Digital Quantum Simulation for Energy Spectroscopy of Schwinger Model

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(本多正純)

based on a collaboration with

[cf. arXiv: 2204.14788]

Dongwook Ghim (RIKEN iTHEMS)

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XVIth Quark Confinement and
the Hadron Spectrum Conference 2024

This talk is on

Application of Quantum Computation to Quantum Field Theory (QFT)

Motivation:

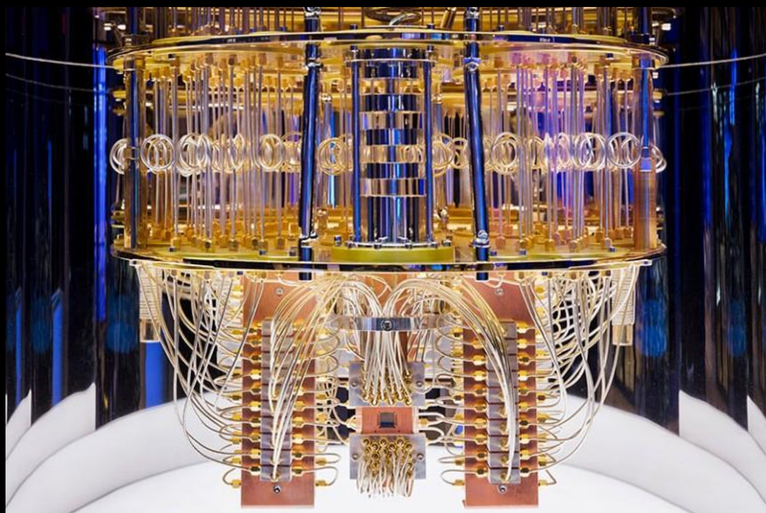
Quantum computation is more natural in operator formalism

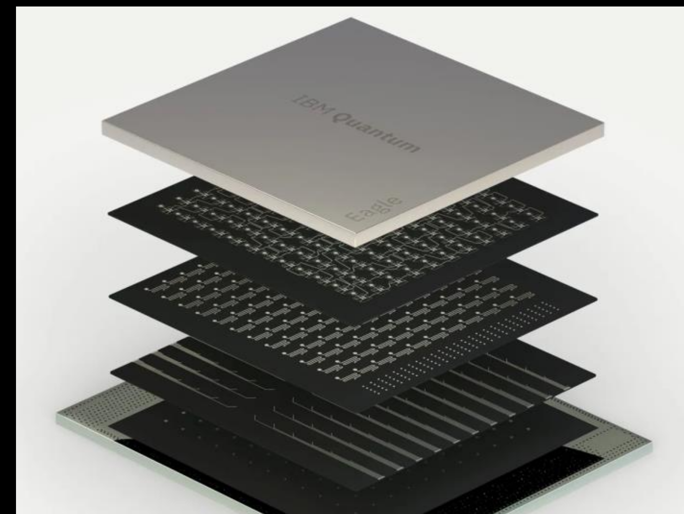
⟶ Liberation from infamous sign problem in Monte Carlo

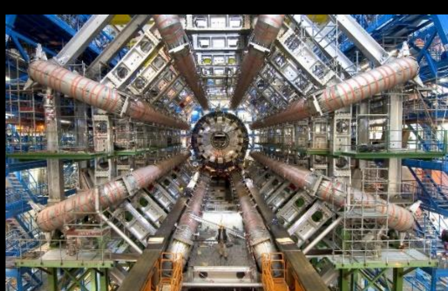
Typical situations w/ sign problem:

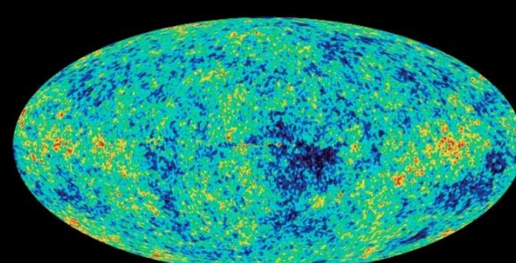
- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " *much worse*

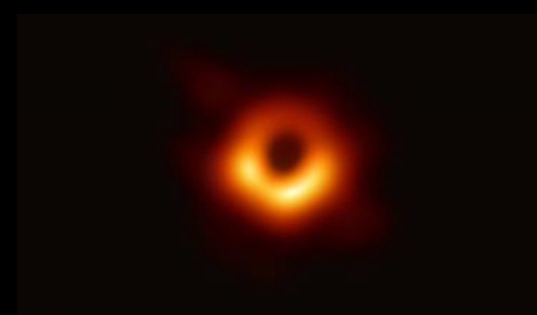
My long term goal











etc…

Today's focus:

Quantum algorithm for energy spectrum in QFT

- degeneracy of ground states
- energy gap between ground & 1st excited states
- distribution of excited states at low levels

⇒ phase structure, mass spectrum of particles

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⇨ phase structure, mass spectrum of particles

Desired algorithm:

efficient computation of spectrum at low levels

(doesn't need ground state energy itself)

For this purpose, it seems inefficient to explicitly construct energy eigenstates one by one and measure their energies

Algorithm: coherent imaging spectroscopy

[Senko-Smith-Richerme-Lee-Campbell-Monroe '14]
[in preparation, MH-Ghim]

We'd like to know spectrum of excited energies:

$$\hat{H}_{\text{target}}\,|n\rangle = E_n|n\rangle$$

Time dependent Hamiltonian:

$$\hat{H}(t;\nu) = \hat{H}_{\text{target}} + B\sin(\nu t)\cdot\hat{O}$$

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Survival probability of ground state after some time:

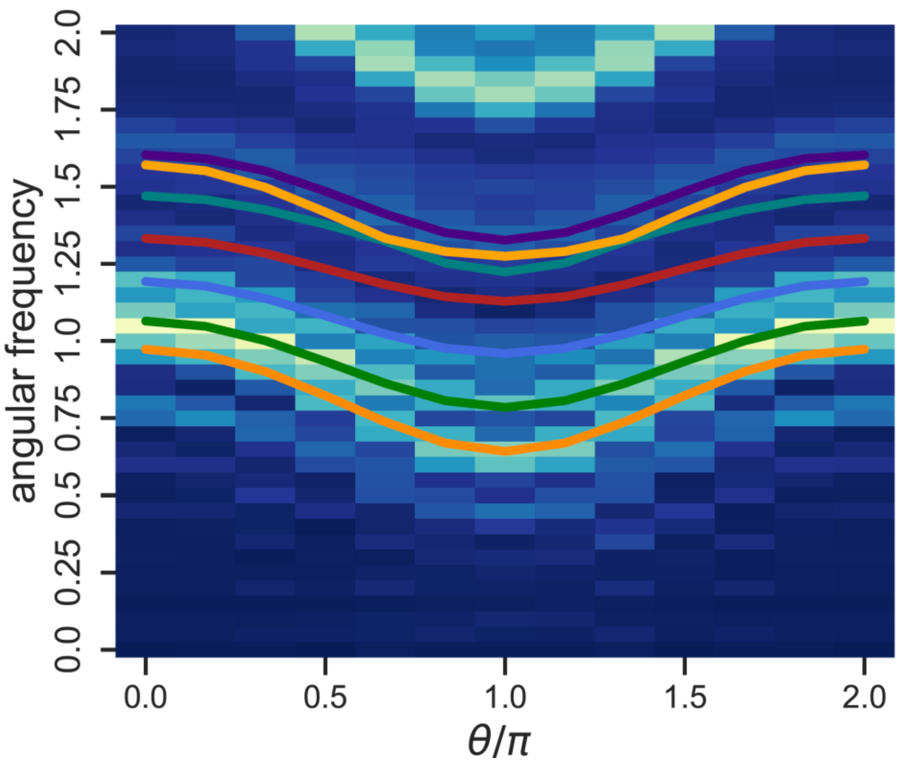
$$P(\nu) := |\langle 0|\mathcal{T}e^{-i\int dt\hat{H}\,(t;\nu)}|0\rangle|^2$$ small if $\nu \sim E_n$

Scanning various values of $\nu \rightarrow$ spectrum!

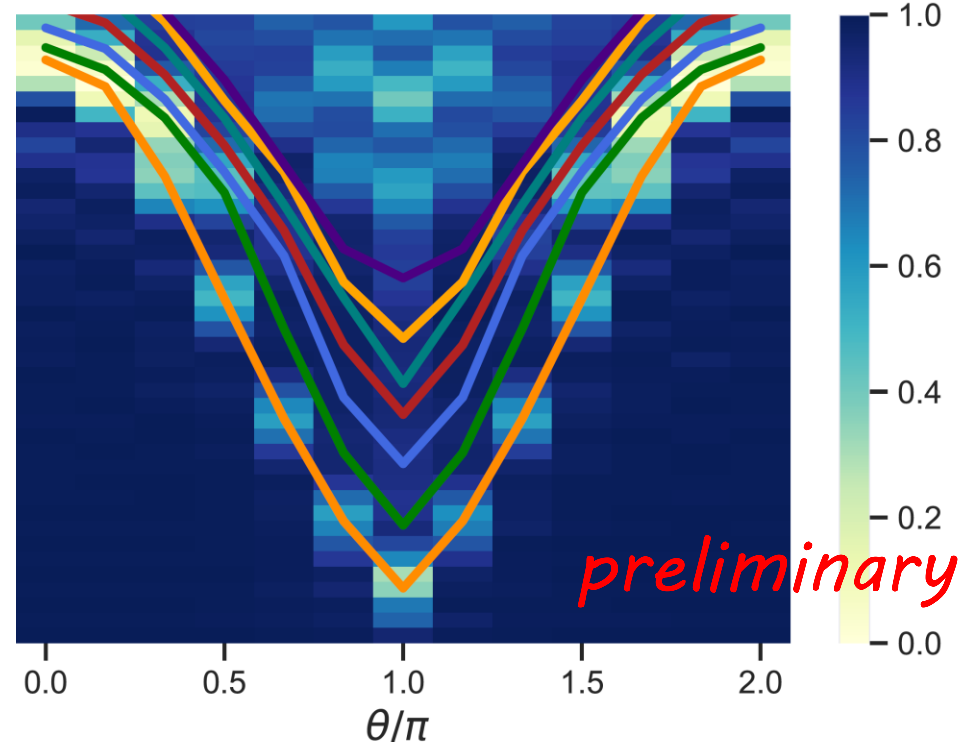
Implementation in Lattice Schwinger model

(details & more plots later)

Gapped for any θ



Parity SSB at $\theta = \pi$ (for ∞-vol.)



- vacuum prepared by adiabatic state preparation
- implemented on simulator
- continuum/thermodynamic limits not taken

Plan

1. Introduction

2. Schwinger model as qubits

3. Algorithm for energy spectrum

4. Summary

QFT as Quantum Bit (=Qubit) ?

Qubit = Quantum system w/ 2-dim. Hilbert space

(ex. up/down spin system)

Quantum computer = a combination of qubits

To put QFT on quantum computer,

1. "Regularize" Hilbert space (make it finite-dim.!)
2. Rewrite the regularized theory in terms of qubits

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Schwinger model = the simplest nontrivial example w/ gauge interaction in this context

— 1+1d gauge field has only 1-dim. physical Hilbert sp.

— Lattice fermion has finite-dim. Hilbert sp.

Schwinger model w/ topological term

Continuum:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_\mu + igA_\mu)\psi - m\bar{\psi}\psi$$

[used in Nagano & Okuda's talks]

Using "chiral anomaly", the same physics can be studied by [Fujikawa'79]

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_\mu + igA_\mu)\psi - m\bar{\psi}e^{i\theta\gamma^5}\psi}$$

used here

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used here

Taking temporal gauge $A_0 = 0$, ($\Pi = \dot{A}^1$)

$$\hat{H} = \int dx\left[-i\bar{\psi}\gamma^{1}(\partial_{1} + igA_{1})\psi + m\bar{\psi}e^{i\theta\gamma^5}\psi + \frac{1}{2}\Pi^2\right]$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1\Pi - g\bar{\psi}\gamma^0\psi$$

Lattice theory w/ staggered fermion

Hamiltonian:

$$\hat{H} = -i \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[\chi_n^\dagger e^{i\phi_n} \chi_n - \text{h.c.} \right]$$

$$+ m \cos\theta \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n + J \sum_{n=1}^{N-1} L_n^2 \qquad \left(w = \frac{1}{2a}, J = \frac{g^2 a}{2} \right)$$

Commutation relation:

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0, \ [\phi_n, L_m] = i\delta_{mn}$$

Gauss law:

$$L_n - L_{n-1} = \chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2}$$

Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = \sum_{\ell=1}^{n-1} \left[\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right] \qquad \text{(took } L_0 = 0\text{)}$$

2. Redefine fermion to absorb ϕ_n:

$$\chi_n \to \prod_{\ell<n} \left[e^{-i\phi_\ell} \right] \chi_n$$

Then,

$$\hat{H} = -i \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[\chi_n^\dagger \chi_{n+1} - \text{h.c.} \right] + m\cos\theta \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n$$
$$+ J \sum_{n=1}^{N-1} \left[\sum_{\ell=1}^{n-1} \left(\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right) \right]^2$$

This acts on finite dimensional Hilbert space

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2} \left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i \right)$$

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

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"Jordan-Wigner transformation"

Now the system is purely a spin system:

[Jordan-Wigner'28]

$$\hat{H} = H_{ZZ} + H_{\pm} + H_Z$$

$$\begin{cases} H_{ZZ} = \frac{J}{2} \sum_{n=2}^{N-1} \sum_{1 \le k < \ell \le n} Z_k Z_\ell, \\ H_{\pm} = \frac{1}{2} \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[X_n X_{n+1} + Y_n Y_{n+1} \right], \\ H_Z = \frac{m \cos\theta}{2} \sum_{n=1}^{N} (-1)^n Z_n - \frac{J}{2} \sum_{n=1}^{N-1} (n \bmod 2) \sum_{\ell=1}^{n} Z_\ell \end{cases}$$

Qubit description of the Schwinger model !!

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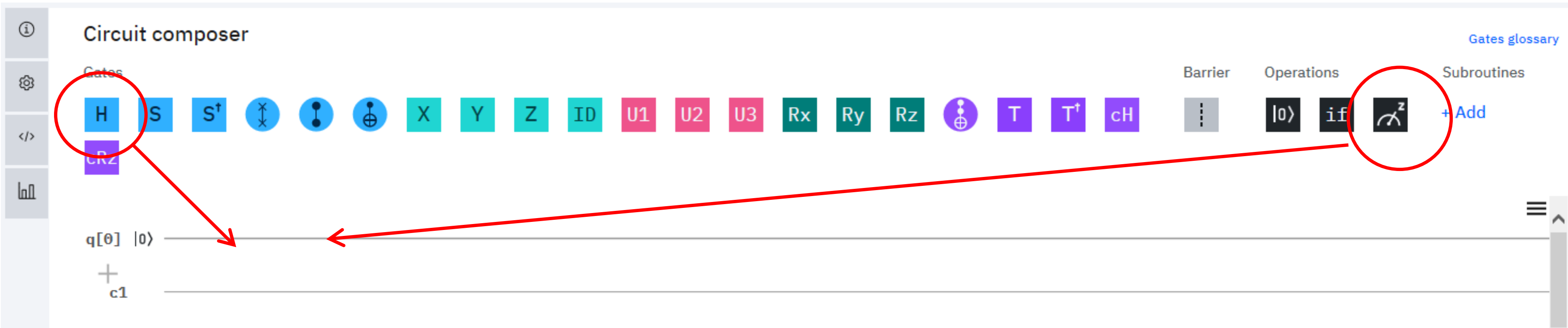
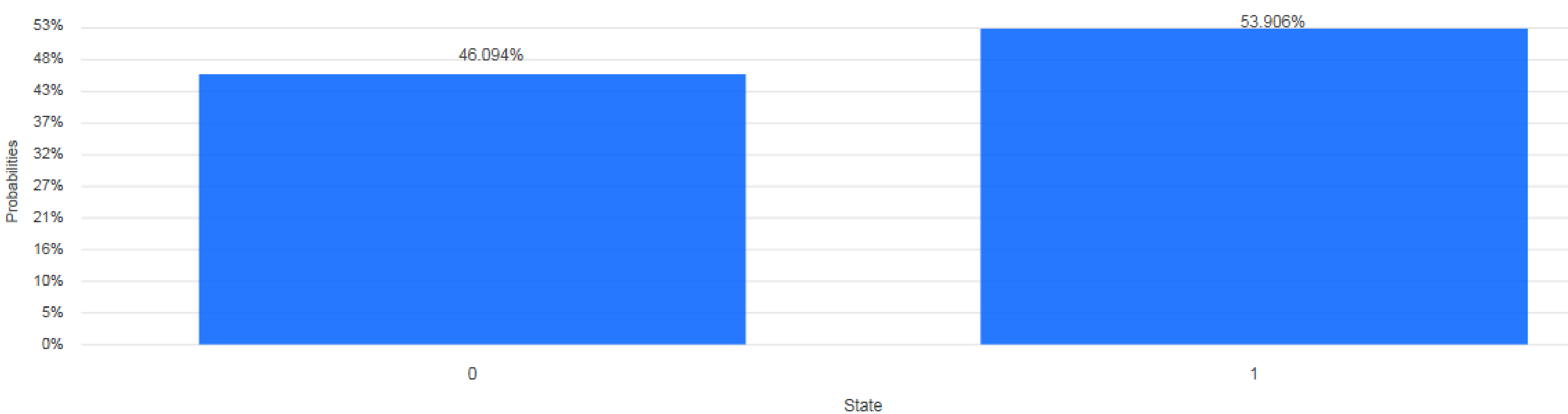
Atmosphere (?) of using quantum computer…

Suppose we'd like to measure the state: $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Screenshot of IBM Quantum Experience:



Output of 1024 times measurements ("shots") :



Idea: express physical quantities in terms of "probabilities"
& measure the "probabilities"

Algorithm: coherent imaging spectroscopy

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Time dependent Hamiltonian:

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$$P(\nu) \coloneqq |\langle 0|\mathcal{T}e^{-i\int dt\hat{H}\,(t;\nu)}|0\rangle|^2$$ small if $\nu \sim E_n$

Scanning various values of $\nu \to$ spectrum!

Features of the algorithm

$$\hat{H}(t;\nu) = \hat{H}_{\text{target}} + \int dx\, B(x)\sin(\nu t)\cdot \hat{O}(x)$$

😄 Advantage:

▪ can directly consider high energy regime

(likely more costly for higher energy)

▪ can filter quantum number by choosing ops.

ex.) if we want to know only mass spectrum for trans. inv. theories

$\rightarrow$ take $B(x)$ to be constant

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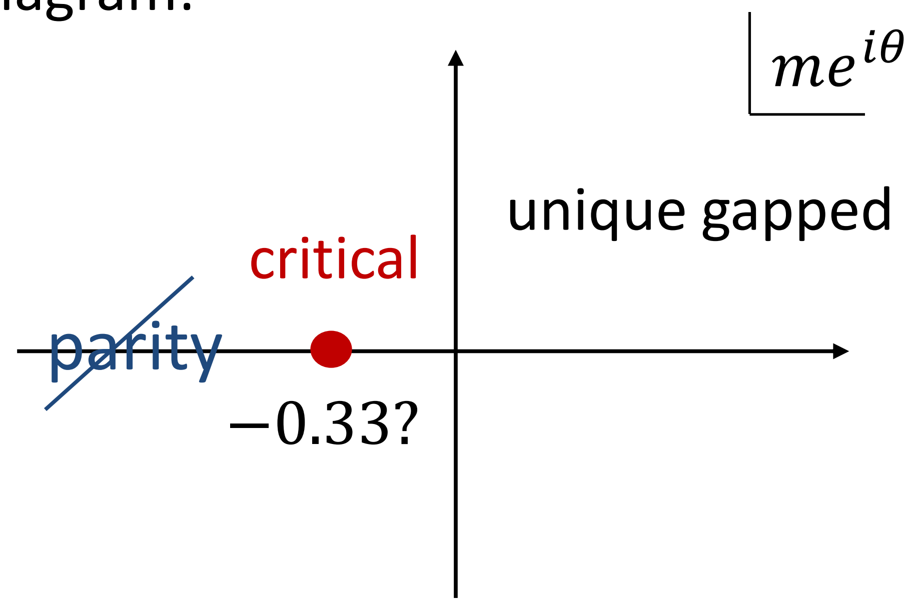
☹️ Disadvantage:

- $\exists$ ambiguity on whether transition occurred
- not good for precise estimation

Coherent imaging spectroscopy in Schwinger model

$$\hat{H} = -i\sum_{n=1}^{N-1}\left(w - (-1)^n \frac{m}{2}\sin\theta\right)\left[\chi_n^\dagger e^{i\phi_n}\chi_n - \text{h.c.}\right] + m\cos\theta\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n + J\sum_{n=1}^{N-1}L_n^2$$

Expected phase diagram:



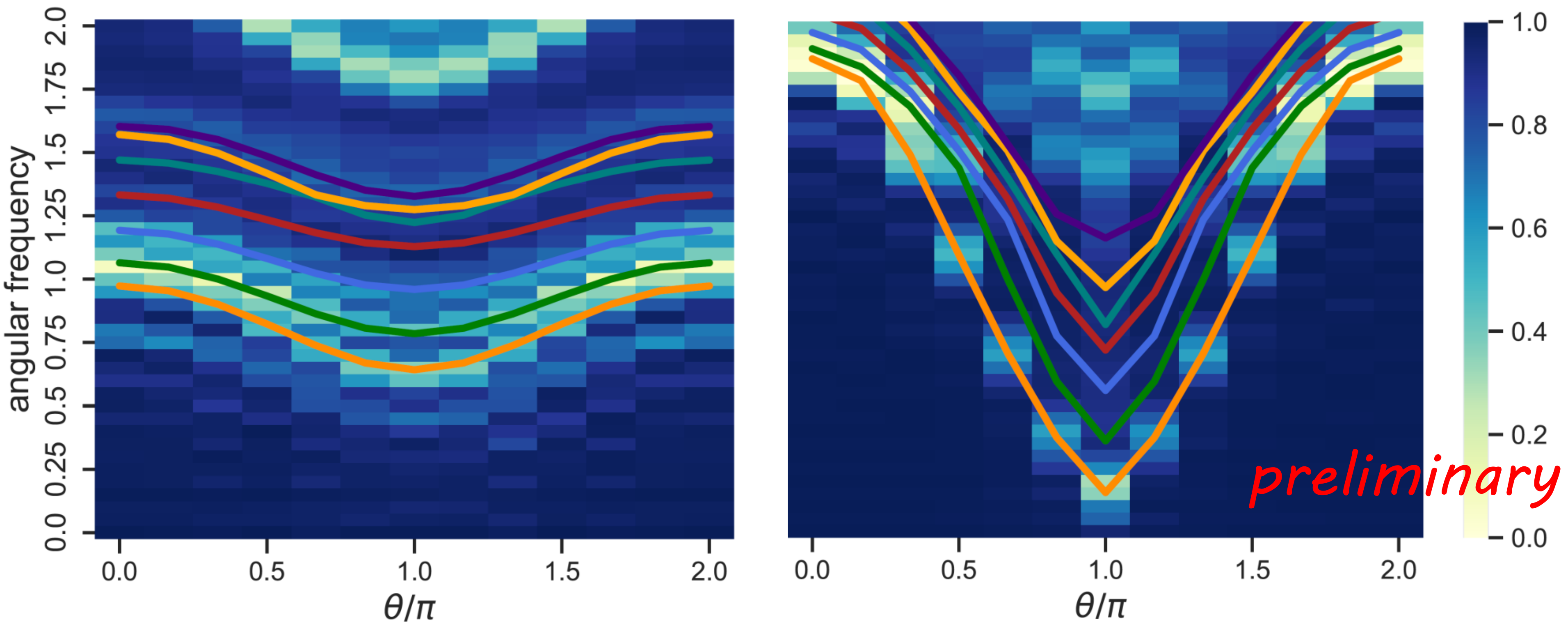
Let's consider time evolution by (perturbed by "$\bar{\psi}\gamma_5\psi$")

$$\Delta H(t) = \frac{B_p}{2}\sum_{n=0}^{N-2}(-1)^{n+1} f_n \sin(\omega t)\left(X_n X_{n+1} + Y_n Y_{n+1}\right)$$

Result of constant perturbation

$g = 1, N = 11, w = 1, B_p = 0.025g, T = 30$, 800 Trotter steps, 20000 shots

$m = 0.1$ (gapped for any θ) $m = 0.6$ (SSB for $\theta = \pi, N \to \infty$)



- vacuum prepared by adiabatic state preparation
- implemented on simulator
- continuum/thermodynamic limits not taken

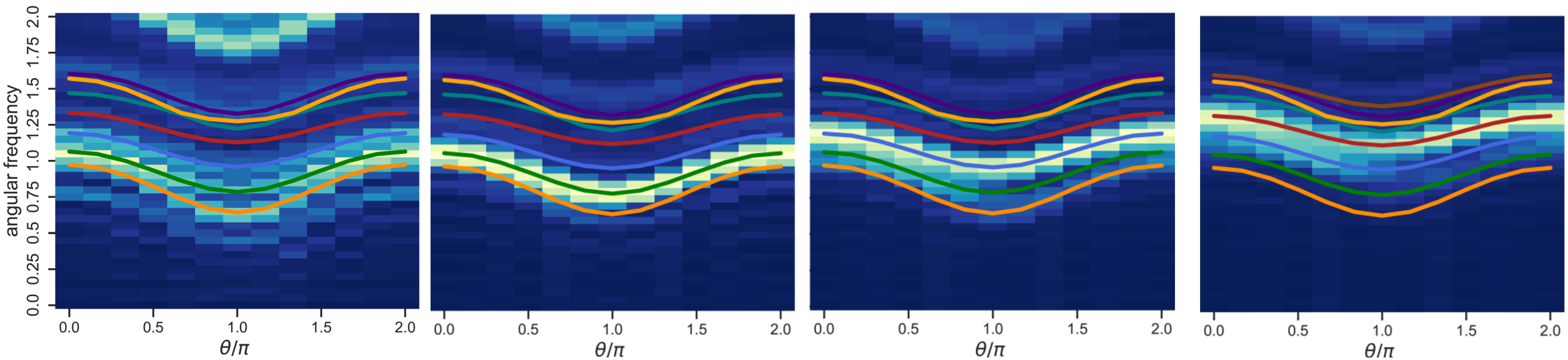
Space modulated perturbation for $m = 0.1$

($g = 1, N = 11, w = 1, B_p = 0.025g, T = 30$, 800 Trotter steps, 20000 shots)

To get moving particles, turn on space modulation:

$$\Delta H(t) = \frac{B_p}{2} \sum_{n=0}^{N-2} (-1)^{n+1} f_n \sin(\omega t) \left(X_n X_{n+1} + Y_n Y_{n+1}\right) \qquad \left\{f_n^{(k)}\right\}_{k=0,1,2\cdots} \equiv \left\{\cos\left(\frac{k\pi n}{N-1}\right)\right\}$$

$k = 0$ $k = 1$ $k = 2$ $k = 3$



capturing the momentum modes

preliminary

Summary

Summary

Quantum algorithm for energy spectrum in QFT

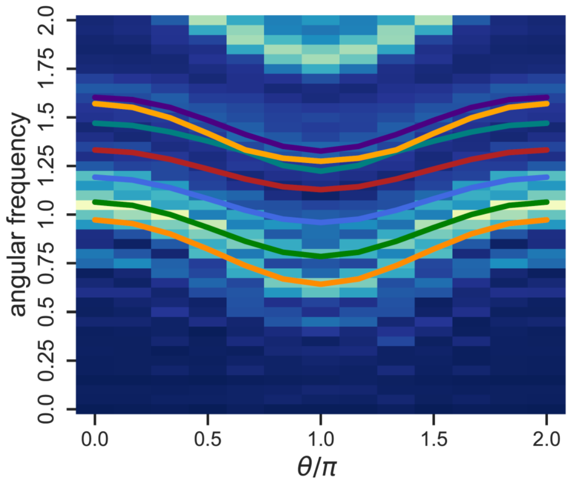
- Algorithm = Coherent imaging spectroscopy:

$$\hat{H} = \hat{H}_{\text{target}} + B\sin(\nu t) \cdot \hat{O}\,(t;\nu)$$

$$P(\nu) \coloneqq |\langle 0|\mathcal{T}e^{-i\int dt \hat{H}\,(t;\nu)}|0\rangle|^2$$

- Implemented in Schwinger model:

Gapped for any θ



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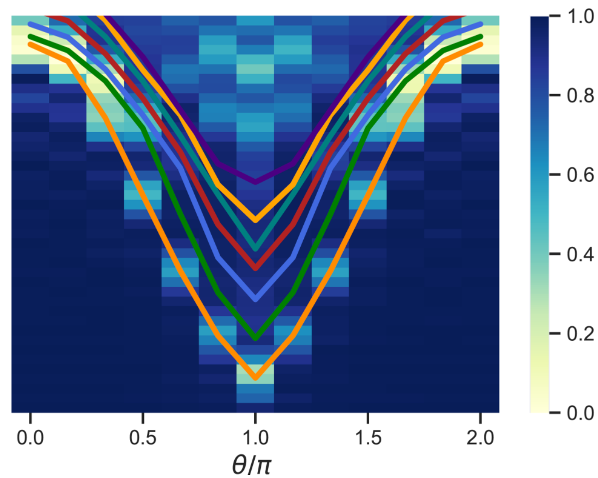


Thanks!

Appendix

"Regularization" of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

$\longrightarrow$ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)

  — Putting on spatial lattice, Hilbert sp. is finite dimensional

- scalar

  — Hilbert sp. at each site is ∞ dimensional

  (need truncation or additional regularization)

- gauge field (w/ kinetic term)

  — no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)

  — ∞ dimensional Hilbert sp. in higher dimensions

Accessible region by analytic computation

- Massive limit:

The fermion can be integrated out

&

the theory becomes effectively pure Maxwell theory w/ θ

- Bosonization:

[Coleman ’76]

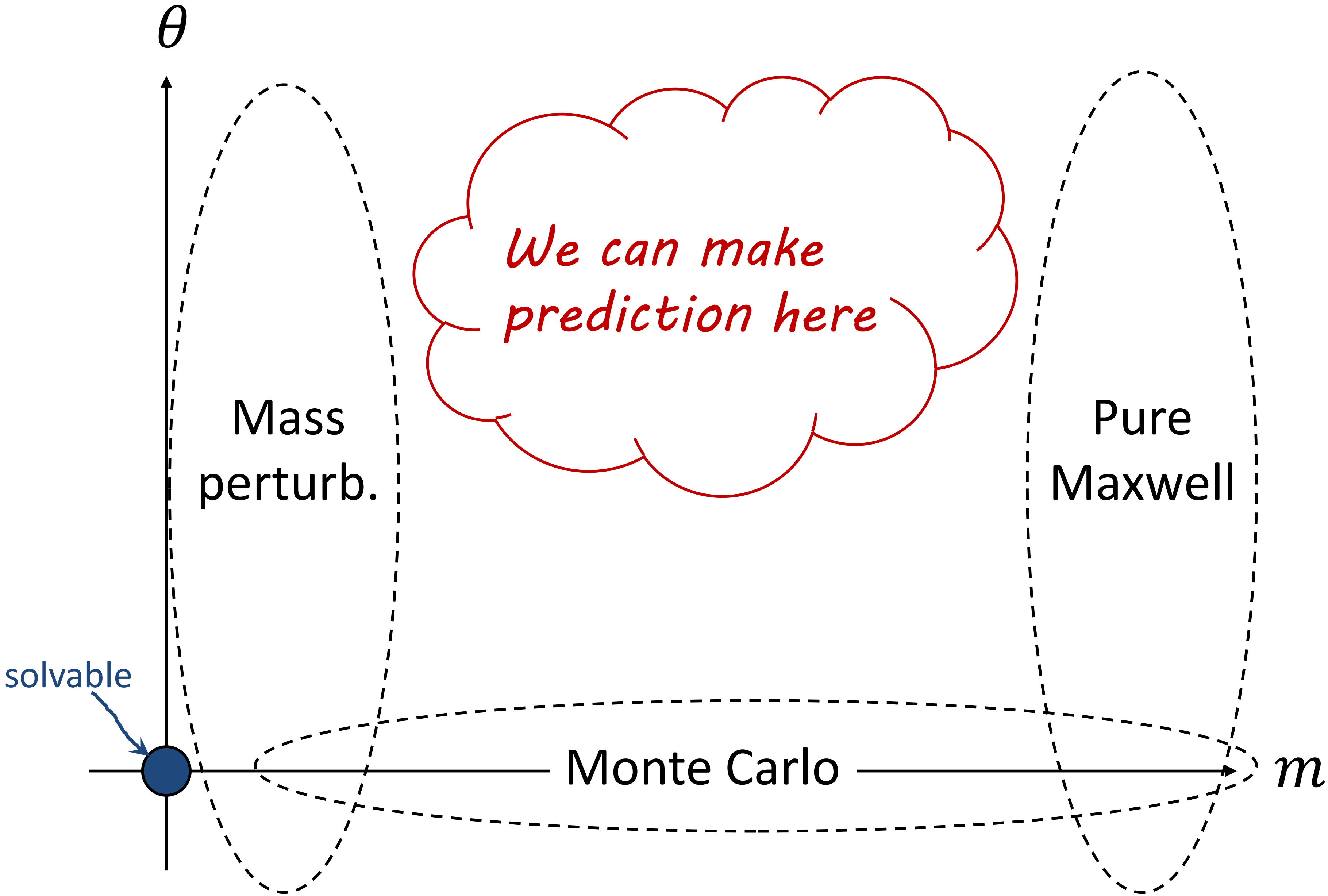
$$\mathcal{L} = \frac{1}{8\pi}\left(\partial_\mu \phi\right)^2 - \frac{g^2}{8\pi^2}\phi^2 + \frac{e^\gamma g}{2\pi^{3/2}} m \cos(\phi + \theta)$$

exactly solvable for $m = 0$

&

small m regime is approximated by perturbation

Map of accessibility/difficulty



Choice of boundary conditions

Gauss law: $L_n - L_{n-1} = q\left[\chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}\right]$

Open b.c.

- $L_n =$ (fermion op.)

$\longrightarrow \quad \dim(\mathcal{H}_{\text{phys}}) < \infty$

- θ-periodicity is lost
- momentum not conserved

Periodic b.c.

- one of L_n's remains

$\longrightarrow \quad \dim(\mathcal{H}_{\text{phys}}) = \infty$

additional truncation needed

- $^\exists\theta$-periodicity
- momentum conserved

Even N or odd N?

$\chi_0 \quad \chi_1 \quad \chi_2 \quad \chi_3 \quad \cdots \quad \chi_{N-2} \quad \chi_{N-1}$

Staggered fermion: $\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$ $\psi_u \longrightarrow$ odd site, $\psi_d \longrightarrow$ even site

- Usually even N is taken (p.b.c. allows only even N)
- Open b.c. allows both but parity is different: $\chi_n \to i(-1)^n \chi_{N-n-1}$

	n mod 2	$\bar{\psi}\psi \sim \sum_n (-1)^n \chi_n^\dagger \chi_n$	$\bar{\psi}\gamma^5\psi \sim \sum_n (-1)^n (\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.})$
even N	changes	flipped	invariant
odd N	invariant	invariant	flipped

Odd N seems more like the continuum theory?

Constructing ground state

$^{\exists}$various quantum algorithms to construct vacuum:

- adiabatic state preparation
- algorithms based on variational method
- imaginary time evolution

etc...

Here, let's apply

adiabatic state preparation

Adiabatic state preparation

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2: Introduce adiabatic Hamiltonian $H_A(t)$ s.t.

- $H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}}$
- $\left|\frac{dH_A}{dt}\right| \ll 1$ for $T \gg 1$

Step 3: Use the adiabatic theorem

If $H_A(t)$ has a unique ground state w/ a finite gap for $\forall t$, then the ground state of H_{target} is obtained by

$$|\mathrm{vac}\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\, H_A(t)\right)|\mathrm{vac}_0\rangle$$

Comment on adiabatic state preparation

$$(\text{"systematic error"}) \sim \frac{1}{T\,(\text{gap})^2}$$

😄 Advantage:

- guaranteed to be correct for $T \gg 1$ & $\delta t \ll 1$
  if $H_A(t)$ has a unique gapped vacuum
- can directly get excited states under some conditions

☹️ Disadvantage:

- doesn’t work for degenerate vacua
- costly — likely requires many gates

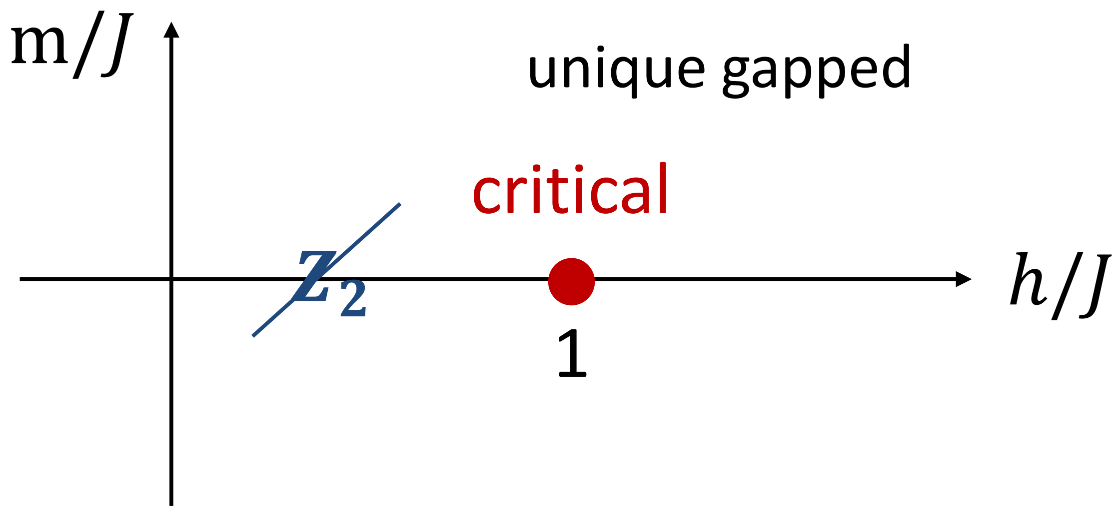
➡ more appropriate for FTQC than NISQ

Coherent imaging spectroscopy in Ising model

[working in progress, MH-Ghim]

$$\hat{H}_{\text{Ising}} = -J\sum_{n=1}^{N-1} Z_n Z_{n+1} - h\sum_{n=1}^{N} X_n - m\sum_{n=1}^{N} Z_n$$

Known phase diagram:



Let's consider time evolution by

$$\hat{H}_{\text{Ising}} + B\sin(\nu t)\sum_{n=1}^{N} Y_n$$

Coherent imaging spectroscopy in Ising model (cont'd)

[working in progress, MH-Ghim]

$N = 8, m/J = 0.1$ ($|0\rangle$ by adiabatic state preparation)