# Implication of Quarkyonic duality to the hyperon puzzle

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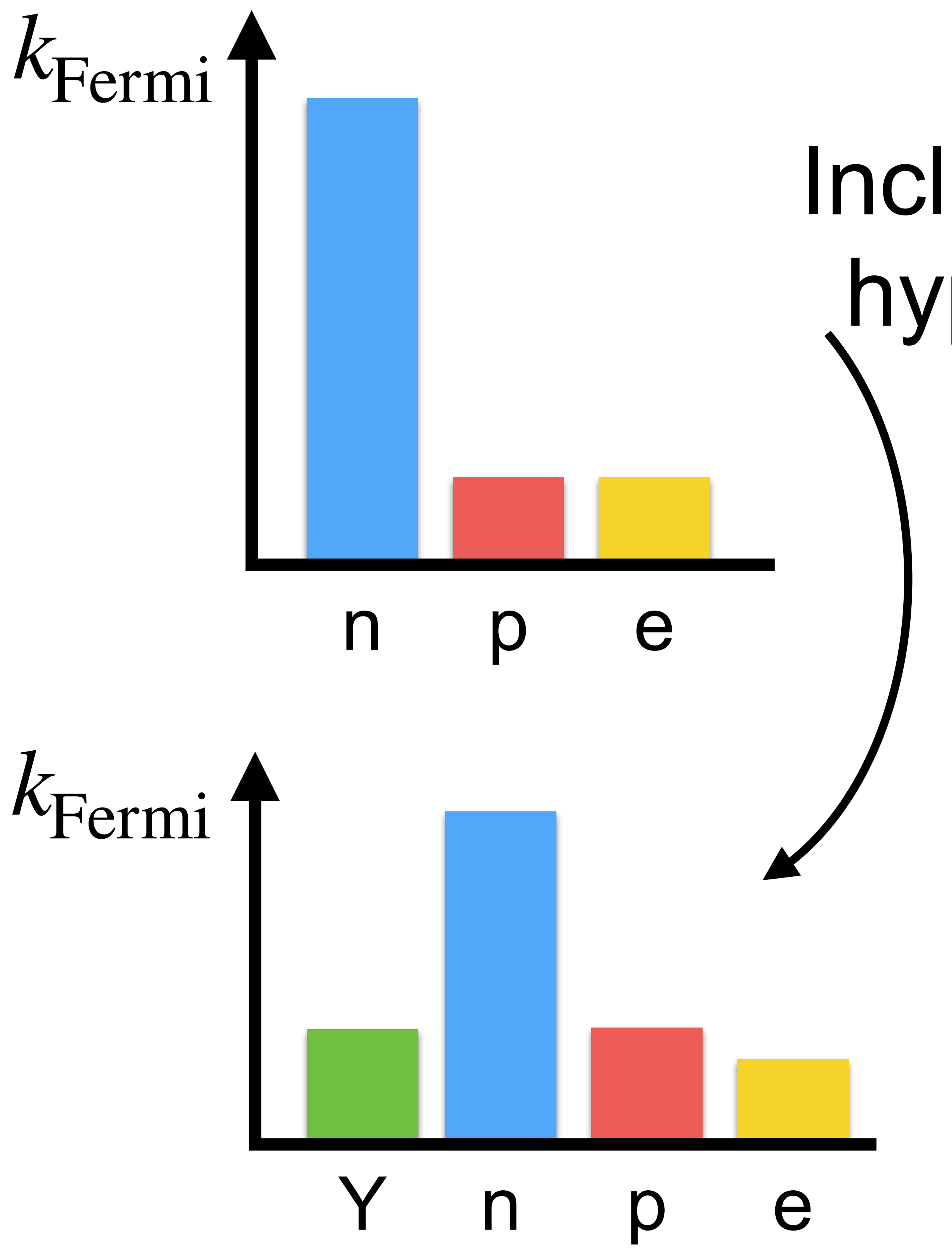
References:
[1] Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024) [2306.04304]
[2] Y. Fujimoto, T. Kojo, L. McLerran, in preparation

August 21, 2024 - Quark Confinement and the Hadron Spectrum XVI, Cairns

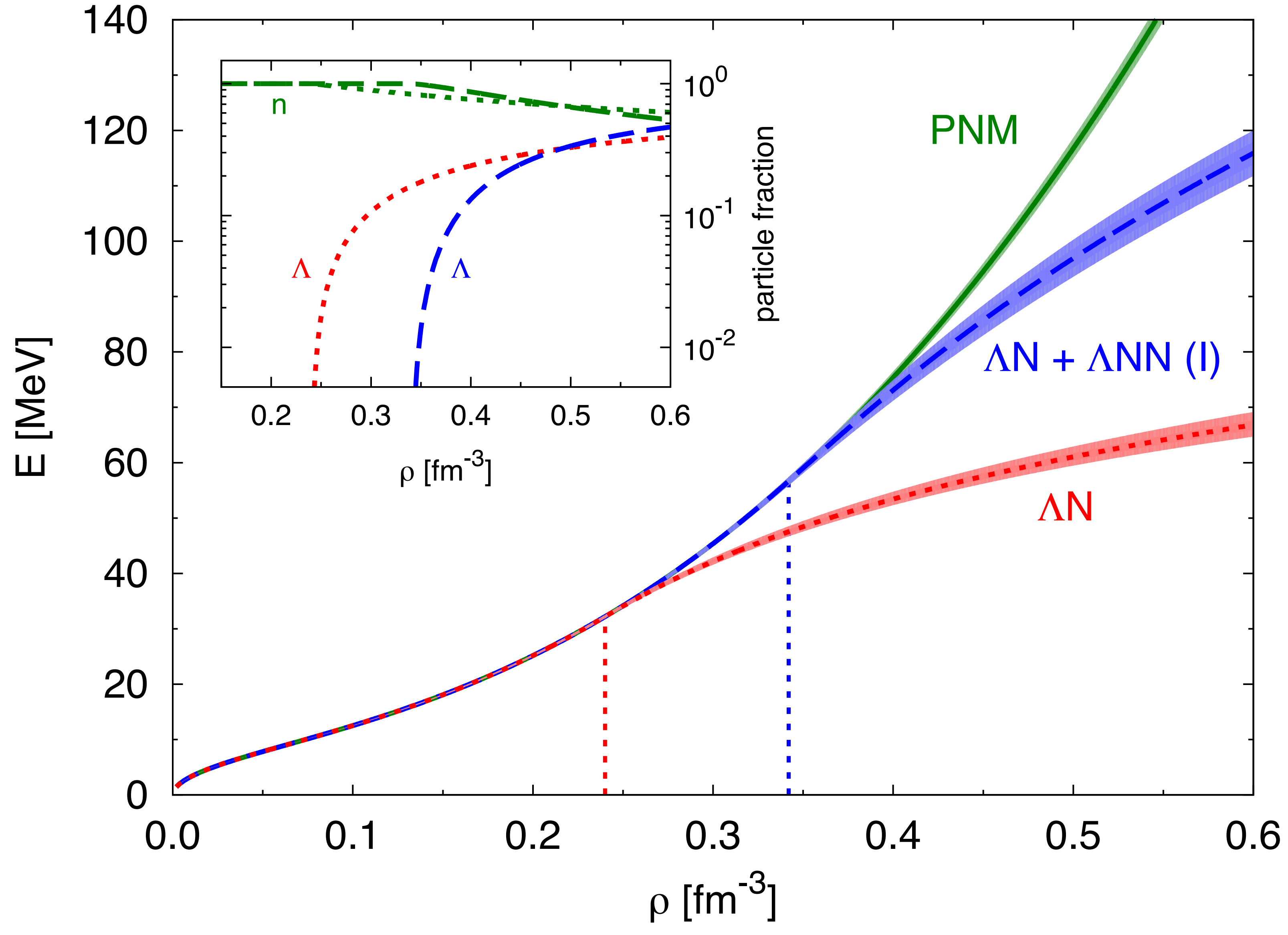
# Strangeness in neutron stars

## Hyperon puzzle




Hyperons (Y) lower the energy density at a given baryon density

Hyperons soften the EoS drastically …




Cannot support heavy neutron stars

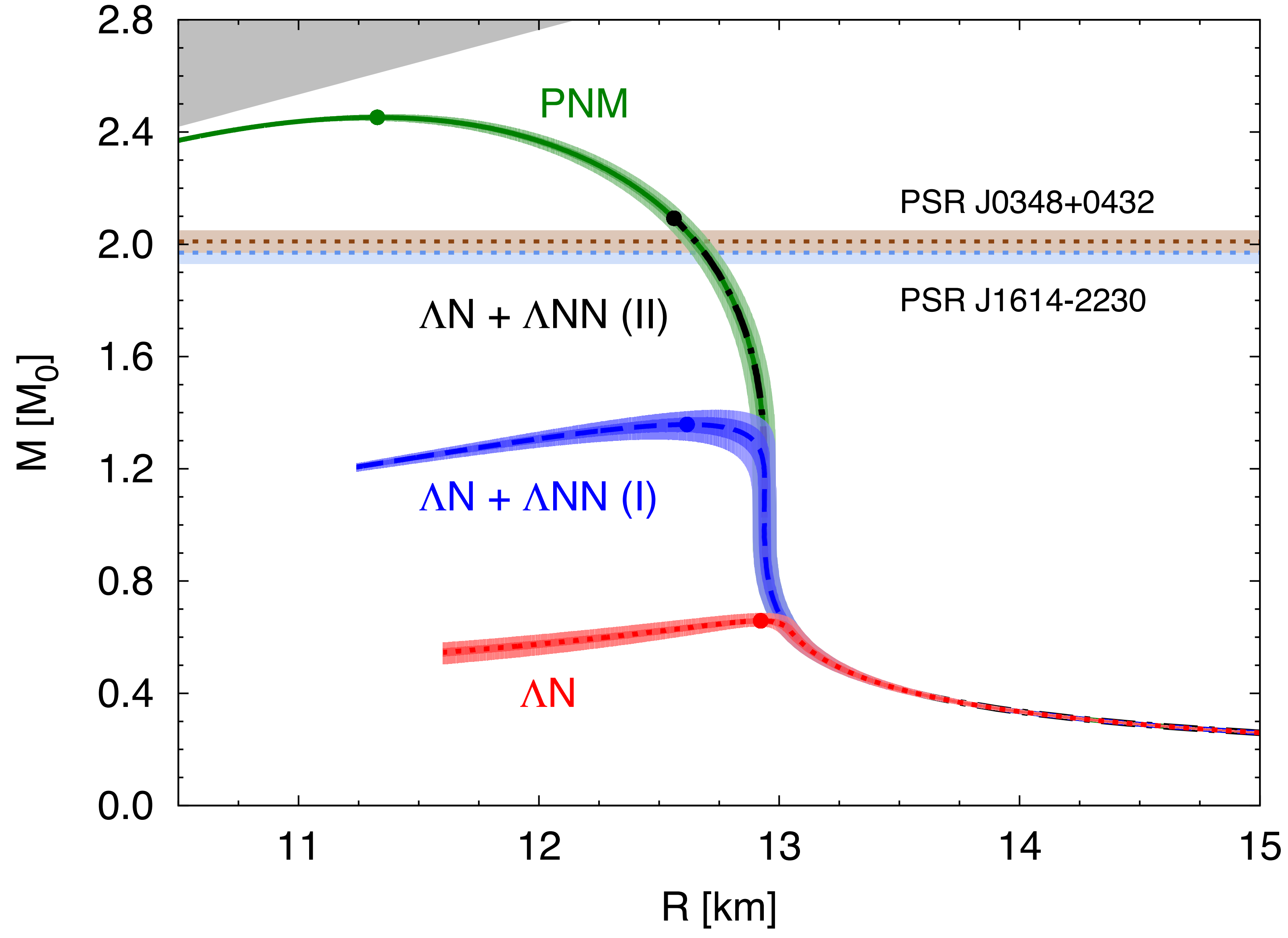



figure from Lonardoni et al. (2014)

In ordinary EoS,
hyperons appear at $n_B \simeq 2 - 3 n_{\text{sat}}$

# Brief summary of the result:

**In Quarkyonic picture…**

- The threshold for $S=-1$ hyperons are shifted to much higher densities:

$$\mu_B = M_Y \to \mu_B = M_Y + (M_Y - M_N)$$

- New threshold is very close to the mass of $\Xi^0$ (uss):

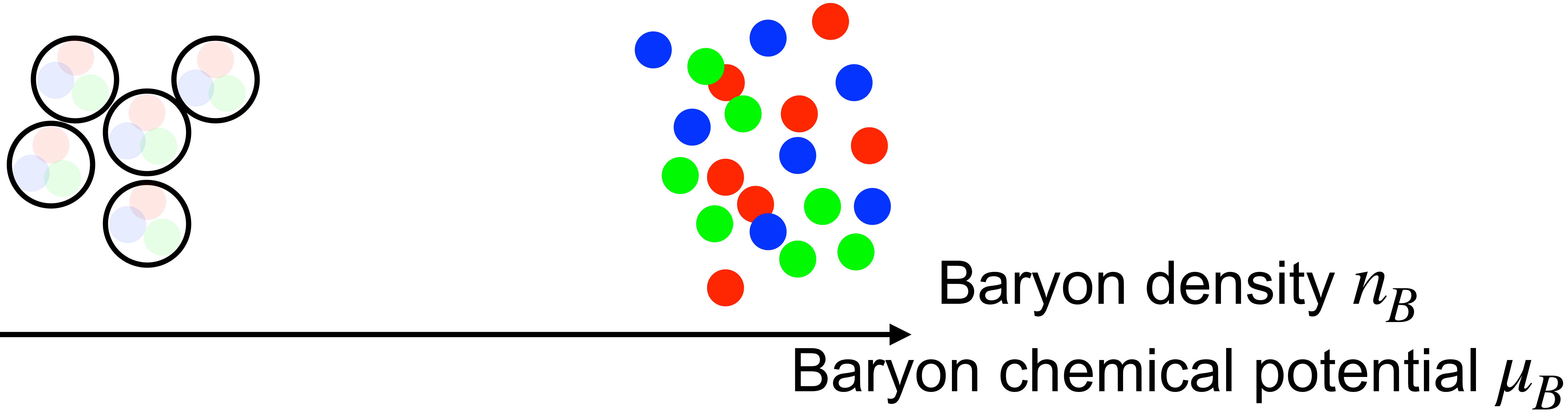
$$2M_Y - M_N \simeq M_{\Xi^0}$$

- $S=-1$ hyperons do not soften the EoS much, but $S=-2$ $\Xi^0$ hyperon does soften the EoS substantially

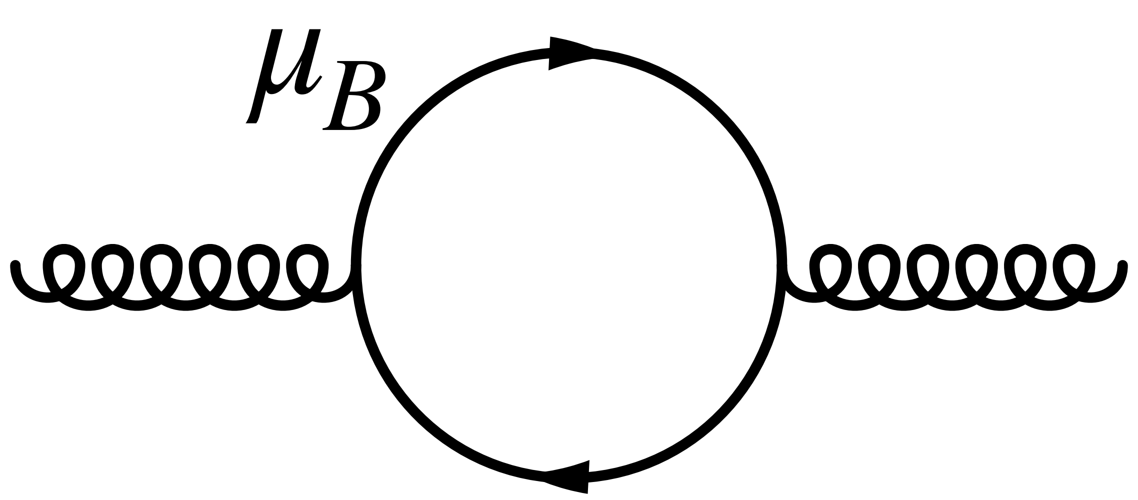
# Confinement at high baryon densities

Collins & Perry (1974): **Naive picture of deconfinement at high density**

In weak-coupling regime at high density, quarks liberate




This is led by screening of the confinement potential

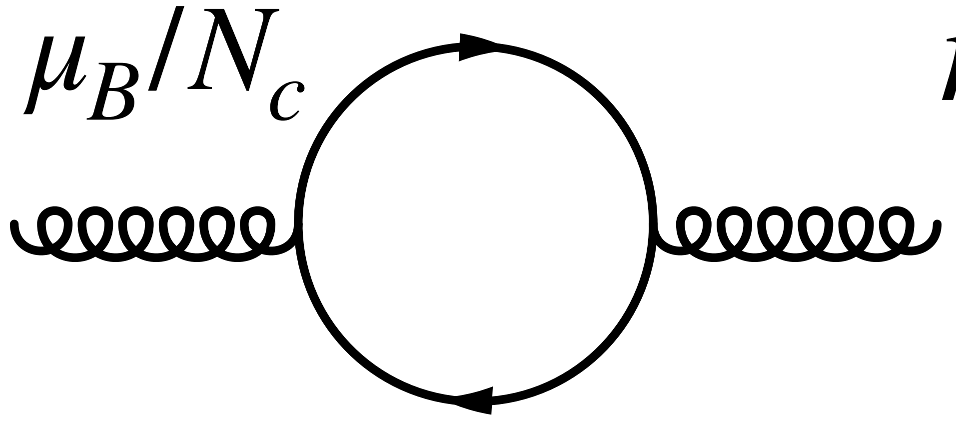


# Confinement at high baryon densities

McLerran & Pisarski (2007): **Quarkyonic duality**

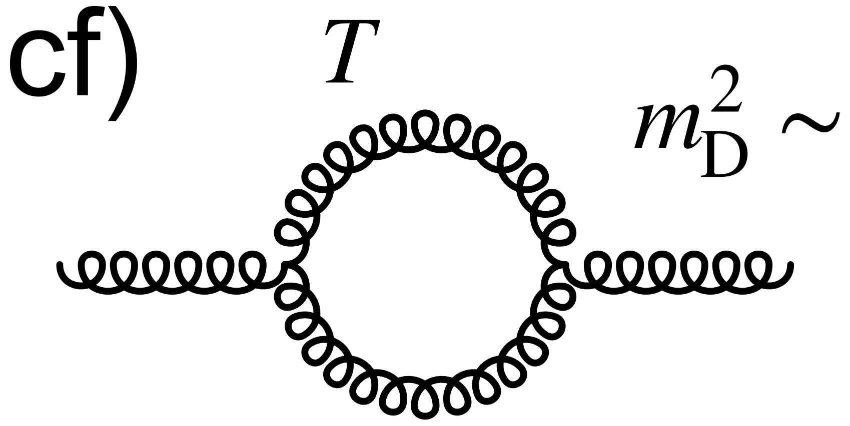
In Large-$N_c$ QCD…

$\mu = \mu_B / N_c$



$$m_\mathrm{D}^2 \sim g^2 \mu^2 \sim \lambda_{'t\ Hooft} \mu^2 / N_c \to 0$$

cf)




$m_\mathrm{D}^2 \sim g^2 N_c T^2$

$\sim \lambda_{'t\ Hooft} T^2$

… confinement is never affected by quarks!

Dense QCD matter can be described either as

- Confined baryons (because confining interaction is never screened)
- (Weakly-coupled) Quarks

→ **implies duality (paradox?) between quark and confined baryonic matter**

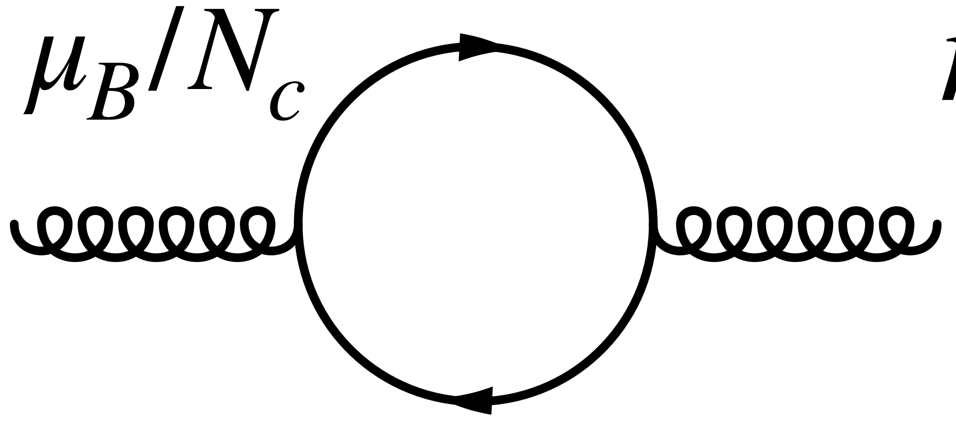
**Quark yonic**

# Confinement at high baryon densities

McLerran & Pisarski (2007): **Quarkyonic duality**

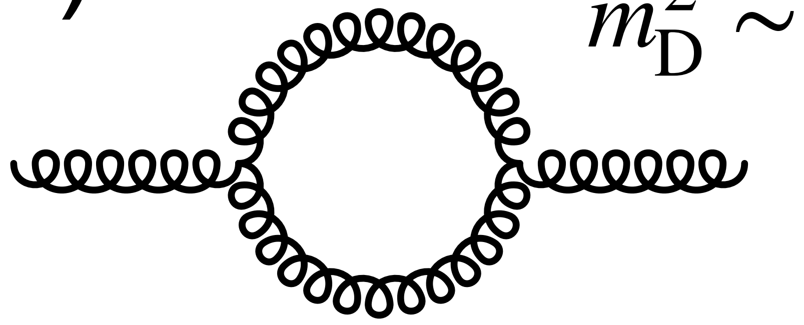
In Large-$N_c$ QCD…

$\mu = \mu_B / N_c$



$$m_D^2 \sim g^2 \mu^2 \sim \lambda_{'t\ Hooft} \mu^2 / N_c \to 0$$

cf) $T$



$m_D^2 \sim g^2 N_c T^2$

$\sim \lambda_{'t\ Hooft} T^2$

… confinement is never affected by quarks!

Dense QCD matter can be described either as $\quad m_D^2 \ll \Lambda_{QCD}^2 \to \mu \ll \sqrt{N_c} \Lambda_{QCD}$

- Confined baryons (because confining interaction is never screened)
- (Weakly-coupled) Quarks $\quad \mu \gg \Lambda_{QCD}$

**→ implies duality (paradox?) between quark and confined baryonic matter**

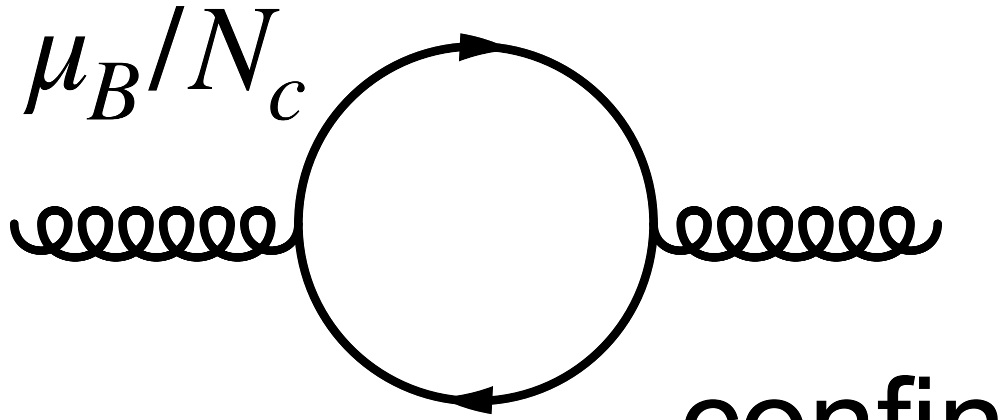
**Quark yonic**

# Confinement at high baryon densities

McLerran & Pisarski (2007): **Quarkyonic duality**

In $N_c = 3$ QCD…

$\mu = \mu_B / N_c$



$$m_{\rm D}^2 \sim g^2 \mu^2 \sim \lambda_{'t\ Hooft} \mu^2 / N_c$$

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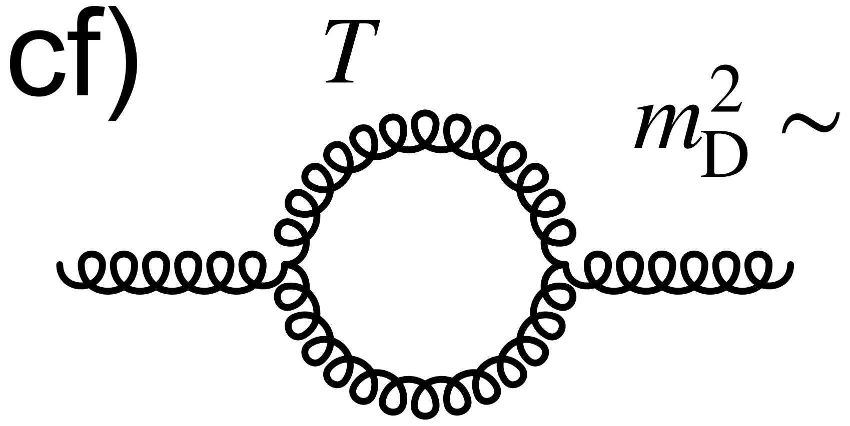



$m_{\rm D}^2 \sim g^2 N_c T^2$

$\sim \lambda_{'t\ Hooft} T^2$

… confinement is **less** affected by quarks!
inefficient deconfinement compared to finite-T case

Speculation:

Duality persists in finite-Nc, i.e.,

Dense QCD matter can be described either as baryons and quarks at

$$\Lambda_{\rm QCD} < \mu < \sqrt{N_c}\Lambda_{\rm QCD}$$ … **Quarkyonic** regime

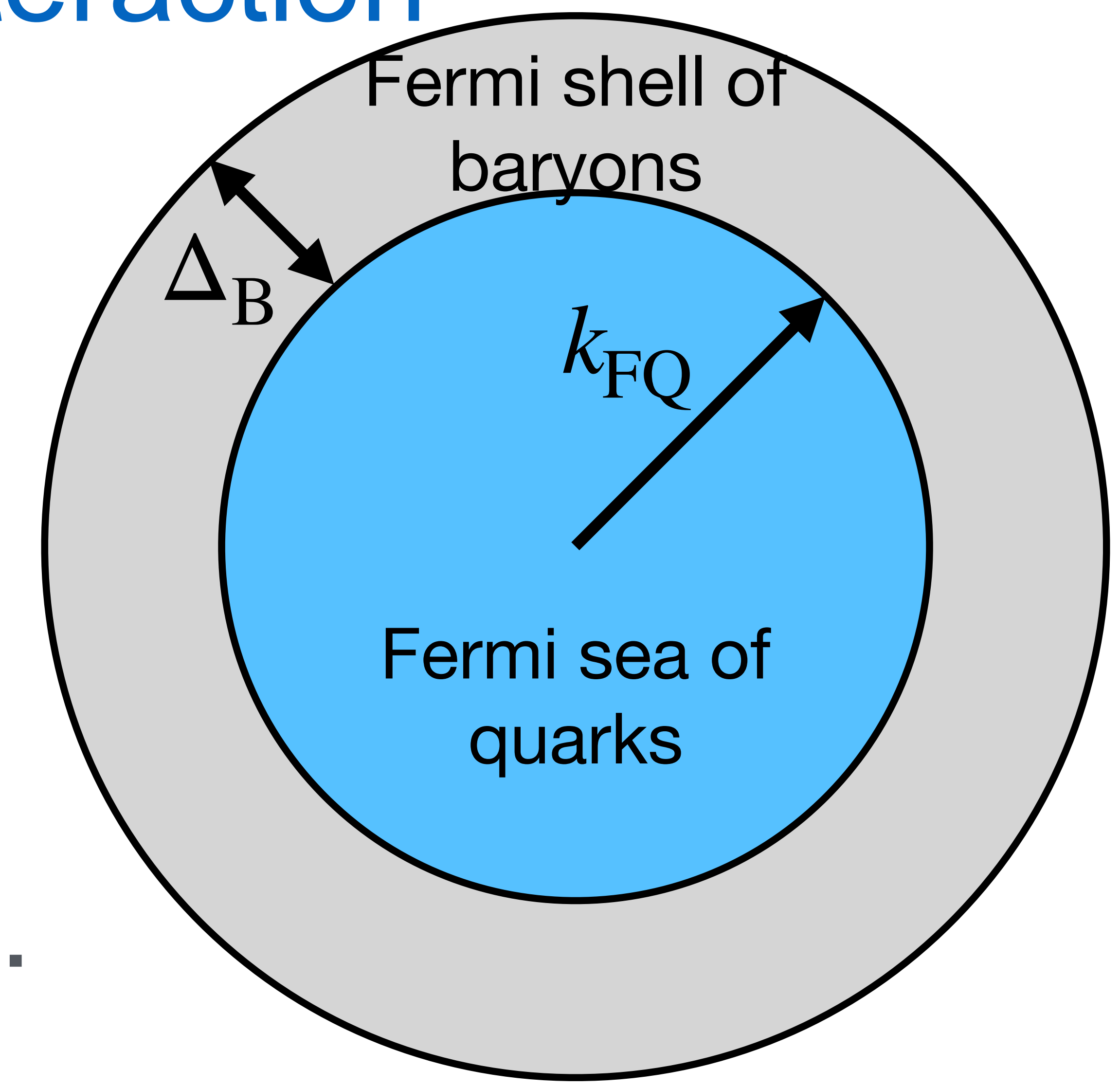
# Quarkyonic “shell” model

McLerran & Pisarski (2007):

To resolve the duality “paradox”, the following picture of Fermi shell of baryons is proposed:

**Fermi sea**: dominated by interaction that is less sensitive to IR
→ quarks

**Fermi shell**: interaction sensitive to IR d.o.f.
→ baryons, mesons, glues…



# Quarkyonic model for neutron stars

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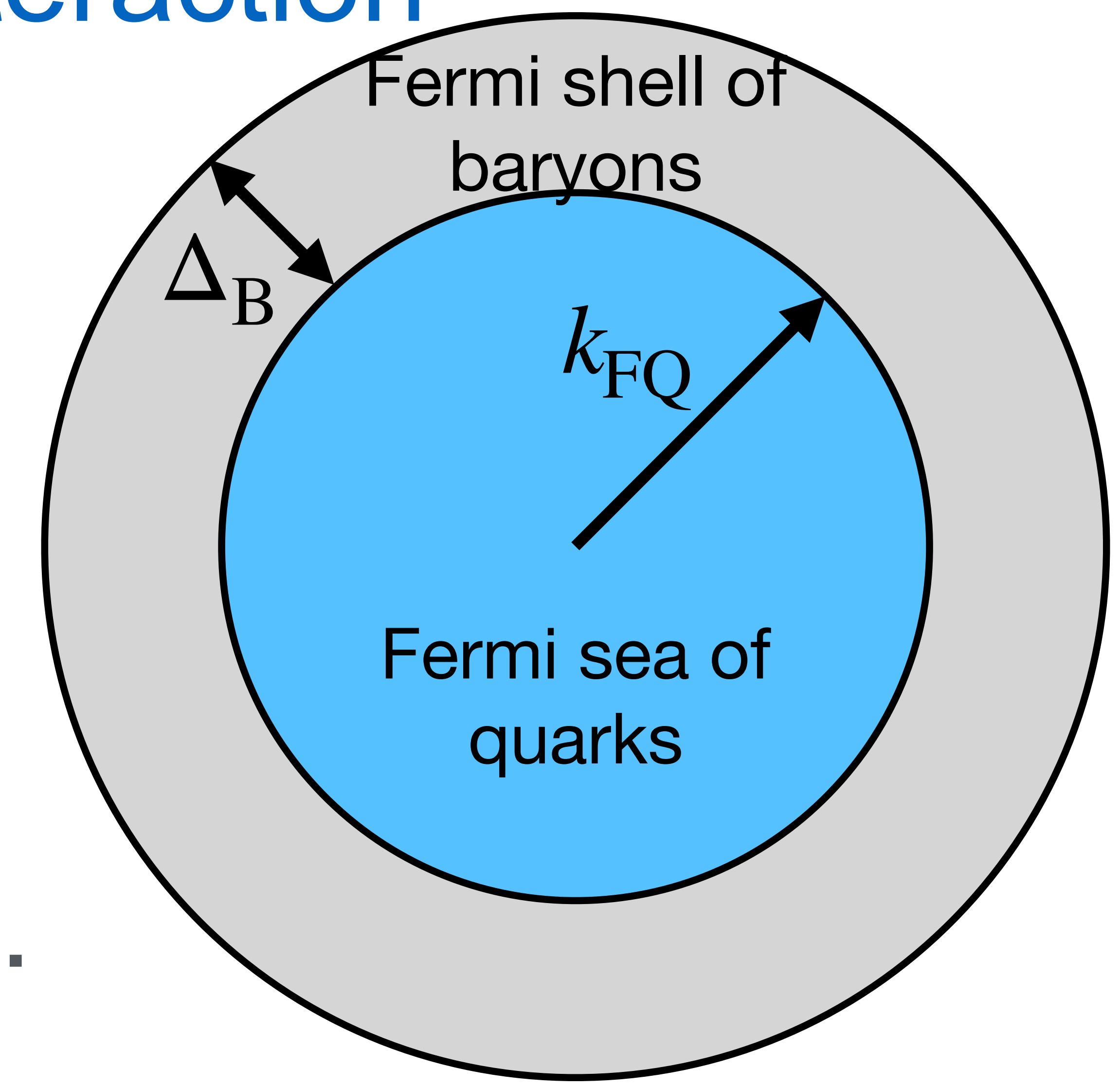
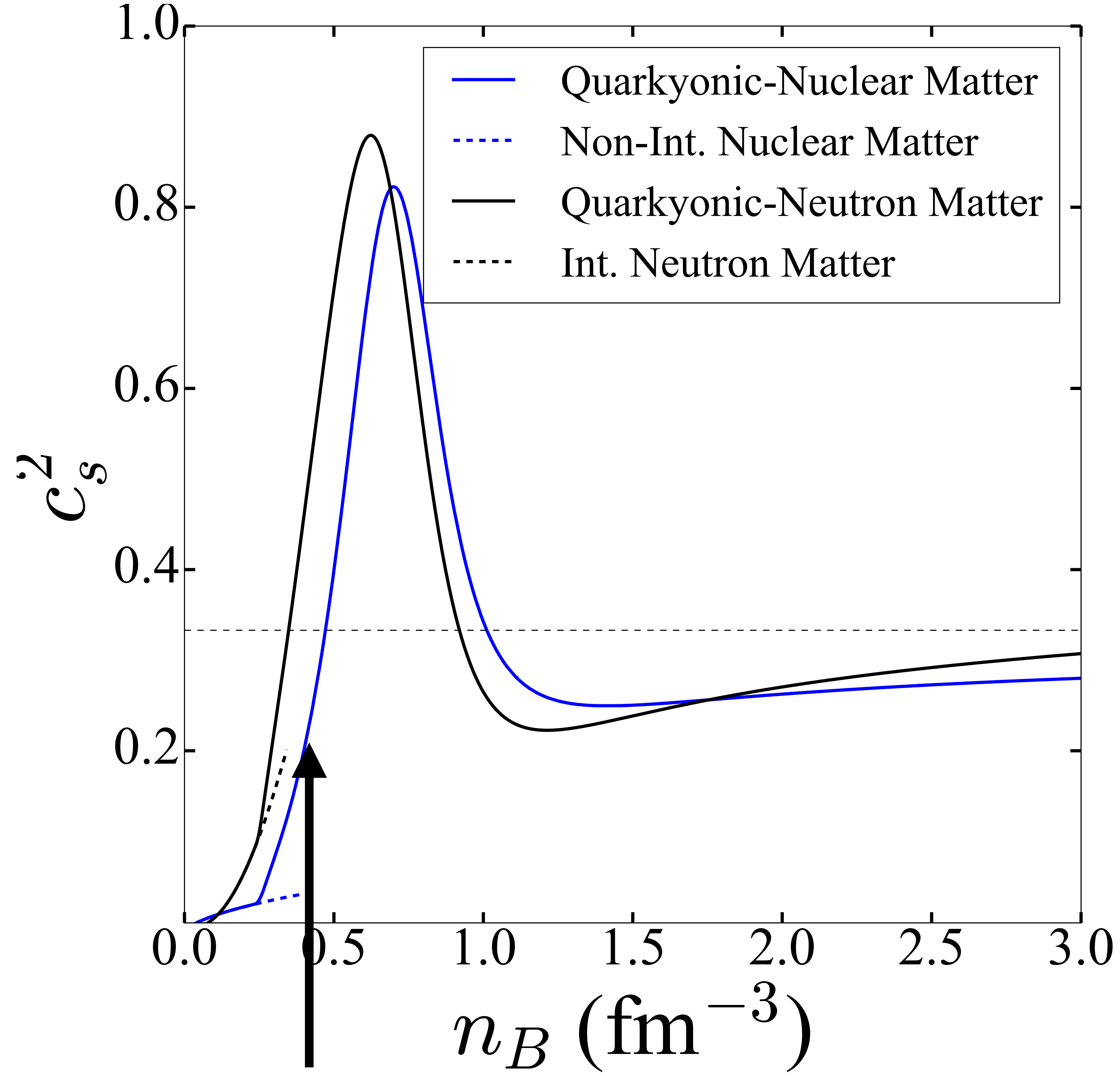



McLerran,Reddy (2018):

Quarkyonic model applied to NS EoS:




can reproduce rapid stiffening in EoS
(the only robust feature confirmed in NS EoS)

# Quarkyonic model for neutron stars

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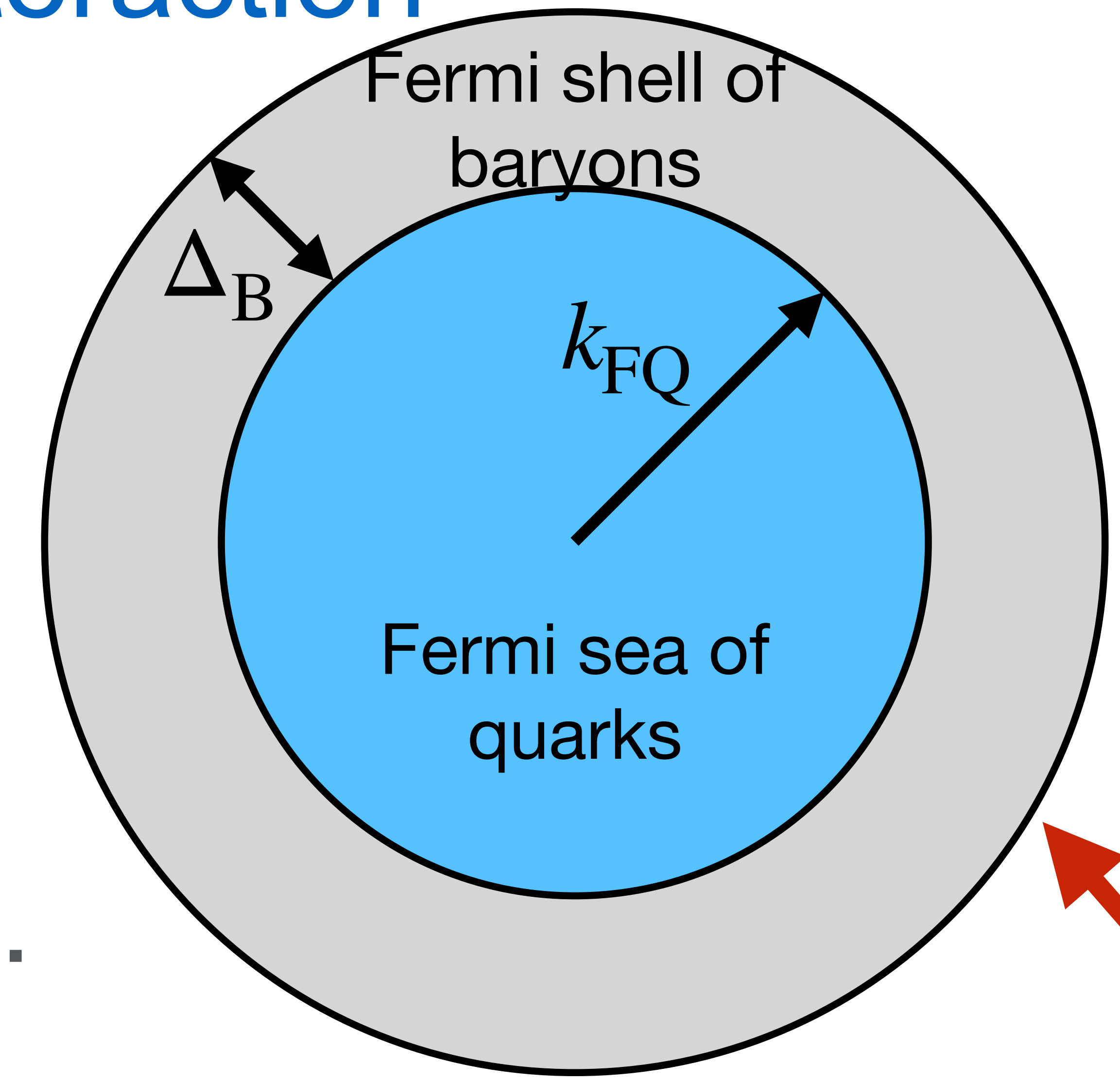


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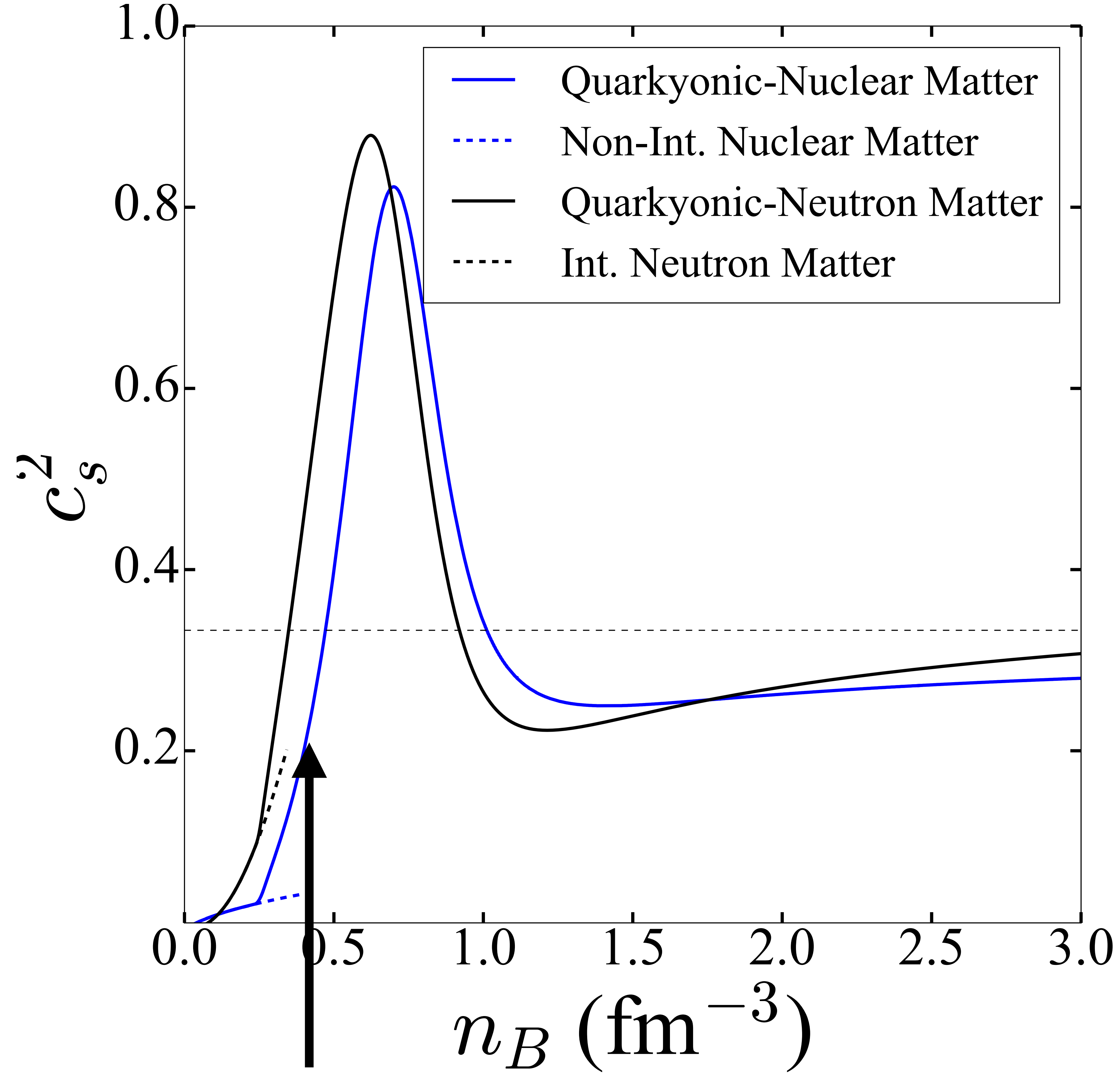



can reproduce rapid stiffening in EoS
nfirmed in NS EoS)

**This talk: reinterpretation of this baryon shell**

# Duality in Fermi gas model

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Implement duality in Fermi gas model
(= simultaneous description in terms of baryons & quarks)

**Fermi gas model w/ an explicit duality:**

$$\varepsilon = \int_{\boldsymbol{k}} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k) = \int_{\boldsymbol{k}} E_{\mathrm{Q}}(q) f_{\mathrm{Q}}(q)$$

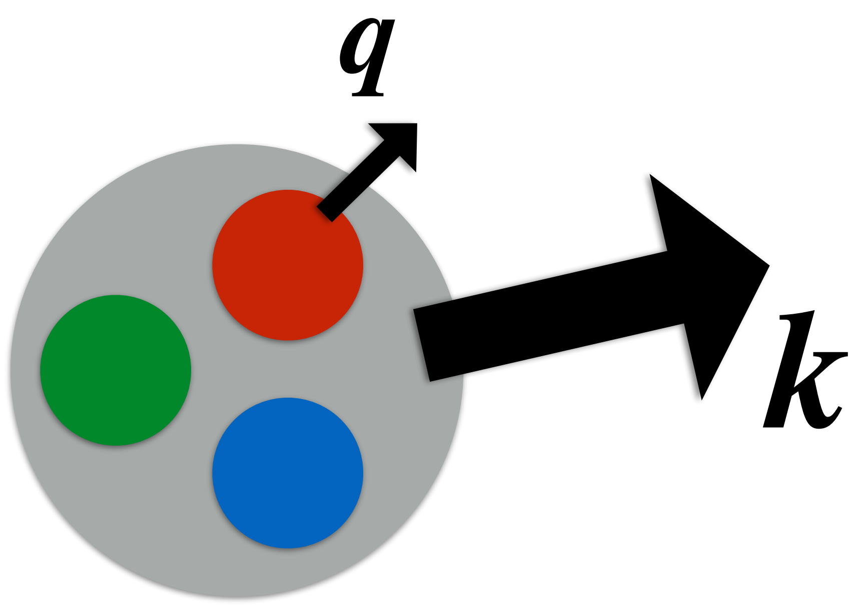
$$n_{\mathrm{B}} = \int_{\boldsymbol{k}} f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} f_{\mathrm{Q}}(q)$$

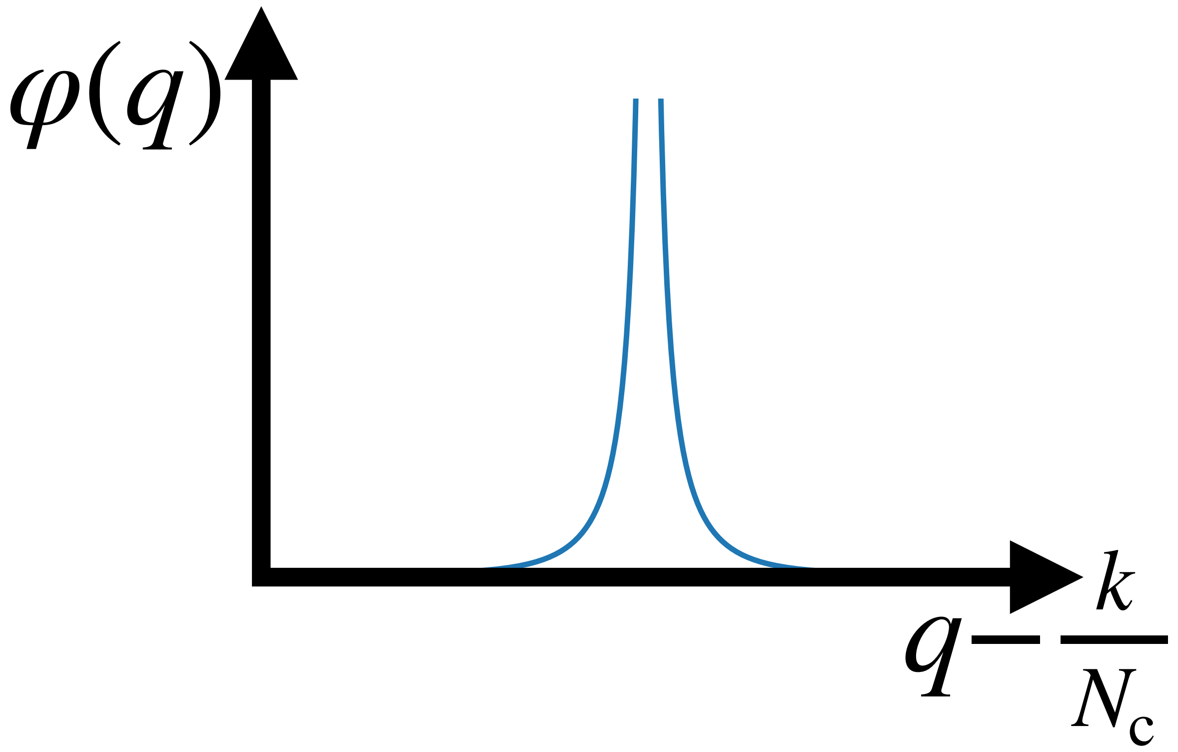
$0 \leq f_{\mathrm{B,Q}} \leq 1$ : Pauli exclusion

$E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}$ : ideal baryon dispersion relation

**Modeling of confinement:**

$$f_{\mathrm{Q}}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$



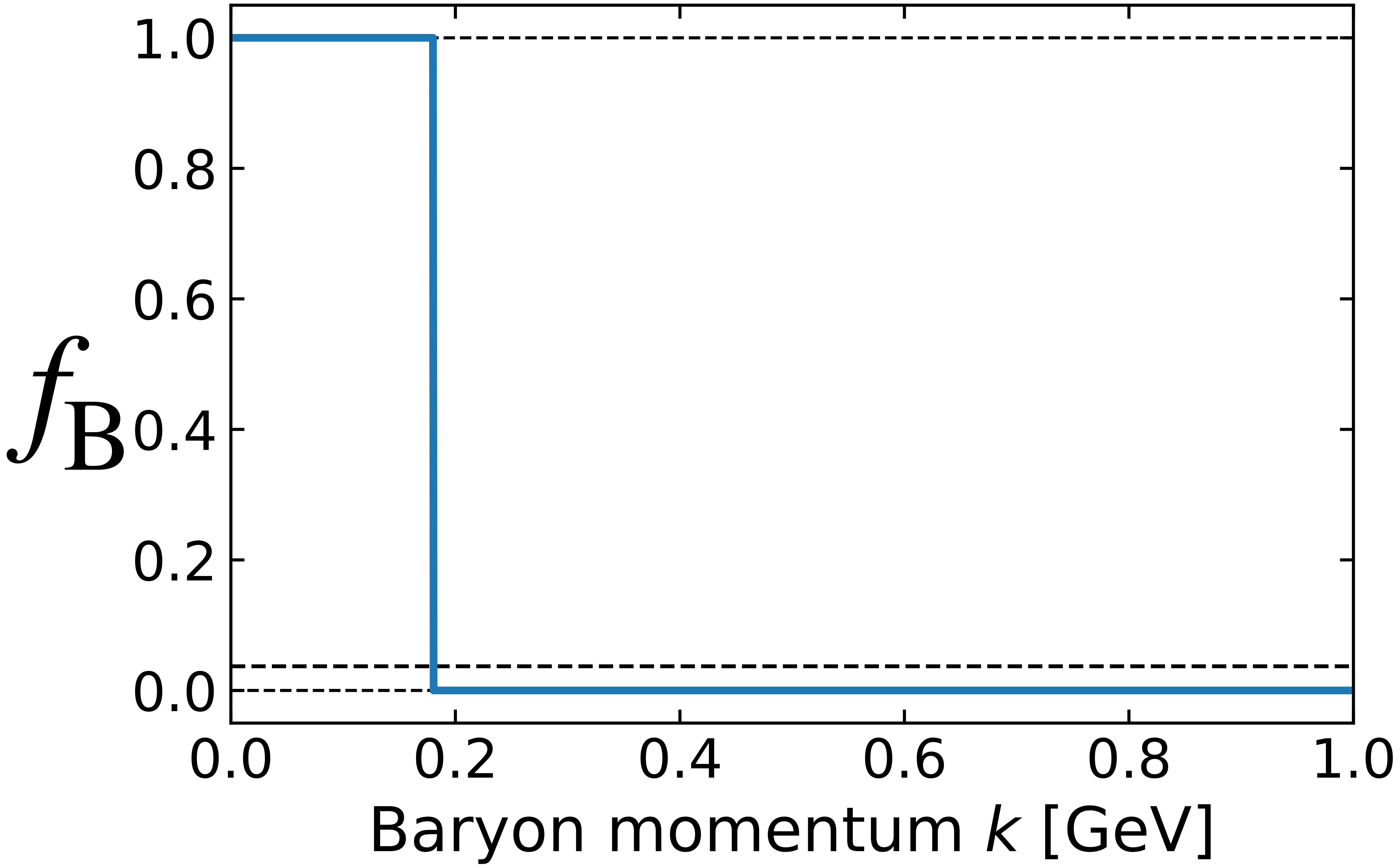

Ideal dual Quarkyonic model (**IdylliQ** model)
→ Find a solution for $f_{\mathrm{B}}$ and $f_{\mathrm{Q}}$ with minimum $\varepsilon$ at a given $n_B$

# Solution of IdylliQ model

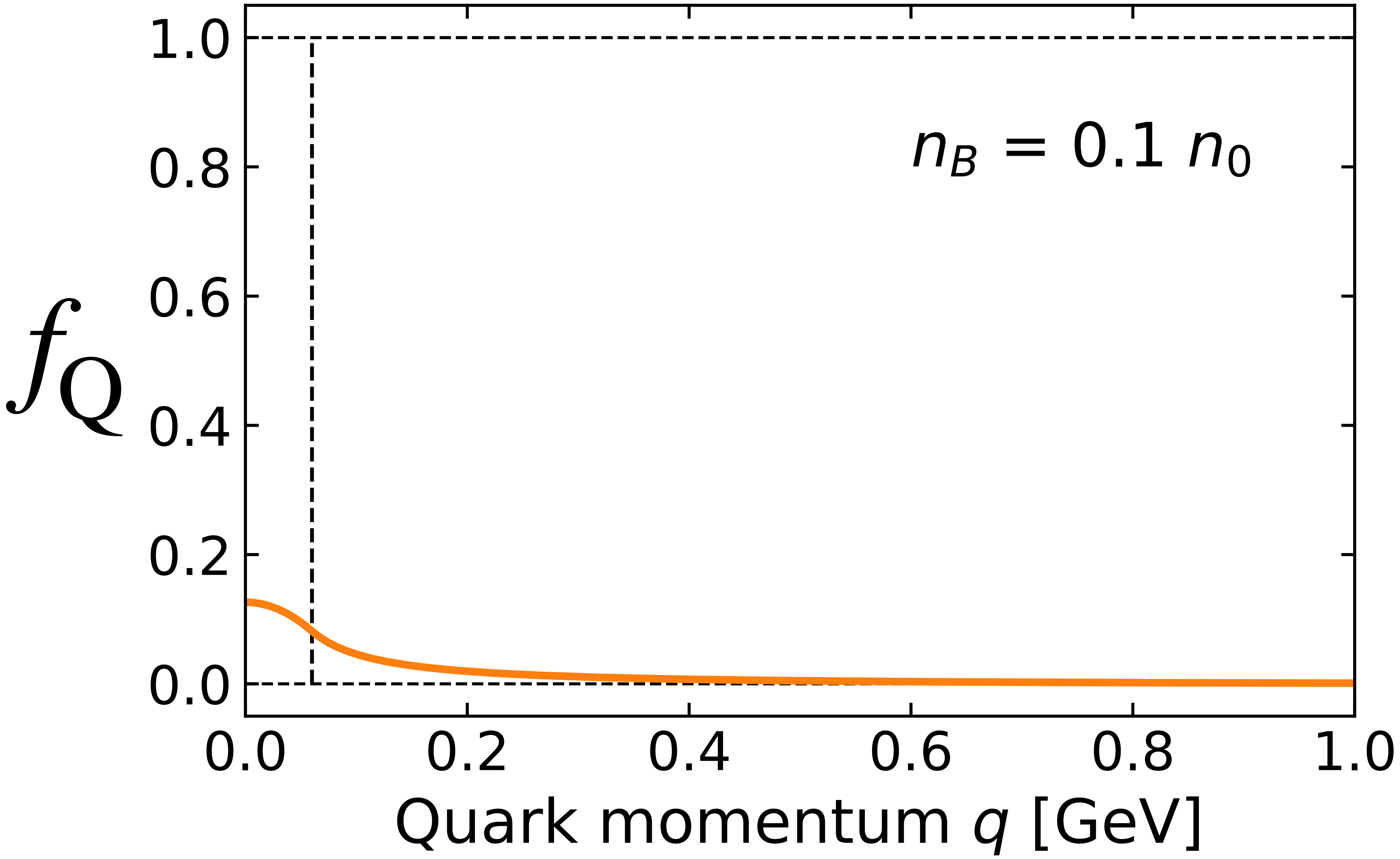
Kojo (2021); Fujimoto,Kojo,McLerran (2023)

At low density…

Fermi-Dirac distribution
for baryons




Quarks do not fill up
the Fermi sea yet

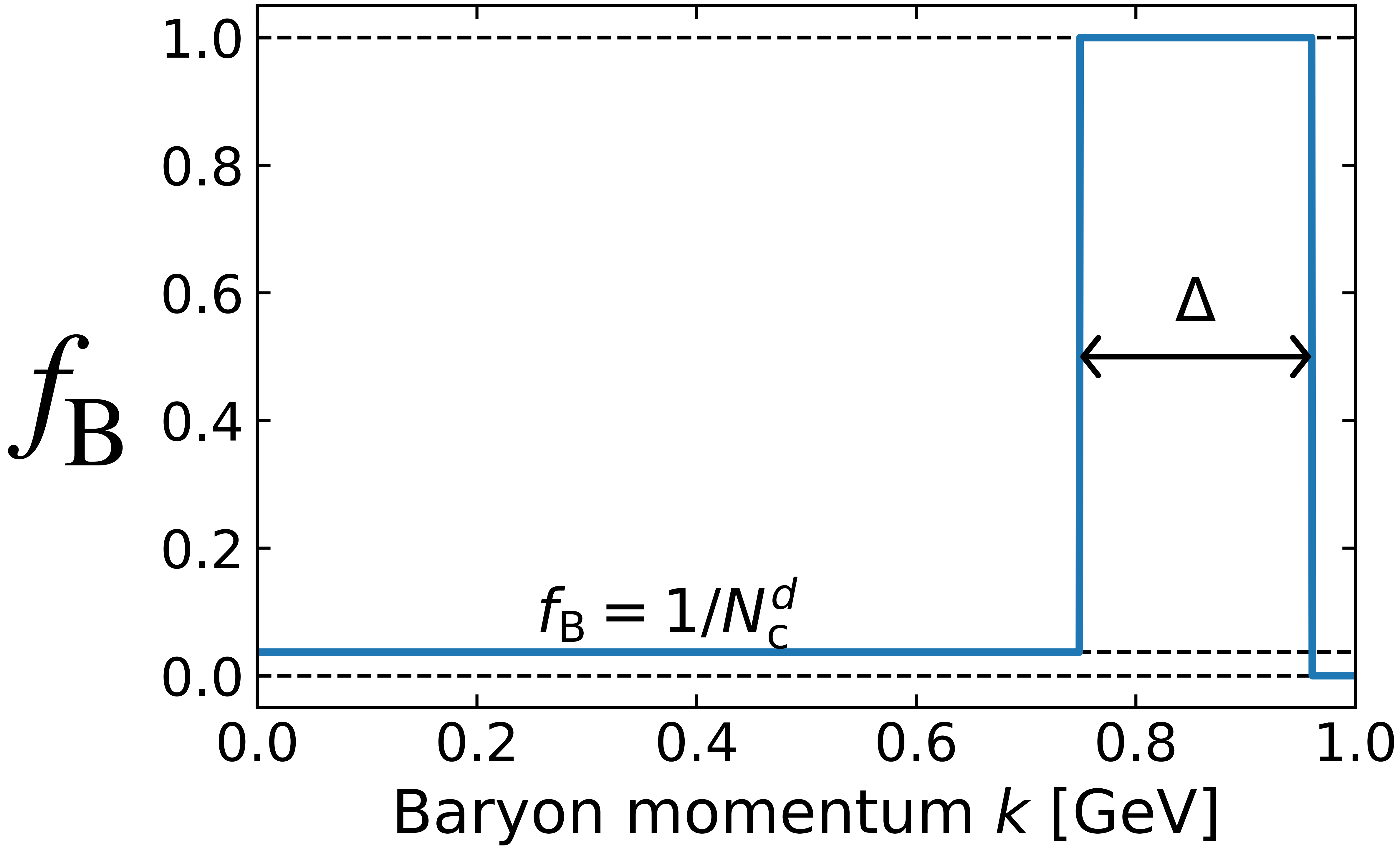


# Solution of IdylliQ model

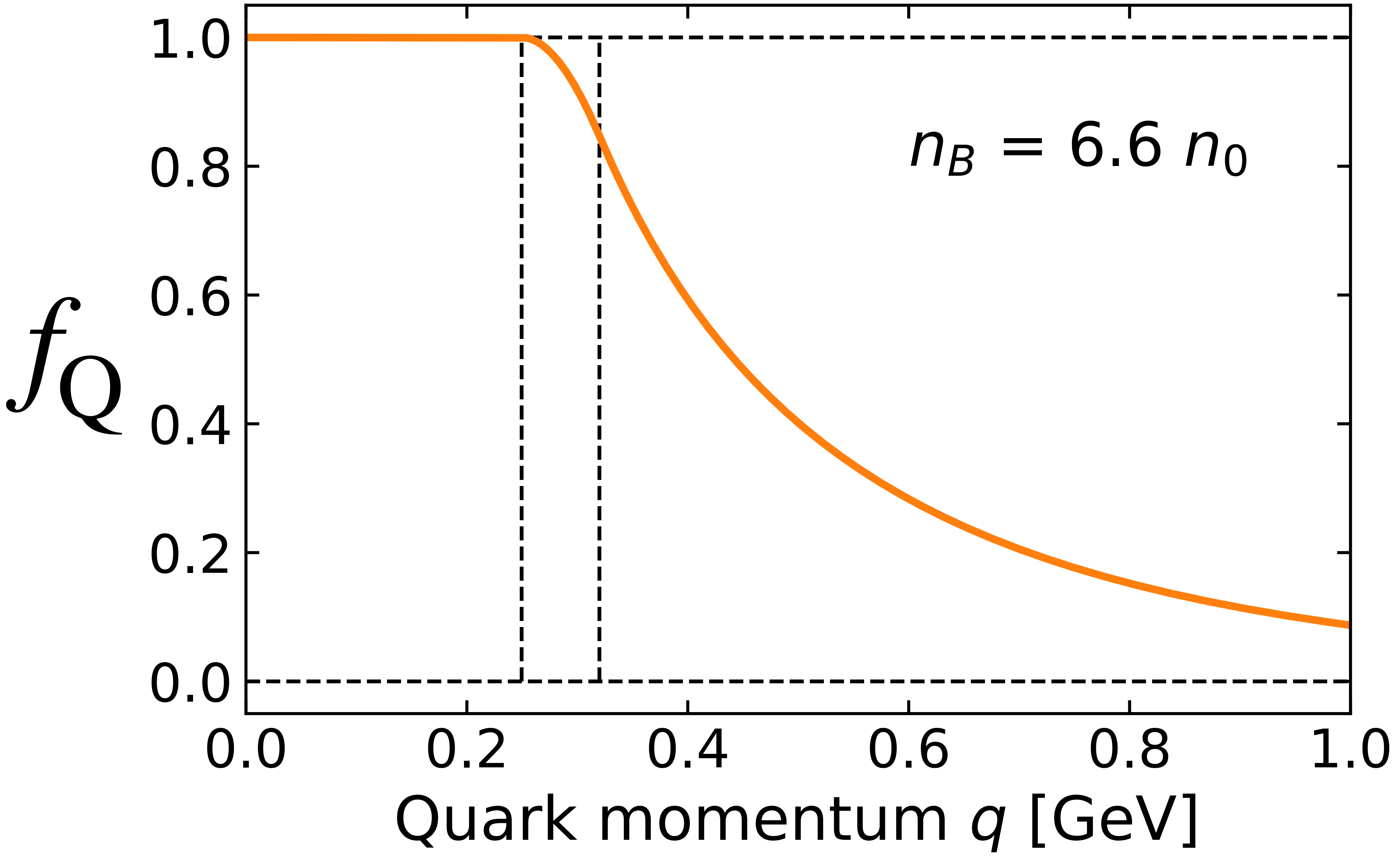
Fujimoto,Kojo,McLerran (2023)

At sufficiently high density…

**Fermi-Dirac distribution for baryons is modified**




Quark obeys the FD distribution
(with a tail from confinement)

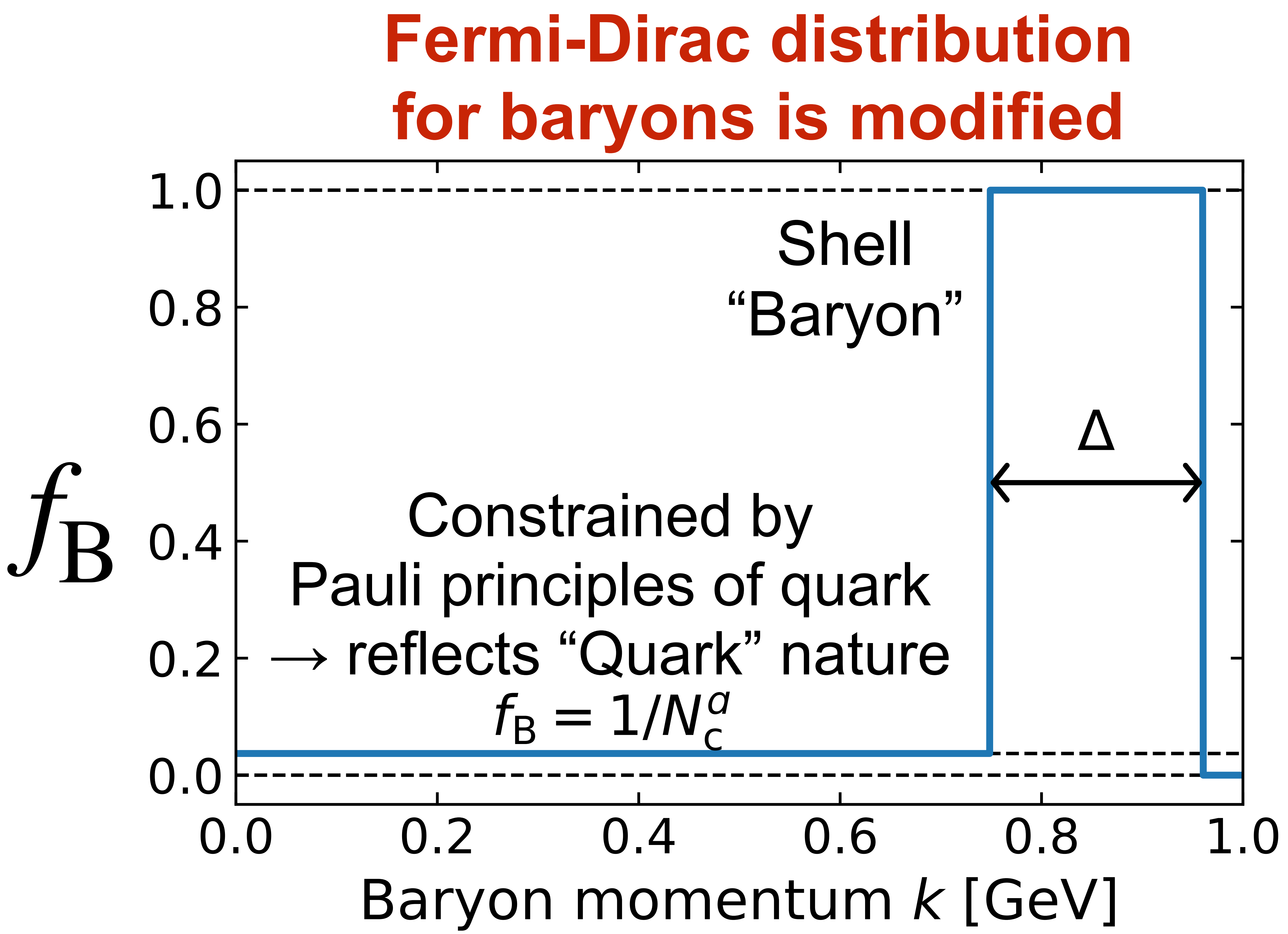


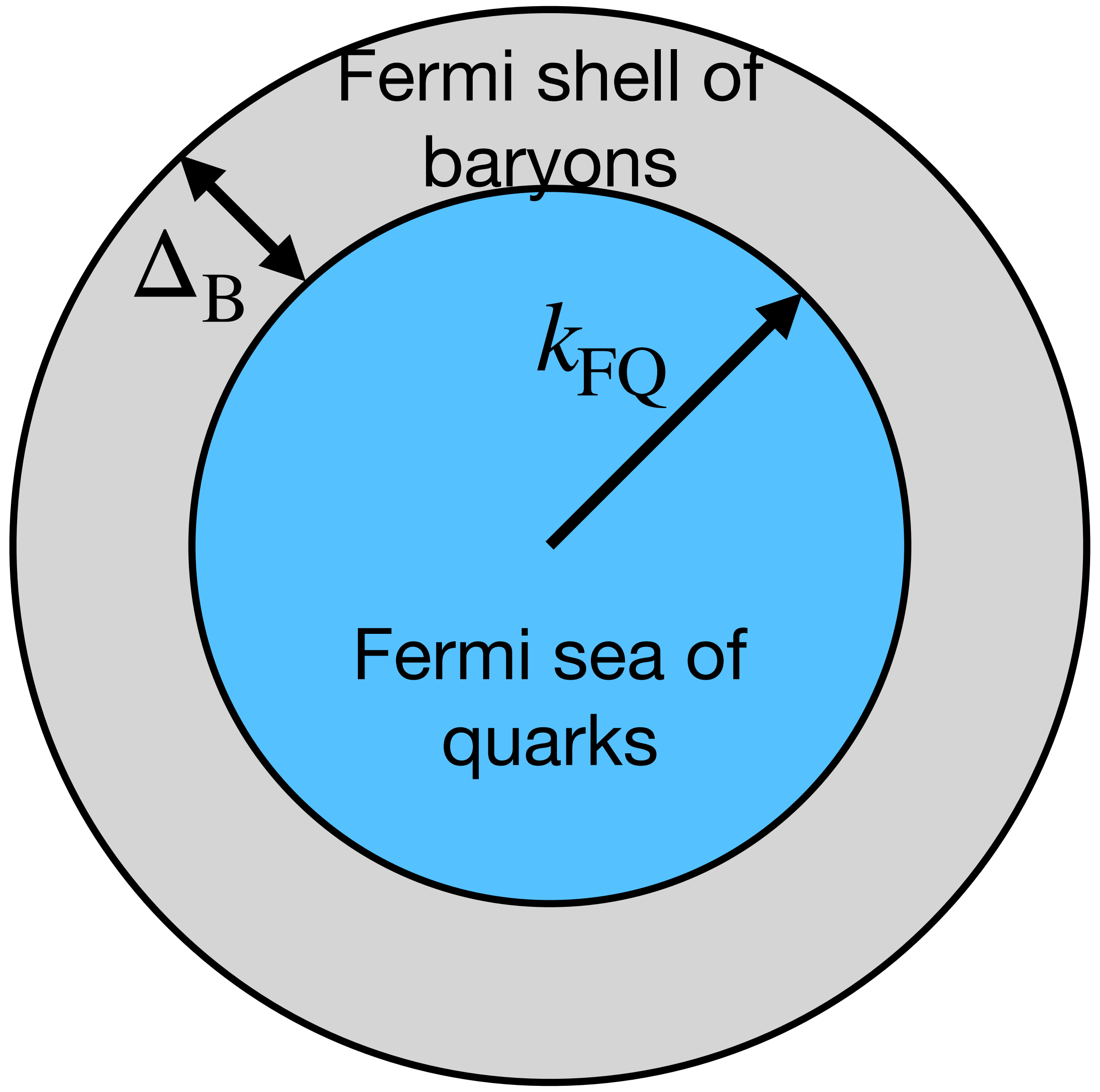

cf. Hayata,Hidaka,Nishimura (2023)

# Equivalence to Quarkyonic model

Fujimoto,Kojo,McLerran (2023)

At sufficiently high density…




McLerran,Pisarski (2007)
McLerran,Reddy (2018)

Fermi shell structure arises in $f_B$
(Note: this is **purely baryonic description**)

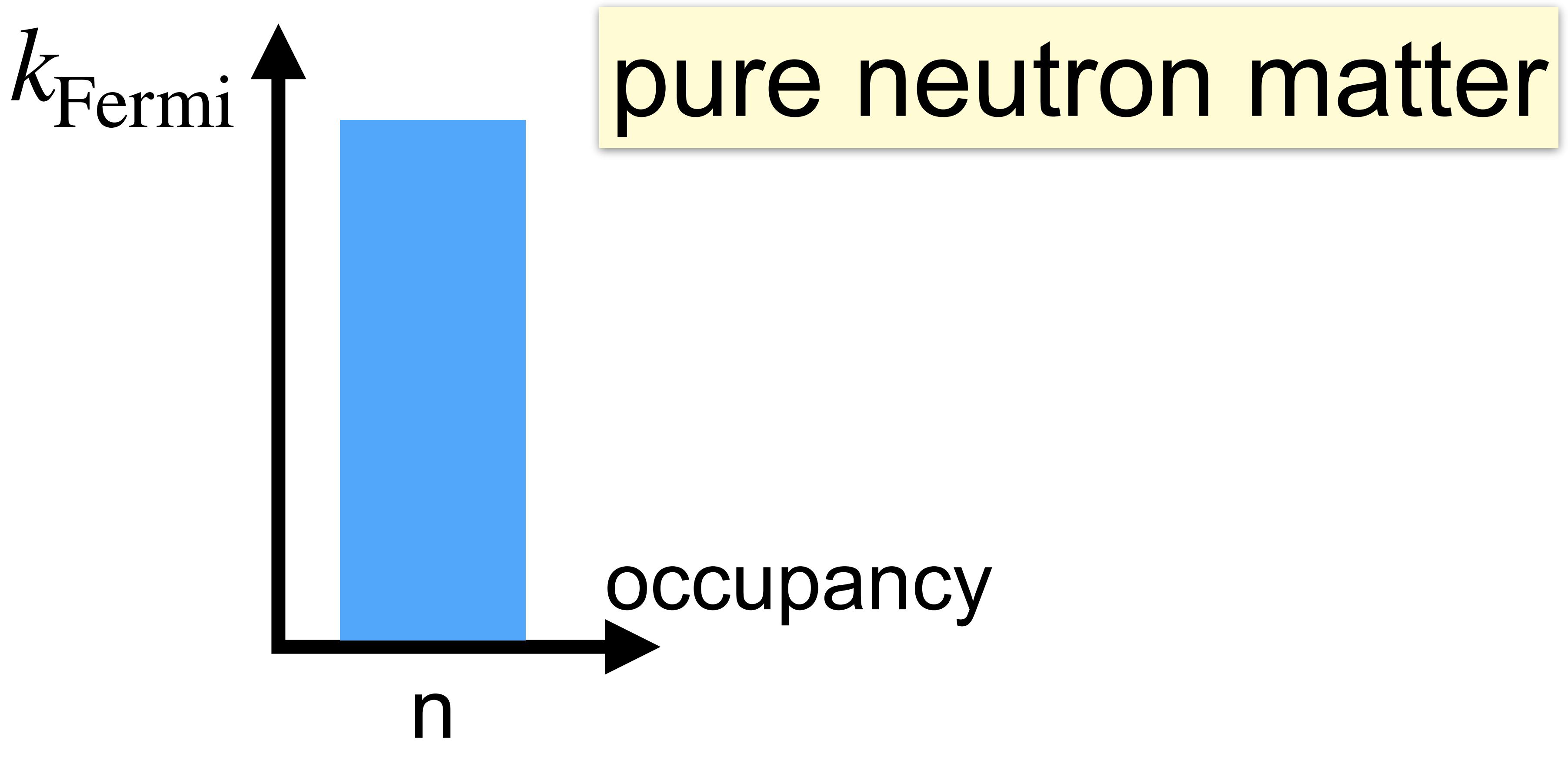
This picture is equivalent to McLerran-Reddy model of the NS based on the McLerran-Pisarski picture

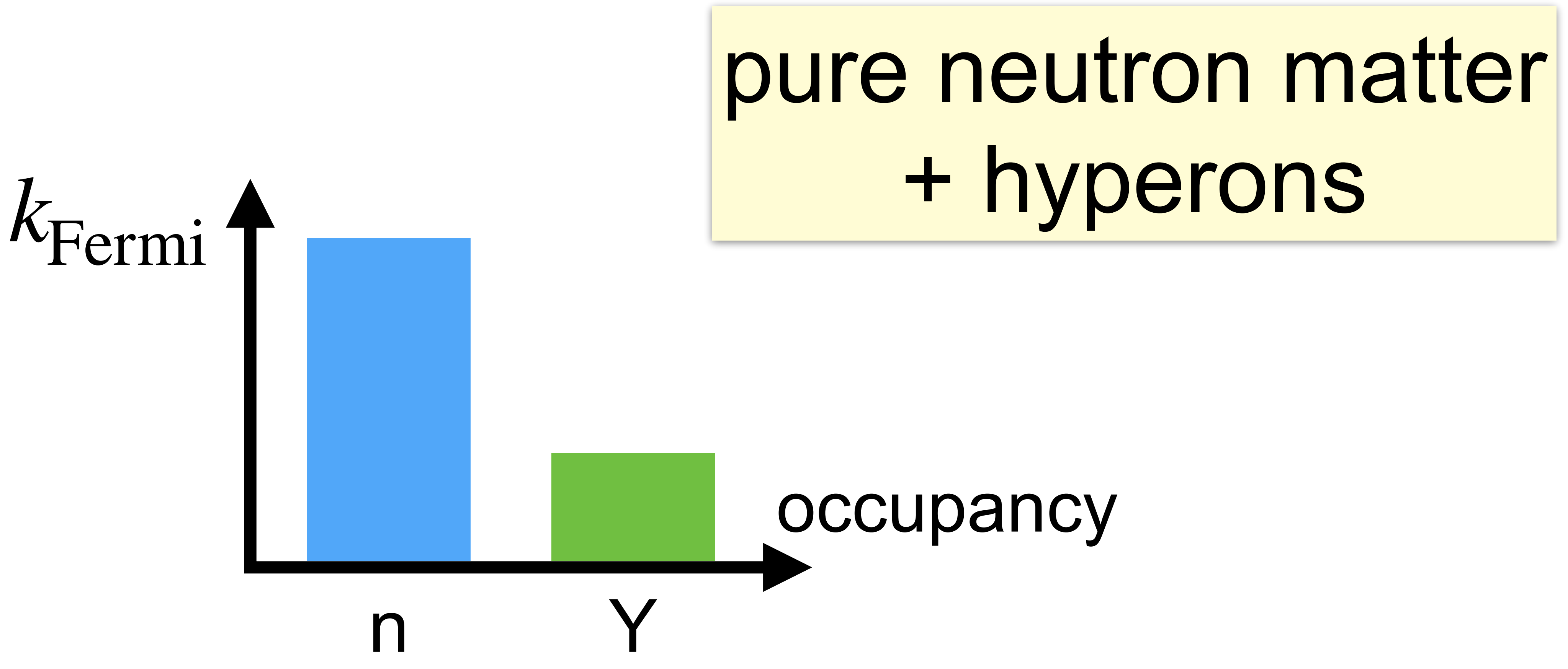
# Including hyperons in IdylliQ model

Fujimoto,Kojo,McLerran, in preparation (2024)

Let us consider pure neutron matter and neglect lepton contributions ($\mu_Q = 0$)

**Conventional picture:**



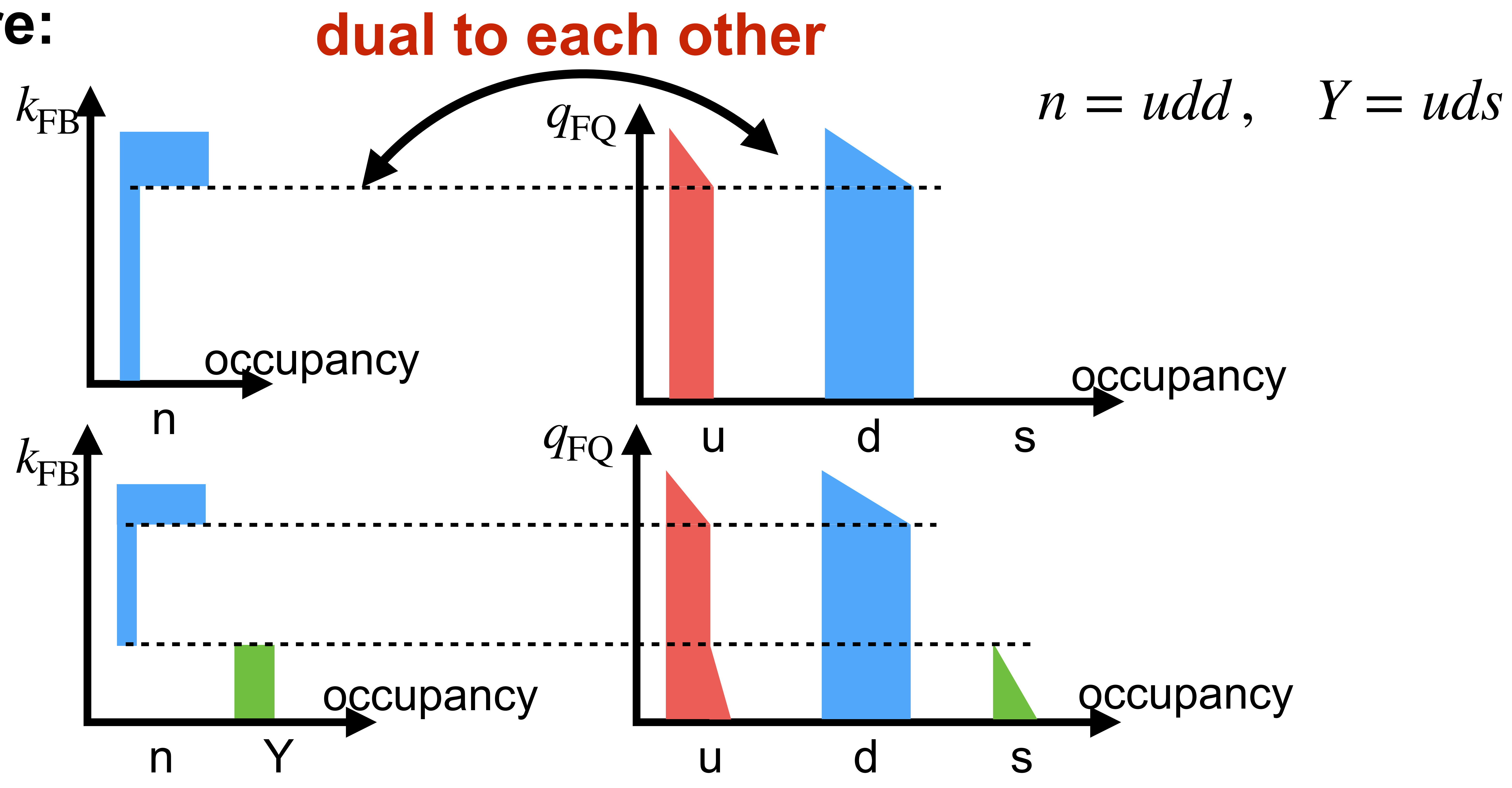

Hyperons (Y) lower the energy density at a given baryon density

**Threshold of Y with $S = -1$: $\mu_B = M_Y$**

# Including hyperons in IdylliQ model

Fujimoto,Kojo,McLerran, in preparation (2024)

**Quarkyonic picture:**

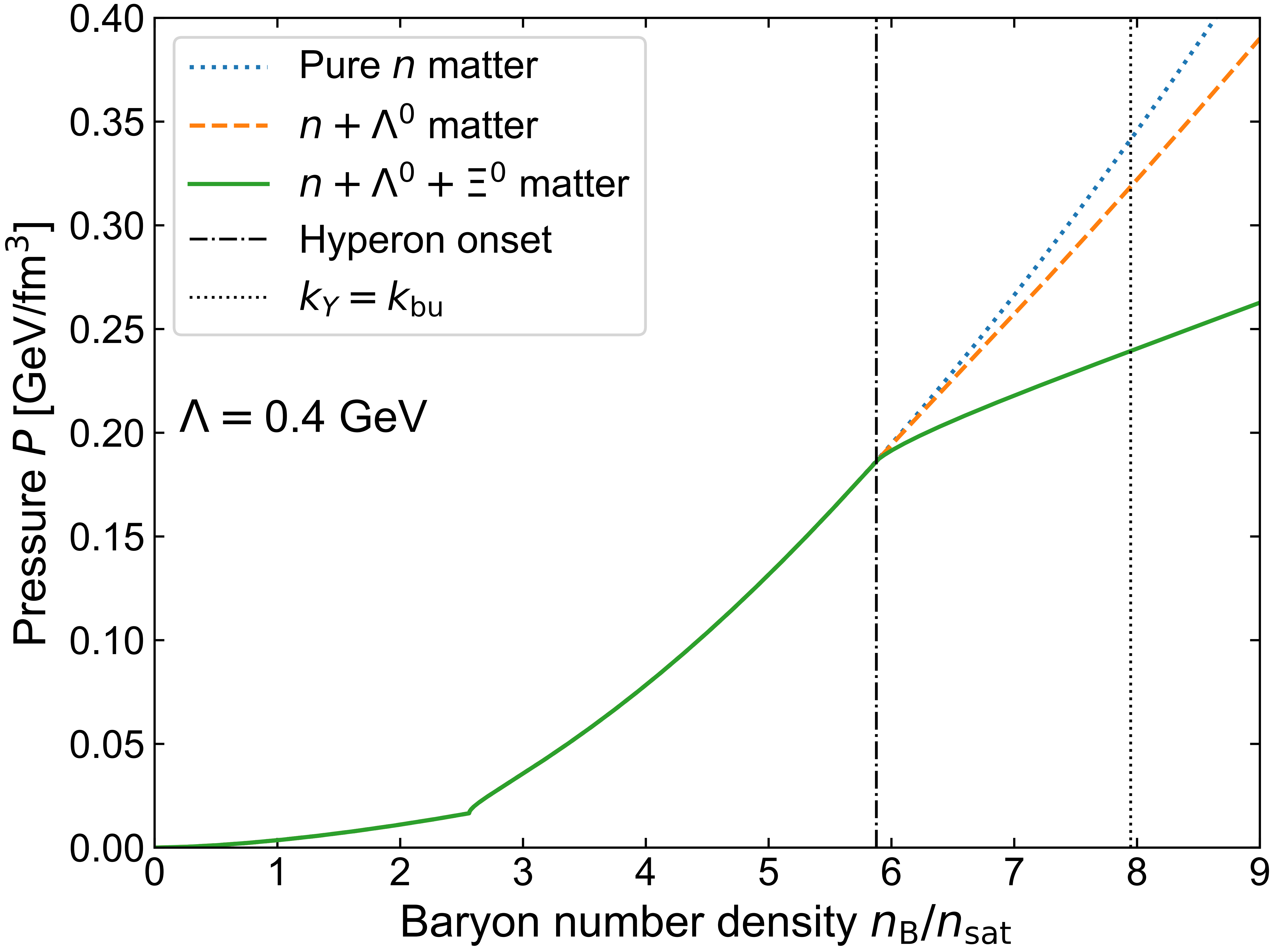



Y has to appear so that d-quark states are kept saturated: n = u**dd** → Y+Y = uu**dd**ss

**Threshold of Y shifted to:** $\mu_B = 2M_Y - M_n$ … close to the value of $M_{\Xi^0}$

# Including hyperons in IdylliQ model

Fujimoto Kojo McLerran, in preparation (2024)




**Due to the saturation of d-quark states, EoS softening by $S = -1$ hyperons is mild**

Bulk of the softening comes from $S = -2$ $\Xi^0$ hyperons

# Summary

- **Quarkyonic matter:** reinterpretation as a duality between confined baryons and weakly-coupled quarks

- **Saturation of quark momentum distribution**
  → under-occupied states in baryonic momentum distribution
  (modification from Fermi-Dirac distribution)

- **Implication to hyperon puzzle:**
  Because of the saturation in d-quark states,
  1) The threshold of hyperons shifted to a higher $\mu_B$
  2) The softening in the EoS is milder

# Supplemental materials

# Underoccupied $f_B$ and occupied $f_Q$

Baryon number in the bulk "quark" region in the quark language:

$$n_B = \int_0^{k_{FQ}} \frac{d^3q}{(2\pi)^3} f_Q(q) \sim k_{FQ}^3 f_Q$$

In the baryon language:

$$n_B = \int_0^{k_{FB}} \frac{d^3k}{(2\pi)^3} f_B(k) \sim k_{FB}^3 f_B \sim N_c^3 k_{FQ}^3 f_B$$

where the Fermi momenta are related as $k_{FB} \sim N_c k_{FQ}$.

Because $f_Q \leq 1, \quad f_B \sim 1/N_c^3$ … composite baryon states are underoccupied

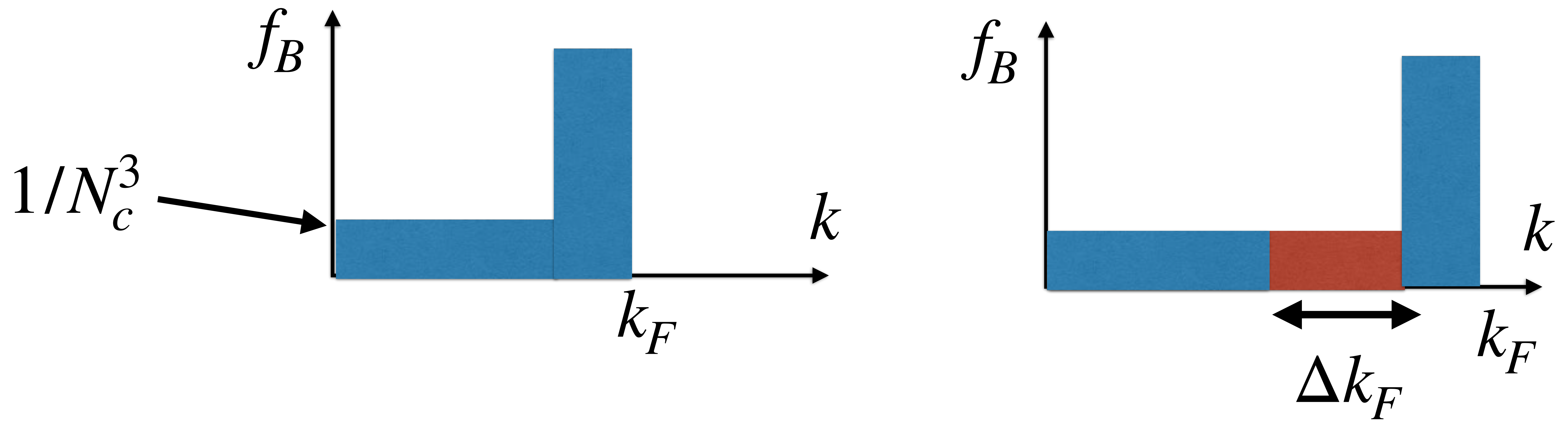
# Rapid stiffening in the EoS

Fujimoto,Kojo,McLerran (2023)

A partial occupation of available baryon phase space leads to **large sound speed:**

$$v_s^2 = \frac{n_B}{\mu_B dn_B/d\mu_B} \to \frac{\delta\mu_B}{\mu_B} \sim v_s^2 \frac{\delta n_B}{n_B}$$

If baryons have underoccupied state, the change in density is small while the change in Fermi energy ( $\sim k_F$) is large




$\to$ Favor the crossover from nucleons to quarks