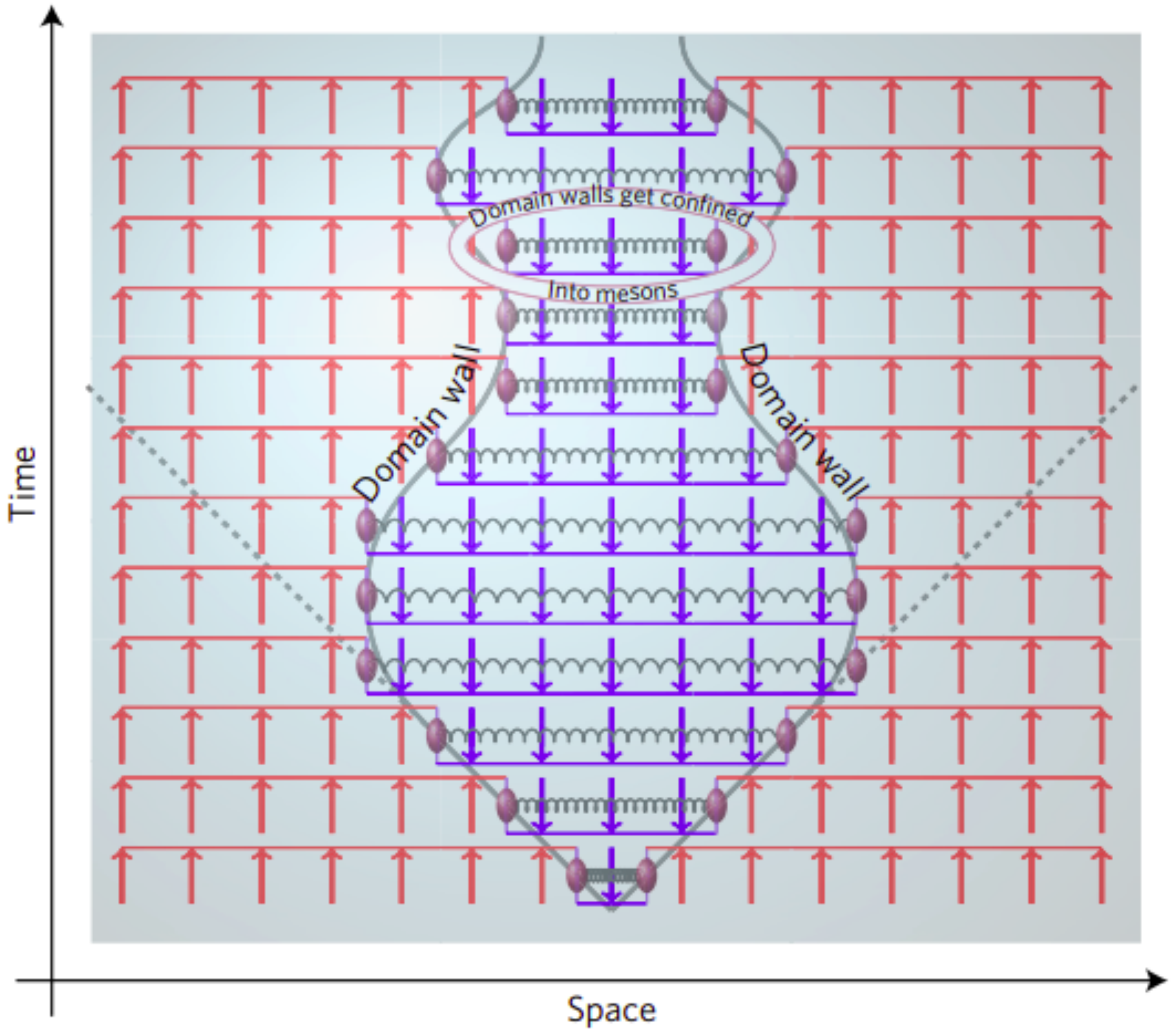
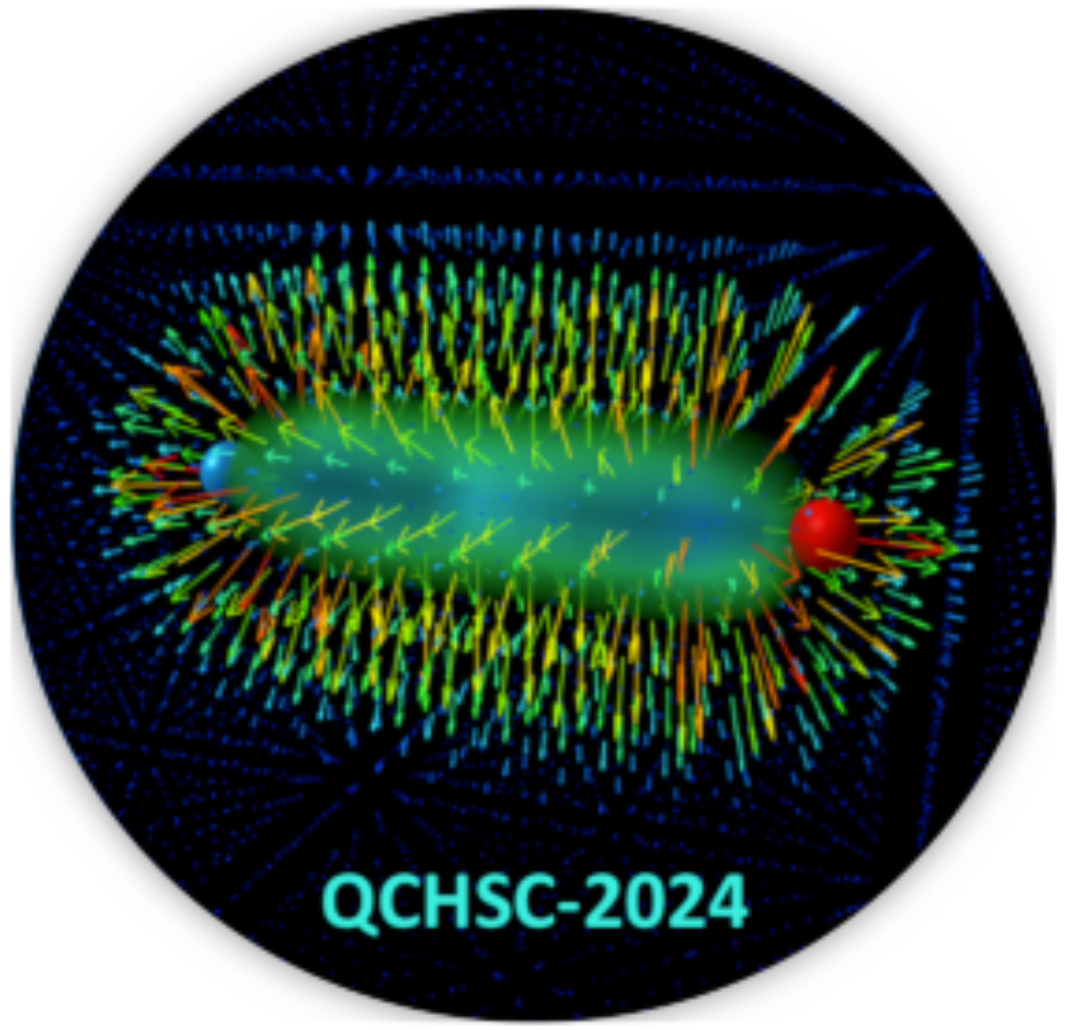
Confinement and False Vacuum Decay in Quantum Spin Chains



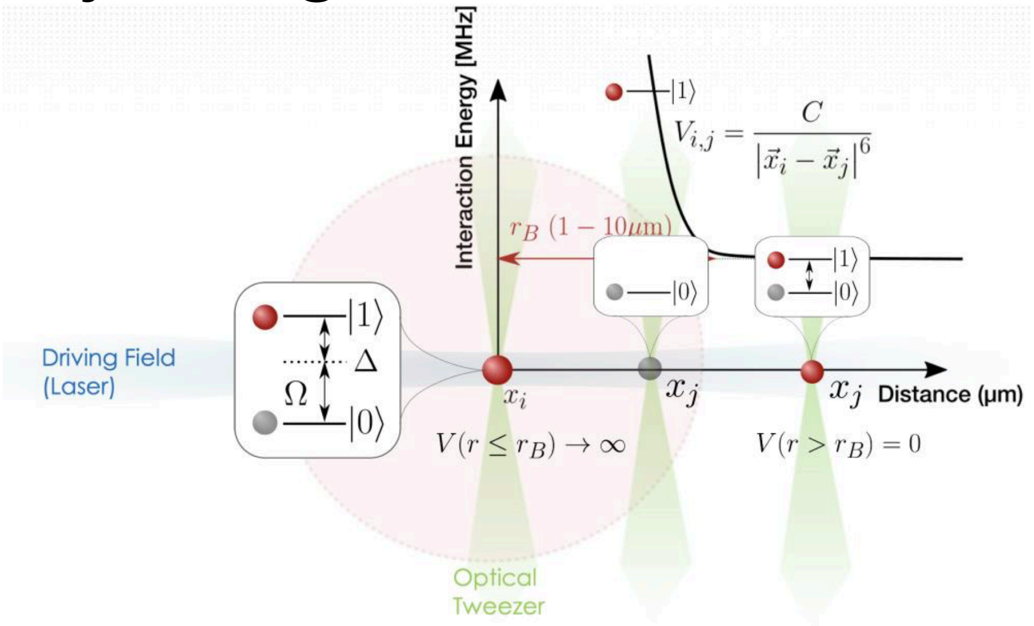


Jožef Stefan Institute, Ljubljana, Slovenia

Gianluca Lagnese

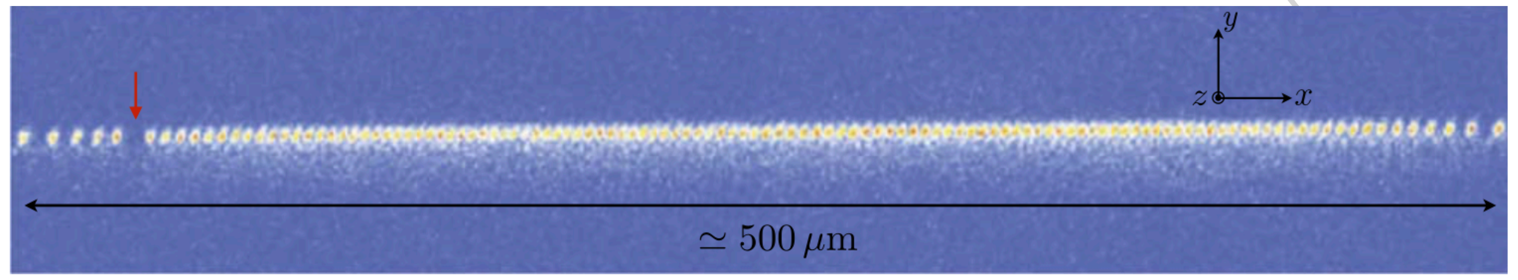
Experiments/Quantum simulators

Rydberg Neutral Atoms



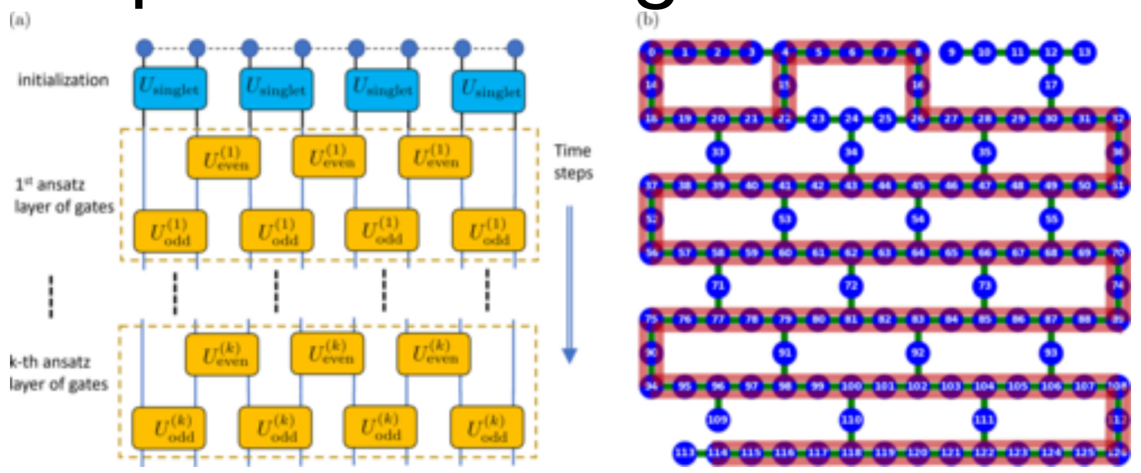
J. Wurtz et al., arXiv:2306.11727

Cryogenic Trapped Ions



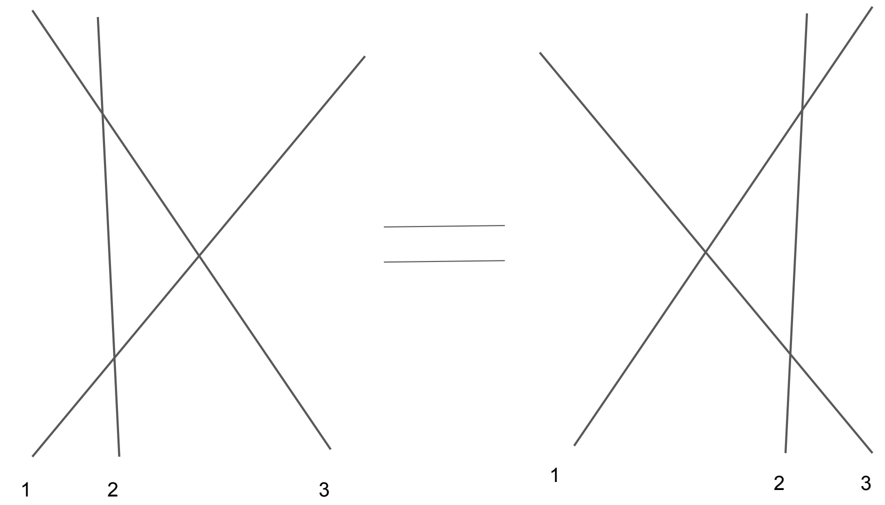
G. Pagano et al., Quantum Sci. Technol. 2019

Superconducting Qubits



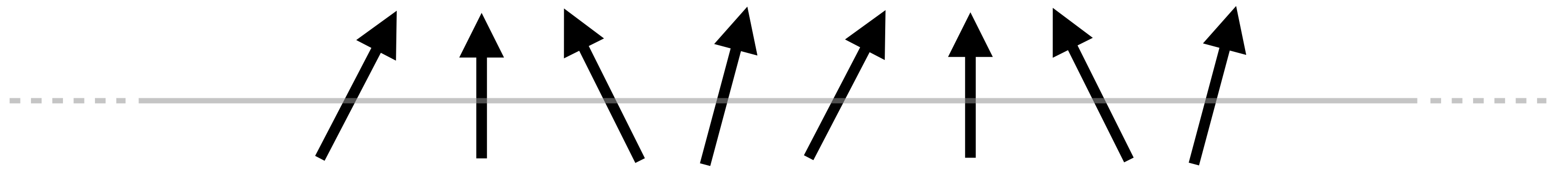
H. Yu, Y. Zhao, Tzu-Chieh Wei, Phys. Rev. Research 2023

Analytical (Integrability, CFT…)

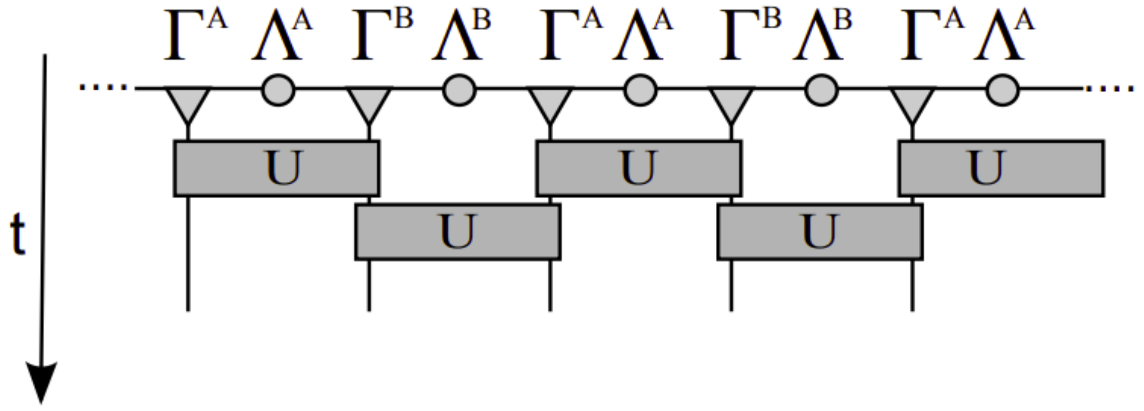


Franchini, Springerlink LNP 940

Quantum Spin Chains



Numerical methods (Tensor Networks, TEBD, DMRG)



J. Hauschild, F. Pollmann, SciPost Phys. 2018

Key concepts

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1) Real-time out-of-equilibrium dynamics of many-body systems

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2) Isolated systems $\leftrightarrow$ Pure unitary time evolution

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3) Zero temperature $\leftrightarrow$ No thermal fluctuations

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“Introduction to 'Quantum Integrability in Out of Equilibrium Systems’”

Calabrese, Essler, Mussardo J. Stat. Mech. 2016

Overview

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1) The Model: Transverse Field Quantum Ising Chain

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2) Confinement:

“Real time confinement following a quantum quench to a non-integrable model”

Kormos, Collura, Takaks, Calabrese, Nature Physics 2017

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Lagnese, Surace, Kormos, Calabrese PRB 2021

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Lagnese, Surace, Kormos, Calabrese PRB 2021

4) Another Model

The Model

The model: Transverse Field Quantum Ising Chain

Classical Ising: $E = -J \sum_{\langle ij \rangle} s_i s_j \qquad s_i \in \{-1,1\}$

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$s_i \rightarrow \sigma_i^z$

1+1 D Quantum Ising: $H = -J \sum_i \left(\sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \right)$

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Transverse Field ↔ Quantum Fluctuations

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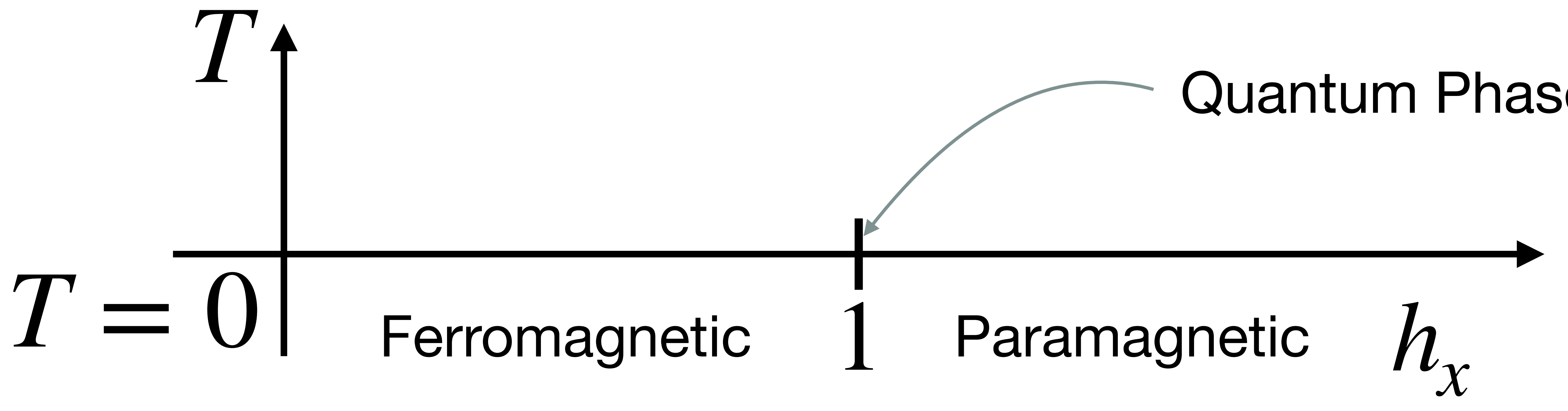
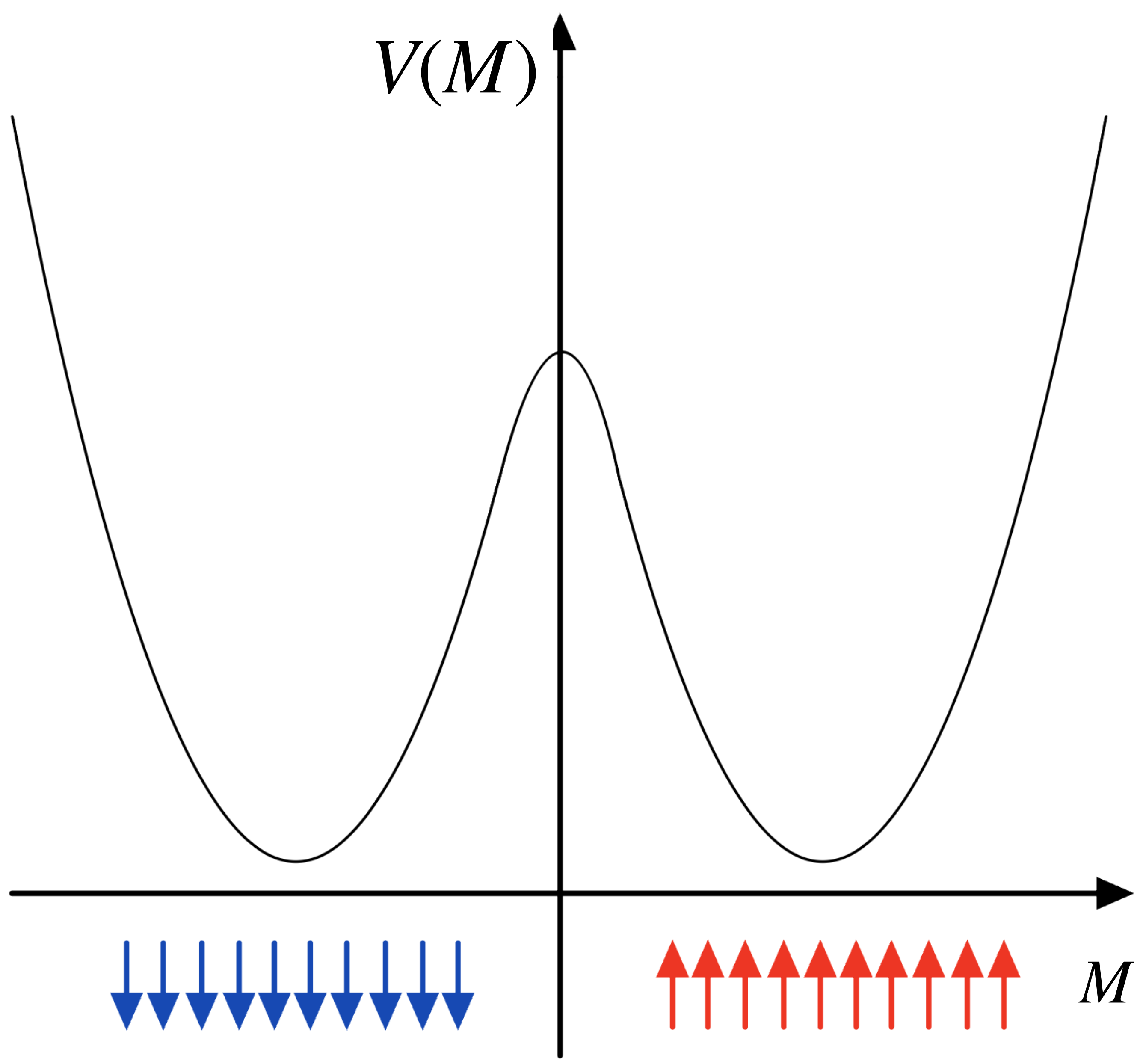


Quantum Phase Transition ∼ 2D Classical Thermal

The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x$$

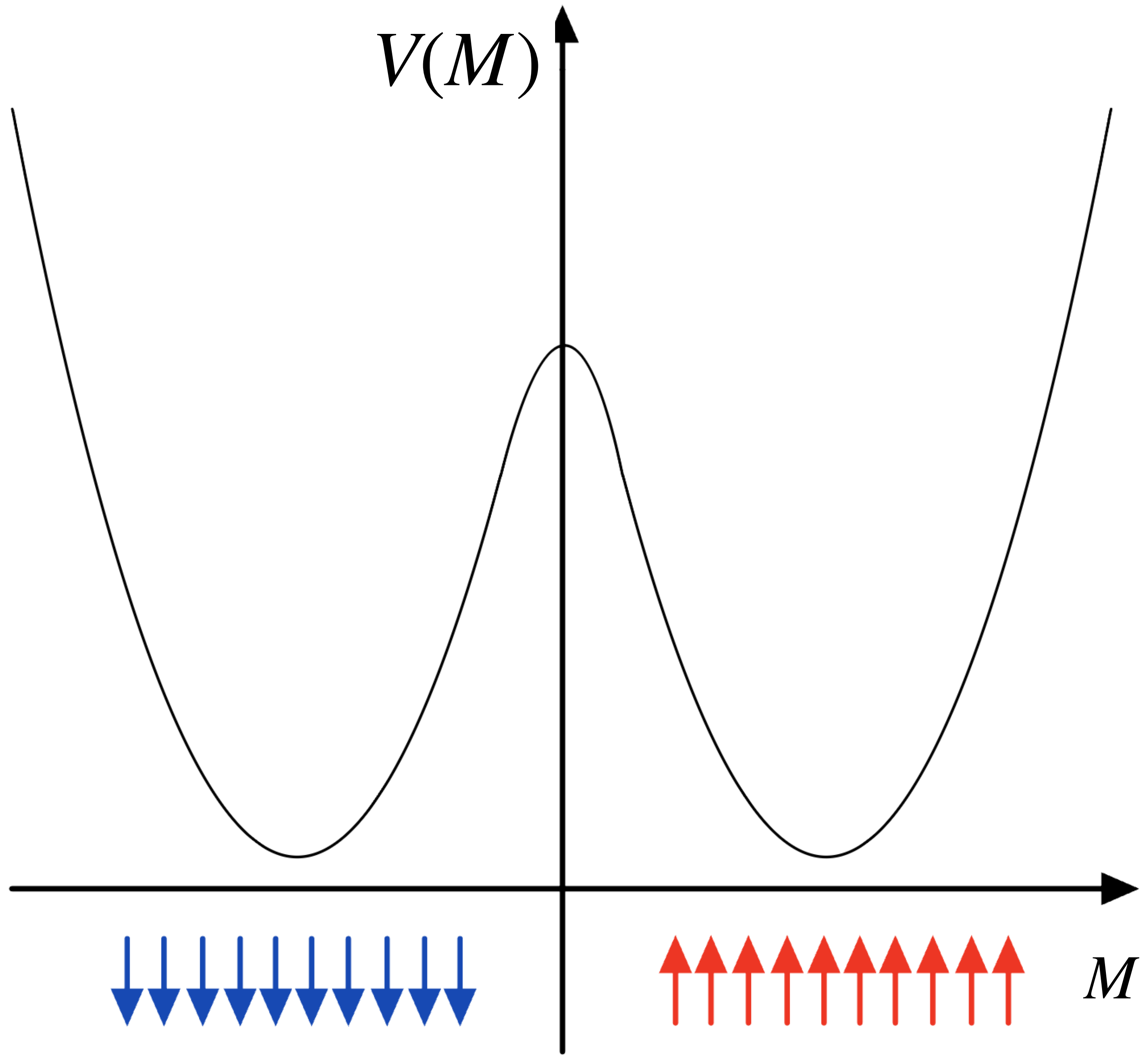
$|h_x| < 1 \Rightarrow$ Two **Degenerate** Ferromagnetic ground states



The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x$$

$$|h_x| \ll 1 \quad \Rightarrow \quad H \approx -\sum_j 2\,|j\rangle\langle j| - h_x\left(|j\rangle\langle j+1| + h.c.\right)$$



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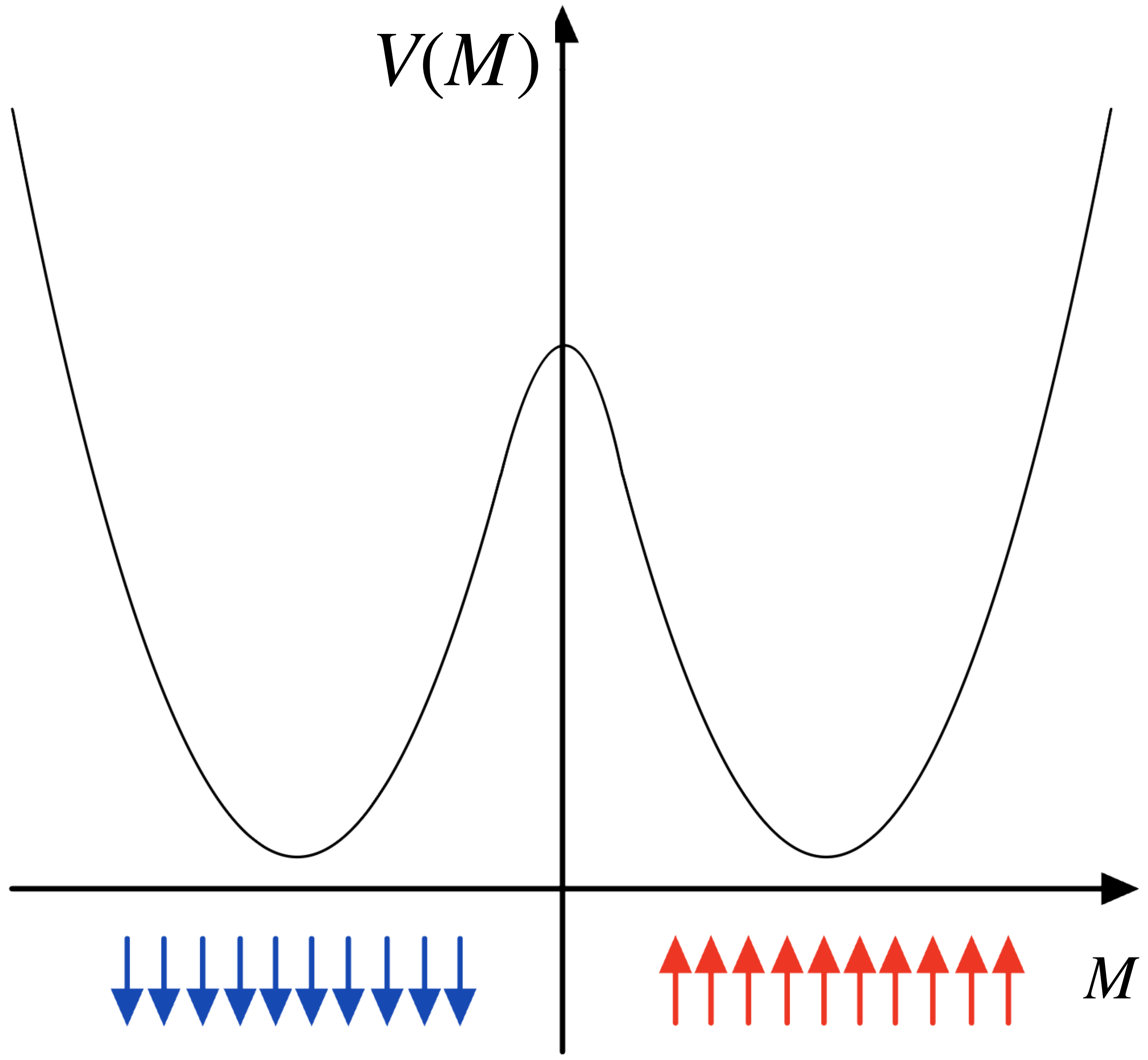
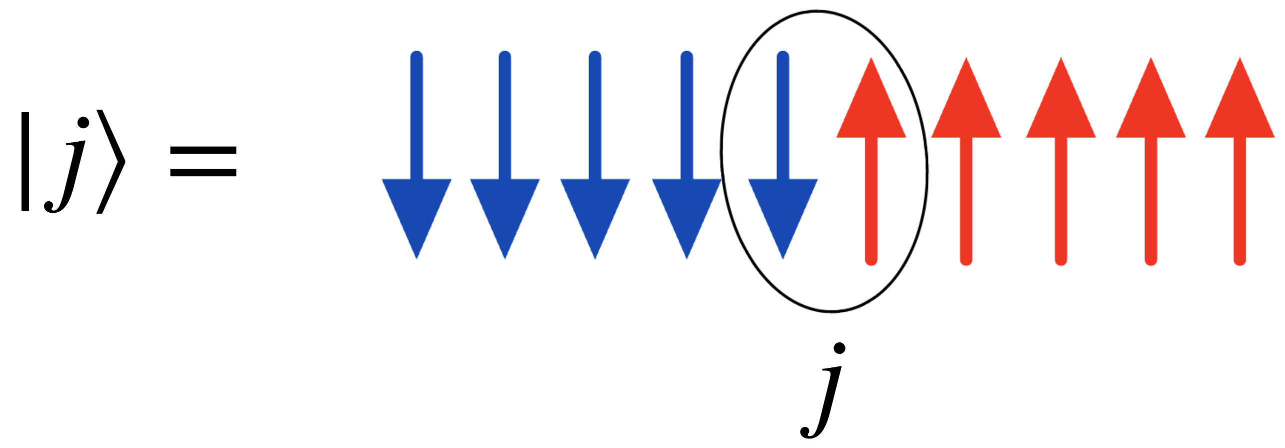


Domain-wall excitations:



The model: Transverse Field Quantum Ising Chain

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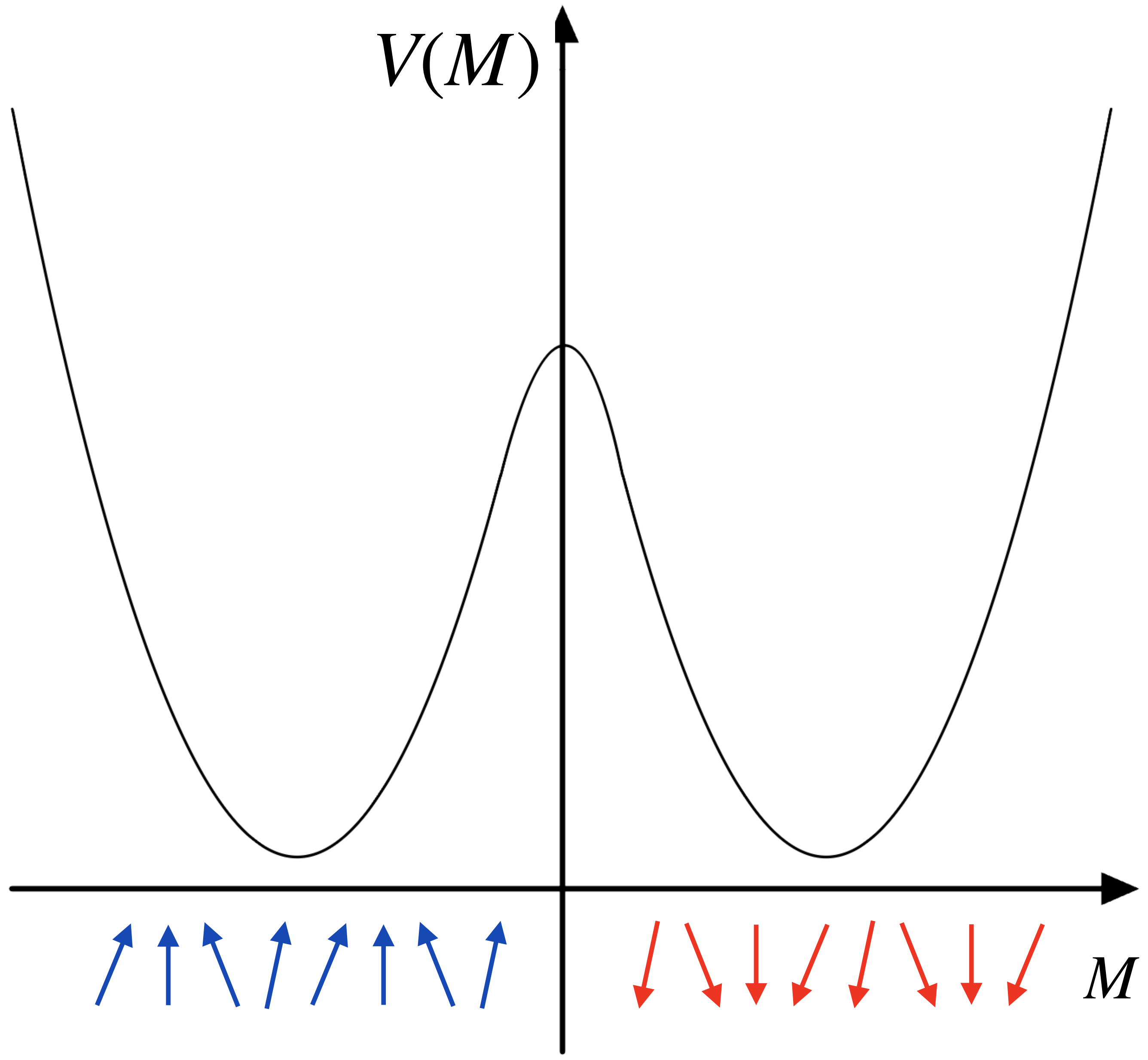
$$|h_x| < 1 \quad \Rightarrow \quad H = \sum_k \varepsilon_k \eta_k^\dagger \eta_k + const$$

Free Fermion mapping (Jordan-Wigner) + Bogoliubov + Fourier



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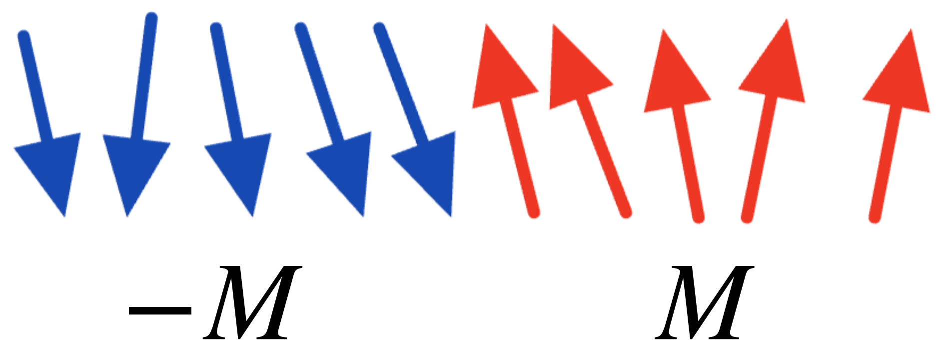
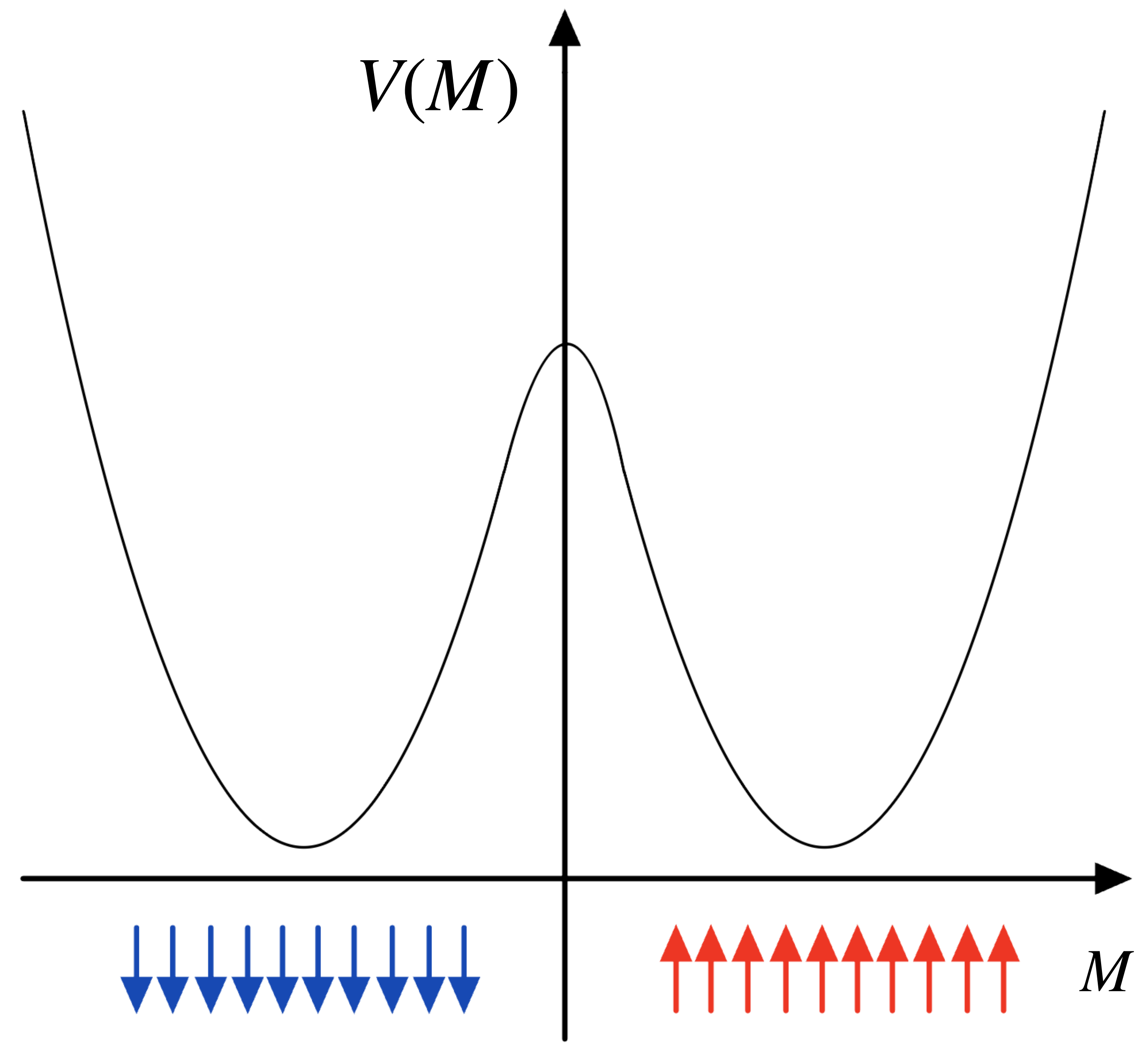


$$M = (1 - h_x)^{1/8}$$

Energy cost / Mass gap: $\Delta = 2(1 - h_x)$

The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \quad - h_z \sigma_i^z$$



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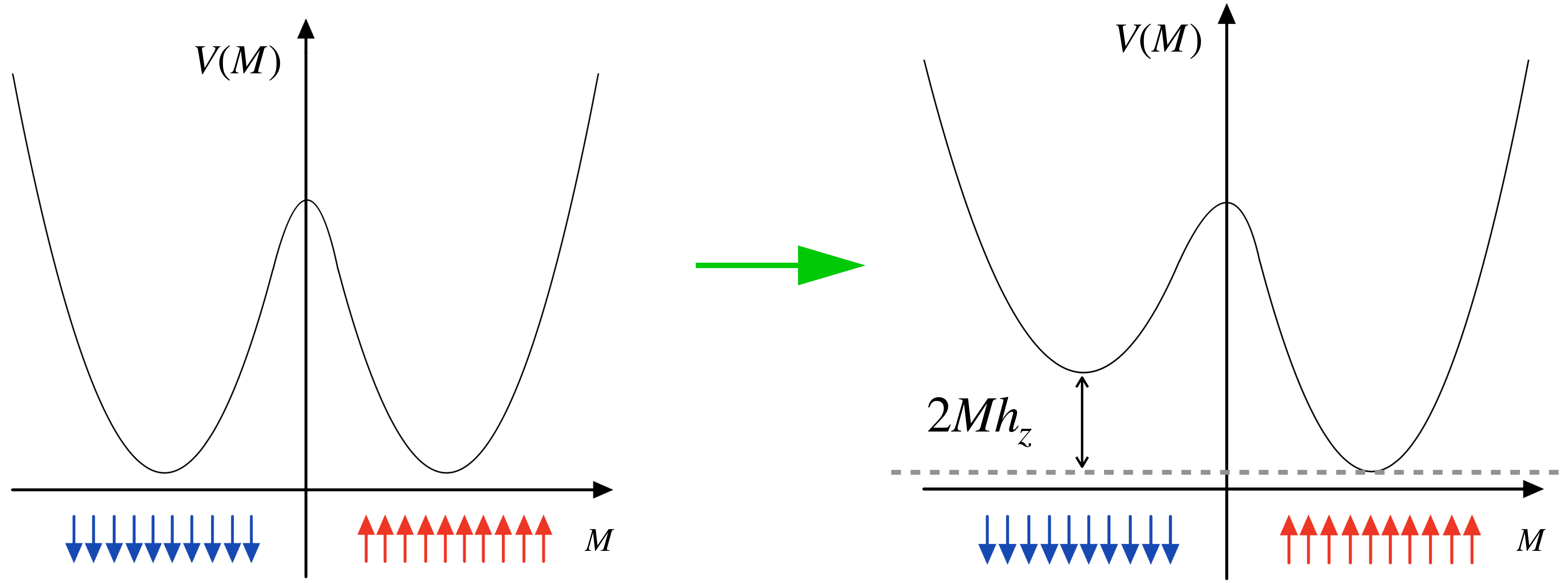
The longitudinal field h_z lifts the degeneracy



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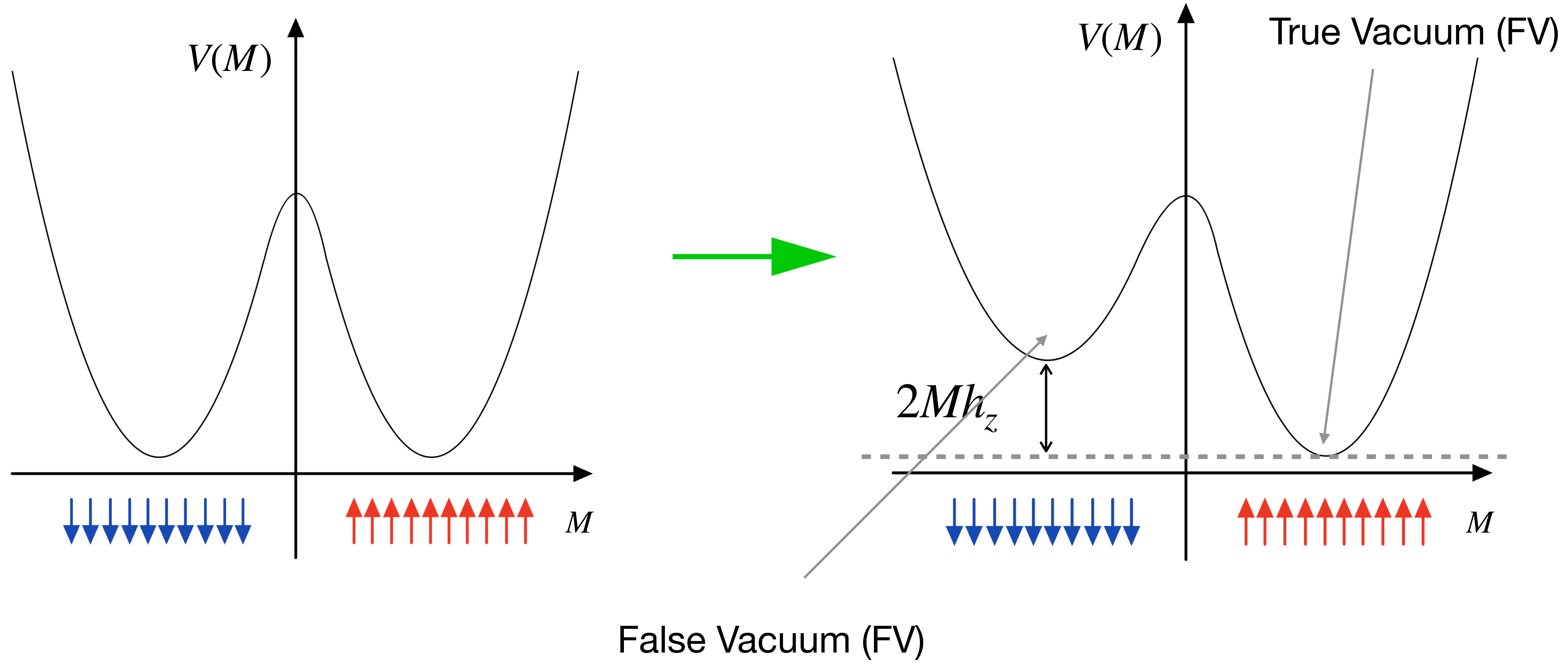

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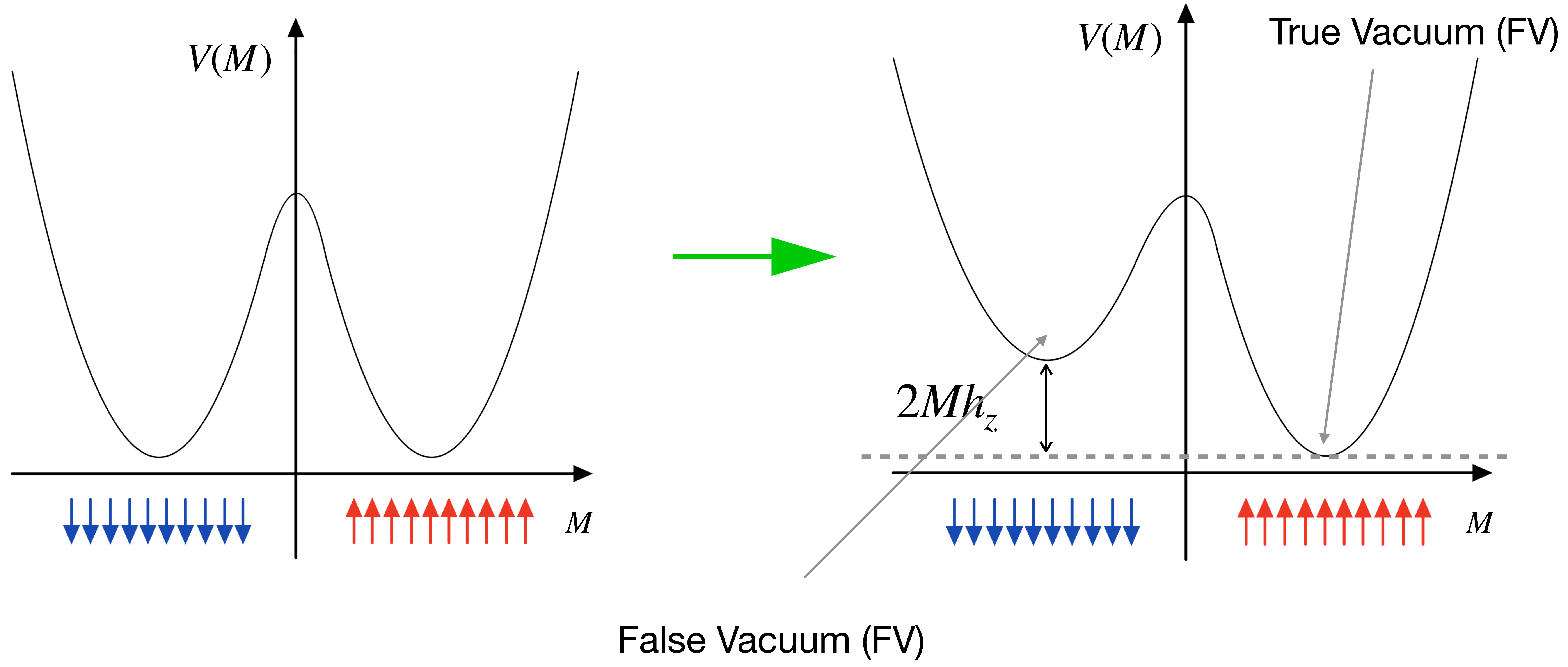
The longitudinal field h_z lifts the degeneracy → Distinctive effects in the dynamics



Confinement

The Quench Protocol

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1 - Prepare ground state $|\psi_0\rangle$ of $H_0 = H(g_0)$.

$g = (h_x, h_z)$

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2 - At $t = 0$ **suddenly** change parameters $g_0 \to g$. $H_1 = H(g)$

The Quench Protocol

1 - Prepare ground state $|\psi_0\rangle$ of $H_0 = H(g_0)$. $g = (h_x, h_z)$

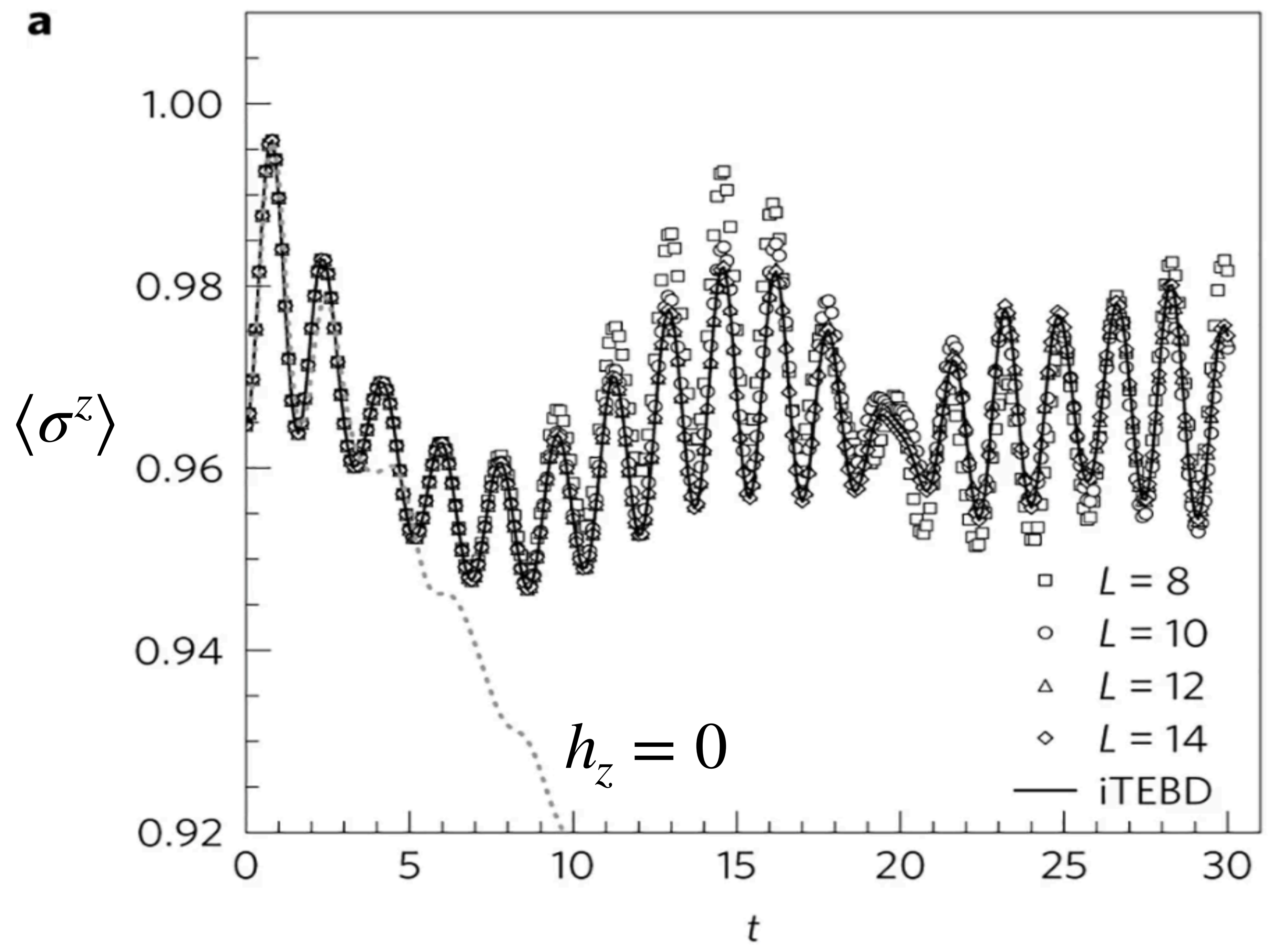
2 - At $t = 0$ **suddenly** change parameters $g_0 \to g$. $H_1 = H(g)$

3 - At $t > 0$ evolve with the new Hamiltonian $|\psi(t)\rangle = \exp(-iH_1 t)|\psi_0\rangle$

Quench Spectroscopy

$$\langle \sigma^z \rangle(t) = \sum_i \langle \psi(t) \,|\, \sigma_i^z \,|\, \psi(t) \rangle / L$$

$$h_x = 0.5, h_z = 0 \rightarrow h_x = 0.25, h_z = 0.1$$



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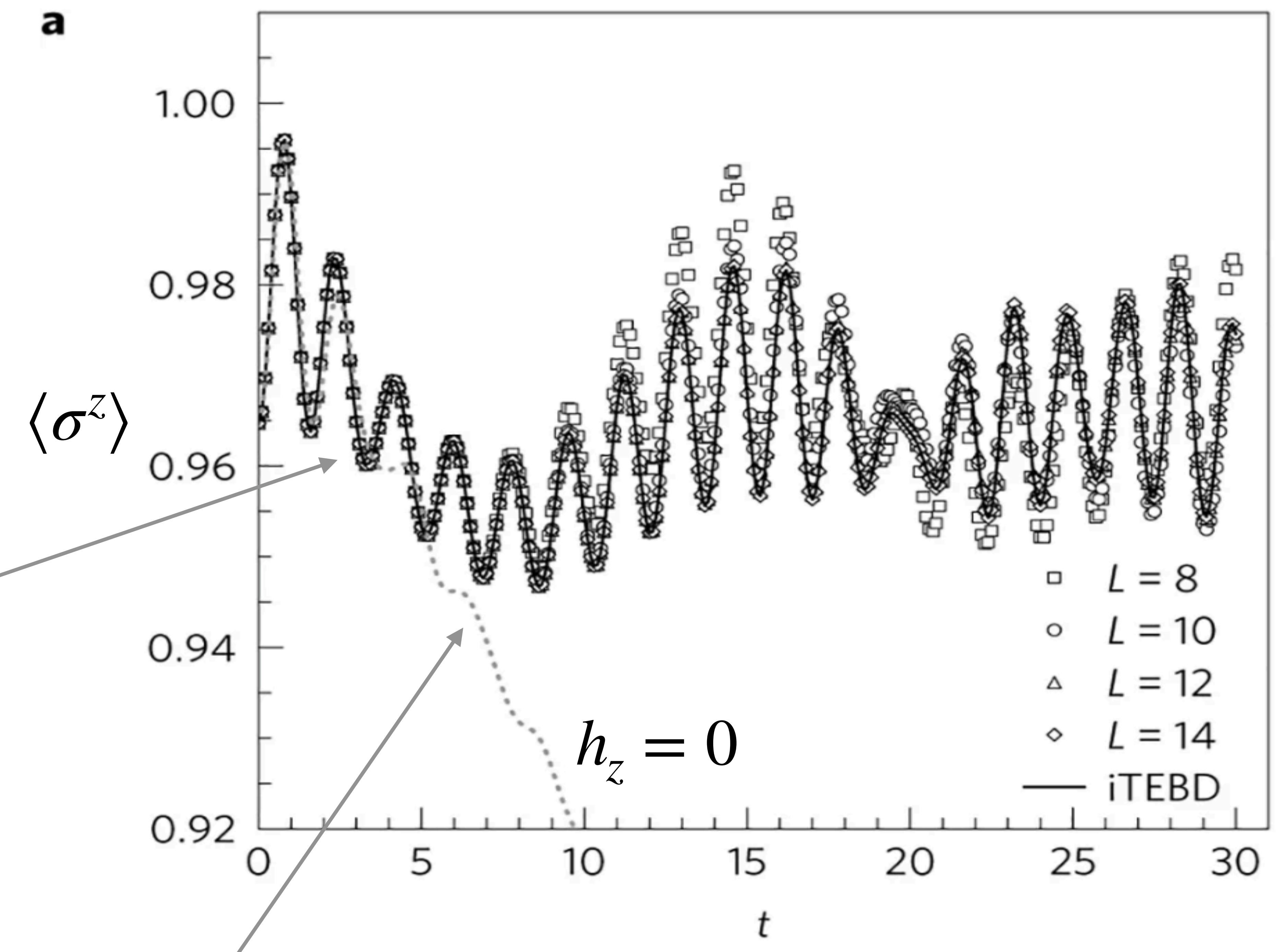


Oscillations + Slow relaxation

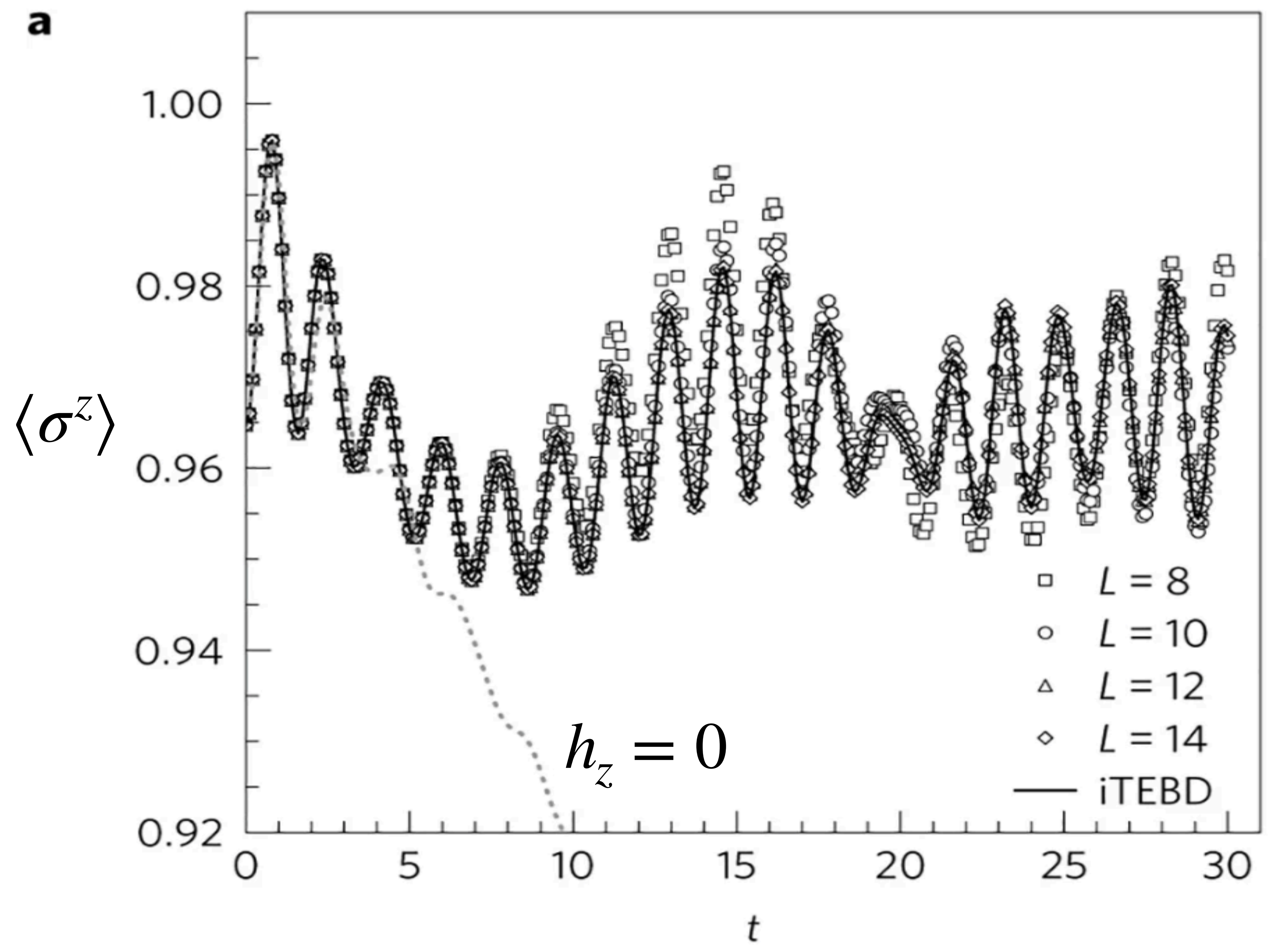
$$\langle \sigma^x \rangle(t) \approx e^{-kt}$$

Calabrese, Essler, Fagotti PRL 2011

Quench Spectroscopy

$$\langle \sigma^z \rangle(t) = \sum_i \langle \psi(t) | \sigma_i^z | \psi(t) \rangle / L$$

$|\Psi_0\rangle$ is not an eigenstate of H_1



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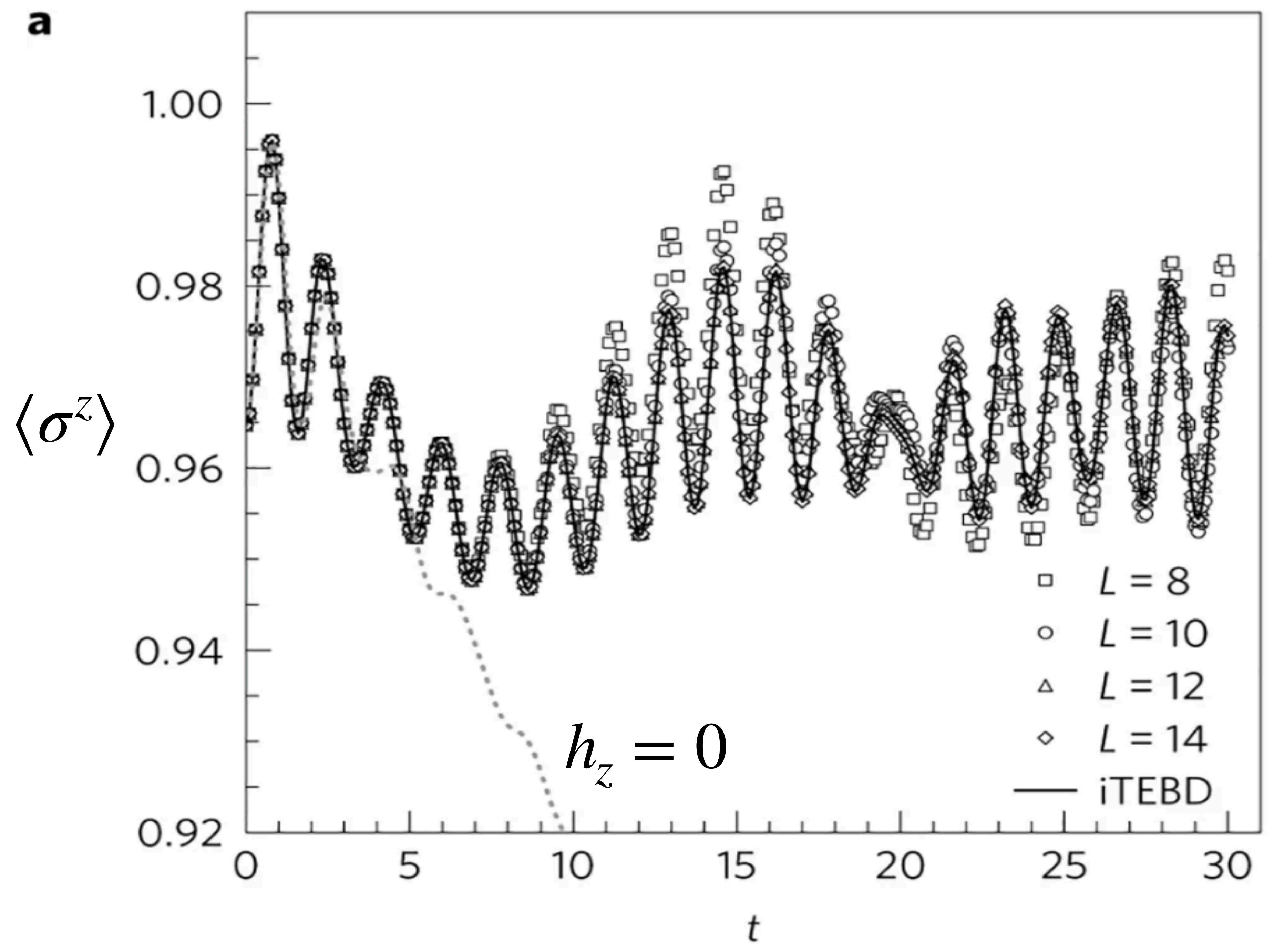

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$$\Rightarrow |\psi(t)\rangle = \exp(-iHt)|\psi_0\rangle = \sum_i a_i e^{-iE_i t} |e_i\rangle \qquad a_i = \langle e_i | \psi \rangle \qquad H_1 |e_i\rangle = E_i |e_i\rangle$$

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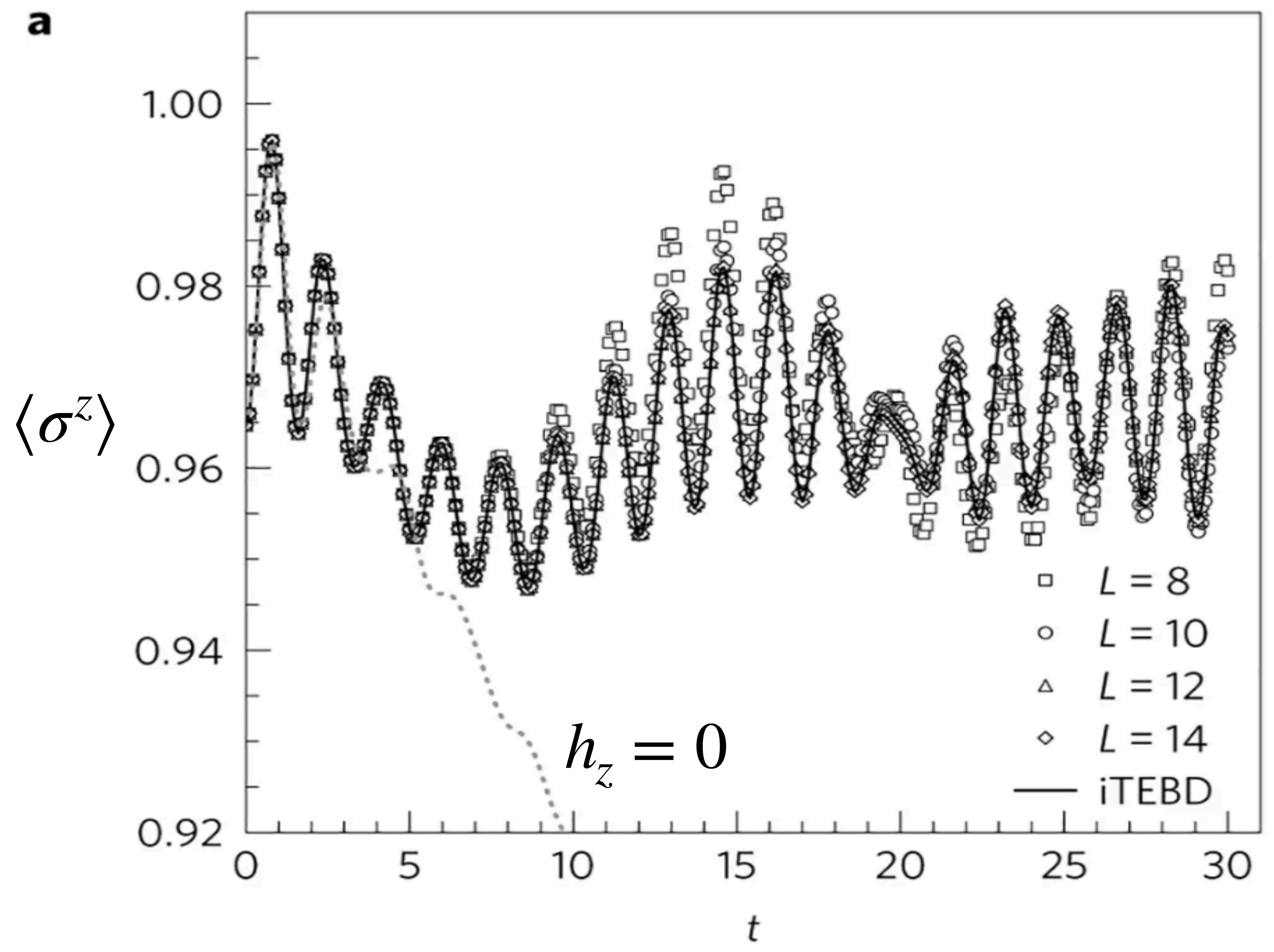

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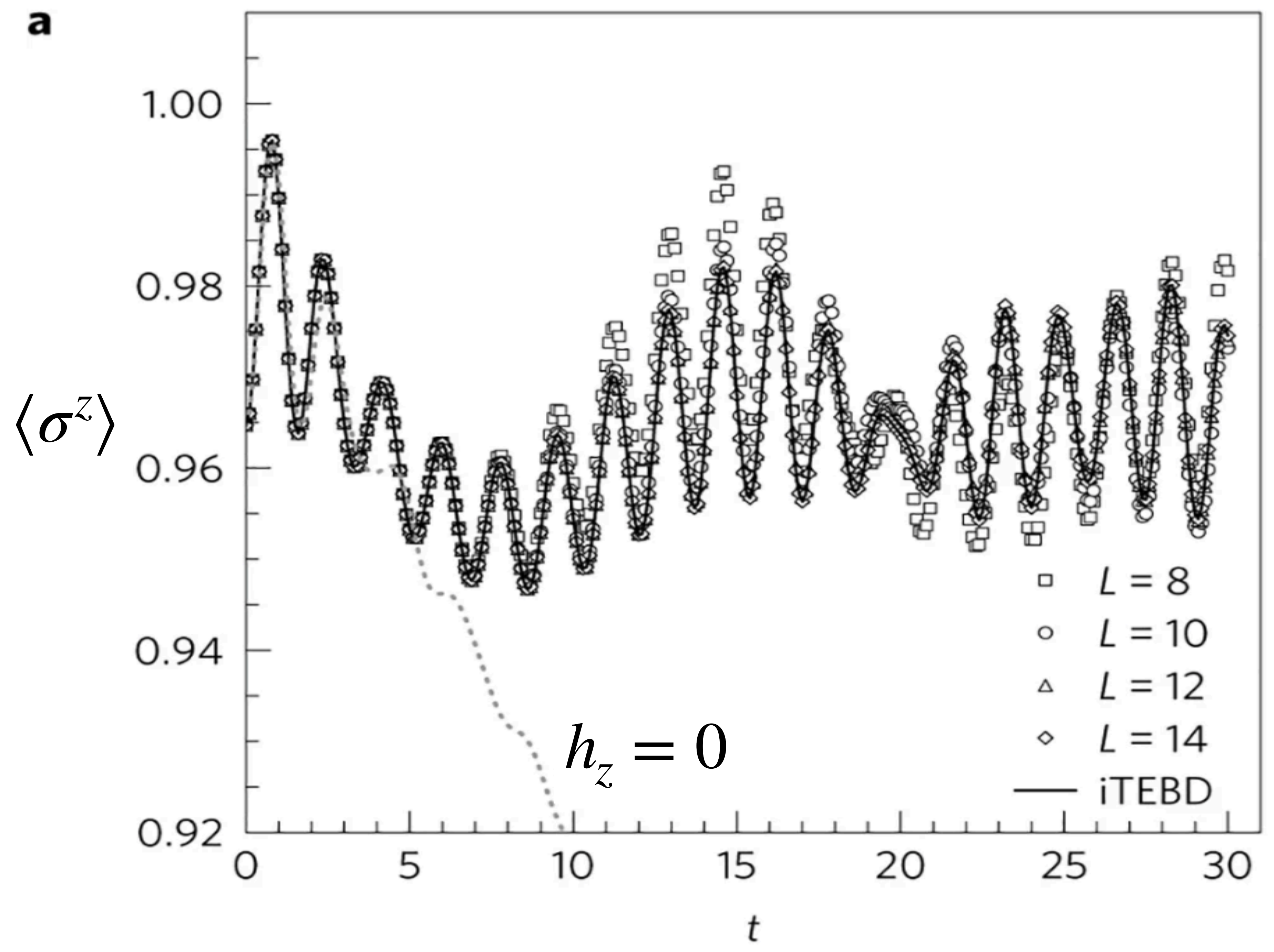
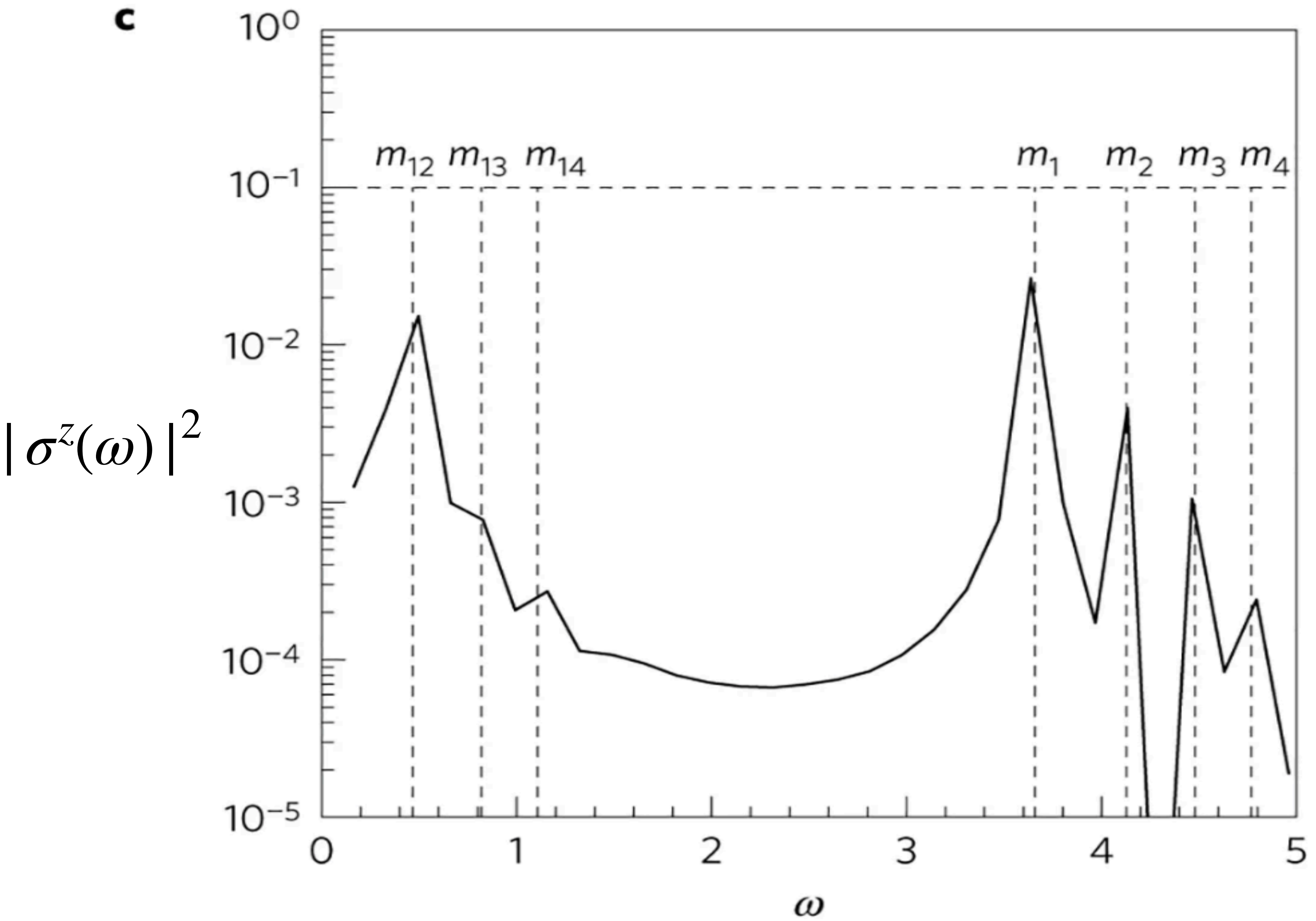
$$\Rightarrow \langle \sigma^z \rangle(t) = \sum_{ij} a_i a_j^* e^{-i(E_i - E_j)} \langle e_j | \sigma^z | e_i \rangle$$

Quench Spectroscopy

$$\langle \sigma^z \rangle(t) = \sum_i \langle \psi(t) | \sigma_i^z | \psi(t) \rangle / L$$

$$|\langle \sigma^z \rangle(\omega)|^2 = \left| \int dt e^{i\omega t} \langle \sigma^z \rangle(t) \right|^2$$





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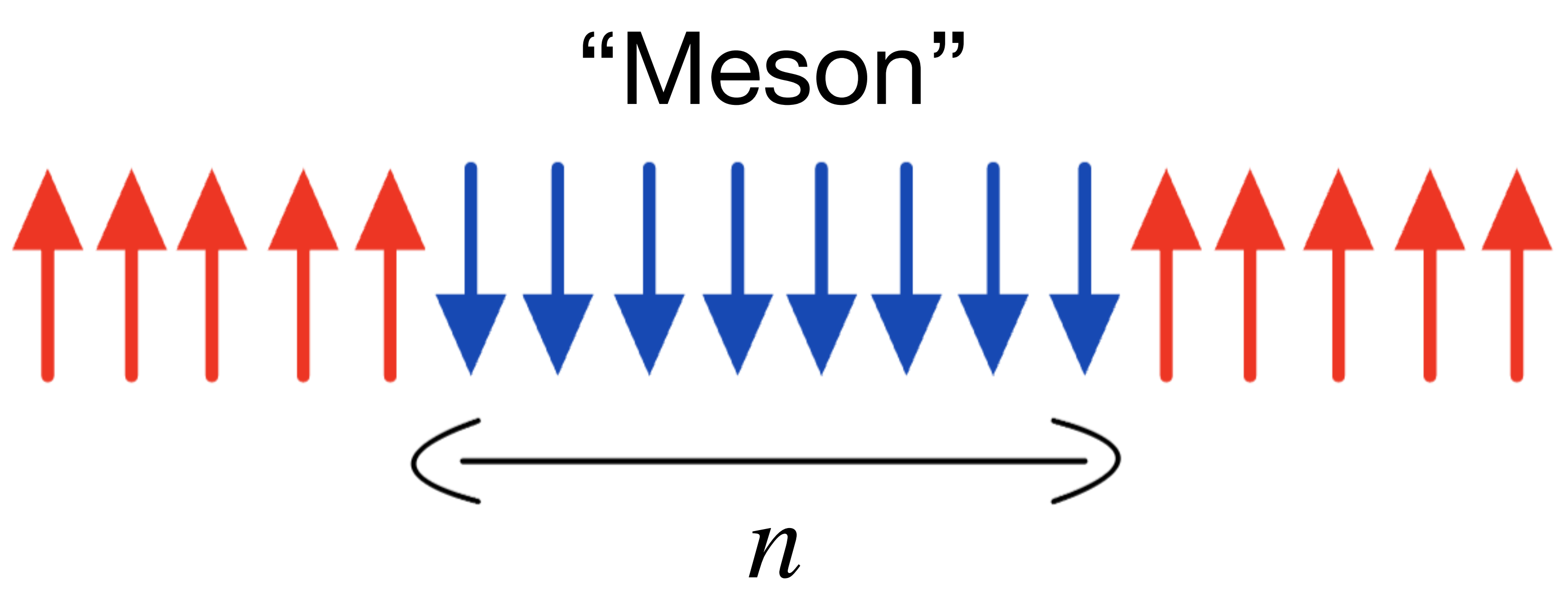
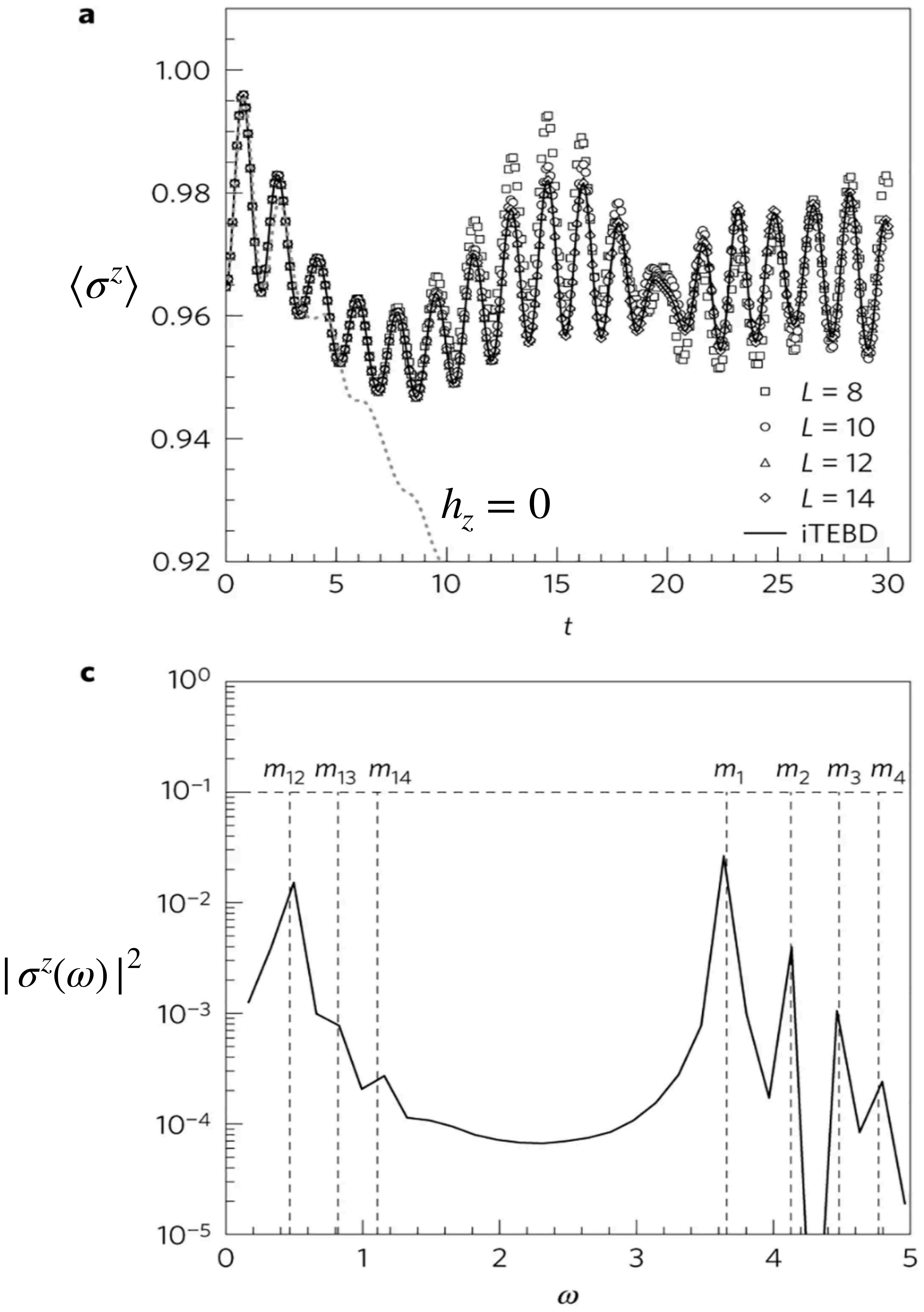


$$E_n = 2\Delta + 2Mh_z\, n$$



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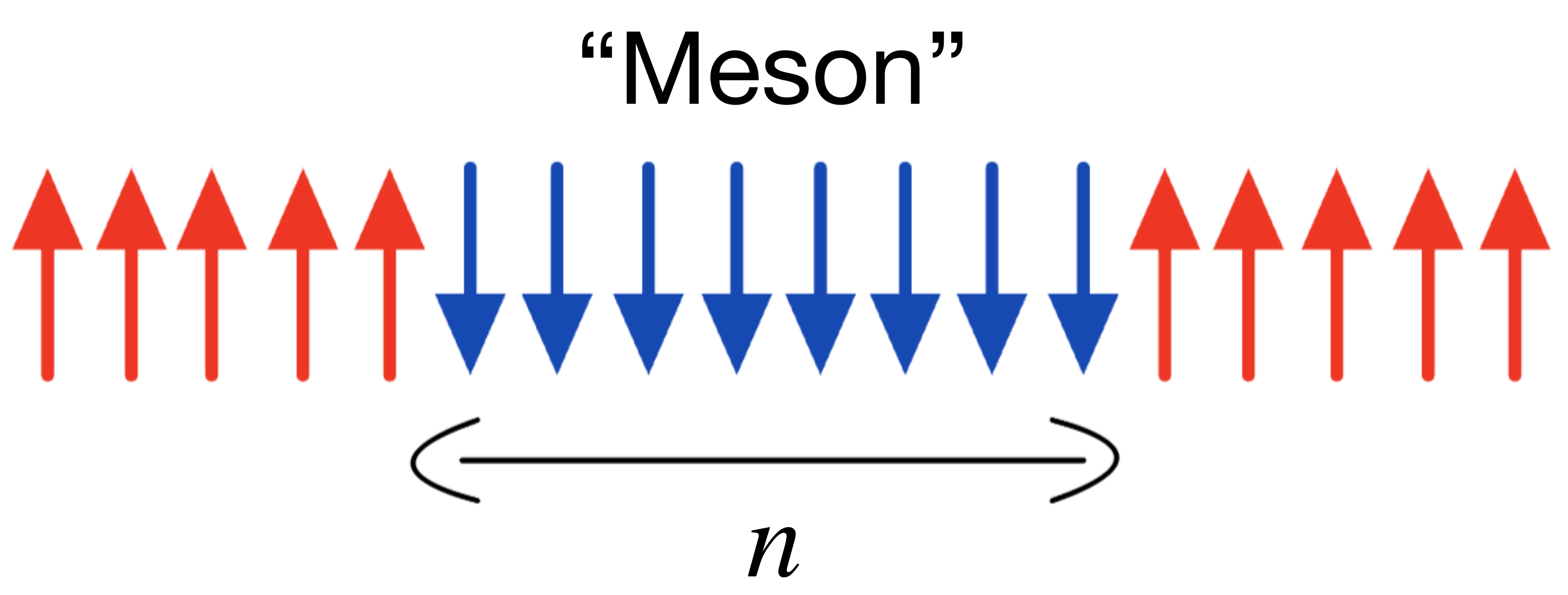

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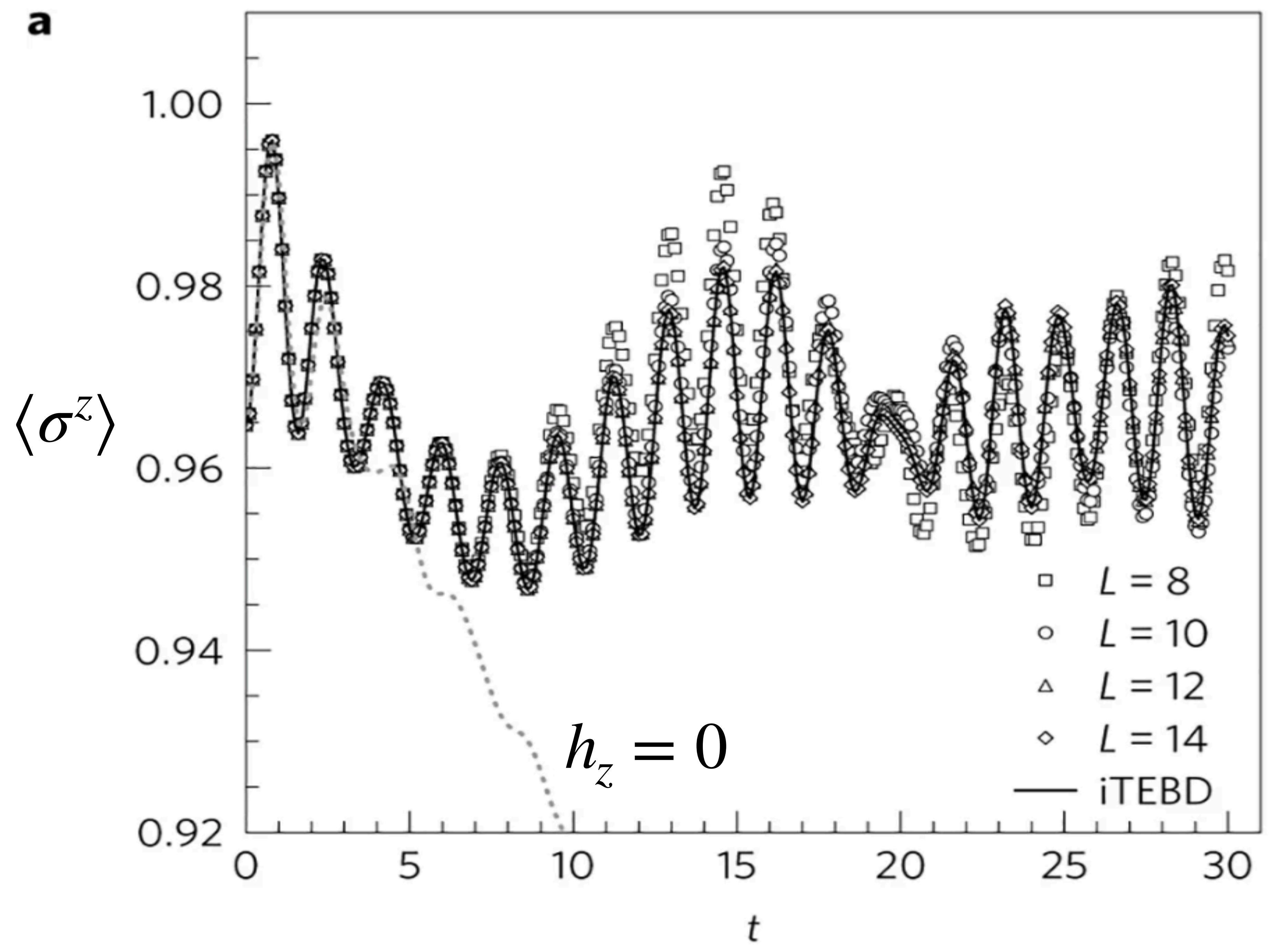
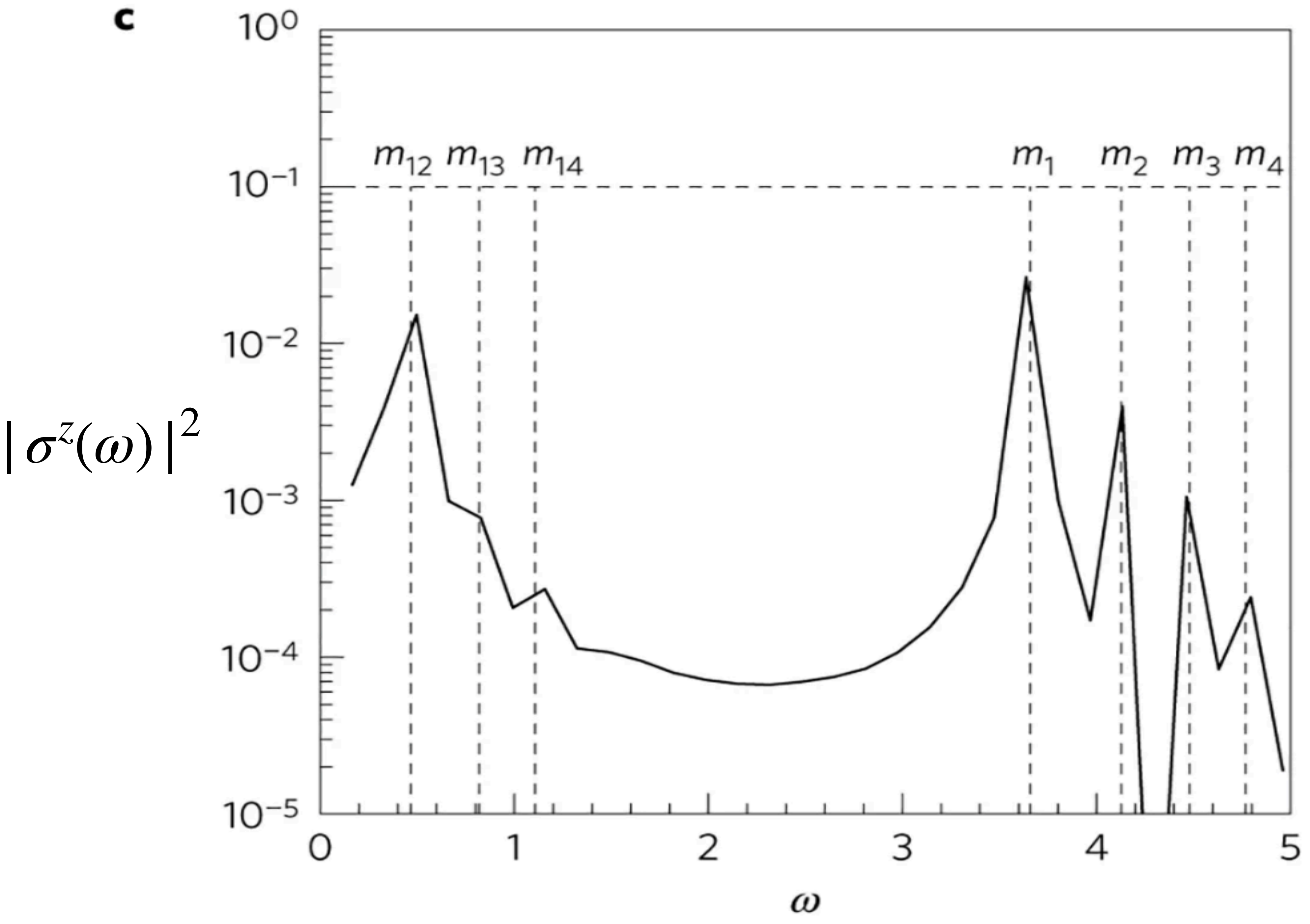
Kink's mass

Energy density





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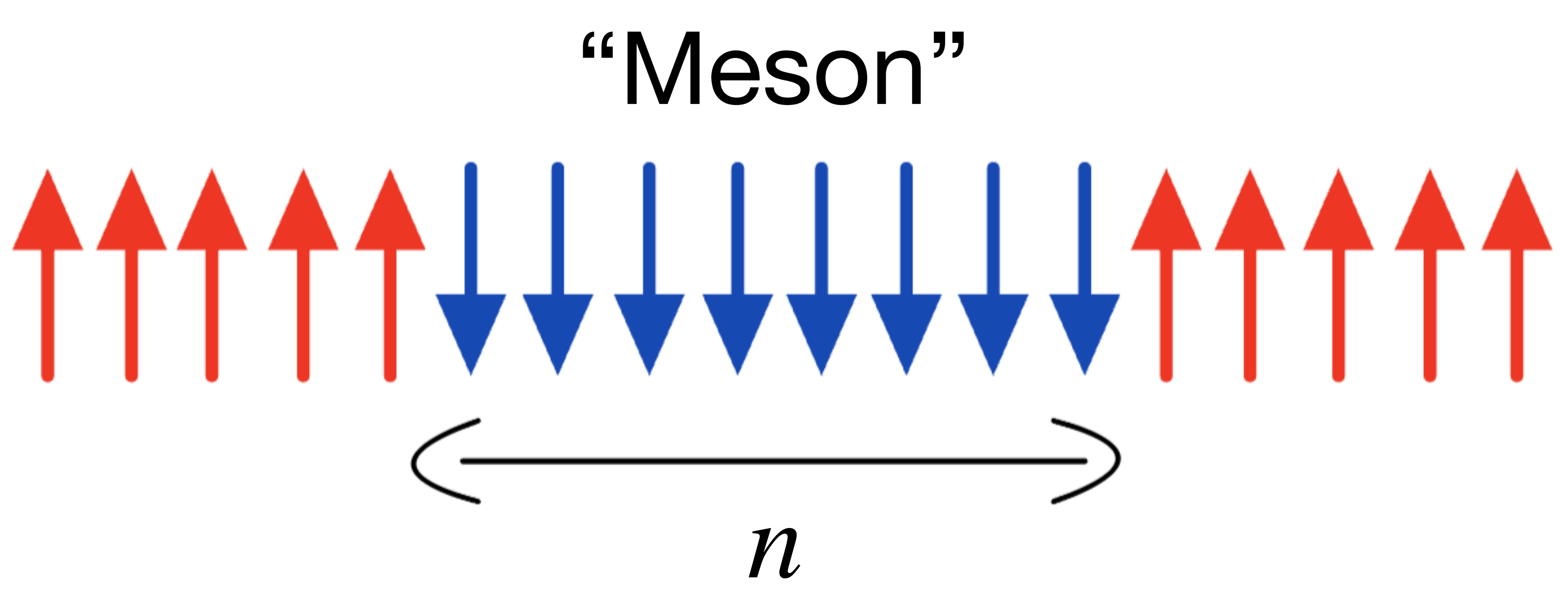
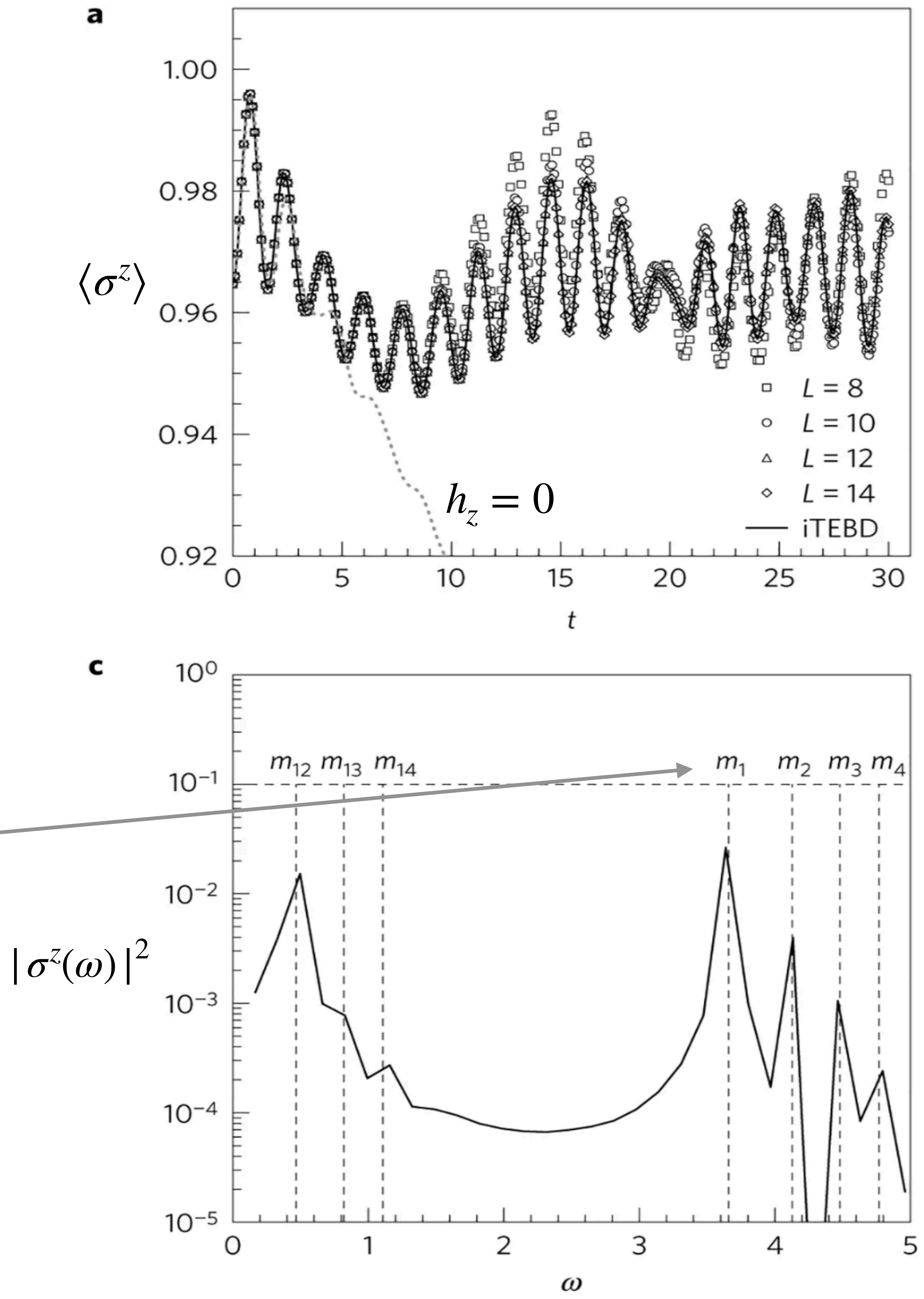


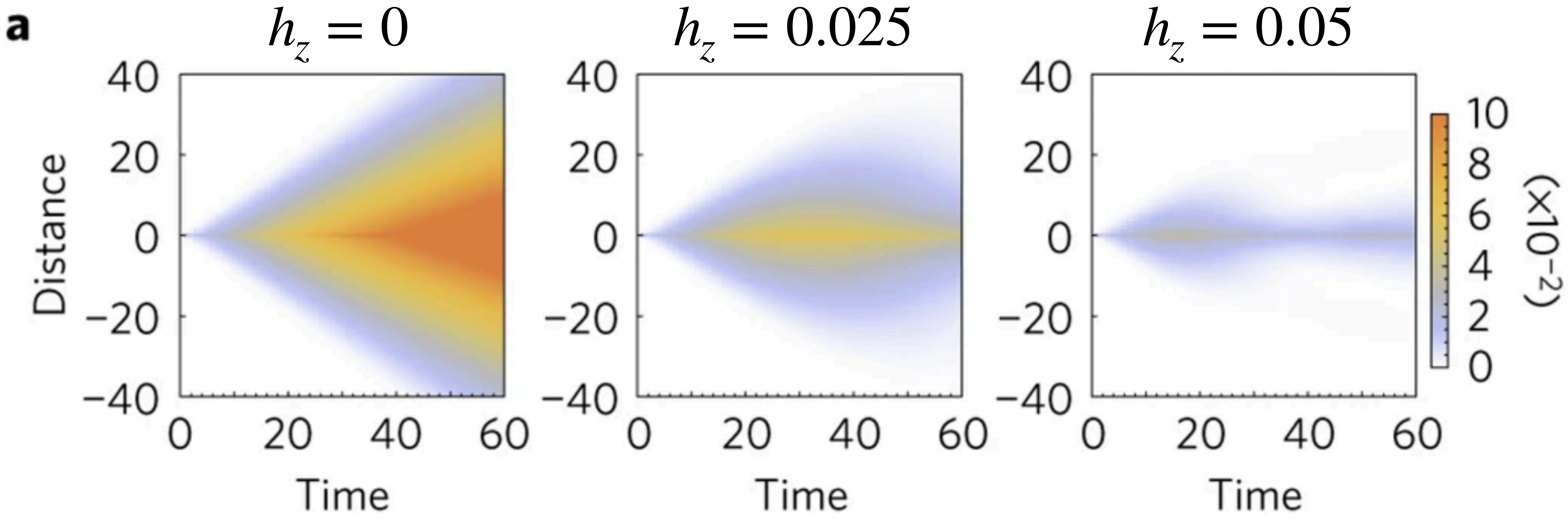
$$E_n = 2\Delta + 2Mh_z\, n$$

Effective two-body model:

$$H_{eff} = \varepsilon_{k_1} + \varepsilon_{k_2} + 2Mh_z\, n$$

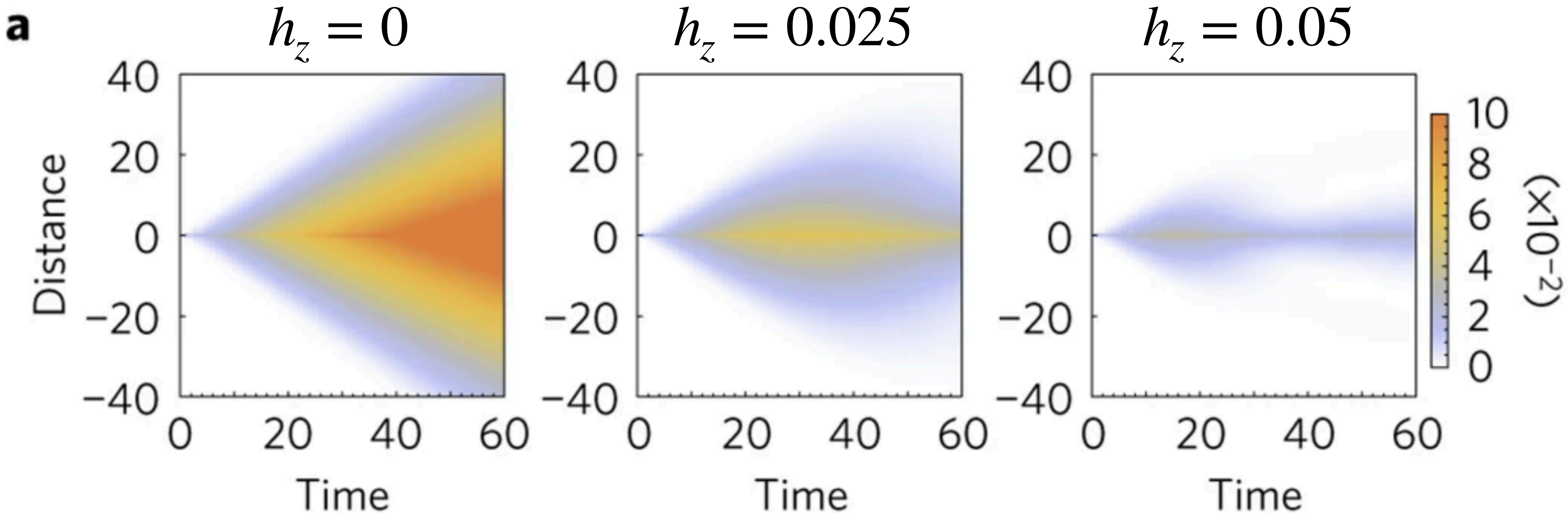


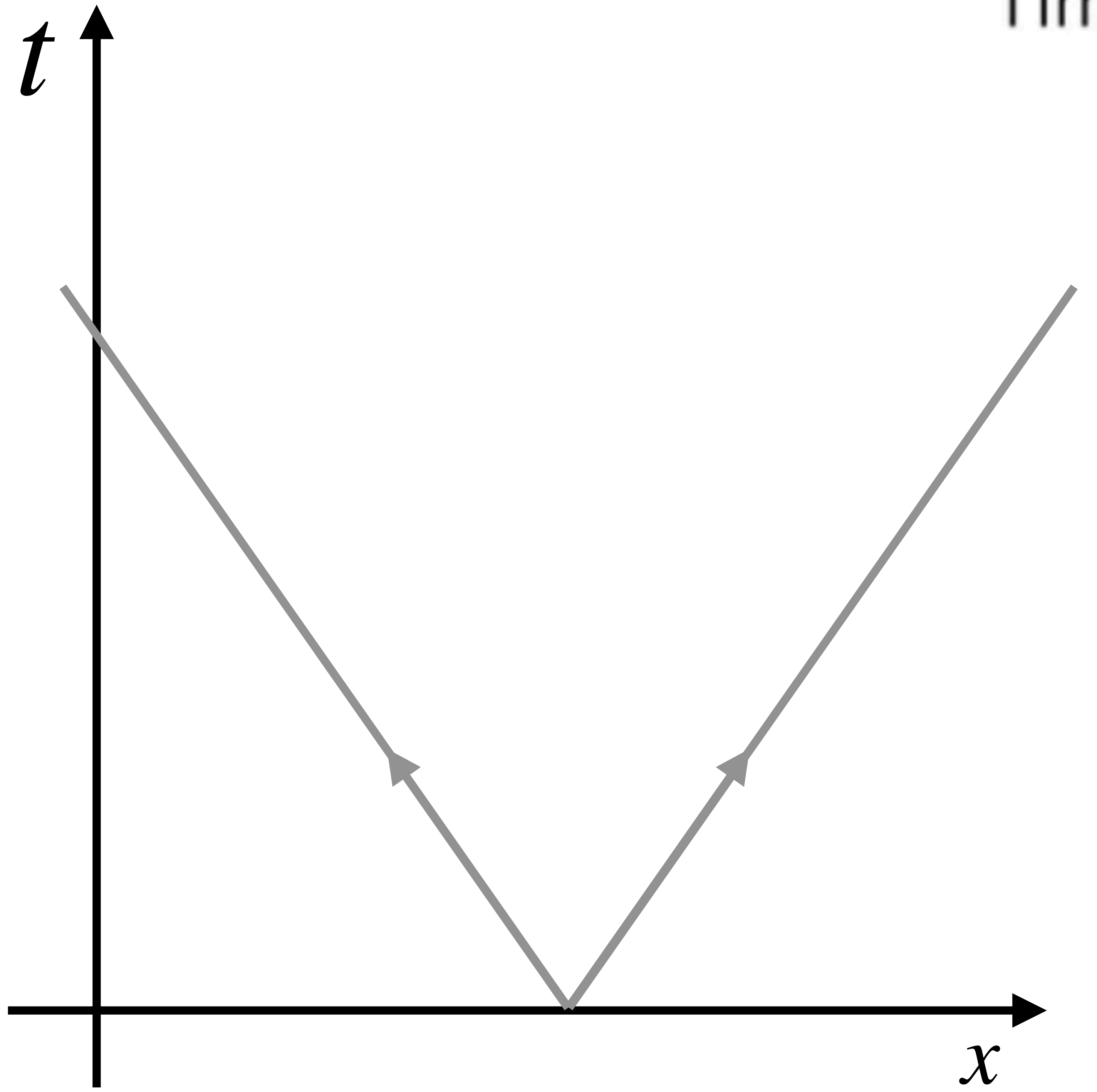
Connected Correlations



$$\langle \sigma_1^z \sigma_{1+d}^z \rangle_c = \langle \sigma_1^z \sigma_{1+d}^z \rangle - \langle \sigma_1^z \rangle \langle \sigma_{1+d}^z \rangle$$

Quasi-particle Picture

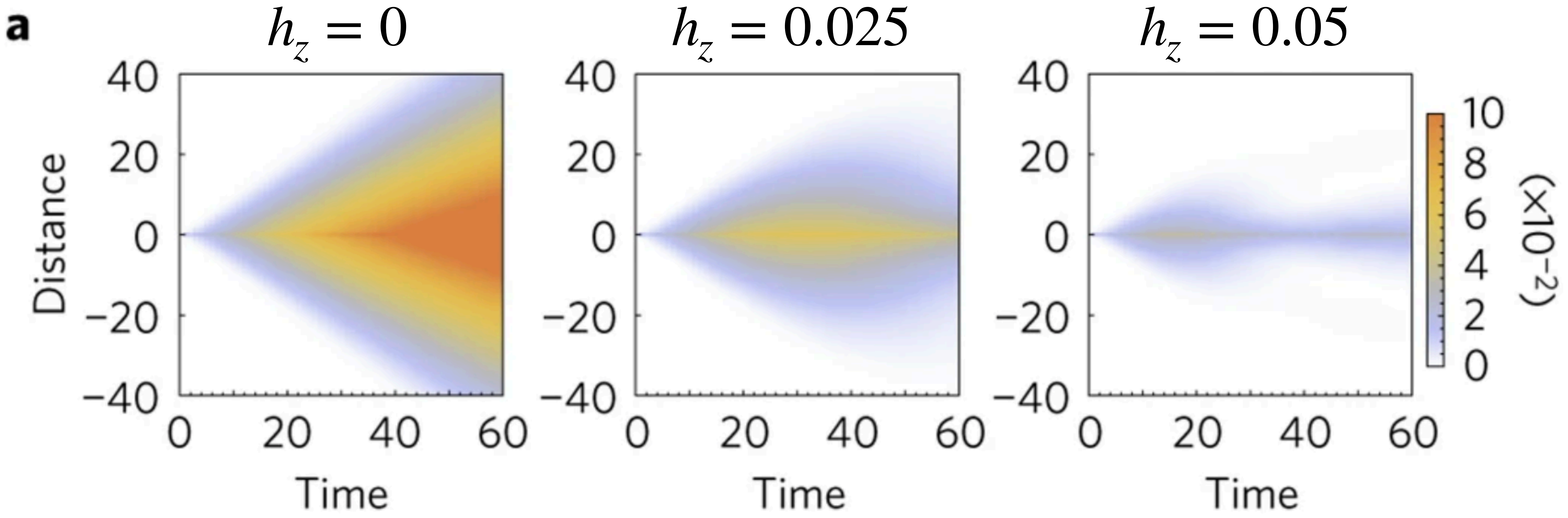


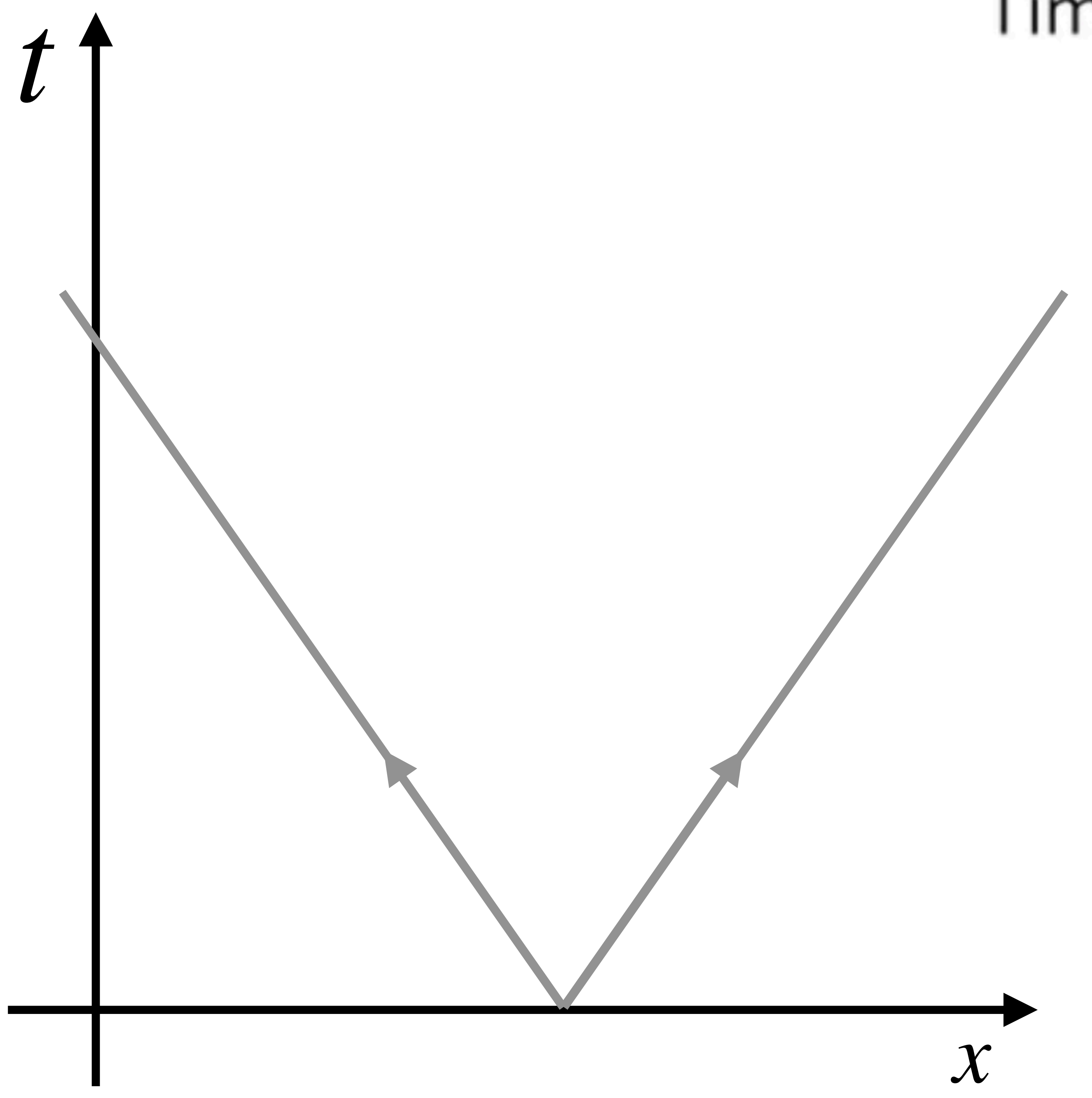


$|\Psi_0\rangle$ acts as a source of excitations

Calabrese, Cardy PRL 2006

Quasi-particle Picture





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And

$$v_k = \frac{d\varepsilon}{dk} \qquad v_{max} \geq v_k \ \forall k$$

Lieb-Robinson bound

Calabrese, Cardy PRL 2006

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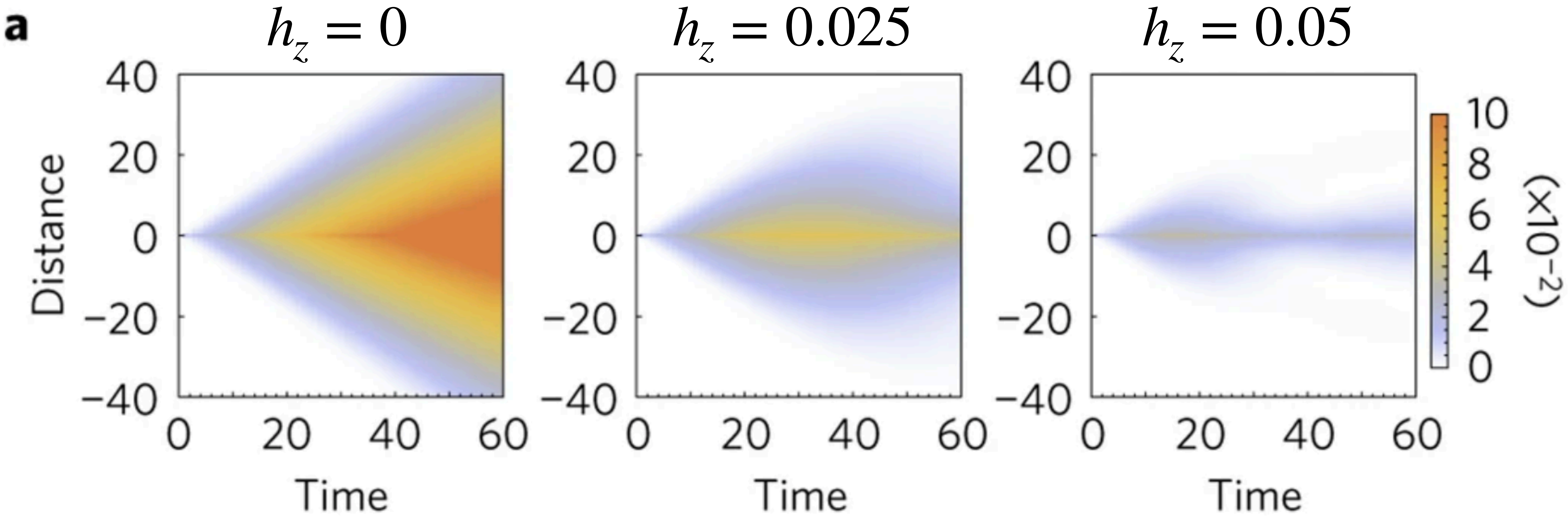

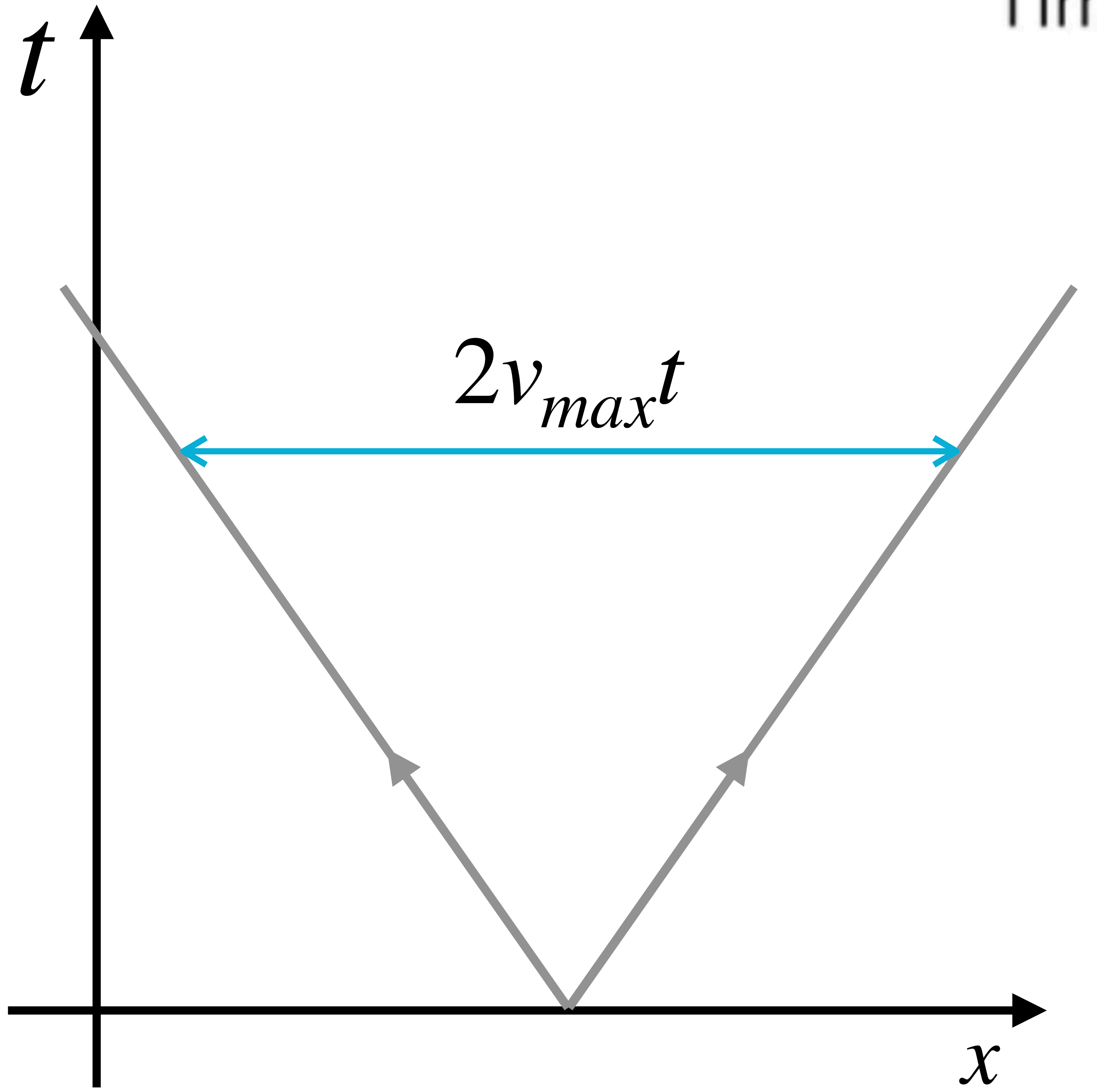

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Lieb-Robinson bound

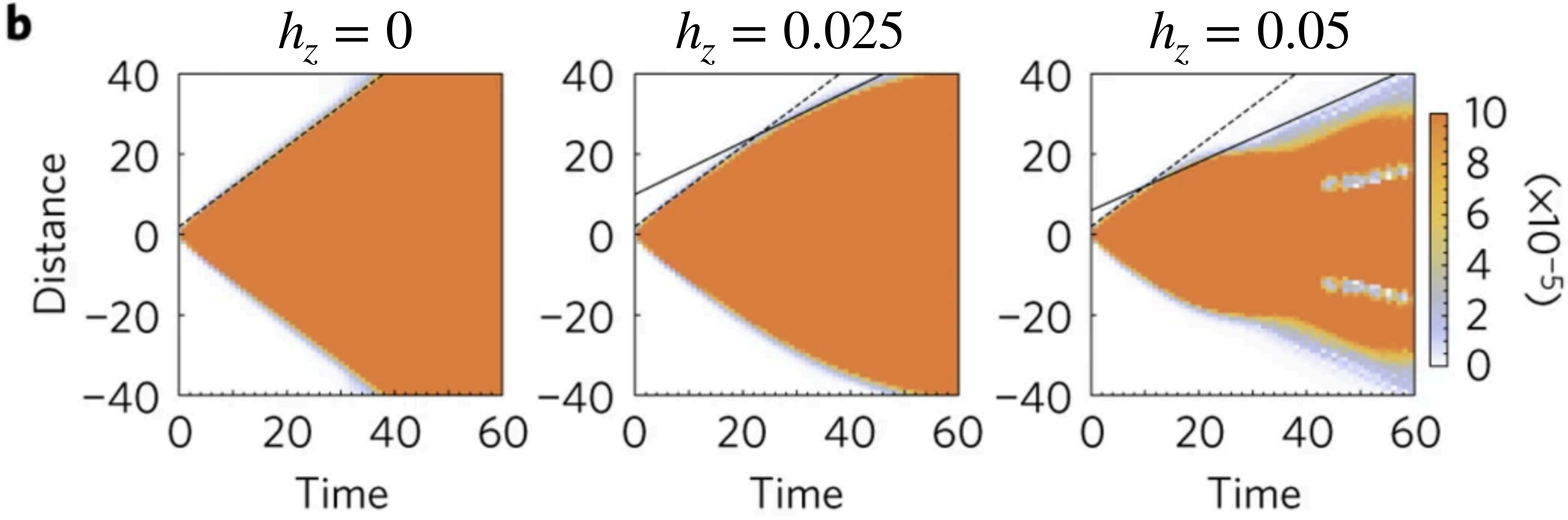
$\Rightarrow$

All correlators vanishing for

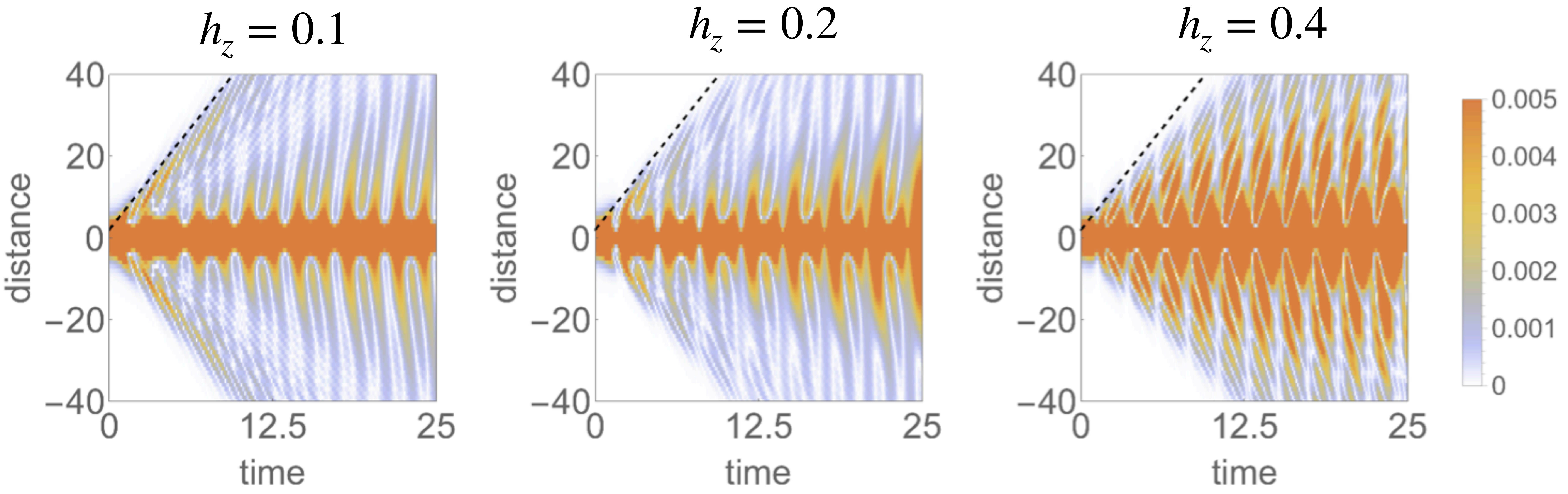
$$d > 2v_{max}t$$

Calabrese, Cardy PRL 2006

Secondary light cone

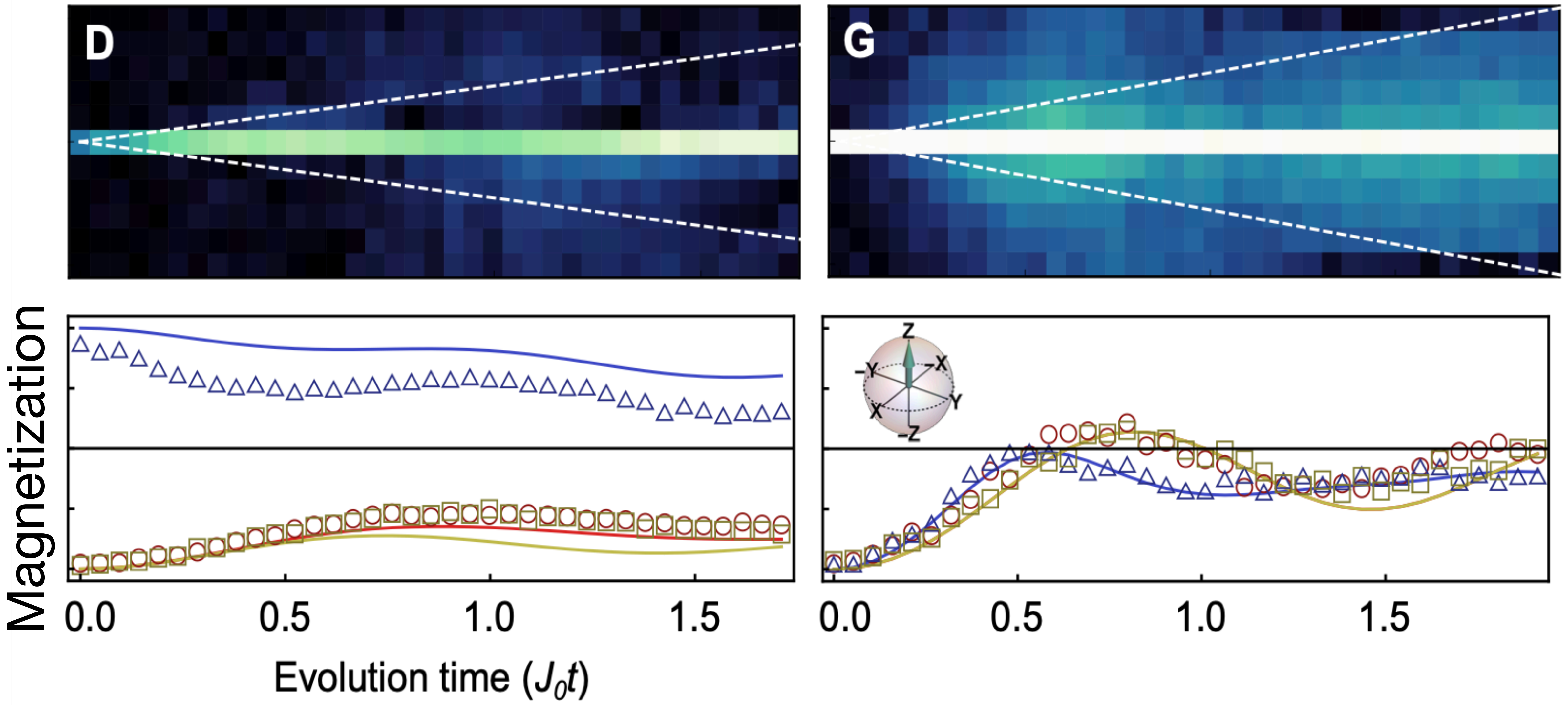


Absence for $h_x > 1$



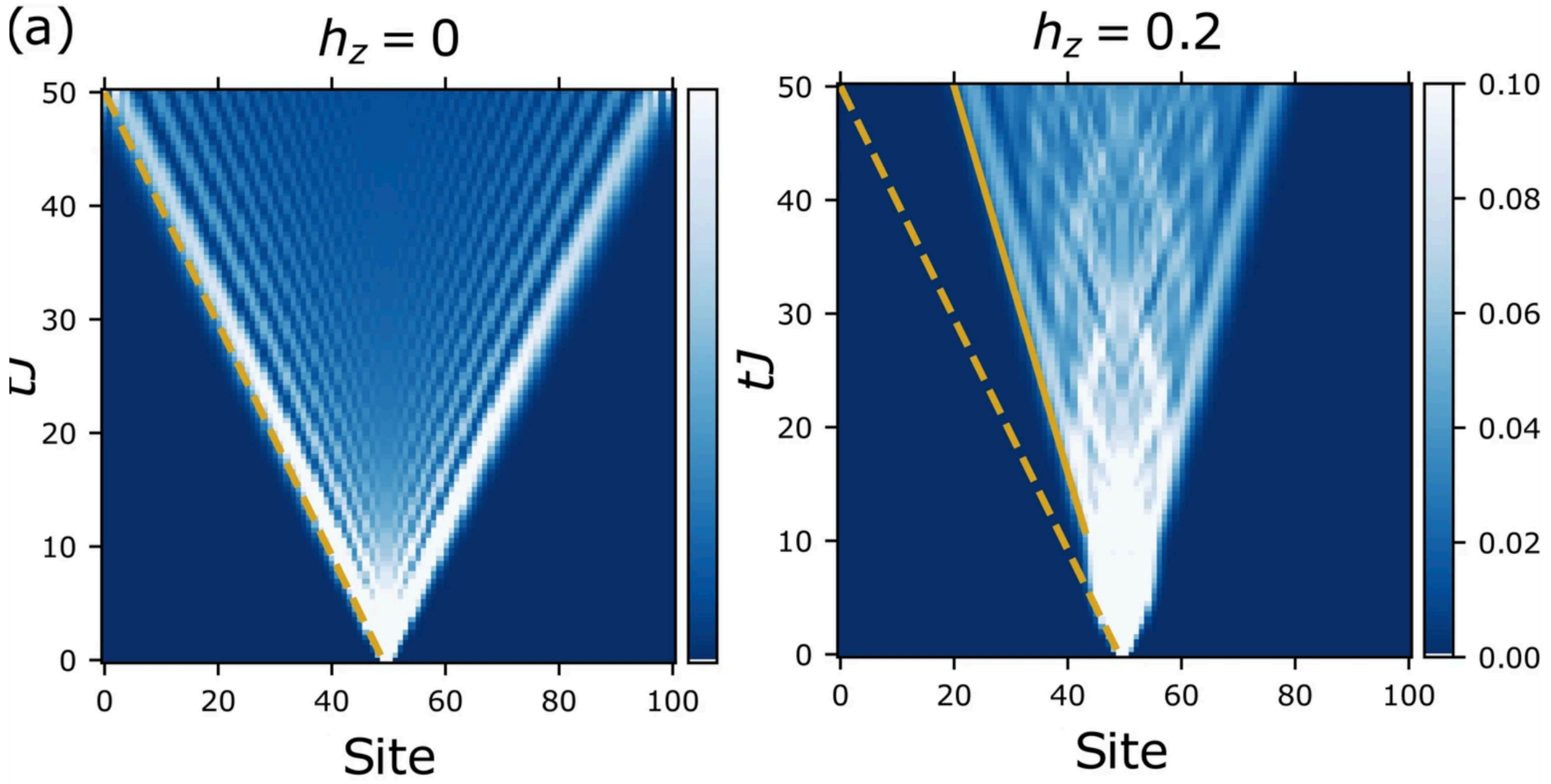
Experiments

Trapped Ions



W.L. Tan et al Nature Physics 2021

IBM Quantum computer



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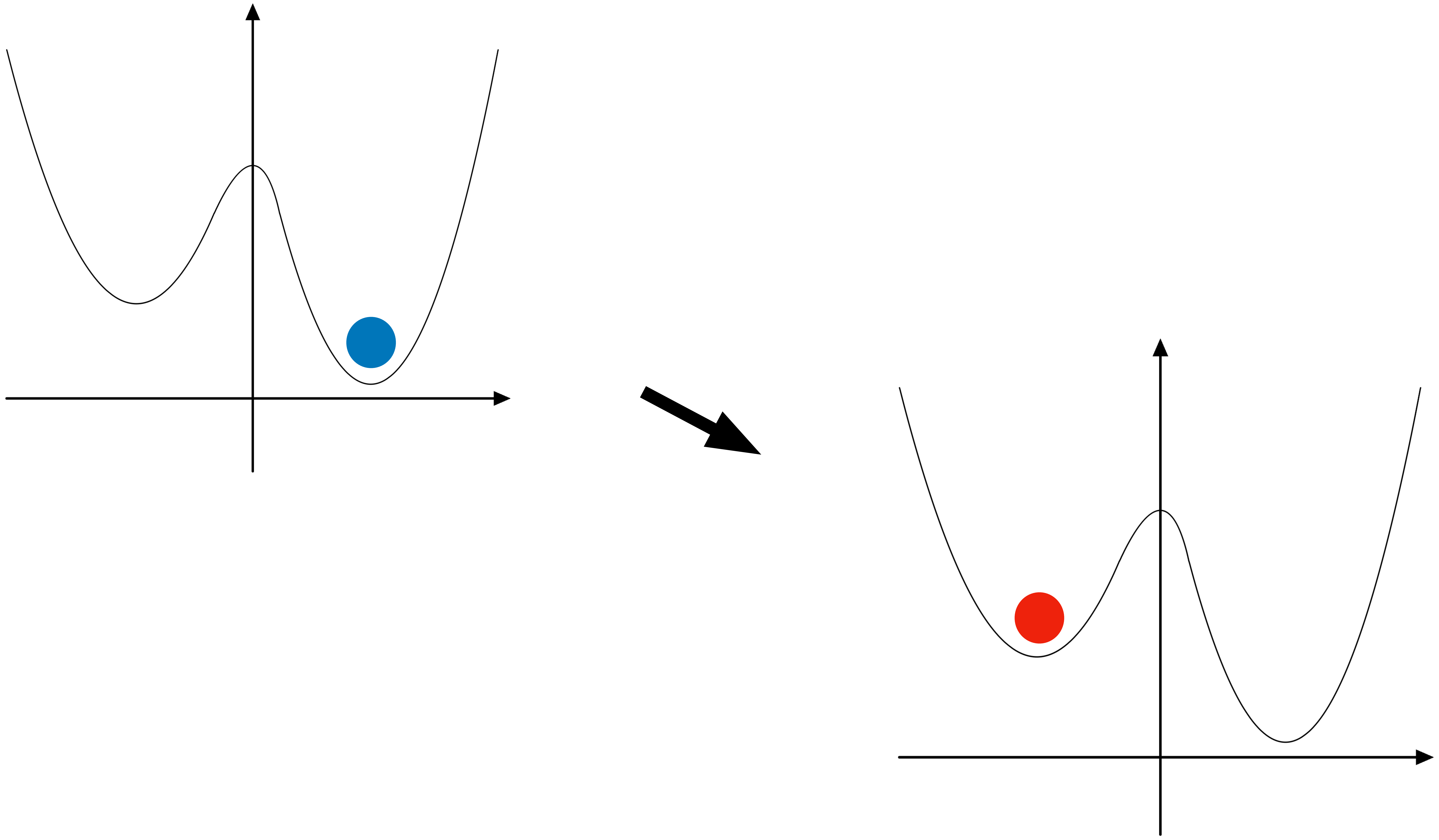
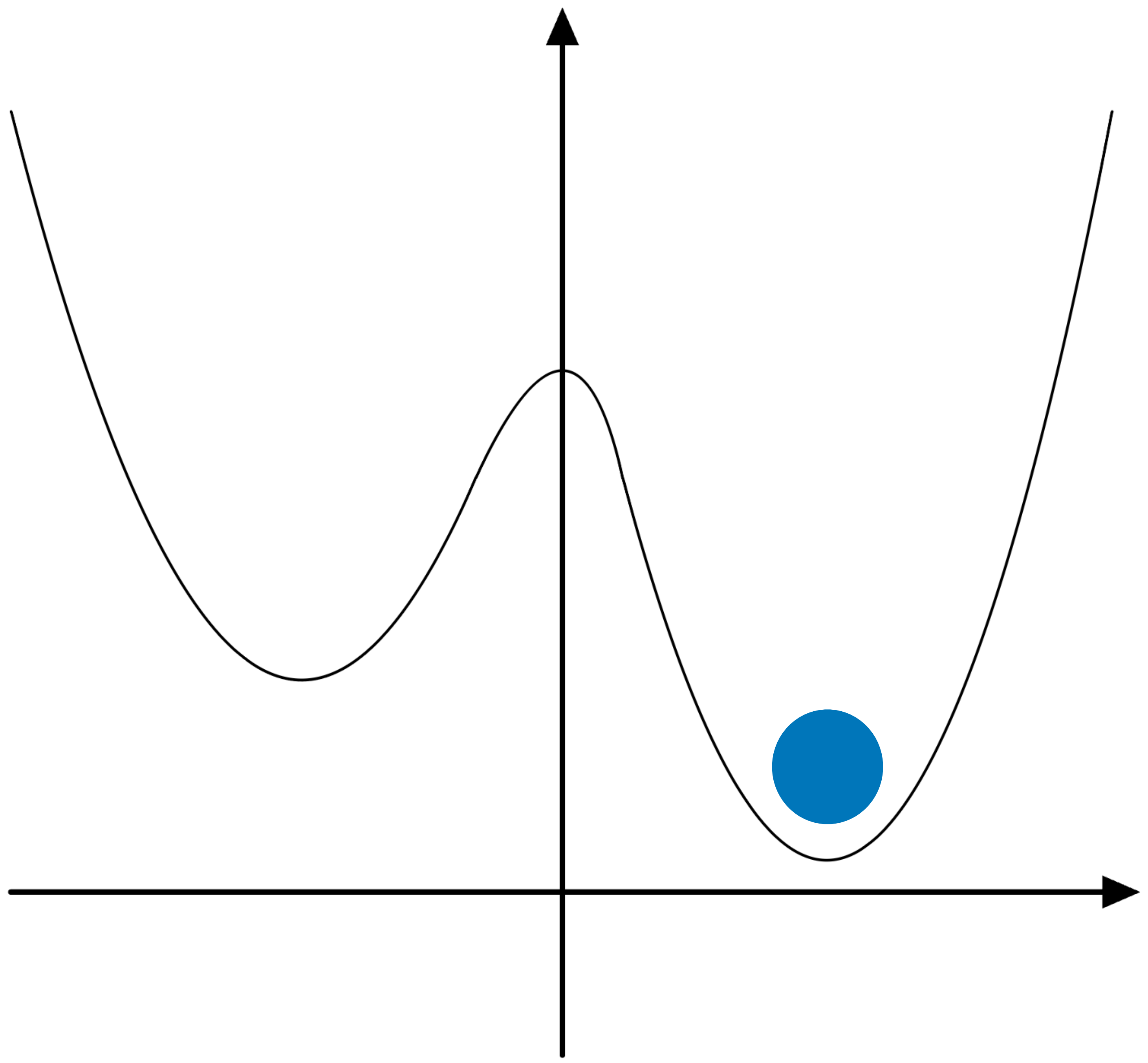
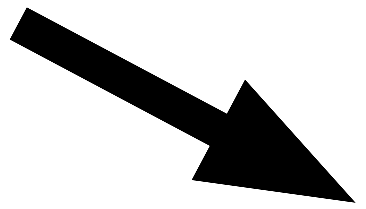
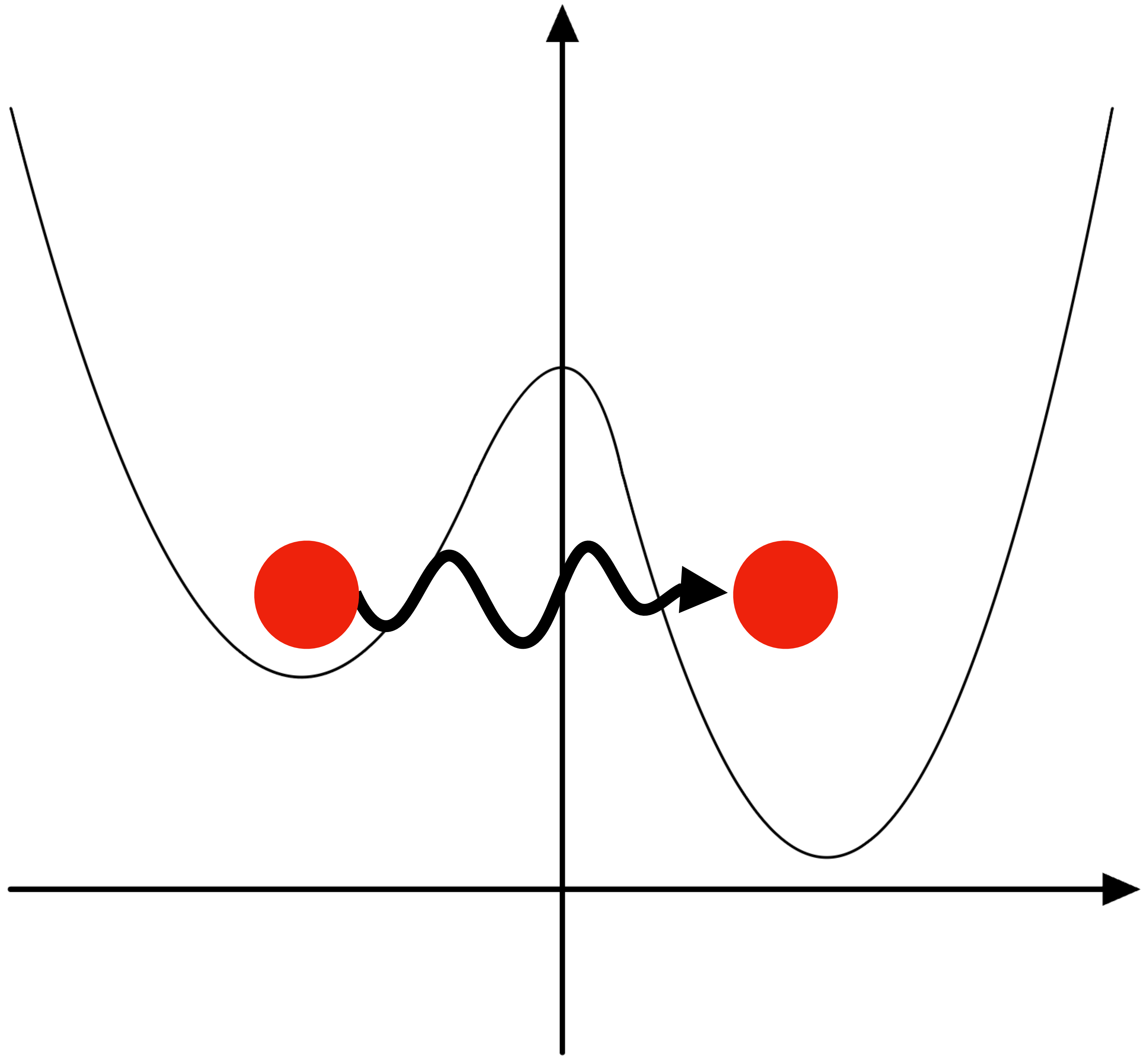
False Vacuum Decay

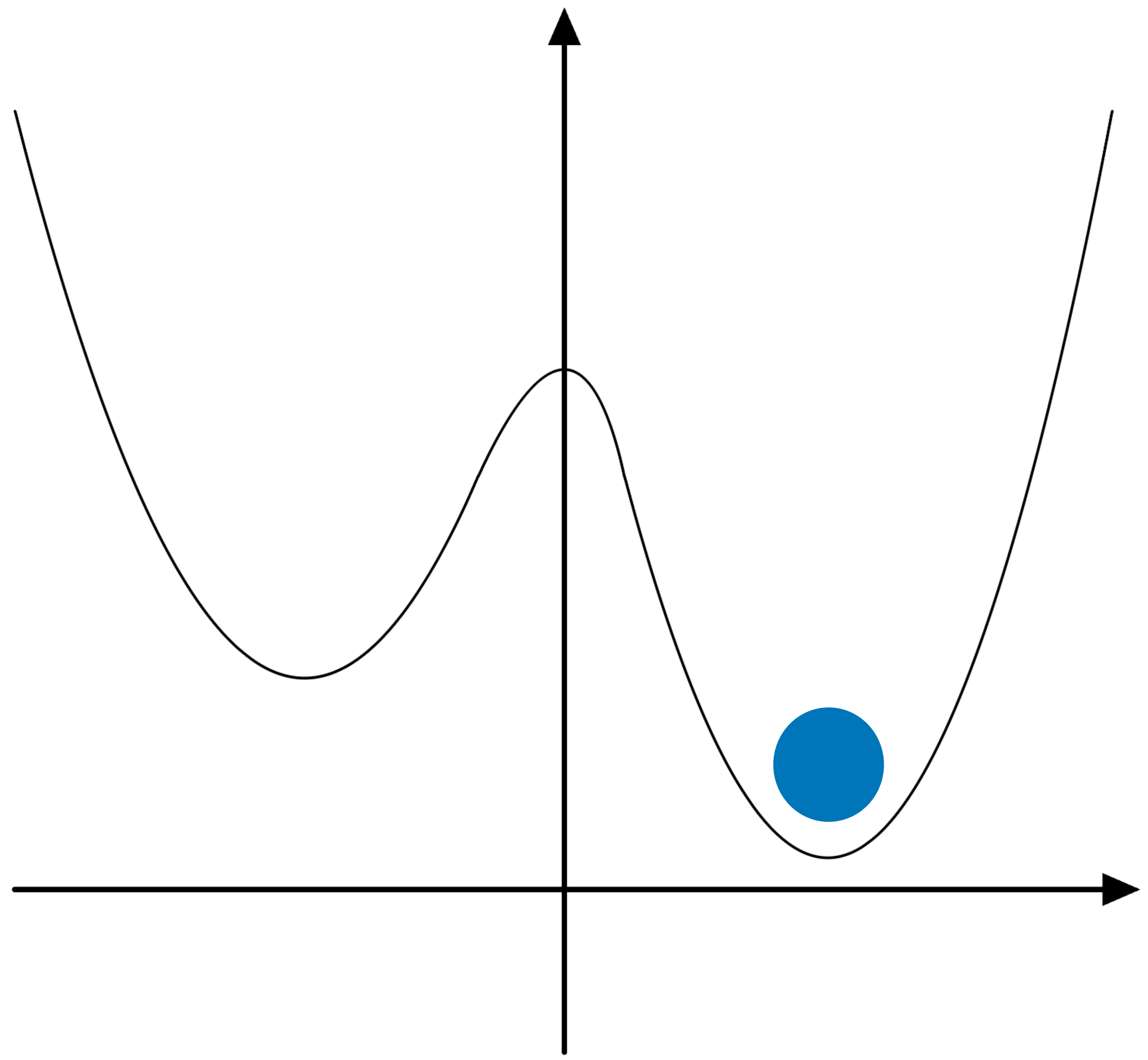


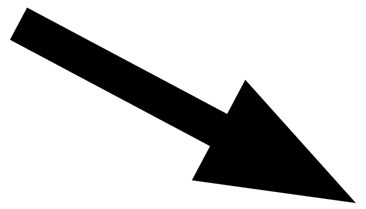




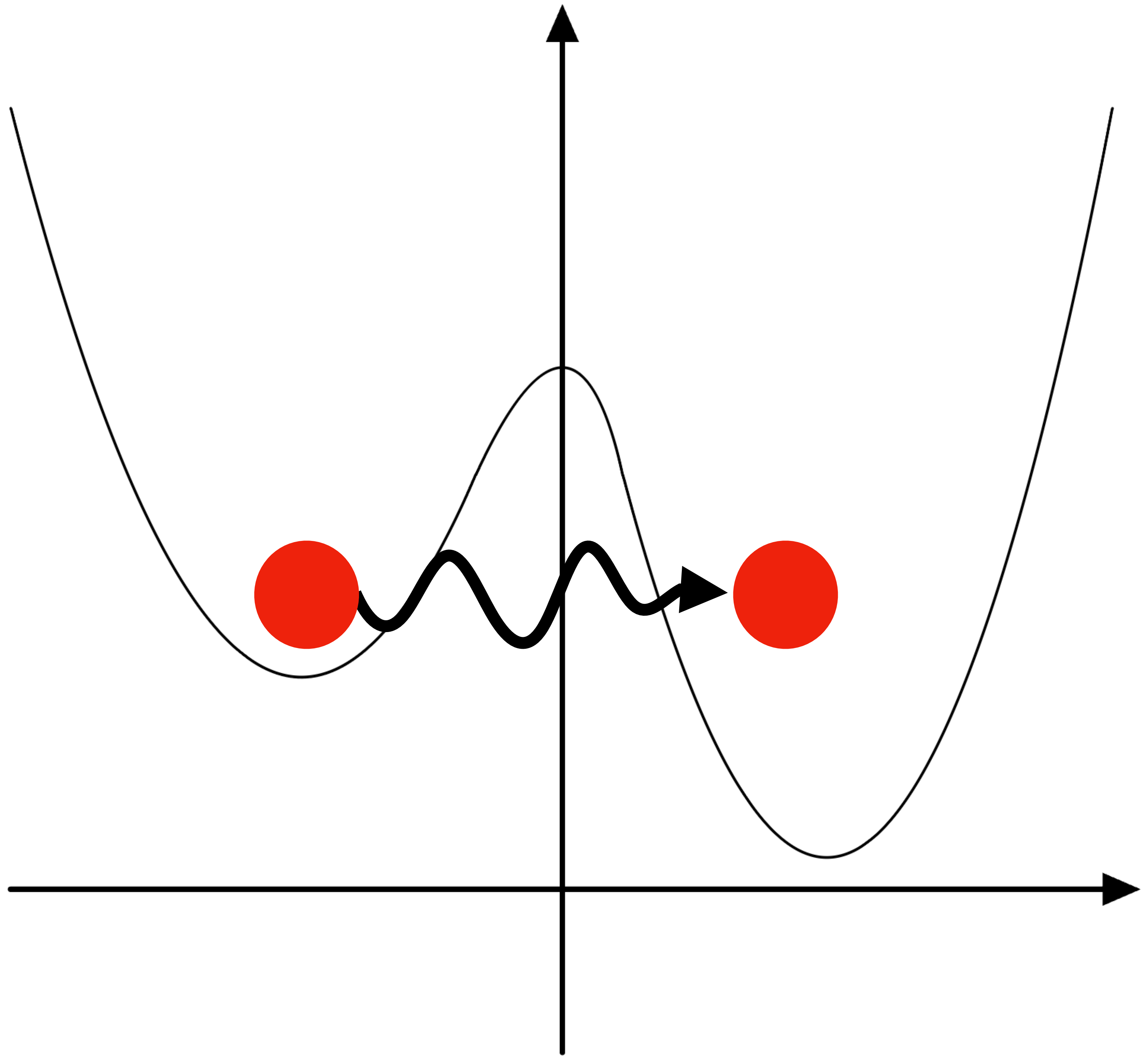
Decay by quantum fluctuations





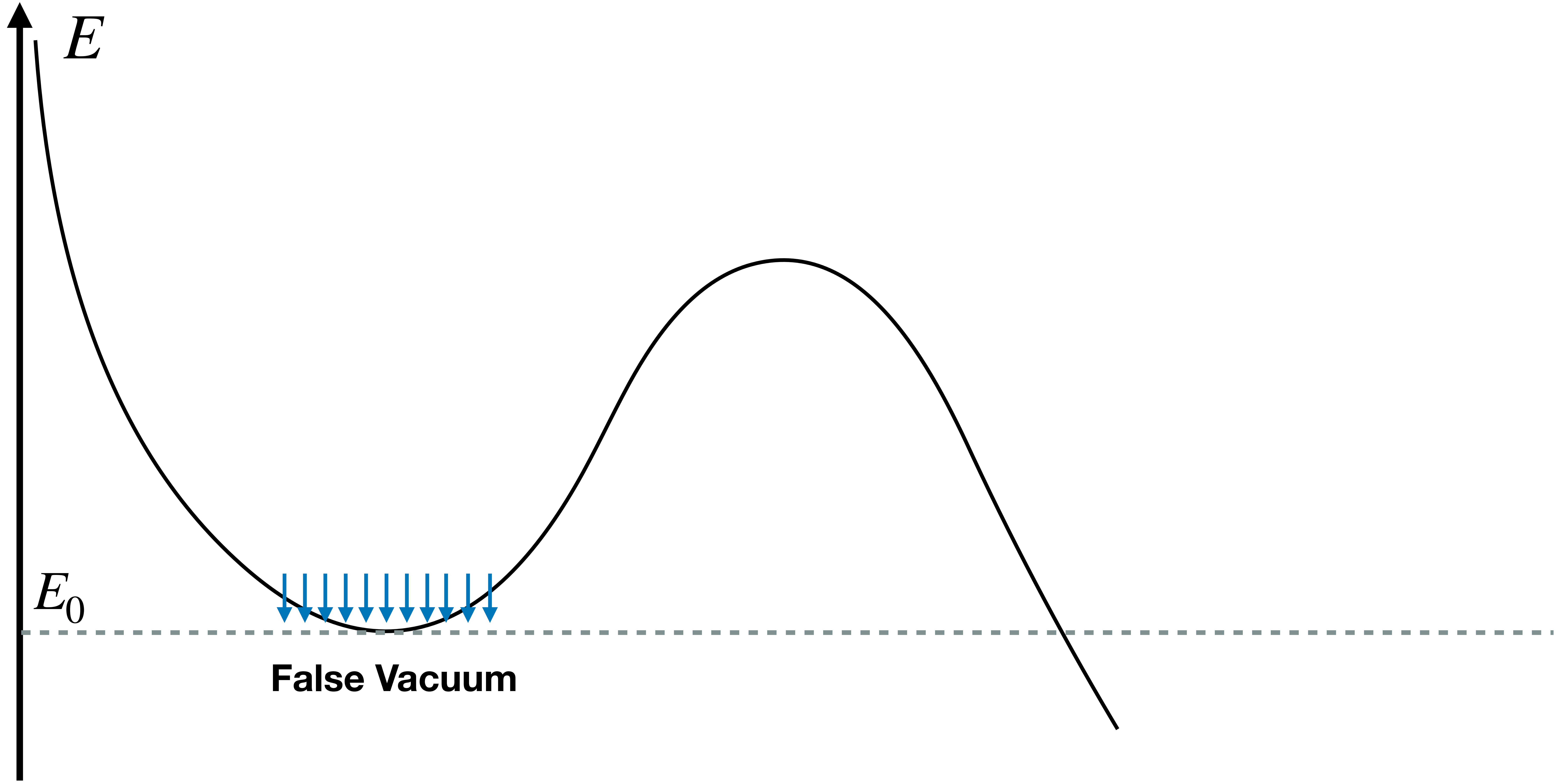


Decay by quantum fluctuations

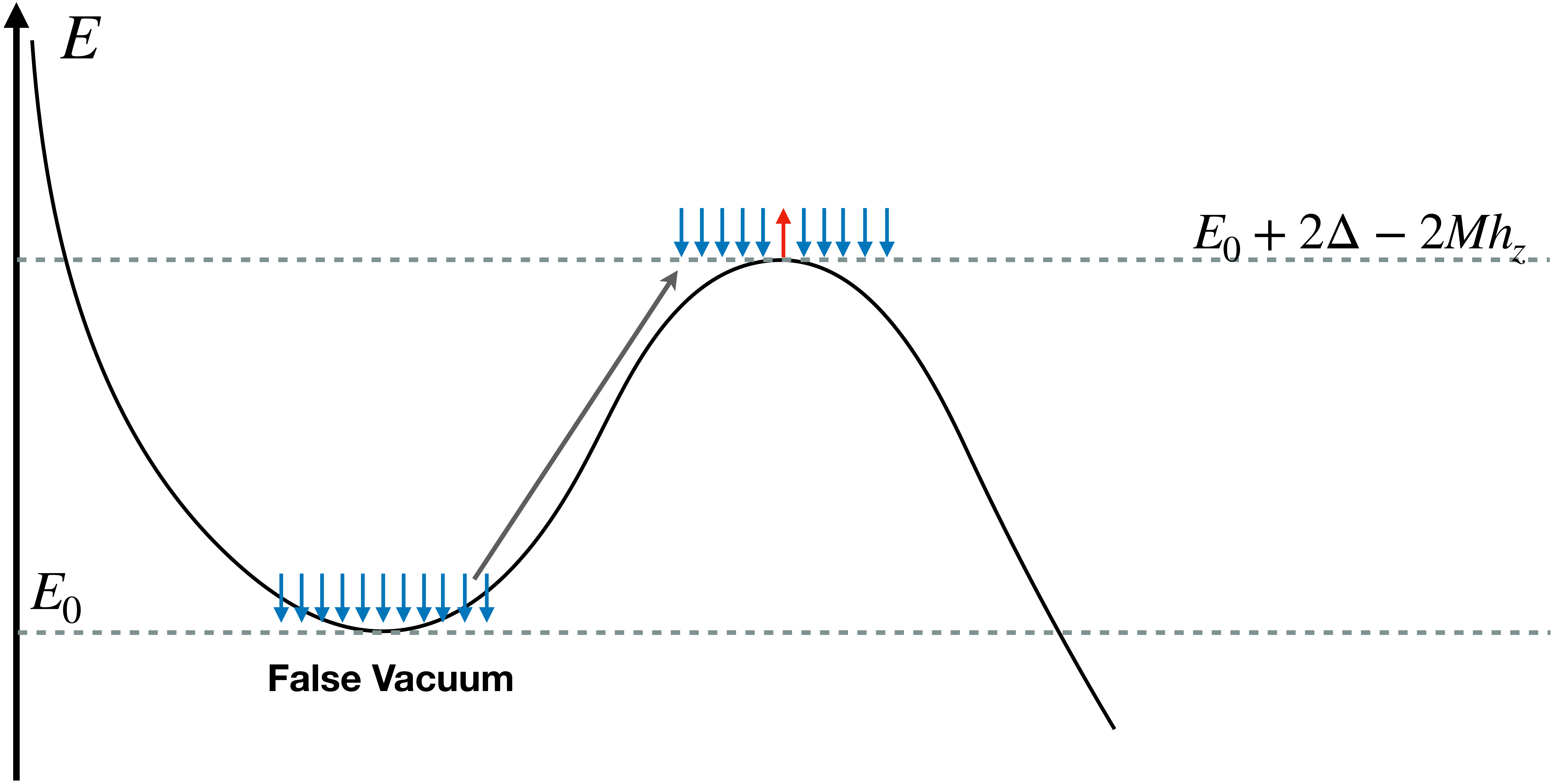


$$|\langle FV|\,U(t)\,|FV\rangle|^2 \approx \exp\left(-L\,\gamma\,t\right)$$

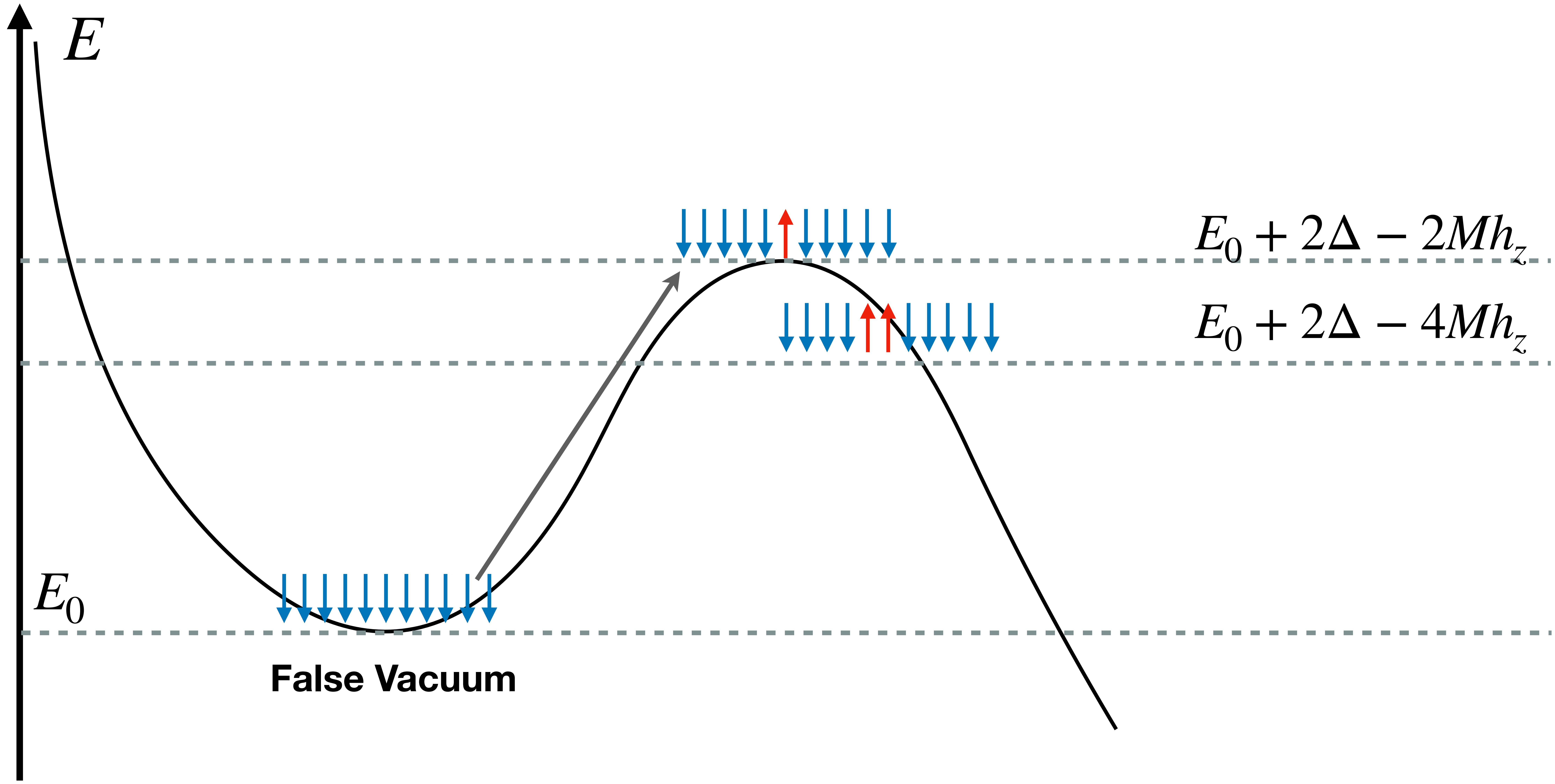
Unitary time evolution



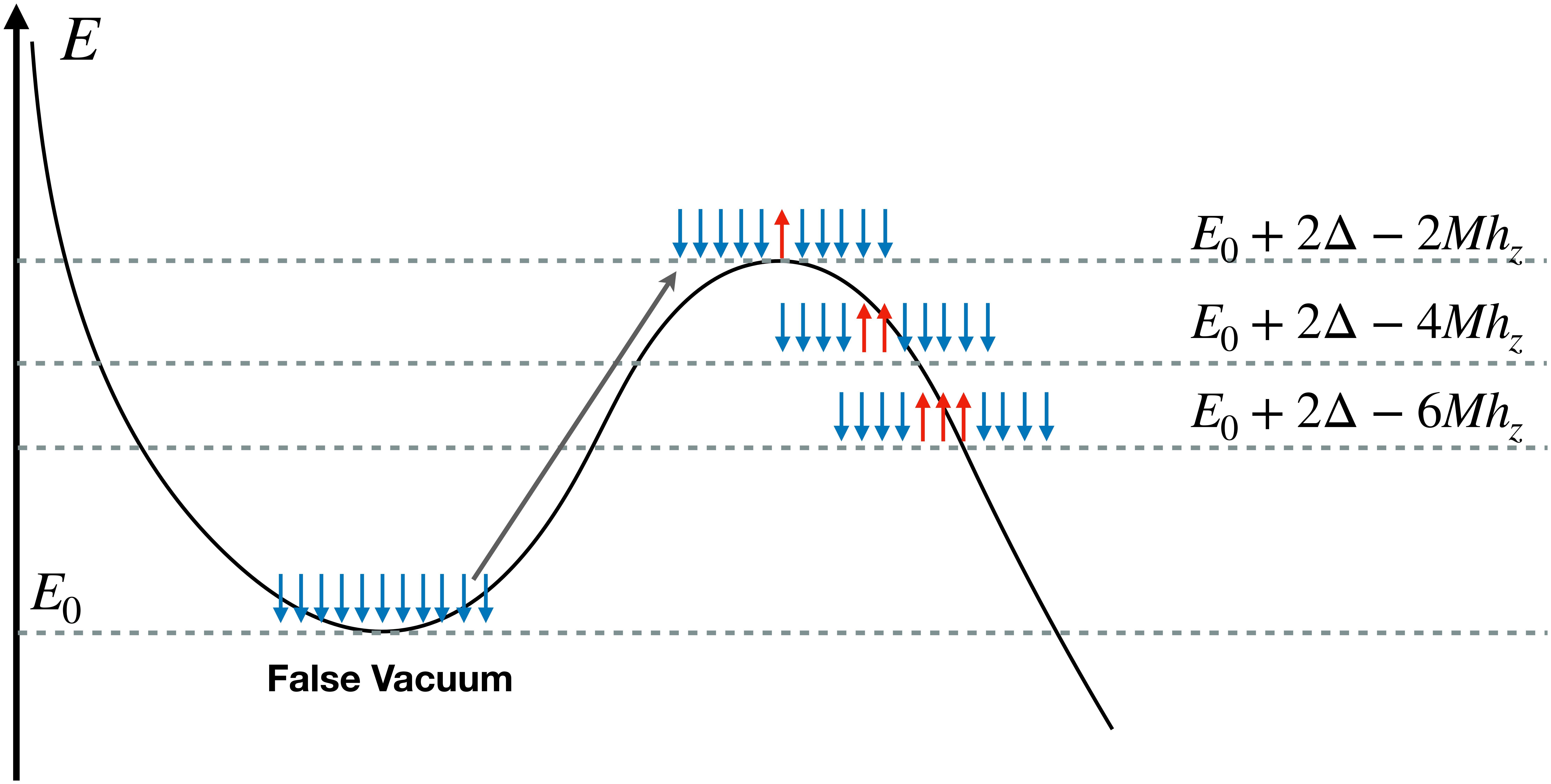
E
E0
False Vacuum



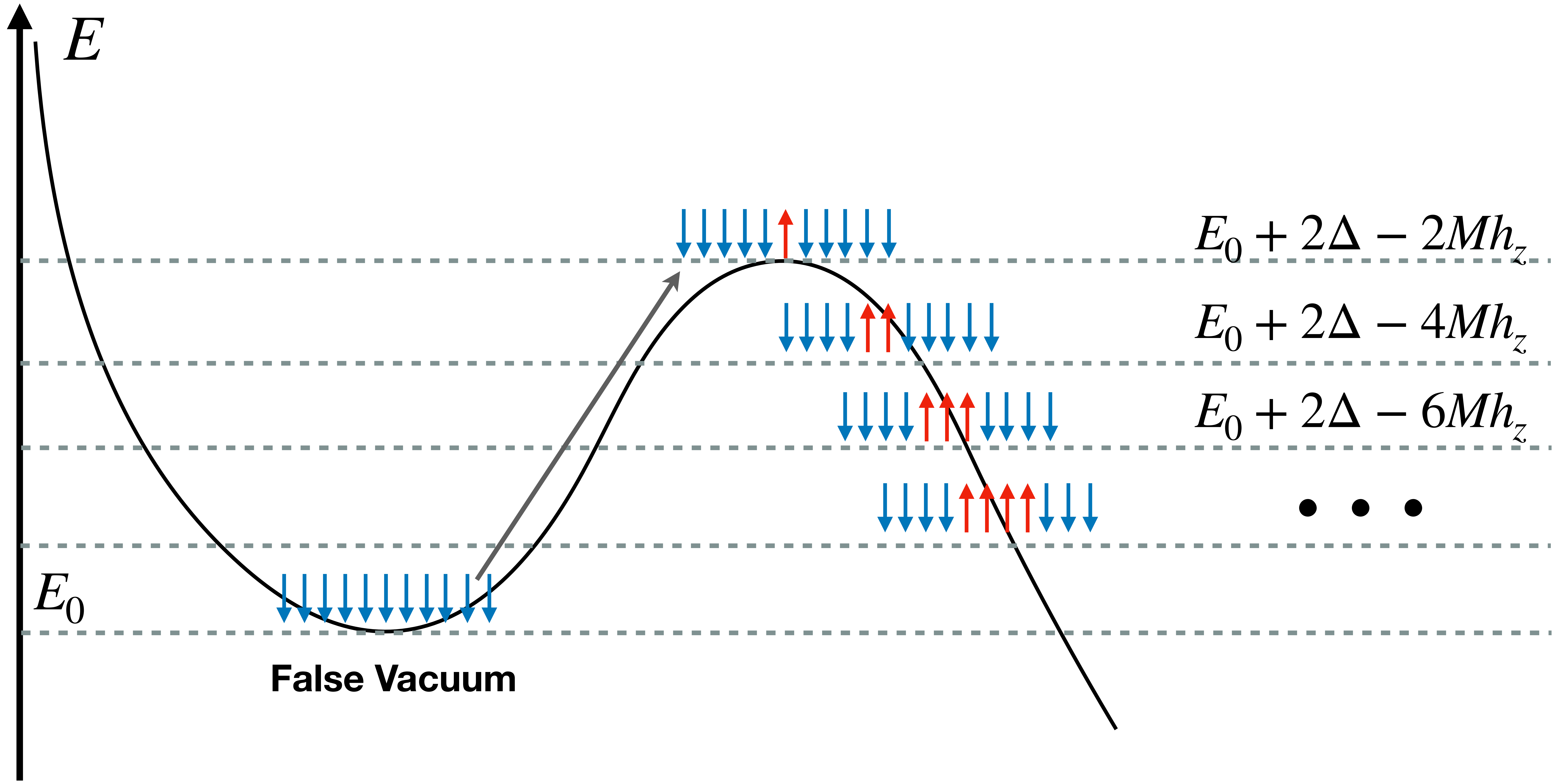
E
$E_0 + 2\Delta - 2Mh_z$
E_0
False Vacuum



E
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
E_0
False Vacuum



E
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
E_0
False Vacuum



E
$E_0 + 2\Delta - 2Mh_z$
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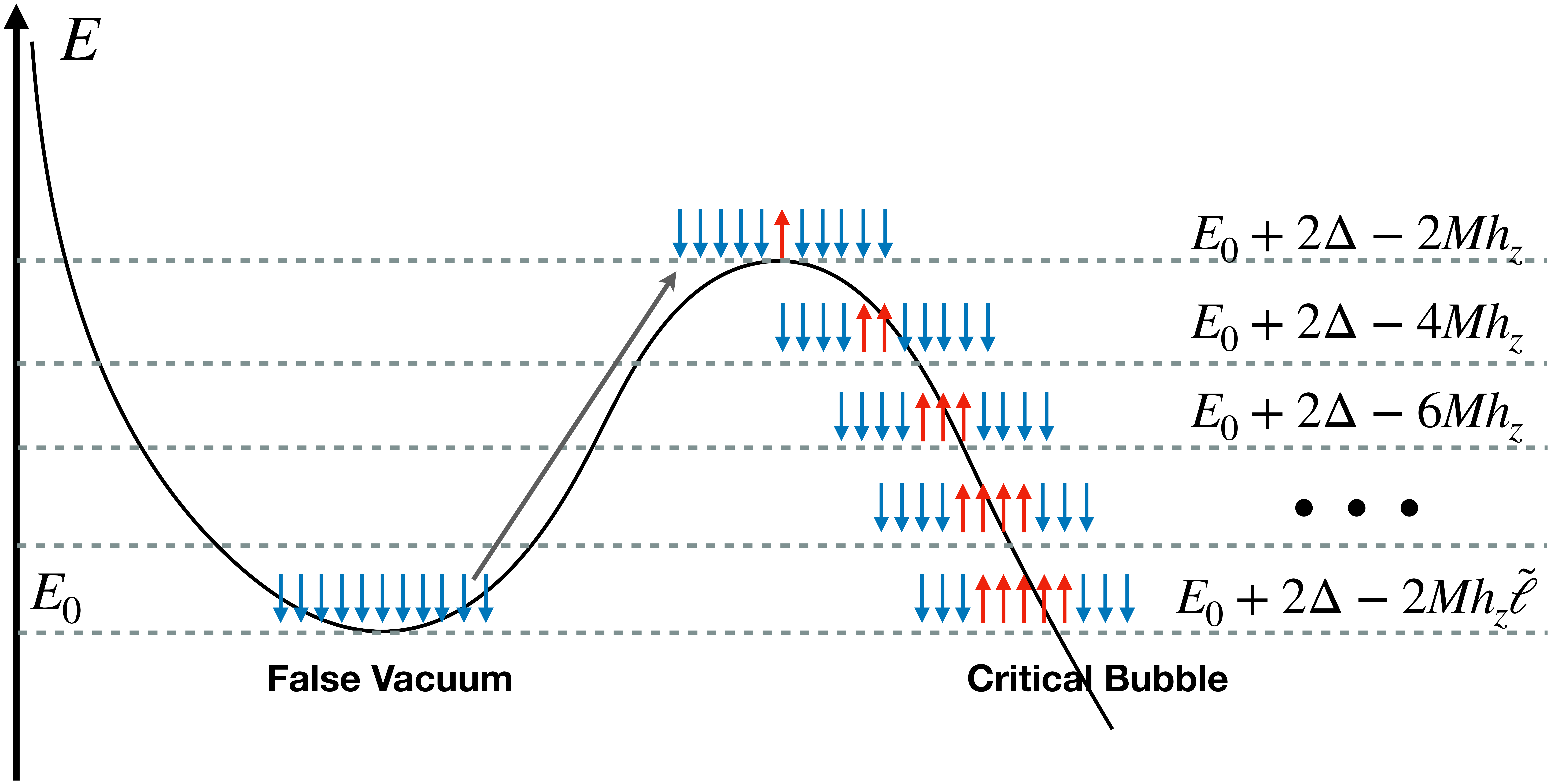
E_0
False Vacuum



E
$E_0 + 2\Delta - 2Mh_z$
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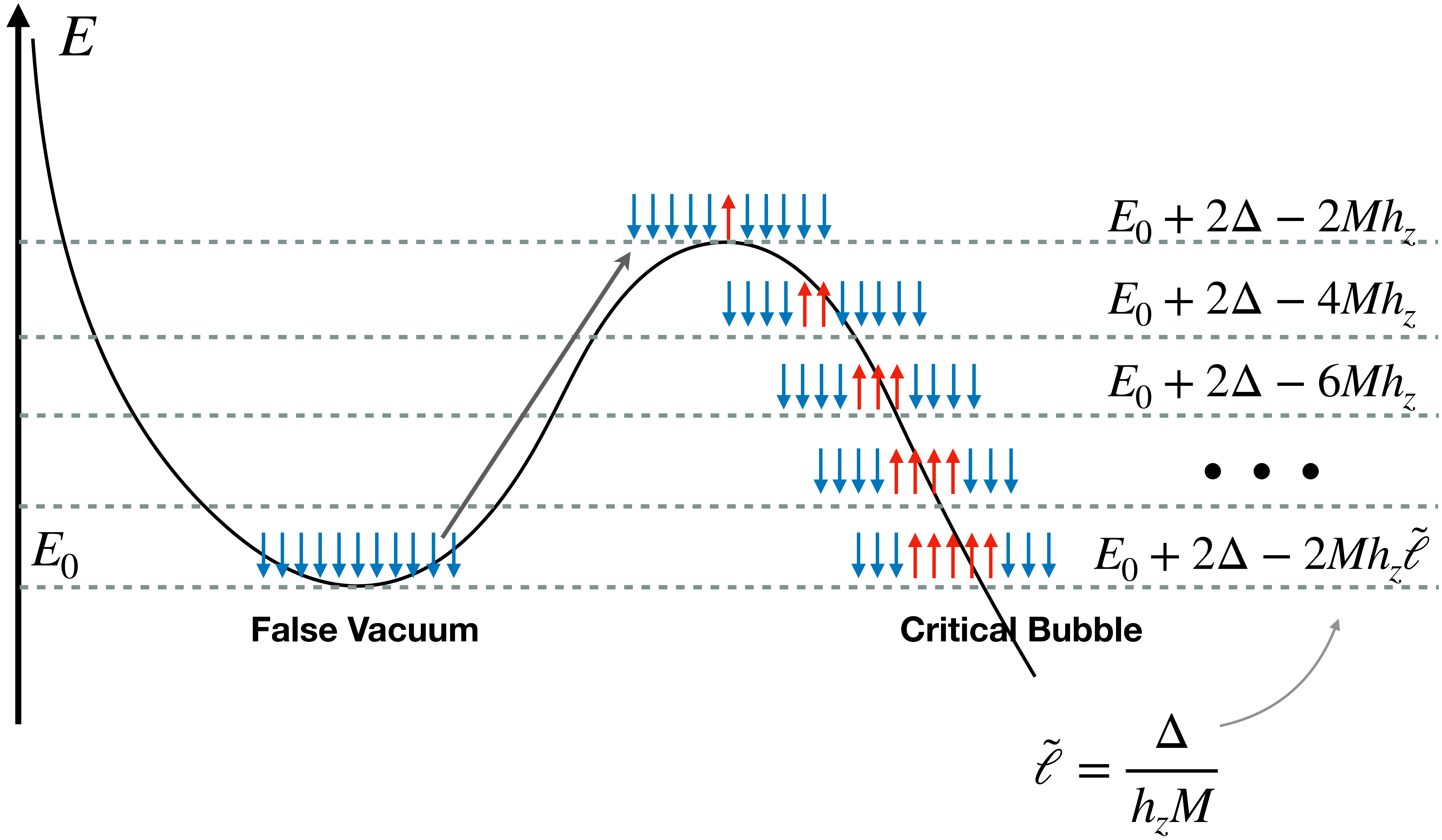
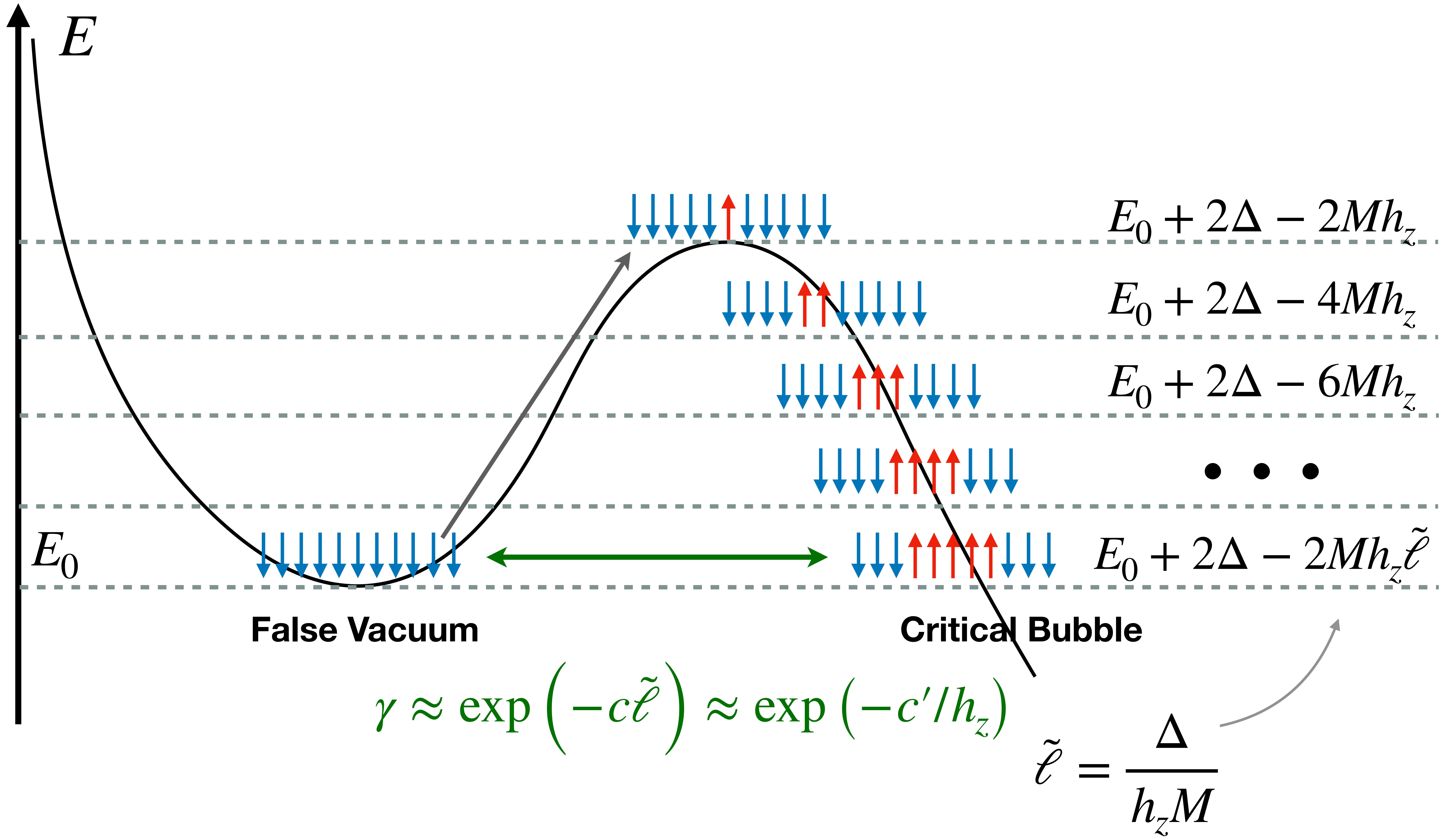
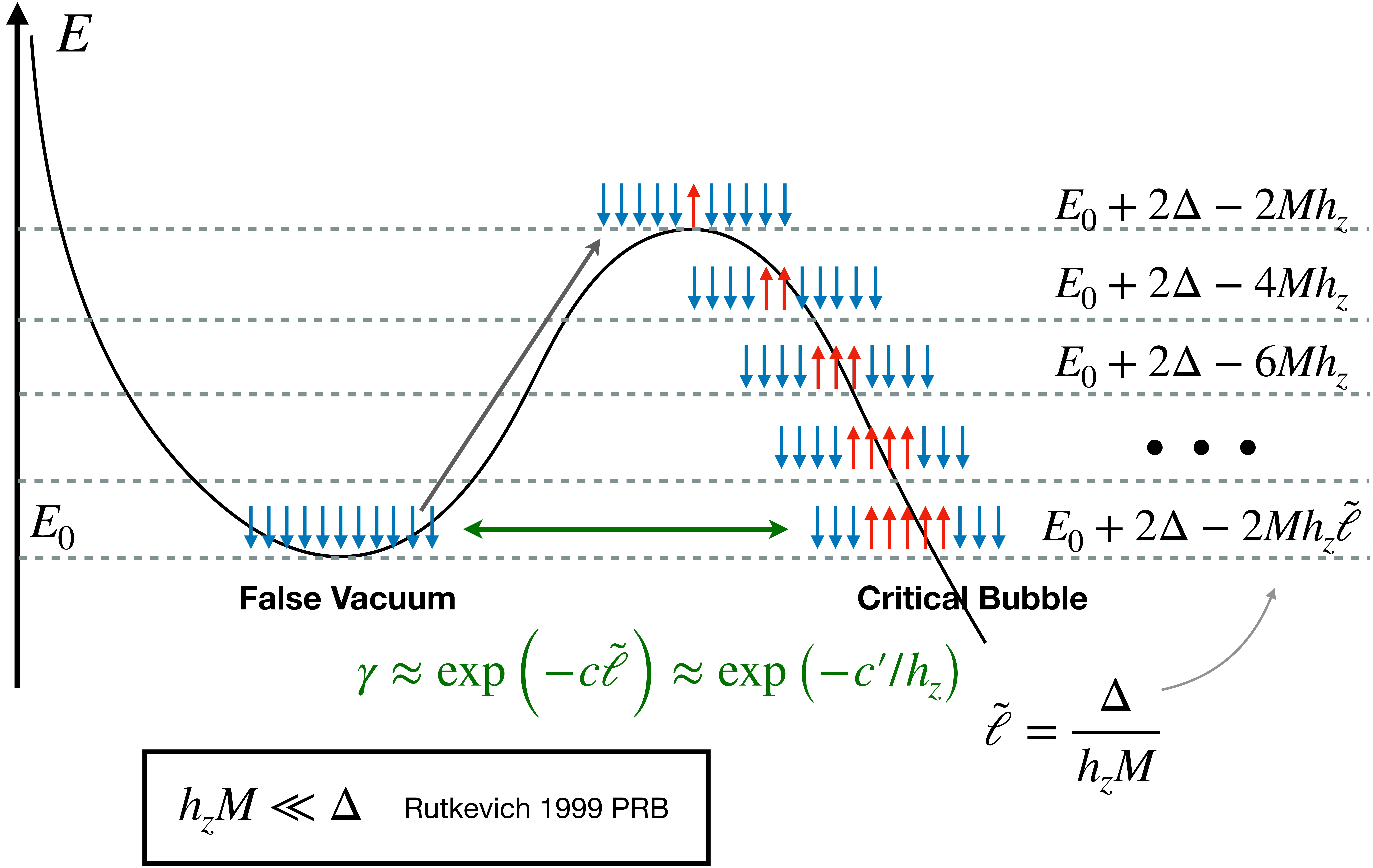
E_0
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
False Vacuum
Critical Bubble



E
E_0
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
• • •
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
False Vacuum
Critical Bubble
$\tilde{\ell} = \frac{\Delta}{h_z M}$



E
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
$\bullet\ \bullet\ \bullet$
E_0
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
False Vacuum
Critical Bubble
$\gamma \approx \exp\left(-c\tilde{\ell}\right) \approx \exp\left(-c'/h_z\right)$
$\tilde{\ell} = \frac{\Delta}{h_z M}$



E
E_0
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
$\bullet\ \bullet\ \bullet$
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False Vacuum
Critical Bubble
$\gamma \approx \exp\left(-c\tilde{\ell}\right) \approx \exp\left(-c'/h_z\right)$
$\tilde{\ell} = \dfrac{\Delta}{h_z M}$
$h_z M \ll \Delta$
Rutkevich 1999 PRB

The Quench Protocol

The Quench Protocol

1 - Prepare $|\psi_{\downarrow}\rangle = \otimes_i |\downarrow\rangle_i$: Ground state of $H(h_x = 0, h_z = 0)$

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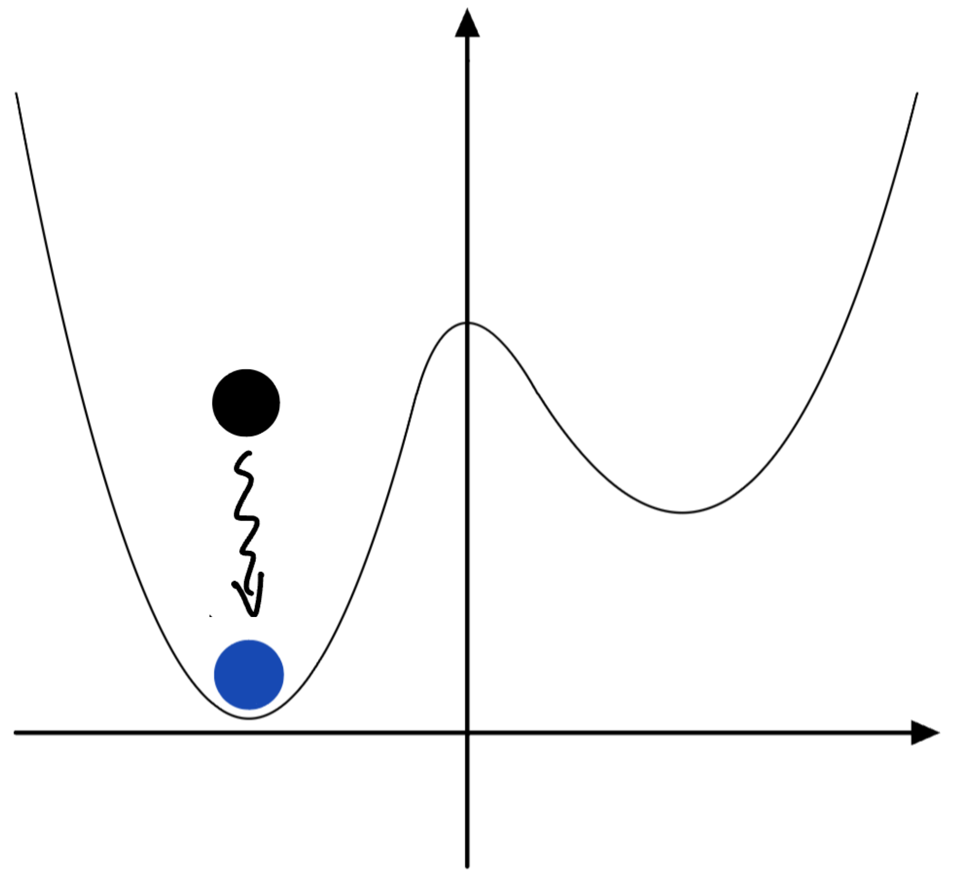


2 - Approach ground state of $H(0 < h_x < 1, -h_z)$:

$$|0\rangle = exp\left[-\tau H(h_x, -h_z)\right]|\psi_\downarrow\rangle$$

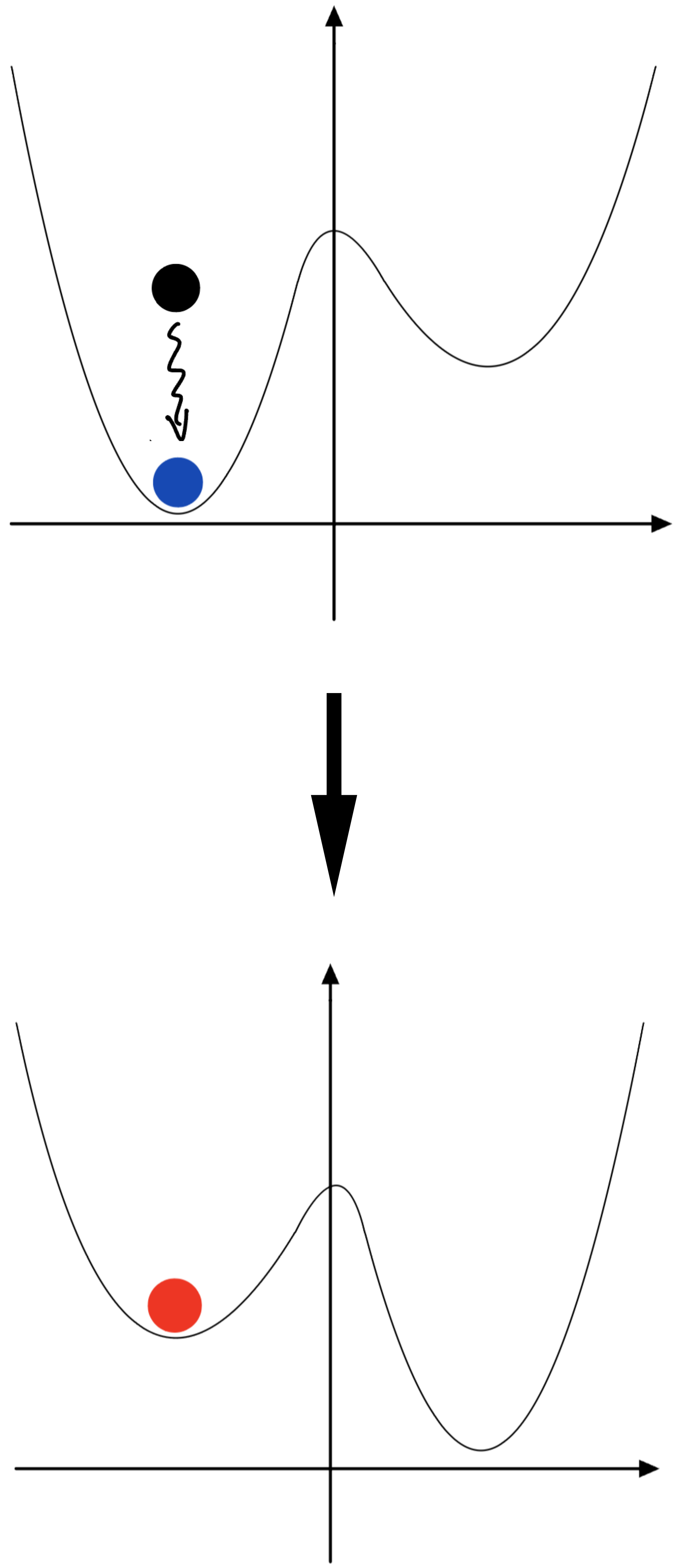
The Quench Protocol

1 - Prepare $|\psi_{\downarrow}\rangle = \otimes_i |\downarrow\rangle_i$: Ground state of $H(h_x = 0, h_z = 0)$

2 - Approach ground state of $H(0 < h_x < 1, -h_z)$:

$$|0\rangle = exp\left[-\tau H(h_x, -h_z)\right]|\psi_{\downarrow}\rangle$$

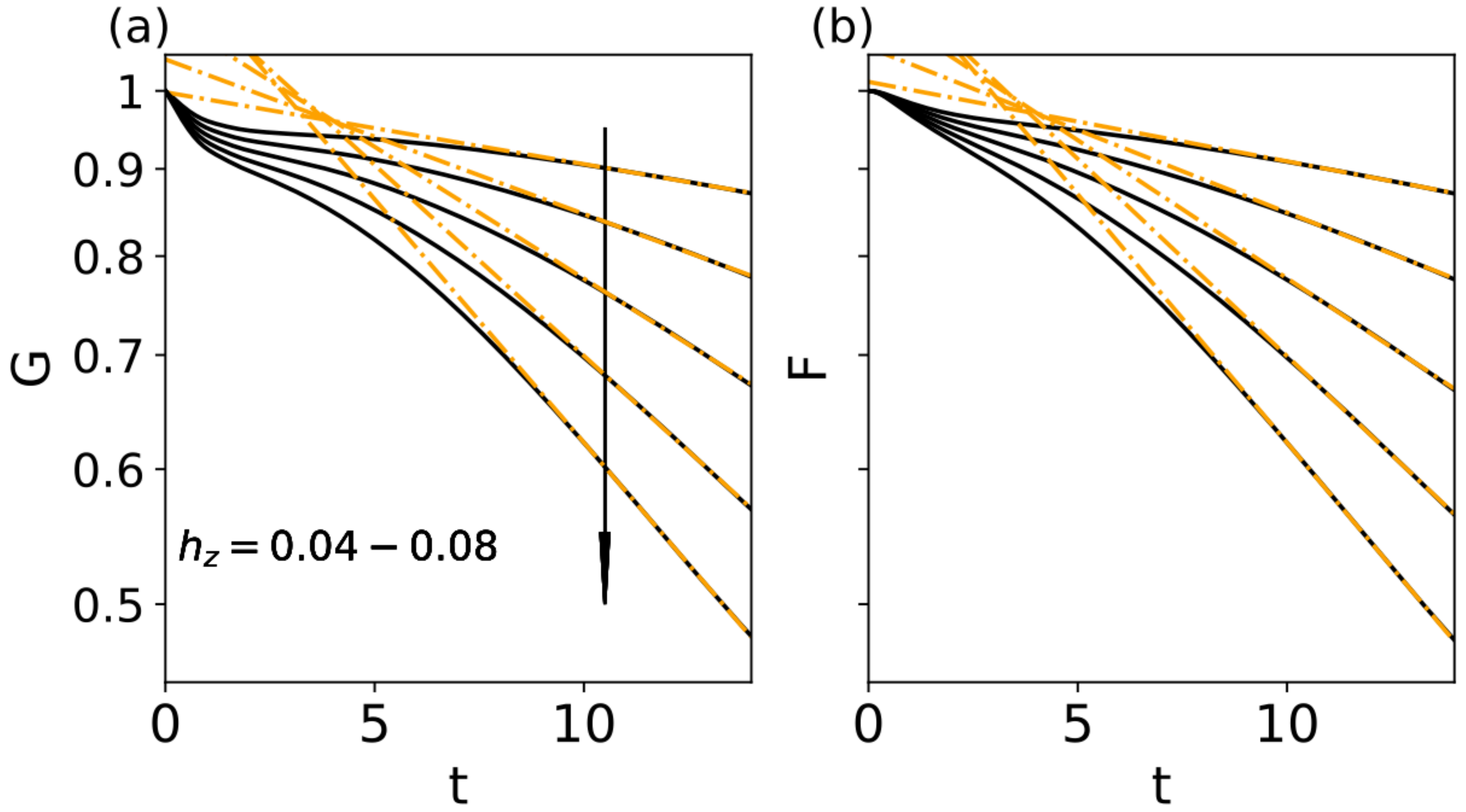
3 - Switch -h_z to $+h_z$ and evolve with $exp\left[-i\,H(h_x, +h_z)\,t\right]$:



Rate extraction

Two-sites observable

$$G(t) = 1 - ||\rho(t) - \rho(0)||_1$$



Rescaled Magnetisation

$$F(t) = \frac{\langle \sigma^z(t) \rangle + \langle \sigma^z(0) \rangle}{2\langle \sigma^z(0) \rangle}$$

Rate extraction

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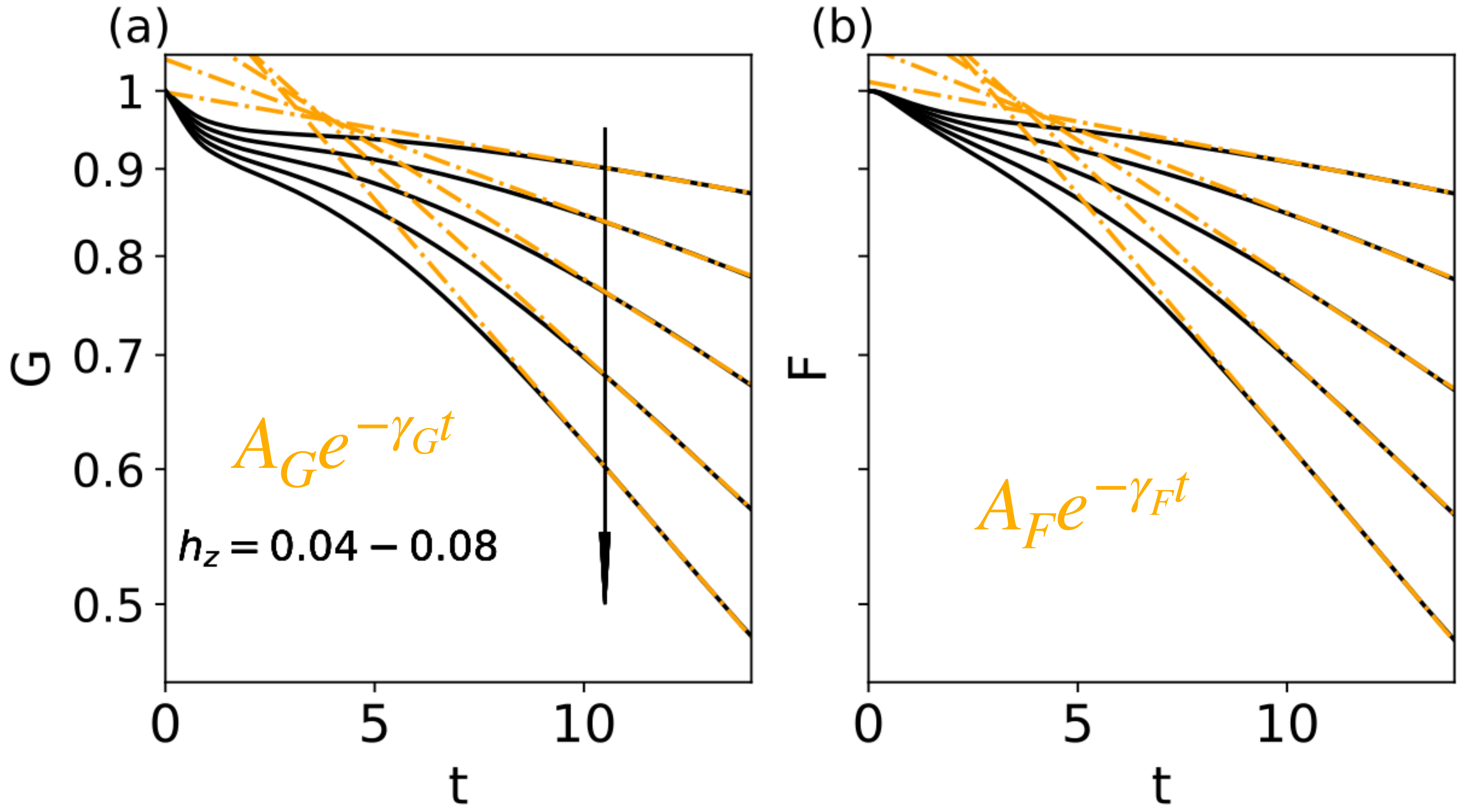

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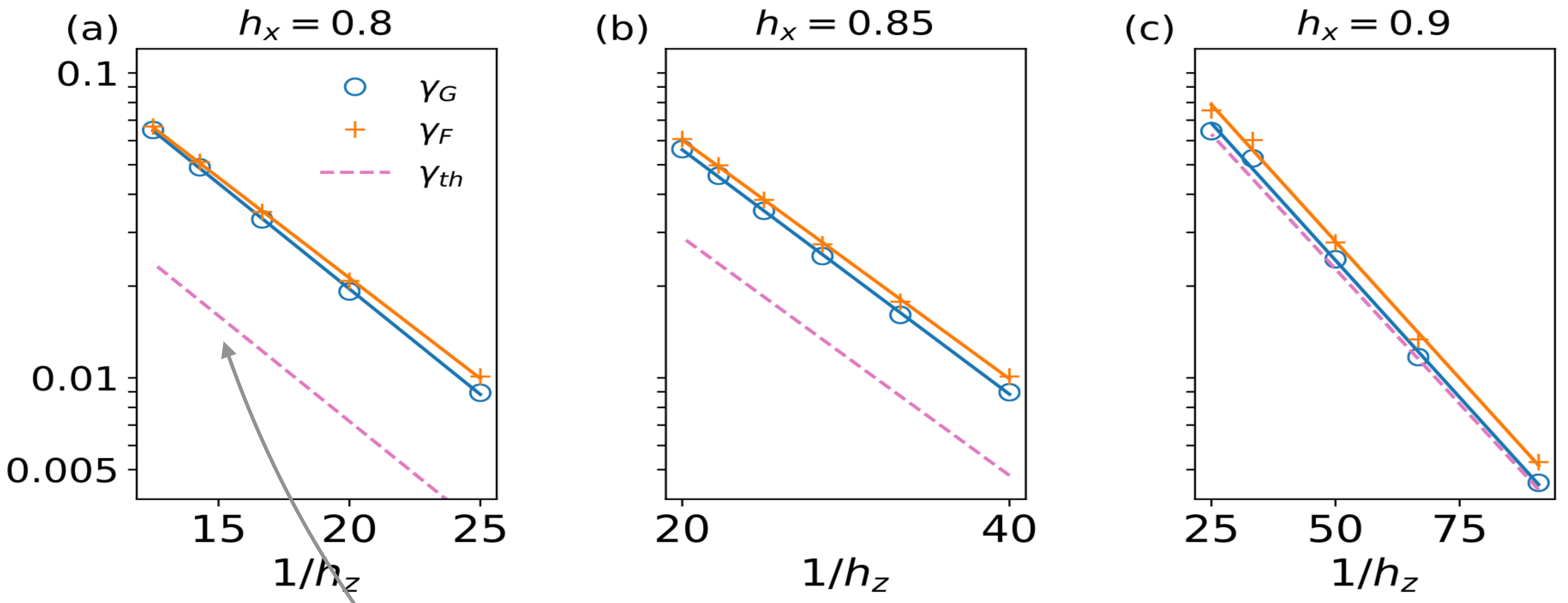
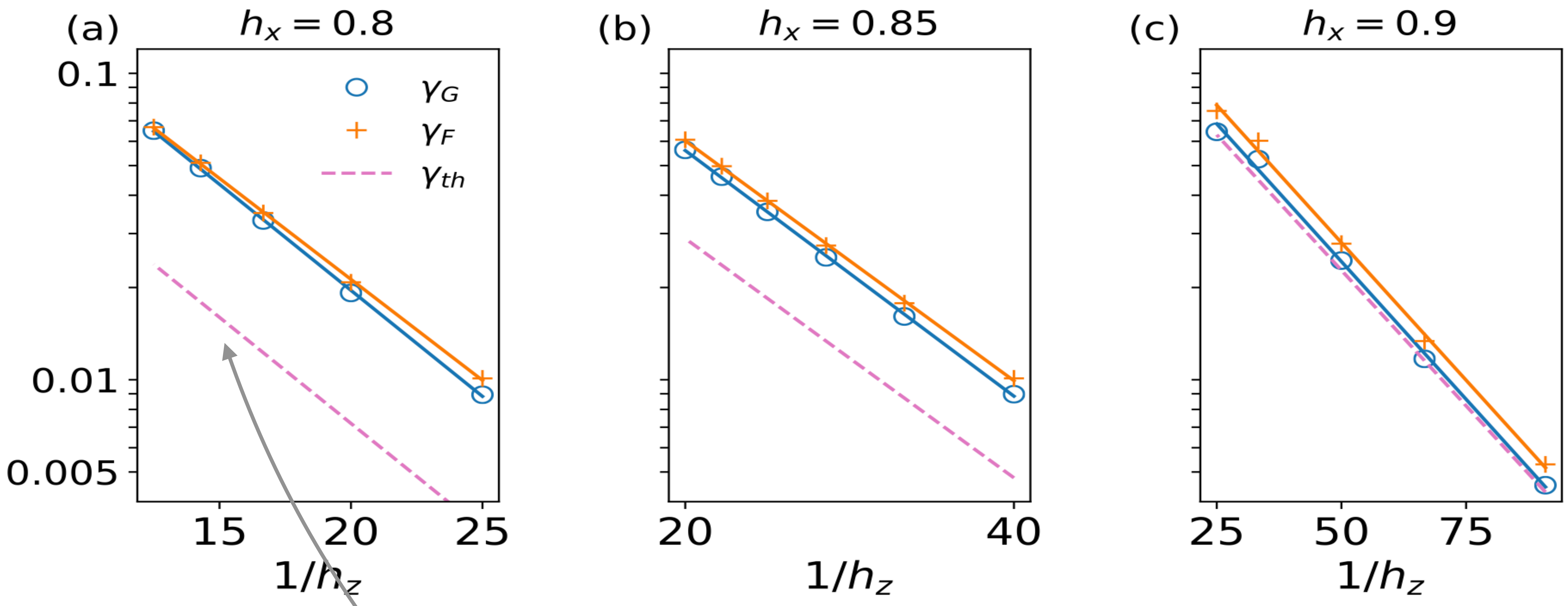
Comparison with theoretical prediction



$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

Comparison with theoretical prediction

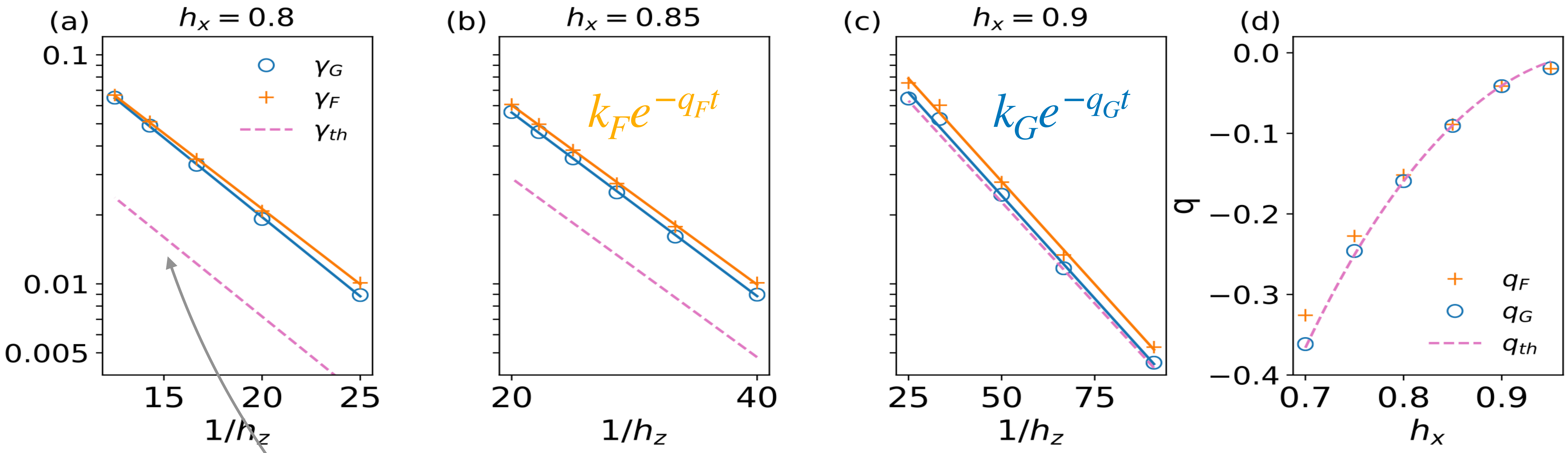


$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

Prefactor is off !

Comparison with theoretical prediction



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Rutkevich 1999 PRB

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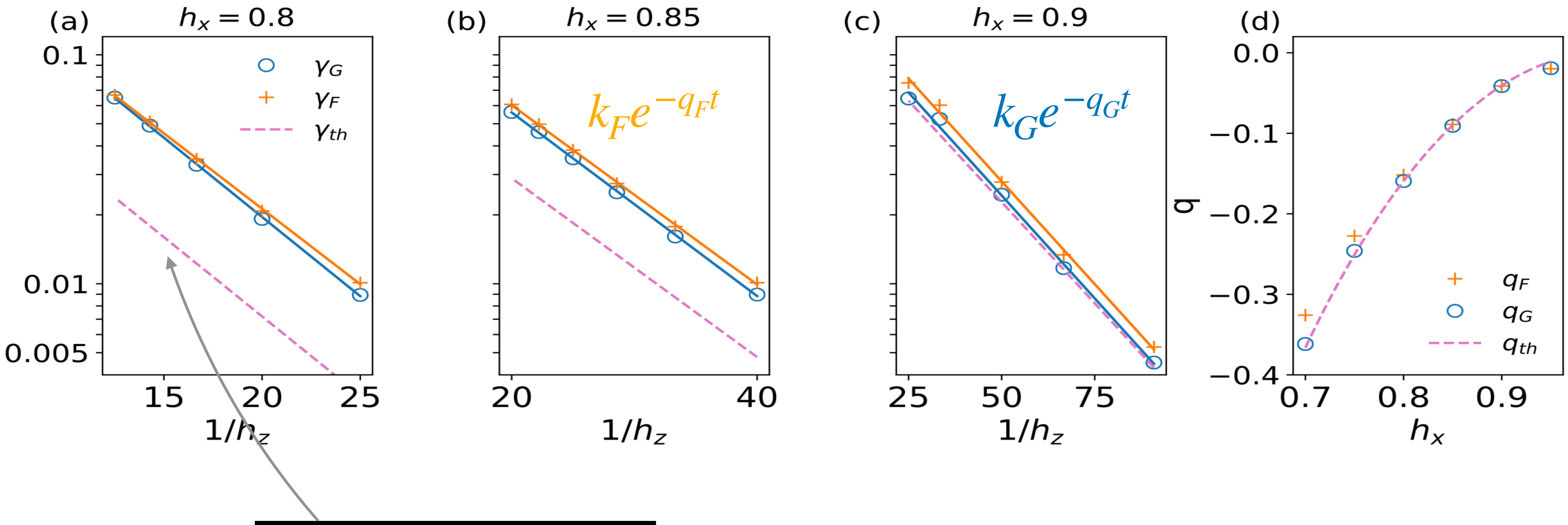
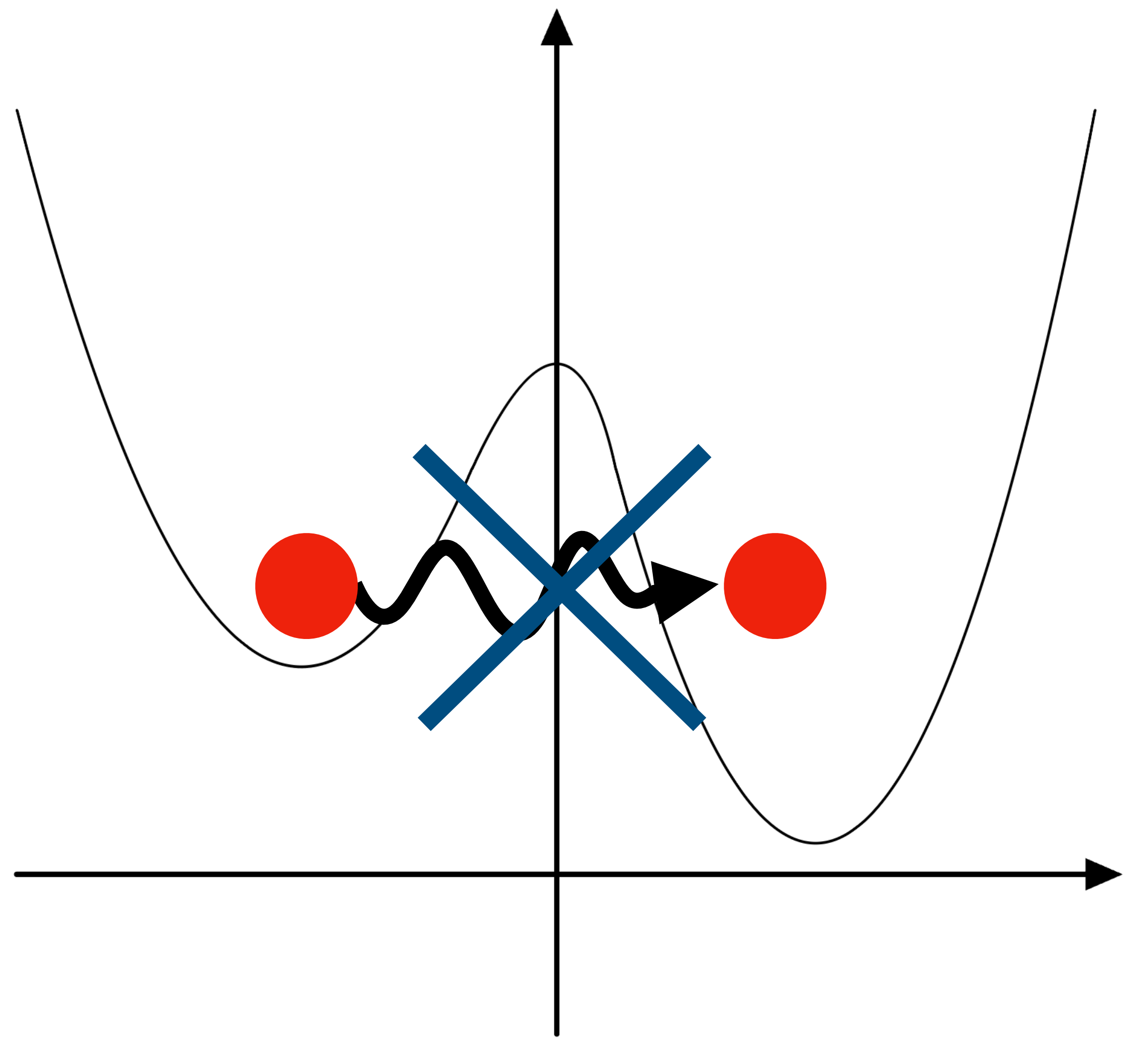


$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

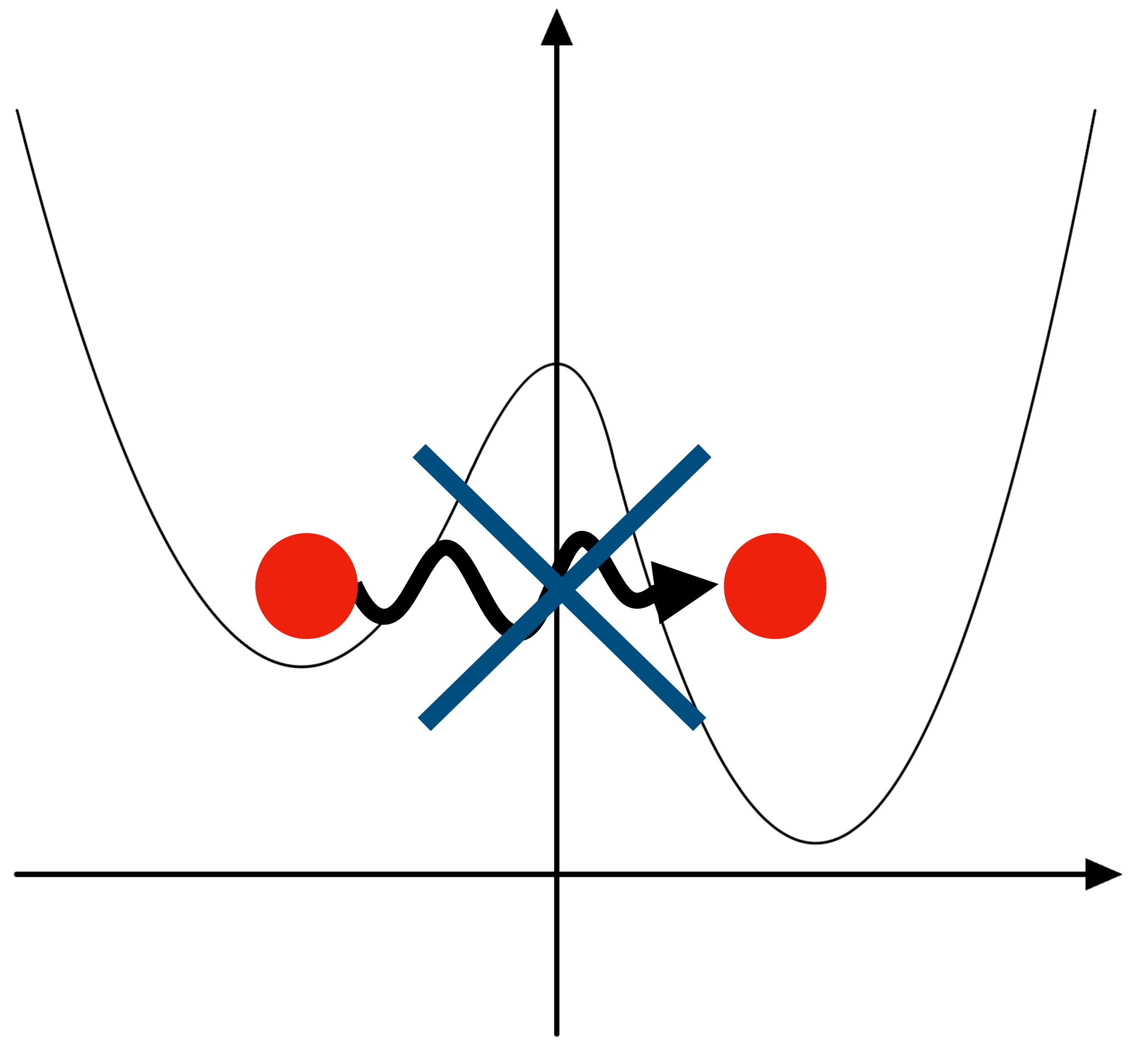
Exponential coefficient is good!

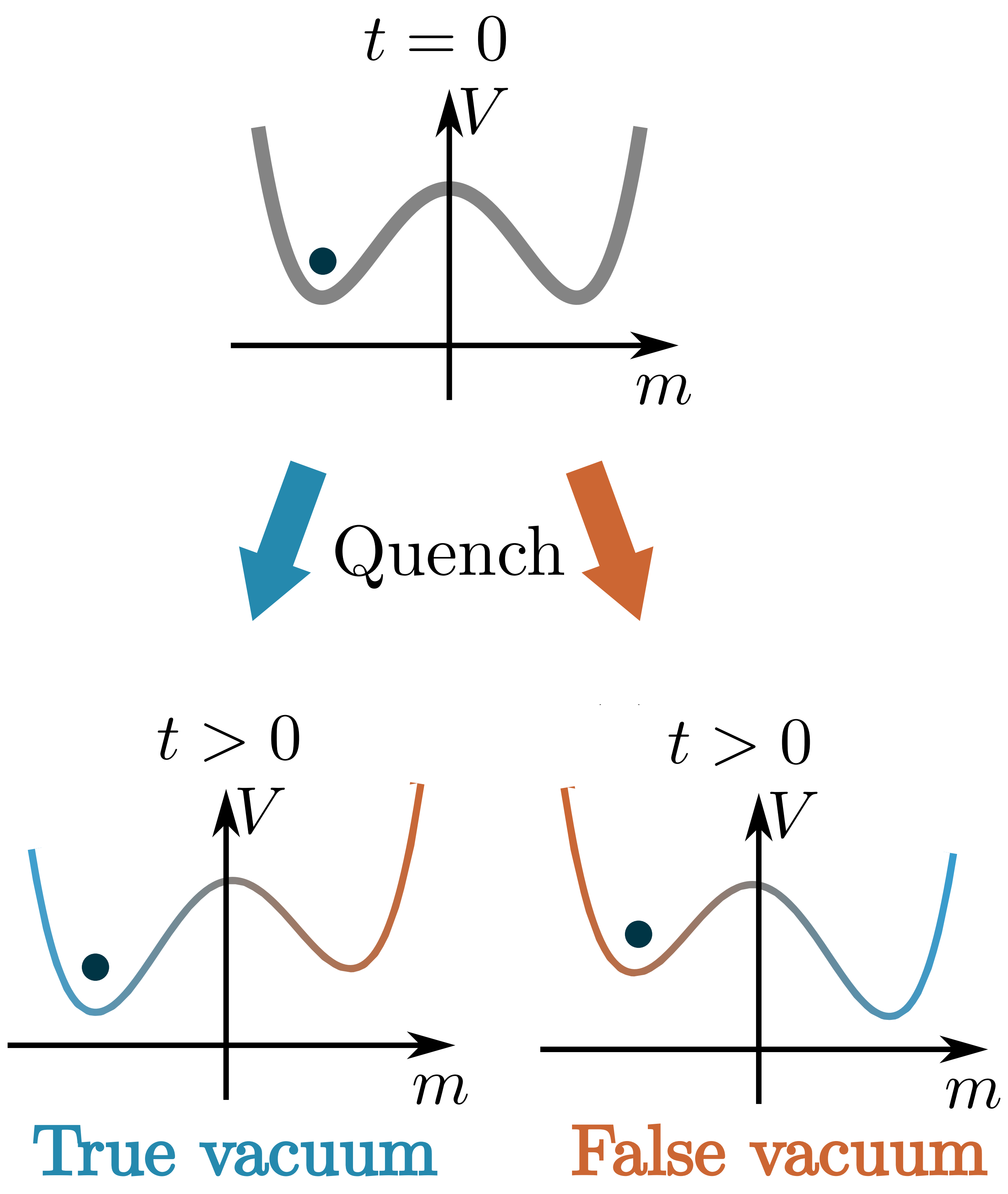
More on the oscillations



Lagnese, Surace, Morampudi, Wilczek, arXiv:2308.08340

More on the oscillations





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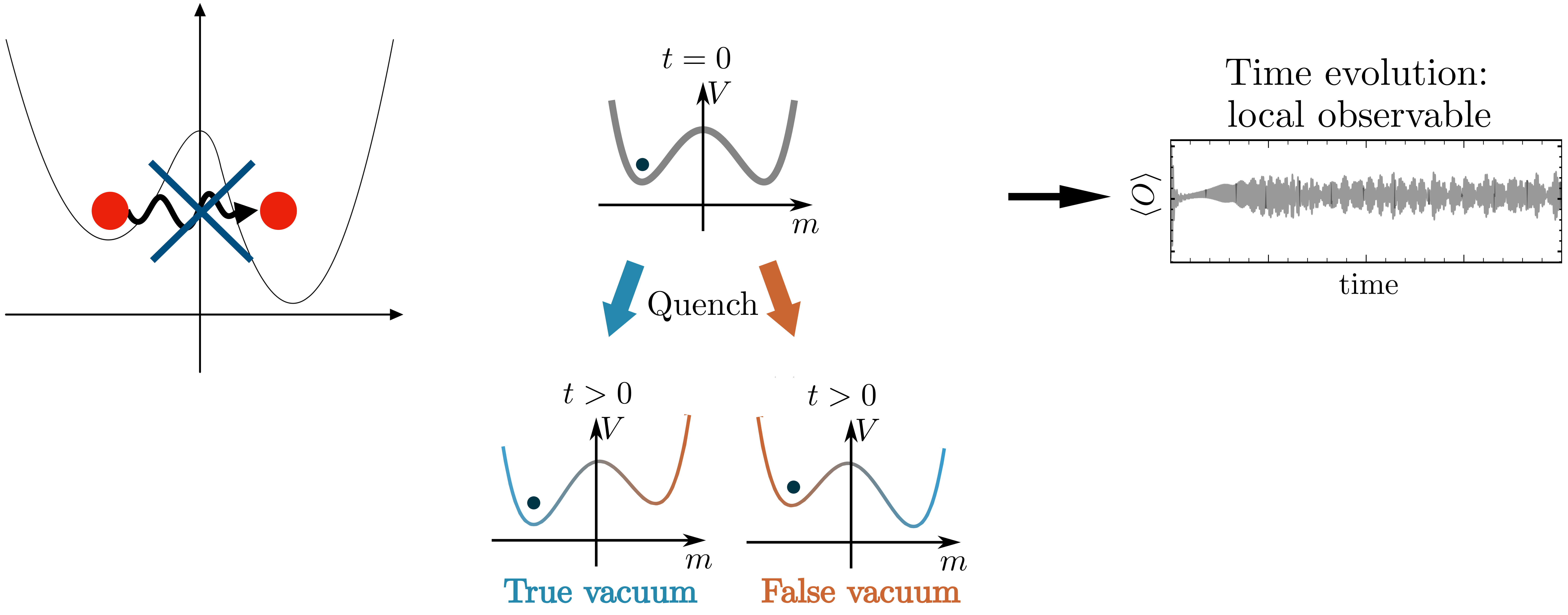

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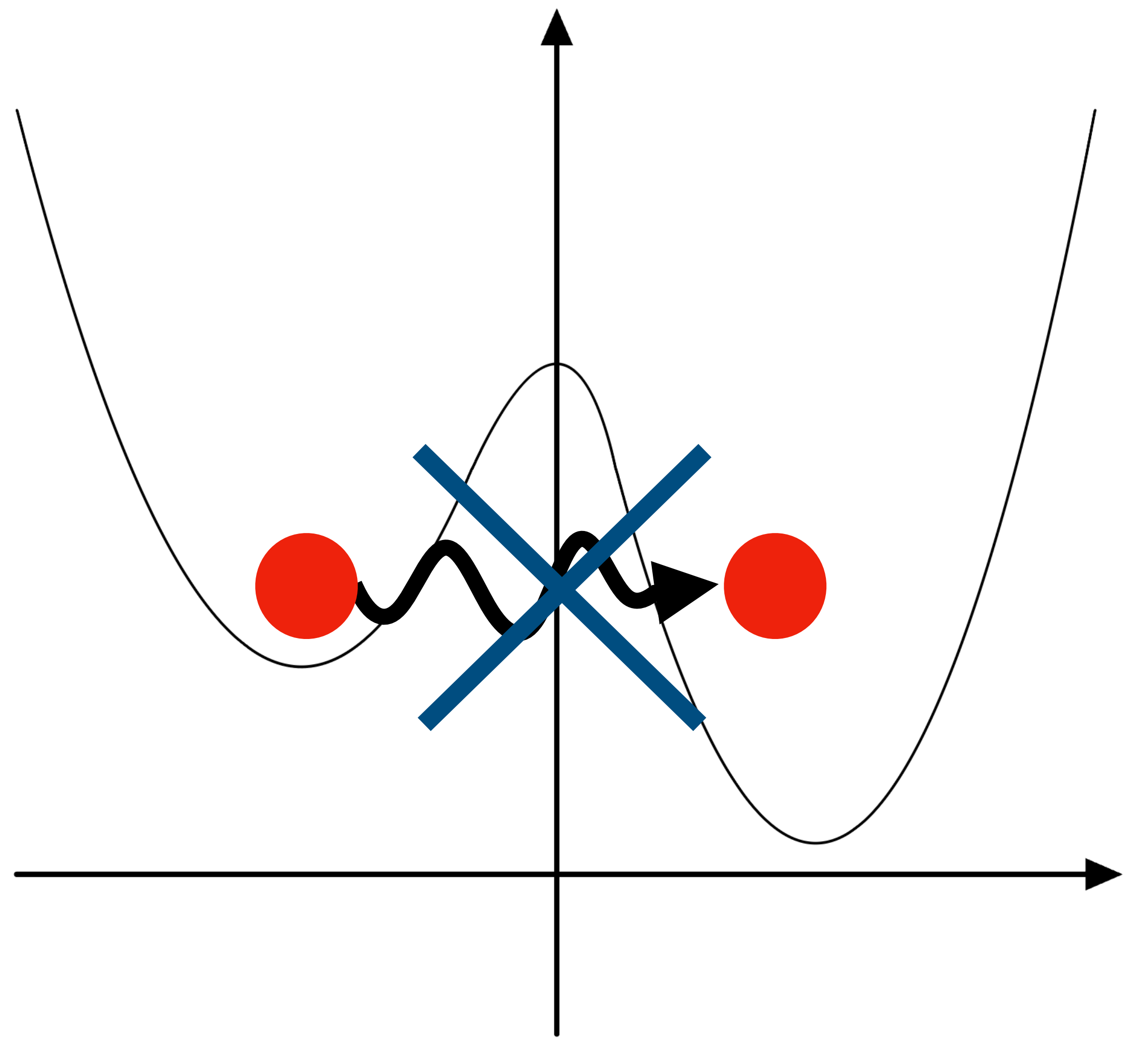

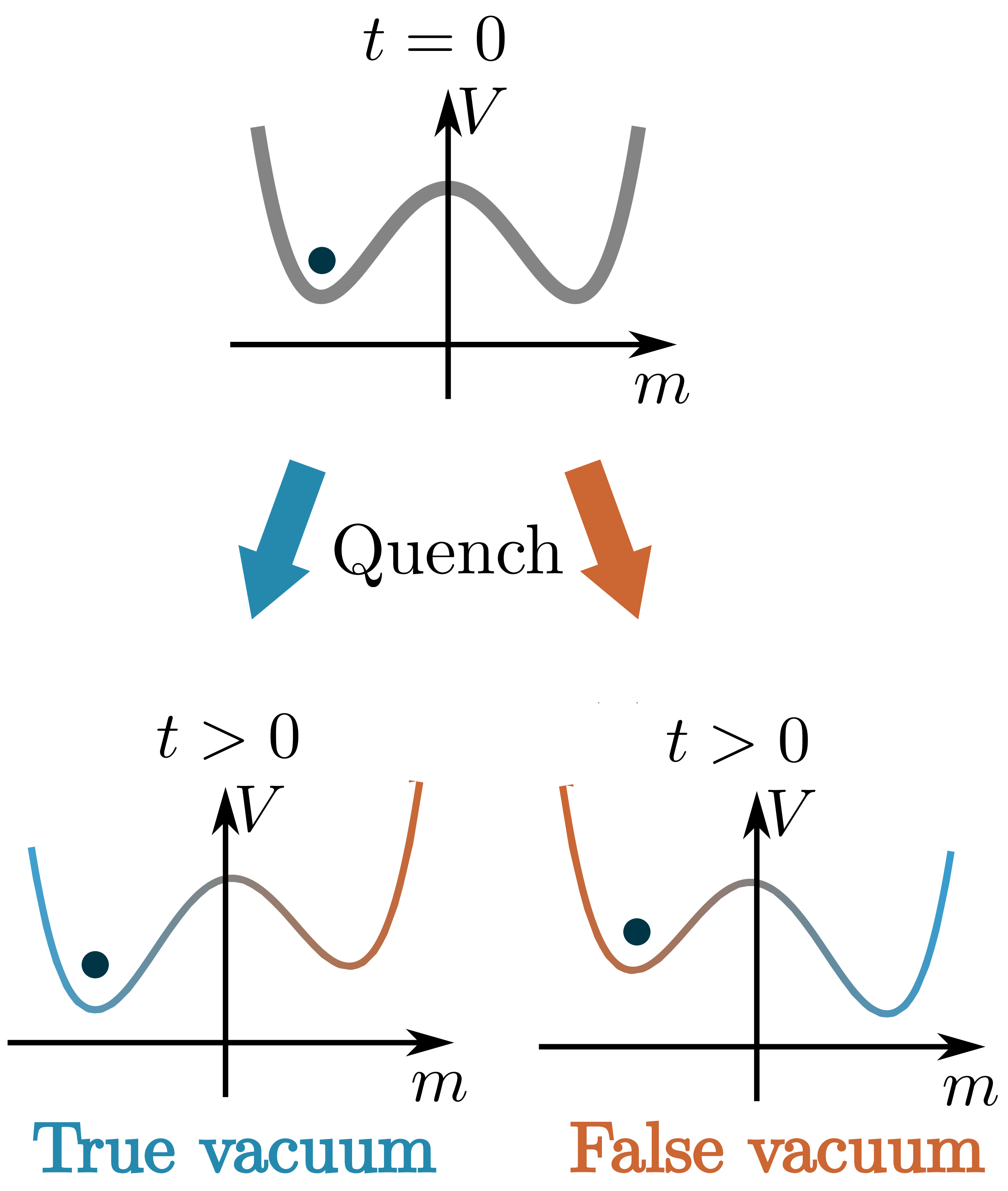

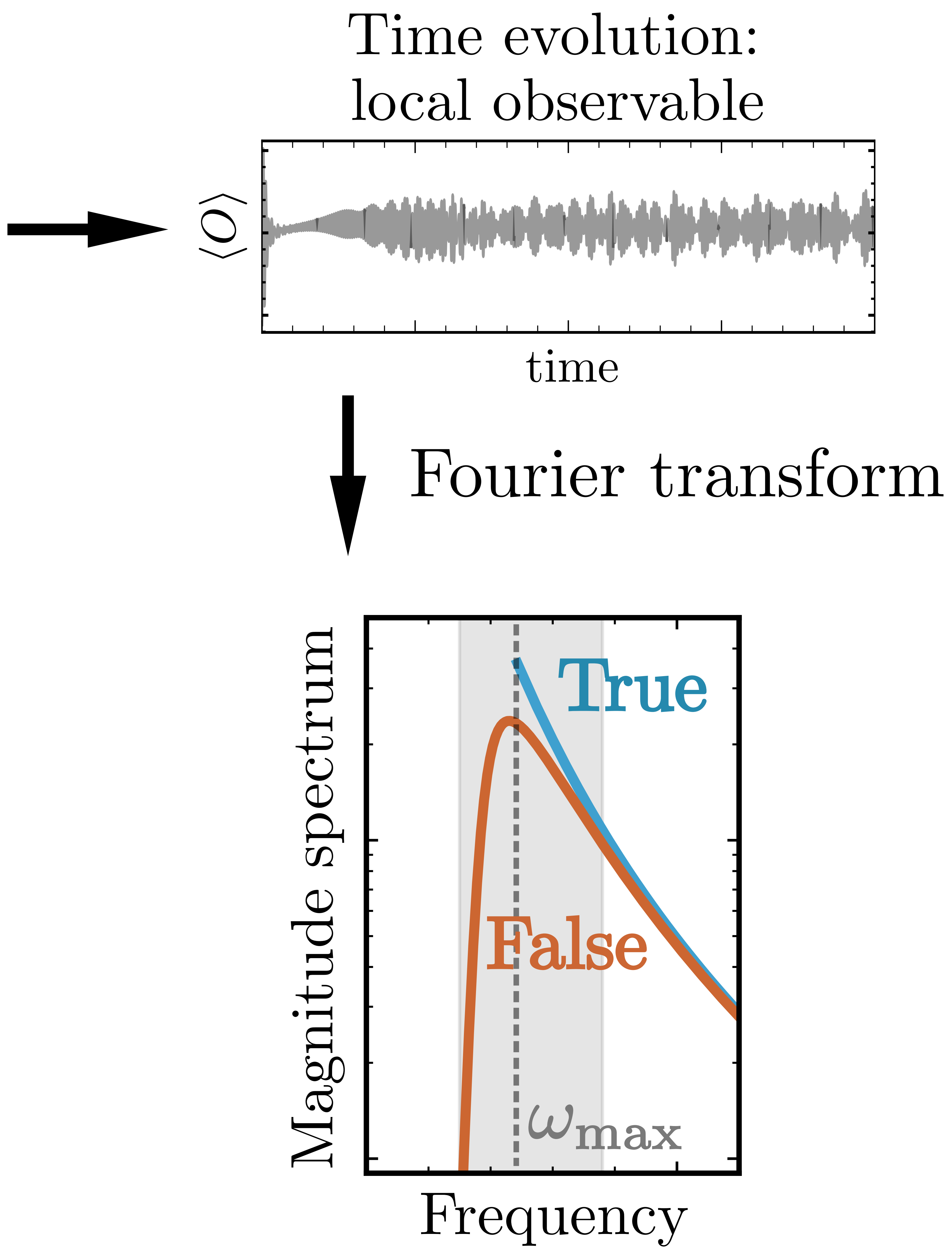

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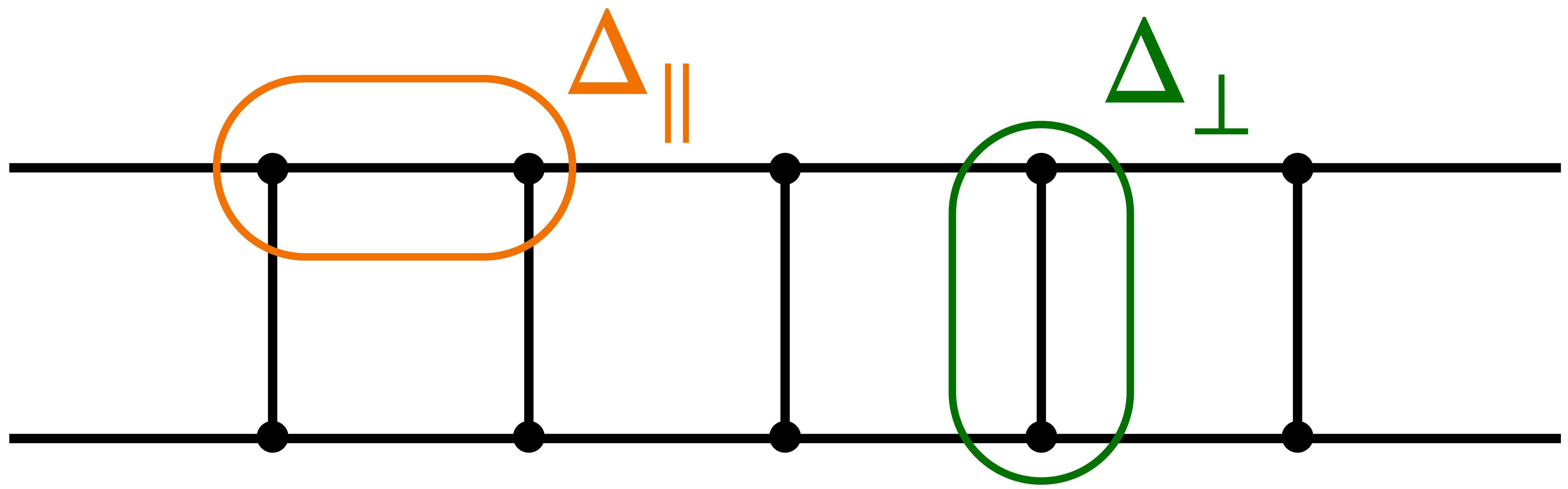
Another Model

XXZ Ladder

$$H = \frac{1}{2} \sum_j \sum_{\alpha=1,2} \left(\sigma^x_{j,\alpha}\sigma^x_{j+1,\alpha} + \sigma^y_{j,\alpha}\sigma^y_{j+1,\alpha} + \Delta_\parallel \sigma^z_{j,\alpha}\sigma^z_{j+1,\alpha} \right) + \Delta_\perp \sigma^z_{j,1}\sigma^z_{j+1,2}$$



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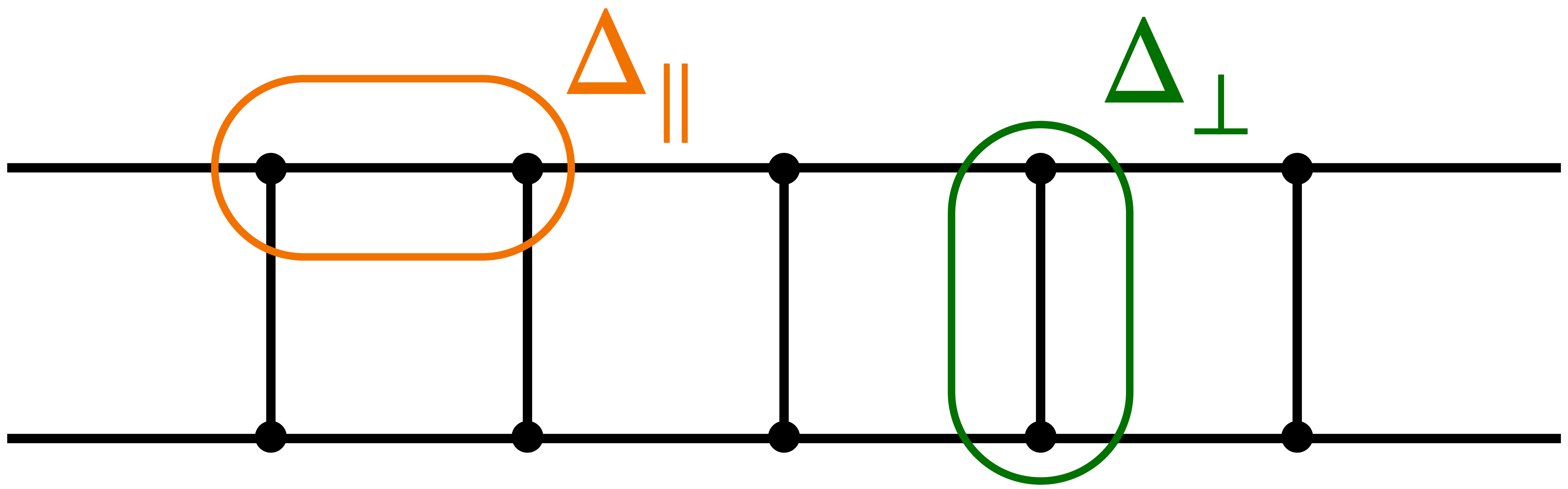

$$\Delta_\perp \leftrightarrow h_z$$

Interacting ↔ No Free Fermion Mapping

XXZ Ladder

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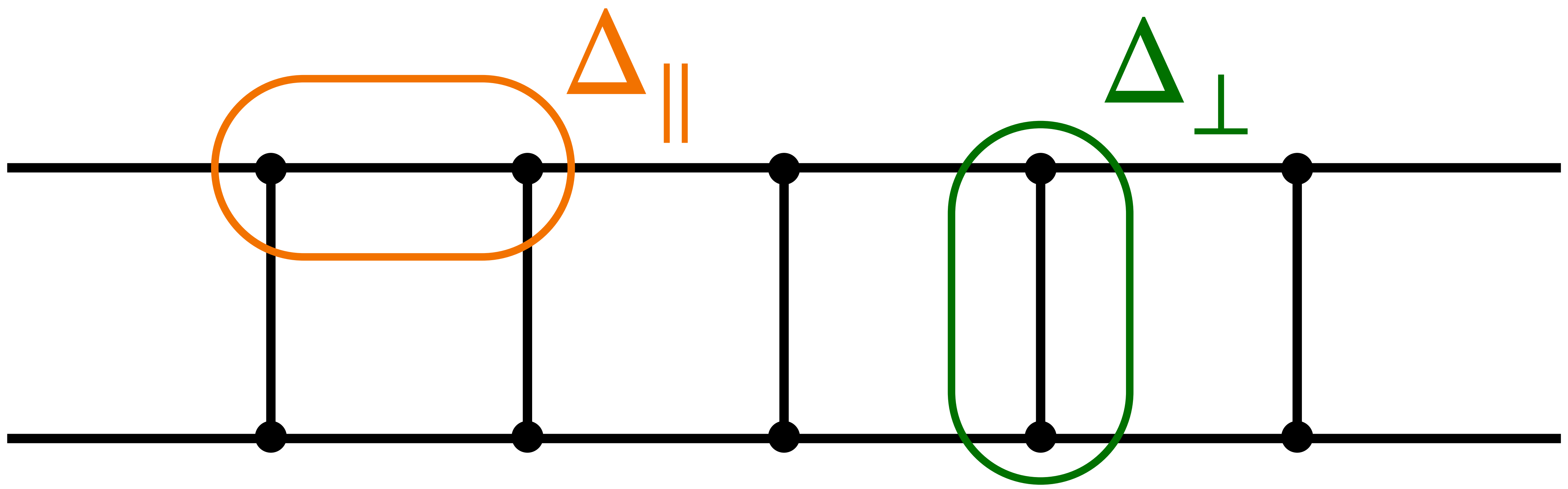
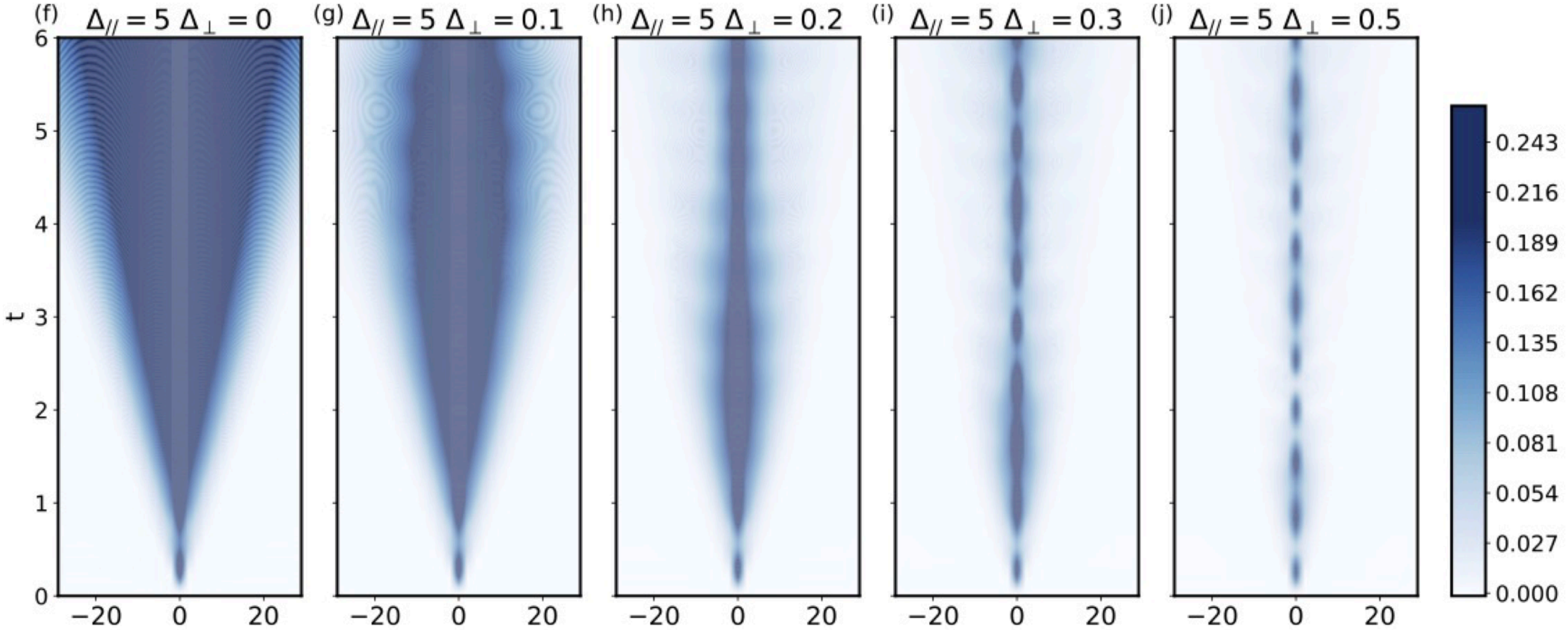


Lagnese, Surace, Kormos, Calabrese J. Phys A 2022

Lagnese, Surace, Kormos, Calabrese J. Stat. Mech. 2022



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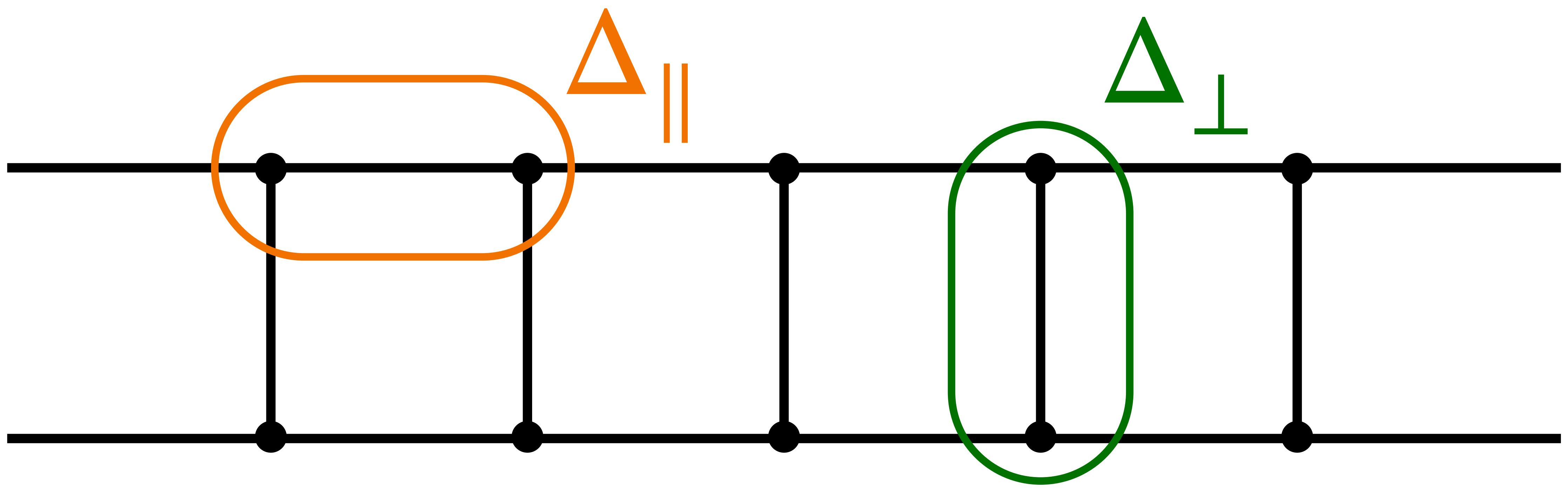

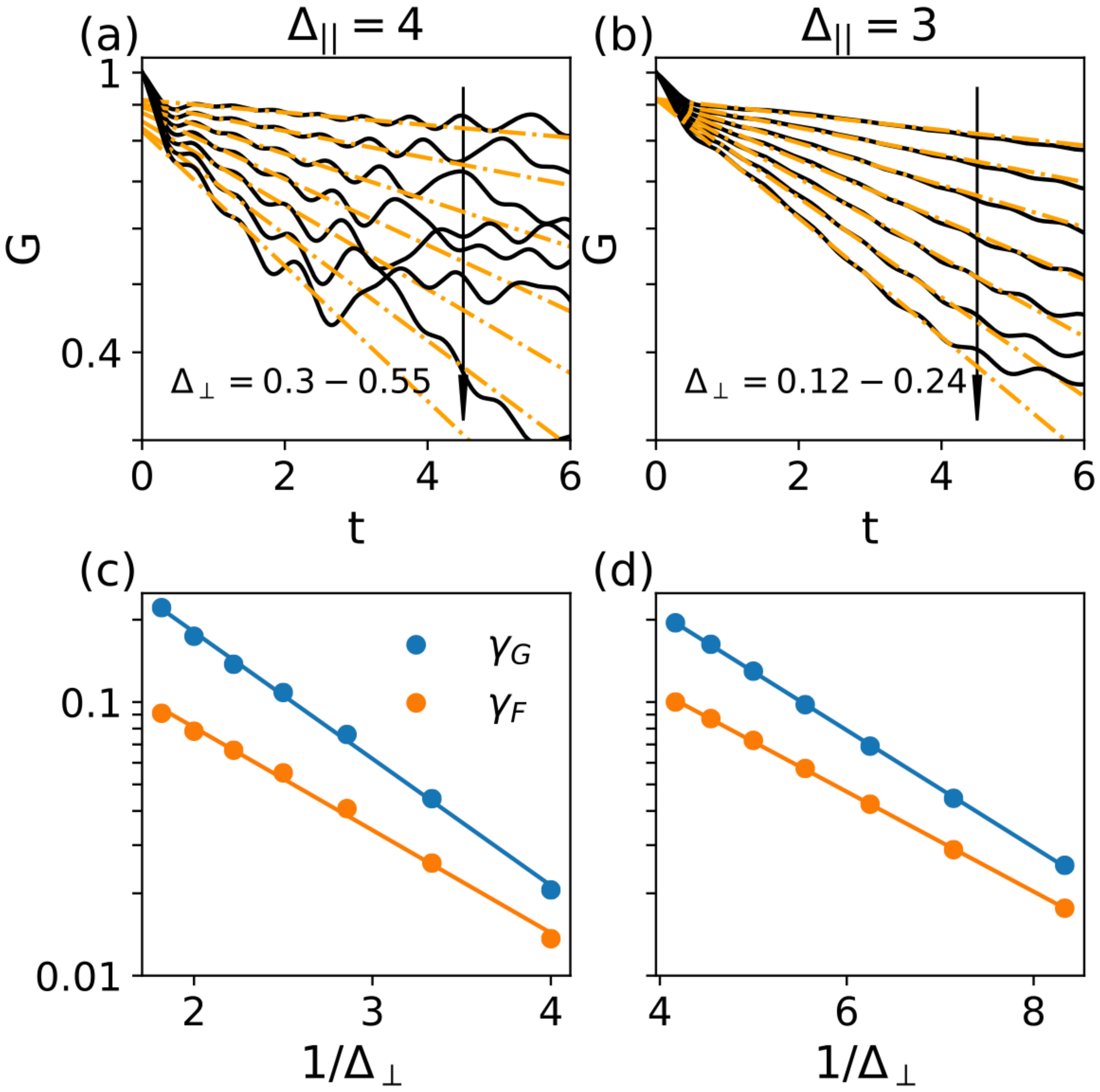

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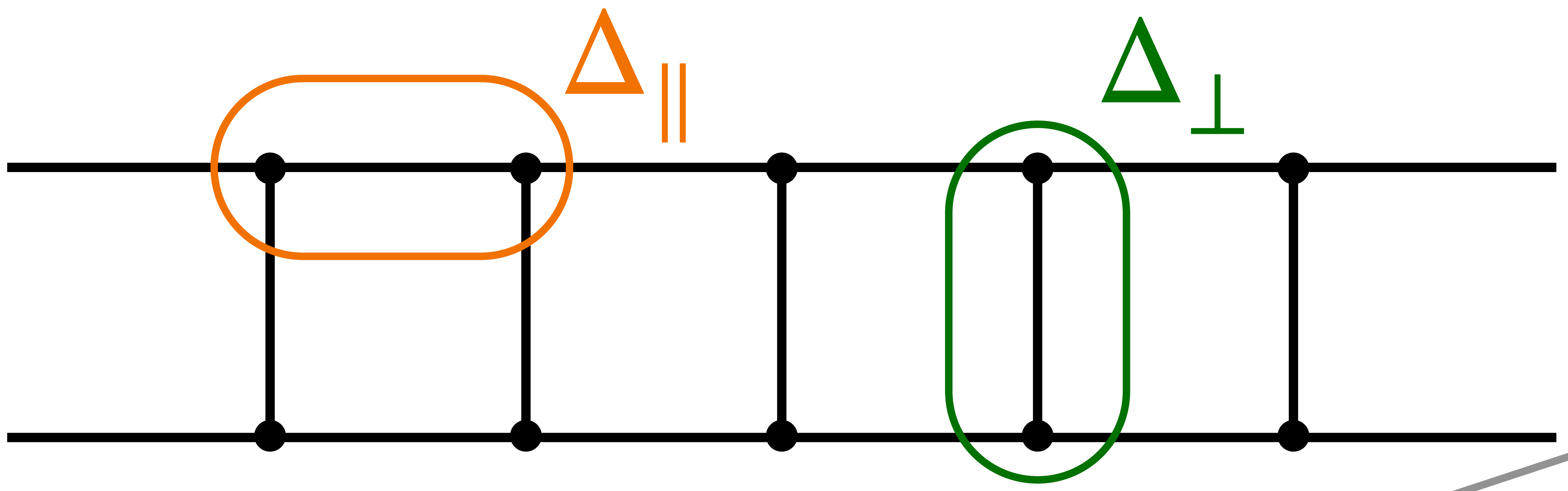
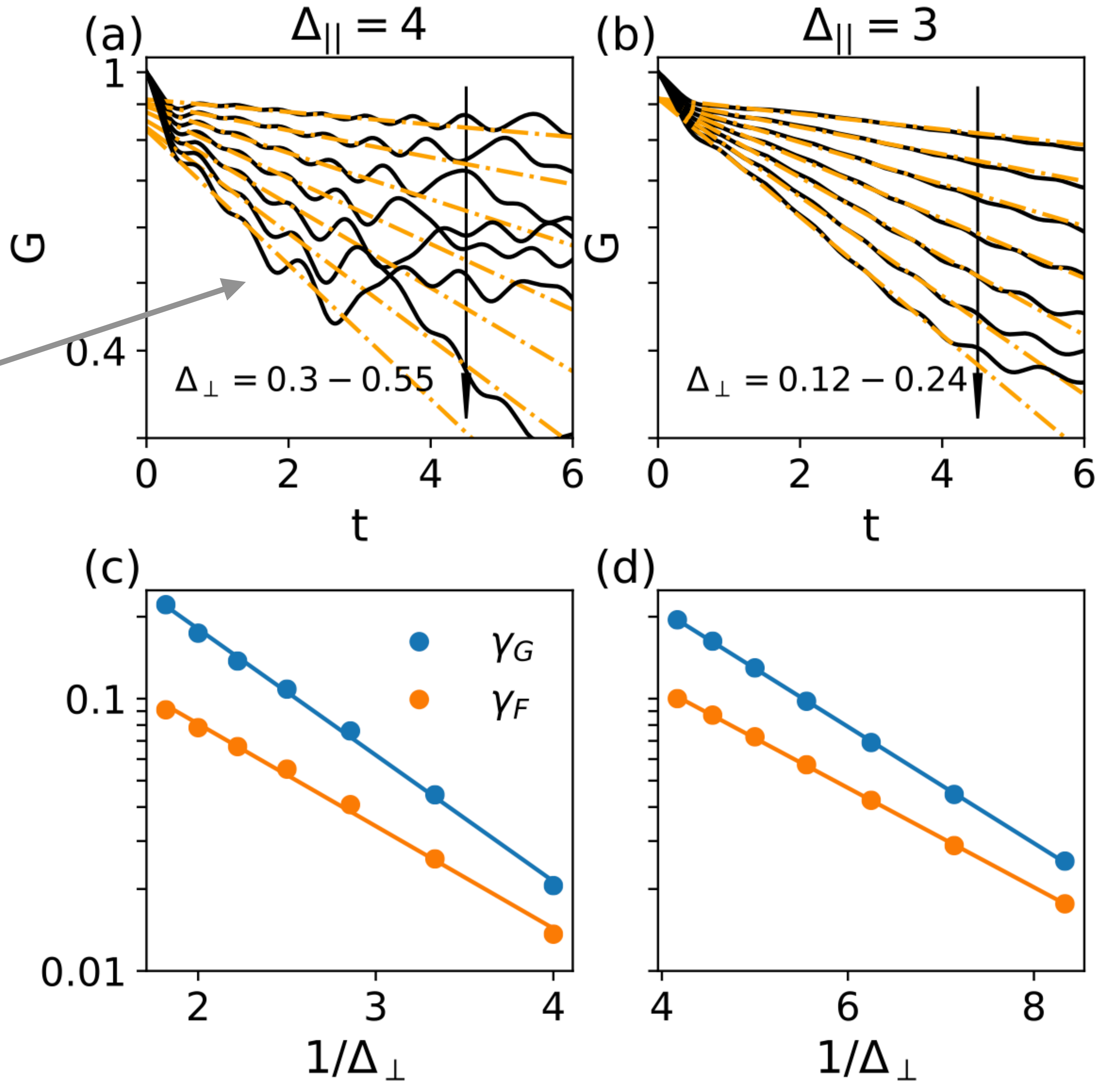


Extract decay rates



XXZ Ladder

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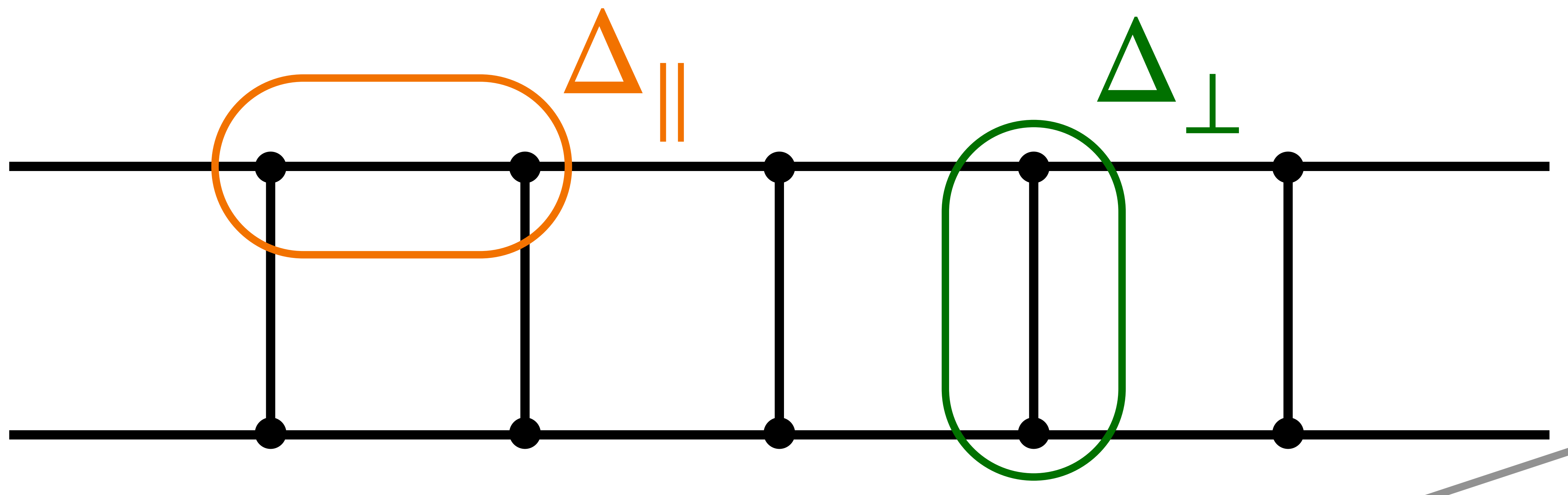
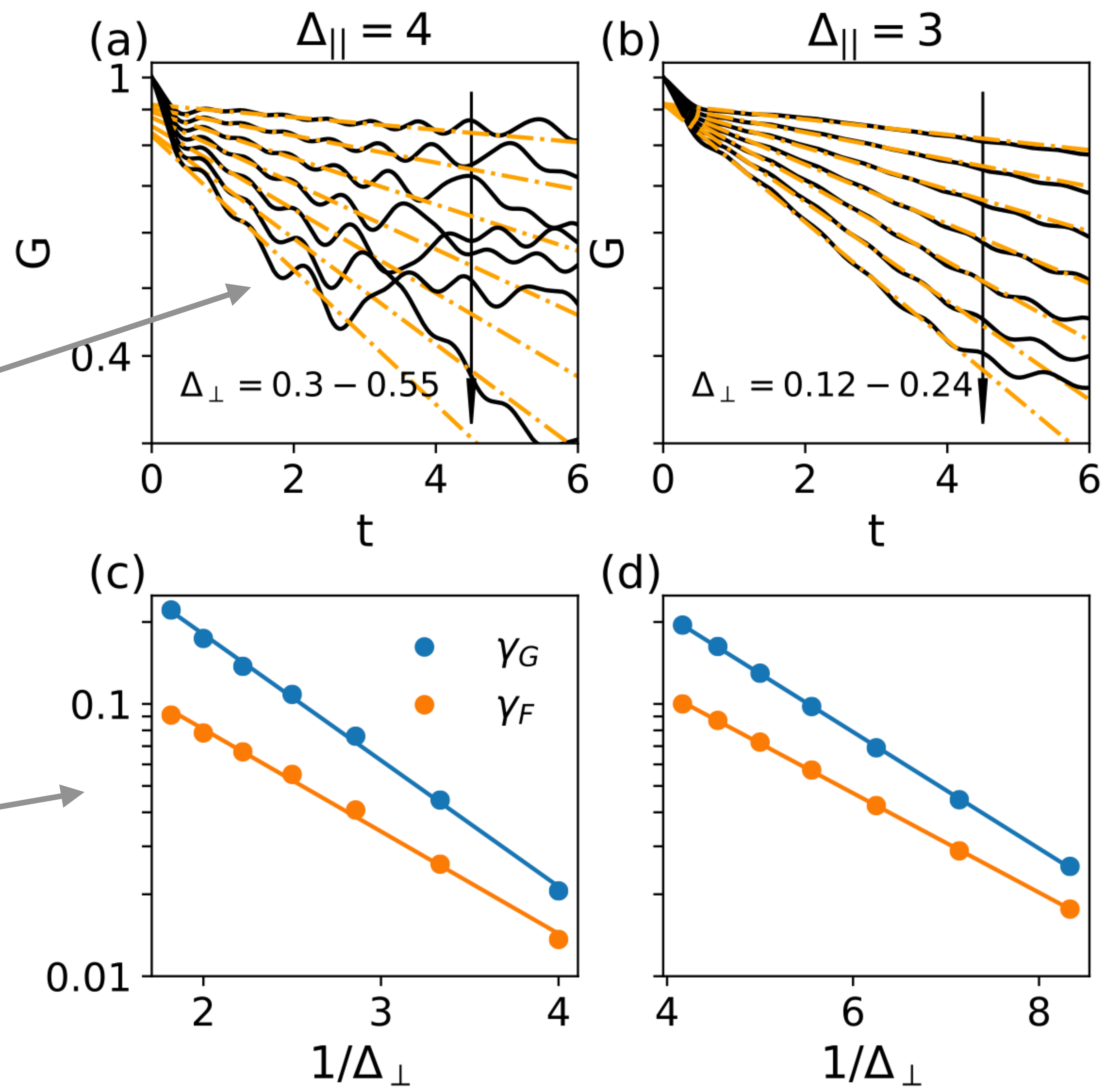




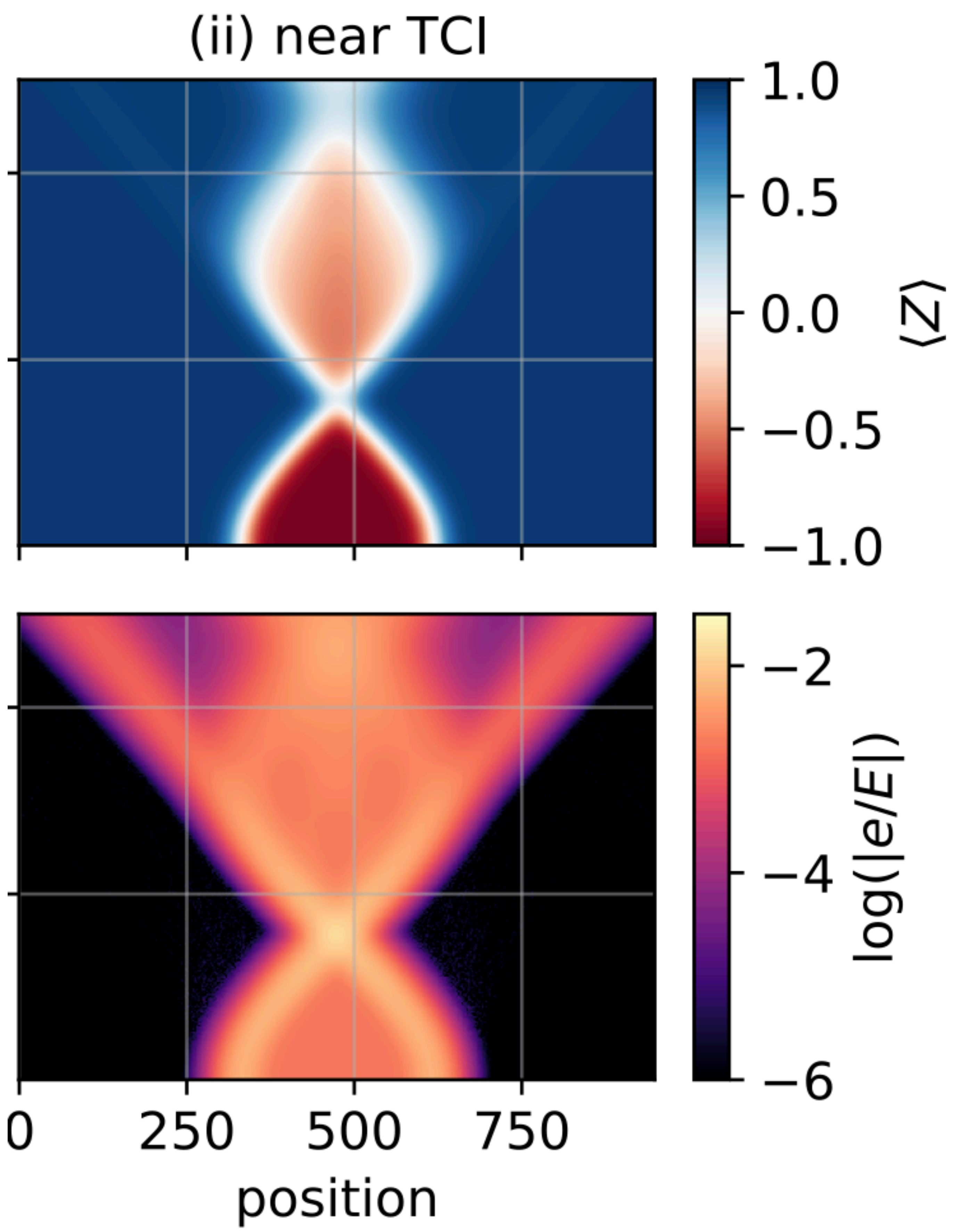
Extract decay rates

Test functional form

$$\gamma = A e^{-c/\Delta_{\perp}}$$

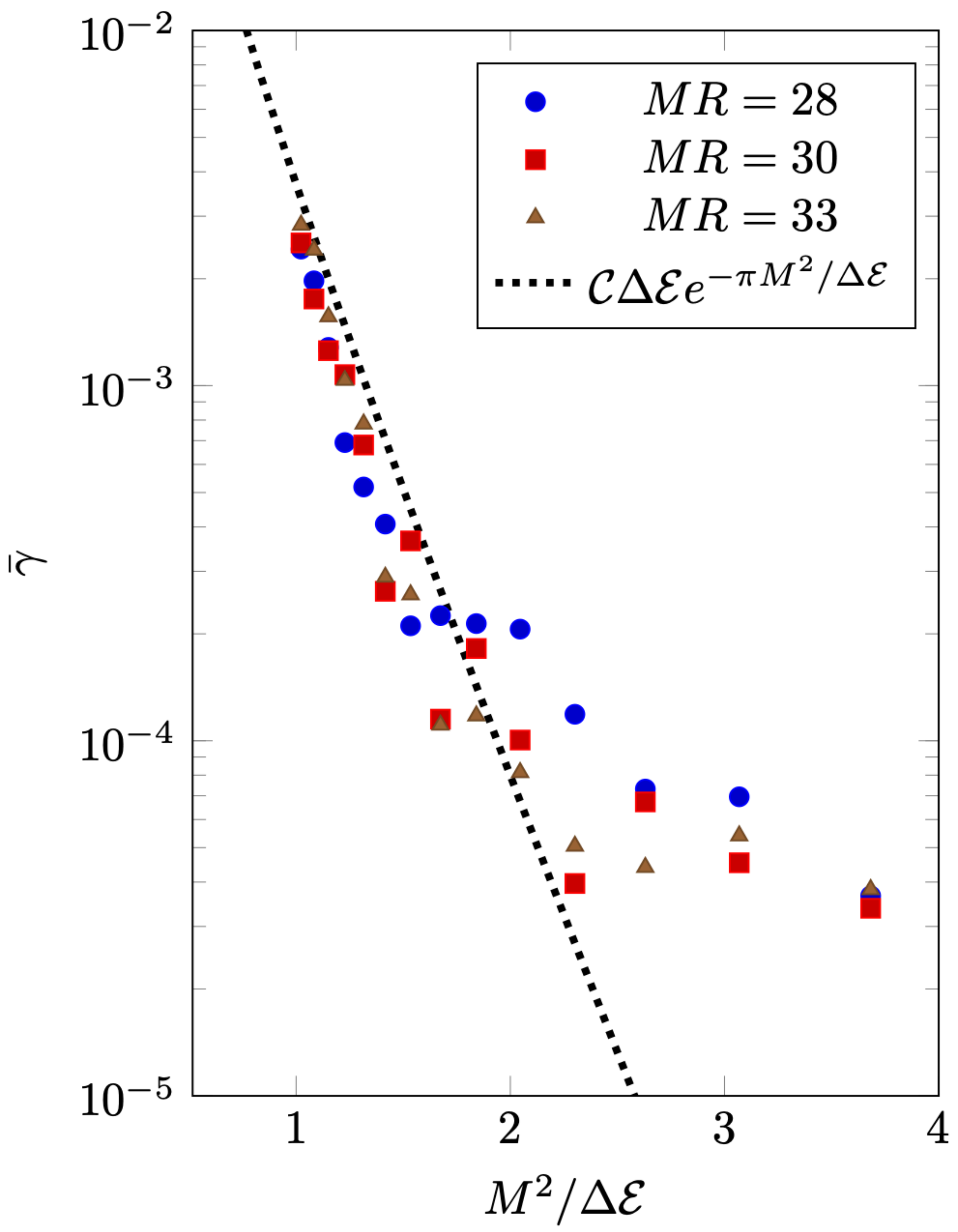
Outlook

More on Bubble Dynamics



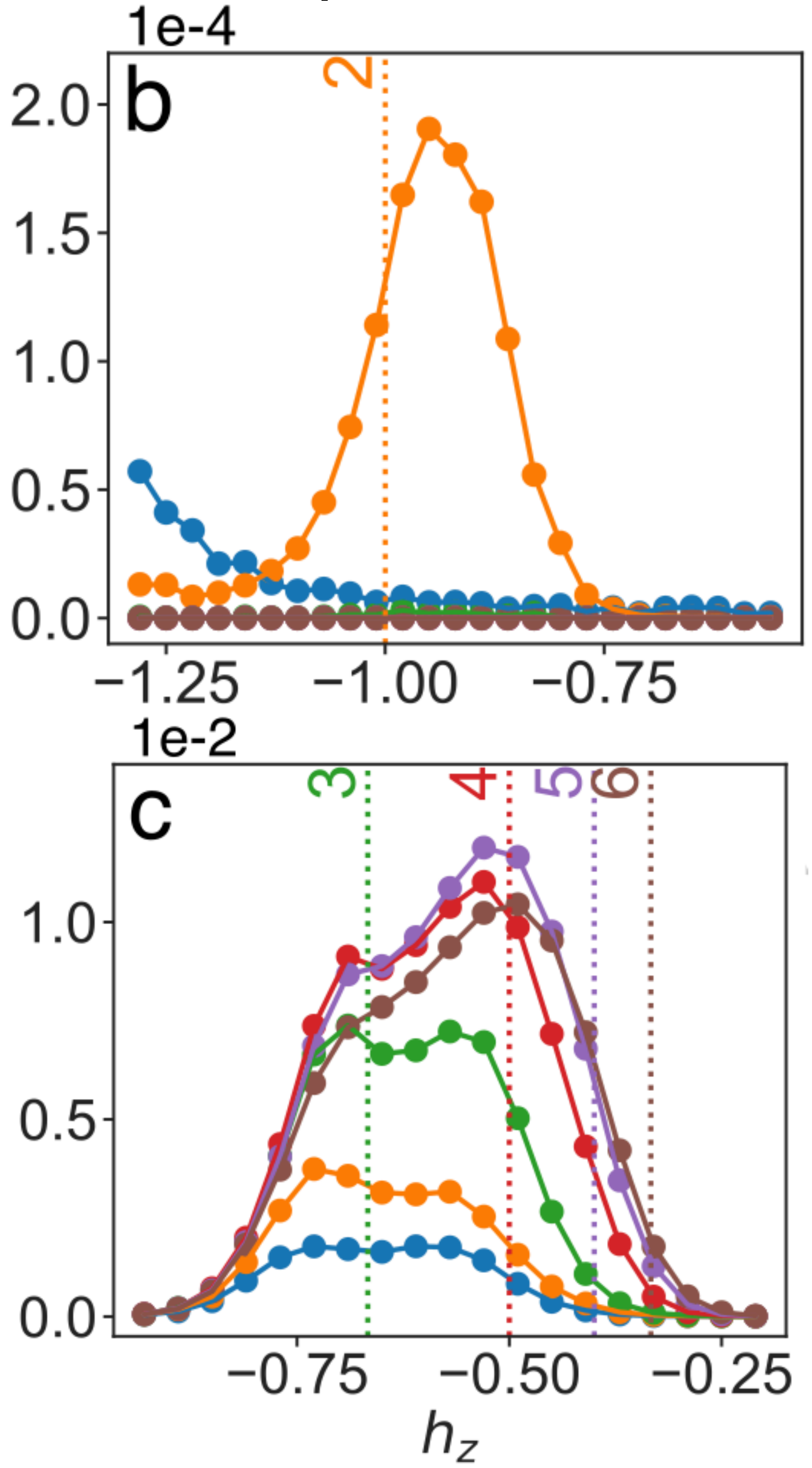
Milsted, Liu, Preskill, Vidal
PRX Quantum 2022

Prefactor in the decay rate



Lencses, Mussardo, Tacaks PRD 2022

Experiments

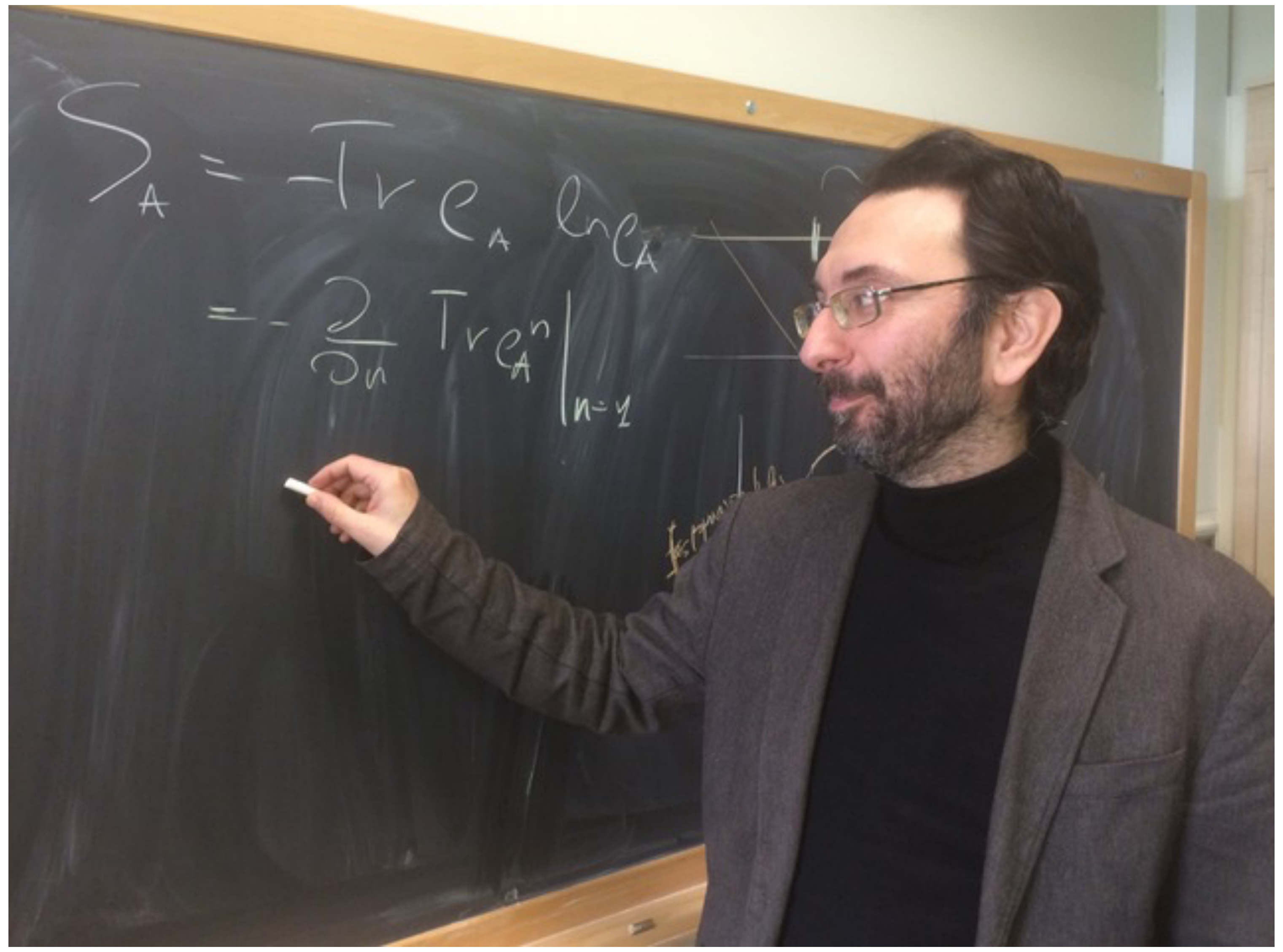


Jaka Vodeb et al. arXiv:2406.14718

Thanks for the attention !

Federica M. Surace

Pasquale Calabrese



Marton Kormos