Narrow Spectra in Heavy Quark Systems

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Fermilab

Overview

What heavy quark states with very narrow widths remain to be observed ?

- Heavy Quark Systems:
  - $\bar{c}c$, $\bar{b}b$, $\bar{b}c$, ccc, ccb, cbb and bbb. Very narrow states (no Zweig allowed strong decays) below threshold. Above threshold resonances and exotic states.
  - Heavy quarks (c or b) plus light degrees of freedom:
    - Heavy-light mesons ($\bar{c}q$, $\bar{b}q$) q = u,d,s. Generally strong decays above ground states.
    - Tetraquark systems - some ground states are actually stable.
    - Doubly heavy baryons : ccq, bbq,{bc}q and [bc]q. Can any excited states be narrow?

Notation : $\mathbf{j}$ = total angular momentum of the light system = $\mathbf{l} + \mathbf{s}$

$\mathbf{J_h}$ = total angular momentum of the heavy quark system

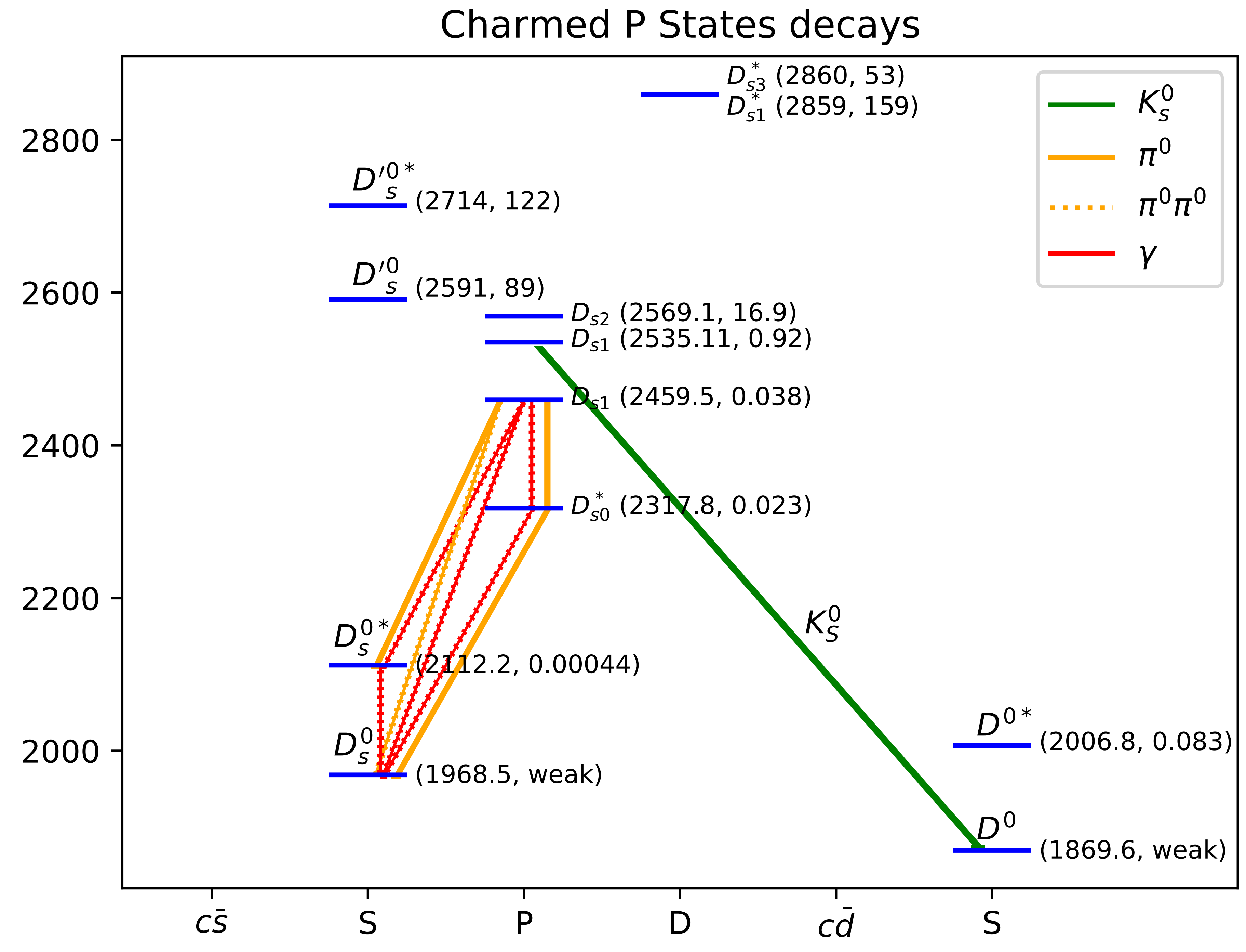
$$\mathbf{J} = \mathbf{j} + \mathbf{J_h}$$

$\mathbf{J_h} = \mathbf{s_Q}$ (static quark) $\mathbf{J_h} = \mathbf{L_h} + \mathbf{S_h}$ (for a multi heavy quark system)

Heavy-Light Mesons

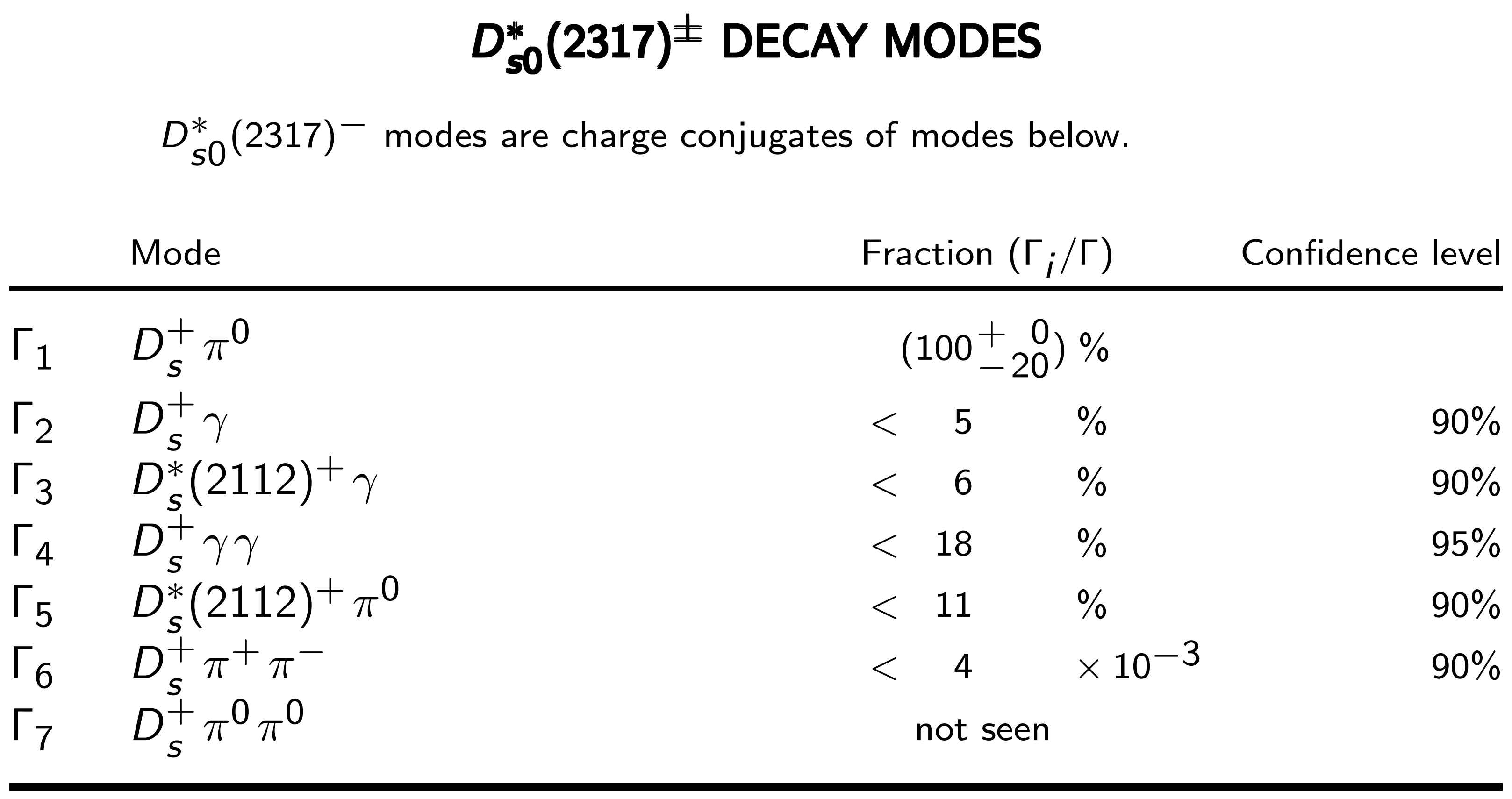
For the P states :	j^P	J^P
	$3/2^+$	2^+
		1^+
	$1/2^+$	1^+
		0^+

- D and B:
  - The excited 1P states have strong decays by pion transitions to the 1S states.
  - Decay widths depend on phase space and partial wave. D*0, D1, D1, D*2: 229, 314, 31, 47 MeV: and B*0, B1, B1, B^*2^+ : wide, wide, 31, 24 MeV
- D_s and B_s :
  - The D_s $j^P = 1/2^+$ P states are very narrow.
  - No allowed strong decays.
  - Independent of their composition.
  - Strong decays $[D^{(*)} + K]$ for the states $j^P = 3/2^+$ (D_s1 and D_s2).
  - For the D_s P-states all are observed and decays measured



Heavy-Light Decays

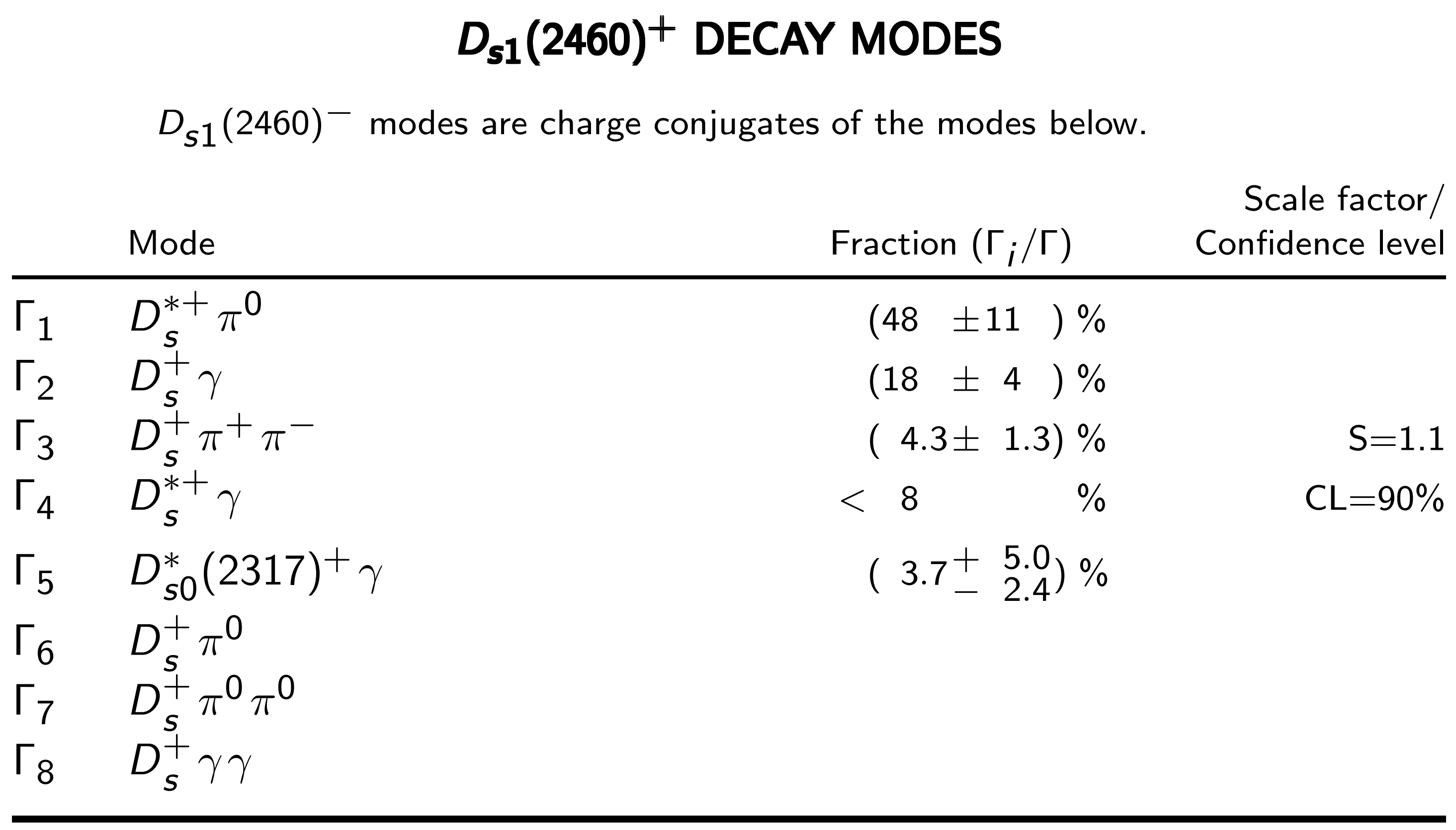
PDG 2024



$D^*_{s0}(2317)^\pm$ DECAY MODES

$D^*_{s0}(2317)^-$ modes are charge conjugates of modes below.

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$D_s^+\pi^0$	$(100^{+\ 0}_{-20})$ %	
Γ_2	$D_s^+\gamma$	$<$ 5 %	90%
Γ_3	$D_s^*(2112)^+\gamma$	$<$ 6 %	90%
Γ_4	$D_s^+\gamma\gamma$	$<$ 18 %	95%
Γ_5	$D_s^*(2112)^+\pi^0$	$<$ 11 %	90%
Γ_6	$D_s^+\pi^+\pi^-$	$<$ 4 $\times 10^{-3}$	90%
Γ_7	$D_s^+\pi^0\pi^0$	not seen	



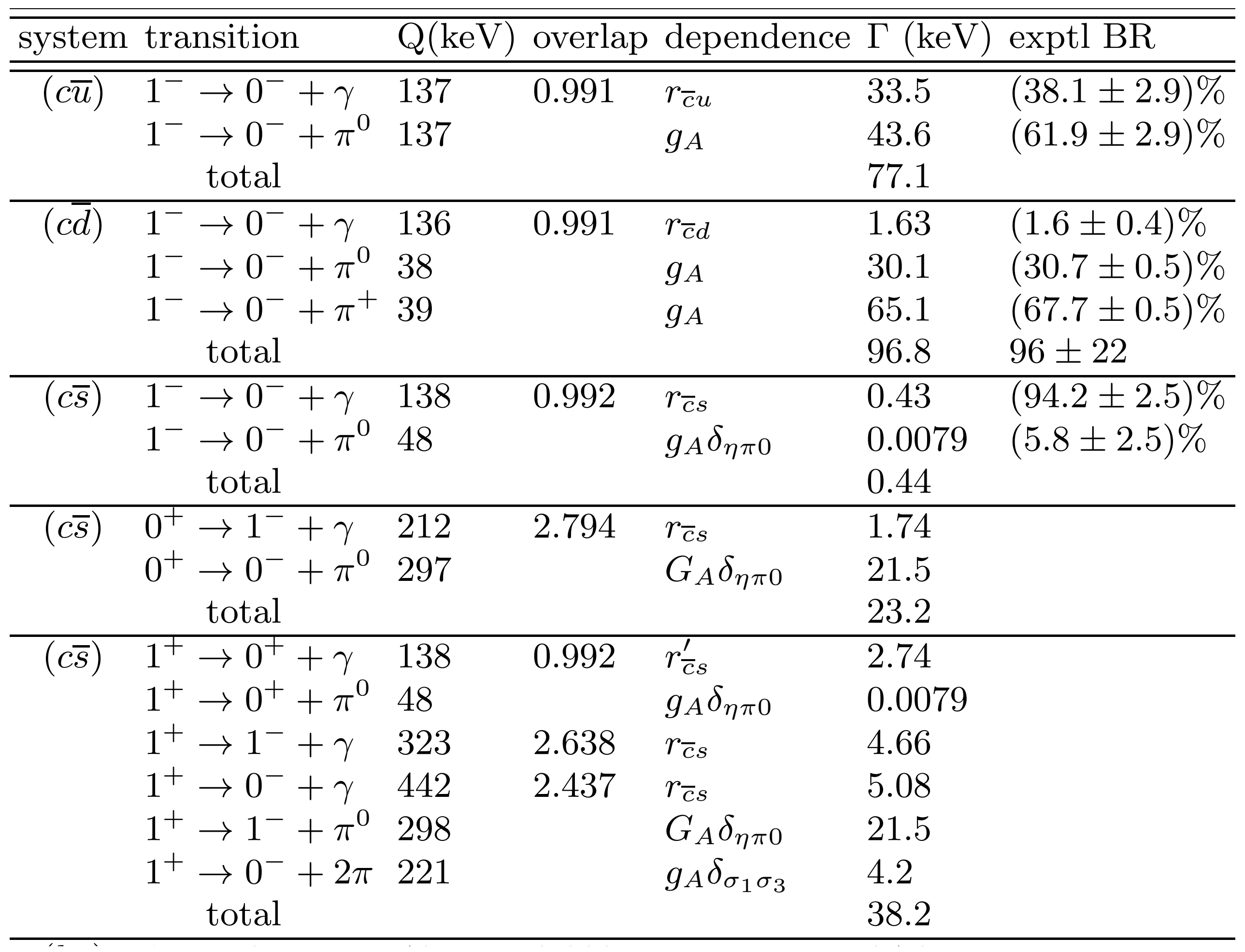
$D_{s1}(2460)^+$ DECAY MODES

$D_{s1}(2460)^-$ modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$D_s^{*+}\pi^0$	$(48\ \pm 11\ )$ %	
Γ_2	$D_s^+\gamma$	$(18\ \pm\ 4\ )$ %	
Γ_3	$D_s^+\pi^+\pi^-$	$(\ 4.3 \pm\ 1.3)$ %	S=1.1
Γ_4	$D_s^{*+}\gamma$	$<$ 8 %	CL=90%
Γ_5	$D^*_{s0}(2317)^+\gamma$	$(\ 3.7^{+\ 5.0}_{-\ 2.4})$ %	
Γ_6	$D_s^+\pi^0$		
Γ_7	$D_s^+\pi^0\pi^0$		
Γ_8	$D_s^+\gamma\gamma$		

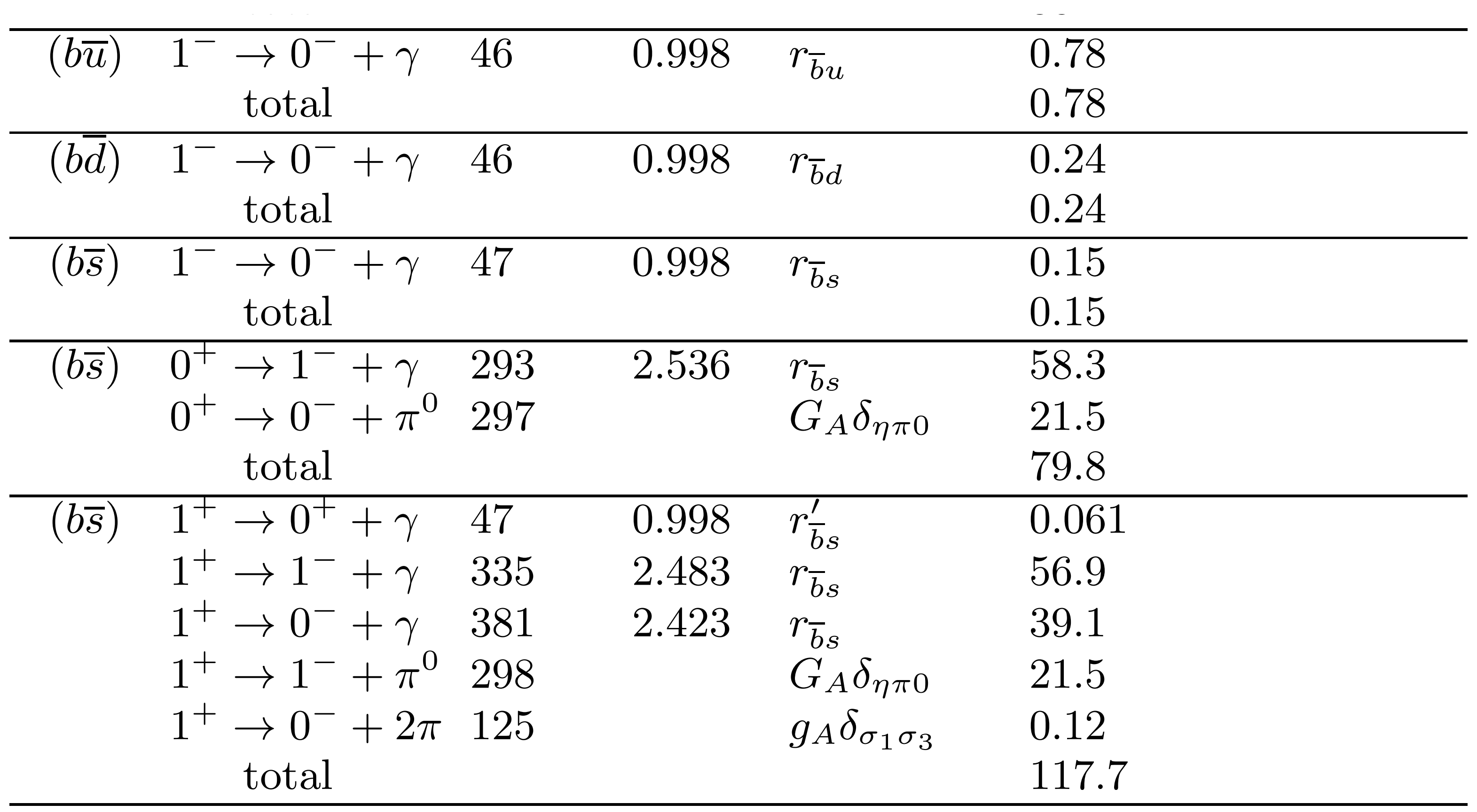
Detailed measurements of the branching ratios can distinguish models. Molecular (D K) or (cs) P state?

E. E. and W. Bardeen
PR D68 054024 2003



system	transition	Q(keV)	overlap	dependence	Γ (keV)	exptl BR
$(c\overline{u})$	$1^- \to 0^- + \gamma$	137	0.991	$r_{\overline{c}u}$	33.5	$(38.1 \pm 2.9)\%$
	$1^- \to 0^- + \pi^0$	137		g_A	43.6	$(61.9 \pm 2.9)\%$
	total				77.1	
$(c\overline{d})$	$1^- \to 0^- + \gamma$	136	0.991	$r_{\overline{c}d}$	1.63	$(1.6 \pm 0.4)\%$
	$1^- \to 0^- + \pi^0$	38		g_A	30.1	$(30.7 \pm 0.5)\%$
	$1^- \to 0^- + \pi^+$	39		g_A	65.1	$(67.7 \pm 0.5)\%$
	total				96.8	96 ± 22
$(c\overline{s})$	$1^- \to 0^- + \gamma$	138	0.992	$r_{\overline{c}s}$	0.43	$(94.2 \pm 2.5)\%$
	$1^- \to 0^- + \pi^0$	48		$g_A\delta_{\eta\pi 0}$	0.0079	$(5.8 \pm 2.5)\%$
	total				0.44	
$(c\overline{s})$	$0^+ \to 1^- + \gamma$	212	2.794	$r_{\overline{c}s}$	1.74	
	$0^+ \to 0^- + \pi^0$	297		$G_A\delta_{\eta\pi 0}$	21.5	
	total				23.2	
$(c\overline{s})$	$1^+ \to 0^+ + \gamma$	138	0.992	$r'_{\overline{c}s}$	2.74	
	$1^+ \to 0^+ + \pi^0$	48		$g_A\delta_{\eta\pi 0}$	0.0079	
	$1^+ \to 1^- + \gamma$	323	2.638	$r_{\overline{c}s}$	4.66	
	$1^+ \to 0^- + \gamma$	442	2.437	$r_{\overline{c}s}$	5.08	
	$1^+ \to 1^- + \pi^0$	298		$G_A\delta_{\eta\pi 0}$	21.5	
	$1^+ \to 0^- + 2\pi$	221		$g_A\delta_{\sigma_1\sigma_3}$	4.2	
	total				38.2	

Note the ratio of
(D1->D* + ɣ)/(D1->D + ɣ).



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	total				0.24
$(b\overline{s})$	$1^- \to 0^- + \gamma$	47	0.998	$r_{\overline{b}s}$	0.15
	total				0.15
$(b\overline{s})$	$0^+ \to 1^- + \gamma$	293	2.536	$r_{\overline{b}s}$	58.3
	$0^+ \to 0^- + \pi^0$	297		$G_A\delta_{\eta\pi 0}$	21.5
	total				79.8
$(b\overline{s})$	$1^+ \to 0^+ + \gamma$	47	0.998	$r'_{\overline{b}s}$	0.061
	$1^+ \to 1^- + \gamma$	335	2.483	$r_{\overline{b}s}$	56.9
	$1^+ \to 0^- + \gamma$	381	2.423	$r_{\overline{b}s}$	39.1
	$1^+ \to 1^- + \pi^0$	298		$G_A\delta_{\eta\pi 0}$	21.5
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PDG 2024

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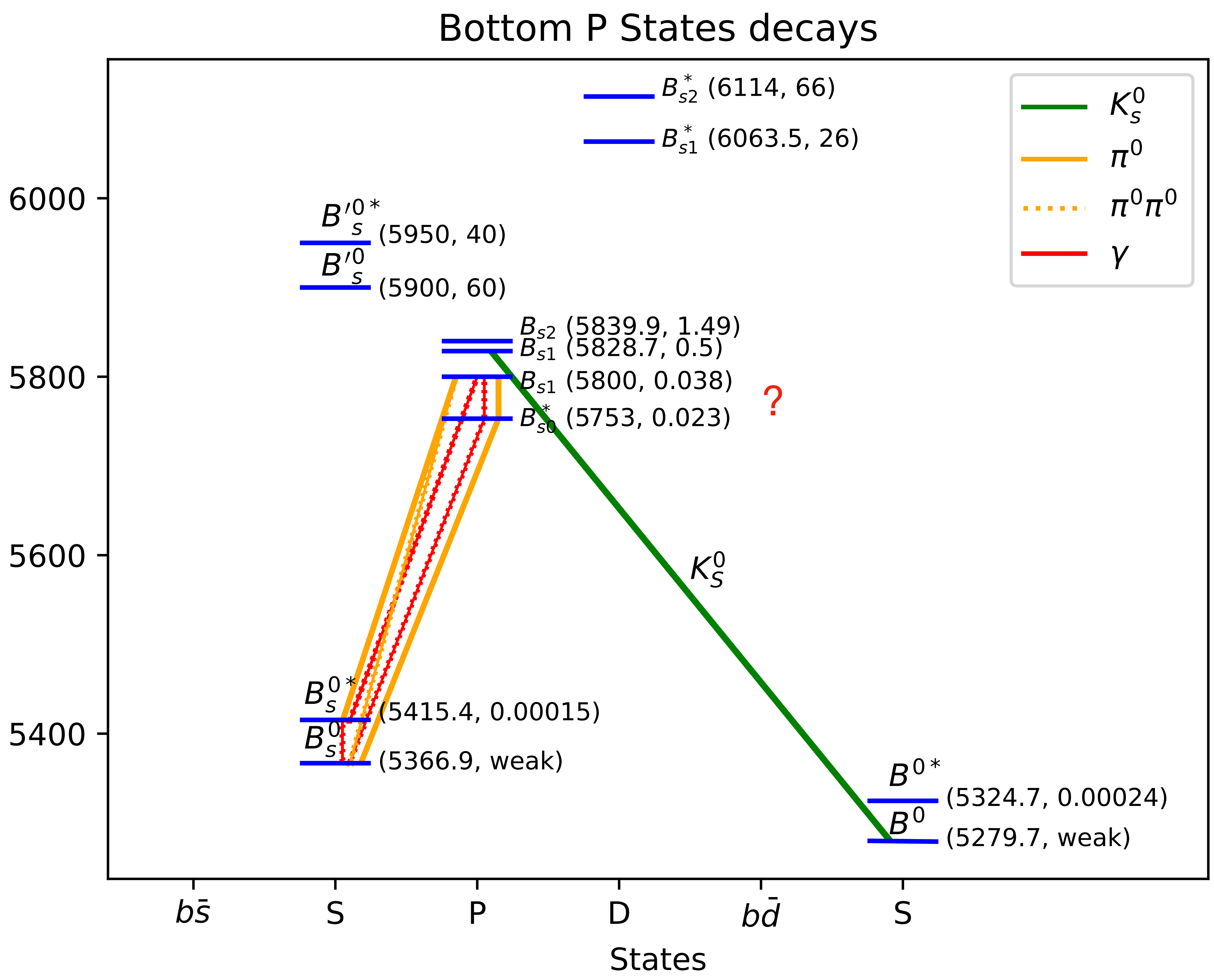

Bottom P States decays
B^{*}_{s2} (6114, 66)
B^{*}_{s1} (6063.5, 26)
B'^{0*}_{s} (5950, 40)
B'^{0}_{s} (5900, 60)
B_{s2} (5839.9, 1.49)
B_{s1} (5828.7, 0.5)
B_{s1} (5800, 0.038)
B^{*}_{s0} (5753, 0.023)
?
K^0_S
B^{0*}_{s} (5415.4, 0.00015)
B^{0}_{s} (5366.9, weak)
B^{0*} (5324.7, 0.00024)
B^{0} (5279.7, weak)
K^0_s
π^0
$\pi^0\pi^0$
γ
6000
5800
5600
5400
$b\bar{s}$
S
P
D
$b\bar{d}$
S
States

BESIII

Study the D_P + $D^{(*)}$ states in e+e- [Y(4260) structure]. Some have S wave thresholds but are very wide (227, 314 MeV). The states produced in D wave are narrow Γ~ 31,47 MeV. All will be seen in $D^{(*)}$ + $D^{(*)}$ + π final states

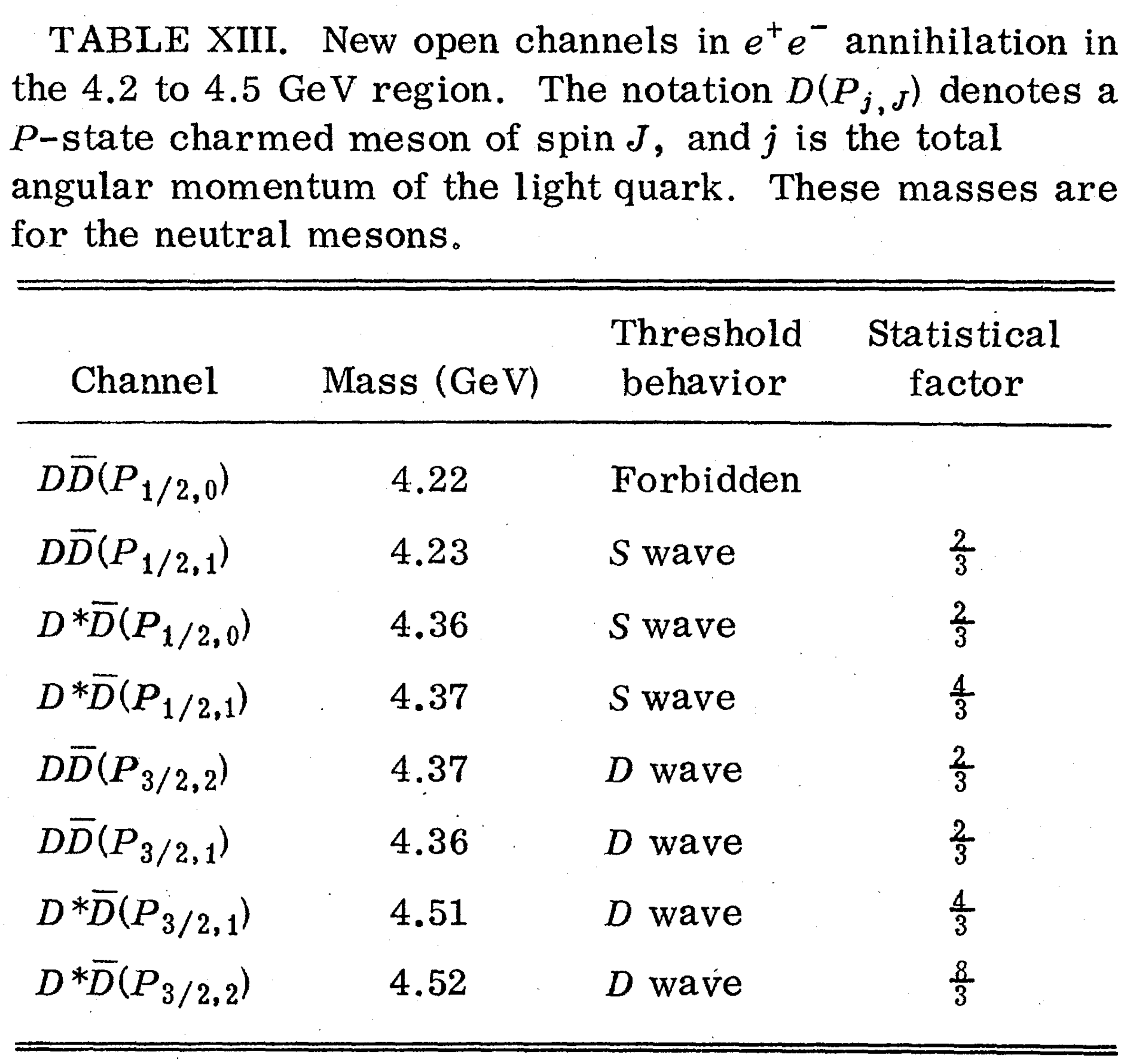


TABLE XIII. New open channels in e^+e^- annihilation in the 4.2 to 4.5 GeV region. The notation $D(P_{j,J})$ denotes a P-state charmed meson of spin J, and j is the total angular momentum of the light quark. These masses are for the neutral mesons.

Channel	Mass (GeV)	Threshold behavior	Statistical factor
$D\bar{D}(P_{1/2,0})$	4.22	Forbidden	
$D\bar{D}(P_{1/2,1})$	4.23	S wave	$\frac{2}{3}$
$D^*\bar{D}(P_{1/2,0})$	4.36	S wave	$\frac{2}{3}$
$D^*\bar{D}(P_{1/2,1})$	4.37	S wave	$\frac{4}{3}$
$D\bar{D}(P_{3/2,2})$	4.37	D wave	$\frac{2}{3}$
$D\bar{D}(P_{3/2,1})$	4.36	D wave	$\frac{2}{3}$
$D^*\bar{D}(P_{3/2,1})$	4.51	D wave	$\frac{4}{3}$
$D^*\bar{D}(P_{3/2,2})$	4.52	D wave	$\frac{8}{3}$

	Threshold
D D_P*0	4295
D D_P1	4307
D* D_P*0	4437
D* D_P1	4326
D D_P*2	4287
D D_P1	4429
D* D_P1	4466
D* D_P*2	

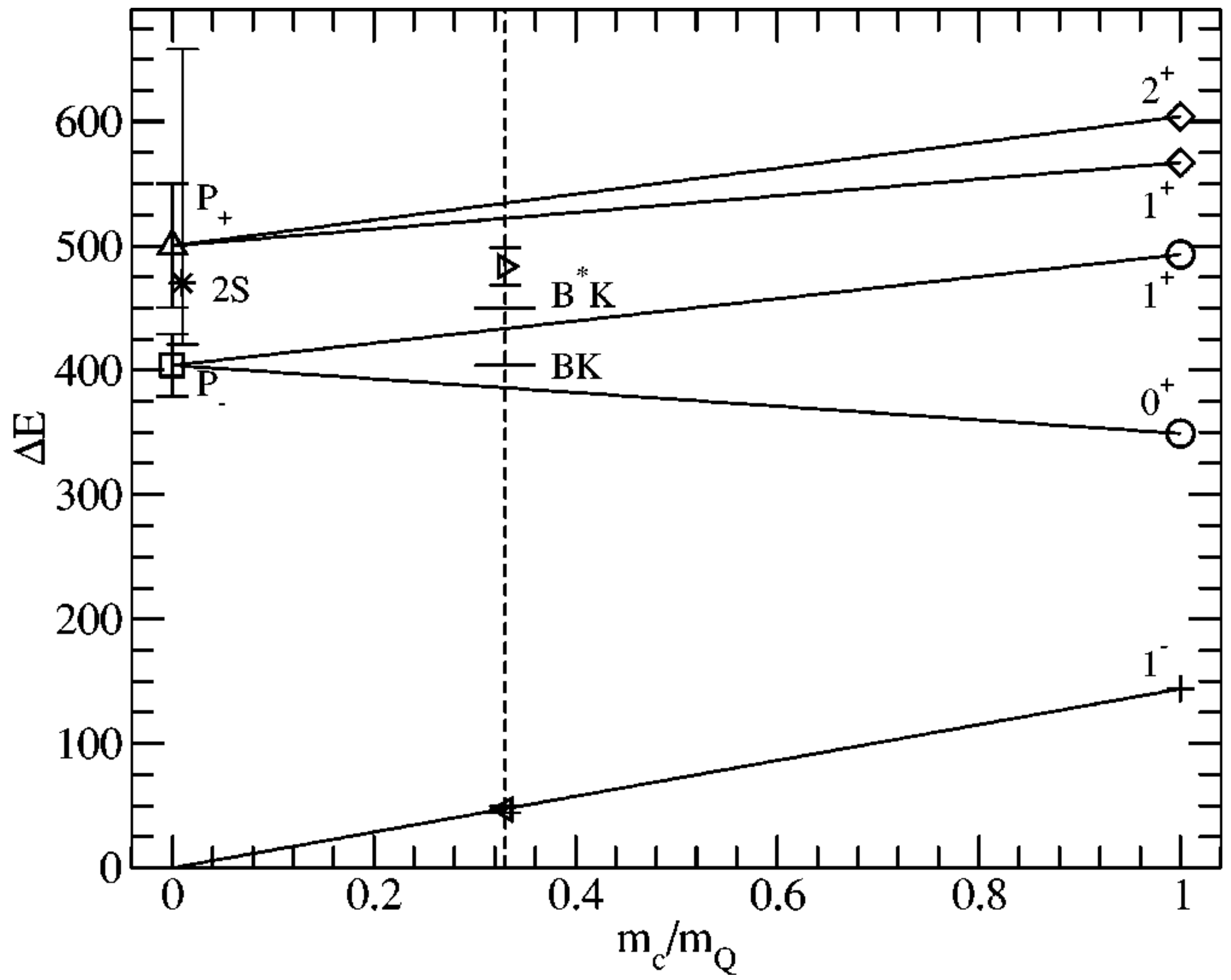
BELLE 2

$B^{(*)}$ + $B_P(3/2^+)$: 11,005; 11,016; 11,051; 11,062

Search for $B_P(j_l=1/2)$ states recoiling against a reconstructed B or B* at the $\Upsilon(6S)$

Are the j_l =1/2 1P(j_l=1/2)(bs) states wide or very narrow ?

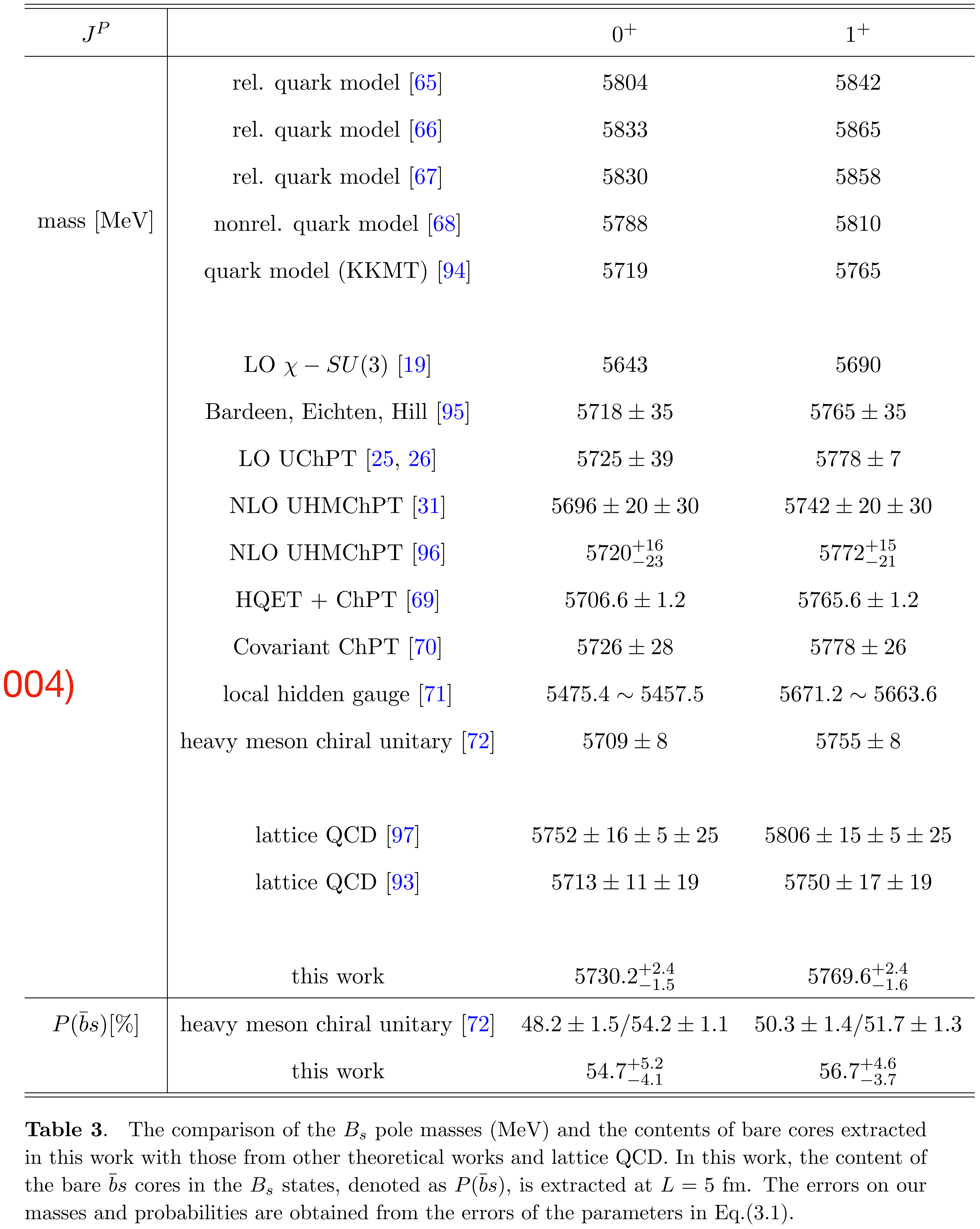
- The (1P) B_s system is not complete experimentally.
- Many approaches RQM, ChPT, Lattice, Coupled channels,…
- HQET in lattice QCD is an interesting approach.



UKQCD Collaboration,
A. M. Green et. al. PRD 69, 094505 (2004)

- Green et. al. concluded that both $B^*_s(0^+)$and $B_s(1^+)$ are below threshold for strong decays

Zhi Yang et.al , arXiv: 2207.07320



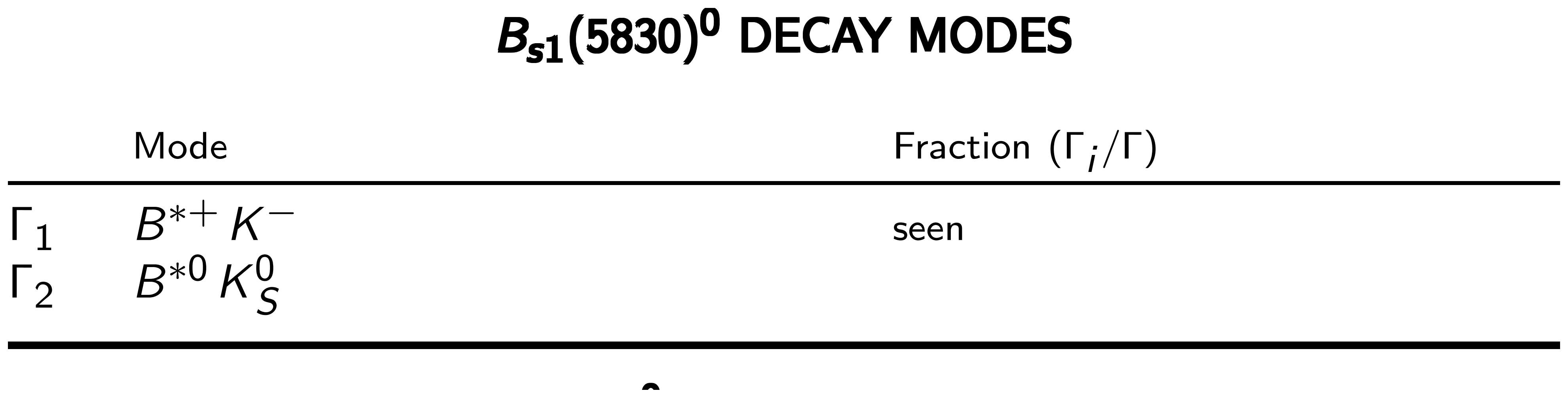
J^P		0^+	1^+
mass [MeV]	rel. quark model [65]	5804	5842
	rel. quark model [66]	5833	5865
	rel. quark model [67]	5830	5858
	nonrel. quark model [68]	5788	5810
	quark model (KKMT) [94]	5719	5765
	LO $\chi - SU(3)$ [19]	5643	5690
	Bardeen, Eichten, Hill [95]	5718 ± 35	5765 ± 35
	LO UChPT [25, 26]	5725 ± 39	5778 ± 7
	NLO UHMChPT [31]	$5696 \pm 20 \pm 30$	$5742 \pm 20 \pm 30$
	NLO UHMChPT [96]	5720^{+16}_{-23}	5772^{+15}_{-21}
	HQET + ChPT [69]	5706.6 ± 1.2	5765.6 ± 1.2
	Covariant ChPT [70]	5726 ± 28	5778 ± 26
	local hidden gauge [71]	$5475.4 \sim 5457.5$	$5671.2 \sim 5663.6$
	heavy meson chiral unitary [72]	5709 ± 8	5755 ± 8
	lattice QCD [97]	$5752 \pm 16 \pm 5 \pm 25$	$5806 \pm 15 \pm 5 \pm 25$
	lattice QCD [93]	$5713 \pm 11 \pm 19$	$5750 \pm 17 \pm 19$
	this work	$5730.2^{+2.4}_{-1.5}$	$5769.6^{+2.4}_{-1.6}$
$P(\bar{b}s)[\%]$	heavy meson chiral unitary [72]	$48.2 \pm 1.5/54.2 \pm 1.1$	$50.3 \pm 1.4/51.7 \pm 1.3$
	this work	$54.7^{+5.2}_{-4.1}$	$56.7^{+4.6}_{-3.7}$

Table 3. The comparison of the B_s pole masses (MeV) and the contents of bare cores extracted in this work with those from other theoretical works and lattice QCD. In this work, the content of the bare $\bar{b}s$ cores in the B_s states, denoted as $P(\bar{b}s)$, is extracted at $L = 5$ fm. The errors on our masses and probabilities are obtained from the errors of the parameters in Eq.(3.1).

B+K, B*+K (thresholds) = 5777, 5793

- Theoretical question: What is mass difference between the 1P ($j^P = 3/2^+$) and ($j^P = 1/2^+$) meson states in QCD? (+,0,or -)?
- For a light quark moving in a static source in a funnel potential: A s·L (A>0 SE A<0 DE)
- HQET on the lattice can resolve this but still results not yet accurate enough.
- Using the form $M(j^P) = M0 + M1/m_Q$: We use the known $j^P = 3/2^+$ states masses, to find as $1/m_Q$ -> 0 $M(3/2^+) - M(1/2^-) = 423$ MeV. (Below threshold for strong decay.)
- Hence for potential models, knowing $M(1/2^+) - M(1/2^-)$ for $1/m_Q$ -> 0 can predict the value M(B1) and M(B*0)
- However, including the effects of coupling to the strong decay channels or in molecular model the distance of the state from the two body strong decay threshold is critical. So would not expect this simple behavior.

$j_l = 3/2$ states

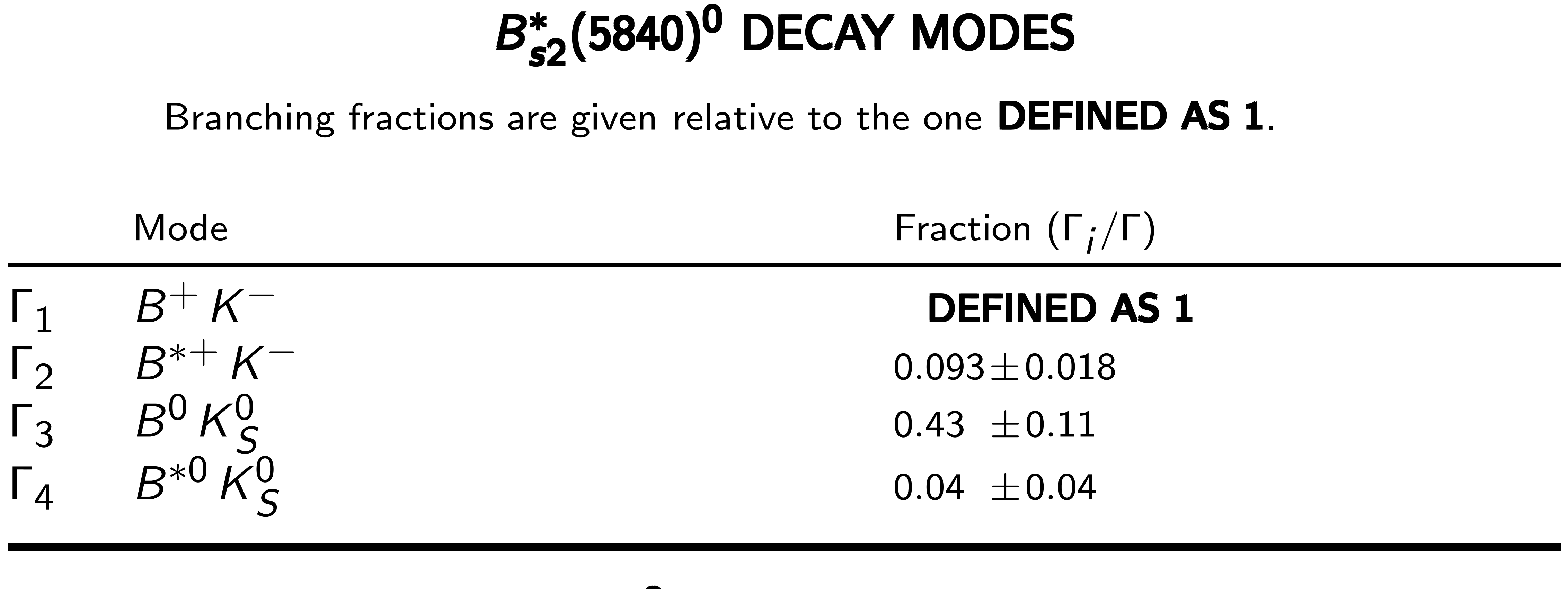


$B_{s1}(5830)^0$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$B^{*+}K^-$	seen
Γ_2	$B^{*0}K_S^0$	

$\Gamma = 0.5 \pm 0.4$ MeV

PDG24



$B_{s2}^*(5840)^0$ DECAY MODES

Branching fractions are given relative to the one **DEFINED AS 1**.

	Mode	Fraction (Γ_i/Γ)
Γ_1	B^+K^-	**DEFINED AS 1**
Γ_2	$B^{*+}K^-$	0.093±0.018
Γ_3	$B^0K_S^0$	0.43 ±0.11
Γ_4	$B^{*0}K_S^0$	0.04 ±0.04

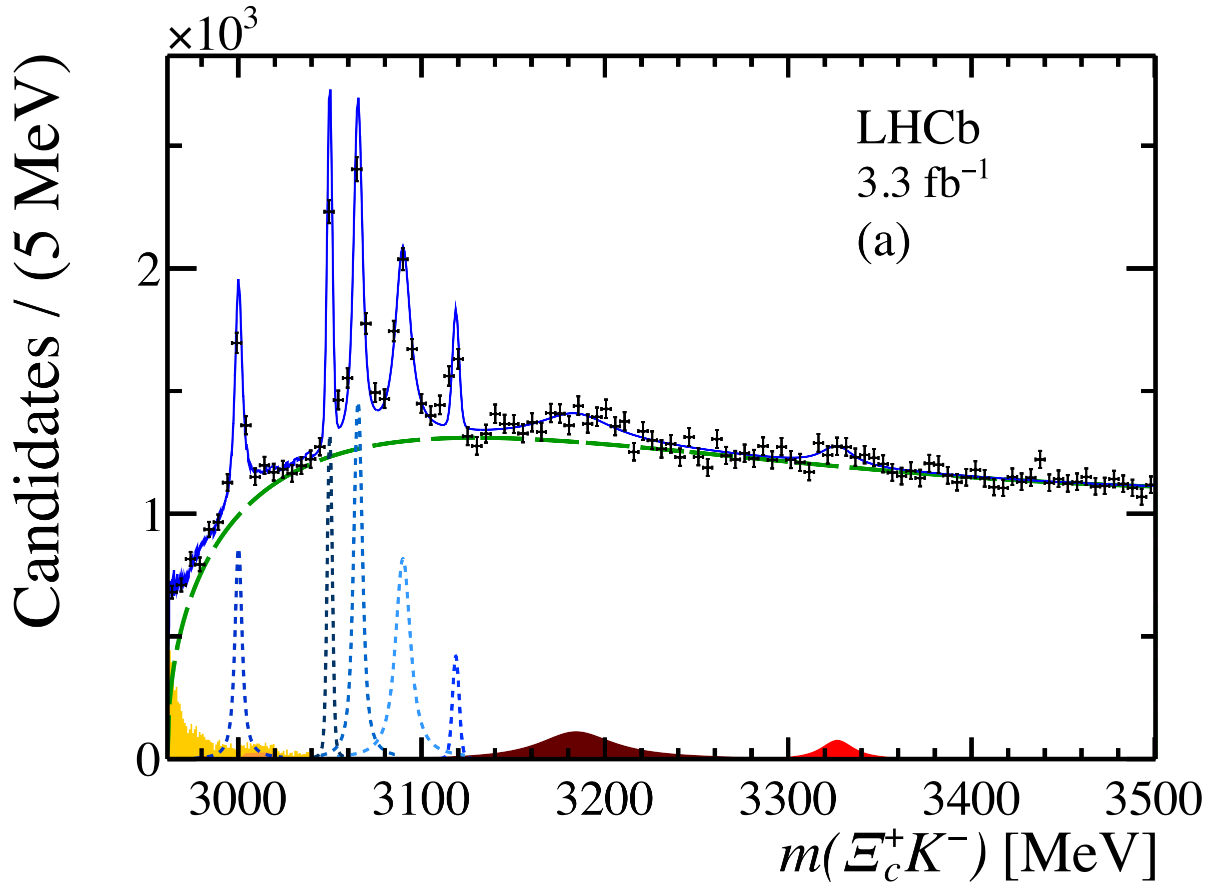
$\Gamma = 1.49 \pm 0.27$ MeV

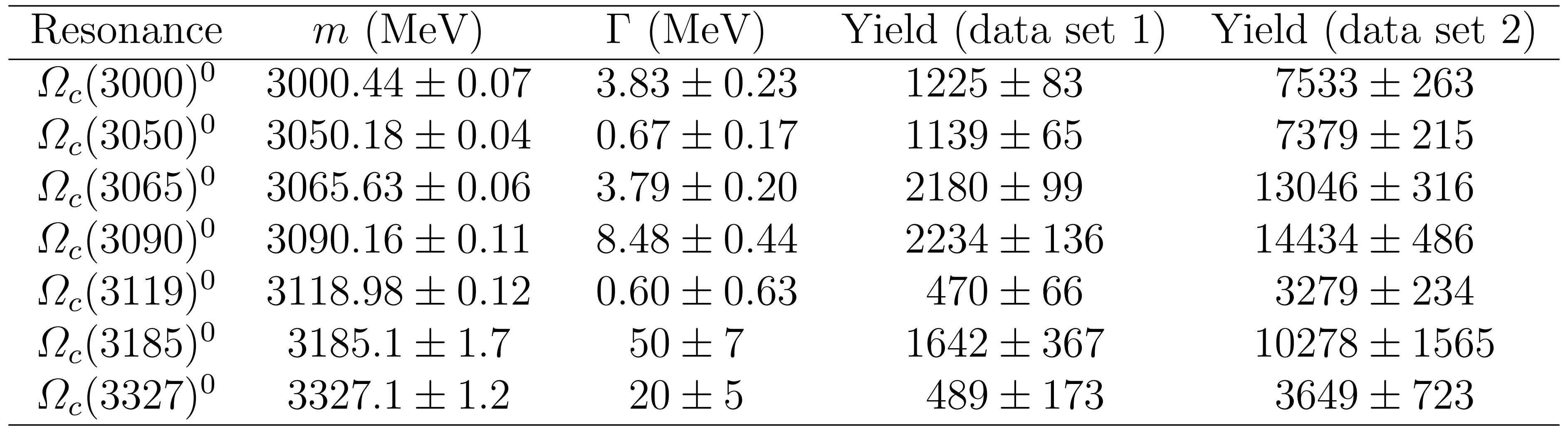
Small widths: D-wave amplitudes and closeness to $B^{(*)}$+K threshold

Heavy Baryons

- Baryons with one heavy quark. Only Ω_c and Ω_b lowest P states might be stable against strong decays. Because, if isospin conserving pion transitions to the ground state can occur, there are strong decays. But the decays Ω_c (excited) -> Ξ_c + K has been observed.

- K^- + Ξ_c^+ decays are observed. Narrow states generally interpreted as the Ω_c(1P) states. The J^P for these states not yet determined.





Resonance	m (MeV)	Γ (MeV)	Yield (data set 1)	Yield (data set 2)
$\Omega_c(3000)^0$	3000.44 ± 0.07	3.83 ± 0.23	1225 ± 83	7533 ± 263
$\Omega_c(3050)^0$	3050.18 ± 0.04	0.67 ± 0.17	1139 ± 65	7379 ± 215
$\Omega_c(3065)^0$	3065.63 ± 0.06	3.79 ± 0.20	2180 ± 99	13046 ± 316
$\Omega_c(3090)^0$	3090.16 ± 0.11	8.48 ± 0.44	2234 ± 136	14434 ± 486
$\Omega_c(3119)^0$	3118.98 ± 0.12	0.60 ± 0.63	470 ± 66	3279 ± 234
$\Omega_c(3185)^0$	3185.1 ± 1.7	50 ± 7	1642 ± 367	10278 ± 1565
$\Omega_c(3327)^0$	3327.1 ± 1.2	20 ± 5	489 ± 173	3649 ± 723

LHCb arXiv:2302.04733

- The situation for Ω_b lowest P states should be similar.

• Doubly Heavy Baryons. with Chris Quigg

- Ground states. Only one state $\Xi_{(cc)}^{+}$ (3621) has been observed to date. Can use lattice calculations to determine the Ω_{cc} states.

- Will use Ω_{cc} (j^P =1/2$^+$) = 3712, Ω_{cc}^*(j^P =3/2$^+$) = 3788),
Ξ_c(j^P =1/2$^+$)=3627, Ξ_c^*(j^P = 3/2$^+$)=3690

N. Mather and M. Padmanath, PRD 99, 031501 (2019),
Rosner and Karliner PRD 90 094007(2014)

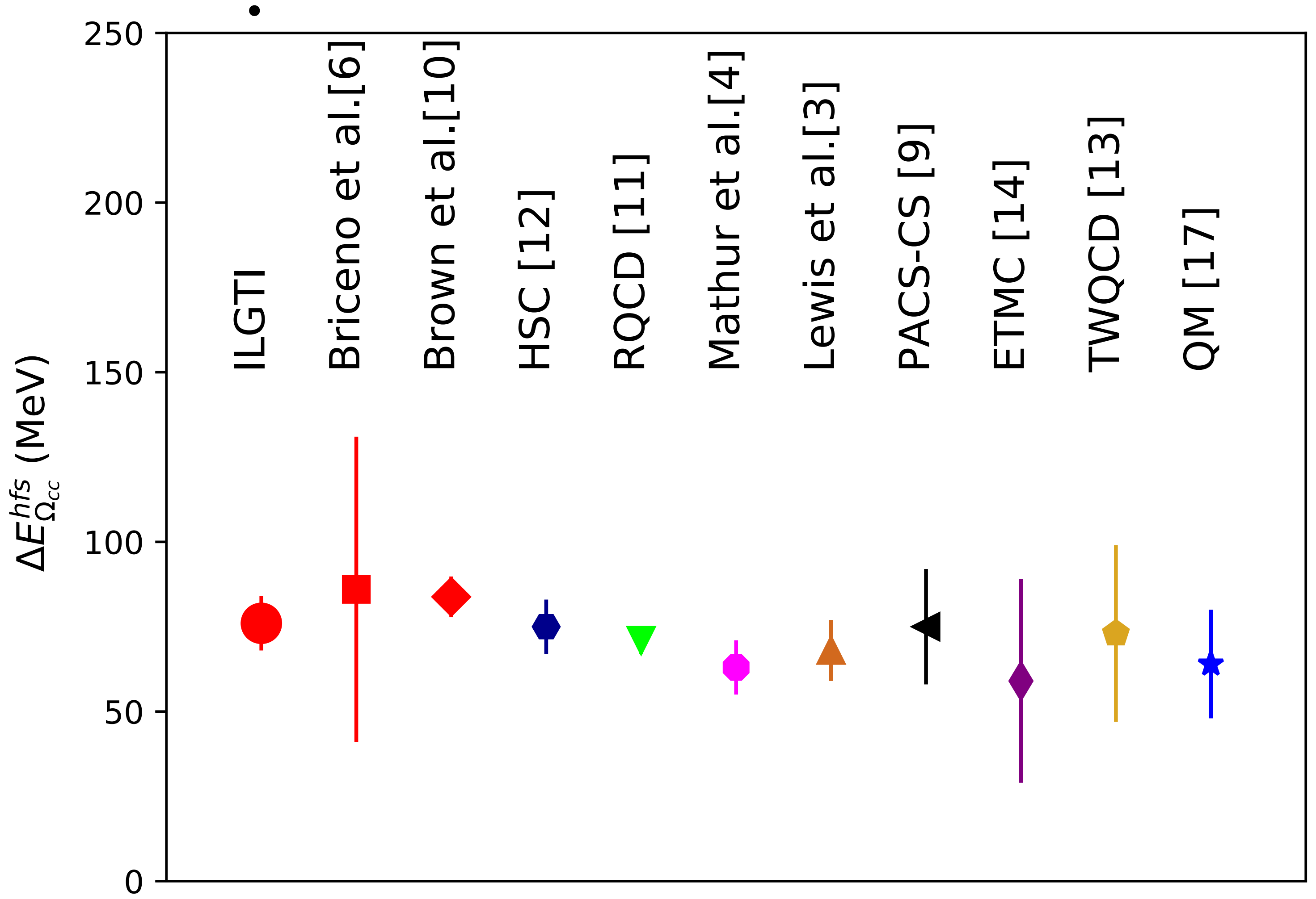


FIG. 4. Comparison of hyperfine splitting between the ground states of $3/2^+$ and $1/2^+$ baryons obtained from various theoretical calculations. Continuum extrapolated results are shown by symbols with red color while symbols with all other colors are obtained only at one lattice spacing.

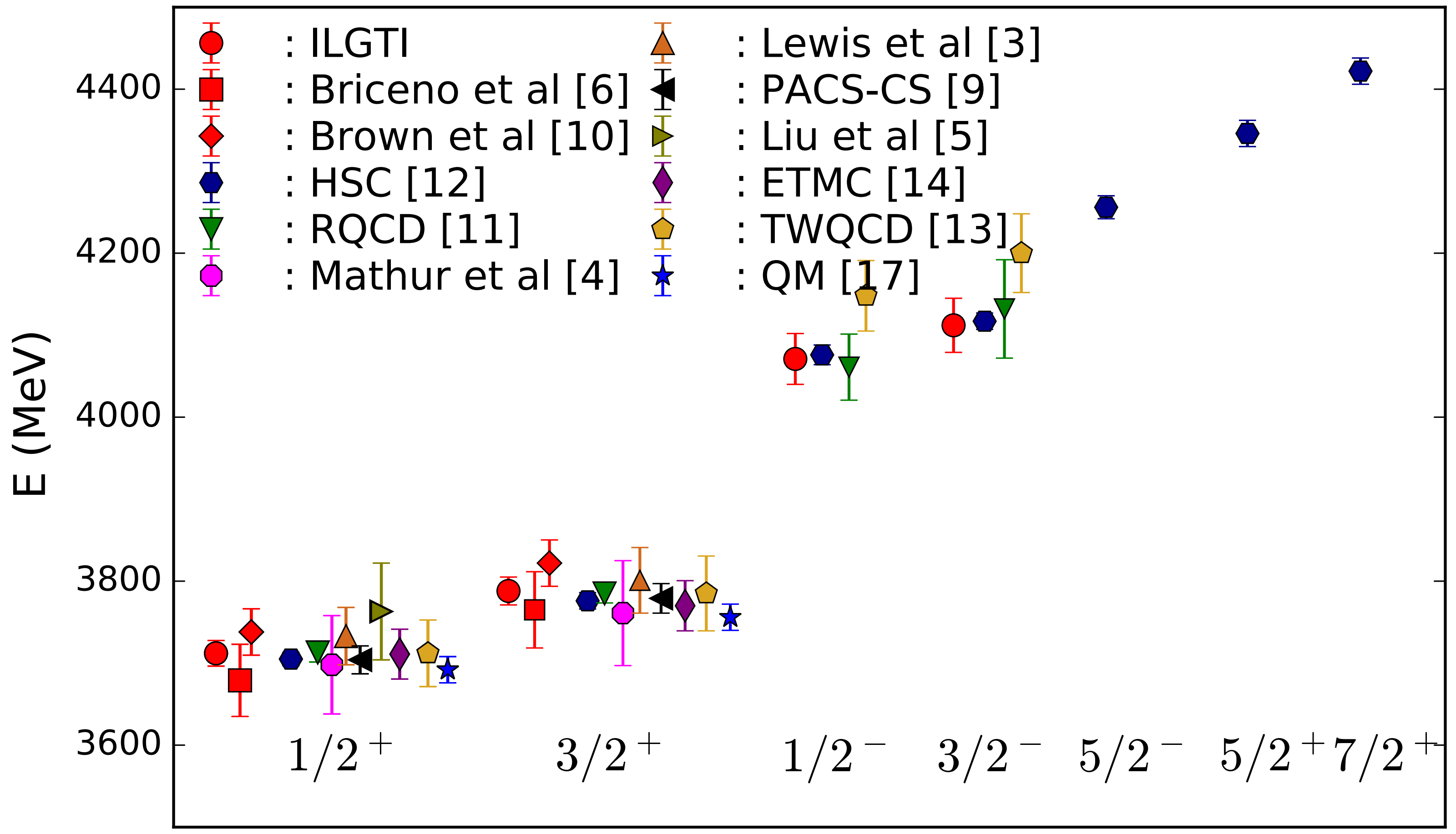
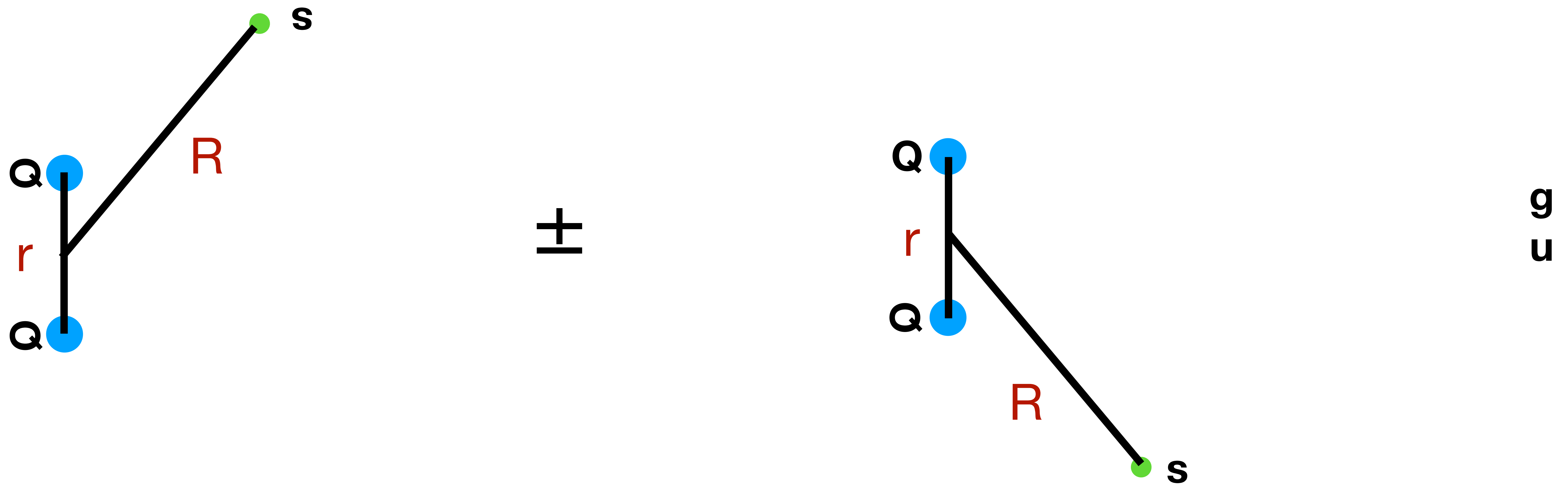


FIG. 6. Energy spectra of the low-lying $\Omega_{cc}(ccs)$ baryons obtained from different lattice calculations and a recent quark model calculation. Our results are represented as ILGTI (this calculation) and HSC (previous calculation). Continuum extrapolated results are shown by symbols with red color while symbols with all other colors are obtained only at one lattice spacing.

A Simple Model

E.E. and Chris Quigg (in progress)

- Two scale model
  - $[(Q(r/2)Q(-r/2)]_{3bar}$: NRQCD with $V(r) = -(2/3)\,\alpha/r + 1/2\; r/a^2$ $= 1/2\; V(r)$(Cornell)
  - $[QQ]_{3bar}(0)\, s(R)$: Dirac equation for light quark motion around a static diquark. Use a Cornell potential.



For the $(Q(r/2)\ Q(-r/2))_{3bar}$ system

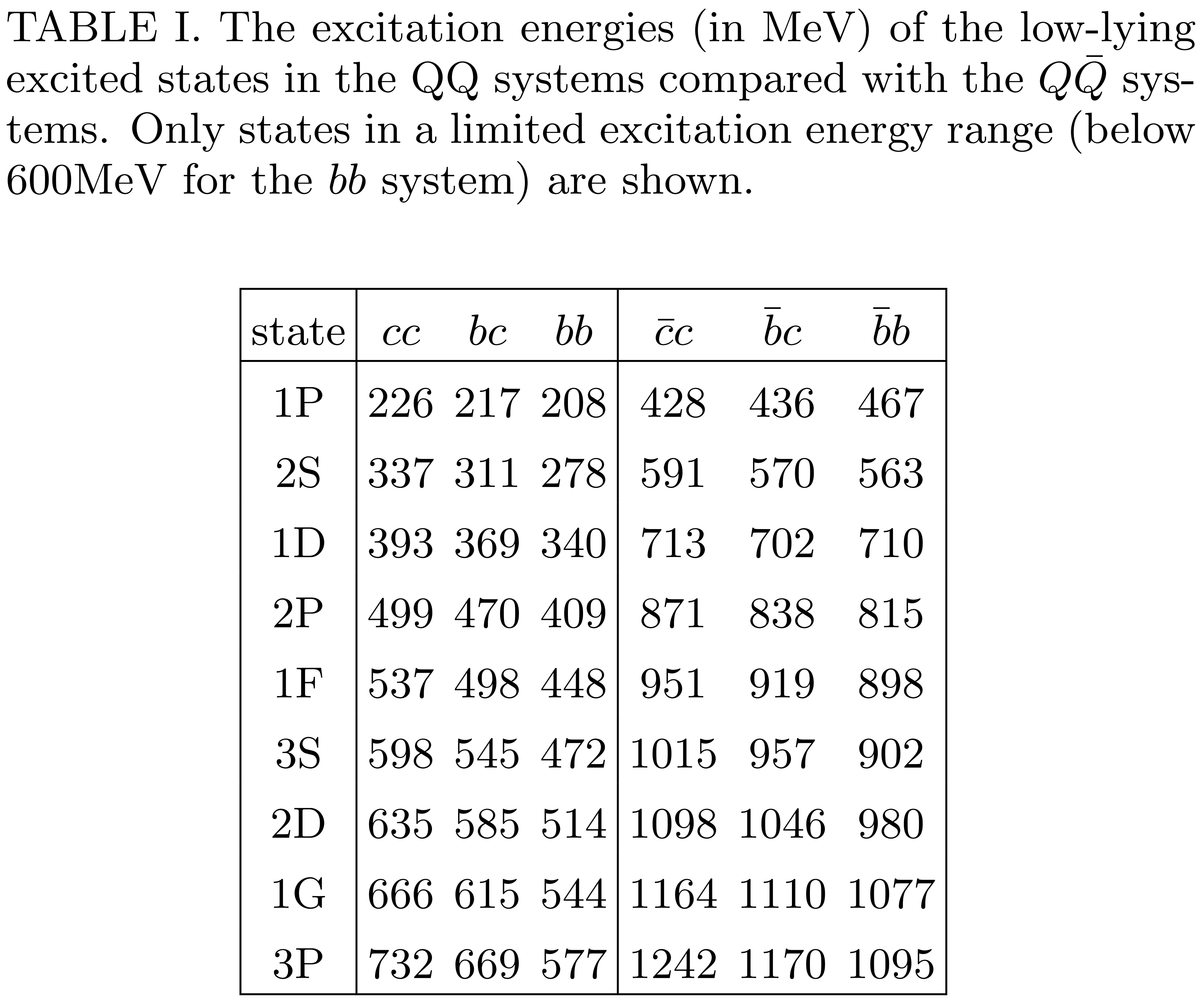
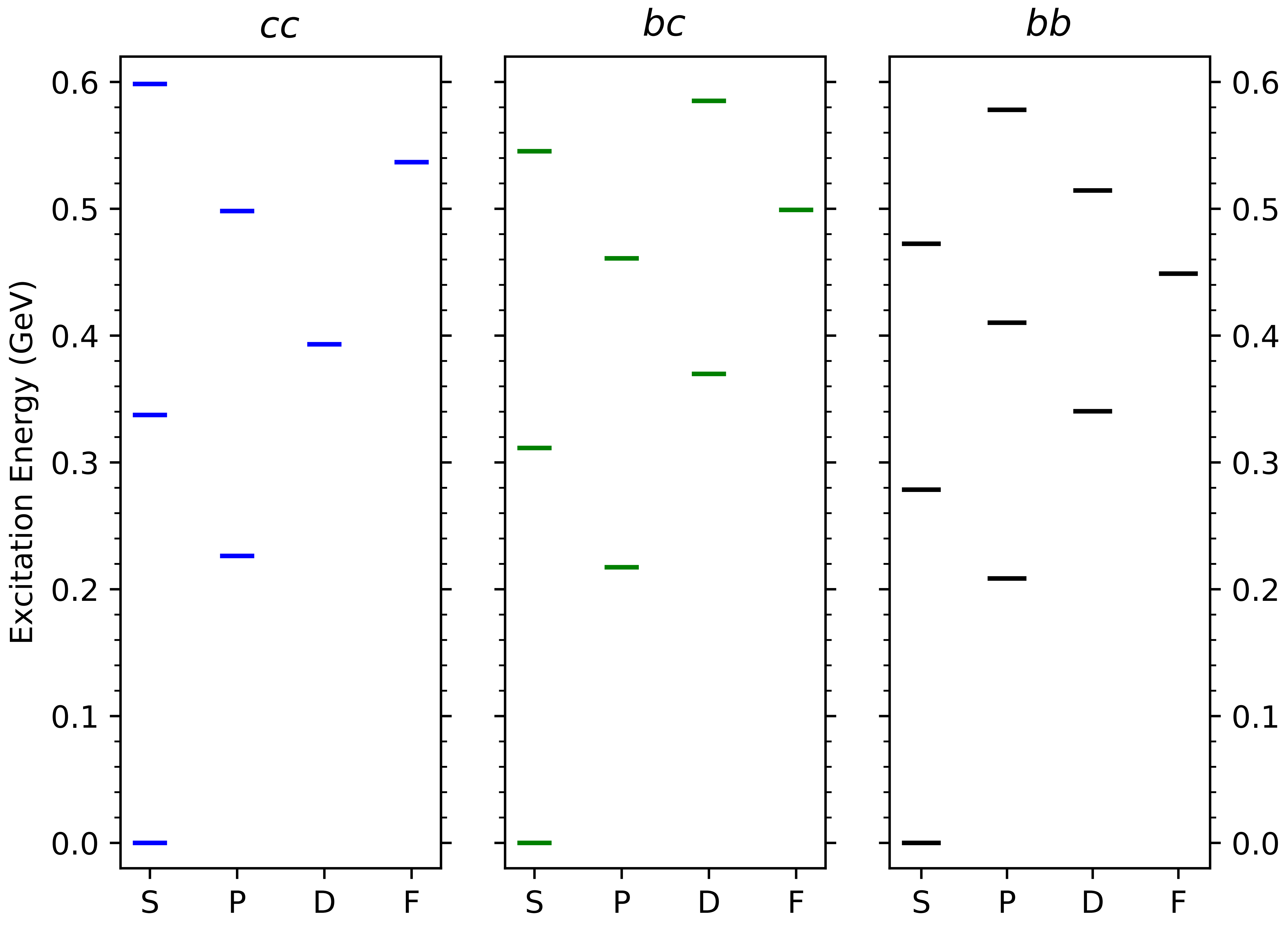


TABLE I. The excitation energies (in MeV) of the low-lying excited states in the QQ systems compared with the $Q\bar{Q}$ systems. Only states in a limited excitation energy range (below 600MeV for the bb system) are shown.

state	cc	bc	bb	$\bar{c}c$	$\bar{b}c$	$\bar{b}b$
1P	226	217	208	428	436	467
2S	337	311	278	591	570	563
1D	393	369	340	713	702	710
2P	499	470	409	871	838	815
1F	537	498	448	951	919	898
3S	598	545	472	1015	957	902
2D	635	585	514	1098	1046	980
1G	666	615	544	1164	1110	1077
3P	732	669	577	1242	1170	1095

Excitation Spectra Q Q



Lowest order

- The heavy-light system assumed governed by the Dirac Hamiltonian.
  - Simply four coupled differential equations, describing a wavefunction ψ with four components. Upper two components (u) and lower two components (v).
  - Neither the orbital angular momentum nor the light quark spin is conserved. But there is a conserved quantity ϰ, a eigenvalue of the wave function

$$\kappa = \begin{pmatrix} \vec{\sigma} \cdot \vec{L} + 1 & 0 \\ 0 & -\vec{\sigma} \cdot \vec{L} + 1 \end{pmatrix}$$

$$\begin{aligned}\frac{df^{(0)}}{dr} + \frac{\kappa}{r} f^{(0)} &= (E - V_v + V_s + m_q) f^{(1)} \\ \frac{df^{(1)}}{dr} - \frac{\kappa}{r} f^{(1)} &= (-E + V_v + V_s + m_q) f^{(0)}\end{aligned}$$

$$\begin{aligned}f^{(0)} &= u \\ f^{(1)} &= v\end{aligned}$$

$$\Psi_{n,l,j,J,M} = \sum_{S=(-\frac{1}{2},+\frac{1}{2})} C^{J,M}_{j,m;\frac{1}{2},S} \psi_{n,l,j,m}(r,\theta,\phi) \otimes \xi_S$$

Light quark wavefunction

Spin of heavy quark

$$\psi_{n,l,j,m}(r,\theta,\phi) = \frac{1}{r} \begin{pmatrix} i\ u_{n,l,j}(r) k^{+}_{l,j,m} Y^{l}_{m-\frac{1}{2}}(\theta,\phi) \\ i\ u_{n,l,j}(r) k^{-}_{l,j,m} Y^{l}_{m+\frac{1}{2}}(\theta,\phi) \\ v_{n,l,j}(r) k^{+}_{2j-l,j,m} Y^{2j-l}_{m-\frac{1}{2}}(\theta,\phi) \\ v_{n,l,j}(r) k^{-}_{2j-l,j,m} Y^{2j-l}_{m+\frac{1}{2}}(\theta,\phi) \end{pmatrix}$$

where the constants k are;

$$k^{\pm}_{l,j,n} = \left\{ \begin{array}{l} +\sqrt{\frac{l \pm m + \frac{1}{2}}{2l+1}} \text{ for j} = \text{l} + \frac{1}{2} \\ \pm\sqrt{\frac{l \pm m + \frac{1}{2}}{2l+1}} \text{ for j} = \text{l} - \frac{1}{2} \end{array} \right\}$$

There are significant $1/m_Q$ corrections in this approach

Heavy-Light Wavefunctions

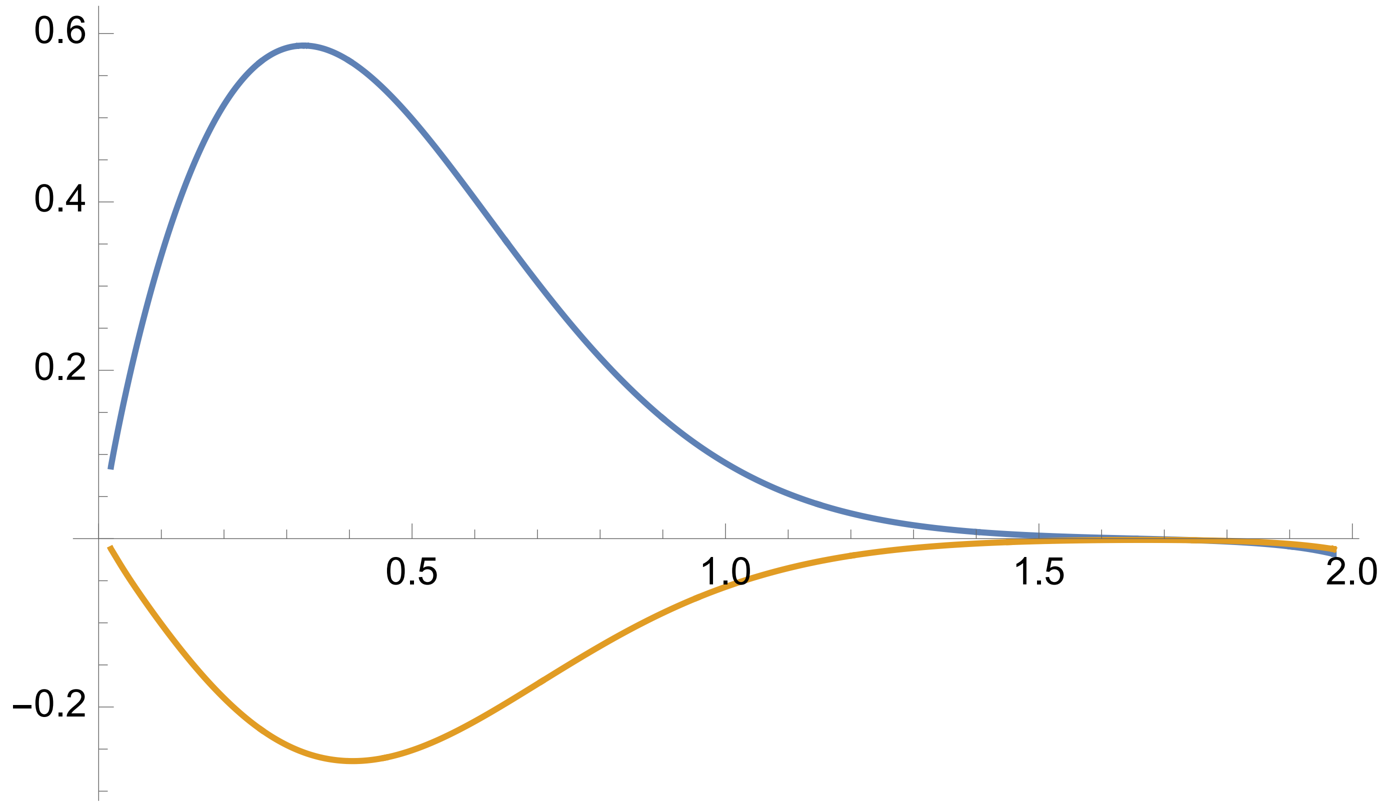


FIG. 2. The 1S state radial wavefunctions.

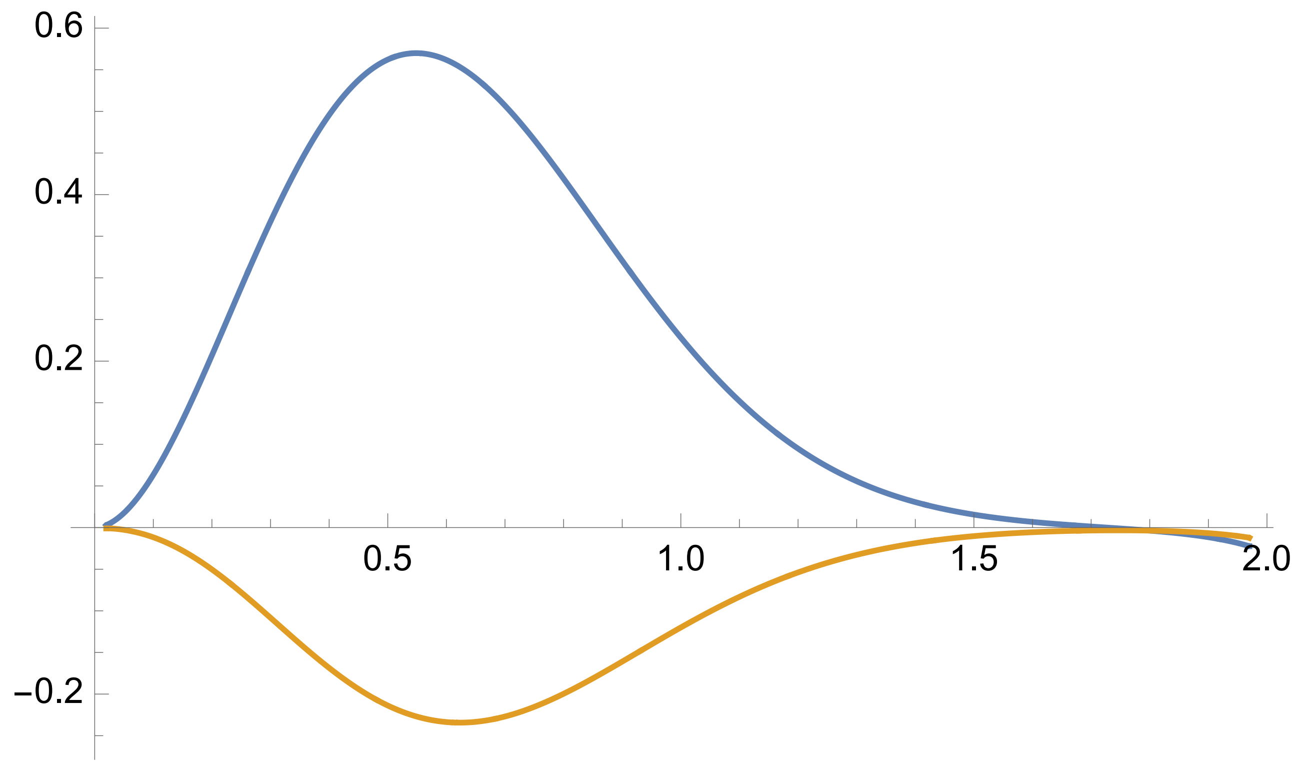


FIG. 4. The $1P^{3/2}$ state radial wavefunctions.

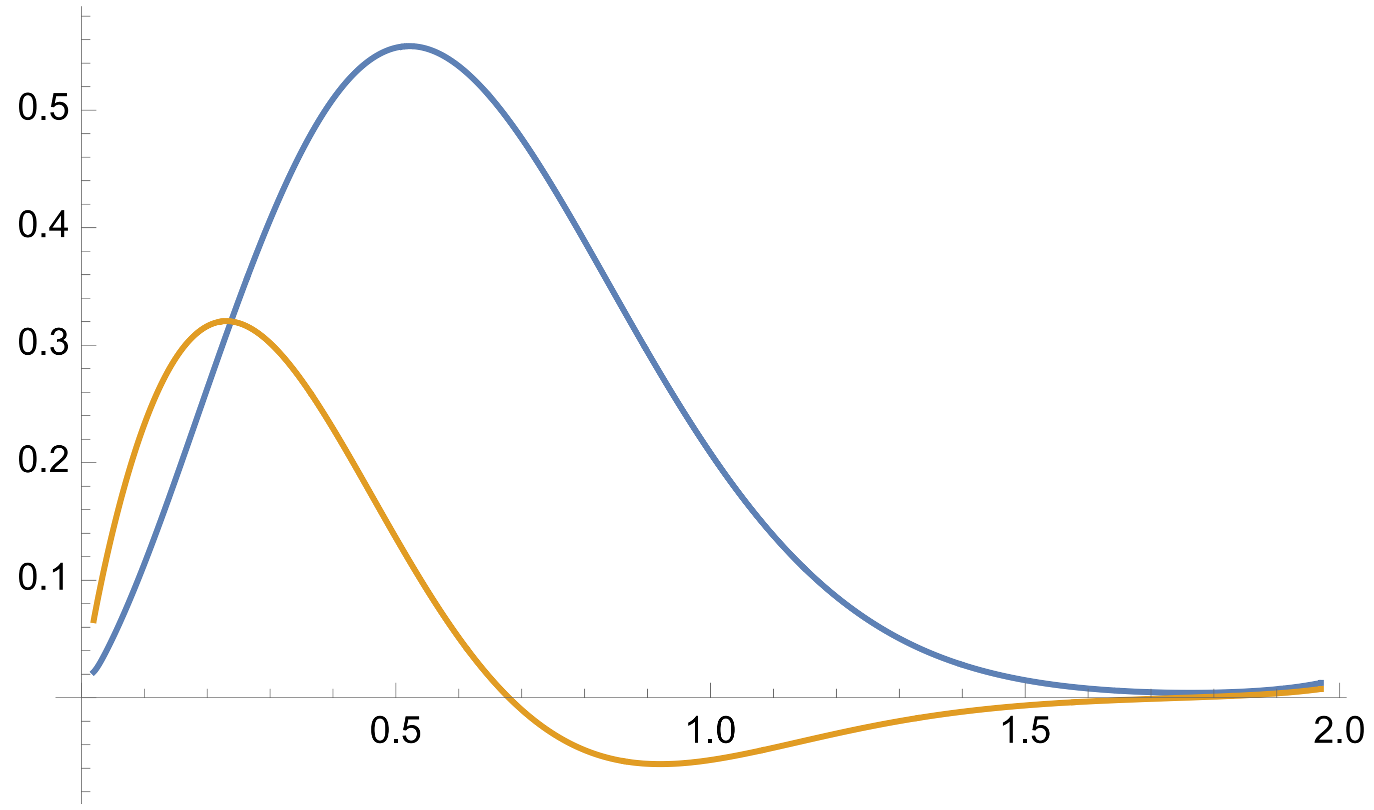


FIG. 3. The $1P^{1/2}$ state radial wavefunctions.

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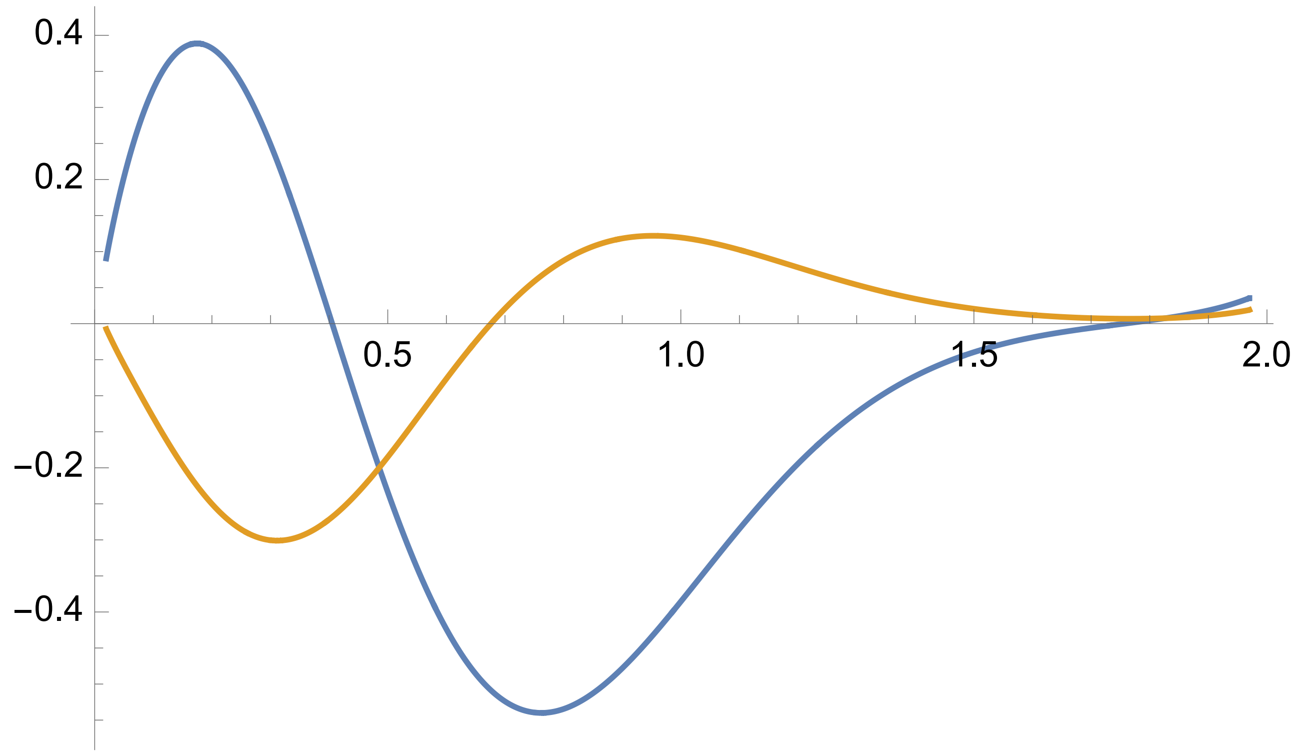


FIG. 5. The 2S state radial wavefunctions

— Upper component

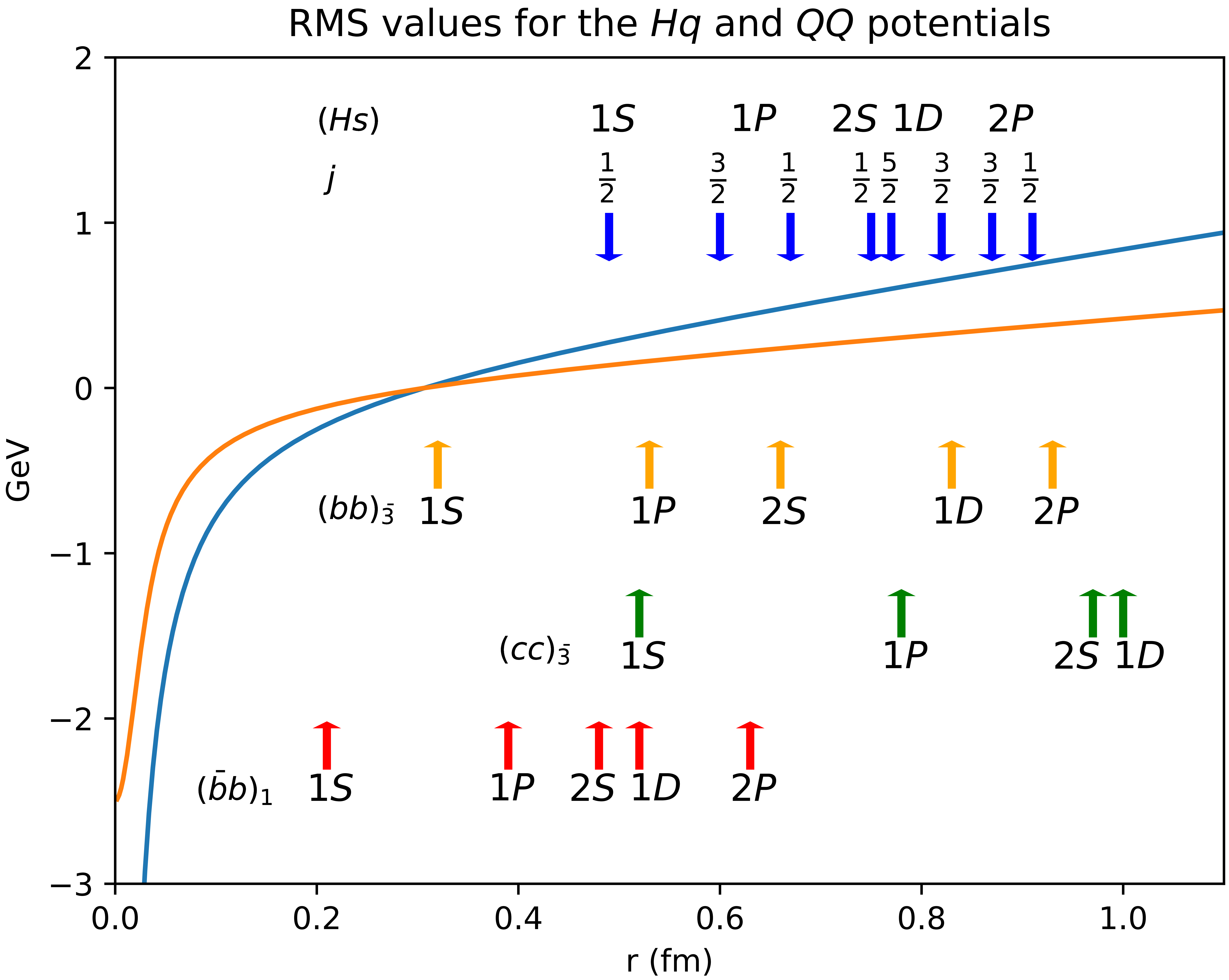
— Lower component

What we learn from this simple model

- The excitation energies of the QQ core and excitations of the light degrees of freedom are comparable. Hence the expected analogy between $(QQ)_3q$ and $\bar{Q}q$ systems is not realized.

- The $\langle R^2\rangle^{1/2}$ distance between the Q and Q in the core compared to the $\langle R^2\rangle^{1/2}$ distance between the center of mass of the QQ state (H) and the light quark is shown below.

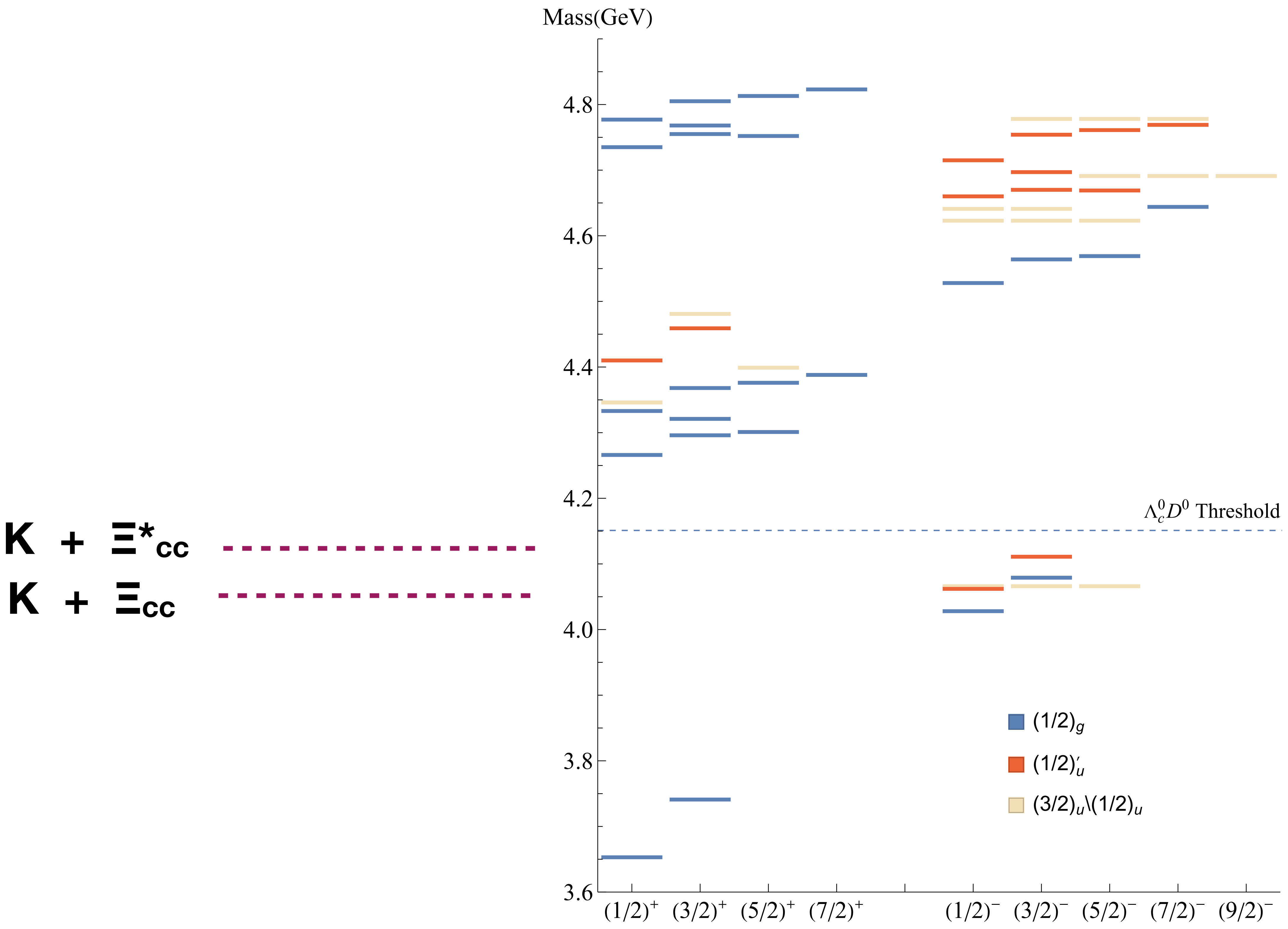
- We see the separation of scales between the two subsystems is not overly large.


RMS values for the Hq and QQ potentials
(Hs)
j
$1S$
$1P$
$2S$
$1D$
$2P$
$\frac{1}{2}$
$\frac{3}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{5}{2}$
$\frac{3}{2}$
$\frac{3}{2}$
$\frac{1}{2}$
$(bb)_{\bar{3}}$
$1S$
$1P$
$2S$
$1D$
$2P$
$(cc)_{\bar{3}}$
$1S$
$1P$
$2S$
$1D$
$(\bar{b}b)_1$
$1S$
$1P$
$2S$
$1D$
$2P$
GeV
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0
r (fm)

Born-Oppenheimer Effective Theory

- One can compute the lowest $(1/2,3/2,3/2,5/2)^{-}$ P wave states directly in LQCD.
- Alternatively:
  - One can compute in Lattice QCD ground state energies E(R) for a system with one dynamic light quark in the static potential of two heavy static quarks separated by a distance R.
  - The potentials associated with $O(1/M_Q)$ corrections can also be calculated in this way.
  - Then the total energy and wavefuction for the heavy quark ground system can be obtained by solving the SE for each distinct set of quantum numbers.
  - This allows the complete solution for the lowest doubly heavy S and P states.

J. Najjar and G. Bali ArXiv:0910.2824
J. Soto and J. Castella, ArXiv:2108.00496



The $1/m_Q$ corrections have also been calculated.
Not shown here.

Summary

- The nature of B_s (1P) $j^P=1/2^+$ states is still an open question. Comparing the decays of the D_s(1P) states to the B_s(1P) states with give valuable information into the microscopic nature of the 1P ($j^P=1/2^+$) states.

- The naive analog between the $\bar{Q}q$ mesons and the QQq baryons fails for the ccq, bbq, bcq systems.

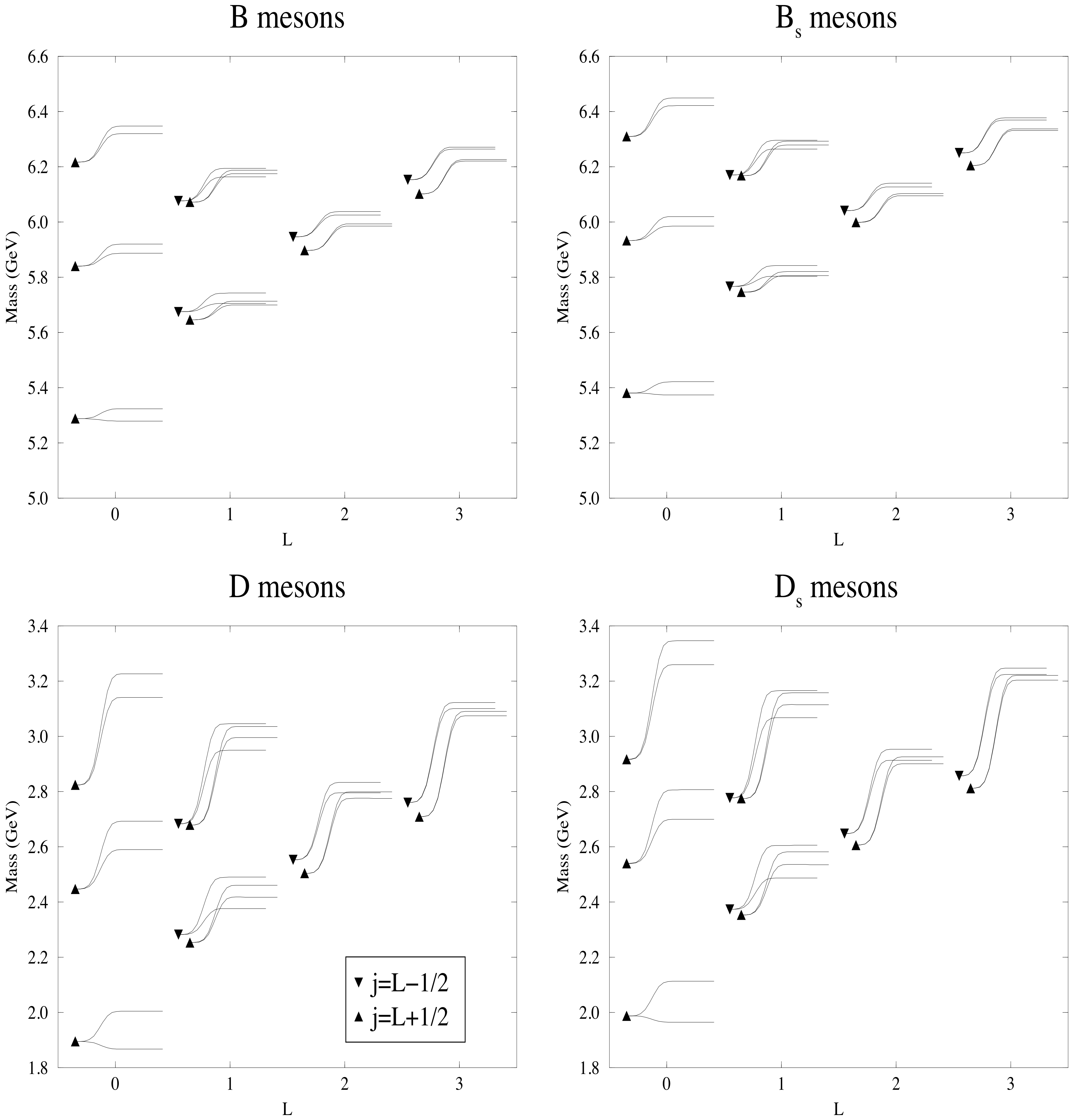
- Low-lying excitations of the QQ core are of the same order as that of the excitations of the light q.

- Expect a complicated spectrum of 1P QQq states.

- Some of the lowest 1P (ccs, cbs, bbs) states may be stable to Zweig allowed decays.

Backup

Significant 1/M_Q corrections.



E.E. and M. di Pierro Phys. Rev. D 64, 114004