Nucleon electric polarizabilities and nucleon-pion scattering from lattice QCD

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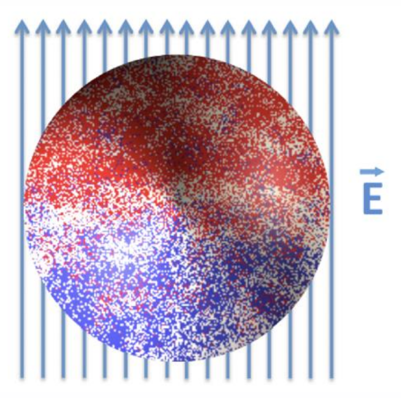
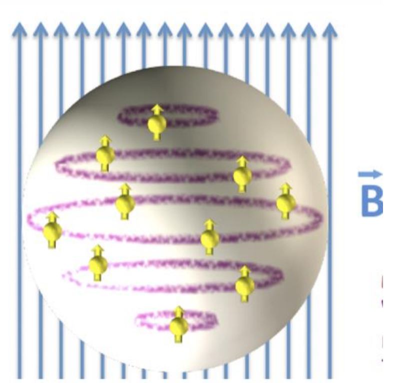
Electromagnetic polarizabilities

- In the realm of nucleon properties, E&M polarizabilities represent crucial fundamental constants akin to the size and shape of the nucleon

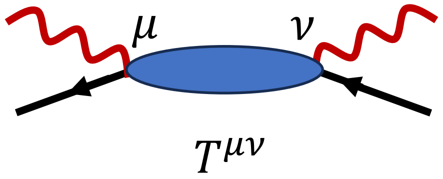




- Polarizabilities offer insights into the distribution of charge and magnetism within nucleons, revealing their response to external electromagnetic fields

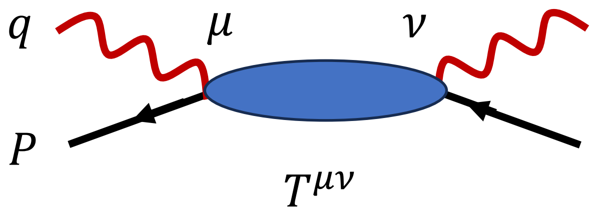
$$H_{eff}^{(2)} = -\frac{4\pi}{2}\alpha_E E^2 - \frac{4\pi}{2}\beta_M B^2$$

- Experimental determination of polarizabilities relies on Compton scattering, wherein external E&M fields polarize the target nucleon or deuteron



Nucleon polarizability and Compton scattering

➢ Nucleon E&M polarizability are most central quantities relevant for Compton scattering



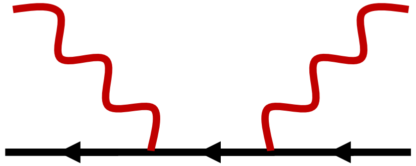
☐ Unpolarized doubly virtual Compton scattering

$$K_1^{\mu\nu} = q^\mu q^\nu - g^{\mu\nu} q^2$$

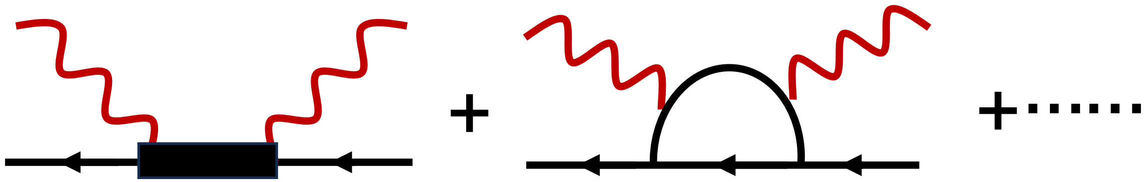
$$T^{\mu\nu}(P,q) = T^{\mu\nu}_{Born} + \frac{8\pi M}{e^2}\left[-(\beta_M + O(q))K_1^{\mu\nu} + (\alpha_E + \beta_M + O(q))K_2^{\mu\nu}\right]$$

$$K_2^{\mu\nu} = \frac{1}{M^2}\left[(P^\mu q^\nu + P^\nu q^\mu)P\cdot q - g^{\mu\nu}(P\cdot q)^2 - P^\mu P^\nu q^2\right]$$

- Born term
- Polarizabilities



Intermeidate states: N

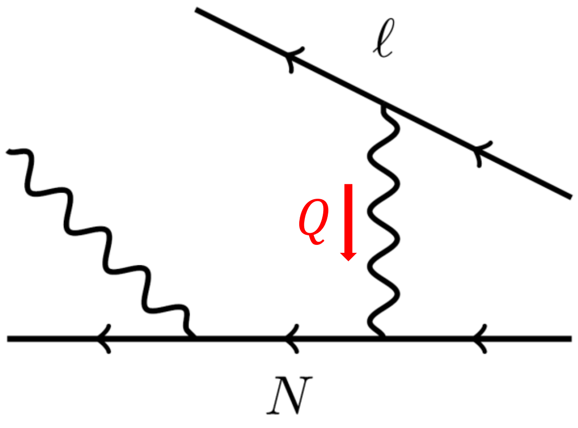


Intermeidate states: $N\pi, N^*, \Delta, \cdots$

Generalized electric polarizabilities

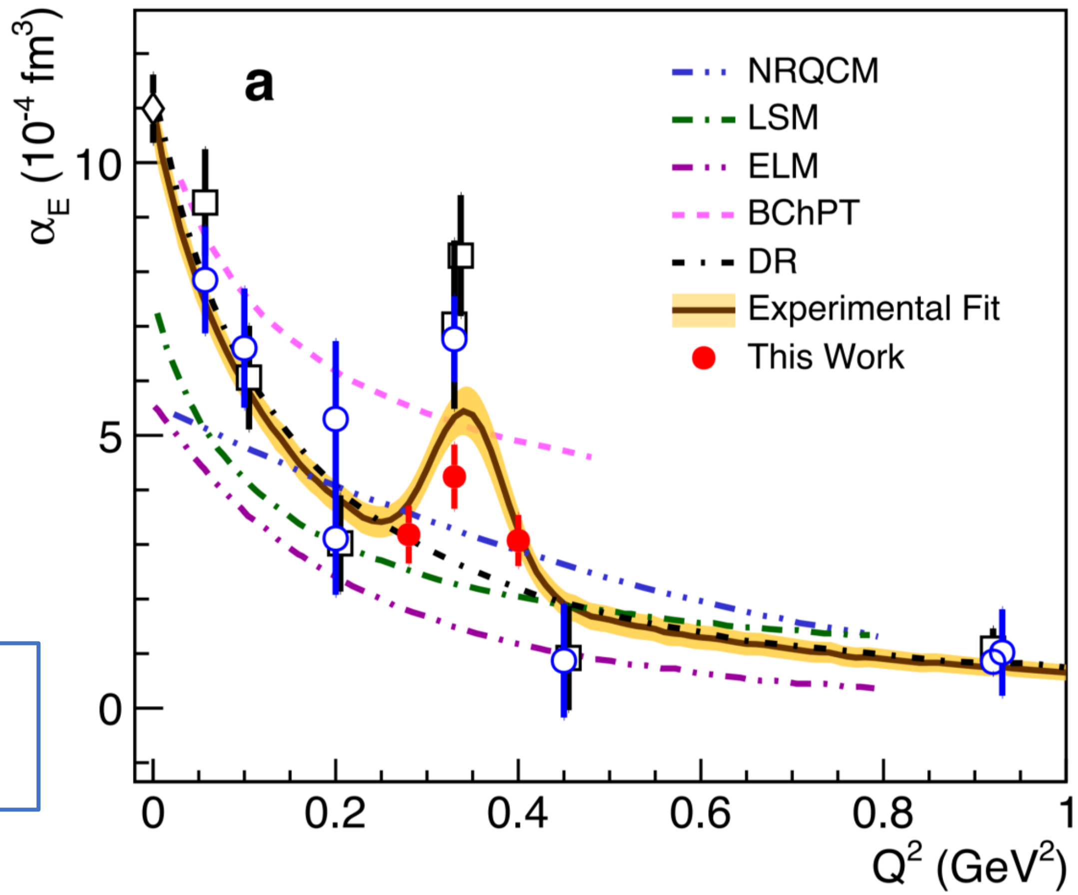
- In real-virtual Compton scattering

$$\alpha_E \Rightarrow \alpha_E(Q^2)$$



R. Li et al., Nature 611 (2022) 7935, 265-270

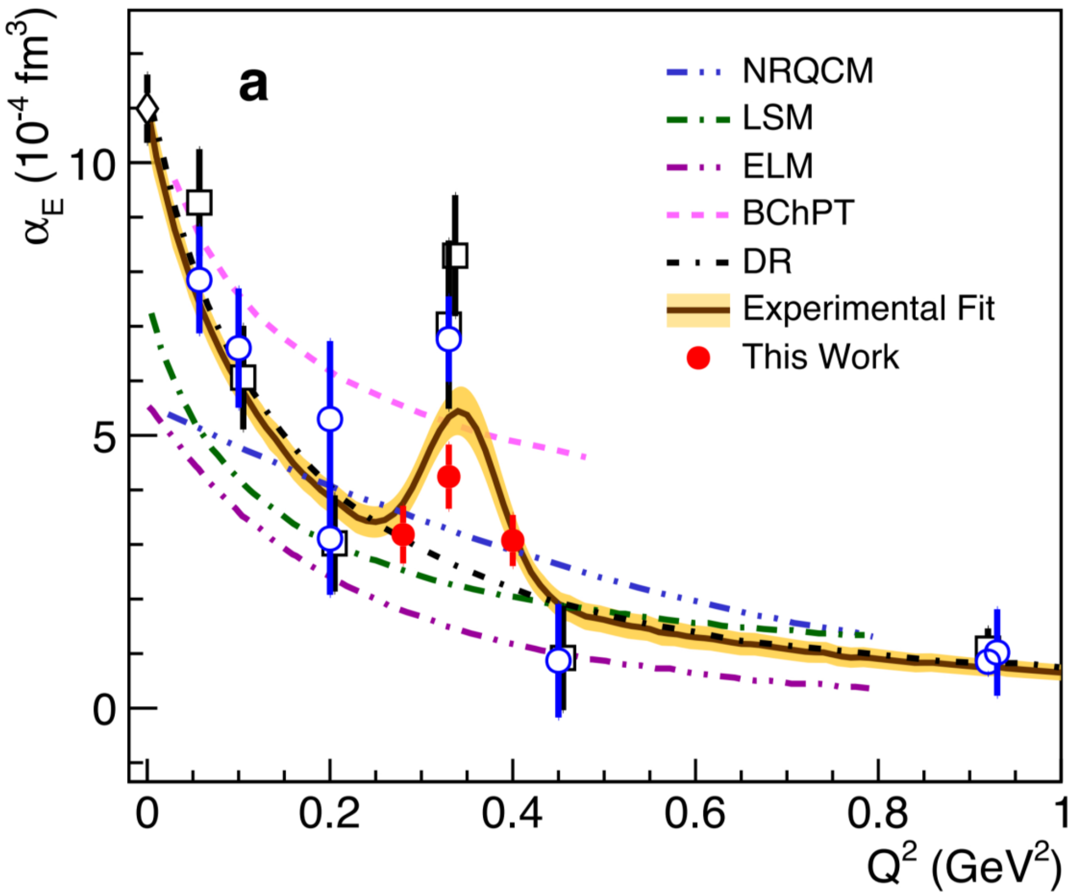
- Measured proton E&M structure deviates from theoretical predictions



Abnormal peak of generalized proton electric polarizability at $Q^2 \approx 0.35\ GeV^2$?

Generalized electric polarizabilities

- Measured proton E&M structure deviates from theoretical predictions



R. Li, N. Sparveris, et. al. Nature 611 (2022) 265

VS

- Different viewpoints

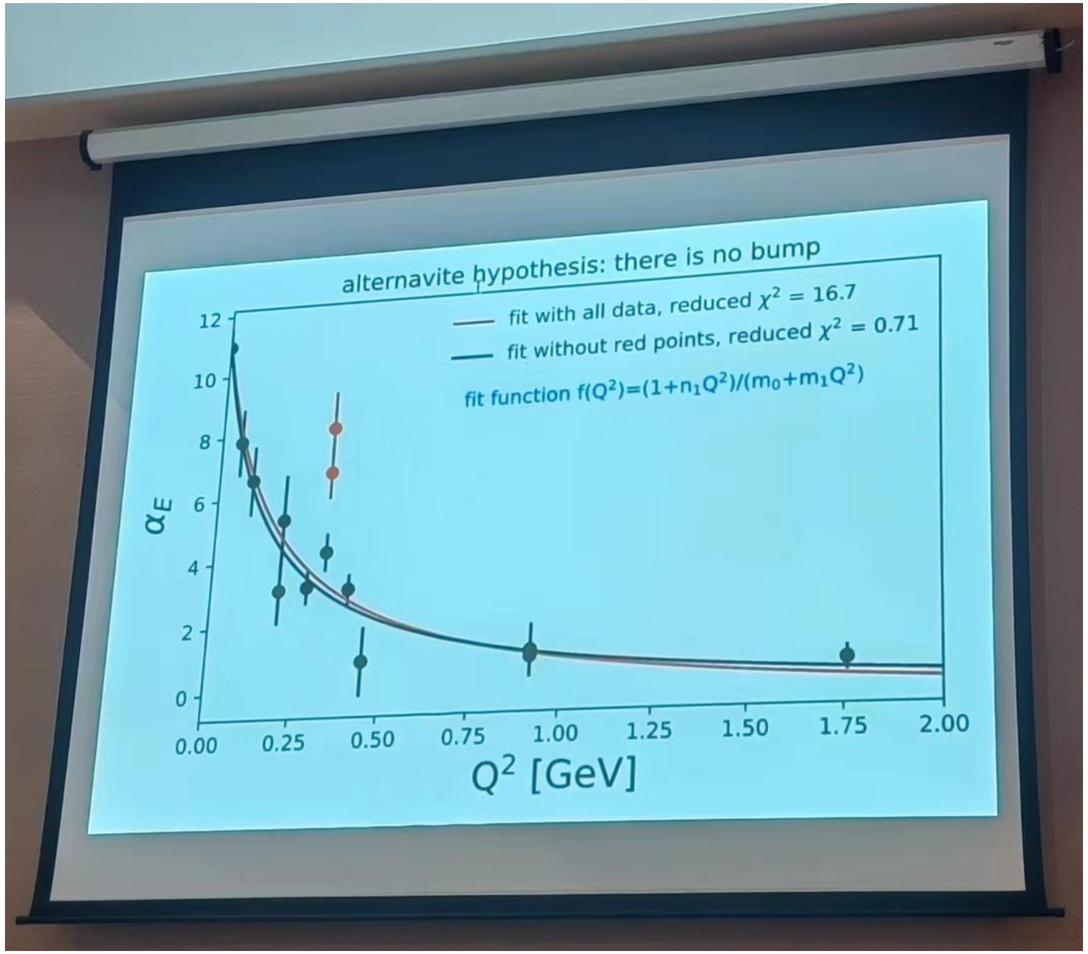
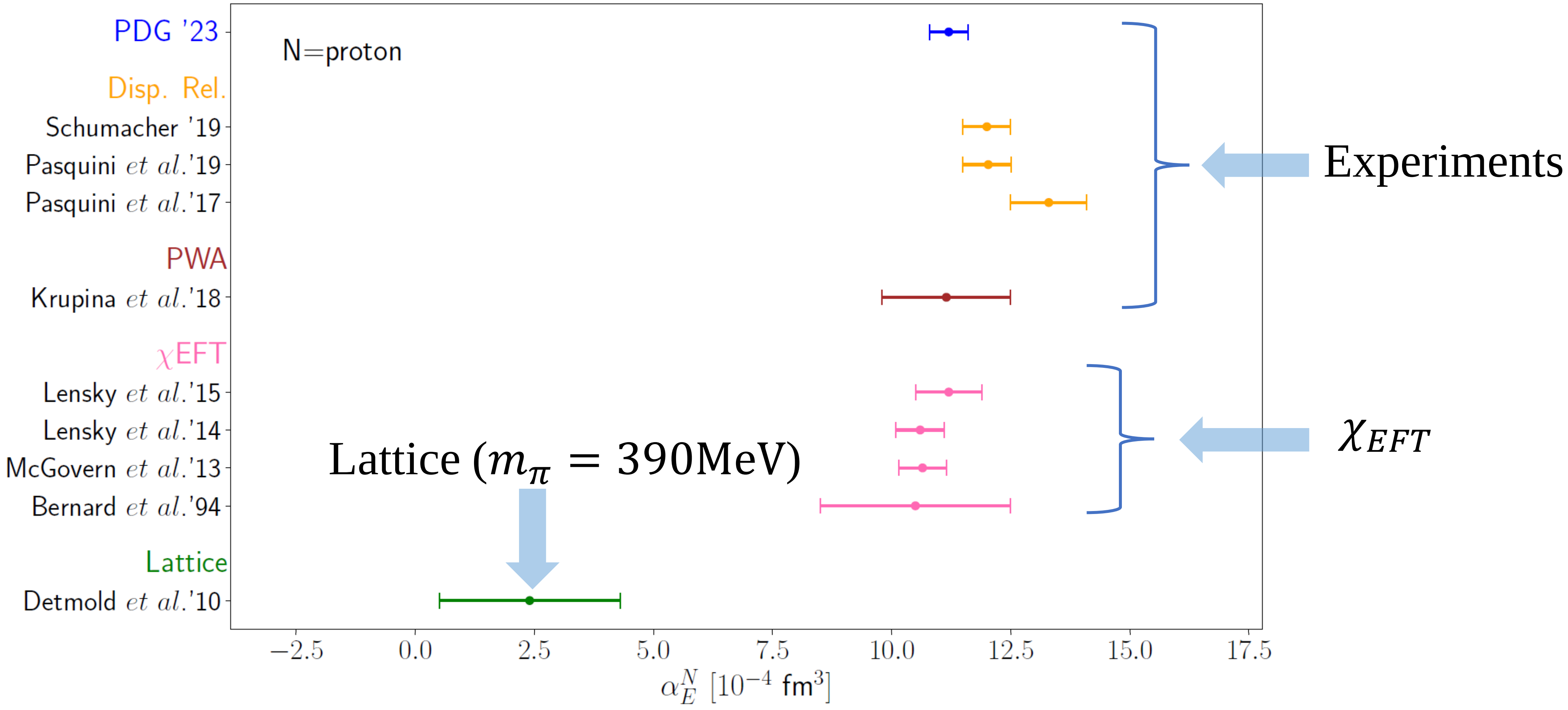


Figure shown by D. Higinbotham
@ 2nd Workshop on Nucleon Structure at Low Q

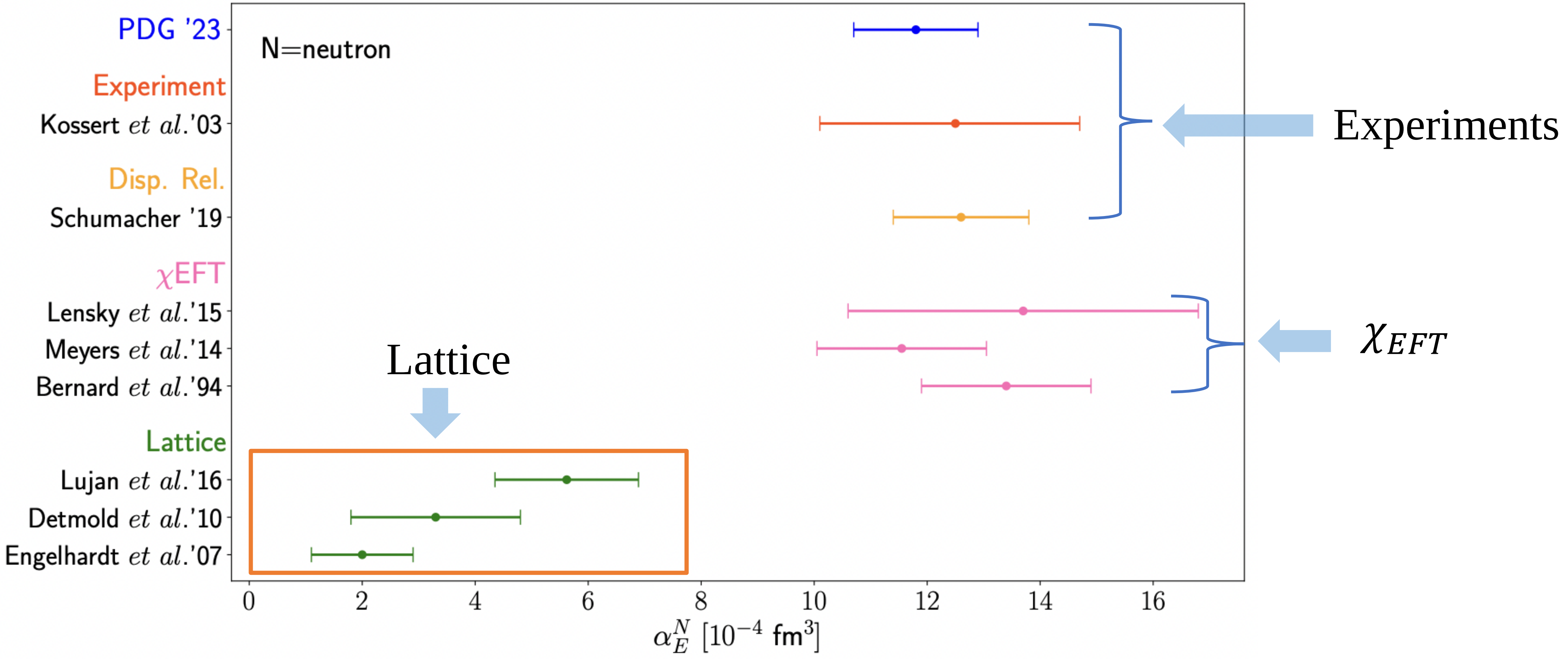
Determination of electric polarizabilities

- For proton



Determination of electric polarizabilities

- For neutron



Determination of electric polarizabilities

➢ What is the primary source of discrepancy between lattice QCD and other studies?

① Lattice calculations are performed at unphysical pion masses, ranging from 227 - 759 MeV

➡ Unphysical quark mass effects

② Background field technique is used, which converts 4pt function to 2pt function using Feynman-Hellman theorem

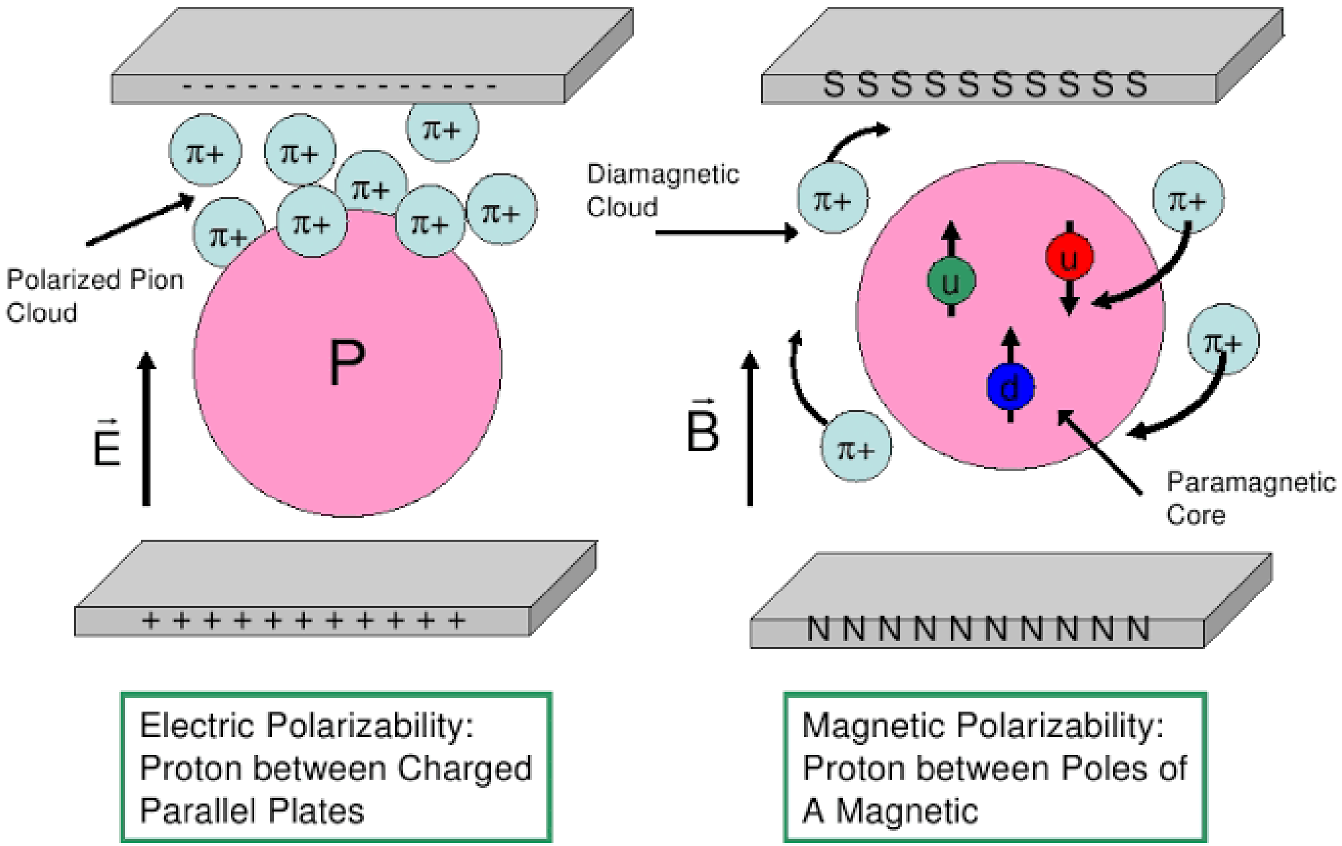
➡ Hard to explore intermediate-state contributions and control systematics

⬇

Perform calculation at physical pion mass, using 4pt function

Why physical pion mass is important

- Pion cloud in nucleon polarizabilities



- LO in χ_{PT}:

$$\alpha_E = \frac{5}{96}\left(\frac{g_A}{f_\pi}\right)^2 \frac{\alpha_{em}}{m_\pi}$$

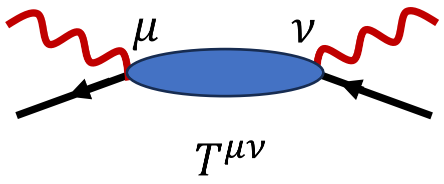
V. Bernard et al., Phys. Rev. Lett. 67 (1991) 1515

α_E is inversely proportional to pion mass

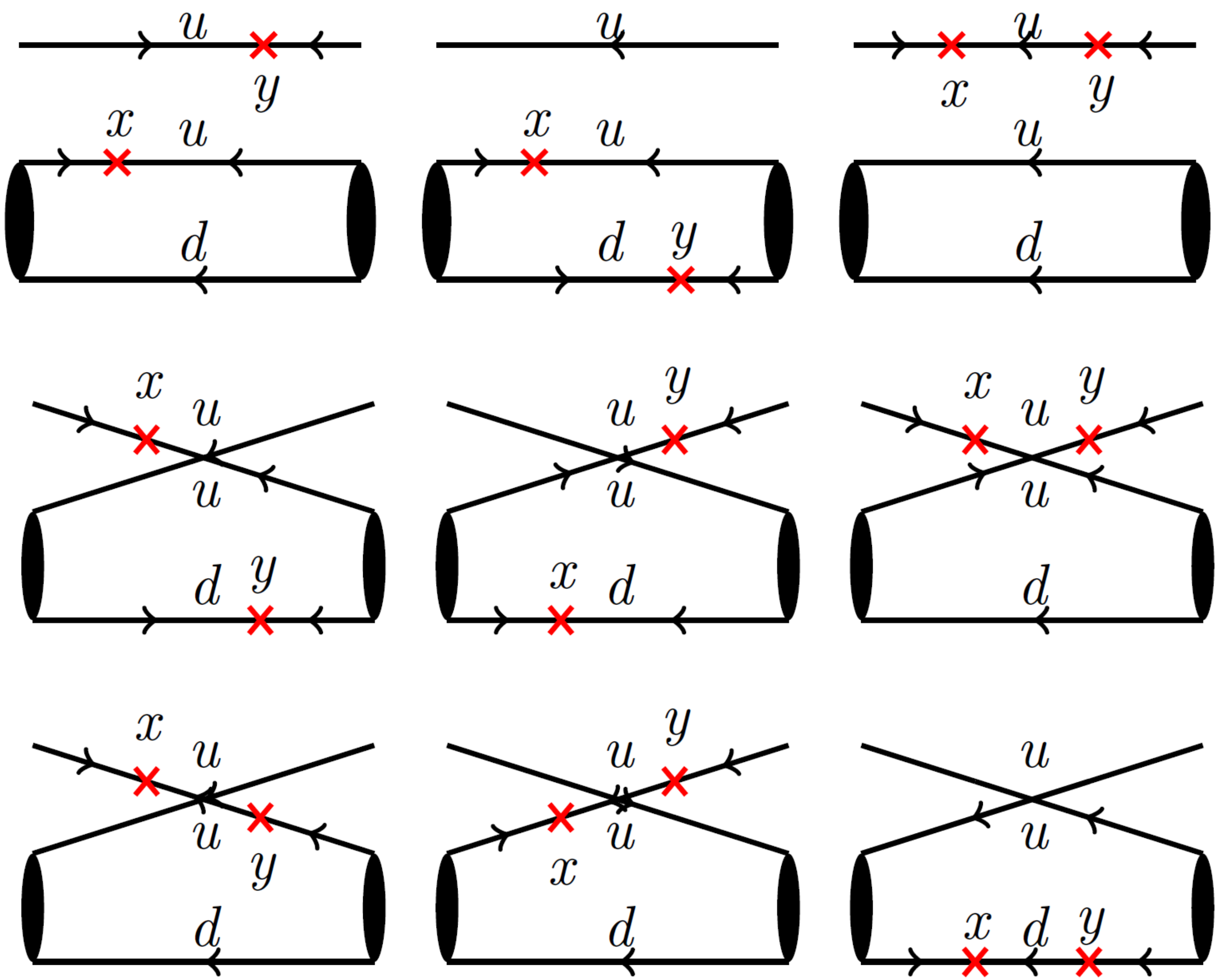
- Use two DWF ensembles @ physical π mass

Ensembles	m_π[MeV]	L/a	T/a	a[fm]	N_{conf}
24D	142.6(3)	24	64	0.1929	207
32Dfine	143.6(9)	32	64	0.1432	82

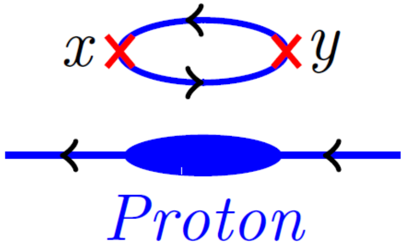
Numerical calculations



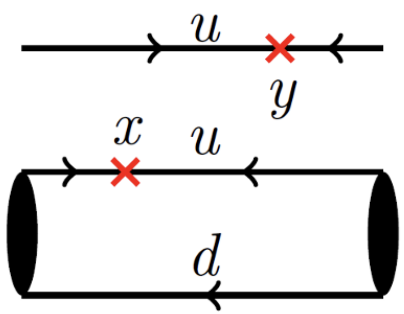
- Complicated quark field contractions with two current insertions



 $= \gamma_0 \sim \gamma_3\ (EM)$,
$\&\ \gamma_5\gamma_0 \sim \gamma_5\gamma_3\ (Weak)$



Examination of 4-pt function: charge conservation

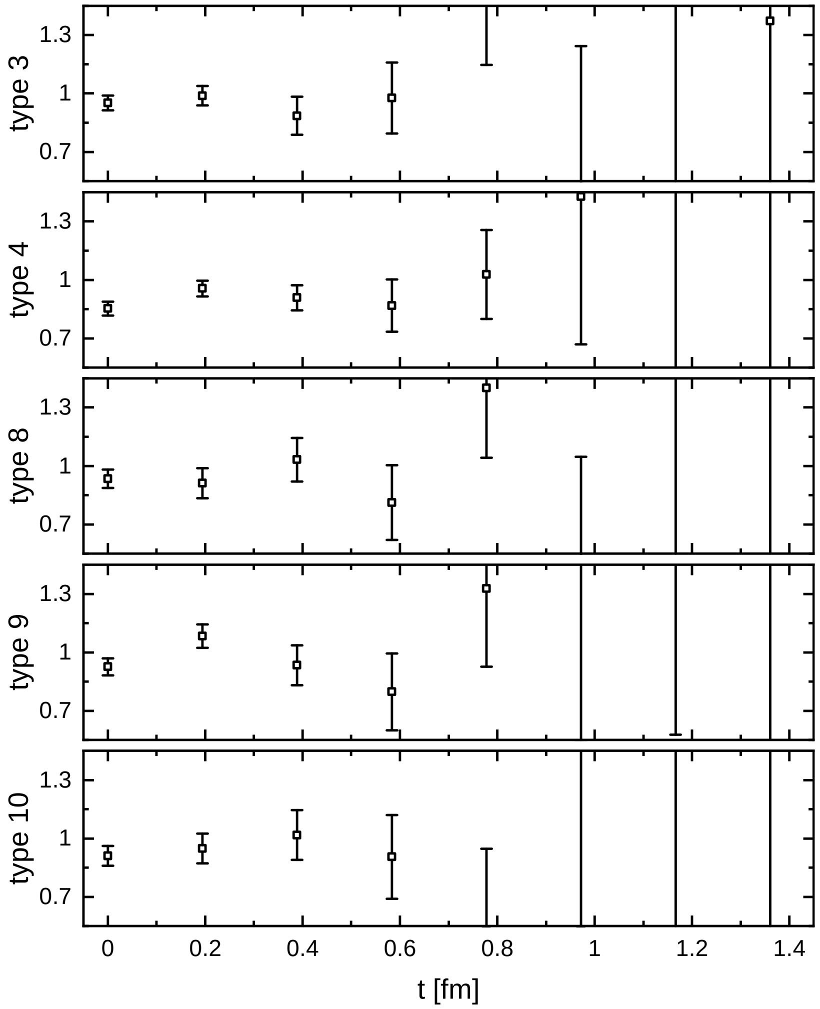


✕ $= \gamma_0$ ⇨ charge operator

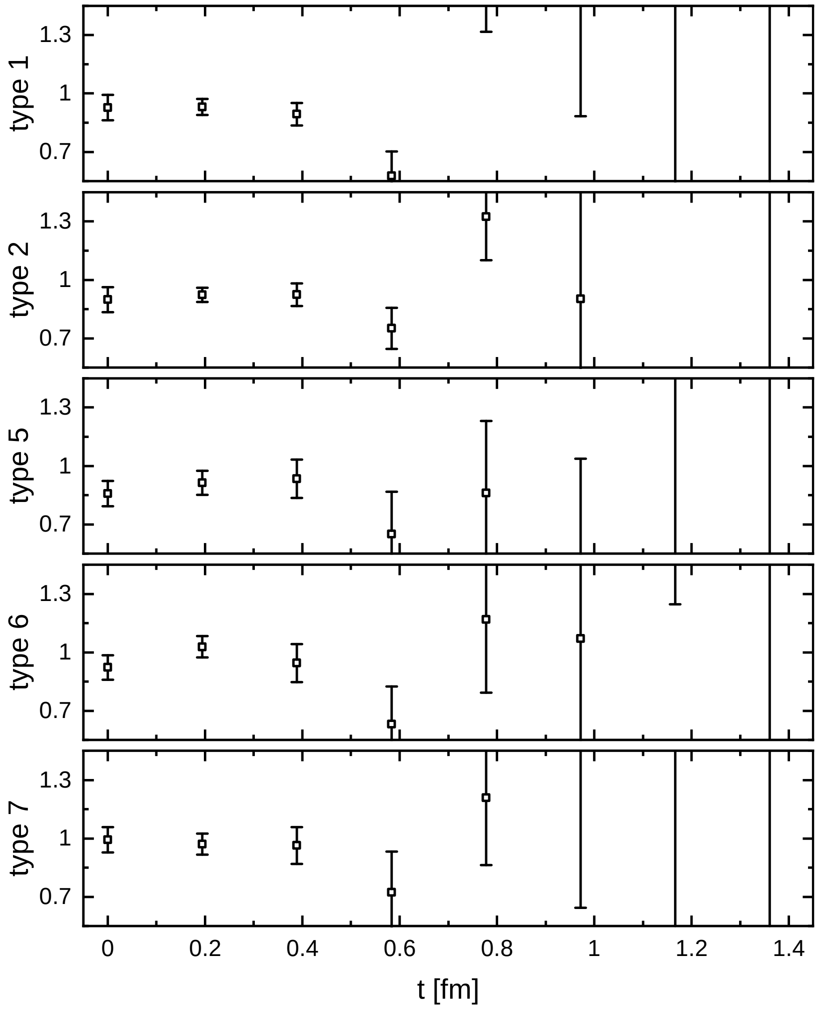
$$\int d^3\vec{x}\, J_0(\vec{x}) = Q$$

$$\frac{1}{2M}\int d^3\vec{x}\, H_{00}(t,\vec{x}) = G_E^2(0)$$

For proton: $G_E(0) = 1$
For neutron: $G_E(0) = 0$



Two currents inserted in one quark line



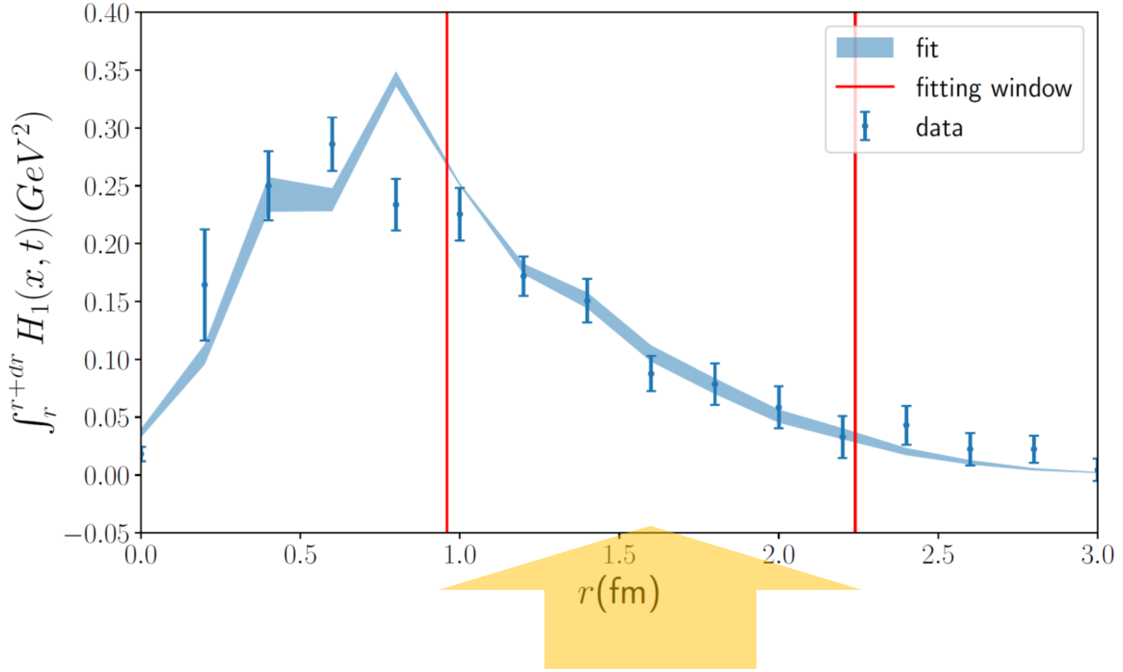
Two currents inserted in two quark lines

Using the charge conservation to verify the contraction code

Examination of 4-pt function: charge radius

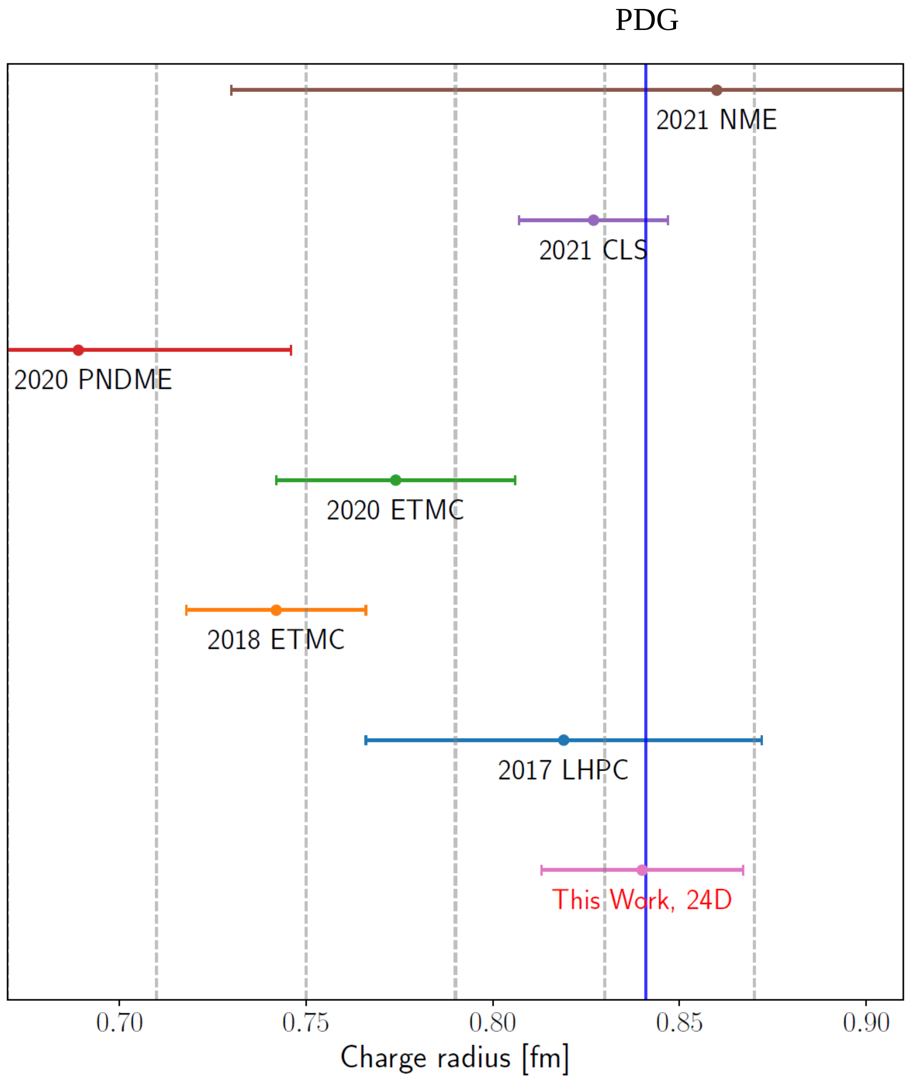
$$\langle N|J^0(x,t)J^0(0)|N\rangle \xrightarrow{\text{long distance}} \int \frac{d^3Q}{(2\pi)^3}\frac{M(E+M)}{E} G_E^2(Q^2) e^{ipx} e^{-(E-M)t}$$

- ☐ $\langle N|J^0(x,t)J^0(0)|N\rangle$ as a function of $|x|$

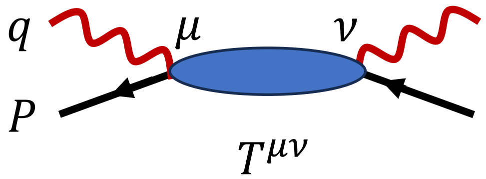


Charge radius fitted at long distance $r > 1fm$, with dipole model:

$$G_E(Q^2) = 1/\left(1 + Q^2\langle r_E^2\rangle/12\right)^2$$



Electric polarizability from 4-pt function



- Compton tensor

Lattice QCD input $H^{\mu\nu}(x)$

$$T^{\mu\nu} = \int d^4x\, e^{iqx} \langle N|J^\mu(x,t)J^\nu(0)|N\rangle = T^{\mu\nu}_{Born} + \frac{8\pi M}{e^2}\left[-(\beta_M + O(q))K_1^{\mu\nu} + (\alpha_E + \beta_M + O(q))K_2^{\mu\nu}\right]$$

- Derive 3 formula to calculate α_E

  - $P = (M, 0), q = (0, \vec{\xi})$: $\quad \alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x\, \vec{x}^2 \left(H^{00}(x) - H^{00}_{GS}(x)\right) + \alpha_E^r$
  - $P = (M, 0), q = (\xi, 0, 0, \xi)$: $\quad \alpha_E = \frac{\alpha_{em}}{4M}\int d^4x\, (t + x_i)^2 \left(H^{0i}(x) - H^{0i}_{GS}(x)\right) + \alpha_E^r$
  - $P = (M, 0), q = (\xi, 0)$: $\quad \alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x\, t^2 H^{ii}(x) + \alpha_E^r$ → Our choice

$H^{ii}(x,t) = \langle N|J^i(x)J^i(0)|N\rangle$

- Residual term α_E^r is analytically known

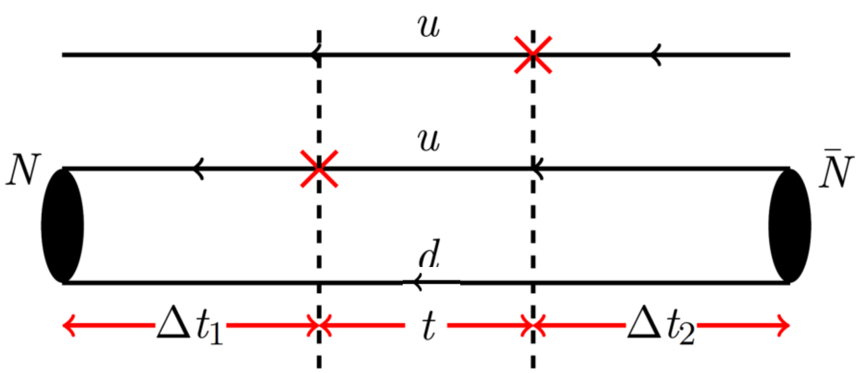
$$\alpha_E^r = \frac{\alpha_{em}}{M}\left(\frac{G_E^2(0) + \kappa^2}{4M^2} + \frac{G_E(0)\langle r_E^2\rangle}{3}\right),$$

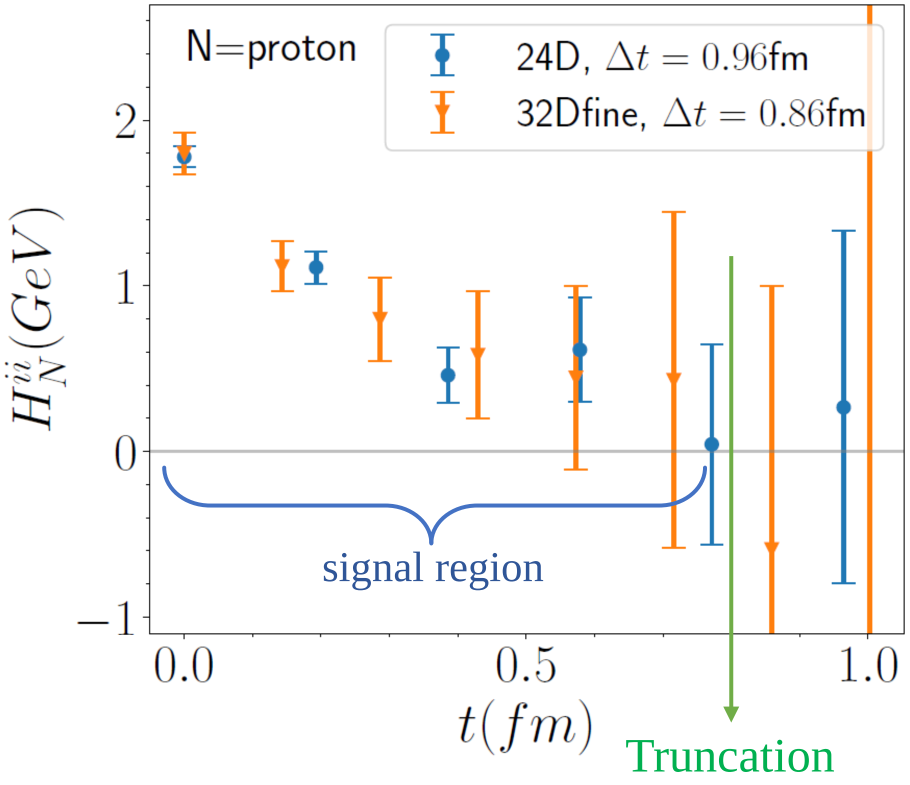
anomalous magnetic moment & charge radius

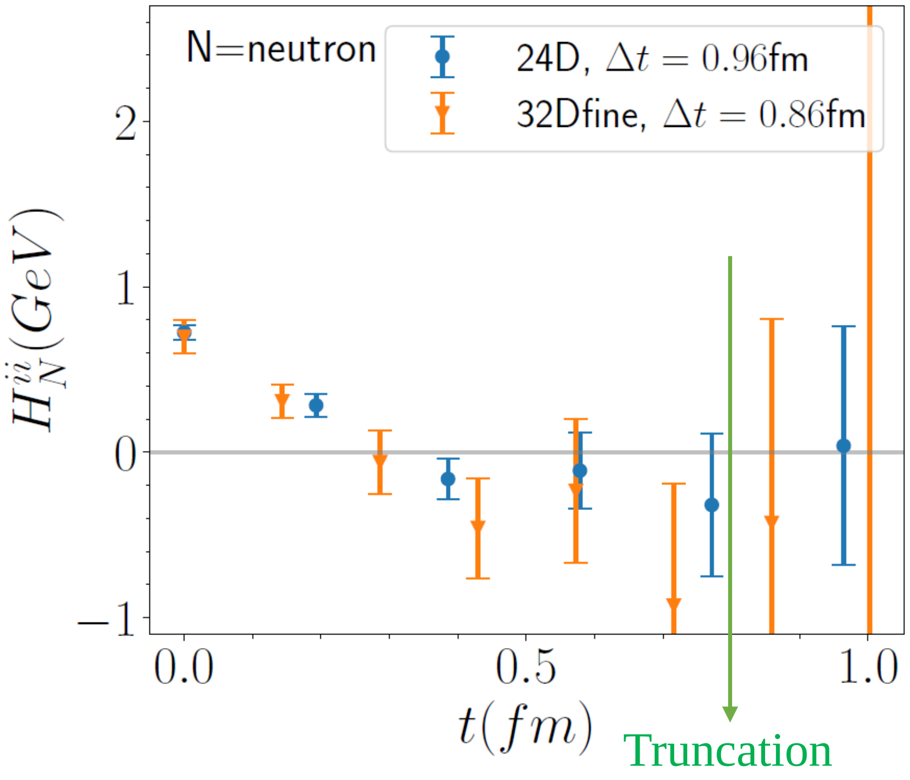
$G_E(0) = 1/0$, for proton/neutron

Signal of hadronic function

$$\sum_{\vec{x}} H^{ii}(t,\vec{x}) = \sum_{k} \langle p|J_i(0)|k\rangle e^{-(E_k-M)t}\langle k|J_i(0)|p\rangle$$



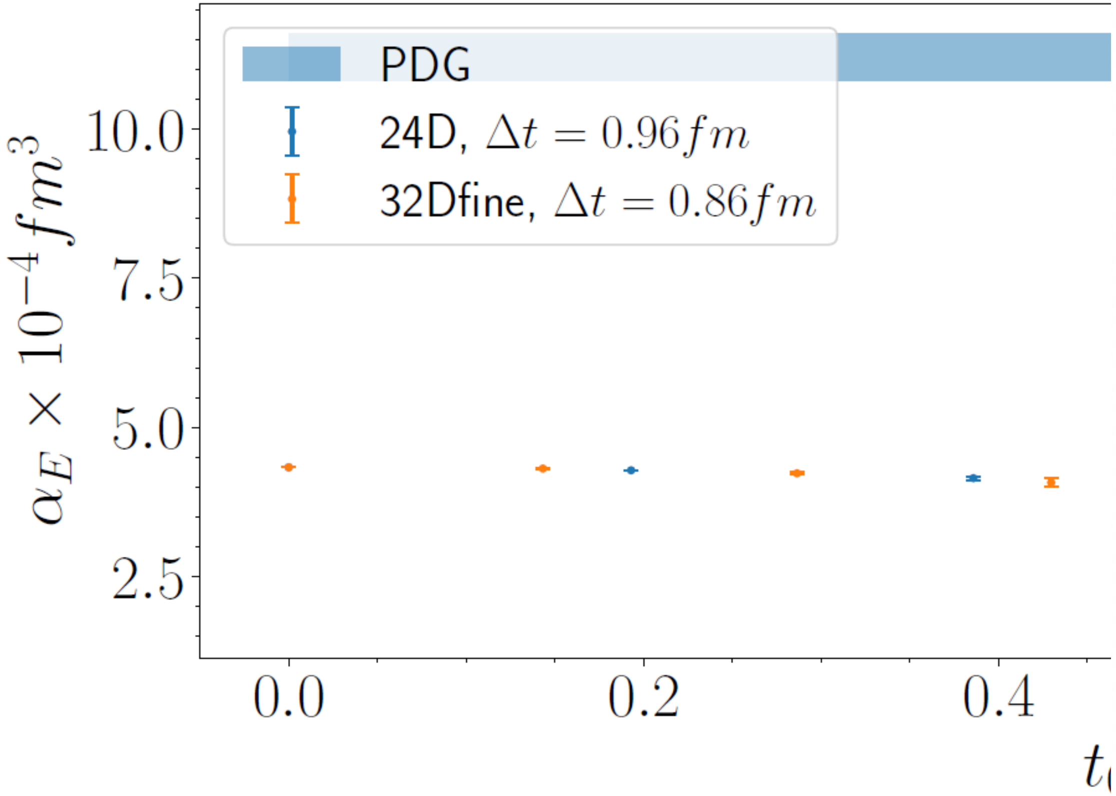


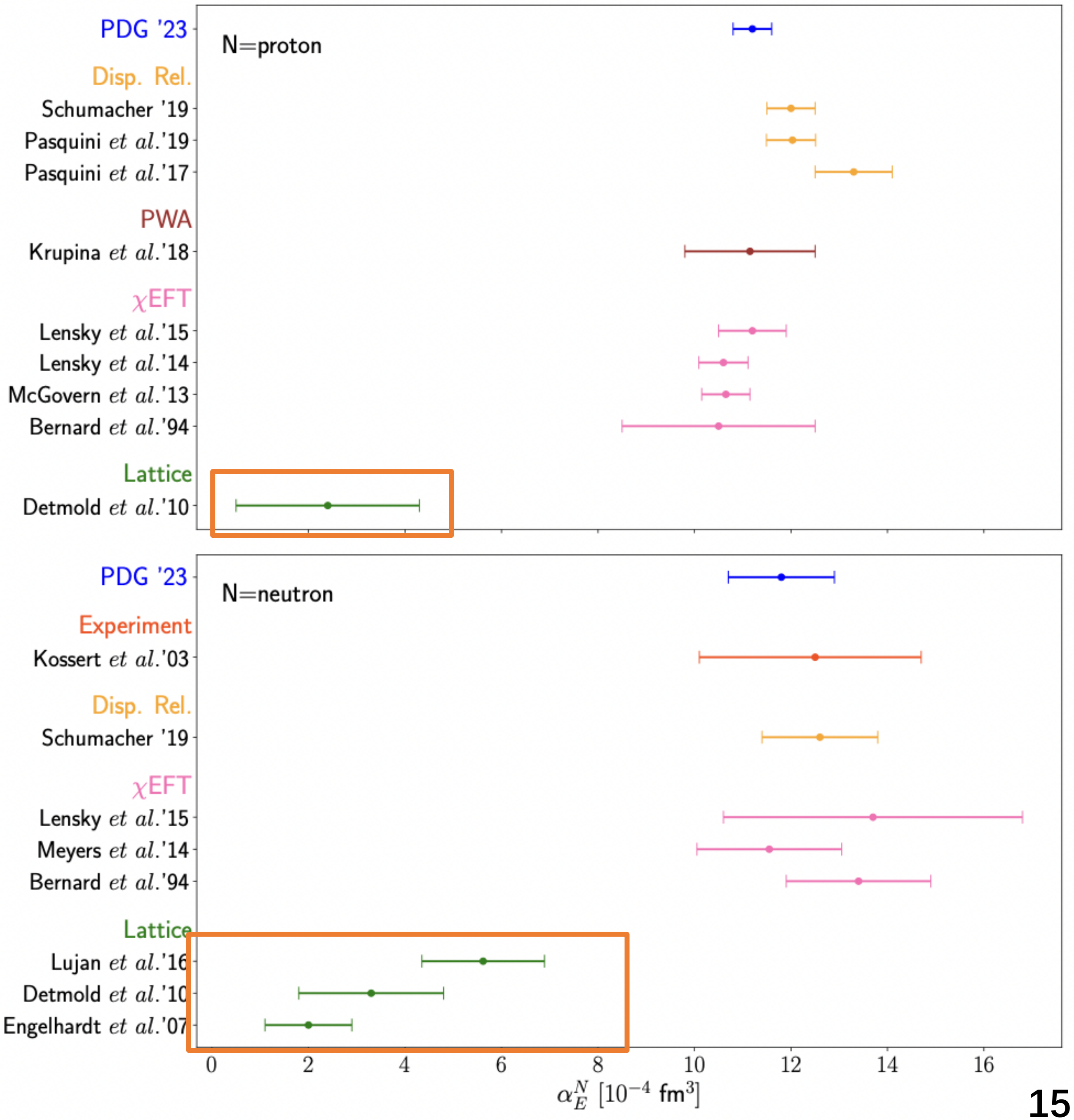


$\Delta t = \Delta t_1 + \Delta t_2 = \begin{cases} 0.96\ fm\ (24D) \\ 0.86\ fm\ (32Dfine) \end{cases}$, truncation at $t_0 = \begin{cases} 0.77\ fm\ (24D) \\ 0.72\ fm\ (32Dfine) \end{cases}$ ➡ In total, $\Delta t_1 + \Delta t_2 + t_0 \sim 1.6\text{-}1.8$ fm

Polarizability α_E from $H_{ii}(x)$

Polarizability extraction $\alpha_E = -\frac{\alpha_{em}}{12M}\int d$

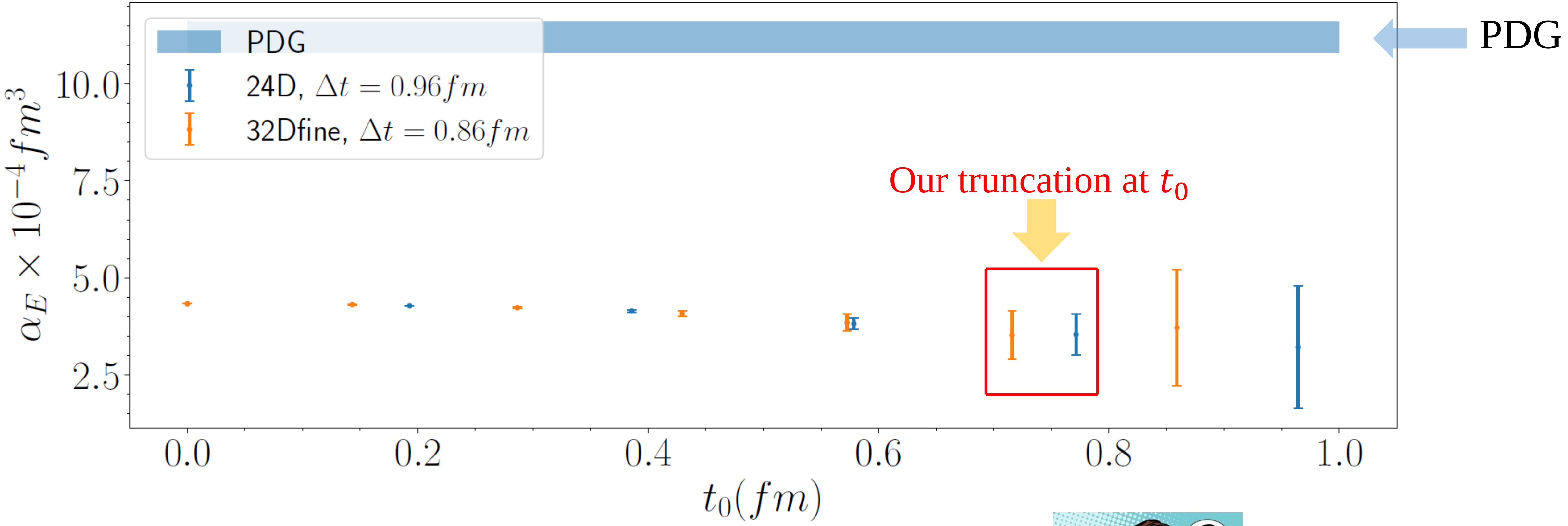




However, lattice results are significantly belo

Polarizability α_E from $H_{ii}(x)$

Polarizability extraction $\alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x t^2 H^{ii}(x) + \alpha_E^r$



However, lattice results are significantly below the PDG value.

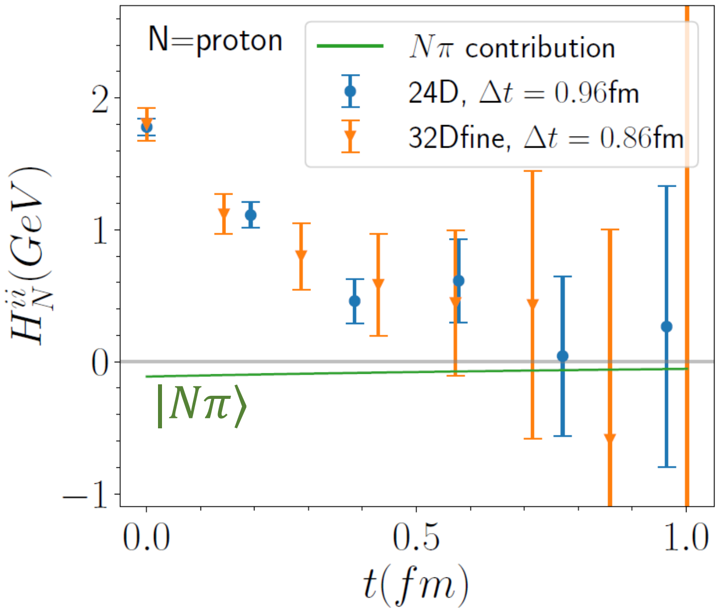
Need new insight to turn the decent to the magic!

Nucleon polarizabilities and $N\pi$ scattering

Structure of hadronic function $\int d^4x\, t^2 H_{ii}(x,t) = \int dt\, t^2 \sum_k \langle p|J_i(0)|k\rangle e^{-(E_k-M)t}\langle k|J_i(0)|p\rangle$

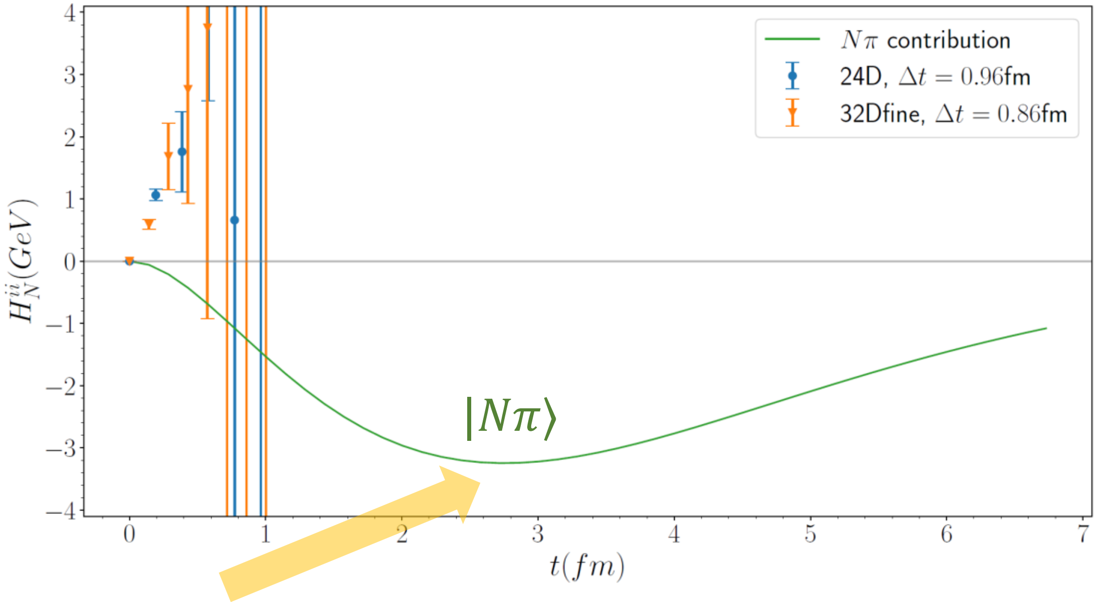
$$= 4\sum_k \frac{\langle p|J_i(0)|k\rangle\langle k|J_i(0)|p\rangle}{(E_k-M)^3}$$

The dominant contribution is given by $|k\rangle = |N\pi\rangle$ states



t^2

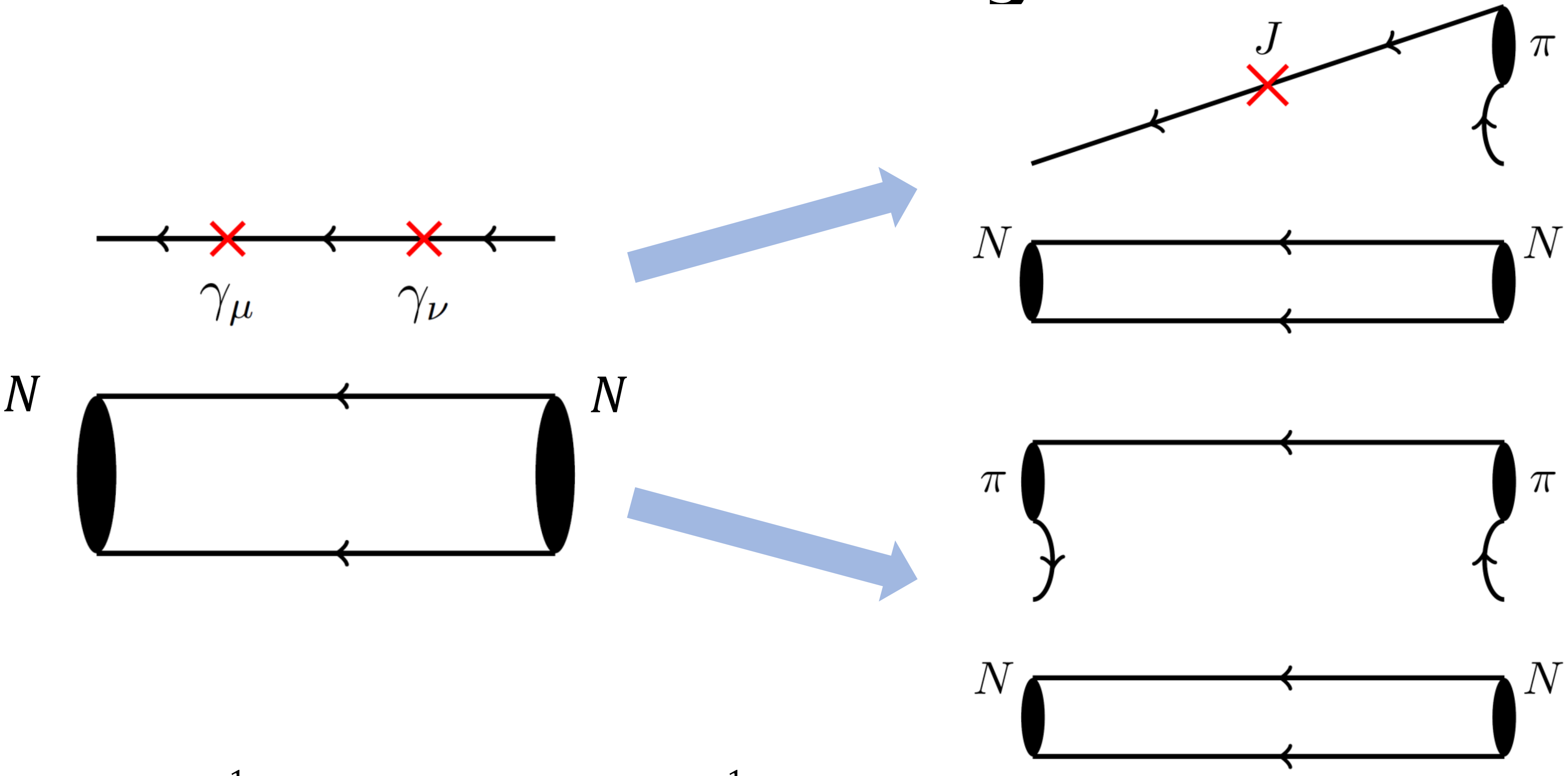
Weight function



$|N\pi\rangle$ states contribution exhibits a peak at $t = 2.8\ fm$, far exceeding our truncation at $t_0 \approx 0.75\ fm$

Must calculate $N\pi$ contribution directly!

Wick contraction of $N\pi$ rescattering



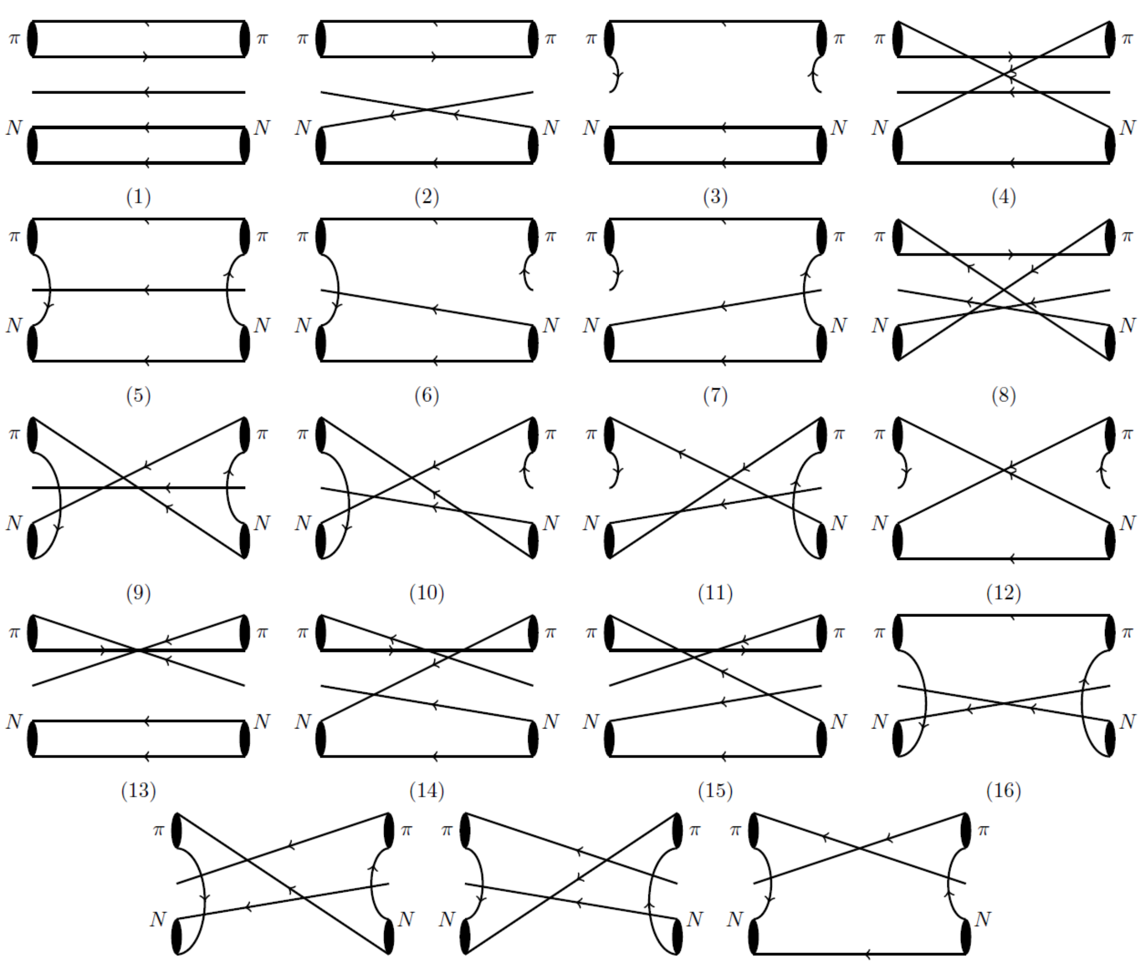
$I=1/2$: $O_{N\pi}^{I_3=+\frac{1}{2}}=O_pO_{\pi^0}-\sqrt{2}O_nO_{\pi^+}, O_{N\pi}^{I_3=-\frac{1}{2}}=\sqrt{2}O_nO_{\pi^0}-O_pO_{\pi^-}$

$I=3/2$: $O_{N\pi}^{I_3=+\frac{1}{2}}=\sqrt{2}O_pO_{\pi^0}+O_nO_{\pi^+}, O_{N\pi}^{I_3=-\frac{1}{2}}=O_nO_{\pi^0}+\sqrt{2}O_pO_{\pi^-}$

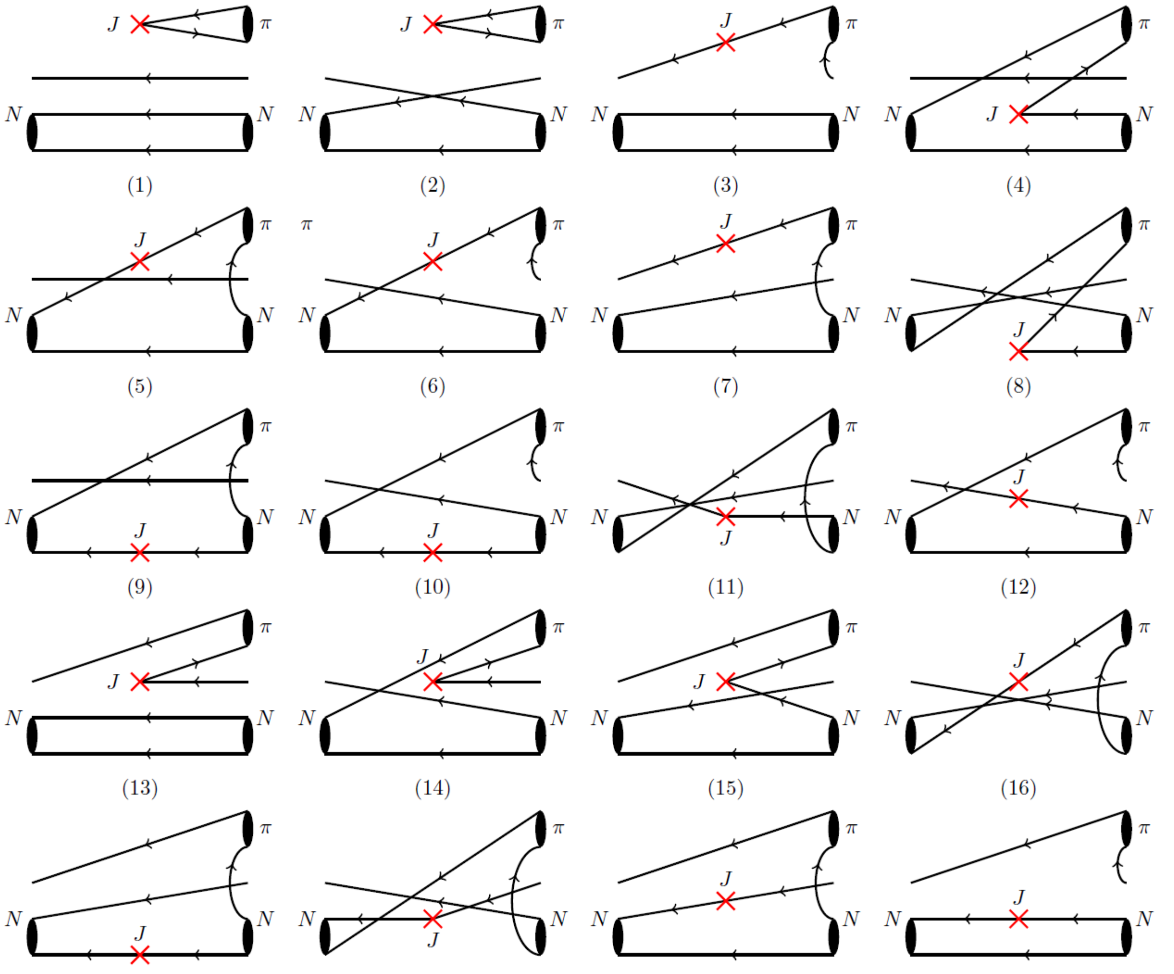
} Operators

Wick contraction of $N\pi$ Rescattering

- 19 diagrams for $N\pi$ rescattering



- 20 diagrams for $N + \gamma \to N\pi$

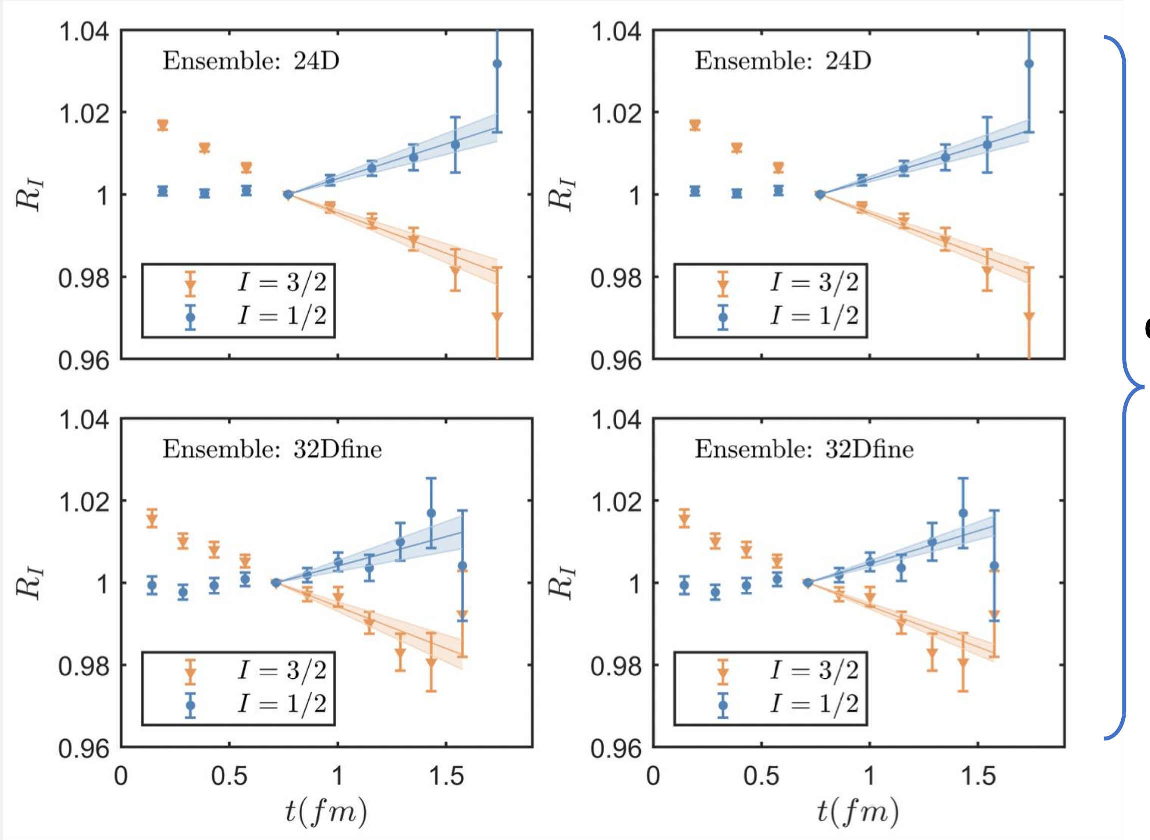


Results of $N\pi$ Scattering

➢ Nπ scattering at m_π=142 MeV

$$R = \frac{C_2^{N\pi}(t)}{C_2^N(t)C_2^\pi(t)} = \frac{A_{N\pi}}{A_N A_\pi}\frac{e^{-E_{N\pi}t}}{e^{-(M_N+M_\pi)t}} \approx R_0(1-\Delta E t)$$

with $\Delta E = E_{N\pi} - M_N - M_\pi$



➢ Scattering for different isospin channel

- $I = 1/2, \Delta E < 0$, attractive interaction
- $I = 3/2, \Delta E > 0$, repulsive interaction

Results using data at threshold

$a_0^{1/2} m_\pi = 0.0127(24)$, $a_0^{3/2} m_\pi = -0.127(14)$

ETMC [arXiv: 2307.12846] $a_0^{3/2} m_\pi = -0.13(4)$

Analysis based on cross section and πH, πD spectrum

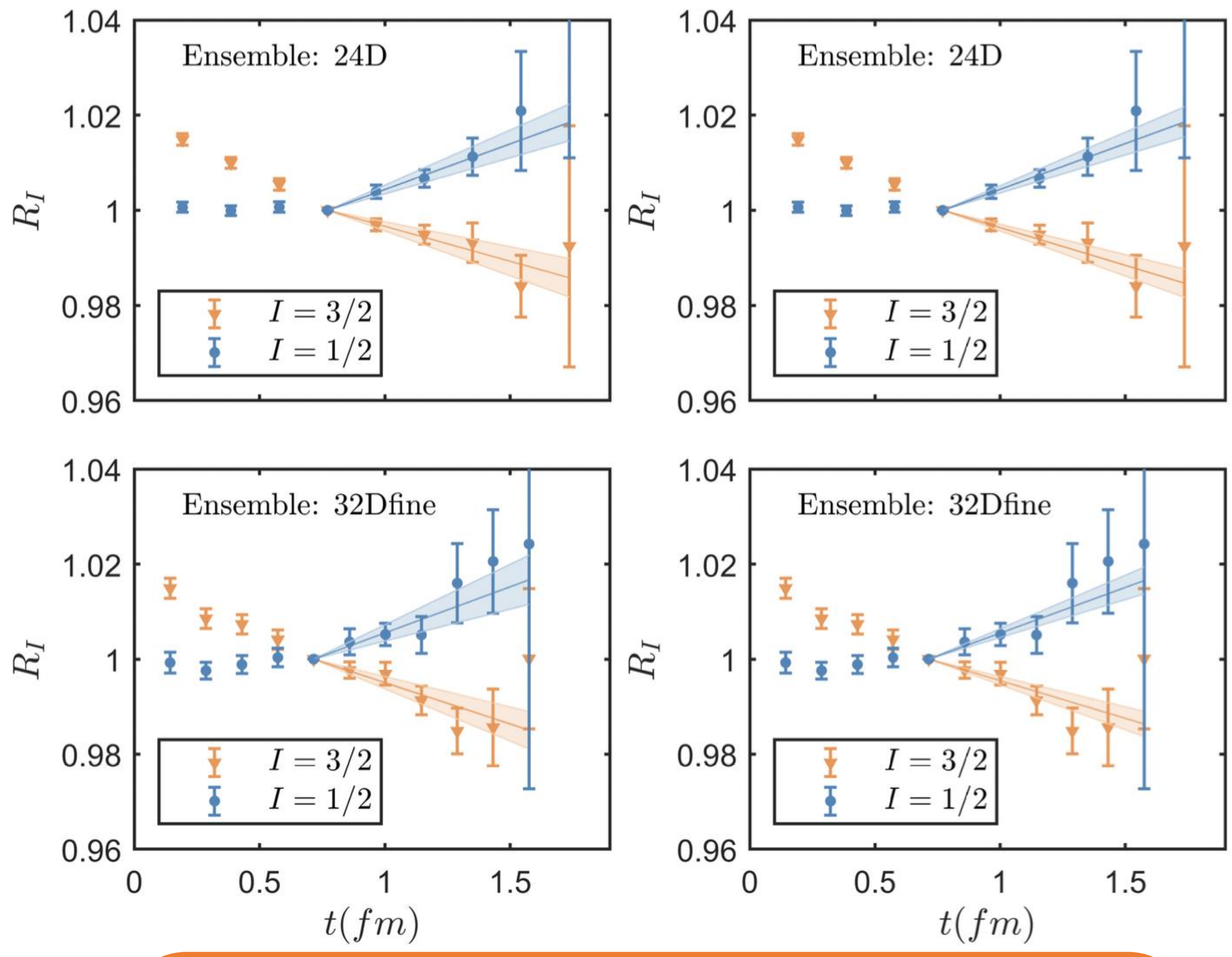
$a_0^{1/2} m_\pi = 0.170(2)$, $a_0^{3/2} m_\pi = -0.087(2)$

M. Hoferichter et al, PLB 843 (2023) 138001

~3σ

Results of $N\pi$ Scattering

- Use $N\pi$ operators with 4 lowest momenta & apply GEVP method



combined fit

- After GEVP, energy eigenvalues shift downwards for both I=1/2 & 3/2

Results using GEVP

$$a_0^{1/2} m_\pi = 0.152(28),\ a_0^{3/2} m_\pi = -0.107(17)$$

~1σ

Analysis based on cross section and πH, πD spectrum

$$a_0^{1/2} m_\pi = 0.170(2),\ a_0^{3/2} m_\pi = -0.087(2)$$

M. Hoferichter et al, PLB 843 (2023) 138001

Matrix elements of $N\gamma \to N\pi$

- Normalization for ${}_I\langle N\pi | J_i^{I'} | N\rangle$

$$R = \frac{C_{NJN\pi}(t_1, t_2)}{C_{N\pi}(t_1 + t_2)} \times \sqrt{\frac{C_N(t_1)C_{N\pi}(t_2)C_{N\pi}(t_1 + t_2)}{C_{N\pi}(t_1)C_N(t_2)C_N(t_1 + t_2)}}$$

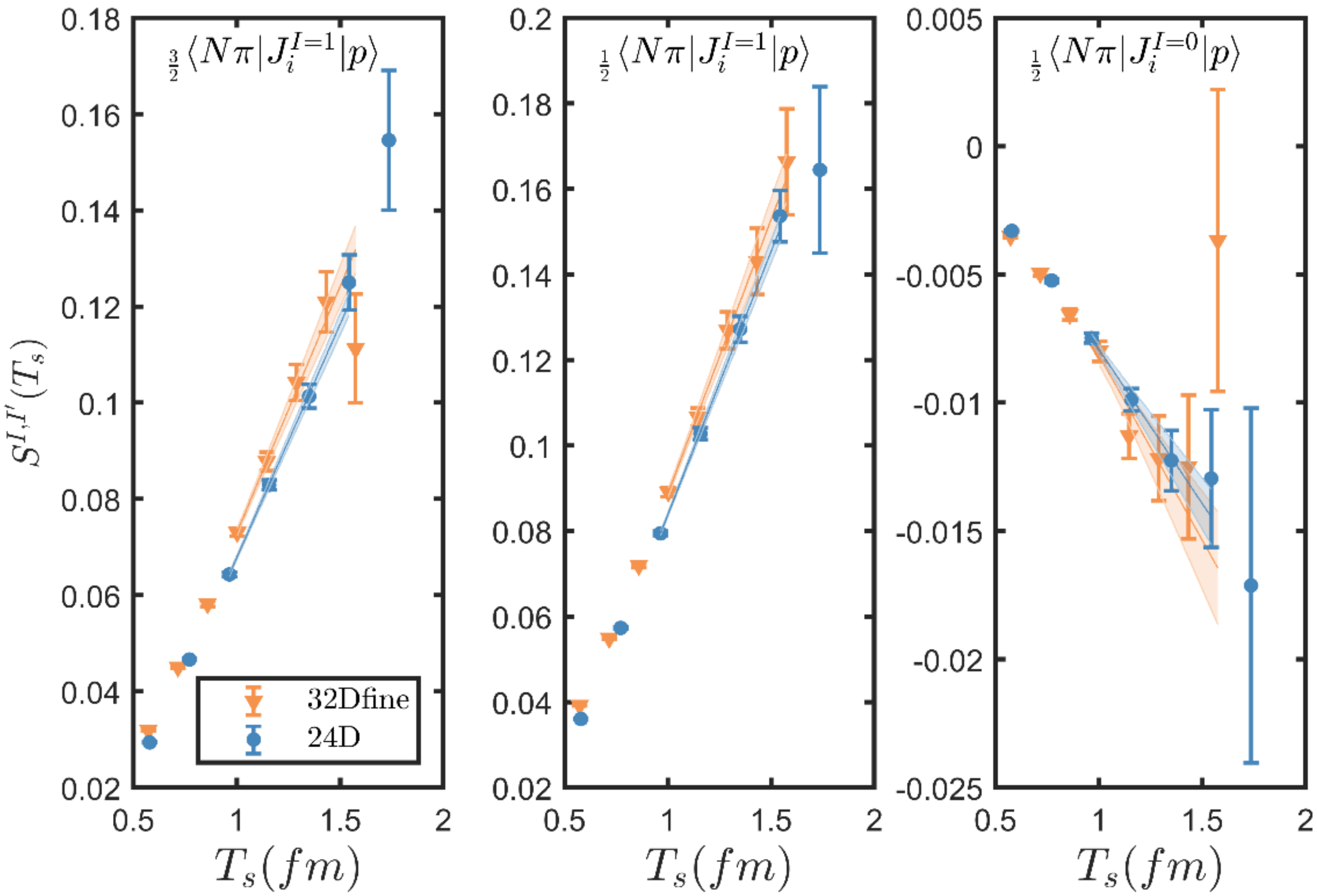
- Summed insertion

Maiani L. NPB, 293, 420(1987)

$$S(T_s) = \sum_{t_1+t_2=T_s} R(t_1, t_2) \xrightarrow{T_s\to\infty} c_0 + \frac{1}{\sqrt{2M}}\, {}_I\langle N\pi | J_i^{I'} | N\rangle \cdot T_s$$

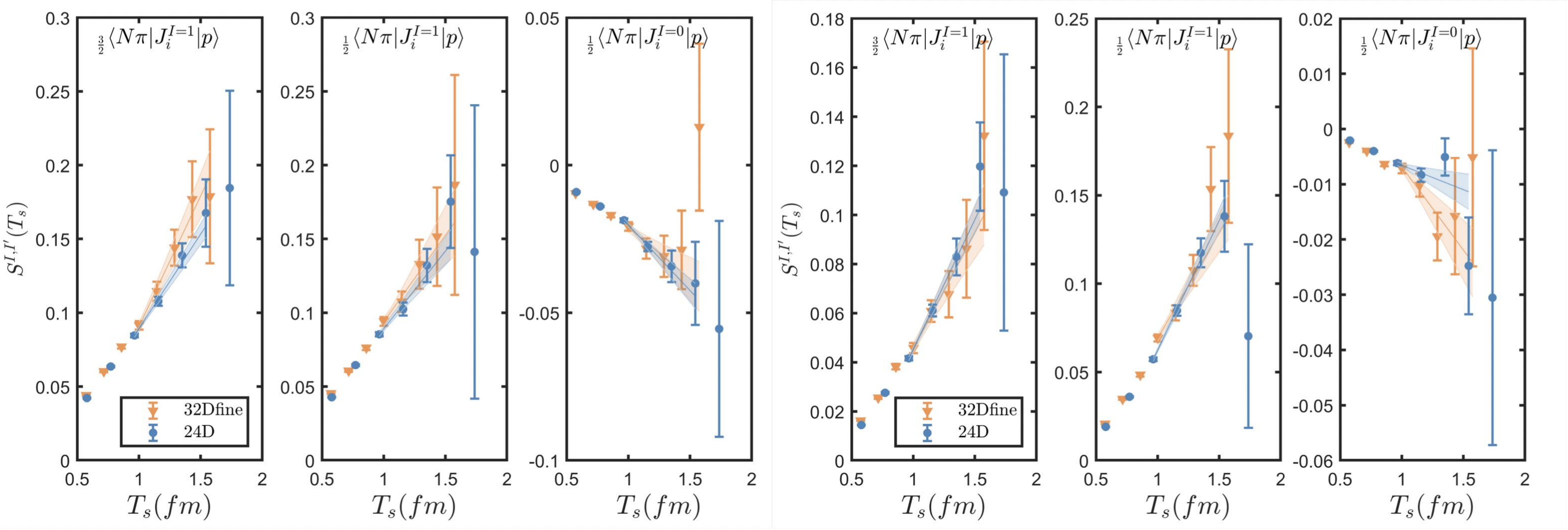
- Linear fit $S(T_s)$ with T_s to extract

Nπ at the threshold



Matrix elements of $N\gamma \rightarrow N\pi$

$N\pi$ in the center of mass frame with mom. mode (100)

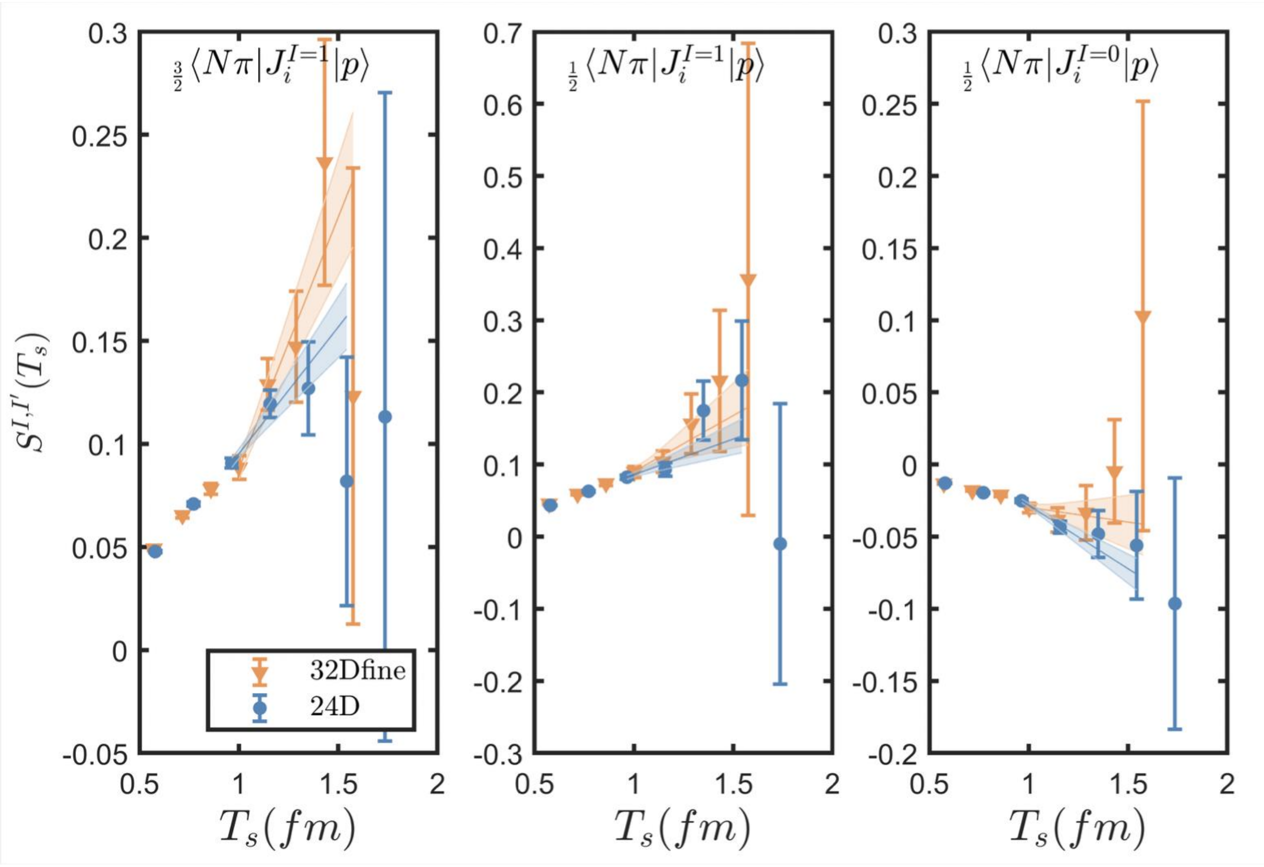


G_1^- representation

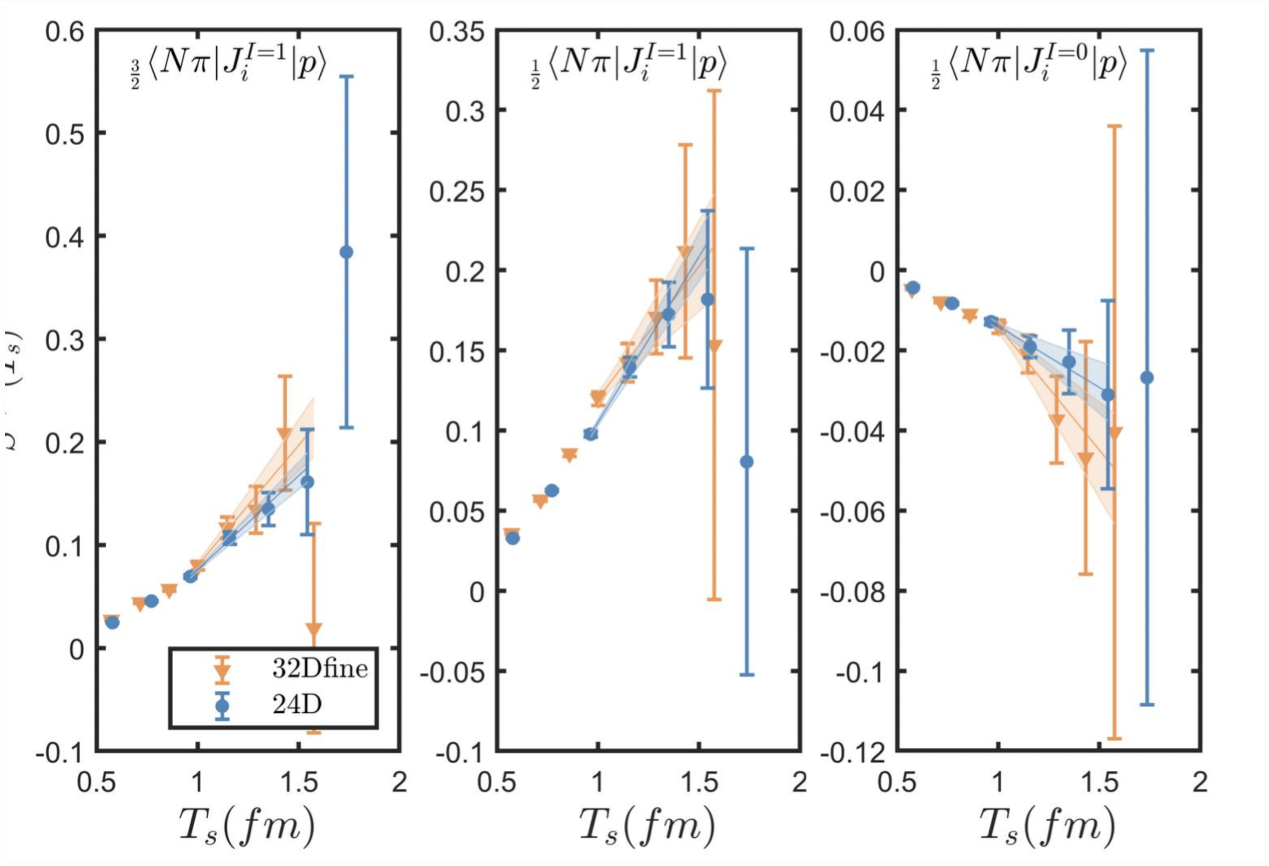
H^- representation

Matrix elements of $N\gamma \rightarrow N\pi$

Nπ in the center of mass frame with mom. mode (110)



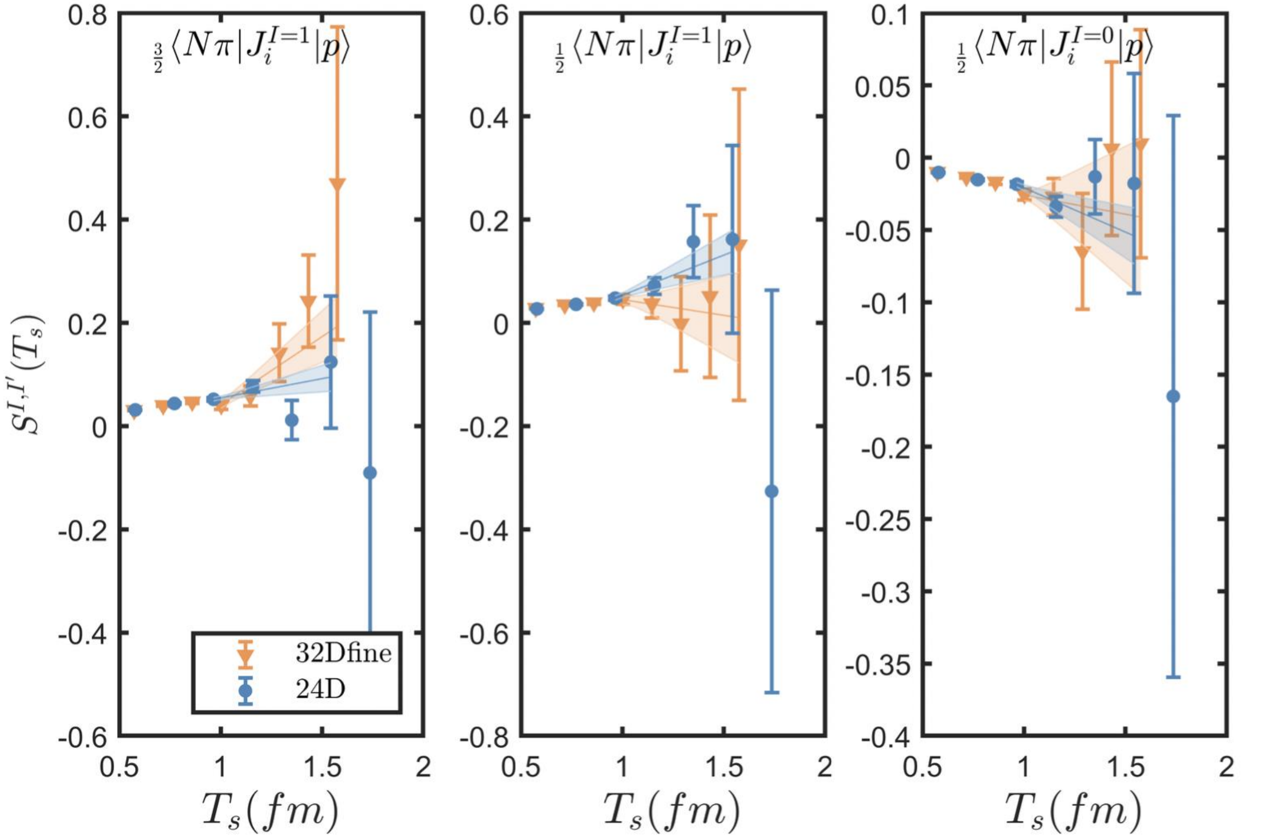
G_1^- representation



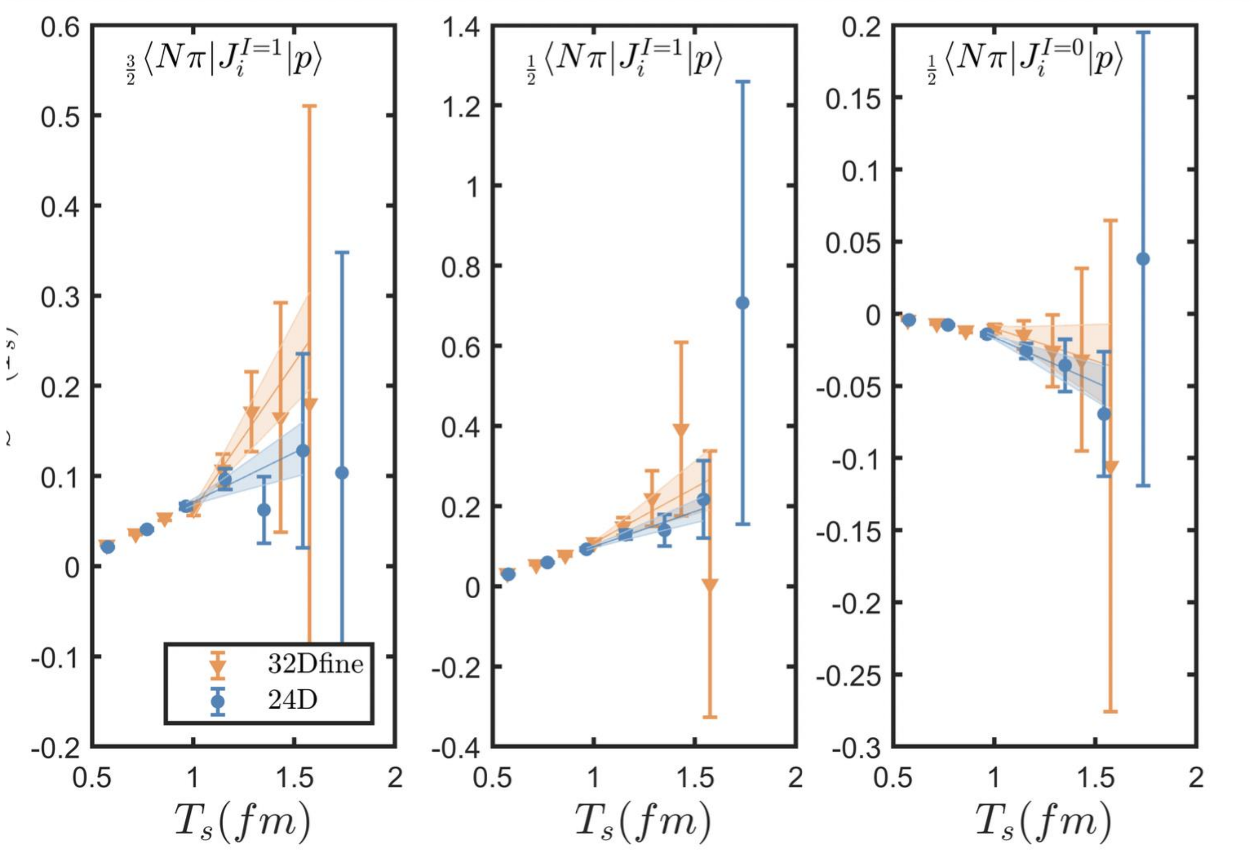
H^- representation

Matrix elements of $N\gamma \rightarrow N\pi$

Nπ in the center of mass frame with mom. mode (111)



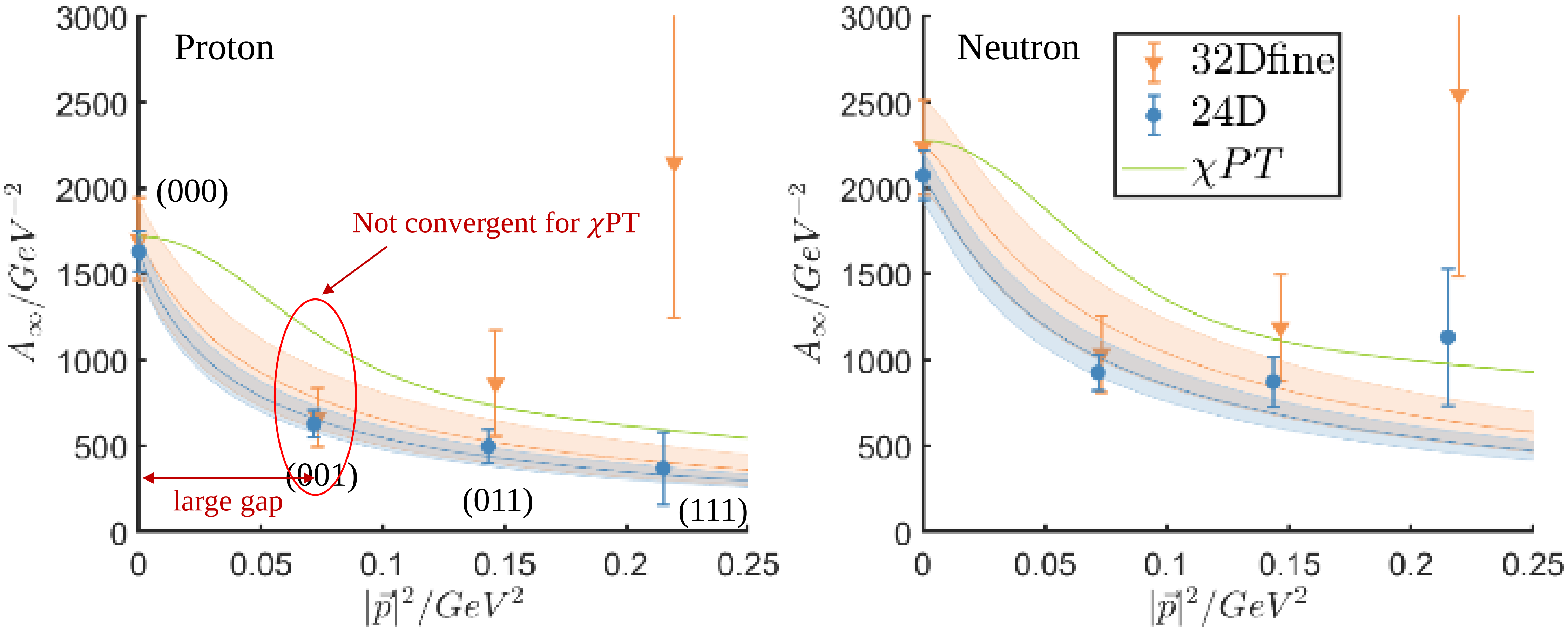
G_1^- representation



H^- representation

Matrix elements of $N\gamma \rightarrow N(\mathrm{p})\pi(\text{-p})$ with 4 lowest mom modes

$$A_\infty = \sum_{I=\frac{1}{2},\frac{3}{2}} \sum_{\Gamma=G_1^-,H^-} \sum_{r=1}^{\dim(\Gamma)} \sum_{i=1,2,3} |_\infty\langle N\pi, I, \Gamma, r, n|J^i|N\rangle|^2$$



Limitations in the comparison between lattice QCD and χPT:

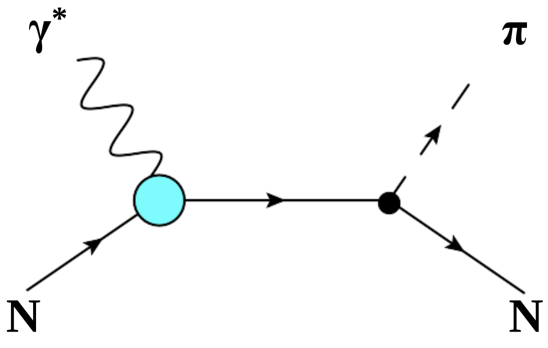
For lattice, momentum modes are limited

For χPT, photon is very timelike & χPT does not work well

Momentum dependence of A_∞

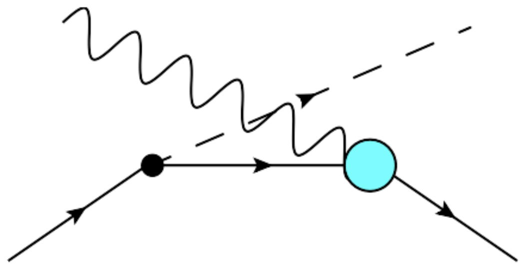
$$N(p_1) + \gamma^*(k) \to \pi(q) + N(p_2)$$

s



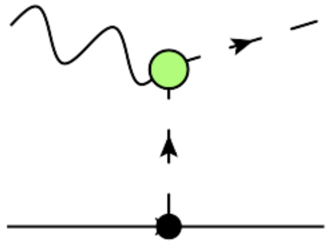
$$\frac{1}{s - M_N^2} = \frac{1}{E_\pi + E_N + M_N}\frac{1}{E_\pi + E_N - M_N}$$

u



$$\frac{1}{u - M_N^2} = -\frac{1}{M_N - E_\pi + E_N}\frac{1}{-M_N + E_\pi + E_N}$$

t



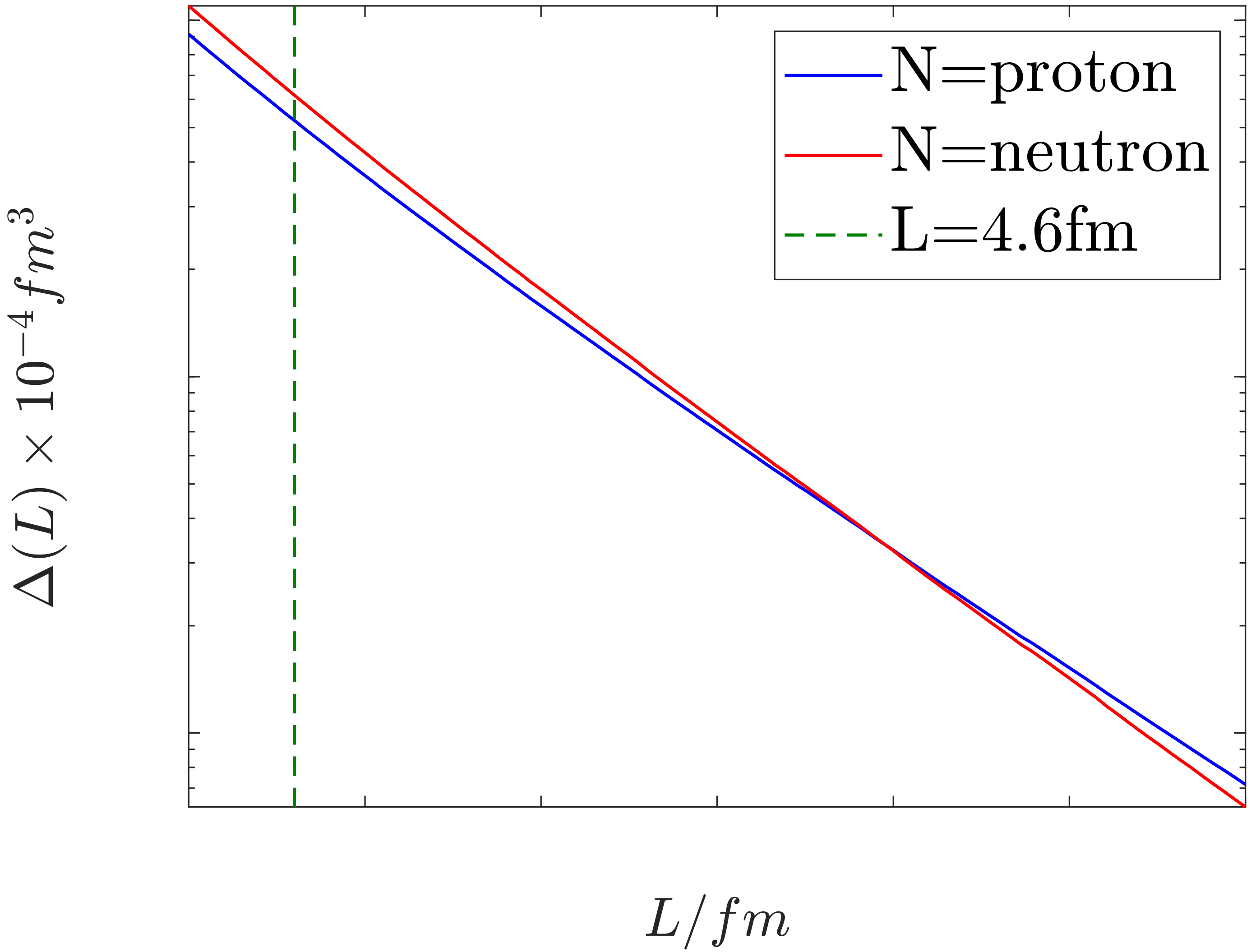
$$\frac{(q + p_1 - p_2)_\mu}{t - M_\pi^2} = -\frac{\delta_{\mu 0}}{-M_N + E_N + E_\pi}$$

$$A_\infty = \frac{\sum_s a_s(\vec{p}^2)^s}{(2E)(2E_\pi)(E_N + E_\pi - M_N)^2}$$

- Keep quasi-singular in denominator
- Taylor expansion in numerator

Finite-volume effects

$$\Delta(L) = \alpha_E^{ii,N\pi}(L) - \alpha_E^{ii,N\pi} = \frac{1}{3}\frac{\alpha_{em}}{M_N}\left(\frac{1}{L^3}\sum_{\vec{p}=\frac{2\pi}{L}\vec{m}} - \int\frac{d^3\vec{p}}{(2\pi)^3}\right)\frac{A_\infty}{(E_N + E_\pi - M_N)^3}$$



- Always positive and thus only exponentially suppressed FV effects
- It is crucial to replace mom. summation by mom. integral

$$\alpha_E^{ii,N\pi} = \frac{1}{3}\frac{\alpha_{em}}{M}\int_{|\vec{p}|<\Lambda}\frac{d^3\vec{p}}{(2\pi)^3}\frac{A_\infty}{(E + E_\pi - M)^3}$$

After replacement, residual FV effects are estimated to be < 10^{-5} fm^3

Numerical results

- Our results of α_E, in units of $10^{-4} fm^3$ X. Wang, Z. Zhang, et. al., arXiv:2310.01168

		24D	32Dfine	PDG
Proton	$\alpha_E^{N\pi}$	5.65(53)	6.5(1.2)	
	α_E	10.0(1.3)	9.3(2.2)	11.2(4)
Neutron	$\alpha_E^{N\pi}$	8.33(75)	9.8(1.5)	
	α_E	9.7(1.4)	10.1(2.4)	11.8(1.1)

- Confirm large contributions of $N\pi$ states
- Develop the methodology for lattice QCD computation of polarizabilities
- More sophisticated study to control systematic effects

→ Larger volume to have more momentum modes

Excited-state contamination from initial and final states

Finer lattice spacing for continuum extrapolations

Extended projects – threshold pion EW production

➢ Consider the process $\gamma^*(k)+N(p_1) \to \pi(q)+N(p_2)$

➢ Threshold pion production means that in the γ^*N center of mass frame, pion is at threshold $q_\mu=(M_\pi,0,0,0)$

V. Bernard, N. Kaiser, T.-S. Lee, U.-G. Meissner, Phys. Rept. 246 (1994) 315

$$\mathcal{V}_\mu = \langle N(\vec{0})\pi(\vec{0})|V_\mu|N(-\vec{k})\rangle$$

$$\vec{\mathcal{V}} = 4\pi i(1+\mu)\chi_f^\dagger\left\{E_{0+}(\mu,\nu)\vec{\sigma} + [L_{0+}(\mu,\nu) - E_{0+}(\mu,\nu)]\hat{k}\,\vec{\sigma}\cdot\hat{k}\right\}\chi_i$$

- Multipole amplitude describes the transverse and longitudinal coupling of γ^* to the nucleon spin

- Parameters are define as $\mu = \frac{M_\pi}{M_N}, \quad \nu = \frac{k^2}{M_N^2}$

Extended projects – threshold pion EW production

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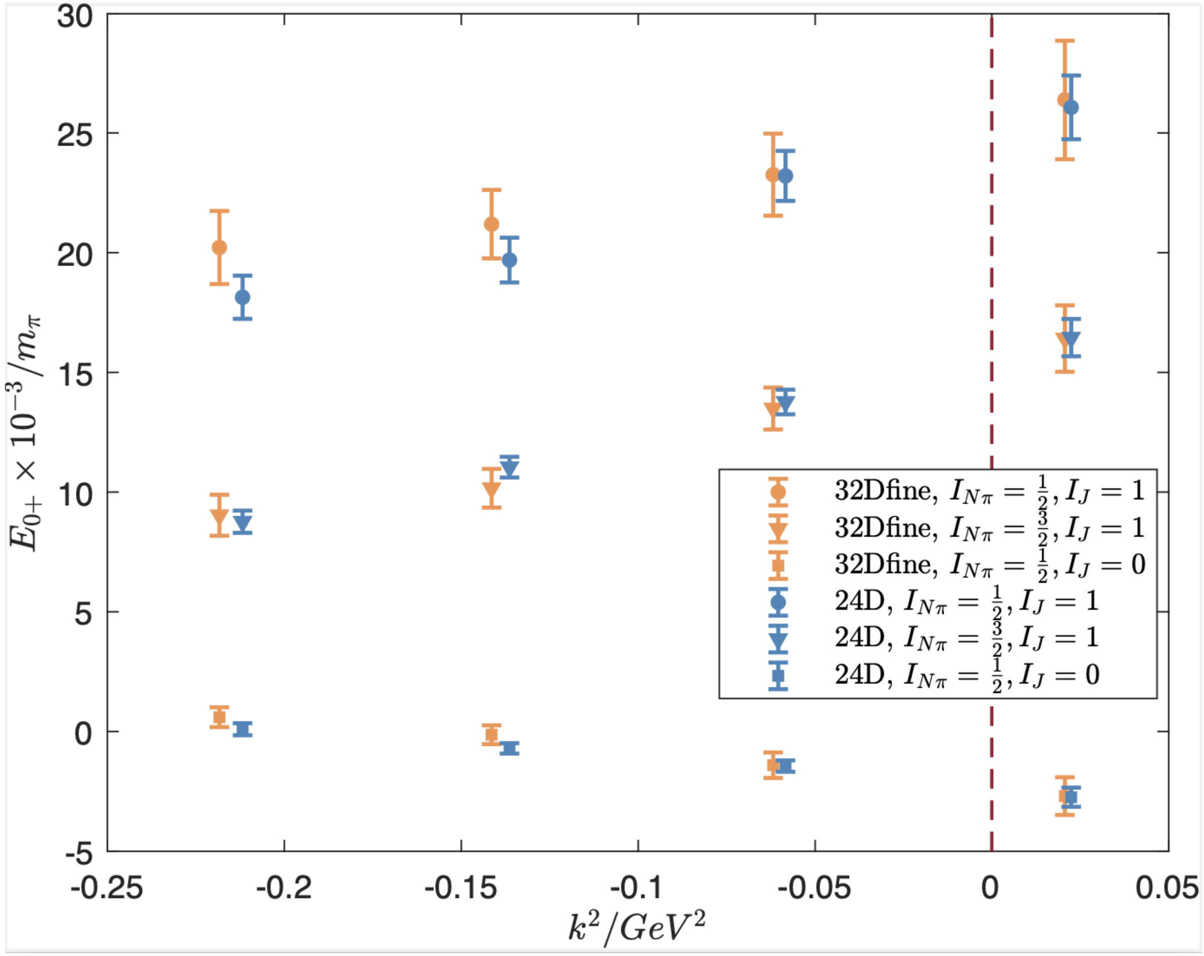
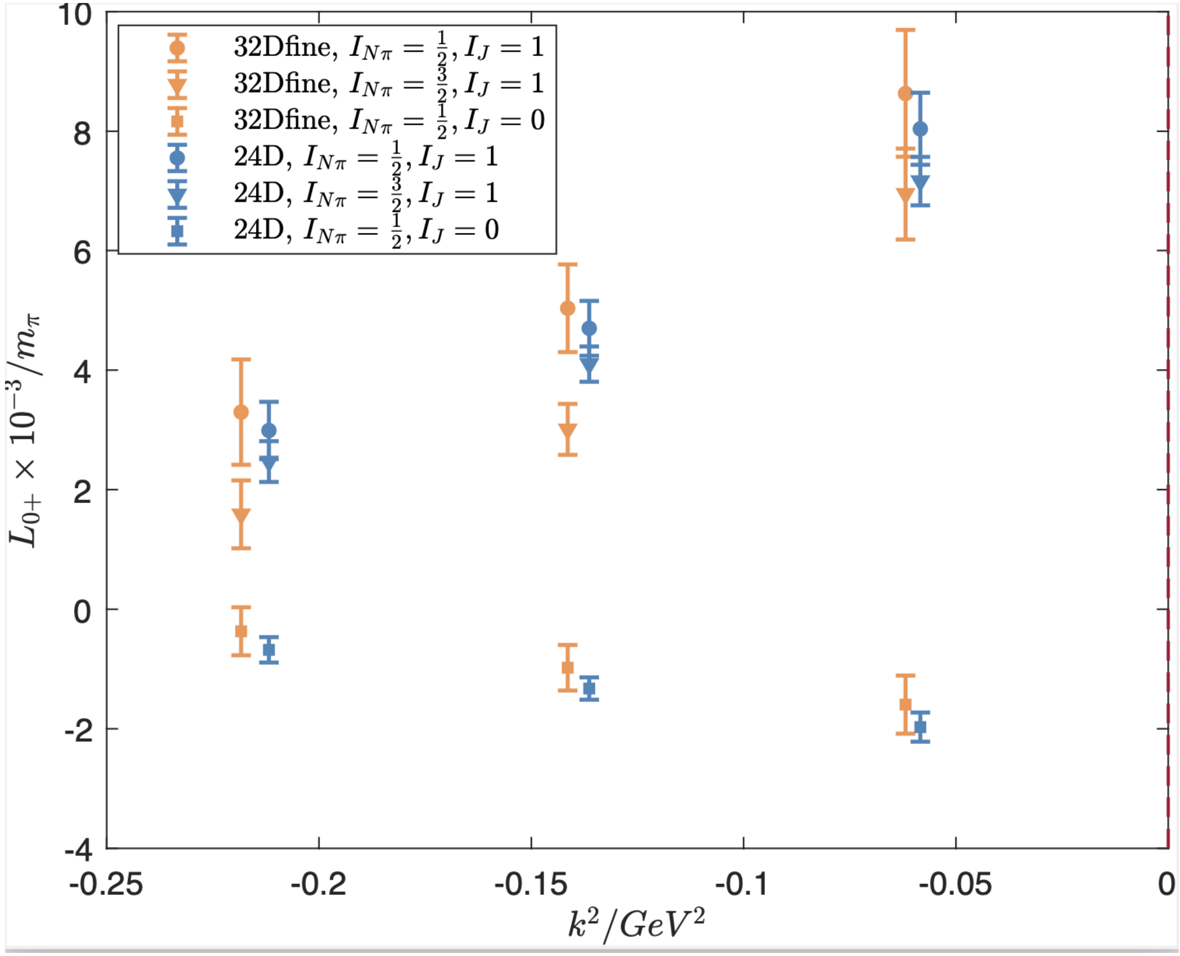
- Threshold pion production means that in the γ^*N center of mass frame, pion is at threshold $q_\mu=(M_\pi,0,0,0)$



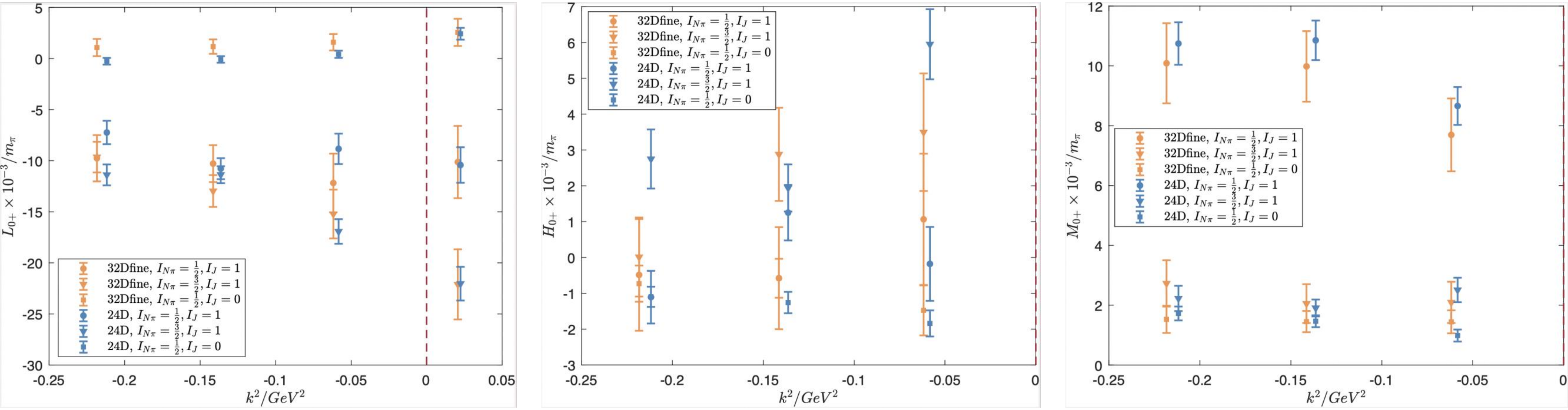


Extended projects – threshold pion EW production

- Consider the process $W^*(k)+N(p_1) \rightarrow \pi(q)+N(p_2)$

$$\mathcal{A}_\mu = \langle N(\vec{0})\pi(\vec{0})|A_\mu|N(-\vec{k})\rangle \qquad \mathcal{A}\cdot\epsilon = 4\pi i(1+\mu)\chi_f^\dagger\left\{\epsilon_0 L_{0+} + \vec{\epsilon}\cdot\hat{k}\,H_{0+} + i\vec{\sigma}\cdot(\hat{k}\times\vec{\epsilon})M_{0+}\right\}\xi_i$$

V. Bernard, N. Kaiser, U.-G. Meissner, PLB 331 (1994) 137



Conclusion

Nucleon E&M polarizability → Nucleon structure

↓

Nπ rescattering → Hadron spectroscopy

↓

Pion EW production → Lepton-nucleon inelastic scattering

An interesting journey to explore nucleon properties!

↓

New frontiers, new methodology and new findings!