phi_1, Q\phi_2] + \frac{1}{2}[\eta\phi_2, Q\phi_1] - \frac{1}{6}\Big([\eta\phi_1, [\phi_1, Q\phi_1]] - \frac{1}{2}[\phi_1, [\eta\phi_1, Q\phi_1]] + \frac{1}{2}[\phi_1, [\phi_1, \eta Q\phi_1]]\Big)$$
$$\cdots \, ,$$

Vacuum shift and LHS perturbation theory

- The equations can be solved fixing a gauge ***outside*** Ker(L_0), analogously to the bosonic case.

$$\phi_1 = -\xi_0 h\, e = -\tilde{h}he$$
$$\phi_2 = \frac{1}{2}\xi h[he,\tilde{h}e] = \frac{1}{2}\tilde{h}h[he,\tilde{h}e],$$
$$\phi_3 = -\tilde{h}h\left(\frac{1}{4}[he,\tilde{h}[he,\tilde{h}e]] + \frac{1}{4}[\tilde{h}e,h[he,\tilde{h}e]] - \frac{1}{6}[he,[\tilde{h}he,\tilde{h}e]] + \frac{1}{12}[\tilde{h}he,[he,\tilde{h}e]]\right)$$
$$\ldots$$

- Dual structure of homotopy operators for the two commuting derivations

$$h = \frac{b_0}{L_0}(1-P_0) = \eta_0(\xi_0 h)$$
$$\tilde{h} = -Q(\xi_0 h) = \left(\xi_0 - X_0\frac{b_0}{L_0}\right)(1-P_0),$$

$$[Q,h] = 1-P_0$$
$$[\eta,h] = 0$$
$$[Q,\tilde{h}] = 0$$
$$[\eta,\tilde{h}] = 1-P_0$$

Cfr with *Kroyter, Okawa, Schnabl, Torii, Zwiebach (2012)*

$$\boxed{\tilde{h}h = \xi h = \xi_0\frac{b_0}{L_0}(1-P_0)}$$ *Full Propagator*

- Obstructions to vacuum shift given by LHS amplitudes between a physical open string and n deforming closed strings. Notice the $h \leftrightarrow \tilde{h}$ symmetry. *"Double" homotopy transfer???*

$$0 = P_0 e$$
$$0 = P_0[he,\tilde{h}e]$$
$$0 = P_0\left(\frac{1}{4}[he,\tilde{h}[he,\tilde{h}e]] + \frac{1}{4}[\tilde{h}e,h[he,\tilde{h}e]] - \frac{1}{6}[he,[\tilde{h}he,\tilde{h}e]] + \frac{1}{12}[\tilde{h}he,[he,\tilde{h}e]]\right)$$
$$\vdots$$

Closed string deformed effective action in LHS

- Even without an homotopy transfer "package" we can integrate out order by order by hand and we find $(\varphi \in \text{Ker}\, L_0)$

$$
\begin{aligned}
S^{(\mu)}_{\text{eff}}[\lambda\varphi] \;=\;& \left(-\frac{\mu^2}{2}\left\langle e, \tilde{h}he \right\rangle + O\left(\mu^3\right)\right) \\
+\;& \lambda\left(\mu\left\langle e, \varphi\right\rangle - \frac{\mu^2}{2}\left\langle [he, \tilde{h}e], \varphi \right\rangle + O\left(\mu^3\right)\right) \\
+\;& \lambda^2\left(\frac{1}{2}\left\langle \eta\varphi, Q\varphi\right\rangle - \frac{\mu}{2}\left\langle \eta\varphi, \left[\tilde{h}he, Q\varphi\right] \right\rangle + O\left(\mu^2\right)\right) \\
+\;& \lambda^3\left(-\frac{1}{6}\left\langle \eta\varphi, [\varphi, Q\varphi]\right\rangle + O\left(\mu\right)\right) \\
+\;& \lambda^4\left(\frac{1}{24}\left\langle \eta\varphi, [\varphi, [\varphi, Q\varphi]]\right\rangle - \frac{1}{8}\left\langle [\eta\varphi, Q\varphi], \tilde{h}h[\eta\varphi, Q\varphi] \right\rangle + O(\mu)\right) \\
+\;& O(\lambda^5).
\end{aligned}
$$

Non-dynamical cosmological constant

Effective tadpole
aka obstructions to vacuum shift

Mass deformations

- These effective couplings can be computed in complete analogy to the bosonic case by "flattenization" of the Witten diagrams to the UHP.

- But the superstring setting sometimes allows for drastic simplification: ***localization at the boundary of the world sheet moduli space.***

Localization of open-closed mass terms

- From now on we will work at strict zero momentum (algebraic effective couplings)
- Often the matter BCFT has a N=2 SCFT structure and the massless states can be decomposed according to their N=2 R-charge (chiral + antichiral ring)

$$\varphi = ce^{-\phi}\mathbb{V}_{\frac{1}{2}} = ce^{-\phi}(\mathbb{V}^+_{\frac{1}{2}} + \mathbb{V}^-_{\frac{1}{2}}) \equiv \varphi^+ + \varphi^-$$

$$G = G^+ + G^-$$

$$G^{\pm}_{\frac{1}{2}} \, |\mathbb{V}^{\mp}_{\frac{1}{2}}\rangle = 0$$

$$e = \varepsilon_{ij}\left[X_0 U^i(i) U^j(-i)\right] I + \varepsilon_{ij}\left[U^i(i) X_0 U^j(-i)\right] I$$

$$U^i = c\mathbb{U}^i_{\frac{1}{2}} e^{-\phi} \qquad \mathcal{U}^i = \xi_0 c\mathbb{U}^i_{\frac{1}{2}} e^{-\phi}$$

$$\mathbb{U}^i_{\frac{1}{2}} = (\mathbb{U}^i_{\frac{1}{2}})^+ + (\mathbb{U}^i_{\frac{1}{2}})^- \qquad e = e^{++} + e^{+-} + e^{-+} + e^{--}$$

- If this decomposition holds then the effective tadpole is vanishing to first order $P_0 e = 0$ and also the cubic open string coupling vanishes $P_0[\eta\varphi, Q\varphi] = 0$. Induced mass term localizes at boundary of ws moduli space

$$\tilde{S}_{1,1}(\eta\varphi) = \frac{1}{2}\varepsilon_{ij}\left\langle [\eta\varphi, Q\varphi], \xi_0 \frac{b_0}{L_0}\bar{P}_0[(\eta\mathcal{U}^i)(i)(Q\mathcal{U}^j)(-i)]I \right\rangle +$$

$$+ \frac{1}{2}\varepsilon_{ij}\left\langle [\eta\varphi, Q\varphi], \xi_0 \frac{b_0}{L_0}\bar{P}_0[(Q\mathcal{U}^i)(i)(\eta\mathcal{U}^j)(-i)]I \right\rangle .$$

$$= -\frac{1}{2}\varepsilon_{ij}\langle([\eta\varphi^+, Q\varphi^-] - [\eta\varphi^-, Q\varphi^+]), P_0\left[(\mathcal{U}^i)^+(\mathcal{U}^j)^- - (\mathcal{U}^i)^-(\mathcal{U}^j)^+\right] I\rangle$$

$$-\frac{1}{2}\varepsilon_{ij}\langle[\varphi^+, Q\varphi^+], P_0\left[(\mathcal{U}^i)^-\eta(\mathcal{U}^j)^- - \eta(\mathcal{U}^i)^-(\mathcal{U}^j)^-\right] I\rangle$$

$$-\frac{1}{2}\varepsilon_{ij}\langle[\varphi^-, Q\varphi^-], P_0\left[(\mathcal{U}^i)^+\eta(\mathcal{U}^j)^+ - \eta(\mathcal{U}^i)^+(\mathcal{U}^j)^+\right] I\rangle$$

The propagator DISAPPEARED and it has been replaced by P_0 (localization at the boundary of moduli space)

$$P_0 \sim \lim_{s\to+\infty} e^{-sL_0} = \lim_{t\to 0^+} t^{L_0}$$

(Deformed) generalized ADHM constraints

- 4 point amplitude—> 2 point function

$$\tilde{S}_{1,1} = 2\langle \mathbb{G}_1^- | \mathbb{H}_1^+ \rangle + 2\langle \mathbb{G}_1^+ | \mathbb{H}_1^- \rangle + \langle \mathbb{G}_0 | \mathbb{H}_0 \rangle$$

$$\mathbb{G}_1^\pm = \varepsilon_{ij} \lim_{z \to z^*} \left[(\mathbb{U}^i_{\frac{1}{2}})^\pm(z)(\mathbb{U}^j_{\frac{1}{2}})^\pm(z^*) \right] ,$$

$$\mathbb{G}_0 = \varepsilon_{ij} \lim_{z \to z^*} \left[2z \left((\mathbb{U}^i_{\frac{1}{2}})^-(z)(\mathbb{U}^j_{\frac{1}{2}})^+(z^*) - (\mathbb{U}^i_{\frac{1}{2}})^+(z)(\mathbb{U}^j_{\frac{1}{2}})^-(z^*) \right) \right]$$

$$\mathbb{H}_1^\pm = \lim_{z \to 0} \left[\mathbb{V}^\pm_{\frac{1}{2}}(z)\mathbb{V}^\pm_{\frac{1}{2}}(-z) \right] ,$$

$$\mathbb{H}_0 = \lim_{z \to 0} \left[2z \left(\mathbb{V}^-_{\frac{1}{2}}(z)\mathbb{V}^+_{\frac{1}{2}}(-z) - \mathbb{V}^+_{\frac{1}{2}}(z)\mathbb{V}^-_{\frac{1}{2}}(-z) \right) \right]$$

- Combine with the quartic open string potential, also localized *(CM, Merlano 2018)*

$$\tilde{S}_{3,0} = \langle \mathbb{H}_1^+ | \mathbb{H}_1^- \rangle + \frac{1}{4} \langle \mathbb{H}_0 | \mathbb{H}_0 \rangle$$

- To get a total potential

$$\tilde{S}^{(\mu)} \supset \tilde{S}_{3,0} + \mu \tilde{S}_{1,1} = \langle \mathbb{H}_1^+ + 2\mu \mathbb{G}_1^+ | \mathbb{H}_1^- + 2\mu \mathbb{G}_1^- \rangle + \frac{1}{4} \langle \mathbb{H}_0 + 2\mu \mathbb{G}_0 | \mathbb{H}_0 + 2\mu \mathbb{G}_0 \rangle \; - 4\mu^2 \Big(\langle \mathbb{G}_1^+ | \mathbb{G}_1^- \rangle + \frac{1}{4} \langle \mathbb{G}_0 | \mathbb{G}_0 \rangle \Big)$$

- Global minima controlled by the *deformed ADHM-like constraints*

$$\mathbb{H}_1^\pm = -2\mu \mathbb{G}_1^\pm + \mathcal{O}(\mu^2)$$

$$\mathbb{H}_0 = -2\mu \mathbb{G}_0 \; + \mathcal{O}(\mu^2) \, ;$$

- Example: Nekrasov non commutative instantons by taking a D3/D(-1) system deformed by a constant Kalb-Ramond (see 2103.04921)

All order results in SHS: A_∞ Munich Theory

- Small Hilbert with space theory with cyclic A_∞ products

$$\mathbf{M}=\sum_{k=1}^{\infty}\mathbf{M}_k=\mathbf{G}^{-1}\mathbf{Q}\mathbf{G}\qquad [\eta,\mathbf{M}]=0\qquad \mathbf{M}^2=0$$

- The microscopic open-closed coupling is non-linear in the open strings (*Vosmera 2020)*

$$\mathbf{E}=\sum_{k=1}^{\infty}\mathbf{E}_k=\mathbf{G}^{-1}\mathbf{e}\mathbf{G}\qquad [\eta,\mathbf{E}]=0\qquad \mathbf{E}^2=0\qquad [\mathbf{M},\mathbf{E}]=0$$

- The effective theory is readily obtained by ***homotopy transfer*** of the nilpotent coderivation

$$\mathbf{M}^{(\mu)}\equiv\mathbf{M}+\mu\mathbf{E}$$

$$S^{(\mu)}(\Psi)=\int_0^1 dt\,\langle\omega_{\rm S}|\pi_1\boldsymbol{\partial}_t\frac{1}{1-\Psi(t)}\otimes\pi_1\mathbf{M}^{(\mu)}\frac{1}{1-\Psi(t)}\longrightarrow \tilde S^{(\mu)}(\psi)=S^{(\mu)}(\Psi_\mu(0))+\int_0^1 dt\,\langle\tilde\omega_{\rm S}|\pi_1\boldsymbol{\partial}_t\frac{1}{1-\psi(t)}\otimes\pi_1\tilde{\mathbf{M}}^{(\mu)}\frac{1}{1-\psi(t)}$$

$$\Psi_\mu(\psi)\equiv\pi_1\frac{1}{\mathbf{1}_{T\mathcal{H}}+\mathbf{h}\delta\mathbf{M}^{(\mu)}}\mathbf{I}_0\frac{1}{1_{T\mathcal{H}}-\psi}\qquad \tilde{\mathbf{M}}^{(\mu)}=\mathbf{\Pi}_0\mathbf{M}^{(\mu)}\frac{1}{\mathbf{1}_{T\mathcal{H}}+\mathbf{h}\delta\mathbf{M}^{(\mu)}}\mathbf{I}_0$$

$$\tilde{\mathbf{M}}^{(\mu)}=\sum^{\infty}\mu^\alpha\tilde{\mathbf{N}}_\alpha\longrightarrow$$

$$\tilde N_{k,\alpha}\equiv\pi_1\tilde{\mathbf{N}}_\alpha\pi_k$$

$$\begin{aligned}
\tilde N_{0,1}&=P_0e\,,\\
\tilde N_{0,2}&=-P_0E_1(he)+P_0M_2(he,he)\,,\\
\tilde N_{1,0}(\psi)&=P_0Q\psi\,,\\
\tilde N_{1,1}(\psi)&=P_0E_1(\psi)-P_0M_2(he,\psi)-P_0M_2(\psi,he)\,,\\
\tilde N_{2,0}(\psi,\psi)&=P_0M_2(\psi,\psi)\,,\\
\tilde N_{3,0}(\psi,\psi)&=P_0M_3(\psi,\psi,\psi)-P_0M_2(\psi,hM_2(\psi,\psi))-P_0M_2(hM_2(\psi,\psi),\psi)\\
&\vdots
\end{aligned}$$

- This effective theory is in general *different* from the previous WZW one, but they have the *same* vacuum structure (in particular same results on localization).

Taming closed string degeneration

Work in progress

- Diagram with two or more closed strings will have regions in moduli space where closed strings can collide (closed string degeneration).
- Similar open string collisions are systematically taken care of by

$$\left.\frac{1}{L_0+\epsilon}\right|_{\epsilon\to 0}$$

- Witten vertex and propagators cover the whole ws moduli space, but it does not give an obvious regularization towards closed string degeneration.

- Take for example the first term in the cosmological constant for the radius deformation

$$\Gamma = \langle\, e, he\,\rangle = -\frac{1}{4\pi^2}\langle 0|V(i,-i)U_1\,\frac{b_0}{L_0+\epsilon}U_1^\dagger V(i,-i)|0\rangle.$$

$$= \frac{1}{\pi^2}\int_0^1 dt\, t^\epsilon(1-t^2)\left(\frac{1}{16t^2}+\frac{1}{(1-t)^4}+\frac{1}{(1+t)^4}\right)$$

- What to do with the $t\to 1$ divergence? This is a closed string degeneration so it should be regulated by a closed string propagator.

- In this case it is "easy" to see what to do. First we prove that

$$\langle 0|\, V_1(i,-i)\frac{b_0}{L_0}V_2(i,-i)\,|0\rangle \;=\; -\langle B|\,\frac{b_0^+}{L_0^+}\,V_1(-1,-\bar{1})V_2(1,\bar{1})\,|0\rangle,$$

$$\updownarrow$$

$$\int_0^1 \frac{dt}{t}\langle 0|\,V_1(i,-i)\,b_0V_2(it,-it)\,|0\rangle \;=\; \int_1^0 \frac{dx}{x}\langle B|\,b_0^+\,V_1(-x,-\bar{x})V_2(x,\bar{x})\,|0\rangle$$

$$x = \frac{1-\sqrt{t}}{1+\sqrt{t}}$$

- This suggests that the term which diverges in $t \to 1$ should be regulated

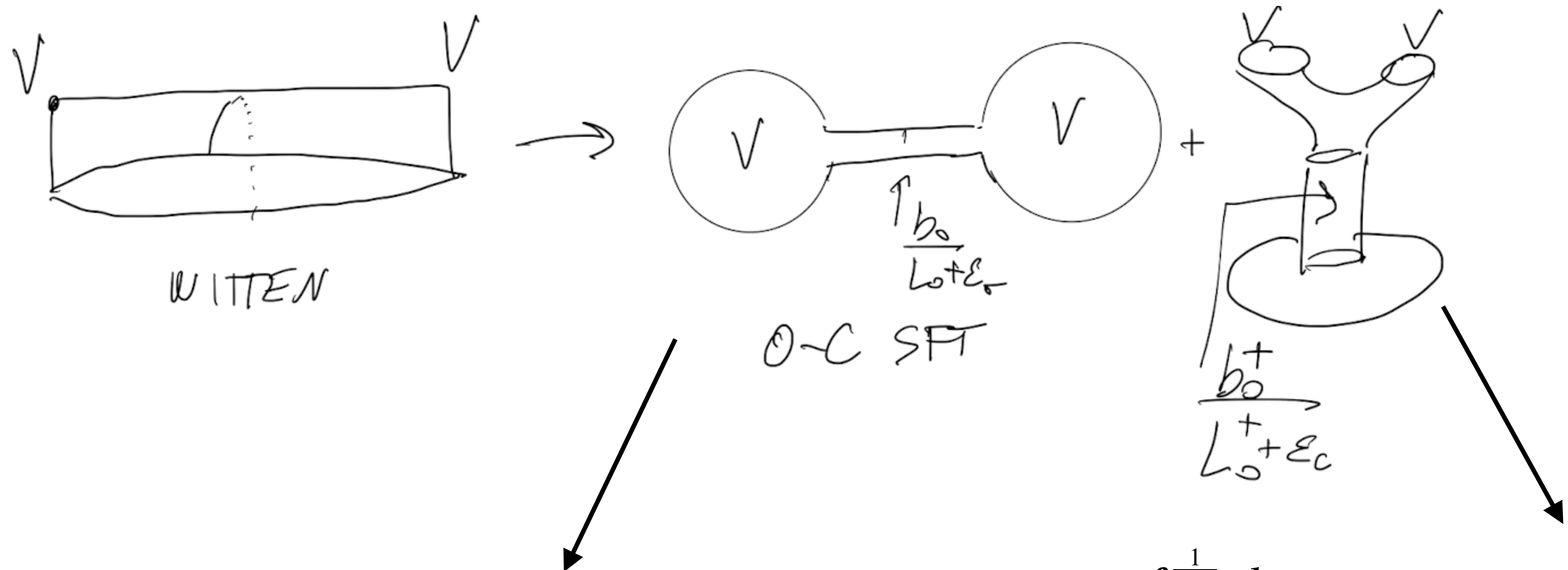
$$\frac{b_0^+}{L_0^+ + \epsilon_{\text{closed}}} = \int_0^1 dx\, x^{\epsilon_{\text{closed}}-1}\, b_0^+\, x^{L_0^+} \qquad \int_0^1 dt\, x(t)^{\epsilon_{\text{closed}}}\,\frac{1+t}{(1-t)^3} = -\frac{1}{8-2\epsilon_{\text{closed}}^2} \quad\to\quad -\frac{1}{8}$$

- In total this gives

$$\Gamma = \langle\, e, he\,\rangle = 0, \qquad \text{(radius deformation)}$$

Work in progress

- To have a more rigorous check, we have computed the same amplitude in O-C SFT with SL(2,C) vertices



$$\int_0^{\frac{1}{\lambda_o^2}} \frac{dt}{t} t^{\epsilon_o}\langle V(i,-i)\, b_0 V(it,-it)\rangle$$

$$-\int_0^{\frac{1}{3\lambda_c^2}} \frac{dx}{x} x^{\epsilon_c}\langle B\,|\, b_0^+ V(-x,-\bar{x}) V(x,\bar{x})\,|\,0\rangle$$

$$x = \frac{1-\sqrt{t}}{1+\sqrt{t}}$$

Integrands are the same!

- The full moduli space is covered by the two diagrams if the open and closed string stubs are related as $\lambda_o = \dfrac{3\lambda_c^2 + 1}{3\lambda_c^2 - 1}$

- The “heuristic” result we got in previous slide seems then to be robust.

- *How does this generalises to superstring? New localization channels?*

Conclusions

- OSFT based on Witten product allow for explicit perturbative and non-perturbative computations involving open and closed strings. All computations are fairly explicit to the very end.

Pressing questions

- *Is there a large Hilbert space homotopy transfer? Positive evidence in the partially gauge fixed theory (work in progress)*
- *Can we systematically control closed string degeneration inside OSFT? Is it useful to understand OSFT as a singular limit of O-C SFT? (work in progress)*
- *Sometimes amplitudes localize on closed string degeneration. Can we understand this in OSFT? Localization in O-C SFT? (work in progress)*

- *Can we consistently couple the ghost dilaton? (it's not a (0,0) primary)*
- *What is the cosmological constant computing?*
- ***So much to do!***
- ***Thanks, comments and suggestions are welcome!!***