Higher moments of parton distribution functions using gradient flow

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In collaboration with:

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Introduction

- Motivation: precise knowledge of parton distribution functions (PDFs) from lattice QCD
  → PDF uncertainties largely affect SM predictions, new physics searches

- x-dependence in lattice QCD:
  → heavy-quark OPEs [Aglietti et al hep-ph/9804416, Detmold Lin hep-lat/0507007]
  → quasi-PDFs [Ji 1305.1539], → pseudo-PDFs [Radyuskin 1612.05170]
  → OPE-based method [Chambers et al 1703.01153],
  → current-current approach [Ma Qiu 1404.6860], → hadron tensor method [χQCD 1906.05312], ...

Tuesday plenary by
Huey-Wen Lin

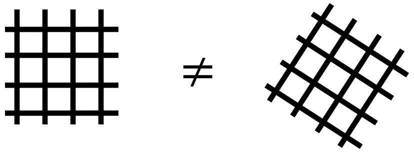
- Operator product expansion → Mellin moments of PDFs $\langle x^{n-1}\rangle = \int_0^1 dx\, x^{n-1}\, (q(x) + (-1)^n \bar{q}(x))$

  e.g., for unpolarized: $\langle h(p)|\mathcal{O}^{\mu_1\cdots\mu_n}|h(p)\rangle = 2\langle x^{n-1}\rangle p^{\mu_1} p^{\mu_2} \ldots p^{\mu_n} - \mathrm{Traces}$

  twist-2 $\mathcal{O}^{\mu_1\cdots\mu_n} = i^{n-1}\bar{\psi}\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n\}}\psi - \mathrm{Traces}$ → transform irreducibly under $O(4)$ group
  → renormalization does not induce mixing with lower dimensional operators

- One can reconstruct $q(x)$ from set of moments
  x-dependence methods have unique challenges → can benefit from complementary $\langle x^{n-1}\rangle$ information
  e.g., combined analyses, cross-checks, etc.

Moments of PDFs on Euclidean lattice



(a lattice spacing)

- Discrete lattice: Lorentz symmetry reduces to hypercubic symmetry
  20 irreducible representations → for $n > 4$ power divergent $\propto \frac{1}{a^m}$ mixing with lower dimensional operators

$$\langle h(p)|\mathcal{O}^{\mu_1 \cdots \mu_n}|h(p)\rangle = 2\langle x^{n-1}\rangle p^{\mu_1} p^{\mu_2} \dots p^{\mu_n} + \cdots$$

Kronfeld Photiadis 1985
Mandula Zweig Govaerts 1983
Göckeler et al, hep-lat/9602029

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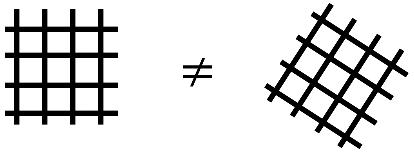

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- $n = 2 : \; \tau_1^{(3)} \; \rightarrow \mathcal{O}_{44} - \text{Traces}, \dots$
  $\tau_3^{(6)} \; \rightarrow \mathcal{O}_{\{14\}}$, … → requires boost in 1 direction

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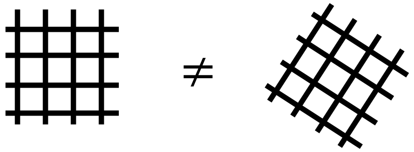

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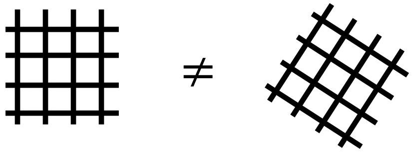
- $n = 3 :\ \tau_1^{(4)} \rightarrow \mathcal{O}_{444} - \mathrm{Traces}, \dots \rightarrow \frac{1}{a^2}$ mixing
  $\tau_2^{(4)} \rightarrow \mathcal{O}_{\{412\}}, \dots$ → requires boost in 2 directions
  $\dots$

Moments of PDFs on Euclidean lattice



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- $n = 4:\ \tau_1^{(1)} \rightarrow \sum_\mu \mathcal{O}_{4444} - \text{Traces}, \ldots \rightarrow \frac{1}{a^3}$ mixing, …
  $\tau_2^{(1)} \rightarrow \mathcal{O}_{\{1234\}}, \ldots$ → requires boost in 3 directions
  $\tau_1^{(3)} \rightarrow \mathcal{O}_{4444} - \cdots, \ldots$ → mixes with $n = 2, \ldots$
  $\tau_3^{(6)} \rightarrow \mathcal{O}_{\{4111\}} - \text{Traces}, \ldots$ → mixes with $n = 2, \ldots$
  …

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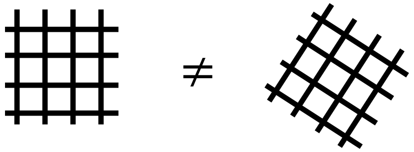

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  …

- $n > 4$: no irrep safe from power-divergent mixing

Gradient flow to the rescue

See Shindler
2311.18704 for details

- Flowed gauge and quark fields $A_\mu(t), \psi(t)$ defined by gradient flow equations and unflowed B.C [Lüscher 1006.4518, 1302.5246]

- Breakthrough applications to lattice QCD: e.g., scale setting, renormalization

- Correlators involving twist-2 operators: flow time t becomes regulator for short-distance singularities

- Take the $a \to 0$ limit at finite t, match to physical $t = 0$ observable using small flow-time expansion (SFTX)

- $\langle h(p)|\mathcal{O}^{\mu_1 \cdots \mu_n}(\mu, \overline{\mathrm{MS}})|h(p)\rangle = \underline{c_n(t,\mu)}\langle h(p)|\mathcal{O}^{\mu_1 \cdots \mu_n}(t)|h(p)\rangle$

- Other proposal for restoration of rotational symmetry: Davoudi Savage 1204.4146

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twist-2 anomalous dimension
Gross Wilczek 1973

$\psi(z)$ digamma function

- $c_n^{(1)}(t,\mu) = C_F(\gamma_n \log(8\pi t\mu^2) + B_n,$

$$\boxed{\gamma_n} = 1 + 4\sum_{j=2}^{n}\frac{1}{j} - \frac{2}{n(n+1)}$$

$$\varphi(z,s,a) = \sum_{k=0}^{\infty}\frac{z^k}{(k+a)^s}$$

$c_2^{(1)}(t,\mu)$ agrees with
Makino Suzuki 1403.4772

$$B_n = \frac{4}{n(n+1)} + 4\frac{n-1}{n}\log 2 + \frac{2-4n^2}{n(n+1)}\gamma_E$$
$$-\frac{2}{n(n+1)}\psi(n+2) + \frac{4}{n}\psi(n+1) - 4\psi(2)$$
$$-4\sum_{j=2}^{n}\frac{1}{j(j-1)}\frac{1}{2^j}\varphi\left(\frac{1}{2,1},j\right) - \log(432)$$

- The above holds for “ringed” quark fields, and requires the renormalization of the flowed fermion fields
  → cancels for ratios of matrix elements of $\mathcal{O}_{\mu_1\dots\mu_n}$ with the same quark content
  → $\mathcal{O}(a^2)$ discretization artifacts for ratios ($\mathcal{O}(am)$ n-independent cancel)

can also consider different hadron in num/denom

$$\left\{\frac{\langle x^{n-1}\rangle(\mu)}{\langle x^{m-1}\rangle(\mu)} = \frac{c_m(t,\mu)}{c_n(t,\mu)}\frac{\langle h(p)|\mathcal{O}_{\mu_1\dots\mu_n}(t)|h(p)\rangle}{\langle h(p)|\mathcal{O}_{\mu_1\dots\mu_m}(t)|h(p)\rangle}\right\}$$

normalize using $\langle x\rangle$ from global analyses, lattice QCD, …

Flowed moments

Shindler 2311.18704

$$\frac{\langle x^{n-1}\rangle(\mu)}{\langle x^{m-1}\rangle(\mu)} = \frac{c_m(t,\mu)}{c_n(t,\mu)} \frac{\langle h(p)|\mathcal{O}_{\mu_1\dots\mu_n}(t)|h(p)\rangle}{\langle h(p)|\mathcal{O}_{\mu_1\dots\mu_m}(t)|h(p)\rangle}$$

$$\mathcal{O}_{\mu_1\dots\mu_n}(t) = \bar{\psi}(t)\gamma_{\mu_1}\overleftrightarrow{D}_{\{\mu_2}(t)\dots\overleftrightarrow{D}_{\mu_n\}}(t)\psi(t) - \text{Traces}$$

Obtain $\langle h(p)|\mathcal{O}_{\mu_1\dots\mu_n}(t)|h(p)\rangle$ on the lattice

Choose $\mu_1 = \mu_2 = \dots = \mu_n = 4$ and $p = 0$ for best STN properties

$$n = 2: \quad \bar{\psi}\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar{\psi}\gamma_i\overleftrightarrow{D}_i\psi$$

$$n = 3: \quad \bar{\psi}\gamma_4\overleftrightarrow{D}_4\overleftrightarrow{D}_4\psi - \frac{1}{12}\sum_{i=1}^{3}(\bar{\psi}\gamma_4\overleftrightarrow{D}_i\overleftrightarrow{D}_i\psi + \bar{\psi}\gamma_i\overleftrightarrow{D}_4\overleftrightarrow{D}_i\psi + \bar{\psi}\gamma_i\overleftrightarrow{D}_i\overleftrightarrow{D}_4\psi)$$

…

n	# of unique ops
2	4
3	10
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$\frac{\langle x^{n-1}\rangle(\mu)}{\langle x^{m-1}\rangle(\mu)}$ from lattice QCD

- 3-point correlation function projected to zero momentum

$$\mathrm{C}^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(t_s,\tau,t) = \sum_{\boldsymbol{x},\boldsymbol{y}}\langle \eta(\boldsymbol{x},t_s)\bar{\psi}(\boldsymbol{y},\tau,t)\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}(\boldsymbol{y},\tau,t)\psi\,(\boldsymbol{y},\tau,t)\bar{\eta}(0)\rangle$$

t: flow time
t_s: sink time
τ: operator insertion time

$\eta(x)$: interpolator overlapping with hadron state

$$\sim S^{seq}(t_s,\tau,t)\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}(\tau,t)\ldots\overleftrightarrow{D}_{\mu_n}(\tau,t)S(\tau,t)$$

sequential propagator

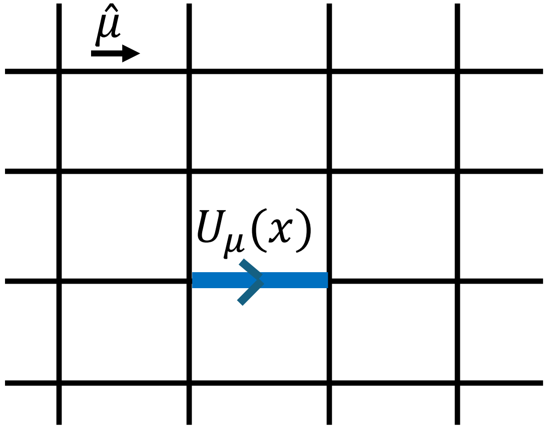
quark propagator

spectral decomposition:

$$\mathrm{C}^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(t_s,\tau) = \textstyle\sum_{n,m}\langle\Omega|\eta|n\rangle\,\langle m|\bar{\eta}|\Omega\rangle e^{-E_n t_s - E_m(t_s-\tau)}\langle n|\mathcal{O}_{\mu_1\ldots\mu_n}|m\rangle \qquad \text{for } \tau \gg 0, t_s-\tau\gg 0$$

$$\frac{\mathrm{C}^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(t_s,\tau,t)}{\mathrm{C}^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_m}(t_s,\tau,t)} \propto \frac{\langle h|\mathcal{O}_{\mu_1\ldots\mu_n}(t)|h\rangle}{\langle h|\mathcal{O}_{\mu_1\ldots\mu_m}(t)|h\rangle} \xrightarrow[\text{factor}]{\times\text{ matching}} (\mathrm{mass})^{m-n}\frac{\langle x^{n-1}\rangle(\mu)}{\langle x^{m-1}\rangle(\mu)} \qquad \text{for } \sqrt{8t} \ll t_s$$

Compute $\boldsymbol{S}^{\mathrm{seq}}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_n}\boldsymbol{S}$: technical challenges



$$\overleftrightarrow{D}_\mu = (\overrightarrow{D}_\mu - \overleftarrow{D}_\mu)/2$$

$$S^{\mathrm{seq}}(x)\overrightarrow{D}_\mu(x)S(x) = \frac{1}{2}(S^{\mathrm{seq}}(x)U_\mu(x)S(x+\hat{\mu}) - S^{\mathrm{seq}}(x)U_\mu^\dagger(x-\hat{\mu})S(x-\hat{\mu}))$$

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$S^{seq}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_n}S$: naively compute 4^{n-1} terms

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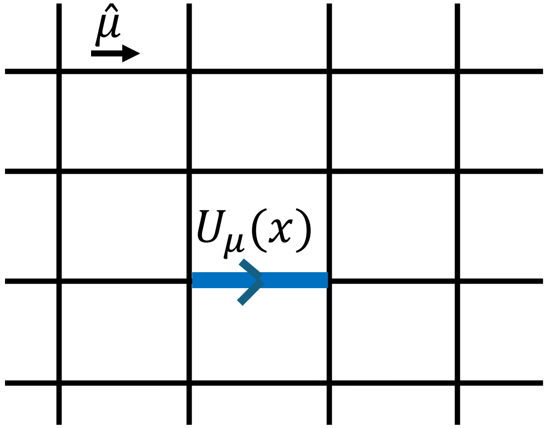

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$$\downarrow \boxed{x \to x - \hat{\mu}} \qquad \downarrow \boxed{x \to x + \hat{\mu}}$$

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$S^{seq}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}S$: naively compute 4^{n-1} terms

→ compute only 2^{n-1} terms (those in $S^{seq}\gamma_{\mu_1}\overrightarrow{D}_{\mu_2}\ldots\overrightarrow{D}_{\mu_n}S$)
and generate the rest by linear combinations and time shifts

Compute $\boldsymbol{S}^{\text{seq}}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}\boldsymbol{S}$: pipeline

$$F_\mu S(x) = U_\mu(x)S(x+\hat{\mu})$$
$$B_\mu S(x) = -U_\mu^\dagger(x-\hat{\mu})S(x-\hat{\mu})$$

$$\begin{tikzcd}
 & & & & F_{\mu_{n-1}}F_{\mu_n}S(x) \arrow[r, "\vec{D}_{\mu_{n-2}}"] & \cdots \\
 & & F_{\mu_n}S(x) \arrow[r, "\vec{D}_{\mu_{n-1}}"] & {} \arrow[ur] \arrow[dr] & & \\
 & & & & B_{\mu_{n-1}}F_{\mu_n}S(x) \arrow[r, "\vec{D}_{\mu_{n-2}}"] & \cdots \\
S(x) \arrow[r, "\vec{D}_{\mu_n}"] & {} \arrow[uur] \arrow[ddr] & & & & \\
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\end{tikzcd}$$

$$\tilde{S}^i_{\mu_2\ldots\mu_n}(x), i = 1,\ldots,2^n$$

t and t_s dependence implicit
(t_s fixed by seq source)

$$C^i_{\mu_1\ldots\mu_n}(\tau) = \sum_{\boldsymbol{x}}\text{Tr}[S^{\text{seq}}(\boldsymbol{x},\tau)\gamma_{\mu_1}\tilde{S}^i_{\mu_2\ldots\mu_n}(\boldsymbol{x},\tau)]$$

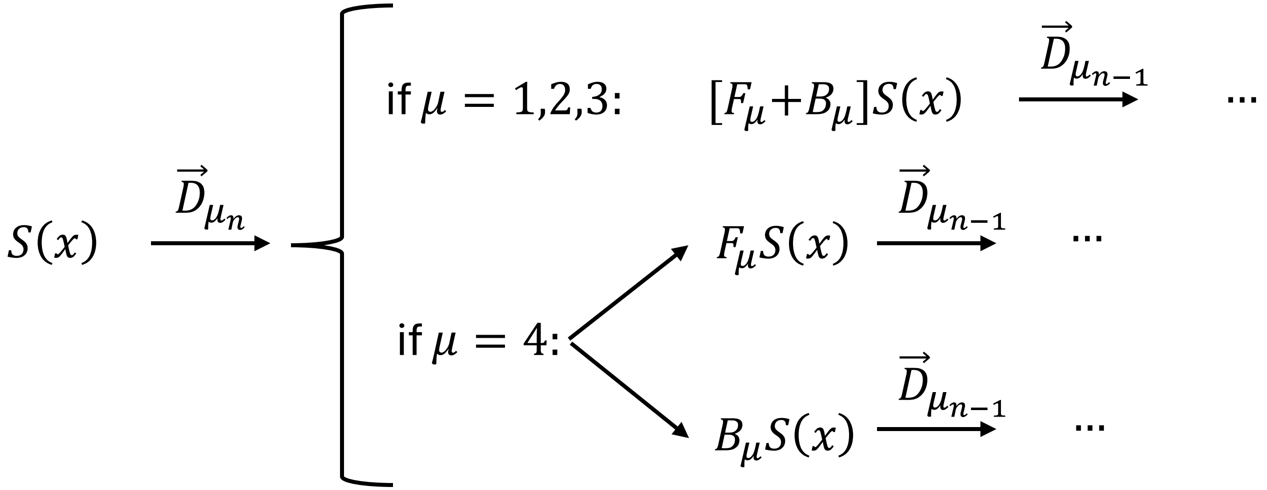
$$C^i_{\mu_1\ldots\mu_n}(\tau) \to \{C_{\mu_1\ldots\mu_n}(\tau;1,1,\ldots,1), C_{\mu_1\ldots\mu_n}(\tau;1,1,\ldots,0)\ ,\ldots, C_{\mu_1\ldots\mu_n}(\tau;0,0,\ldots,0)\}$$

$$\rightarrow S^{\text{seq}}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}S = \sum_{\ell_i\epsilon\{0,1\}}\frac{1}{2^{n+n_t-1}}\sum_{k=0}^{n_t}\binom{n_t}{k}C_{\mu_1\ldots\mu_n}(t-\Delta t(\mu_2,\ldots,\mu_n;\ell_2,\ell_3,\ldots,\ell_n)+k;\ \ell_2,\ell_3,\ldots,\ell_n)$$

$$\Delta t(\mu_2,\ldots,\mu_n;\ell_2,\ell_3,\ldots,\ell_n) = \sum_{i \text{ with } \mu_i=4}\ell_i$$

Compute $\boldsymbol{S}^{\text{seq}}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}\boldsymbol{S}$: pipeline

$S^{\text{seq}}\gamma_{\mu_1}\vec{D}_{\mu_2}\ldots\vec{D}_i S = S^{\text{seq}}\gamma_{\mu_1}\vec{D}_{\mu_2}\ldots\overleftarrow{D}_i S$, for $i = \{1,2,3\}$
→ further reduces total #



$S(x) \xrightarrow{\vec{D}_{\mu_n}}$

if $\mu = 1,2,3$: $[F_\mu + B_\mu]S(x) \xrightarrow{\vec{D}_{\mu_{n-1}}} \ldots$

if $\mu = 4$:

$F_\mu S(x) \xrightarrow{\vec{D}_{\mu_{n-1}}} \ldots$

$B_\mu S(x) \xrightarrow{\vec{D}_{\mu_{n-1}}} \ldots$

Compute $\boldsymbol{S}^{\mathrm{seq}}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2} \ldots \overleftrightarrow{D}_{\mu_n}\boldsymbol{S}$: pipeline

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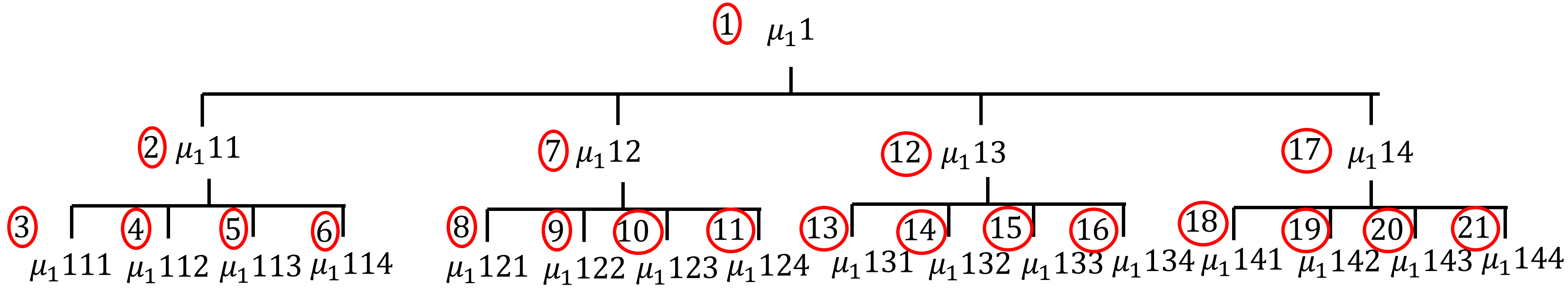
→ If ordered in a tree-search manner, the shifted propagators $\tilde{S}^{i}_{\mu_2 \ldots \mu_n}$ can be reused instead of computed for each unique op starting from S

e.g. for $n = \{2,3,4\}$:

μ_1: γ-index



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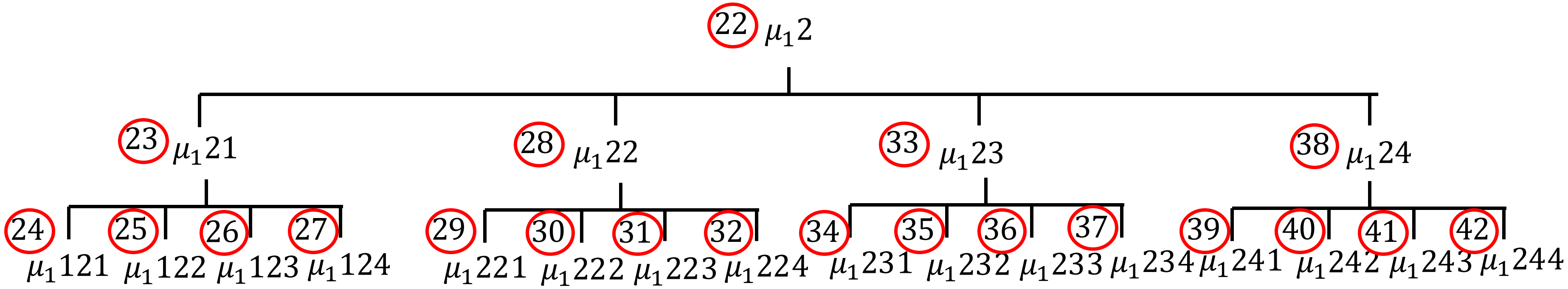
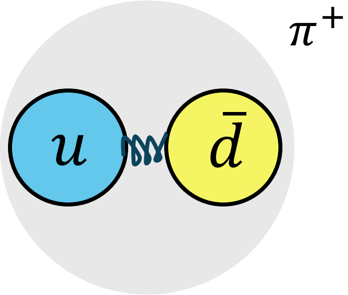


Test case: Pion at SU(3)



- Consider purely connected light-quark contribution u^+
  → if $m_u = m_d = m_s$, corresponds to non-singlet $\bar{u}u + \bar{d}d - 2\bar{s}s$

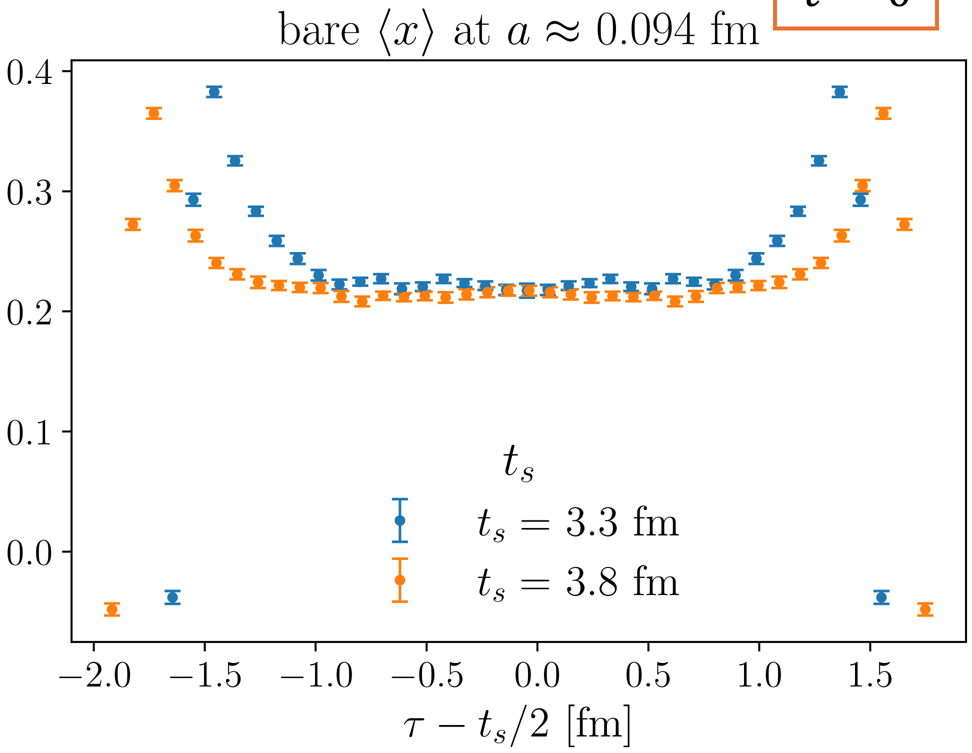
- $m_\pi \approx 410$ MeV, $m_u = m_d = m_s$
  $a \approx 0.12$ fm, 0.094 fm, 0.077 fm, 0.064 fm

- One stochastic point-source per configuration
  $a \approx 0.12$ fm: $N_{cfg} = 119$, $t_s/a \in \{25,30,35,40\}$
  $a \approx 0.094$ fm: $N_{cfg} = 210$, $\frac{t_s}{a} \in \{35,40\}$
  $a \approx 0.077$ fm: $N_{cfg} = 20$, $\frac{t_s}{a} \in \{40\}$
  $a \approx 0.064$ fm: $N_{cfg} = 80$, $t_s/a \in \{40\}$

- Flow integration step 0.01
  Measurements equally spaced in t up to flow radius
  $\sqrt{8t} \approx 0.6$ fm, $t/t_0 \approx 2.5$

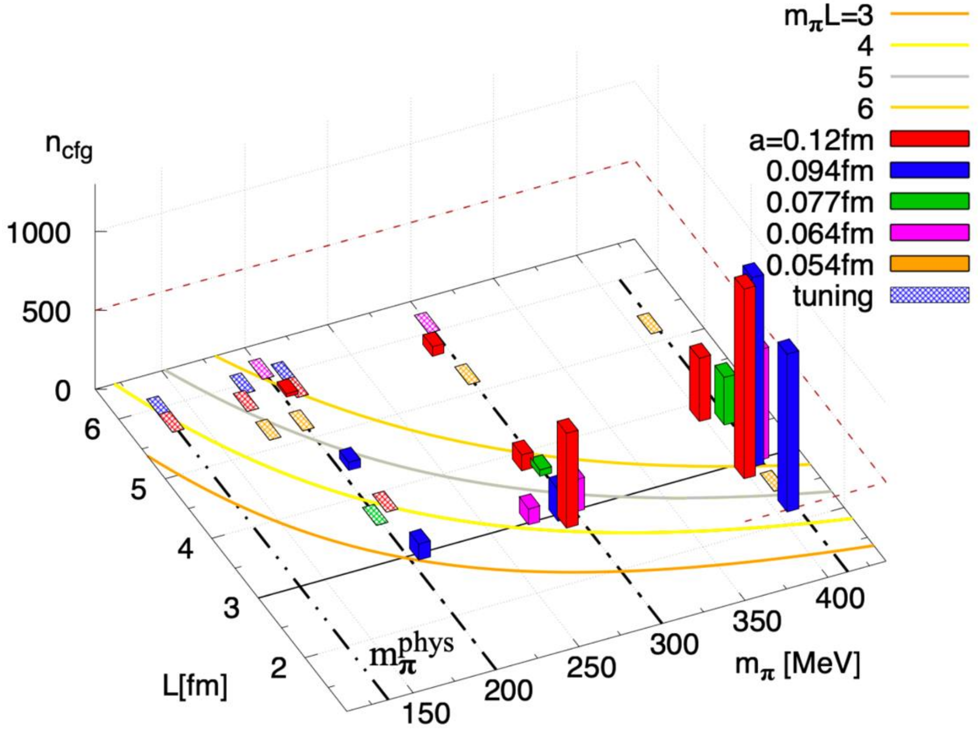
- Stabilized Wilson fermion (SWF) ensembles generated by OpenLat

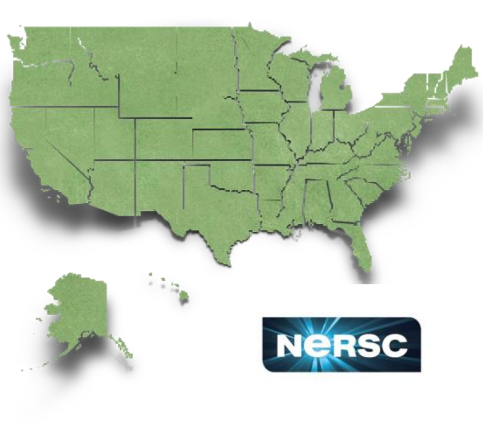
$t = 0$



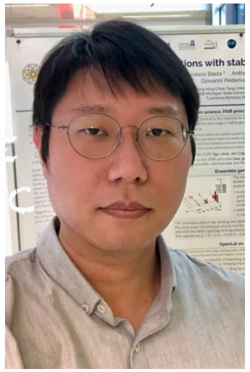
OpenLat Initiative

OpenLat: open science initiative.
Gauges with SWF open to the whole community





PRACE
EuroHPC



Jangho Kim

Francesca Cuteri

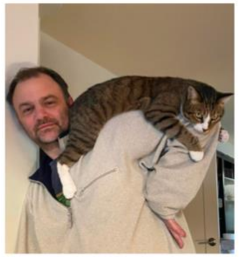
Anthony Francis

Patrick Fritzsch

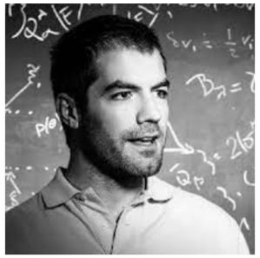
Giovanni Pederiva

Antonio Rago

Andrea Shindler

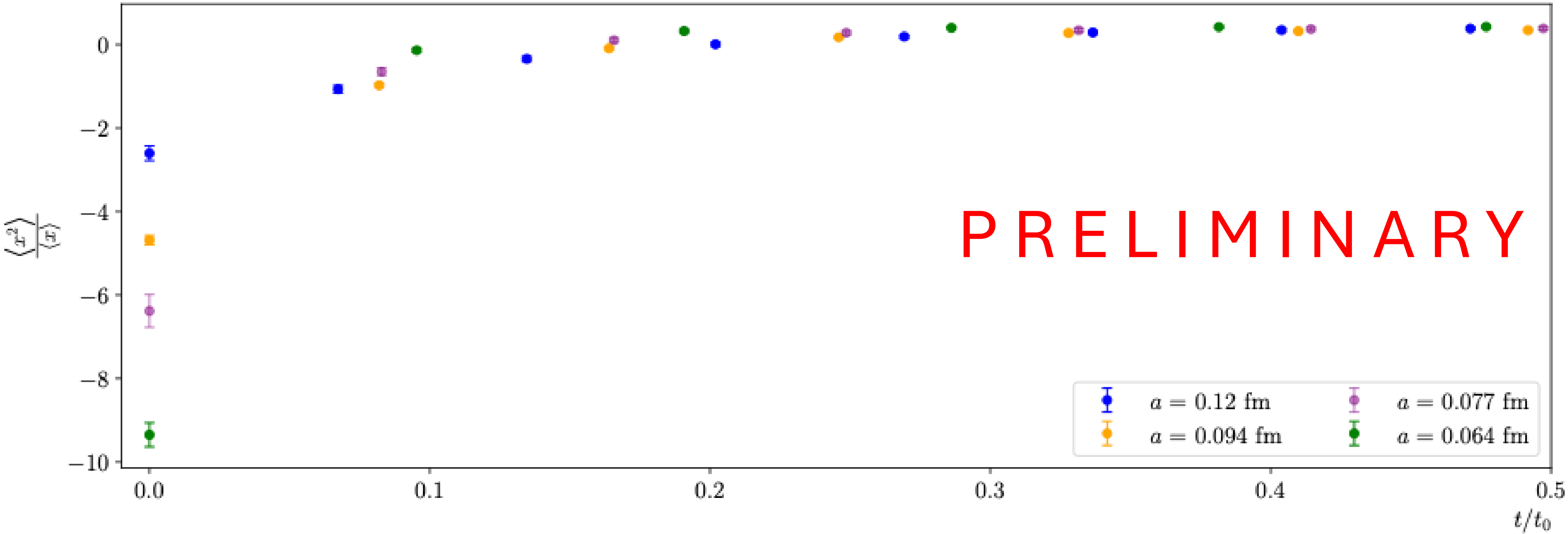


André Walker-Loud

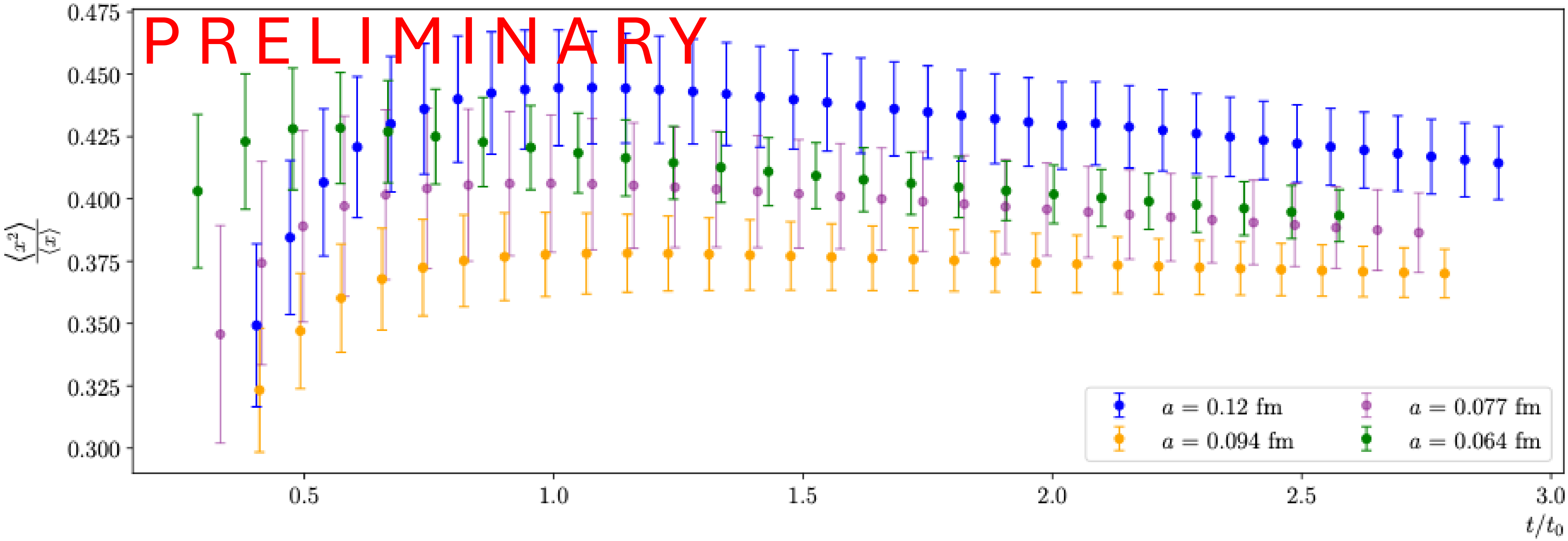


Savvas Zafeiropoulos

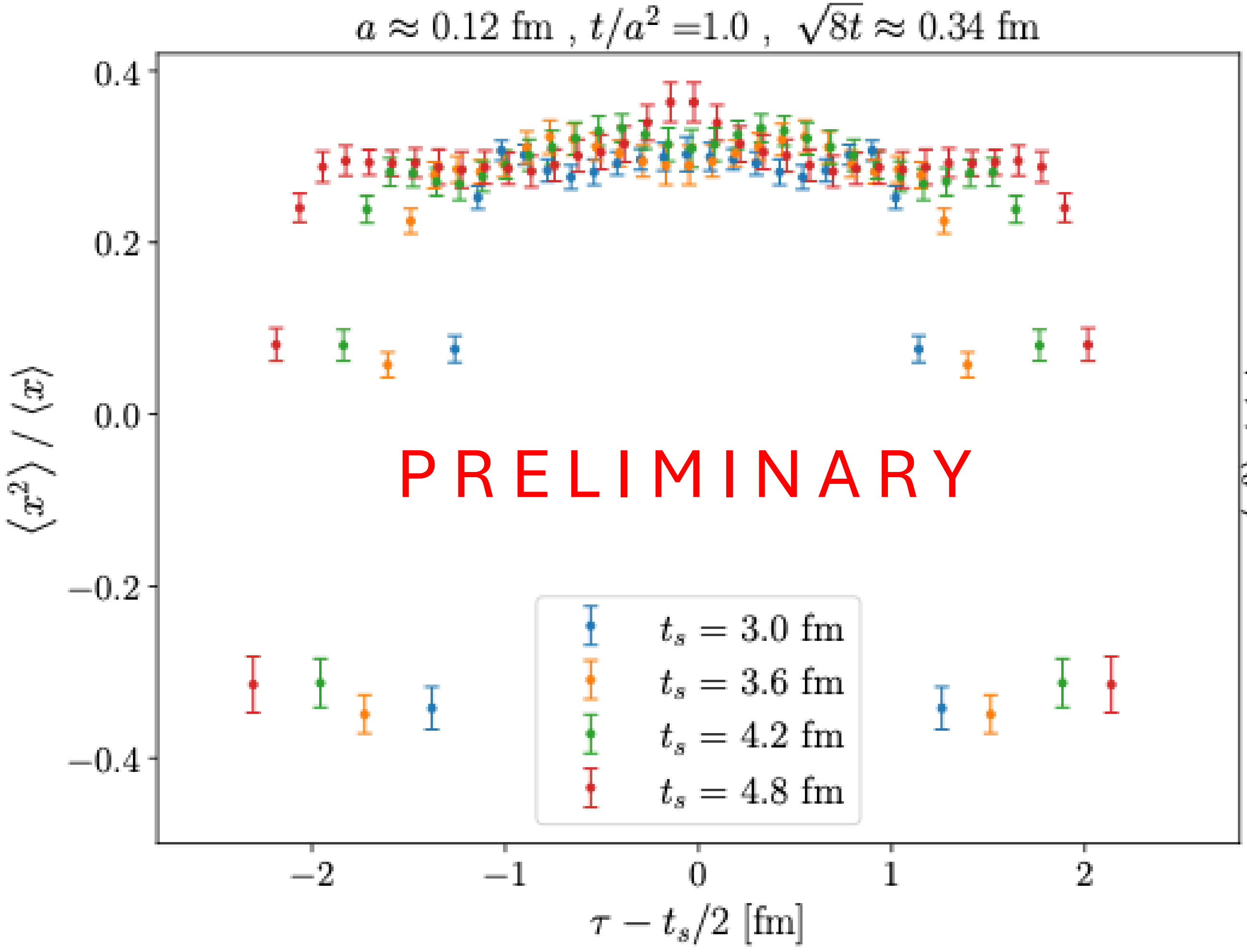
$$\frac{\langle x^2\rangle_{\overline{\mathrm{MS}}}}{\langle x\rangle_{\overline{\mathrm{MS}}}}(\mu = 2\ \mathrm{GeV})\ \text{at}\ t_s = 40a, \tau = 20a$$

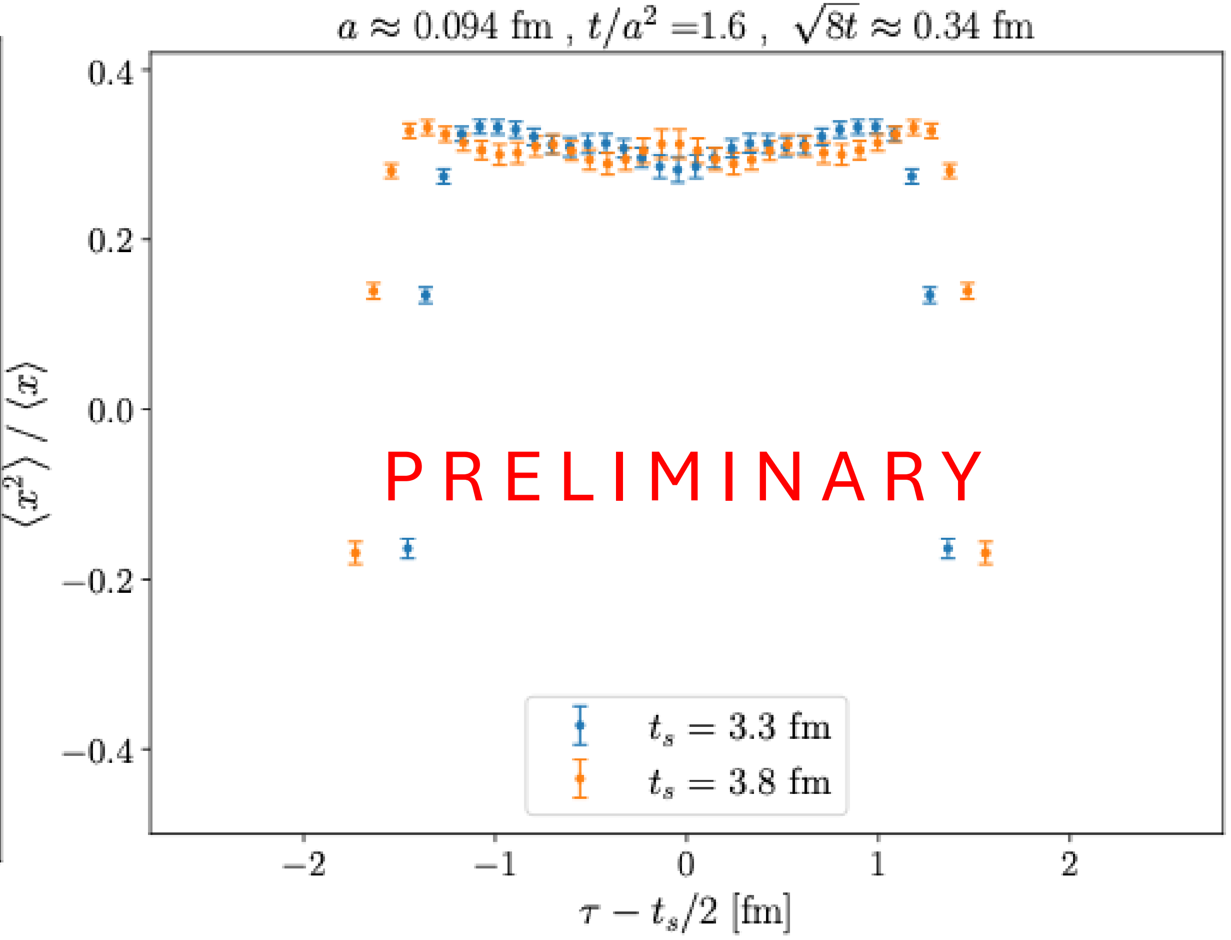


$\frac{\langle x^2 \rangle_{\overline{\text{MS}}}}{\langle x \rangle_{\overline{\text{MS}}}}(\mu = 2\text{ GeV})$ at $t_s = 40a, \tau = 20a$

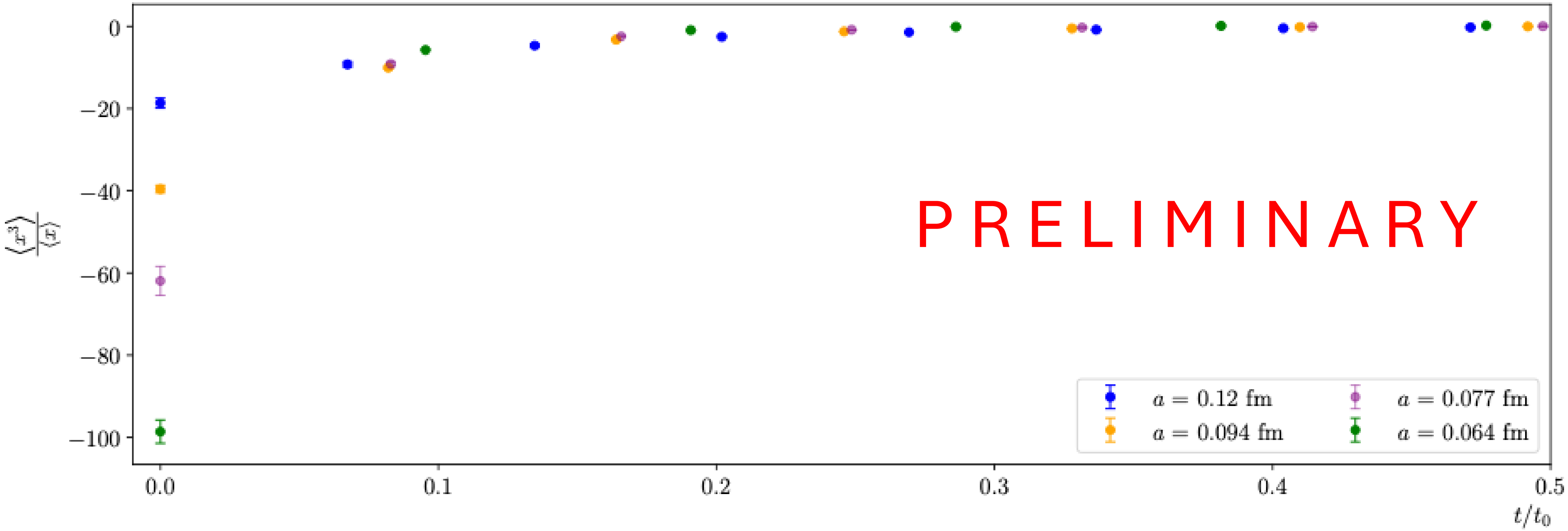


$\langle x^2\rangle_{\overline{\mathrm{MS}}}/\langle x\rangle_{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV})$ at fixed $\sqrt{8t} \approx 0.34$ fm





$\frac{\langle x^3\rangle_{\overline{\mathrm{MS}}}}{\langle x\rangle_{\overline{\mathrm{MS}}}}(\mu = 2\ \mathrm{GeV})$ at $t_s = 40a, \tau = 20a$



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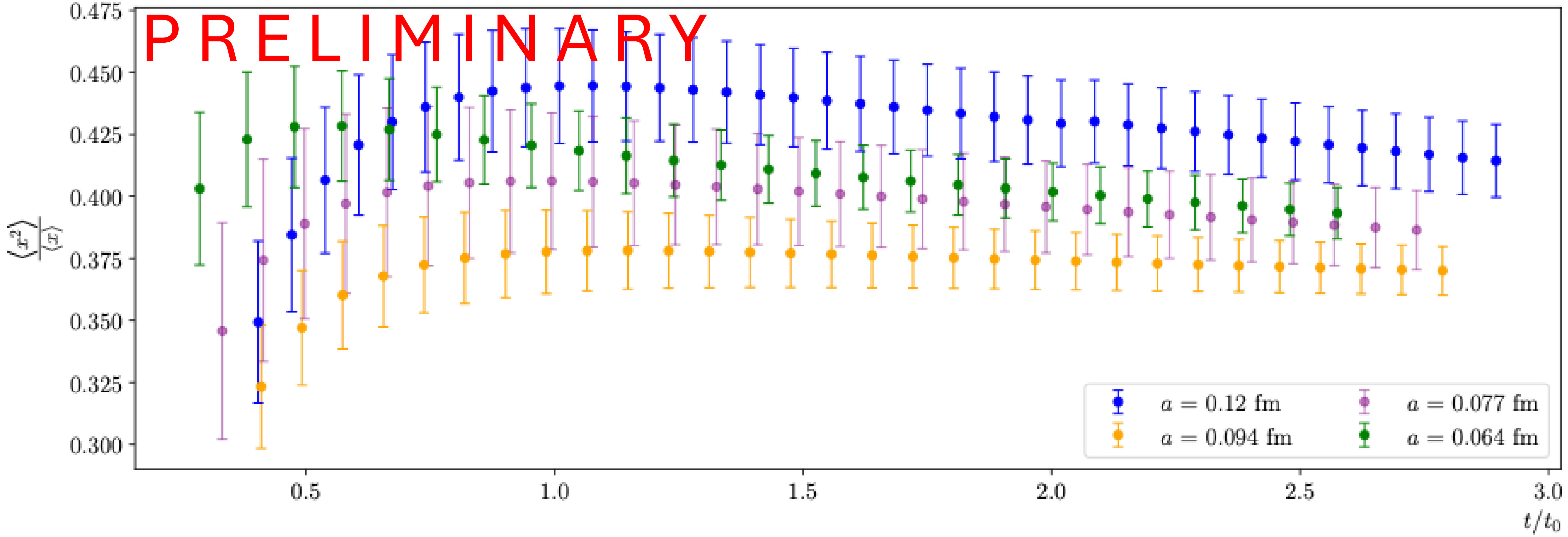
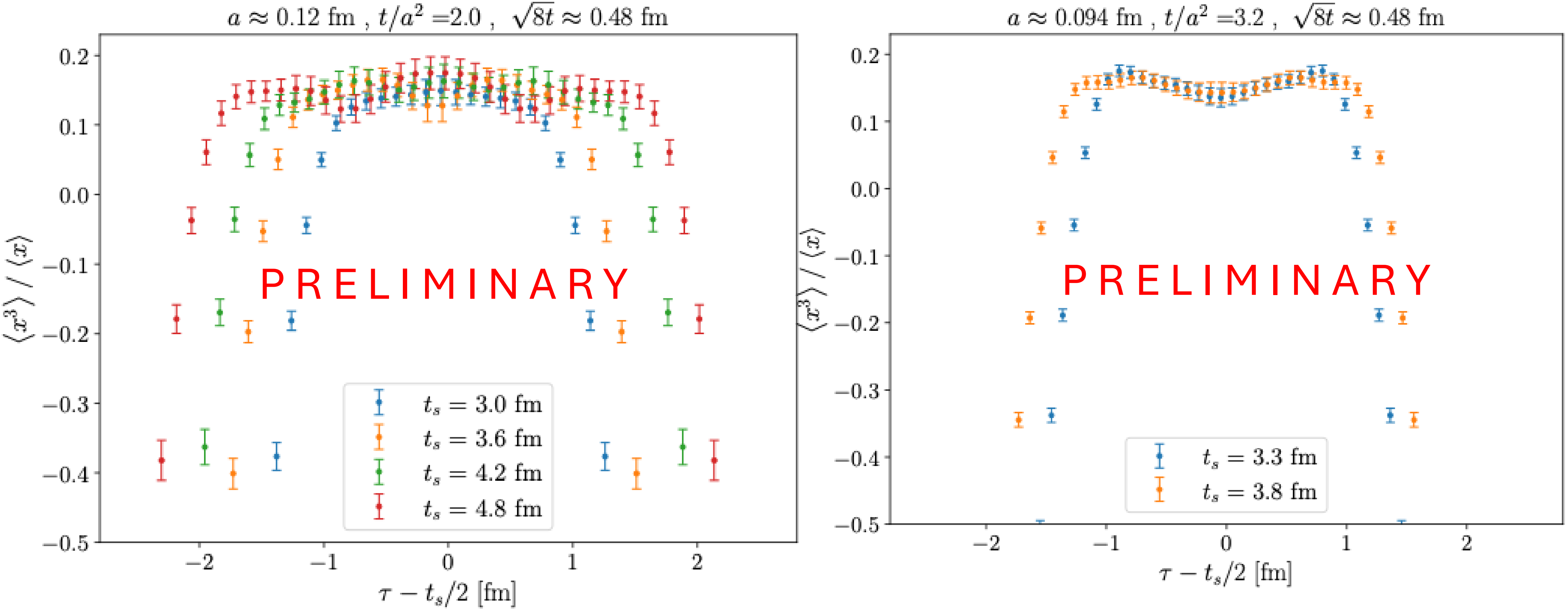
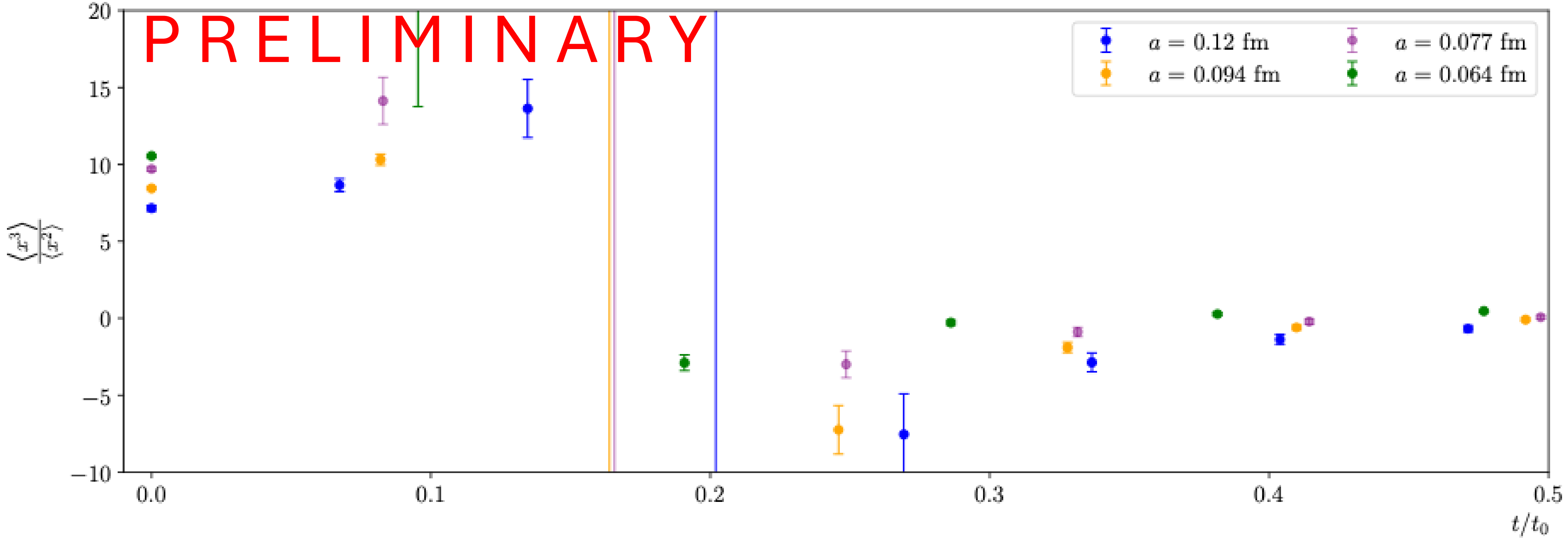


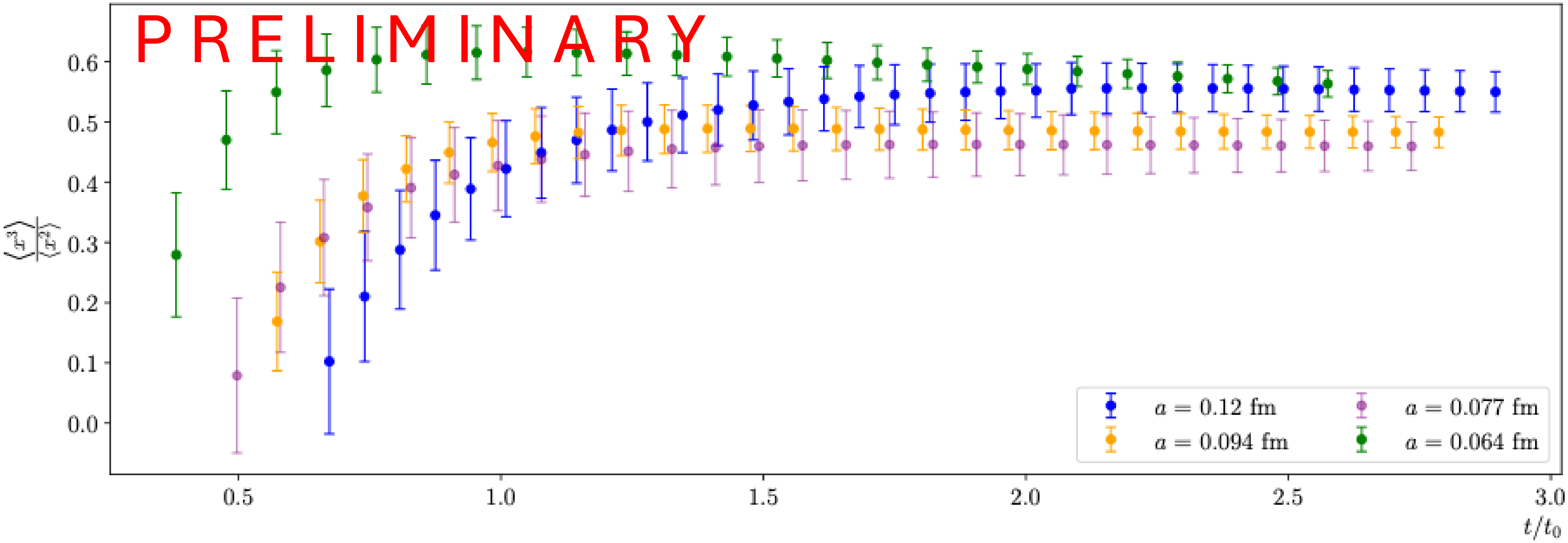
$\langle x^3\rangle_{\overline{\mathrm{MS}}}/\langle x\rangle_{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV})$ at fixed $\sqrt{8t} \approx 0.48$ fm



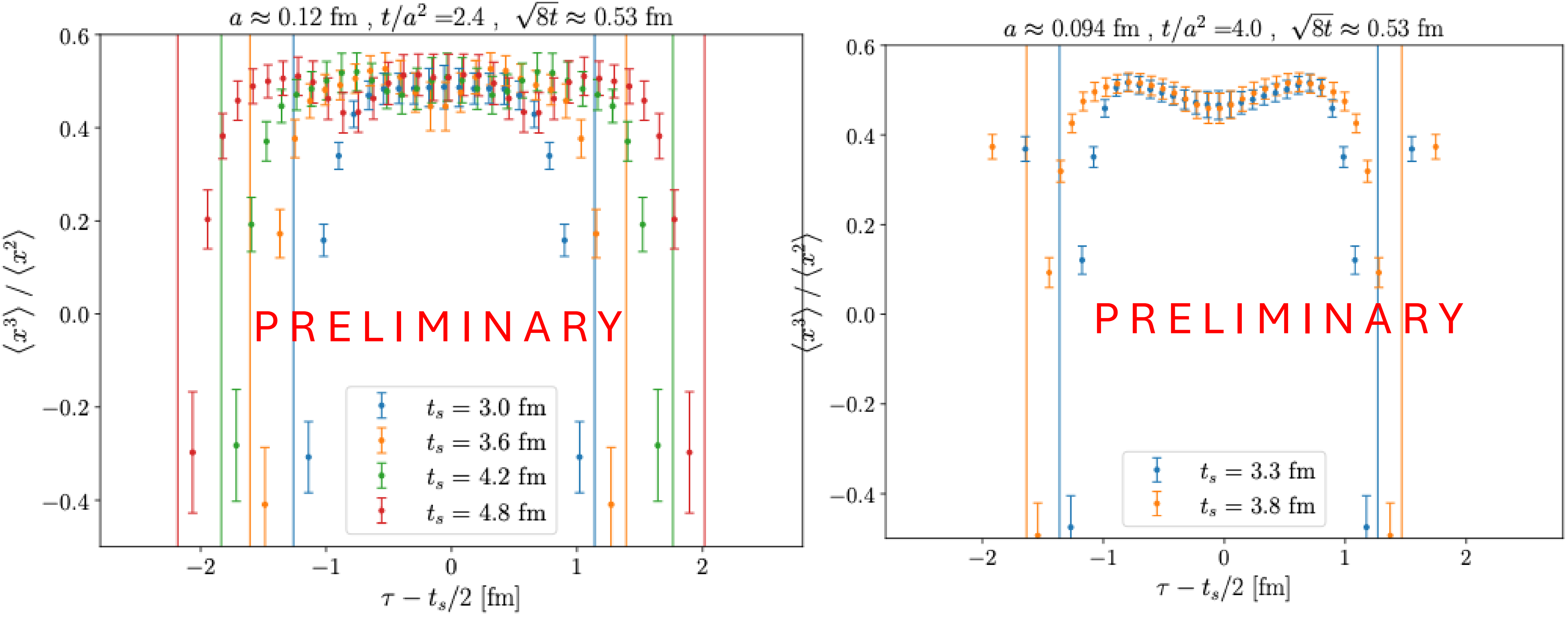
$\frac{\langle x^3 \rangle_{\overline{\mathrm{MS}}}}{\langle x^2 \rangle_{\overline{\mathrm{MS}}}}(\mu = 2\ \mathrm{GeV})$ at $t_s = 40a, \tau = 20a$



$\frac{\langle x^3\rangle_{\overline{\mathrm{MS}}}}{\langle x^2\rangle_{\overline{\mathrm{MS}}}}(\mu = 2\text{ GeV})$ at $t_s = 40a, \tau = 20a$



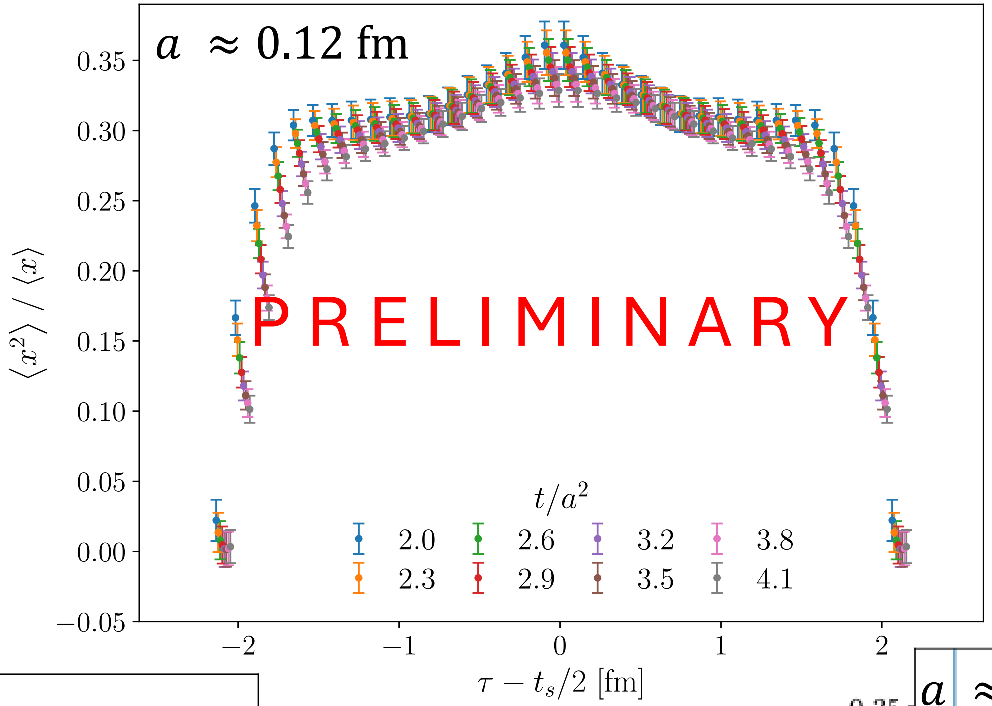
$\langle x^3\rangle_{\overline{\mathrm{MS}}}/\langle x^2\rangle_{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV})$ at fixed $\sqrt{8t} \approx 0.53$ fm

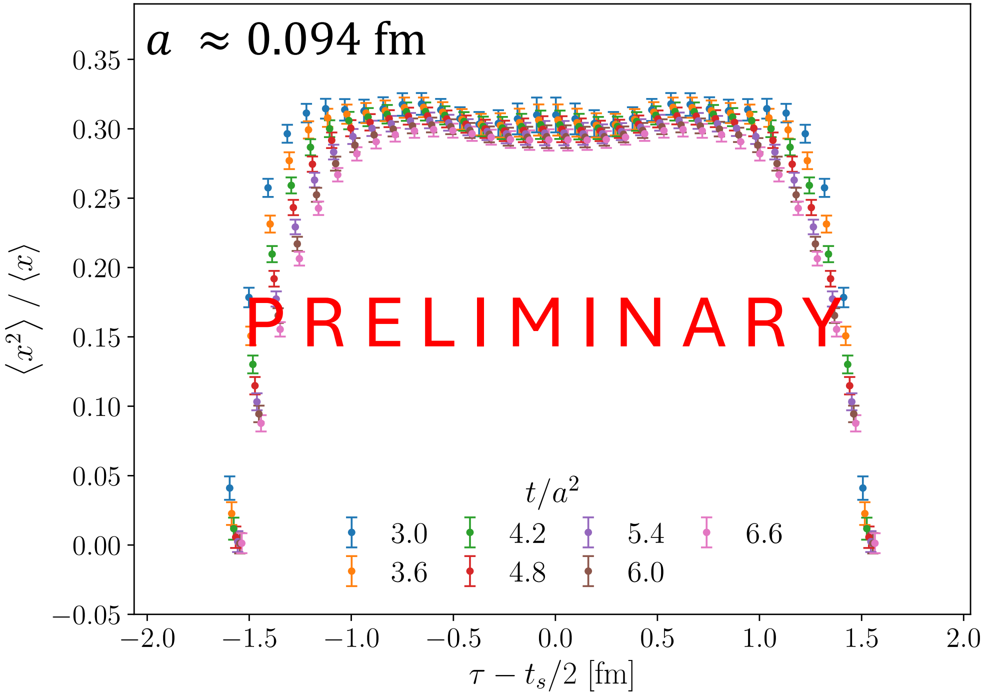


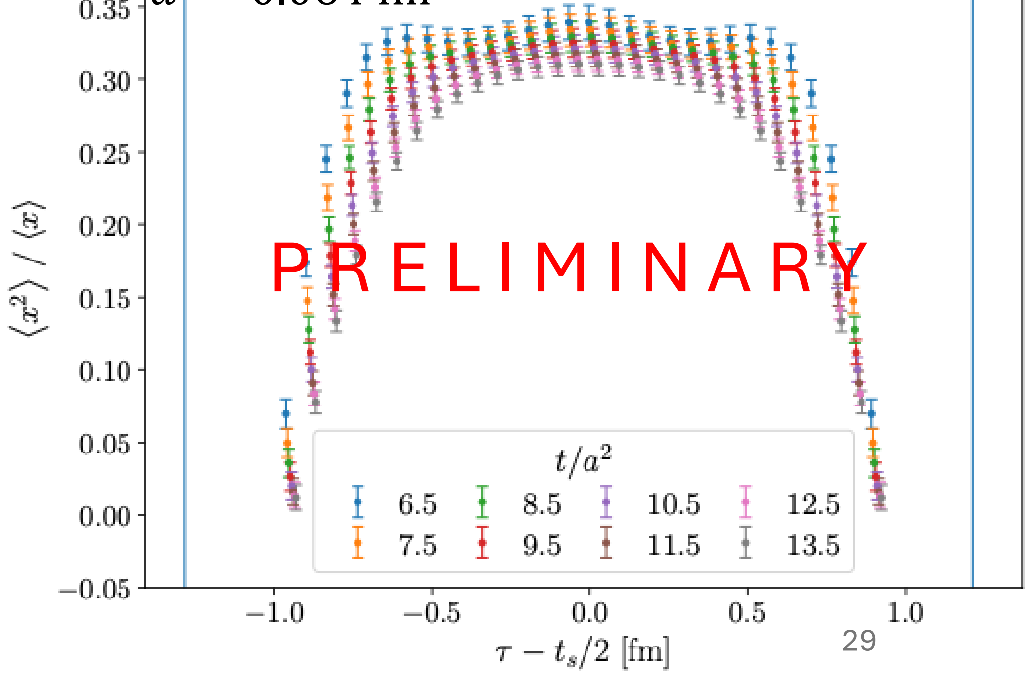
flow-dependent plateaus

$\langle x^2 \rangle_{\overline{\mathrm{MS}}} / \langle x \rangle_{\overline{\mathrm{MS}}} (\mu = 2\ \mathrm{GeV})$

at fixed $t_s \approx 40a$

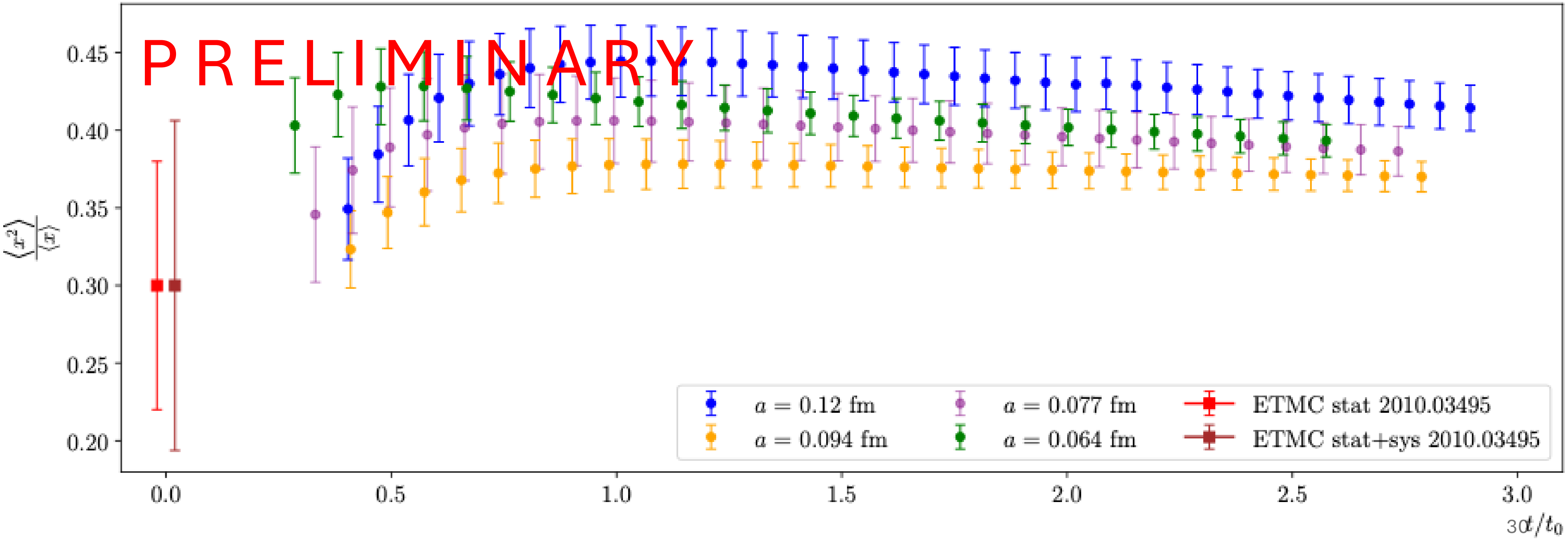






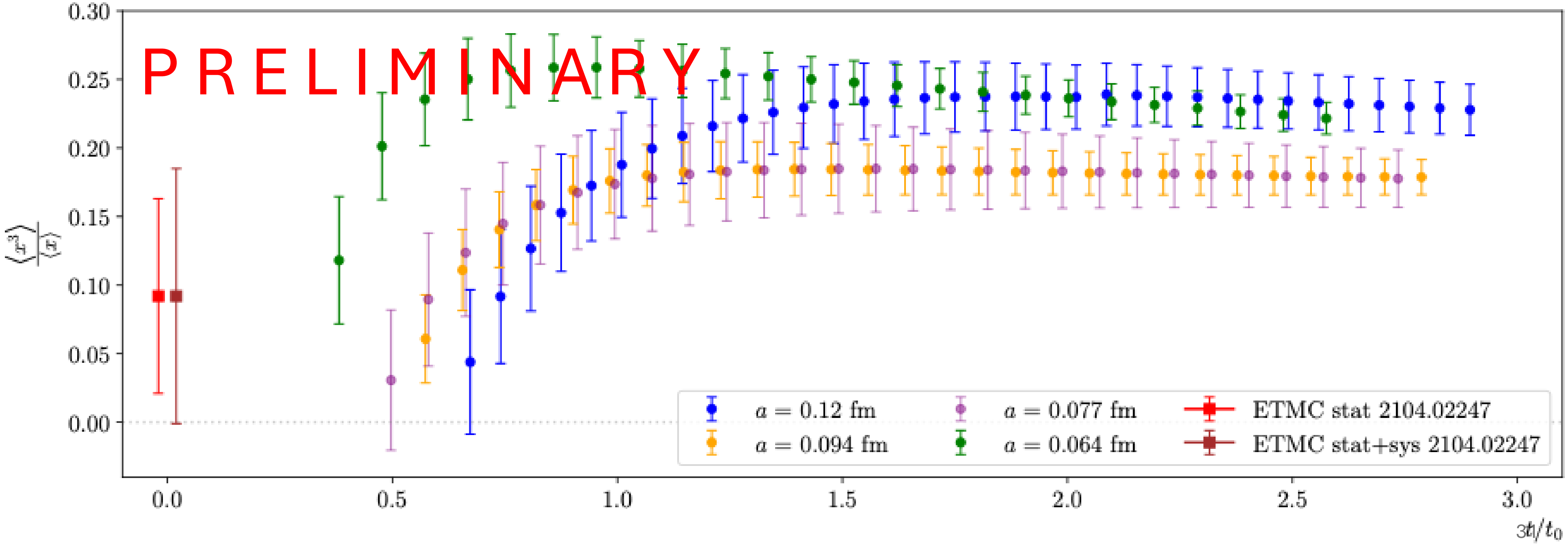
$\frac{\langle x^2\rangle_{\overline{\mathrm{MS}}}}{\langle x\rangle_{\overline{\mathrm{MS}}}}$ ($\mu = 2$ GeV) comparison with ETMC

- 2010.03495: $\mathrm{N_{cfgs}}$ x $\mathrm{N_{source}}$ = 3904 versus 20-210



$\frac{\langle x^3\rangle_{\overline{\mathrm{MS}}}}{\langle x\rangle_{\overline{\mathrm{MS}}}}(\mu = 2\ \mathrm{GeV})$ comparison with ETMC

- 2104.02247: N_{cfgs} x N_{source} x N_{boost} ~ 15k-70k versus 20-210



$\frac{\langle x^3\rangle_{\overline{\mathrm{MS}}}}{\langle x^2\rangle_{\overline{\mathrm{MS}}}}(\mu = 2\ \mathrm{GeV})$ comparison with ETMC

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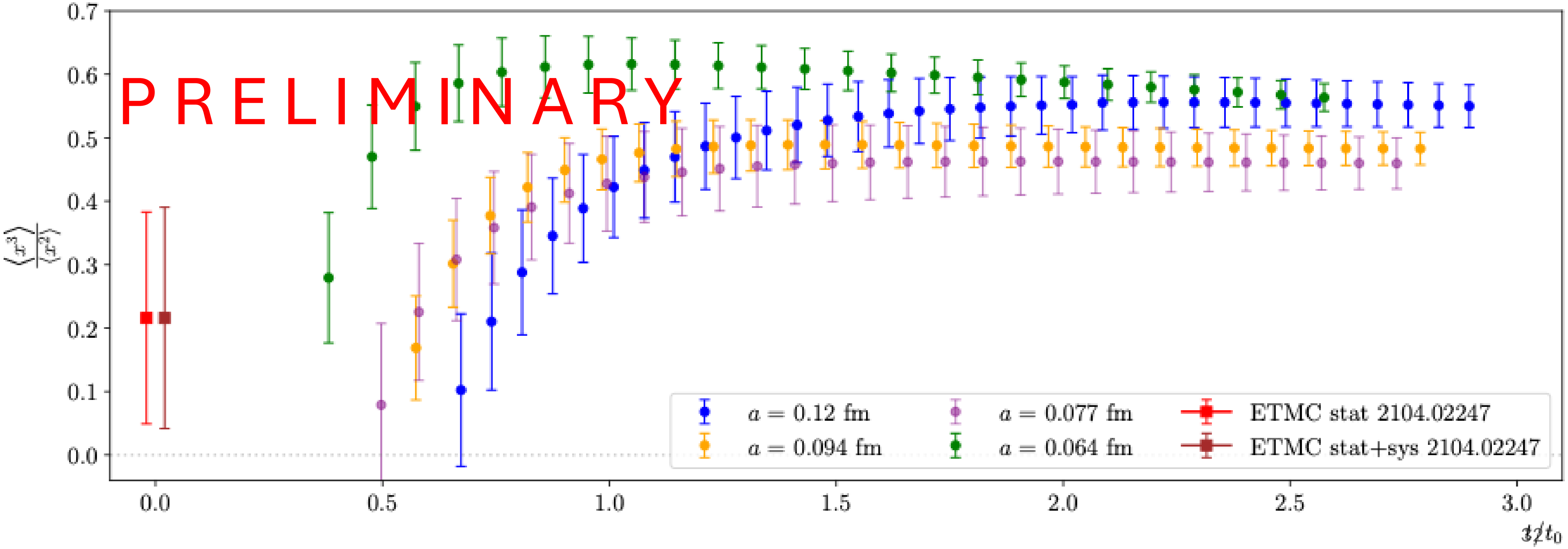
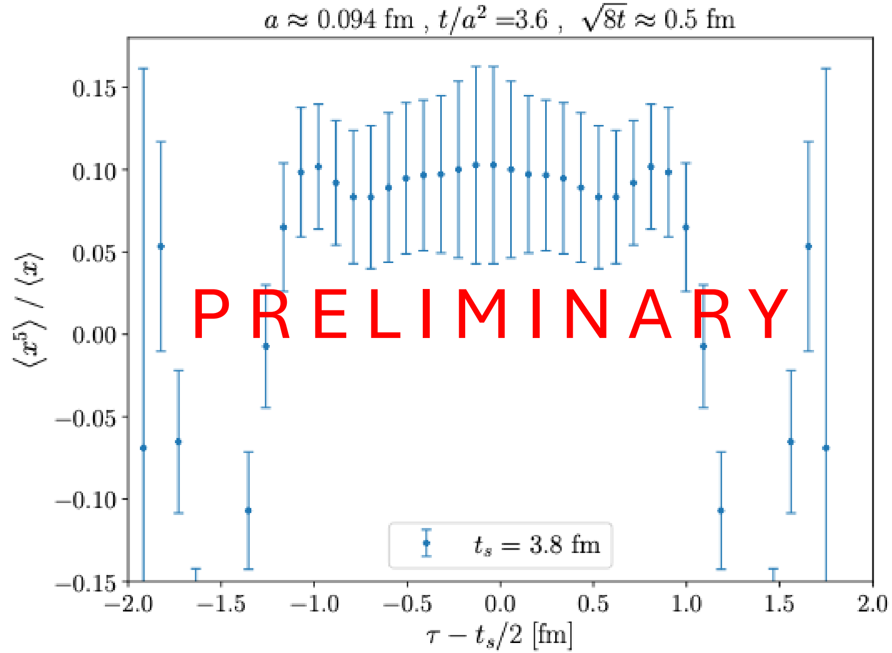


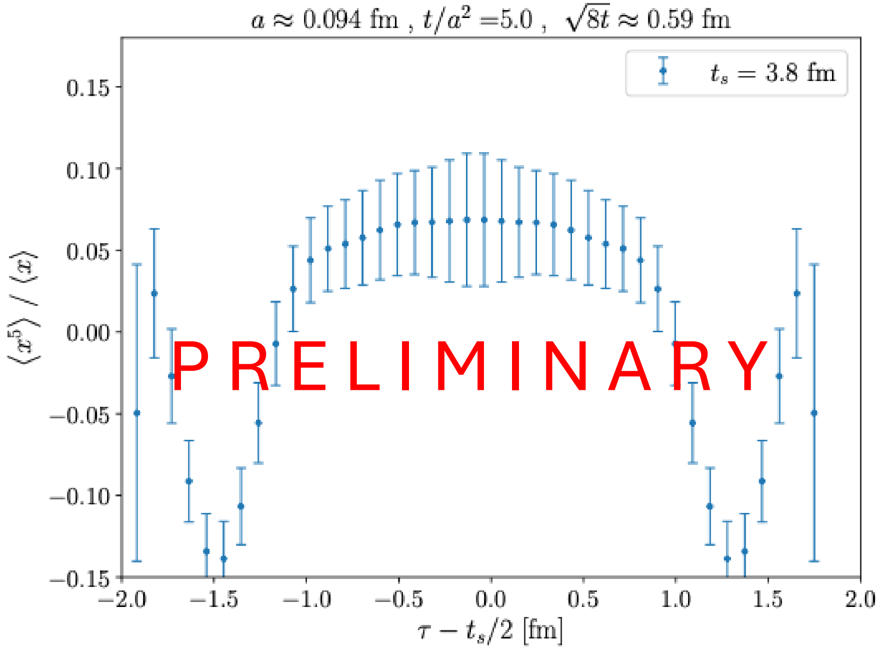
higher moments?

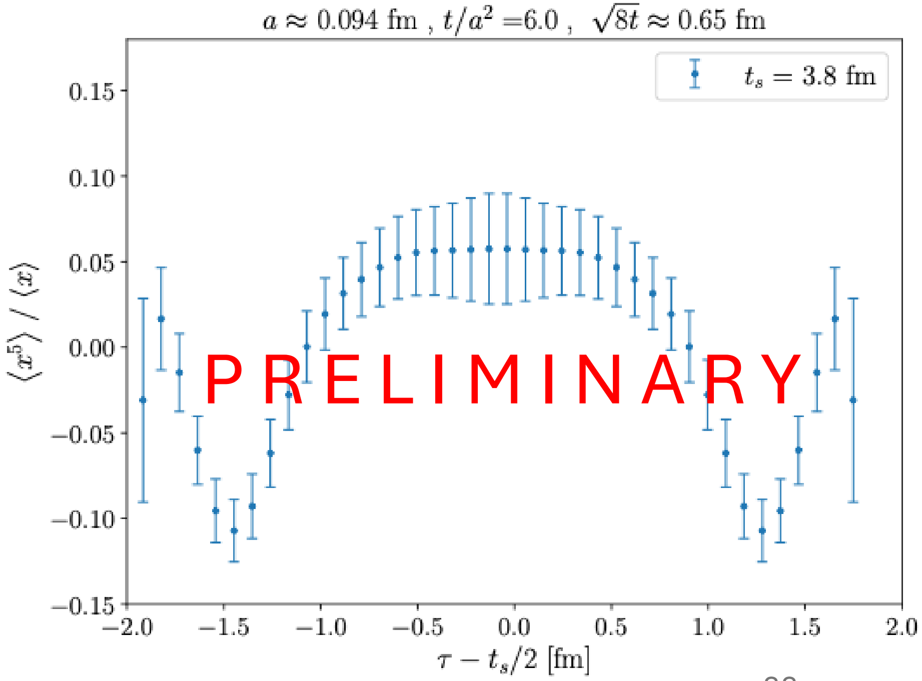
$\langle x^5\rangle_{\overline{\text{MS}}}/\langle x\rangle_{\overline{\text{MS}}}(\mu = 2\text{ GeV})$

at $a \approx 0.094$ fm, vary t


$a \approx 0.094$ fm , $t/a^2 = 3.6$, $\sqrt{8t} \approx 0.5$ fm

n	# of unique ops
2	4
3	10
4	40
5	136
6	544
7	2080

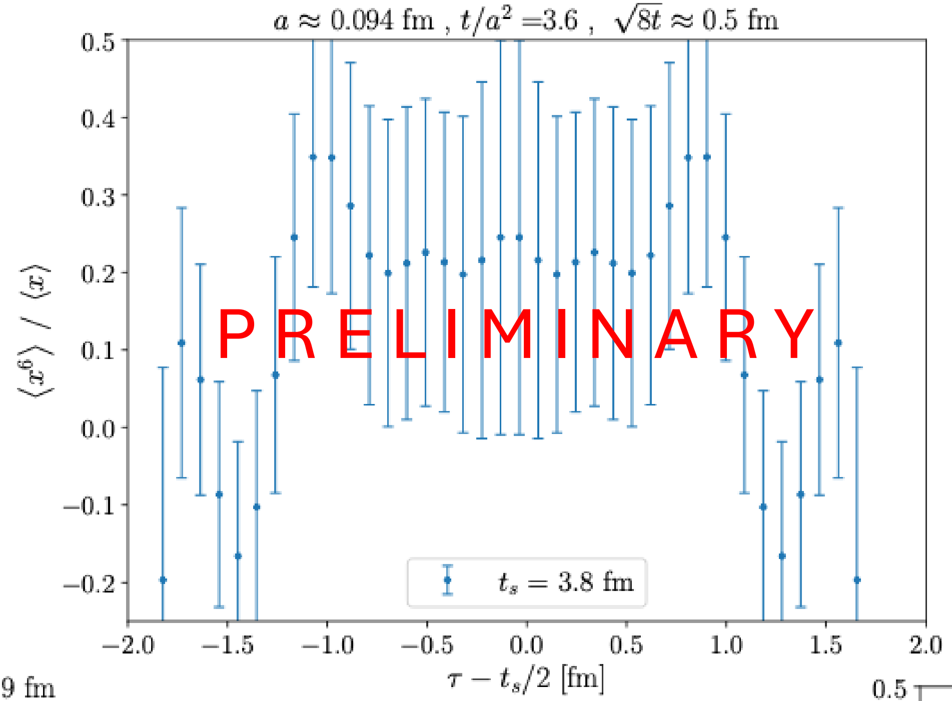

$a \approx 0.094$ fm , $t/a^2 = 5.0$, $\sqrt{8t} \approx 0.59$ fm


$a \approx 0.094$ fm , $t/a^2 = 6.0$, $\sqrt{8t} \approx 0.65$ fm

higher moments?

$\langle x^6\rangle_{\overline{\mathrm{MS}}}/\langle x\rangle_{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV})$

at $a \approx 0.094$ fm, vary t



n	# of unique ops
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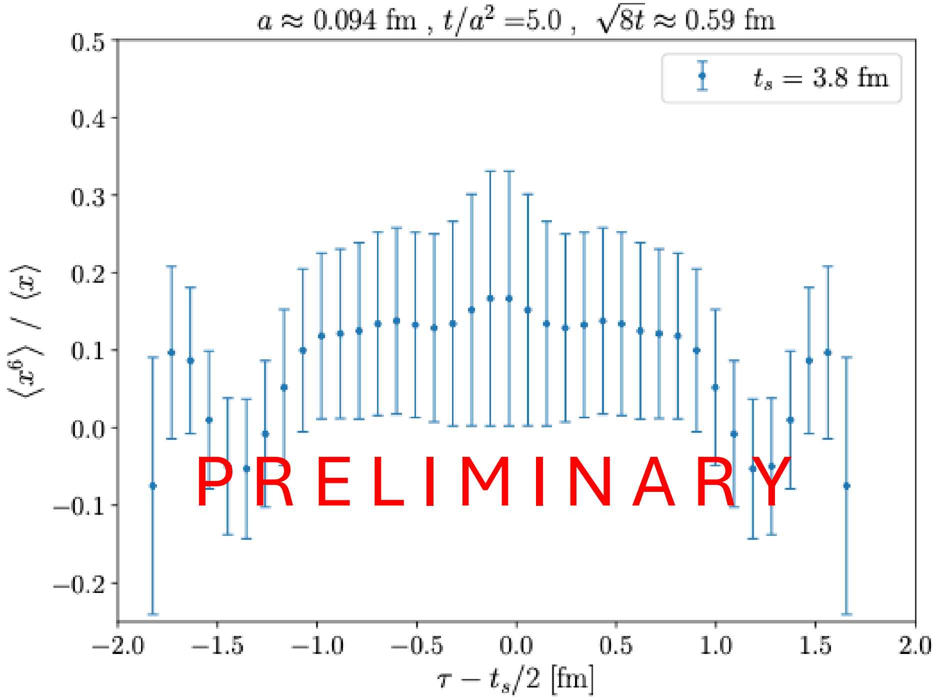
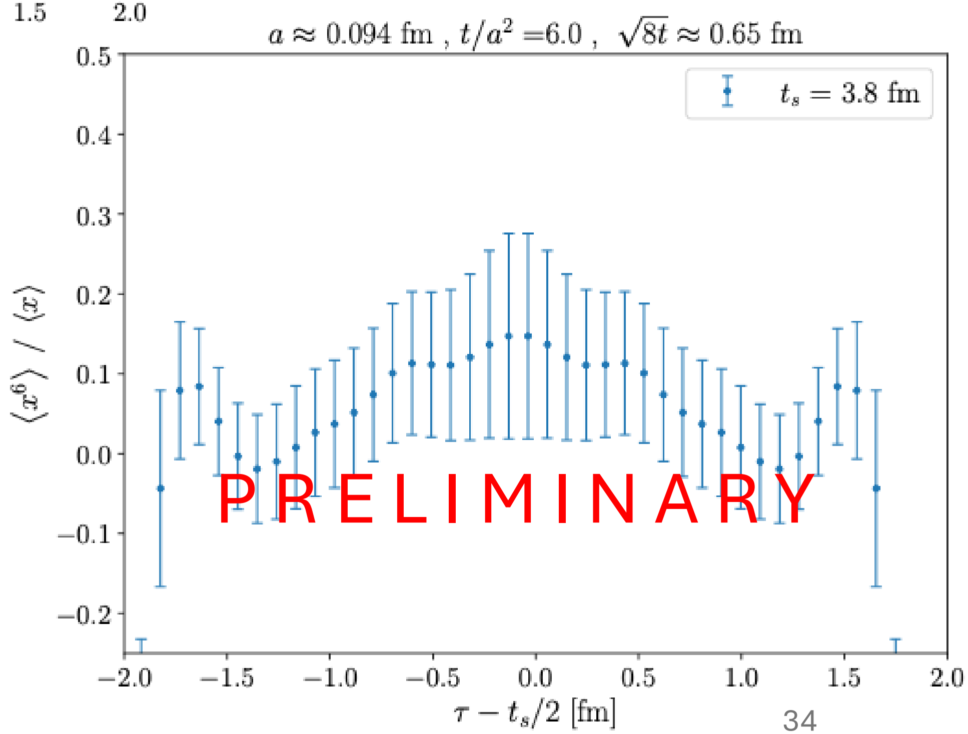


message: obtainable to resolve up to $\langle x^6\rangle$ with a moderate increase in statistics



Summary and conclusion

- Preliminary investigation of proposal to use the gradient flow for obtaining precise, higher moments of PDFs

- Promising results for the pion flavor non-singlet moments using four SWF ensembles generated by OpenLat at $m_\pi \approx 410\ \mathrm{MeV}$ and $a \approx 0.064, 0.077, 0.094, 0.12\ \mathrm{fm}$

- This work: increase of statistics to resolve up to $n = 6$ or 7, careful investigation of systematics

- Future: nucleon
  flavor singlet (gluon)
  off-forward: generalized form factors– moments of GPDs

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- Preliminary investigation of proposal to use the gradient flow for obtaining precise, higher moments of PDFs
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  flavor singlet (gluon)
  off-forward: generalized form factors– moments of GPDs

Thank you!